INTERNATIONAL STANDARD

ISO 22514-4

First edition
2016-08-01

Statistical methods in process management — Capability and performance —

Part 4: Process capability estimates and performance measures

Méthodes statistiques dans la gestion de processus — Aptitude et performance —

Partie 4: Estimations de l'aptitude de processus et mesures de performance

Reference number
ISO 22514-4:2016(E)

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary information.

The committee responsible for this document is Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

This first edition of ISO 22514-4 cancels and replaces ISO/TR 22514-4:2007, which has been technically revised.

ISO 22514 consists of the following parts, under the general title *Statistical methods in process management — Capability and performance*:

— *Part 1: General principles and concepts*

— *Part 2: Process capability and performance of time-dependent process models*

— *Part 3: Machine performance studies for measured data on discrete parts*

— *Part 4: Process capability estimates and performance measures*

— *Part 5: Process capability estimates and performance for attributive characteristics*

— *Part 6: Process capability statistics for characteristics following a multivariate normal distribution*

— *Part 7: Capability of measurement processes*

— *Part 8: Machine performance of a multi-state production process*

Introduction

Many organizations have embarked upon a continuous improvement strategy. To comply with such a strategy, any organization will need to evaluate the capability and performance of its key processes. The methods described in this part of ISO 22514 are intended to assist any management in this respect. These evaluations need to be constantly reviewed by the management so that actions compatible with continuous improvement can be taken when required.

The content of this part of ISO 22514 has been subject to large shifts of opinion during recent times. The most fundamental shift has been to philosophically separate what is named in this part of ISO 22514 as capability conditions from performance conditions, the primary difference being whether statistical stability has been obtained (capability) or not (performance). This naturally leads onto the two sets of indices that are to be found in their relevant clauses. It has become necessary to draw a firm distinction between these since it has been observed in the industry that companies have been deceived about their true capability position due to inappropriate indices being calculated and published.

The progression of this part of ISO 22514 is from the general condition to the specific and this approach leads to general formulae being presented before their more usual, but specific manifestations.

There exist numerous references that describe the importance of understanding the processes at work within any organization, be it a manufacturing process or an information handling process. As organizations compete for sales with each other, it has become increasingly apparent that it is not only the price paid for a product or service that matters so much, but also what costs will be incurred by the purchaser from using such a product or service. The objective for any supplier is to continually reduce variability and not to just satisfy specification.

Continual improvement leads to reductions in the costs of failure and assists in the drive for survival in an increasingly more competitive world. There will also be savings in appraisal costs for as variation is reduced, the need to inspect product might disappear or the frequency of sampling might be reduced.

Process capability and performance evaluations are necessary to enable organizations to assess the capability and performance of their suppliers. Those organizations will find the indices contained within this part of ISO 22514 useful in this endeavour.

Quantifying the variation present within a process enables judgement of its suitability and ability to meet some given requirement. The following paragraphs and clauses provide an outline of the philosophy required to be understood to determine the capability or performance of any process.

All processes will be subject to certain inherent variability. This part of ISO 22514 does not attempt to explain what is meant by inherent variation, why it exists, where it comes from nor how it affects a process. This part of ISO 22514 starts from the premise that it exists and is stable.

Process owners should endeavour to understand the sources of variation in their processes. Methods such as flowcharting the process and identifying the inputs and outputs from a process assist in identification of these variations together with the appropriate use of cause and effect (fishbone) diagrams.

It is important for the user of this part of ISO 22514 to appreciate that variations exist that will be of a short-term nature, as well as those that will be of a long-term nature and that capability determinations using only the short-term variation might be greatly different to those which have used the long-term variability.

When considering short-term variation, a study that uses only the shortest-term variation, sometimes known as a machine study and described in ISO 22514-3, might be carried out. The method required to carry out such a study will be outside the scope of this part of ISO 22514; however, it should be noted that such studies are important and useful.

It should be noted that where the capability indices given in this part of ISO 22514 are computed, they only form point estimates of their true values. It is therefore recommended that, wherever possible, the indices' confidence intervals are computed and reported. This part of ISO 22514 describes methods by which these can be computed.

Statistical methods in process management — Capability and performance —

Part 4:
Process capability estimates and performance measures

1 Scope

This part of ISO 22514 describes process capability and performance measures that are commonly used.

2 Symbols and abbreviated terms

2.1 Symbols

In addition to the symbols listed below, some symbols are defined where they are used within the text.

α	fraction or proportion
β	shape parameter in a Weibull distribution
β_2	coefficient of kurtosis
c_4	constant based on subgroup size, n (see ISO 7870-2)
C_{p}	process capability index
C_{pk}	minimum process capability index
C_{pk_L}	lower process capability index
C_{pk_U}	upper process capability index
C_{pm}	alternative process capability index
C_{R}	process capability fraction (PCF)
d_2	constant based on subgroup size, n (see ISO 7870-2)
e	Eulers's number (approximately 2,718), mathematical constant
Φ	distribution function of the standard normal distribution
γ	location parameter in a Weibull distribution
γ_1	coefficient of skewness
m	number of subgroups
K_{l}, K_{u}	multipliers for estimating the confidence limits for a process capability index
L	lower specification limit

$P_{0,135\ \%}$ lower 0,135 % percentile

μ location of the process; population mean value

N total sample size

n number of values or subgroup size (for a control chart)

$P_{\alpha\ \%}$ α percentile

p_L lower fraction nonconforming

P_{p} process performance index

P_{pk} minimum process performance index

P_{pk_L} lower process performance index

P_{pk_U} upper process performance index

p_{t} total fraction nonconforming

p_U upper fraction nonconforming

$P_{99,865\ \%}$ upper 99,865 % percentile

π geometric constant

Q_{k} process variation index

θ parameter required for the Rayleigh distribution

$\overline{R}$ average subgroup range

S standard deviation, sample statistic

S_{t} standard deviation, with the subscript ‘t’ indicating total

$\overline{S}$ average sample standard deviation

S_j observed sample standard deviation of the j^{th} subgroup

σ standard deviation, population

$\hat{\sigma}_{\mathrm{t}}$ estimated standard deviation, total

T target value

U upper specification limit

$X_{\alpha\ \%}$ α % percentile

X_i i^{th} value in a sample

$\overline{X}$ arithmetic mean value, sample

$\overline{\overline{X}}$ arithmetic mean, of a number of sample arithmetic means

ξ scale parameter in a Weibull distribution

Y_1, Y_2 values read from a graph

z_α quantile of the standardized normal distribution from $-\infty$ to α

2.2 Abbreviated terms

MSE mean square error

PCF process capability fraction

PCI process capability index

3 Basic concepts used for process capability and performance

3.1 General

The measures referred to in 4.2 to 4.6 refer only to measured data. They are unsuitable for count or attributes data and information concerning the expression of measures for such data will be found in ISO 22514-5.

3.2 Location

The characterization of location is the mean, μ, or the median, $X_{50\ \%}$. Although for symmetric distributions the mean is the most natural selection, with non-symmetric distributions the median is the preferred selection.

3.3 Dispersion

3.3.1 Inherent dispersion

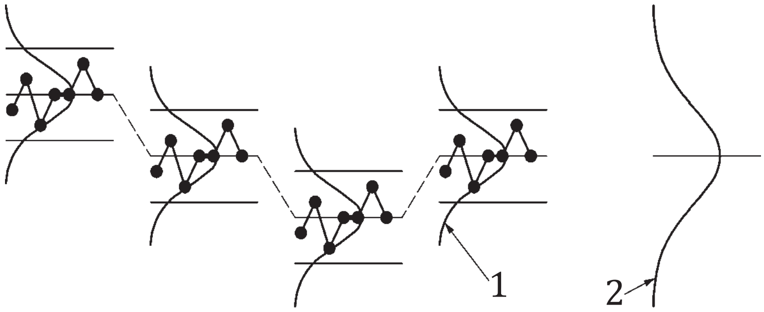
The preferred selection to quantify inherent dispersion is the standard deviation σ. This is often estimated from the mean range value, $\bar{R}$, taken from a range (R) chart or $\bar{S}$ from a standard deviation (S) chart when the process is stable and in a state of statistical control as indicated in 4.1. Methods used to estimate the process standard deviation are given in Annex A.

3.3.2 Total dispersion

It is necessary to differentiate between a standard deviation that measures only short-term variation and that which measures longer-term variation. The total dispersion is the dispersion that is inherent in the long-term variation. Methods of calculating the standard deviations representing these variations are given in Annex A. Very often, when data are gathered over a long period of time, the standard deviation is made larger by the effects of fluctuations in the process, σ_t.

3.3.3 Short-term dispersion

A process may have a short-term dispersion effect that is a part of the total dispersion. Figure 1 illustrates this. The short-term dispersion includes the inherent dispersion and can also include some short-term instability effect.



Key

1 short-term dispersion

2 overall dispersion

Figure 1 — Short-term dispersion and its relationship to the total dispersion

The total dispersion can be any shape and not necessarily normal as illustrated here.

3.4 Mean square error (MSE)

When minimizing variation, some practitioners use the mean square error as the preferred measure. It is compatible with the methods used in off-line quality techniques.

3.5 Reference limits

The lower and upper reference limits are respectively defined as the 0,135 % and the 99,865 % percentiles of the distribution that describe the output of the process characteristic. They are written as $X_{0,135\,5\,\%}$ and $X_{99,865\,\%}$.

3.6 Reference interval (also known as process spread)

The reference interval is the interval between the upper and the lower reference limits. The reference interval includes 99,73 % of the individuals in the population from a process that is in a state of statistical control.

4 Capability

4.1 General

Process capability is a measure of inherent process variability. The variability that is inherent in a process when operating in a state of statistical control is known as the inherent process variability. It represents the variation that remains after all known removable *assignable causes* have been eliminated. If the process is monitored using a control chart, the control chart will show an *in control* state.

Capability is often regarded as being related to the proportion of output that will occur within the product specification tolerances. Since a process in statistical control should be described by a predictable distribution, the proportion of out-of-specification outputs can be estimated. As long as the process remains in statistical control, it will continue to produce the same proportion out-of-specification.

Management actions to reduce the variation from *random causes* are required to improve the process' ability to consistently meet the specification requirements.

In short, the following will be necessary:

— define the process and its operating conditions. If there is a change to those conditions, it will necessitate a new process study;

— assess the short-term and long-term measurement variabilities as percentages of the total variability and minimize them;

— preserve the process stability and maintain its statistical control;

— estimate the remaining inherent variation;

— select an appropriate measure of capability.

The following are the conditions that will apply for capability:

— all technical conditions, e.g. temperature and humidity, shall be clearly stated;

— the uncertainty of the measurement system shall be estimated and judged appropriate (see ISO 22514-7);

— multi-factor, multi-level aspects of the process should be allowed;

— the duration over which the data has been gathered shall be recorded;

— the frequency of sampling and sample size shall be specified and the start and finish dates of data collection;

— the process shall be controlled with a control chart;

— the process shall be in a state of statistical control.

It is necessary to check the control chart from which the data have been taken for statistical control and to examine a histogram of the data with any specification limits superimposed upon it. A valid test for normality should be used in assessing the data such as the Anderson-Darling test[15] or any other suitable method. This test is powerful in detecting departures from normality in the tails of the distribution and is suggested here as this is the region of interest for capability and performance indices. Additionally, a normal probability plot can be used to look for the following:

a) verification of normality;

b) outliers;

c) data beyond any specification limit;

d) whether the data are well inside the specification limit(s);

e) evidence of asymmetry (i.e. skewness);

f) evidence of “long tails” in the data (i.e. kurtosis);

g) off-centre distribution;

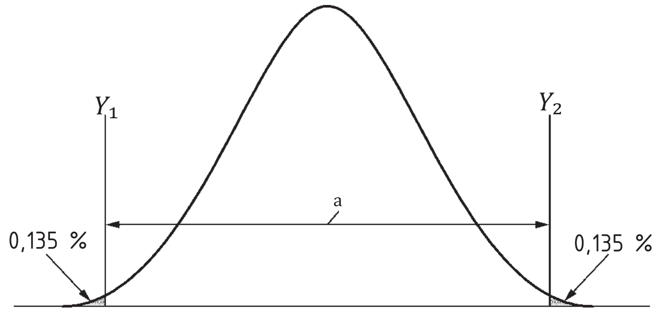
h) any unusual patterns.

Explanations of anomalies should be sought in relation to these mentioned features and appropriate action taken on the data prior to the calculation of any measure. It would be inappropriate to just discard data that do not appear to fit any preconceived pattern. Such departures might be very revealing about the process’ behaviour and should be thoroughly investigated.

4.2 Process capability

4.2.1 Normal distribution

Process capability is defined as a statistical measure of inherent process variability for a given characteristic. The conventional method is to take the reference interval that describes 99,73 % of the individual values from a process that is in a state of statistical control with the 0,135 % remaining on each side. This applies even if the population of individual values is not normally distributed. For a normal distribution, this process interval is represented by six standard deviations (see Figure 2).



Key

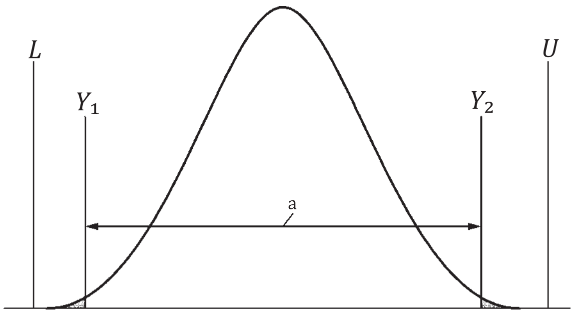
a Reference interval 99,73 %.

Figure 2 — Normal distribution

On occasions, process capability is taken to account for extra sources of variation such as a multiple stream process, for example, output from a multi-cavity injection moulding press. Under these circumstances, the distribution of all values from all cavities could still be approximately normal, but with extra variability so that the standard deviation shall represent the total variation, σ_t. It is important to state how the standard deviation has been calculated, as well as the sampling strategy used, sample size and the quantity and variability of output produced between samples as these will, in practice, affect the validity of the capability assessment (see ISO 22514-2 for further information).

Data will usually be taken from a control chart. If the control chart had relaxed control lines or modified control lines, the real process standard deviation will be larger than that estimated from data taken from a control chart with standard control lines. Issues such as these and those given earlier will influence the reference interval and it is important that they are stated in any capability assessment.

“Capable” processes will be those whose reference intervals are less than any specified tolerance by a particular amount. An example of this is shown in Figure 3.



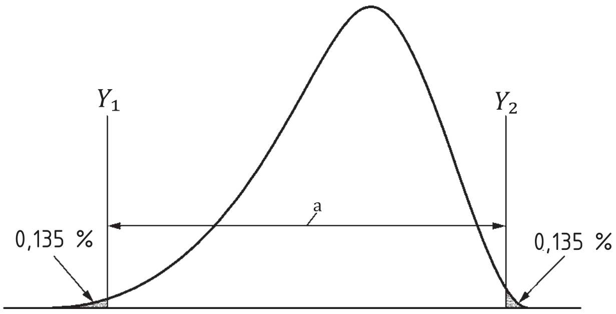
Key

a Reference interval 99,73 %.

Figure 3 — Normal distribution with specification limits

4.2.2 Non-normal distribution

If the distribution of individual values does not form a normal distribution, but is skewed, then the reference interval may appear as in Figure 4. The values Y_1 and Y_2, which will usually be the 0,135 % and the 99,865 % percentiles, can be estimated using a suitable probability paper (see Figure 5 for an example using an extreme value distribution probability paper) or by the use of suitable computer software. They can also be computed using tabular values (see Annex B) or using the particular probability function as suggested in Annex C.



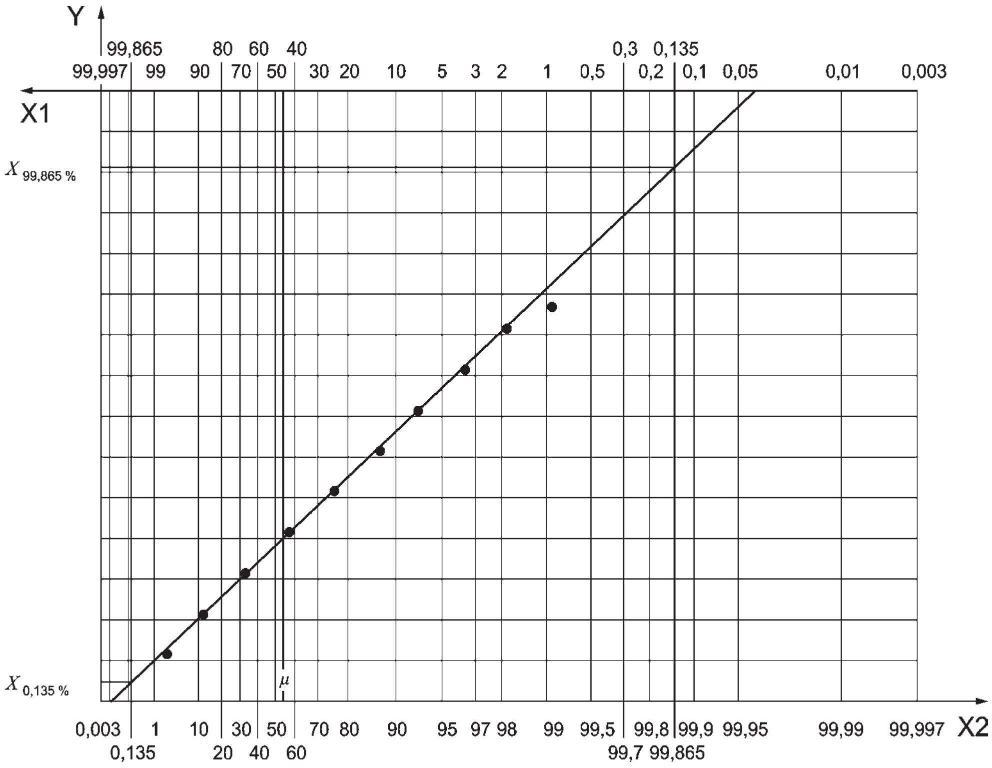
Key

a Reference interval 99,73 %.

Figure 4 — Non-normal distribution

4.3 Process location

Even if a process can be deemed capable by the above definition (in 4.2.1), if the process distribution has been poorly centred relative to the specification limits, out-of-specification items might be produced. For this reason, it is necessary to assess the location in addition to the process interval.



Key

———	best fit line
●	cumulative percent

Figure 5 — Example using an extreme value distribution probability paper

4.4 Process capability indices for measured data

4.4.1 General

It should be noted that when the capability indices given in this part of ISO 22514 are computed, they only form point estimates of their true values. It is therefore recommended that wherever possible, the indices' confidence intervals are computed and reported. Methods by which these can be computed are described in Annex D.

It is effective to express the capability of a process with the use of an index number. Several indices are given. Care shall be taken when handling non-normal distributions.

The process capability indices are only established for a process that is statistically "in control".

The process capability index often used is the ratio of a specified tolerance to the reference interval and is known as C_p. Thus,

$$C_p = \frac{U - L}{X_{99,865\,\%} - X_{0,135\,\%}} \tag{1}$$

There are other indices that incorporate both the location and the variation. Of these, the most widely used is the C_{pk} index. If the observed index is less than a specified value, the process is deemed unacceptable and might lead to the shipment of a proportion of items outside of the specification or that function and fit might be compromised.

The C_{pk} index is the ratio of the difference between a specified tolerance limit and the process location to the difference between the corresponding natural process limit and the process location.

$$C_{pk_U} = \frac{U - X_{50\,\%}}{X_{99,865\,\%} - X_{50\,\%}} \tag{2}$$

and

$$C_{pk_L} = \frac{X_{50\,\%} - L}{X_{50\,\%} - X_{0,135\,\%}}$$

The C_{pk} index is reported as the smaller value of these.

NOTE Some practitioners report both of the above values (that are also known as CPU and CPL, respectively). This provides information about both sides of the process.

These indices will provide information about whether a process is poorly centred and whether it will possibly produce out-of-specification items. Even if the C_p index is high, a low value of the C_{pk} index will reveal a poorly centred process and a high probability of producing out-of-specification items.

4.4.2 C_p index (for the normal distribution)

If the individual values form a normal distribution and come from a statistically stable process, the length of the reference interval is equal to 6σ, where σ is the inherent process standard deviation. Therefore, the C_p index can be expressed as:

$$C_p = \frac{U - L}{6\sigma}$$

An estimate ($\hat{\sigma}$) of the inherent process standard deviation (σ) is required to obtain an estimate of the C_p index. When this has been obtained, usually with data from a control chart once the process is shown to be statistically stable (see 4.1), the index is estimated:

$$\hat{C}_p = \frac{U - L}{6\hat{\sigma}}$$

4.4.3 C_{pk} index (for the normal distribution)

When the distribution of individual values forms a normal distribution, the median $X_{50\ \%}$ is equal to the mean (μ). Further, $X_{99,865\ \%} - X_{50\ \%}$ and $X_{50\ \%} - X_{0,135\ \%}$ are each equal to 3σ. Therefore, the C_{pk} index can be expressed as the minimum of:

$$C_{pk_U} = \frac{U - \mu}{3\sigma}$$

or

$$C_{pk_L} = \frac{\mu - L}{3\sigma}$$

The estimated C_{pk}, (using $\overline{\overline{X}}$ to estimate μ instead of $X_{50\ \%}$) will be the minimum of:

$$\hat{C}_{pk_U} = \frac{U - \overline{\overline{X}}}{3\hat{\sigma}}$$

or

$$\hat{C}_{pk_L} = \frac{\overline{\overline{X}} - L}{3\hat{\sigma}}$$

In computing a capability index, thought has to be given to the measure of the process variation used in the denominator. Here, σ is given to represent the variation when the data comes from a process that is in a state of statistical control.

The data might come from a multiple stream process such as a multi-headed filling machine or a multi-spindle machine where the total output is treated together, where data from all streams are simultaneously considered. The lower the index, the greater the proportion of items produced out-of-specification.

4.4.4 C_{pk} index for unilateral tolerances

When there is only one specification limit given, it is only possible to calculate a C_{pk} index. The index will be calculated using the appropriate limit, either an L or a U.

4.5 Process capability indices for measured data (non-normal)

4.5.1 General

If the distribution of individual values is non-normal, the expressions in Formulae (1) and (2) still apply, but the estimation of the indices becomes more complicated. Three approaches to estimate the reference limits are given.

The probability paper method described in 4.5.2 is fairly simple and requires little computation, but is somewhat crude. The approach given in 4.5.4 is computationally more involved, but is superior to any other method as far as accuracy is concerned.

4.5.2 Probability paper method

From graphs similar to that shown in Figure 4, estimates of the percentiles $X_{0,135\,\%}$ and $X_{99,865\,\%}$ can be obtained. The estimates are denoted by Y_1 and Y_2, respectively, and Formula (1) becomes:

$$\hat{C}_p = \frac{U - L}{Y_2 - Y_1}$$

In a similar way, the C_{pk} formulae become:

$$\hat{C}_{pk_U} = \frac{U - X_{50\,\%}}{Y_2 - X_{50\,\%}}$$

or

$$\hat{C}_{pk_L} = \frac{X_{50\,\%} - L}{X_{50\,\%} - Y_1}$$

whichever gives the lower value.

If the observed index is less than a specified value, the process is deemed unacceptable and might lead to the shipment of a proportion of items outside of the specification or that function and fit might be compromised. The proportion nonconforming depends upon the distribution and the value of the index. The link between the index and the proportion of nonconforming items produced depends on the class of distributions. Care should be taken not to interpret indices on the basis of cut-off points that have been derived for the normal distribution and, hence, are only applicable for that distribution.

Note that the probability paper method directly estimates fairly extreme percentiles and this can be inaccurate.

4.5.3 Pearson curves method

As an alternative to using probability paper, standardized Pearson curves can be used. The method is described by way of an example (see Annex B). The index is computed using:

$$\hat{C}_p = \frac{U - L}{\hat{X}_{99,865\,\%} - \hat{X}_{0,135\,\%}}$$

where $\hat{X}_{0,135\,\%}$ and $\hat{X}_{99,865\,\%}$ are the 0,135 % and 99,865 % percentiles estimated from the standardized Pearson curves.

Also, we have the formulae:

$$\hat{C}_{pk_U} = \frac{U - \hat{X}_{50\,\%}}{\hat{X}_{99,865\,\%} - \hat{X}_{50\,\%}}$$

or

$$\hat{C}_{pk_L} = \frac{\hat{X}_{50\,\%} - L}{\hat{X}_{50\,\%} - \hat{X}_{0,135\,\%}}$$

where $\hat{X}_{50\,\%}$ is the estimated median.

In order to use this method, it is necessary to establish skewness and kurtosis values in addition to the mean and standard deviation for the data set upon which the index is to be computed.

This method is not preferred, but is presented here for completeness due to its occasional use.

This approach, and a similar one based on Johnson curves, should be regarded with considerable caution, especially when it is a procedure within a “black box” computer program used to analyse large sets of data. Some of the potential difficulties are as follows:

— within a system of distributions, some distributions are more difficult to fit than others. The method of moments can yield unstable or inefficient curve parameters in some cases;

— unless the estimation technique is applied skilfully, it is possible to obtain fitted curves that are meaningless over certain ranges of the data. For example, with the method of moments, an easily made mistake is to fit a Pearson Type III distribution whose estimated threshold is less than the lower bound for the process output, thereby, invalidating the estimates of $X_{0,135\ \%}$ and C_{pk};

— the method of moments does not yield estimates of the variability in the estimated indices. Likewise, these methods do not yield confidence intervals for the indices;

— not every data distribution can be described adequately with a Pearson or Johnson curve;

— goodness-of-fit tests are limited to the chi-squared test since more powerful tests are not generally available for the Pearson and Johnson systems;

— the “black box” approach tends to displace basic practices, such as plotting the data and applying simple normalizing transformations, that provide genuine understanding of the process.

4.5.4 Distribution identification method

Annex C describes certain families of distribution functions (such as the log-normal distribution, the Rayleigh and the Weibull distributions) that are commonly found when investigating process capability. The method is first to identify the appropriate family of distributions, secondly to estimate the parameters of the distribution of the family that best explain the data by some efficient estimation method and, finally, to express the quantiles in terms of the parameters of that distribution.

This is analogous to the procedure adopted in the case of the normal distribution where σ is estimated and 6σ is represented by $(X_{99,865\ \%} - X_{0,135\ \%})$.

Various types of probability paper might be useful to identify the appropriate family of distributions.

4.6 Alternative method for describing and calculating process capability estimates

The bases for this method are the widely used definitions of C_{p} and C_{pk} for the “ideal process” with a normally distributed characteristic, X, where the expectation, μ, and variance, σ^2, are constant with time and the corresponding estimates are $\bar{\bar{X}}$ and S^2.

Table 1 — Process capability indices and estimates for the normal distribution

Index	Estimate
$C_{\text{p}} = \dfrac{U-L}{6\sigma}$	$\hat{C}_{\text{p}} = \dfrac{U-L}{6S}$
$C_{\text{pk}_U} = \dfrac{U-\mu}{3\sigma}$	$\hat{C}_{\text{pk}_U} = \dfrac{U-\bar{X}}{3S}$
$C_{\text{pk}_L} = \dfrac{\mu-L}{3\sigma}$	$\hat{C}_{\text{pk}_L} = \dfrac{\bar{X}-L}{3S}$
$C_{\text{pk}} = \min(C_{\text{pk}_L}, C_{\text{pk}_U})$	$\hat{C}_{\text{pk}} = \min(\hat{C}_{\text{pk}_L}, \hat{C}_{\text{pk}_U})$

This “ideal process” implies that the long-term standard deviation is equal to the short-term standard deviation.

For the normal distribution, there is an exact relation between the lower fraction nonconforming units and C_{pk_L} and between the upper fraction nonconforming and C_{pk_U}. This relation is exploited in 4.8 to calculate the proportion out-of-specification from lower and upper process capability indices. The relationship is displayed in Table 2 for easy reference.

When these measures of process capability have to be extended to characteristics that are not normally distributed, the fraction nonconforming item can be transformed to a capability index using the relationships in Table 2. This method can be applied in particular if the product characteristic is qualitative.

Table 2 — Process capability indices and estimates for the normal distribution — Equivalent formulae

Index	Estimate
$C_p = \dfrac{C_{pk_U} + C_{pk_L}}{2}$ $C_{pk_U} = \dfrac{z_{1-p_U}}{3}$ $C_{pk_L} = \dfrac{z_{1-p_L}}{3}$	$\hat{C}_p = \dfrac{\hat{C}_{pk_U} + \hat{C}_{pk_L}}{2}$ $\hat{C}_{pk_U} = \dfrac{z_{1-\hat{p}_U}}{3}$ $\hat{C}_{pk_L} = \dfrac{z_{1-\hat{p}_L}}{3}$

where p_U and p_L are the fractions nonconforming at the upper and lower specification limits and $\hat{p}_U$, $\hat{p}_L$ are the corresponding estimates. The formulae in the above table can be applied to any distribution.

It is assumed that the user has knowledge of the shape of the distribution because of what is known about the manufacturing process or by some evaluation of a sample by an appropriate probability paper.

For those distributions that are frequently observed (normal, log-normal, Rayleigh and Weibull), the required relations and formulae are given in Annex C.

4.7 Other capability measures for continuous data

4.7.1 Process capability fraction (PCF)

The PCF is the inverse of the C_p index:

$$\frac{6\sigma}{U - L} = \frac{1}{C_p}$$

It can be expressed as a percentage value and occasionally named C_R (%).

4.7.2 Indices when the specification limit is one-sided or no specification limit is given

4.7.2.1 General

Sometimes, a specification is given that has only one limit, e.g. a maximum value. In these circumstances, it will only be possible to compute a C_{pk} or a P_{pk} index.

There will also be situations when specification limits are not given or not known. However, if a target or nominal value is given for the product characteristic or process parameter, the following measures might be appropriate. They present a special appeal to those engaged in minimizing process variation around a target value.

4.7.2.2 Mean square error (MSE)

The mean square error provides a measure that involves both location and variation. It is computed as follows:

$$\sigma^2 + (\mu - T)$$

In deriving this measure from data, it is necessary to provide estimates of the process standard deviation and μ using sample data from a control chart.

4.7.2.3 Q_k index

This index uses the mean square error given in 4.7.2.2, but expresses the whole value as a coefficient of variation and is computed as follows:

$$Q_k = \frac{100\sqrt{\sigma^2 + (\mu - T)^2}}{T}(\%)$$

for $T \neq 0$.

An interesting property of this index is if the process drifts from its target, the index will increase in value and if the process variation increases, it will also increase the value of the index. The smaller this index becomes, the better the process is deemed to have performed.

4.7.2.4 C_{pm} index

The C_{pm} index, like the Q_k index, incorporates the target value and the MSE into the calculation.

In its simplest form, the index is:

$$C_{pm} = \frac{U - L}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

This calculation implies that T is the midpoint between U and L and so a refinement was later introduced that allowed for a non-central T value:

$$C_{pm}^* = \frac{\min(U - T, T - L)}{3\sqrt{\sigma^2 + (\mu - T)^2}}$$

Compared to the usual indices, C_p and C_{pk}, here only one index is needed to describe a situation.

The C_{pm} index is sometimes referred to as a Taguchi index because of the incorporation of the MSE in the denominator.

4.8 Assessment of proportion out-of-specification (normal distribution)

The proportion of out-of-specification items (p_L and p_U) that fall below L and above U can be estimated using properties of the standard normal distribution. Standardized deviates can be calculated as follows:

$$z_{\hat{p}_U} = 3\hat{C}_{\text{pk}_U}$$

and

$$z_{\hat{p}_L} = 3\hat{C}_{\text{pk}_L}$$

$\hat{p}_U$ and $\hat{p}_L$ are found as the proportions exceeding z_{p_U} and z_{p_L}, respectively, in a standard normal distribution.

Additionally, the process yield can be computed as 100 % minus the total percentage nonconforming in the case of a controlled process.

If a characteristic, in statistical control and stable, has a C_{pk_U} of 0,86 and a C_{pk_L} of 0,91, the proportion of out-of-specification can be determined using the method given above as follows.

a) Calculate the “upper” standardized deviate.

$$\begin{aligned} z_{\hat{p}_L} &= 3\hat{C}_{\text{pk}_L} \\ &= 3 \times 0{,}91 \\ &= 2{,}73 \end{aligned}$$

b) Calculate the “lower” standardized deviate.

$$\begin{aligned} z_{\hat{p}_U} &= 3\hat{C}_{\text{pk}_U} \\ &= 3 \times 0{,}86 \\ &= 2{,}58 \end{aligned}$$

c) Using the standard normal distribution, look up or calculate the values $\hat{p}_U$ and $\hat{p}_L$ for the proportions of the distribution beyond the specification limits U and L, z_{p_U} and z_{p_L}, respectively.

For convenience and ease of use, Table 3 gives look-up values for the estimated proportion out-of-specification. Table 3 is indexed by C_{pk_U} or C_{pk_L} the process capability indices (PCI). Table 3 should not be used to derive C_p nor C_pk values for attributes data.

Using the above example of a C_{pk_U} of 0,86 and a C_{pk_L} of 0,91, the estimated proportion beyond the specification limits U and L can be read directly from Table 3 as 0,004 9 and 0,003 2.

Table 3 — C_{pk_U} or C_{pk_L} (PCI in the table) and proportion of the normal distribution remaining in the tails of the distribution beyond a specification limit

PCI	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
1,6	$7{,}9 \times 10^{-07}$	$6{,}8 \times 10^{-07}$	$5{,}9 \times 10^{-07}$	$5{,}0 \times 10^{-07}$	$4{,}3 \times 10^{-07}$	$3{,}7 \times 10^{-07}$	$3{,}2 \times 10^{-07}$	$2{,}7 \times 10^{-07}$	$2{,}3 \times 10^{-07}$	$2{,}0 \times 10^{-07}$
1,5	$3{,}4 \times 10^{-06}$	$3{,}0 \times 10^{-06}$	$2{,}6 \times 10^{-06}$	$2{,}2 \times 10^{-06}$	$1{,}9 \times 10^{-06}$	$1{,}7 \times 10^{-06}$	$1{,}4 \times 10^{-06}$	$1{,}2 \times 10^{-06}$	$1{,}1 \times 10^{-06}$	$9{,}2 \times 10^{-07}$
1,4	$1{,}3 \times 10^{-05}$	$1{,}2 \times 10^{-05}$	$1{,}0 \times 10^{-05}$	$8{,}9 \times 10^{-06}$	$7{,}8 \times 10^{-06}$	$6{,}8 \times 10^{-06}$	$5{,}9 \times 10^{-06}$	$5{,}2 \times 10^{-06}$	$4{,}5 \times 10^{-06}$	$3{,}9 \times 10^{-06}$
1,3	$4{,}8 \times 10^{-05}$	$4{,}2 \times 10^{-05}$	$3{,}7 \times 10^{-05}$	$3{,}3 \times 10^{-05}$	$2{,}9 \times 10^{-05}$	$2{,}6 \times 10^{-05}$	$2{,}3 \times 10^{-05}$	$2{,}0 \times 10^{-05}$	$1{,}7 \times 10^{-05}$	$1{,}5 \times 10^{-05}$
1,2	0,000 2	0,000 1	0,000 1	0,000 1	0,000 1	0,000 1	0,000 1	0,000 1	0,000 1	0,000 1
1,1	0,000 5	0,000 4	0,000 4	0,000 3	0,000 3	0,000 3	0,000 3	0,000 2	0,000 2	0,000 2

Table 3 *(continued)*

PCI	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
1,0	0,001 3	0,001 2	0,001 1	0,001 0	0,000 9	0,000 8	0,000 7	0,000 7	0,000 6	0,000 5
0,9	0,003 5	0,003 2	0,002 9	0,002 6	0,002 4	0,002 2	0,002 0	0,001 8	0,001 6	0,001 5
0,8	0,008 2	0,007 5	0,006 9	0,006 4	0,005 9	0,005 4	0,004 9	0,004 5	0,004 1	0,003 8
0,7	0,017 9	0,016 6	0,015 4	0,014 3	0,013 2	0,012 2	0,011 3	0,010 4	0,009 6	0,008 9
0,6	0,035 9	0,033 6	0,031 4	0,029 4	0,027 4	0,025 6	0,023 9	0,022 2	0,020 7	0,019 2
0,5	0,066 8	0,063 0	0,059 4	0,055 9	0,052 6	0,049 5	0,046 5	0,043 6	0,040 9	0,038 4
0,4	0,115 1	0,109 3	0,103 8	0,098 5	0,093 4	0,088 5	0,083 8	0,079 3	0,074 9	0,070 8
0,3	0,184 1	0,176 2	0,168 5	0,161 1	0,153 9	0,146 9	0,140 1	0,133 5	0,127 1	0,121 0
0,2	0,274 3	0,264 3	0,254 6	0,245 1	0,235 8	0,226 6	0,217 7	0,209 0	0,200 5	0,192 2
0,1	0,382 1	0,370 7	0,359 4	0,348 3	0,337 2	0,326 4	0,315 6	0,305 0	0,294 6	0,284 3
0,0	0,500 0	0,488 0	0,476 1	0,464 1	0,452 2	0,440 4	0,428 6	0,416 8	0,405 2	0,393 6

5 Performance

5.1 General

Process performance for a characteristic is the achieved distribution of results. The single important difference between *performance* and *capability* is that for performance, there is no requirement for the process to be in statistical control or for the process to be controlled using a control chart. The following are the conditions that will apply for performance:

— all technical conditions, e.g. temperature and humidity, shall be clearly stated;

— the uncertainty of the measurement system shall be estimated and judged appropriate (see ISO 22514-7);

— multi-factor, multi-level aspects of the process should be allowed;

— the duration over which the data has been gathered shall be recorded;

— the frequency of sampling shall be specified and the start and finish dates of data collection;

— the process need not be controlled with a control chart;

— the process need not be in a state of statistical control, in particular, historical data where the sequence is unknown can be used to evaluate process performance.

Indices are given below to express process performance. Their form is similar to those already given in the clause on capability and the general relationships given in Formulae (1) and (2) for measured data are used, except they are named P_{p}, P_{pk_U} and P_{pk_L}, respectively.

5.2 Process performance indices for measured data (normal distribution)

5.2.1 P_p index

When the individual values form a normal distribution, the length of the reference interval is equal to $6\sigma_t$ where σ_t is the total standard deviation. Therefore, the P_p index can be expressed as:

$$P_p = \frac{U - L}{6\sigma_t}$$

An estimate, $\hat{\sigma}_t$, of the total standard deviation (σ_t) is required to obtain an estimate of the P_p index. In practice, $\hat{\sigma}_t$ will be the standard deviation (S_t) of all of the data. When this has been obtained, the index is estimated.

5.2.2 P_{pk} index

When the distribution of individual values forms a normal distribution, the median $X_{50\ \%}$ is equal to the mean (μ). Further, $X_{99,865\ \%} - X_{50\ \%}$ and $X_{50\ \%} - X_{0,135\ \%}$ are each equal to $3\sigma_t$. Therefore, the P_{pk} index is the smaller of the two values:

$$P_{pk_U} = \frac{U - \mu}{3\sigma_t}$$

or

$$P_{pk_L} = \frac{\mu - L}{3\sigma_t}$$

where the indices are estimated by:

$$\hat{P}_{pk_U} = \frac{U - \overline{X}}{3\hat{\sigma}_t}$$

or

$$\hat{P}_{pk_L} = \frac{\overline{X} - L}{3\hat{\sigma}_t}$$

A lower index means greater proportion of items produced out-of-specification.

5.3 Process performance indices for measured data (non-normal distribution)

5.3.1 General

The approaches adopted in this clause for non-normal data is the same as that given earlier in 4.5 for capability indices.

5.3.2 Probability paper method

From graphs similar to those given in Figure 5, estimates of the percentiles $X_{0,135\ \%}$ and $X_{99,865\ \%}$ can be obtained. The estimates are denoted by Y_1 and Y_2, respectively, and the formula becomes:

$$\hat{P}_{\mathrm{p}} = \frac{U - L}{Y_2 - Y_1}$$

In a similar way, the P_{pk} formulae become:

$$\hat{P}_{\mathrm{pk}_U} = \frac{U - \hat{X}_{50\ \%}}{Y_2 - \hat{X}_{50\ \%}}$$

or

$$\hat{P}_{\mathrm{pk}_L} = \frac{\hat{X}_{50\ \%} - L}{\hat{X}_{50\ \%} - Y_1}$$

whichever of them gives the least value.

If the index is less than a given value, the process is deemed to have produced an excessive proportion of items outside of the specification. The proportion nonconforming depends upon the distribution and the value of the index. The link between the index and the proportion of nonconforming items produced depends on the class of distributions. Care should be taken not to interpret indices on the basis of cut-off points that have derived for the normal distribution and, hence, are only applicable for that distribution.

The probability paper method directly estimates fairly extreme percentiles and that this can be inaccurate. Additionally, the estimation method using probability paper, although very simple to use, is nevertheless somewhat crude and computational procedures are preferred (see Annex C).

5.3.3 Pearson curves method

As an alternative to using probability paper, standardized Pearson curves are sometimes used. The method is described by way of an example (see Annex B). The index is computed using:

$$\hat{P}_{\mathrm{p}} = \frac{U - L}{\hat{X}_{99,865\ \%} - \hat{X}_{0,135\ \%}}$$

where $\hat{X}_{0,135\ \%}$ and $\hat{X}_{99,865\ \%}$ are the estimated 0,135 % and 99,865 % percentiles from the standardized Pearson curves.

Also, we have the formulae:

$$\hat{P}_{\mathrm{pk}_U} = \frac{U - \hat{X}_{50\ \%}}{\hat{X}_{99,865\ \%} - \hat{X}_{50\ \%}}$$

and

$$\hat{P}_{\mathrm{pk}_L} = \frac{\hat{X}_{50\ \%} - L}{\hat{X}_{50\ \%} - \hat{X}_{0,135\ \%}}$$

where $\hat{X}_{50\ \%}$ is the estimated median value.

In order to use this method, the user will need to establish skewness and kurtosis values in addition to the mean and standard deviation for the data set upon which the index is to be computed.

This method is not preferred, but is presented here due to its occasional use for completeness (see 4.5.3 for further comments about the use of this method).

5.3.4 Distribution identification method

See Annex C for a description of certain families of distribution functions such as the log-normal distribution, the Rayleigh and the Weibull distributions that are commonly found when investigating process performance. See also 4.5.4 for further comments about the method.

5.4 Other performance indices for measured data

All of the indices given earlier for capability will have counterparts when considering performance. Any standard deviation will represent the total variation (σ_t) instead of the inherent variation (σ).

5.5 Assessment of proportion out-of-specification for a normal distribution of the total distribution

The same method to estimate the proportion of out-of-specification is used here as in 4.8. The reader should substitute $\hat{P}_{\text{pk}_U}$ and $\hat{P}_{\text{pk}_L}$ respectively for C_{pk_U} and C_{pk_L}. Table 3 can also be used to determine the proportion out-of-specification and the reader should enter the table with $\hat{P}_{\text{pk}_U}$ or $\hat{P}_{\text{pk}_L}$ instead of C_{pk_U} or C_{pk_L}.

6 Reporting process capability and performance indices

If process capability (or performance) statistics are used for process qualification, they should be reported in relation to this part of ISO 22514. The calculation method and the number of values used as basis for the calculation shall be stated.

An example is given in Table 4.

Table 4 — Example of report of calculated process capability indices

Process capability (or performance) index	C_p = 2,01
Minimum process capability (or performance) index	C_{pk} = 1,90
Confidence interval	1,54 < C_{pk} < 2,26
Number of values used for the calculation	100
Measurement uncertainty	0,002 mm
Optional:	
— frequency of sampling;	30 min.
— time and duration of data taking;	
— distribution model;	Normal
— technical conditions (batches, operation, tools).	

Additionally, the study report shall contain the following information:

a) the place where the study was performed and the type of process the machine is part of;

b) the persons who performed the study and who took the measurements;

c) when the study was performed, including the date, times of start and finish, log of any interruptions;

d) any machine and process reference numbers;

e) the component’s name and reference number;

f) the component characteristic(s) measured;

g) the specification for the characteristic(s) and what factors were held constant;

h) ambient conditions;

i) the raw data;

j) non-standard conditions.

For each characteristic measured, the following shall be reported (or provided):

— the distribution model estimated;

— the calculated indices.

The following should be reported (or provided):

— a control chart of the data;

— a tally chart or histogram of the data;

— a probability plot of the data;

— the mean value from the data;

— the standard deviation from the data;

— the estimated percentage out of specification;

— the calculated indices' confidence intervals;

— the measurement uncertainty/measurement process capability.

Annex A
(informative)

Estimating standard deviations

A.1 General

It is necessary to estimate the standard deviation in order to calculate the indices referred to in this part of ISO 22514. There are two types of standard deviation to consider. The first is what might be described as the short-term standard deviation or instantaneous (inherent) standard deviation. It is often calculated from statistics taken from a control chart and this is shown in A.2. The other is the estimation of the total standard deviation and this is described in A.3.

If the process has more than one mode or state, the spread should be calculated by following the method given in ISO 22514-2 or ISO 22514-8.

A.2 Inherent standard deviation

A.2.1 Estimation using the mean range value

The inherent (process) standard deviation (the data will be taken from an "in control" control chart) can be estimated from a range control chart using:

$$\hat{\sigma} = \frac{\overline{R}}{d_2}$$

where d_2 is a factor taken from Table A.1.

Table A.1 — Factors for the estimation of process standard deviation

Subgroup size (n)	d_2	c_4
2	1,128	0,797 9
3	1,693	0,886 2
4	2,059	0,921 3
5	2,326	0,940 0
6	2,534	0,951 5
7	2,704	0,959 4
8	2,847	0,965 0
9	2,970	0,969 3
10[a]	3,078	0,972 7

[a] Values for d_2 and c_4 may be found in textbooks for sample sizes greater than 10.

A.2.2 Estimation using the mean standard deviation value

If a standard deviation control chart has been used to monitor the within subgroup variation, the inherent (process) standard deviation can be estimated using:

$$\hat{\sigma} = \frac{\bar{S}}{c_4}$$

where c_4 is a control chart factor taken from Table A.1.

A.2.3 Estimation using subgroup standard deviations

If the within subgroup standard deviation is calculated for every subgroup, the inherent (process) standard deviation can be estimated using:

$$\hat{\sigma} = \sqrt{\frac{\sum_{j=1}^{m} S_j^2}{m}}$$

where there are m subgroups of n observations in each.

A.3 Estimation of total standard deviation

When data are generated from a process which is “out-of-control” or if no control chart has been used, it is inappropriate to compute the standard deviation using the methods of A.2. The following formula should be used instead:

$$\hat{\sigma}_t = S_t = \sqrt{\frac{\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2}{N-1}}$$

The circumstances that lead to the use of this formula are when fluctuations exist in the process mean caused by assignable causes that might not be removable and this extra variation is to be incorporated with the remaining random cause variation. It is the appropriate measure of variation for use in calculating the performance indices.

When considering multiple stream processes, such as a multiple cavity injection moulding press, it is often desired to treat the data from all the cavities as if they came from a single process. The data from each cavity might form a single normal distribution. However, the reality is often that each cavity produces a slightly different distribution because either the means or the variabilities or both are different. If the data from all of the process streams can be considered to yield a normal distribution, the best estimate of the process variation will be given by this formula.

Annex B
(informative)

Estimating capability and performance measures using Pearson curves — Procedure and example[1)]

B.1 Record specification limits

Upper limit, $U = 0{,}30$

Lower limit, $L = 0{,}20$

B.2 Record process statistics

The process is shown to be statistically “in control”.

Mean, $\bar{x} = 0{,}235$

Standard deviation, $\hat{\sigma} = 0{,}012\ 2$

Skewness, $\hat{\gamma}_1 = 0{,}7$ (rounding to one decimal place)

Kurtosis, $\hat{\beta}_2 = 3{,}5$ (rounding to one decimal place)

B.3 Look up standardized 0,135 percentile

For positive skewness, use Table B.1. For negative skewness, use Table B.2.

0,135 percentile, $P_{0{,}135\ \%} = 3{,}056$ by interpolation.

B.4 Look up standardized 99,865 percentile

For positive skewness, use Table B.2. For negative skewness, use Table B.1.

99,865 percentile, $P_{99{,}865\ \%} = 4{,}656$ by interpolation.

B.5 Look up standardized median in Table B.3

For positive skewness, reverse the sign. For negative skewness, leave positive.

Standardized median, $P_{50\ \%} = -0{,}067\ 5$ by interpolation.

B.6 Compute estimated 0,135 % percentile

$$\begin{aligned}\hat{X}_{0{,}135\ \%} &= \bar{x} - \hat{\sigma} P_{0{,}135\ \%} \\ &= 0{,}235 - (0{,}012\ 2 \times 3{,}056) \\ &= 0{,}197\ 7\end{aligned}$$

1) Procedure based on that in Reference [10].

B.7 Compute estimated 99,865 % percentile

$$\begin{aligned} X_{99{,}865\ \%} &= \overline{x} + \hat{\sigma} P_{99{,}865\ \%} \\ &= 0{,}235 + (0{,}012\ 2 \times 4{,}656) \\ &= 0{,}291\ 8 \end{aligned}$$

B.8 Compute estimated median

$$\begin{aligned} \hat{X}_{50\ \%} &= \overline{x} + \hat{\sigma} P_{50\ \%} \\ &= 0{,}235 + (0{,}012\ 2 \times -0{,}067\ 5) \\ &= 0{,}234\ 2 \end{aligned}$$

B.9 Compute process capability indices

$$\begin{aligned} \hat{C}_{\mathrm{p}} &= \frac{U - L}{\hat{X}_{99{,}865\ \%} - \hat{X}_{0{,}135\ \%}} \\ &= \frac{0{,}30 - 0{,}20}{0{,}291\ 8 - 0{,}197\ 7} \\ &= 1{,}06 \end{aligned}$$

$$\begin{aligned} \hat{C}_{\mathrm{pk}_U} &= \frac{U - \hat{X}_{50\ \%}}{\hat{X}_{99{,}865\ \%} - \hat{X}_{50\ \%}} \\ &= \frac{0{,}30 - 0{,}234\ 2}{0{,}291\ 8 - 0{,}234\ 2} \\ &= 1{,}14 \end{aligned}$$

$$\begin{aligned} \hat{C}_{\mathrm{pk}_L} &= \frac{\hat{X}_{50\ \%} - L}{\hat{X}_{50\ \%} - \hat{X}_{0{,}135\ \%}} \\ &= \frac{0{,}234\ 2 - 0{,}20}{0{,}234\ 2 - 0{,}197\ 7} \\ &= 0{,}94 \end{aligned}$$

Table B.1

PEARSON CURVES (STANDARDIZED TAILS)

$P_{0,135\ \%}$ (0,135 percentile) for $\gamma_1 > 0$. $P_{99,865\ \%}$ (99,865 percentile) for $\gamma_1 < 0$.

Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
−1,4	1,512	1,421	1,317	1,206	1,092	0,979	0,868	0,762														−1,4
−1,2	1,727	1,619	1,496	1,364	1,230	1,100	0,975	0,858	0,747													−1,2
−1,0	1,966	1,840	1,696	1,541	1,384	1,232	1,089	0,957	0,836													−1,0
−0,8	2,210	2,072	1,912	1,736	1,555	1,377	1,212	1,062	0,927	0,804	0,692											−0,8
−0,6	2,442	2,298	2,129	1,941	1,740	1,539	1,348	1,175	1,023	0,887	0,766	0,656										−0,6
−0,4	2,653	2,506	2,335	2,141	1,930	1,711	1,496	1,299	1,125	0,974	0,841	0,723	0,616									−0,4
−0,2	2,839	2,692	2,522	2,329	2,116	1,887	1,655	1,434	1,235	1,065	0,919	0,791	0,677	0,574								−0,2
0,0	3,000	2,856	2,689	2,500	2,289	2,059	1,817	1,578	1,356	1,163	1,000	0,861	0,739	0,630	0,531							0,0
0,2	3,140	2,986	2,834	2,653	2,447	2,220	1,976	1,726	1,485	1,269	1,086	0,933	0,801	0,686	0,583							0,2
0,4	3,261	3,088	2,952	2,785	2,589	2,368	2,127	1,873	1,619	1,382	1,178	1,008	0,865	0,742	0,634	0,536						0,4
0,6	3,366	3,164	3,045	2,896	2,714	2,502	2,267	2,015	1,754	1,502	1,277	1,087	0,931	0,799	0,685	0,583	0,489					0,6
0,8	3,458	3,222	3,118	2,986	2,821	2,622	2,396	2,148	1,887	1,625	1,381	1,172	1,000	0,857	0,736	0,629	0,533					0,8
1,0	3,539	3,266	3,174	3,058	2,910	2,727	2,512	2,271	2,013	1,748	1,491	1,262	1,072	0,917	0,787	0,675	0,575	0,484				1,0
1,2	3,611	3,300	3,218	3,115	2,983	2,817	2,616	2,385	2,132	1,876	1,602	1,357	1,149	0,979	0,840	0,721	0,617	0,524				1,2
1,4	3,674	3,327	3,254	3,161	3,043	2,893	2,708	2,488	2,243	1,981	1,713	1,456	1,230	1,045	0,894	0,768	0,659	0,562	0,475			1,4
1,6	3,731	3,349	3,282	3,199	3,092	2,957	2,787	2,581	2,345	2,089	1,821	1,556	1,316	1,113	0,950	0,815	0,701	0,600	0,510			1,6
1,8	3,782	3,367	3,306	3,229	3,133	3,011	2,855	2,664	2,438	2,189	1,925	1,664	1,404	1,185	1,008	0,863	0,743	0,638	0,546	0,461		1,8
2,0	3,828	3,382	3,325	3,255	3,167	3,055	2,914	2,736	2,524	2,283	2,023	1,755	1,494	1,261	1,068	0,913	0,785	0,676	0,580	0,494		2,0
2,2	3,870	3,395	3,342	3,277	3,196	3,093	2,964	2,800	2,600	2,369	2,116	1,850	1,584	1,339	1,132	0,964	0,828	0,714	0,615	0,526	0,445	2,2
2,4	3,908	3,405	3,356	3,295	3,220	3,126	3,006	2,855	2,669	2,448	2,202	1,940	1,673	1,420	1,198	1,018	0,873	0,752	0,649	0,557	0,475	2,4
2,6	3,943	3,415	3,367	3,311	3,241	3,153	3,043	2,904	2,730	2,521	2,283	2,026	1,760	1,501	1,267	1,073	0,918	0,791	0,683	0,589	0,504	2,6
2,8	3,975	3,423	3,378	3,324	3,259	3,177	3,075	2,946	2,784	2,586	2,358	2,107	1,844	1,581	1,338	1,131	0,965	0,830	0,717	0,620	0,533	2,8

Table B.1 *(continued)*

PEARSON CURVES (STANDARDIZED TAILS) $P_{0,135\ \%}$ (0,135 percentile) for $\gamma_1 > 0$. $P_{99,865\ \%}$ (99,865 percentile) for $\gamma_1 < 0$.																						
Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
3,0	4,004	3,430	3,387	3,326	3,274	3,198	3,103	2,983	2,831	2,646	2,427	2,183	1,924	1,661	1,410	1,191	1,013	0,870	0,752	0,651	0,562	3,0
3,2	4,031	3,436	3,395	3,346	3,288	3,216	3,127	3,015	2,874	2,699	2,491	2,254	2,000	1,738	1,483	1,253	1,063	0,911	0,787	0,681	0,590	3,2
3,4	4,056	3,441	3,402	3,356	3,300	3,233	3,149	3,043	2,911	2,747	2,549	2,321	2,072	1,813	1,555	1,317	1,115	0,953	0,822	0,712	0,618	3,4
3,6	4,079	3,446	3,408	3,364	3,311	3,247	3,168	3,069	2,945	2,790	2,602	2,383	2,140	1,884	1,626	1,381	1,169	0,996	0,858	0,744	0,646	3,6
3,8	4,101	3,450	3,414	3,371	3,321	3,259	3,184	3,091	2,974	2,829	2,651	2,440	2,205	1,953	1,695	1,446	1,224	1,041	0,895	0,775	0,674	3,8
4,0	4,121	3,454	3,419	3,378	3,329	3,271	3,200	3,111	3,001	2,864	2,695	2,494	2,265	2,018	1,762	1,510	1,281	1,088	0,932	0,807	0,702	4,0
4,2	4,140	3,458	3,423	3,384	3,337	3,281	3,213	3,129	3,025	2,895	2,735	2,543	2,321	2,080	1,827	1,574	1,338	1,135	0,971	0,839	0,730	4,2
4,4	4,157	3,461	3,428	3,389	3,344	3,290	3,225	3,145	3,047	2,923	2,771	2,588	2,374	2,138	1,889	1,636	1,396	1,184	1,011	0,872	0,758	4,4
					v																	
4,6	4,174	3,464	3,431	3,394	3,350	3,299	3,236	3,160	3,066	2,949	2,805	2,629	2,424	2,194	1,948	1,697	1,453	1,234	1,052	0,905	0,786	4,6
4,8	4,189	3,466	3,435	3,399	3,356	3,306	3,246	3,173	3,084	2,972	2,835	2,668	2,470	2,246	2,005	1,756	1,510	1,285	1,094	0,939	0,815	4,8
5,0	4,204	3,469	3,438	3,403	3,362	3,313	3,256	3,186	3,100	2,994	2,863	2,703	2,513	2,296	2,059	1,813	1,566	1,336	1,137	0,975	0,844	5,0
5,2	4,218	3,471	3,441	3,406	3,367	3,320	3,264	3,197	3,114	3,013	2,888	2,735	2,562	2,342	2,111	1,867	1,621	1,387	1,181	1,010	0,874	5,2
5,4	4,231	3,473	3,444	3,410	3,371	3,326	3,272	3,207	3,128	3,031	2,911	2,765	2,589	2,386	2,160	1,920	1,675	1,438	1,225	1,047	0,904	5,4
5,6	4,243	3,475	3,446	3,413	3,375	3,331	3,279	3,216	3,140	3,047	2,933	2,793	2,624	2,427	2,206	1,970	1,727	1,489	1,270	1,085	0,935	5,6
5,8	4,255	3,477	3,448	3,416	3,379	3,336	3,286	3,225	3,152	3,062	2,952	2,818	2,656	2,465	2,250	2,019	1,778	1,539	1,316	1,123	0,966	5,8
6,0	4,266	3,478	3,451	3,419	3,383	3,341	3,292	3,233	3,162	3,076	2,970	2,841	2,685	2,501	2,292	2,065	1,827	1,588	1,361	1,162	0,999	6,0
6,2	4,276	3,480	3,453	3,422	3,386	3,345	3,297	3,240	3,172	3,089	2,987	2,863	2,713	2,535	2,332	2,109	1,874	1,635	1,407	1,202	1,031	6,2
6,4	4,286	3,481	3,454	3,424	3,389	3,349	3,303	3,247	3,181	3,100	3,003	2,883	2,739	2,567	2,369	2,151	1,919	1,682	1,452	1,242	1,065	6,4
6,6	4,296	3,483	3,456	3,426	3,392	3,353	3,308	3,254	3,189	3,111	3,017	2,902	2,763	2,597	2,405	2,191	1,962	1,727	1,496	1,282	1,099	6,6
6,8	4,305	3,484	3,458	3,429	3,395	3,357	3,312	3,260	3,197	3,122	3,030	2,919	2,785	2,624	2,438	2,229	2,004	1,771	1,540	1,323	1,134	6,8
7,0	4,313	3,485	3,459	3,431	3,398	3,360	3,316	3,265	3,204	3,131	3,043	2,936	2,806	2,651	2,469	2,265	2,044	1,814	1,583	1,363	1,169	7,0
7,2	4,322	3,486	3,461	3,432	3,400	3,363	3,321	3,270	3,211	3,140	3,054	2,951	2,825	2,675	2,499	2,300	2,083	1,855	1,625	1,403	1,204	7,2
7,4	4,330	3,487	3,462	3,434	3,403	3,366	3,324	3,275	3,218	3,148	3,065	2,965	2,843	2,698	2,527	2,333	2,120	1,895	1,666	1,443	1,240	7,4

Table B.1 *(continued)*

PEARSON CURVES (STANDARDIZED TAILS) $P_{0,135\ \%}$ (0,135 percentile) for $\gamma_1 > 0$. $P_{99,865\ \%}$ (99,865 percentile) for $\gamma_1 < 0$.																						
Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
7,6	4,337	3,488	3,464	3,436	3,405	3,369	3,328	3,280	3,224	3,156	3,075	2,978	2,860	2,720	2,554	2,364	2,155	1,933	1,706	1,482	1,276	7,6
7,8	4,344	3,489	3,465	3,437	3,407	3,372	3,331	3,284	3,229	3,164	3,085	2,990	2,876	2,740	2,579	2,394	2,189	1,970	1,744	1,521	1,311	7,8
8,0	4,351	3,490	3,466	3,439	3,409	3,374	3,335	3,289	3,235	3,171	3,094	3,002	2,891	2,759	2,603	2,422	2,221	2,005	1,782	1,559	1,347	8,0
8,2	4,358	3,491	3,467	3,440	3,411	3,377	3,338	3,292	3,240	3,177	3,103	3,013	2,906	2,777	2,625	2,449	2,252	2,040	1,818	1,596	1,382	8,2
8,4	4,365	3,492	3,468	3,442	3,412	3,379	3,340	3,296	3,244	3,183	3,111	3,023	2,919	2,794	2,646	2,475	2,282	2,073	1,854	1,632	1,418	8,4
8,6	4,371	3,492	3,469	3,443	3,414	3,381	3,343	3,300	3,249	3,189	3,118	3,033	2,932	2,810	2,666	2,499	2,310	2,104	1,888	1,667	1,452	8,6
8,8	4,377	3,493	3,470	3,444	3,416	3,383	3,346	3,303	3,253	3,195	3,125	3,042	2,943	2,825	2,685	2,522	2,337	2,135	1,921	1,702	1,486	8,8
9,0	4,382	3,494	3,471	3,445	3,417	3,385	3,348	3,306	3,257	3,200	3,132	3,051	2,955	2,839	2,703	2,544	2,363	2,164	1,953	1,736	1,520	9,0
9,2	4,388	3,495	3,472	3,447	3,418	3,387	3,351	3,309	3,261	3,205	3,138	3,059	2,965	2,853	2,720	2,565	2,388	2,192	1,984	1,768	1,553	9,2
9,4	4,393	3,495	3,473	3,448	3,420	3,388	3,353	3,312	3,265	3,209	3,144	3,067	2,975	2,866	2,736	2,585	2,411	2,219	2,014	1,800	1,586	9,4
9,6	4,398	3,496	3,473	3,449	3,421	3,390	3,355	3,315	3,268	3,214	3,150	3,075	2,985	2,878	2,752	2,604	2,434	2,245	2,042	1,831	1,617	9,6
9,8	4,403	3,496	4,474	3,450	3,422	3,392	3,357	3,317	3,272	3,218	3,156	3,082	2,994	2,890	2,766	2,622	2,456	2,271	2,070	1,861	1,648	9,8
10,0	4,408	3,497	3,475	3,451	3,424	3,393	3,359	3,320	3,275	3,222	3,161	3,088	3,003	2,901	2,780	2,639	2,476	2,295	2,097	1,890	1,679	10,0
10,2					3,425	3,395	3,361	3,322	3,278	3,226	3,166	3,095	3,011	2,911	2,793	2,655	2,496	2,318	2,123	1,918	1,708	10,2
10,4						3,396	3,363	3,325	3,281	3,230	3,171	3,101	3,019	2,921	2,806	2,671	2,515	2,340	2,148	1,945	1,737	10,4
10,6							3,364	3,327	3,283	3,233	3,175	3,107	3,026	2,930	2,818	2,686	2,533	2,361	2,172	1,972	1,765	10,6
10,8								3,329	3,286	3,237	3,179	3,112	3,033	2,940	2,829	2,700	2,551	2,382	2,196	1,998	1,793	10,8
11,0									3,289	3,240	3,184	3,118	3,040	2,948	2,840	2,714	2,567	2,401	2,218	2,023	1,819	11,0
11,2										3,243	3,188	3,123	3,046	2,956	2,851	2,727	2,583	2,420	2,240	2,047	1,845	11,2
11,4											3,191	3,128	3,053	2,964	2,861	2,739	2,598	2,438	2,261	2,070	1,870	11,4
11,6											3,195	3,132	3,058	2,972	2,870	2,751	2,613	2,456	2,281	2,093	1,895	11,6

Table B.1 *(continued)*

PEARSON CURVES (STANDARDIZED TAILS)

$P_{0,135\,\%}$ (0,135 percentile) for $\gamma_1 > 0$. $P_{99,865\,\%}$ (99,865 percentile) for $\gamma_1 < 0$.

Kurtosis (β_2)	Skewness (γ_1) 0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	(β_2)
11,8												3,137	3,064	2,979	2,879	2,762	2,627	2,473	2,301	2,115	1,919	11,8
12,0												3,141	3,070	2,986	2,888	2,773	2,641	2,489	2,320	2,136	1,942	12,0
12,2													3,075	2,993	2,896	2,784	2,653	2,505	2,338	2,157	1,965	12,2

Table B.2

PEARSON CURVES (STANDARDIZED TAILS) $P_{99,865\ \%}$ (99,865 percentile) for $\gamma_1 > 0$. $P_{0,135\ \%}$ (0,135 percentile) for $\gamma_1 < 0$.																						
Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
−1,4	1,512	1,584	1,632	1,655	1,653	1,626	1,579	1,516														−1,4
−1,2	1,727	1,813	1,871	1,899	1,895	1,861	1,803	1,726	1,636													−1,2
−1,0	1,966	2,065	2,134	2,170	2,169	2,131	2,061	1,966	1,856													−1,0
−0,8	2,210	2,320	2,400	2,446	2,454	2,422	2,349	2,241	2,108	1,965	1,822											−0,8
−0,6	2,442	2,560	2,648	2,704	2,726	2,708	2,646	2,540	2,395	2,225	2,052	1,885										−0,6
−0,4	2,653	2,774	2,869	2,934	2,969	2,968	2,926	2,837	2,699	2,518	2,314	2,114	1,928									−0,4
−0,2	2,839	2,961	3,060	3,133	3,179	3,194	3,173	3,109	2,993	2,824	2,608	2,373	2,152	1,952								−0,2
0,0	3,000	3,123	3,224	3,303	3,358	3,387	3,385	3,345	3,259	3,116	2,914	2,665	2,405	2,169	1,960							0,0
0,2	3,140	3,261	3,364	3,447	3,510	3,550	3,564	3,546	3,488	3,378	3,206	2,970	2,690	2,412	2,167							0,2
0,4	3,261	3,381	3,484	3,570	3,639	3,688	3,715	3,715	3,681	3,603	3,468	3,264	2,993	2,687	2,398	2,149						0,4
0,6	3,366	3,485	3,588	3,676	3,749	3,805	3,843	3,858	3,844	3,793	3,693	3,529	3,290	2,984	2,658	2,366	2,119					0,6
0,8	3,458	3,575	3,678	3,768	3,844	3,905	3,951	3,978	3,981	3,953	3,883	3,758	3,561	3,283	2,945	2,609	2,322					0,8
1,0	3,539	3,654	3,757	3,847	3,926	3,991	4,044	4,080	4,096	4,087	4,043	3,952	3,797	3,561	3,243	2,881	2,547	2,269				1,0
1,2	3,611	3,724	3,826	3,917	3,997	4,066	4,124	4,167	4,194	4,208	4,177	4,115	3,998	3,808	3,529	3,172	2,798	2,476				1,2
1,4	3,674	3,786	3,887	3,978	4,060	4,131	4,193	4,243	4,278	4,296	4,290	4,252	4,168	4,020	3,789	3,463	3,075	2,705	2,399			1,4
1,6	3,731	3,842	3,942	4,033	4,115	4,189	4,253	4,308	4,351	4,378	4,386	4,367	4,311	4,200	4,015	3,736	3,364	2,961	2,609			1,6
1,8	3,782	3,891	3,990	4,081	4,164	4,239	4,307	4,365	4,414	4,449	4,468	4,472	4,431	4,352	4,209	3,979	3,646	3,238	2,840	2,511		1,8
2,0	3,828	3,936	4,034	4,125	4,208	4,285	4,354	4,416	4,468	4,511	4,539	4,549	4,532	4,479	4,372	4,189	3,907	3,522	3,095	2,719		2,0
2,2	3,870	3,976	4,073	4,164	4,248	4,325	4,396	4,460	4,517	4,564	4,600	4,620	4,619	4,587	4,510	4,369	4,137	3,796	3,370	2,949	2,603	2,2
2,4	3,908	4,013	4,109	4,199	4,283	4,361	4,433	4,500	4,559	4,611	4,653	4,682	4,693	4,678	4,627	4,521	4,336	4,047	3,648	3,201	2,808	2,4
2,6	3,943	4,046	4,142	4,231	4,315	4,394	4,467	4,535	4,597	4,653	4,700	4,736	4,757	4,756	4,725	4,649	4,506	4,269	3,916	3,471	3,033	2,6
2,8	3,975	4,077	4,172	4,261	4,344	4,423	4,498	4,567	4,631	4,690	4,741	4,783	4,812	4,824	4,809	4,758	4,650	4,460	4,160	3,745	3,280	2,8

Table B.2 *(continued)*

PEARSON CURVES (STANDARDIZED TAILS)

$P_{99,865\ \%}$ (99,865 percentile) for $\gamma_1 > 0$. $P_{0,135\ \%}$ (0,135 percentile) for $\gamma_1 < 0$.

Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
3,0	4,004	4,105	4,199	4,287	4,371	4,450	4,525	4,596	4,662	4,723	4,777	4,824	4,860	4,882	4,881	4,850	4,771	4,623	4,376	4,007	3,544	3,0
3,2	4,031	4,131	4,224	4,312	4,396	4,475	4,550	4,622	4,689	4,752	4,810	4,861	4,903	4,932	4,944	4,929	4,875	4,762	4,563	4,247	3,813	3,2
3,4	4,056	4,155	4,247	4,335	4,418	4,498	4,573	4,645	4,714	4,779	4,839	4,893	4,940	4,976	4,997	4,996	4,963	4,880	4,723	4,461	4,072	3,4
3,6	4,079	4,177	4,269	4,356	4,439	4,518	4,594	4,667	4,737	4,803	4,865	4,922	4,973	5,015	5,044	5,055	5,038	4,980	4,859	4,647	4,311	3,6
3,8	4,101	4,197	4,288	4,375	4,458	4,537	4,614	4,687	4,757	4,825	4,888	4,948	5,002	5,049	5,085	5,106	5,103	5,066	4,976	4,806	4,524	3,8
4,0	4,121	4,217	4,307	4,393	4,476	4,555	4,631	4,705	4,776	4,845	4,910	4,972	5,029	5,080	5,122	5,150	5,159	5,139	5,075	4,943	4,712	4,0
4,2	4,140	4,234	4,324	4,410	4,492	4,571	4,648	4,722	4,794	4,863	4,929	4,993	5,052	5,107	5,153	5,189	5,208	5,202	5,159	5,059	4,873	4,2
4,4	4,157	4,251	4,340	4,425	4,508	4,587	4,663	4,737	4,809	4,879	4,947	5,012	5,074	5,131	5,181	5,223	5,250	5,257	5,232	5,159	5,012	4,4
4,6	4,174	4,267	4,355	4,440	4,522	4,601	4,677	4,752	4,824	4,895	4,963	5,029	5,093	5,152	5,207	5,253	5,288	5,305	5,295	5,244	5,131	4,6
4,8	4,189	4,281	4,369	4,454	4,535	4,614	4,691	4,765	4,838	4,909	4,978	5,045	5,110	5,172	5,229	5,280	5,321	5,346	5,349	5,318	5,233	4,8
5,0	4,204	4,295	4,383	4,467	4,548	4,627	4,703	4,778	4,851	4,922	4,992	5,060	5,126	5,190	5,249	5,303	5,350	5,383	5,396	5,381	5,320	5,0
5,2	4,218	4,308	4,395	4,479	4,560	4,638	4,715	4,789	4,862	4,934	5,004	5,073	5,141	5,206	5,267	5,325	5,376	5,415	5,437	5,436	5,395	5,2
5,4	4,231	4,321	4,407	4,490	4,571	4,649	4,725	4,800	4,873	4,945	5,016	5,086	5,154	5,220	5,284	5,344	5,399	5,443	5,474	5,483	5,460	5,4
5,6	4,243	4,332	4,418	4,501	4,581	4,659	4,736	4,810	4,884	4,956	5,027	5,097	5,166	5,233	5,299	5,361	5,418	5,468	5,505	5,525	5,516	5,6
5,8	4,255	4,343	4,429	4,511	4,591	4,669	4,745	4,820	4,893	4,966	5,037	5,108	5,177	5,246	5,312	5,376	5,436	5,491	5,533	5,561	5,565	5,8
6,0	4,266	4,354	4,439	4,521	4,600	4,678	4,754	4,829	4,902	4,975	5,046	5,117	5,188	5,257	5,325	5,390	5,452	5,511	5,558	5,593	5,608	6,0
6,2	4,276	4,364	4,448	4,530	4,609	4,695	4,763	4,837	4,911	4,983	5,055	5,126	5,197	5,267	5,336	5,403	5,467	5,529	5,581	5,621	5,645	6,2
6,4	4,286	4,373	4,457	4,538	4,618	4,703	4,771	4,845	4,919	4,991	5,063	5,135	5,206	5,276	5,346	5,414	5,480	5,542	5,600	5,646	5,678	6,4
6,6	4,296	4,382	4,466	4,547	4,626	4,710	4,778	4,853	4,926	4,999	5,071	5,143	5,214	5,285	5,356	5,425	5,492	5,557	5,618	5,669	5,706	6,6
6,8	4,305	4,391	4,474	4,554	4,633	4,717	4,785	4,860	4,933	5,006	5,078	5,150	5,222	5,293	5,364	5,434	5,503	5,569	5,634	5,688	5,732	6,8
7,0	4,313	4,399	4,481	4,562	4,640	4,724	4,792	4,867	4,940	5,013	5,085	5,157	5,229	5,301	5,372	5,443	5,513	5,581	5,648	5,706	5,754	7,0
7,2	4,322	4,406	4,489	4,569	4,647	4,730	4,799	4,873	4,946	5,019	5,091	5,164	5,236	5,308	5,380	5,451	5,522	5,591	5,658	5,722	5,775	7,2
7,4	4,330	4,414	4,496	4,576	4,654	4,736	4,805	4,879	4,952	5,025	5,097	5,170	5,242	5,314	5,387	5,459	5,530	5,601	5,669	5,736	5,792	7,4

Table B.2 *(continued)*

PEARSON CURVES (STANDARDIZED TAILS)

$P_{99,865\ \%}$ (99,865 percentile) for $\gamma_1 > 0$.$P_{0,135\ \%}$ (0,135 percentile) for $\gamma_1 < 0$.

Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
7,6	4,337	4,421	4,503	4,582	4,660	4,742	4,811	4,885	4,958	5,031	5,103	5,175	5,248	5,320	5,393	5,466	5,538	5,609	5,679	5,749	5,808	7,6
7,8	4,344	4,428	4,509	4,588	4,666	4,747	4,817	4,890	4,963	5,036	5,109	5,181	5,253	5,326	5,399	5,472	5,545	5,617	5,688	5,760	5,823	7,8
8,0	4,351	4,434	4,515	4,594	4,672	4,753	4,822	4,896	4,969	5,041	5,114	5,186	5,259	5,331	5,404	5,478	5,551	5,624	5,696	5,771	5,836	8,0
8,2	4,358	4,441	4,521	4,600	4,677	4,758	4,827	4,901	4,974	5,046	5,118	5,191	5,263	5,336	5,410	5,483	5,557	5,631	5,704	5,775	5,847	8,2
8,4	4,365	4,447	4,527	4,605	4,682	4,762	4,832	4,905	4,978	5,051	5,123	5,195	5,268	5,341	5,414	5,488	5,562	5,637	5,710	5,783	5,858	8,4
8,6	4,371	4,452	4,532	4,611	4,687	4,767	4,837	4,910	4,983	5,055	5,127	5,200	5,272	5,345	5,419	5,493	5,567	5,642	5,717	5,790	5,867	8,6
8,8	4,377	4,458	4,538	4,616	4,692	4,772	4,841	4,914	4,987	5,059	5,132	5,204	5,276	5,349	5,423	5,497	5,572	5,647	5,722	5,797	5,875	8,8
9,0	4,382	4,463	4,543	4,621	4,697	4,776	4,845	4,918	4,991	5,063	5,135	5,208	5,280	5,353	5,427	5,501	5,576	5,652	5,727	5,803	5,883	9,0
9,2	4,388	4,468	4,548	4,625	4,701	4,780	4,850	4,923	4,995	5,067	5,139	5,211	5,284	5,357	5,431	5,505	5,580	5,656	5,732	5,808	5,883	9,2
9,4	4,393	4,473	4,552	4,630	4,705	4,784	4,854	4,926	4,999	5,071	5,143	5,215	5,287	5,361	5,434	5,509	5,584	5,660	5,736	5,813	5,889	9,4
9,6	4,398	4,478	4,557	4,634	4,710	4,788	4,857	4,930	5,002	5,074	5,146	5,218	5,291	5,364	5,437	5,512	5,587	5,663	5,740	5,817	5,894	9,6
9,8	4,403	4,483	4,561	4,638	4,714	4,791	4,861	4,934	5,006	5,078	5,149	5,222	5,294	5,367	5,440	5,515	5,590	5,667	5,744	5,821	5,898	9,8
10,0	4,408	4,487	4,565	4,642	4,717	4,795	4,865	4,937	5,009	5,081	5,153	5,225	5,297	5,370	5,443	5,518	5,593	5,670	5,747	5,825	5,903	10,0
10,2					4,721	4,798	4,868	4,940	5,012	5,084	5,156	5,228	5,300	5,373	5,446	5,521	5,596	5,673	5,750	5,828	5,906	10,2
10,4							4,871	4,943	5,015	5,087	5,158	5,230	5,303	5,375	5,449	5,523	5,599	5,675	5,753	5,831	5,910	10,4
10,6							4,874	4,947	5,018	5,090	5,161	5,233	5,305	5,378	5,451	5,526	5,601	5,678	5,755	5,834	5,913	10,6
10,8								4,949	5,021	5,092	5,164	5,236	5,308	5,380	5,454	5,528	5,603	5,680	5,757	5,836	5,915	10,8
11,0									5,024	5,095	5,166	5,238	5,310	5,383	5,456	5,530	5,605	5,682	5,760	5,838	5,918	11,0
11,2										5,098	5,169	5,240	5,312	5,385	5,458	5,532	5,607	5,684	5,762	5,840	5,920	11,2
11,4											5,171	5,243	5,314	5,387	5,460	5,534	5,609	5,686	5,763	5,842	5,922	11,4
11,6											5,173	5,245	5,316	5,389	5,462	5,536	5,611	5,687	5,765	5,844	5,924	11,6

Table B.2 *(continued)*

PEARSON CURVES (STANDARDIZED TAILS)

$P_{99,865\ \%}$ (99,865 percentile) for $\gamma_1 > 0$.$P_{0,135\ \%}$ (0,135 percentile) for $\gamma_1 < 0$.

Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
11,8												5,247	5,318	5,391	5,464	5,538	5,613	5,689	5,767	5,845	5,925	11,8
12,0												5,249	5,320	5,393	5,465	5,539	5,614	5,690	5,768	5,847	5,927	12,0
12,2													5,322	5,394	5,467	5,541	5,616	5,692	5,769	5,848	5,928	12,2

Table B.3

PEARSON CURVES (STANDARDIZED MEDIAN) $P_{50\,\%}$ (50 percentile). Change sign for $\gamma_1 > 0$.																						
Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
−1,4	0,000	0,053	0,111	0,184	0,282	0,424	0,627	0,754														−1,4
−1,2	0,000	0,039	0,082	0,132	0,196	0,284	0,412	0,591	0,727													−1,2
−1,0	0,000	0,031	0,065	0,103	0,151	0,212	0,297	0,419	0,586													−1,0
−0,8	0,000	0,026	0,054	0,085	0,123	0,169	0,231	0,317	0,439	0,598	0,681											−0,8
−0,6	0,000	0,023	0,047	0,073	0,104	0,142	0,190	0,254	0,343	0,468	0,616	0,653										−0,6
−0,4	0,000	0,020	0,041	0,064	0,091	0,122	0,161	0,212	0,280	0,375	0,504	0,633	0,616									−0,4
−0,2	0,000	0,018	0,037	0,058	0,081	0,108	0,141	0,183	0,237	0,311	0,413	0,542	0,638	0,574								−0,2
0,0	0,000	0,017	0,034	0,053	0,073	0,097	0,126	0,161	0,206	0,266	0,347	0,456	0,579	0,621	0,531							0,0
0,2	0,000	0,015	0,032	0,049	0,068	0,089	0,114	0,145	0,183	0,233	0,299	0,388	0,501	0,605	0,582							0,2
0,4	0,000	0,014	0,029	0,045	0,063	0,082	0,105	0,132	0,165	0,208	0,263	0,336	0,433	0,545	0,607	0,536						0,4
0,6	0,000	0,013	0,028	0,043	0,059	0,077	0,097	0,122	0,151	0,188	0,235	0,297	0,379	0,481	0,579	0,579	0,489					0,6
0,8	0,000	0,013	0,026	0,040	0,055	0,072	0,091	0,113	0,140	0,172	0,213	0,266	0,336	0,425	0,527	0,590	0,533					0,8
1,0	0,000	0,012	0,025	0,038	0,053	0,068	0,086	0,106	0,130	0,159	0,196	0,242	0,301	0,379	0,474	0,563	0,569	0,484				1,0
1,2	0,000	0,011	0,024	0,036	0,050	0,065	0,082	0,100	0,122	0,148	0,181	0,222	0,274	0,341	0,426	0,520	0,576	0,524				1,2
1,4	0,000	0,011	0,023	0,035	0,048	0,062	0,078	0,095	0,116	0,140	0,169	0,206	0,252	0,310	0,385	0,474	0,554	0,555	0,475			1,4
1,6	0,000	0,010	0,022	0,034	0,046	0,060	0,074	0,091	0,110	0,132	0,159	0,192	0,233	0,285	0,351	0,432	0,518	0,564	0,510			1,6
1,8	0,000	0,010	0,021	0,032	0,044	0,057	0,072	0,087	0,105	0,126	0,151	0,180	0,217	0,264	0,323	0,396	0,480	0,549	0,540	0,461		1,8
2,0	0,000	0,009	0,020	0,031	0,043	0,055	0,069	0,084	0,101	0,120	0,143	0,171	0,204	0,246	0,299	0,365	0,443	0,521	0,552	0,494		2,0
2,2	0,000	0,009	0,020	0,030	0,042	0,054	0,067	0,081	0,097	0,115	0,137	0,162	0,193	0,231	0,279	0,338	0,410	0,488	0,544	0,522	0,445	2,2
2,4	0,000	0,009	0,019	0,029	0,040	0,052	0,065	0,078	0,094	0,111	0,131	0,155	0,183	0,218	0,261	0,315	0,381	0,456	0,524	0,538	0,475	2,4
2,6	0,000	0,008	0,018	0,029	0,039	0,051	0,063	0,076	0,091	0,107	0,126	0,148	0,175	0,207	0,246	0,295	0,355	0,426	0,498	0,539	0,503	2,6
2,8	0,000	0,008	0,018	0,028	0,038	0,049	0,061	0,074	0,088	0,104	0,122	0,143	0,167	0,197	0,233	0,278	0,333	0,398	0,470	0,526	0,522	2,8

Table B.3 *(continued)*

PEARSON CURVES (STANDARDIZED MEDIAN)

$P_{50\ \%}$ (50 percentile). Change sign for $\gamma_1 > 0$.

Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
3,0	0,000	0,008	0,017	0,027	0,037	0,048	0,059	0,072	0,085	0,101	0,118	0,138	0,161	0,189	0,222	0,263	0,313	0,374	0,443	0,506	0,530	3,0
3,2	0,000	0,008	0,017	0,027	0,037	0,047	0,058	0,070	0,083	0,098	0,114	0,133	0,155	0,181	0,212	0,250	0,296	0,352	0,417	0,483	0,525	3,2
3,4	0,000	0,008	0,017	0,026	0,036	0,046	0,057	0,068	0,081	0,095	0,111	0,129	0,150	0,174	0,203	0,239	0,281	0,333	0,394	0,460	0,513	3,4
3,6	0,000	0,007	0,016	0,025	0,035	0,045	0,056	0,067	0,079	0,093	0,108	0,125	0,145	0,168	0,196	0,228	0,268	0,316	0,373	0,437	0,495	3,6
3,8	0,000	0,007	0,016	0,025	0,034	0,044	0,054	0,066	0,078	0,091	0,105	0,122	0,141	0,163	0,188	0,219	0,256	0,301	0,354	0,415	0,475	3,8
4,0	0,000	0,007	0,015	0,025	0,034	0,043	0,053	0,064	0,076	0,089	0,103	0,119	0,137	0,158	0,182	0,211	0,246	0,288	0,337	0,395	0,455	4,0
4,2	0,000	0,007	0,015	0,024	0,033	0,043	0,053	0,063	0,075	0,087	0,101	0,116	0,133	0,153	0,176	0,204	0,236	0,276	0,322	0,376	0,435	4,2
4,4	0,000	0,007	0,015	0,024	0,033	0,042	0,052	0,062	0,073	0,085	0,099	0,113	0,130	0,149	0,171	0,197	0,228	0,265	0,308	0,359	0,416	4,4
4,6	0,000	0,007	0,015	0,023	0,032	0,041	0,051	0,061	0,072	0,084	0,097	0,111	0,127	0,145	0,167	0,191	0,220	0,255	0,296	0,344	0,399	4,6
4,8	0,000	0,006	0,015	0,023	0,032	0,041	0,050	0,060	0,071	0,082	0,095	0,109	0,124	0,142	0,162	0,186	0,213	0,246	0,285	0,330	0,382	4,8
5,0	0,000	0,006	0,014	0,023	0,031	0,040	0,049	0,059	0,070	0,081	0,093	0,107	0,122	0,139	0,158	0,181	0,207	0,238	0,274	0,317	0,367	5,0
5,2	0,000	0,006	0,014	0,022	0,031	0,040	0,049	0,058	0,069	0,080	0,092	0,105	0,119	0,136	0,155	0,176	0,201	0,231	0,265	0,306	0,353	5,2
5,4	0,000	0,006	0,014	0,022	0,030	0,039	0,048	0,057	0,068	0,078	0,090	0,103	0,117	0,133	0,151	0,172	0,196	0,224	0,257	0,295	0,340	5,4
5,6	0,000	0,006	0,014	0,022	0,030	0,039	0,047	0,057	0,067	0,077	0,089	0,101	0,115	0,131	0,148	0,168	0,191	0,218	0,249	0,285	0,328	5,6
5,8	0,000	0,006	0,014	0,022	0,030	0,038	0,047	0,056	0,066	0,076	0,087	0,100	0,113	0,128	0,145	0,164	0,186	0,212	0,242	0,277	0,317	5,8
6,0	0,000	0,006	0,014	0,021	0,029	0,038	0,046	0,055	0,065	0,075	0,086	0,098	0,111	0,126	0,142	0,161	0,182	0,207	0,235	0,268	0,307	6,0
6,2	0,000	0,006	0,013	0,021	0,029	0,037	0,046	0,055	0,064	0,074	0,085	0,097	0,110	0,124	0,140	0,158	0,178	0,202	0,229	0,261	0,298	6,2
6,4	0,000	0,006	0,013	0,021	0,029	0,037	0,045	0,054	0,063	0,073	0,084	0,096	0,108	0,122	0,137	0,155	0,175	0,197	0,223	0,254	0,289	6,4
6,6	0,000	0,006	0,013	0,021	0,028	0,037	0,045	0,054	0,063	0,073	0,083	0,094	0,107	0,120	0,135	0,152	0,171	0,193	0,218	0,247	0,281	6,6
6,8	0,000	0,006	0,013	0,021	0,028	0,036	0,044	0,053	0,062	0,072	0,082	0,093	0,105	0,118	0,133	0,150	0,168	0,189	0,213	0,241	0,273	6,8
7,0	0,000	0,005	0,013	0,020	0,028	0,036	0,044	0,053	0,061	0,071	0,081	0,092	0,104	0,117	0,131	0,147	0,165	0,185	0,209	0,236	0,267	7,0
7,2	0,000	0,005	0,013	0,020	0,028	0,036	0,044	0,052	0,061	0,070	0,080	0,091	0,103	0,115	0,129	0,145	0,162	0,182	0,205	0,230	0,260	7,2
7,4	0,000	0,005	0,013	0,020	0,027	0,035	0,043	0,052	0,060	0,070	0,079	0,090	0,101	0,114	0,128	0,143	0,160	0,179	0,201	0,226	0,254	7,4

Table B.3 *(continued)*

PEARSON CURVES (STANDARDIZED MEDIAN) $P_{50\,\%}$ (50 percentile). Change sign for $\gamma_1 > 0$.																						
Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
7,6	0,000	0,005	0,012	0,020	0,027	0,035	0,043	0,051	0,060	0,069	0,079	0,089	0,100	0,113	0,126	0,141	0,157	0,176	0,197	0,221	0,249	7,6
7,8	0,000	0,005	0,012	0,020	0,027	0,035	0,043	0,051	0,059	0,068	0,078	0,088	0,099	0,111	0,124	0,139	0,155	0,173	0,193	0,217	0,243	7,8
8,0	0,000	0,005	0,012	0,019	0,027	0,034	0,042	0,050	0,059	0,068	0,077	0,087	0,098	0,110	0,123	0,137	0,153	0,170	0,190	0,213	0,238	8,0
8,2	0,000	0,005	0,012	0,019	0,027	0,034	0,042	0,050	0,058	0,067	0,076	0,086	0,097	0,109	0,121	0,135	0,151	0,168	0,187	0,209	0,234	8,2
8,4	0,000	0,005	0,012	0,019	0,026	0,034	0,042	0,050	0,058	0,067	0,076	0,086	0,096	0,108	0,120	0,134	0,149	0,165	0,184	0,205	0,229	8,4
8,6	0,000	0,005	0,012	0,019	0,026	0,034	0,041	0,049	0,057	0,066	0,075	0,085	0,095	0,107	0,119	0,132	0,147	0,163	0,181	0,202	0,225	8,6
8,8	0,000	0,005	0,012	0,019	0,026	0,033	0,041	0,049	0,057	0,066	0,075	0,084	0,094	0,106	0,118	0,131	0,145	0,161	0,179	0,199	0,221	8,8
9,0	0,000	0,005	0,012	0,019	0,026	0,033	0,041	0,049	0,057	0,065	0,074	0,084	0,094	0,105	0,116	0,129	0,143	0,159	0,176	0,196	0,218	9,0
9,2	0,000	0,005	0,012	0,019	0,026	0,033	0,040	0,048	0,056	0,065	0,073	0,083	0,093	0,104	0,115	0,128	0,142	0,157	0,174	0,193	0,214	9,2
9,4	0,000	0,005	0,012	0,019	0,026	0,033	0,040	0,048	0,056	0,064	0,073	0,082	0,092	0,103	0,114	0,127	0,140	0,155	0,172	0,190	0,211	9,4
9,6	0,000	0,005	0,012	0,019	0,025	0,033	0,040	0,048	0,055	0,064	0,072	0,082	0,091	0,102	0,113	0,125	0,139	0,153	0,170	0,188	0,208	9,6
9,8	0,000	0,005	0,012	0,018	0,025	0,032	0,040	0,047	0,055	0,063	0,072	0,081	0,091	0,101	0,112	0,124	0,137	0,152	0,168	0,185	0,205	9,8
10,0	0,000	0,005	0,011	0,018	0,025	0,032	0,040	0,047	0,055	0,063	0,071	0,080	0,090	0,100	0,111	0,123	0,136	0,150	0,166	0,183	0,202	10,0
10,2	0,000					0,032	0,039	0,047	0,054	0,063	0,071	0,080	0,089	0,099	0,110	0,122	0,135	0,149	0,164	0,181	0,200	10,2
10,4	0,000					0,032	0,039	0,047	0,054	0,062	0,071	0,079	0,089	0,099	0,109	0,121	0,133	0,147	0,162	0,179	0,197	10,4
10,6	0,000						0,039	0,046	0,054	0,062	0,070	0,079	0,088	0,098	0,109	0,120	0,132	0,146	0,160	0,177	0,195	10,6
10,8	0,000							0,046	0,054	0,061	0,070	0,078	0,088	0,097	0,108	0,119	0,131	0,144	0,159	0,175	0,192	10,8
11,0	0,000								0,053	0,061	0,069	0,078	0,087	0,097	0,107	0,118	0,130	0,143	0,157	0,173	0,190	11,0
11,2	0,000									0,061	0,069	0,078	0,087	0,096	0,106	0,117	0,129	0,142	0,156	0,171	0,188	11,2
11,4	0,000										0,069	0,077	0,086	0,095	0,105	0,116	0,128	0,141	0,154	0,169	0,186	11,4
11,6	0,000										0,068	0,077	0,086	0,095	0,104	0,116	0,127	0,139	0,153	0,168	0,184	11,6

Table B.3 *(continued)*

PEARSON CURVES (STANDARDIZED MEDIAN) $P_{50\,\%}$ (50 percentile). Change sign for $\gamma_1 > 0$.																						
Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
11,8	0,000											0,076	0,085	0,094	0,104	0,115	0,126	0,138	0,152	0,166	0,182	11,8
12,0	0,000											0,076	0,085	0,094	0,104	0,114	0,125	0,137	0,150	0,165	0,181	12,0
12,2	0,000												0,084	0,093	0,103	0,113	0,124	0,136	0,149	0,163	0,179	12,2

Annex C
(informative)

Distribution identification

C.1 General

Sometimes, the form of the distribution is known or can be reasonably assumed and can be verified by goodness-of-fit tests. The approach is to estimate the parameters of that distribution and to use them to derive the relevant quantiles from which the capability estimates are obtained. The proportion out-of-specification can be directly estimated.

The method is illustrated below with some often encountered distributions.

C.2 Normal distribution

If X_1, ..., X_N is a sample from a normal distribution with mean, μ, and variance, σ^2, the estimates of μ and σ^2 are

$$\hat{\mu} = \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

and

$$\hat{\sigma}^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2$$

The capability indices can, in turn, be estimated using the formulae given in this part of ISO 22514. Thus,

$$\hat{C}_{\mathrm{p}} = \frac{U-L}{6\hat{\sigma}}$$

$$\hat{C}_{\mathrm{pk}_U} = \frac{U-\hat{\mu}}{3\hat{\sigma}}$$

$$\hat{C}_{\mathrm{pk}_L} = \frac{\hat{\mu}-L}{3\hat{\sigma}}$$

and, finally,

$$\hat{C}_{\mathrm{pk}} = \min(\hat{C}_{\mathrm{pk}_L}, \hat{C}_{\mathrm{pk}_U})$$

The estimated proportion out-of-specification items below L is calculated as

$$\hat{p}_L = 1 - \Phi\left(3\hat{C}_{\mathrm{pk}_L}\right)$$

and the estimated proportion out-of-specification items above U is calculated as

$$\hat{p}_U = 1 - \Phi\left(3\hat{C}_{\mathrm{pk}_U}\right)$$

Here, Φ denotes the distribution function of the standard normal distribution. The actual calculations of $\hat{p}_L$ and $\hat{p}_U$ may be performed as outlined in 4.8.

C.3 Log-normal distribution

C.3.1 General

The log-normal distribution with parameters μ and σ has probability density function

$$f(x) = \frac{1}{X\sigma\sqrt{2\pi}} \exp\left(-\frac{\left(\log X - \mu\right)^2}{2\sigma^2} \right)$$

where $X > 0$, and log denotes the natural logarithm, i.e. the logarithm to base e. When X has a log-normal distribution with parameters μ and σ, then $\log X$ has a normal distribution with mean, μ, and variance σ^2.

If X_1, ..., X_N is a sample from a log-normal distribution, the data can be transformed to normality by taking the logarithms $\log X_1$, ..., $\log X_N$. The calculations in C.2 can then be used. Alternatively, the calculations can be made directly in the original scale of the measurements. The two methods are given below. In both cases, the parameters are estimated as

$$\hat{\mu} = \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

and

$$\hat{\sigma}^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2$$

C.3.2 Log-normal distribution — Transformation to normality

The upper and lower specification limits shall be transformed so they become $\log U$ and $\log L$. The formulae in C.2 can then be applied. For example, the estimates of C_p, C_{pk_L} and C_{pk_U} are calculated as

$$\hat{C}_p = \frac{\log U - \log L}{6\hat{\sigma}}$$

$$\hat{C}_{pk_U} = \frac{\log U - \hat{\mu}}{3\hat{\sigma}}$$

$$\hat{C}_{pk_L} = \frac{\hat{\mu} - \log L}{3\hat{\sigma}}$$

C_{pk_L} and C_{pk_U} may, in turn, be inserted in the appropriate formulae in C.2 to give the estimated proportions out-of-specification.

C.3.3 Log-normal distribution — Original scale

The quantiles of the log-normal distribution are

$$X_\alpha = \exp\left(\sigma\Phi^{-1}\left(\alpha\right) + \mu\right)$$

where Φ^{-1} is the inverse of the distribution function of the standard normal distribution. In particular,

$$X_{0,135\,\%} = \mathrm{e}^{-3\sigma+\mu}$$

$$X_{50\,\%} = \mathrm{e}^{\mu}$$

$$X_{99,865\,\%} = \mathrm{e}^{3\sigma+\mu}$$

and the estimated indices are

$$\hat{C}_\mathrm{p} = \frac{U - L}{\mathrm{e}^{3\hat{\sigma}+\hat{\mu}} - \mathrm{e}^{-3\hat{\sigma}+\hat{\mu}}}$$

$$\hat{C}_{\mathrm{pk}_U} = \frac{U - \mathrm{e}^{\hat{\mu}}}{\mathrm{e}^{3\hat{\sigma}+\hat{\mu}} - \mathrm{e}^{\hat{\mu}}}$$

$$\hat{C}_{\mathrm{pk}_L} = \frac{e^{\hat{\mu}} - L}{\mathrm{e}^{\hat{\mu}} - \mathrm{e}^{-3\hat{\sigma}+\hat{\mu}}}$$

These indices will be numerically different from those obtained using the transformation approach of C.3.2. A process owner with log-normally distributed items could well develop a “feel” for the indices, but their interpretation should not routinely be based on the cut-off points used for indices calculated for normal data.

The estimated proportion of out-of-specification items is calculated using the specification limits and the distribution function of the log-normal distribution. Thus,

$$\hat{p}_L = \Phi\left(\frac{\log L - \hat{\mu}}{\hat{\sigma}}\right)$$

and

$$\hat{p}_U = 1 - \Phi\left(\frac{\log U - \hat{\mu}}{\hat{\sigma}}\right)$$

These estimates are exactly the same as those obtained with the transformation approach in C.3.2.

C.4 Rayleigh distribution

This distribution is used almost exclusively to describe position, eccentricity and run-off in two-dimensional problems. In these situations, it is typical for there to be only a one-sided specification limit, U.

The Rayleigh distribution has distribution function:

$$F(x) = 1 - \exp\left(-\frac{X^2}{2\theta^2}\right)$$

where $X > 0$ and θ is a positive parameter. If X_1, ..., X_N is a sample from a Rayleigh distribution, the parameter θ is estimated by

$$\hat{\theta} = \sqrt{\frac{\sum_{i=1}^{N} X_i^2}{2N}}$$

and the estimated proportion out-of-specification is given by

$$\hat{p}_U = \exp\left(-\frac{NU^2}{\sum_{i=1}^{N} X_i^2}\right)$$

C.5 Weibull distribution

This is a distribution of great versatility. It is often used in analysing data gathered during reliability studies when the patterns that describe the data are often irregular and non-normal. The distribution has three parameters:

a) ξ a scale parameter;

b) β a shape parameter;

c) γ a location parameter, frequently assumed to be 0.

There are occasions when a capability study will not produce data that follow a normal distribution pattern, but the Weibull distribution can be used to describe the data and, thereafter, provide a way to calculate capability or performance measures.

The distribution function of the Weibull distribution is

$$F(X) = 1 - \exp\left(-\left(\frac{X-\gamma}{\xi}\right)^{\beta}\right)$$

and, hence, the percentiles of the Weibull distribution are

$$X_{\alpha\,\%} = Y - \xi\left(-\log\left(1-p\right)\right)^{-\beta}$$

In particular, the percentiles $X_{0,135\,\%}$, $X_{50\,\%}$ and $X_{99,865\,\%}$ and, subsequently, the capability indices can be calculated. More importantly, the proportions out-of-specification items are

$$p_L = F(L) = 1 - \exp\left(-\left(\frac{L-\gamma}{\xi}\right)^{\beta}\right)$$

and

$$p_U = 1 - F(U) = \exp\left(-\left(\frac{U-\gamma}{\xi}\right)^{\beta}\right)$$

The estimated proportions out-of-specification are obtained using estimates of the parameters.

C.6 Folded half-normal distribution

The folded half-normal distribution is often used to describe the variation in a characteristic whose specification includes geometrical tolerances. This situation gives a one-sided specification. It typically applies where geometrical characteristics, form and orientation are specified.

The folded half-normal distribution, with parameters μ and σ, has probability density function:

$$f\left(x\right)=\frac{2}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left[\frac{\left|X-\mu\right|}{\sigma}\right]^{2}\right\}$$

where $0 \le X < \infty$.

The folded half-normal distribution is proportional to the normal distribution. The distribution's estimated proportions can therefore be found using a standard normal distribution table with the appropriate table value multiplied by 2.

C.7 Other distributions

The distributions given above are those frequently to be encountered. There exist many others that the reader will find documented in the numerous text books on statistics.

Annex D
(informative)

Confidence intervals

D.1 Normal distribution

D.1.1 General

It is important for any person having calculated a capability index to realize that the computed value is only an estimate of the true value of that index. Usually, the greater the amount of data used to compute the index, the better the estimate will be. The following paragraphs are intended to alert the reader to this fact and to provide methods that confidence intervals for the indices can be calculated.

The confidence interval calculations are only appropriate when the mean and not the median has been the chosen measure of location.

D.1.2 Normal distribution — Formula method

The 1 − α confidence intervals are

$$\hat{C}_{\mathrm{p}} \pm z_{1-\alpha/2} \frac{\hat{C}_{\mathrm{p}}}{\sqrt{2N-2}}$$

$$\hat{C}_{\mathrm{pk}_U} \pm z_{1-\alpha/2} \sqrt{\frac{1}{9N} + \frac{\hat{C}^2_{\mathrm{pk}_U}}{2N\text{-}2}}$$

$$\hat{C}_{\mathrm{pk}_L} \pm z_{1-\alpha/2} \sqrt{\frac{1}{9N} + \frac{\hat{C}^2_{\mathrm{pk}_L}}{2N\text{-}2}}$$

where z is the standardized variate for the normal distribution. These calculations should be done with at least 50 readings.

NOTE These formulae assume the estimated indices have been computed using a standard deviation based on the total sample size (N).

D.1.3 Normal distribution — Tabular method for the C_{p} index

D.1.3.1 General

An example of the method to estimate a confidence interval for a C_{p} index is shown in D.1.3.2.

The method requires multipliers K_{l} and K_{u} are read from Table D.1. These differ according to how many readings the estimated C_{p} has been based on. The index is multiplied by these multipliers to give the confidence interval. This calculation should be done with at least 50 readings.

As an example, the 95 % confidence interval would be

$$K_{\mathrm{l}_{95\,\%}} \hat{C}_{\mathrm{p}} \le C_{\mathrm{p}} \le K_{\mathrm{u}_{95\,\%}} \hat{C}_{\mathrm{p}}$$

D.1.3.2 Procedure and example

a) Record estimated C_p value and total sample size:

$\hat{C}_p = 1,20$ and $N = 100$

b) Select the required conficence level.

Confidence level = 95 %

c) Read multipliers from Table D.1.

$K_{l_{95\,\%}} = 0,86$

$K_{u_{95\,\%}} = 1,1,4$

d) Calculate confidence interval.

$$K_{l_{95\,\%}} \hat{C}_p \le C_p \le K_{u_{95\,\%}} \hat{C}_p$$
$$0,86 \times 1,20 \le C_p \le 1,14 \times 1,20$$
$$1,03 \le C_p \le 1,37$$

Table D.1 — Multipliers for the confidence interval of the C_p index

Confidence interval	Multiplier	Total sample size, *N*				
		50	75	100	150	300
90 %	K_l	0,83	0,86	0,88	0,90	0,93
	K_u	1,17	1,14	1,12	1,10	1,07
95 %	K_l	0,80	0,84	0,86	0,89	0,92
	K_u	1,20	1,16	1,14	1,11	1,08
99 %	K_l	0,74	0,79	0,82	0,85	0,89
	K_u	1,25	1,21	1,18	1,15	1,10

D.2 Other confidence intervals

Confidence intervals for other calculated indices (not graphical estimates) can be found for non-normal distributions, as well as for normal distributions.

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ICS 03.120.30
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