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Mechanical vibration — Rotor balancing —

Part 31:
Susceptibility and sensitivity of machines to unbalance

Vibrations mécaniques — Équilibrage des rotors —

Partie 31: Susceptibilité et sensibilité des machines aux balourds

Reference number
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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2, www.iso.org/directives.

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The committee responsible for this document is ISO/TC 108, *Mechanical vibration, shock and condition monitoring*, Subcommittee SC 2, *Measurement and evaluation of mechanical vibration and shock as applied to machines, vehicles and structures*.

This first edition of ISO 21940-31 cancels and replaces ISO 10814:1996, of which it constitutes a technical revision. The main change is modification to the modal amplification factors to make this part of ISO 21940 more consistent with relevant parts of ISO 7919, e.g. machines predicted to operate in ISO 7919-2[2] zone A would be classified as very low (range A) and machines predicted to operate in ISO 7919-2[2] zone B would be classified as low (range B).

ISO 21940 consists of the following parts, under the general title *Mechanical vibration — Rotor balancing*:

— *Part 1: Introduction*1)

— *Part 2: Vocabulary*2)

— *Part 11: Procedures and tolerances for rotors with rigid behaviour*3)

— *Part 12: Procedures and tolerances for rotors with flexible behaviour*4)

— *Part 13: Criteria and safeguards for the in-situ balancing of medium and large rotors*5)

— *Part 14: Procedures for assessing balance errors*6)

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1) Revision of ISO 19499:2007, *Mechanical vibration — Balancing — Guidance on the use and application of balancing standards*

2) Revision of ISO 1925:2001, *Mechanical vibration — Balancing — Vocabulary*

3) Revision of ISO 1940-1:2003 + Cor.1:2005, *Mechanical vibration — Balance quality requirements for rotors in a constant (rigid) state — Part 1: Specification and verification of balance tolerances*

4) Revision of ISO 11342:1998 + Cor.1:2000, *Mechanical vibration — Methods and criteria for the mechanical balancing of flexible rotors*

5) Revision of ISO 20806:2009, *Mechanical vibration — Criteria and safeguards for the in-situ balancing of medium and large rotors*

6) Revision of ISO 1940-2:1997, *Mechanical vibration — Balance quality requirements of rigid rotors — Part 2: Balance errors*

— *Part 21: Description and evaluation of balancing machines*[7)]

— *Part 23: Enclosures and other protective measures for the measuring station of balancing machines*[8)]

— *Part 31: Susceptibility and sensitivity of machines to unbalance*[9)]

— *Part 32: Shaft and fitment key convention*[10)]

7) Revision of ISO 2953:1999, *Mechanical vibration — Balancing machines — Description and evaluation*

8) Revision of ISO 7475:2002, *Mechanical vibration — Balancing machines — Enclosures and other protective measures for the measuring station*

9) Revision of ISO 10814:1996, *Mechanical vibration — Susceptibility and sensitivity of machines to unbalance*

10) Revision of ISO 8821:1989, *Mechanical vibration — Balancing — Shaft and fitment key convention*

Introduction

Rotor balancing during manufacture (e.g. as described in ISO 1940-1[1] and ISO 11342[4]) is normally sufficient to attain acceptable in-service vibration magnitudes if other sources of vibration are absent. However, additional balancing during commissioning may become necessary and after commissioning, some machines may require occasional or even frequent rebalancing *in situ*.

If vibration magnitudes are unsatisfactory during commissioning, the reason may be inadequate balancing or assembly errors. Another important cause may be that an assembled machine is especially sensitive to relatively small residual unbalances which are well within normal balance tolerances.

If vibration magnitudes are unsatisfactory, the first step often is an attempt to reduce the vibration by balancing *in situ*. If high vibration magnitudes can be reduced by installing relatively small correction masses, high sensitivity to unbalance is indicated. This can arise, for example, if a resonance rotational speed is close to the normal operating speed and the damping in the system is low.

A sensitive machine which is also highly susceptible to its unbalance changing, may require frequent rebalancing *in situ*. This may be caused, for example, by changes in wear, temperature, mass, stiffness, and damping during operation.

If the unbalance and other conditions of the machine are essentially constant, occasional trim balancing may be sufficient. Otherwise it may be necessary to modify the machine to change the resonance speed, damping or other parameters to obtain acceptable vibration magnitudes. Therefore, there is a need to consider permissible sensitivity values of the machine.

The repeatability of the unbalance sensitivity of a machine is influenced by several factors and may change during operation. Some thermal machines, especially those with sleeve bearings, have modal vibration characteristics which vary with particular operational parameters (e.g. steam pressure and temperature, partial steam admission or oil temperature). For electrical machines, other parameters such as the excitation current may influence the vibration behaviour. In general, the machine vibration characteristics are influenced by the design features of the machine, including coupling of the rotor and its support conditions including the foundation. It should be noted that the rotor support conditions may vary with time (e.g. wear and tear).

This part of ISO 21940 is only concerned with once-per-revolution vibration caused by unbalance; however, it should be recognized that unbalance is not the only cause of once-per-revolution vibration.

Mechanical vibration — Rotor balancing —

Part 31: Susceptibility and sensitivity of machines to unbalance

1 Scope

This part of ISO 21940 specifies methods for determining machine vibration sensitivity to unbalance and provides evaluation guidelines as a function of the proximity of relevant resonance rotational speeds to the operating speed. This part of ISO 21940 is only concerned with once-per-revolution vibration caused by unbalance. It also makes recommendations on how to apply the numerical sensitivity values in some particular cases.

It includes a classification system that can be applied to machines which is related to their susceptibility to a change in unbalance. Machines are classified into three types of susceptibility and five ranges of sensitivity. The sensitivity values are intended for use on simple machine systems, preferably with rotors having only one resonance speed over their entire operating speed range. The sensitivity values can also be used for machines that have more resonance speeds in their operating speed range if the resonance speeds are widely separated (e.g. by more than 20 %).

The sensitivity values given are not intended to serve as acceptance specifications for any machine group, but rather to give indications regarding how to avoid gross deficiencies as well as specifying exaggerated or unattainable requirements. They can also serve as a basis for more involved investigations (e.g. when in special cases a more exact determination of the required sensitivity is necessary). If due regard is paid to the values given, satisfactory running conditions can be expected in most cases.

The consideration of the sensitivity values alone does not guarantee that a given magnitude of vibration in operating is not exceeded. Many other sources of vibration can occur which lie outside the scope of this part of ISO 21940.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 1925, *Mechanical vibration — Balancing — Vocabulary*[11)]

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 1925 apply.

NOTE Some of the terms used are explained in Annex A.

4 Machine susceptibility classification

4.1 General

Machine susceptibility classification is based on the likelihood of a machine experiencing significant unbalance during operation. Machines with low susceptibility are allowed higher sensitivity values

11) To become ISO 21940-2 when revised.

(require less damping), and machines with high susceptibility are restricted to lower sensitivity values (require more damping).

4.2 Type I: Low susceptibility

Machines of this type have a low likelihood of experiencing significant unbalance changes during operation. Typically they have a large rotor mass in comparison to their support housing and operate in a clean environment, have negligible wear and exhibit minimal rotor distortion caused by temperature change.

EXAMPLES Paper machine rolls, printing rolls, and high-speed vacuum pumps.

4.3 Type II: Moderate susceptibility

Machines of this type have a moderate likelihood of experiencing significant unbalance changes during operation. Typically they are machines which operate in environments with large temperature changes or experience moderate wear.

EXAMPLES Pumps in clean media, electric armatures, gas and steam turbines, generators, and turbo compressors.

4.4 Type III: High susceptibility

Machines of this type have a high likelihood of experiencing significant unbalance changes during operation. Typically they are machines which run in deposit producing (e.g. pumps operating in sludge) or corrosive environments.

EXAMPLES Centrifuges, fans, screw conveyors, and hammer mills.

4.5 Machine susceptibility correction factors

The remainder of this part of ISO 21940 focuses on moderate susceptibility classification machines (type II). For evaluation of low susceptibility or high susceptibility machines, a correction factor can be applied to adjust the sensitivity range. Table 1 shows correction factors that are applied to the sensitivity values (see Clause 5) based on machine susceptibility type (see 4.2 to 4.4).

Table 1 — Correction factors

Machine type	Machine susceptibility classification	Correction factor
I	Low susceptibility	$\frac{4}{3}$
II	Moderately susceptibility	1 (Base)
III	High susceptibility	$\frac{2}{3}$

5 Modal sensitivity

5.1 General

Modal sensitivity is given in terms of the modal amplification factor, M_n, which is a constant value defining the quality range for each resonance rotational speed of a machine. For machines to achieve low unbalance sensitivity, there needs to be adequate separation between their operating and resonance speeds or sufficient damping.

Modal sensitivity at any or each resonance speed is also important to avoid excessive vibration when passing through them to reach the operating speed or speed range.

5.2 Modal sensitivity ranges

Allowable modal amplification factors, which vary with machine rotational speed, make up the modal sensitivity ranges used to classify machines with respect to their expected operating conditions. Table 2 defines the ranges of modal sensitivity.

Table 2 — Modal sensitivity range

Range designation	Description	Expected operating conditions
A	Very low sensitivity	Very smooth
B	Low sensitivity	Smooth
C	Moderate sensitivity	Acceptable
D	High sensitivity	Sensitive to unbalance
E	Very high sensitivity	Too sensitive to unbalance

5.3 Characteristics of modal sensitivity ranges

While range A (see Table 2) theoretically appears to be the most desirable, considerations of cost and feasibility may often make it necessary to operate with higher modal sensitivities.

For high-performance machines (e.g. those that have a short period between planned maintenance cycles), it may be permissible to allow for higher values of modal sensitivity.

For machines for which balancing *in situ* is not practical or not economical, smaller values of modal sensitivity may have to be selected.

Consideration of the sensitivity does not always give sufficient assurance that, at all parts of the machine, vibration limits are not exceeded (see Clauses 7 and 8).

5.4 Values of modal sensitivity

5.4.1 General

Values of modal sensitivity in terms of modal amplification factors, M_n, are constants that are used with a series of formulae to define the modal sensitivity ranges. These values have been derived from permissible eccentricity as defined in ISO 1940-1[1] and allowable vibration amplitude established in ISO 7919-2[2] and ISO 7919-4.[3] Together these documents can be used to develop values of modal sensitivity for operation at operational speed.

5.4.2 Permissible eccentricity

The permissible residual unbalance of a rotor, U_{per}, is

$$U_{\text{per}} = e_{\text{per}} m \qquad (1)$$

where

e_{per} is the permissible residual eccentricity;

m is the rotor mass.

ISO 1940-1[1] establishes balance quality grades, G, that permit a classification of balance quality based on the rotor type. The established grades are based on the machine operating speed Ω:

$$G = e_{\mathrm{per}}\Omega \quad (2)$$

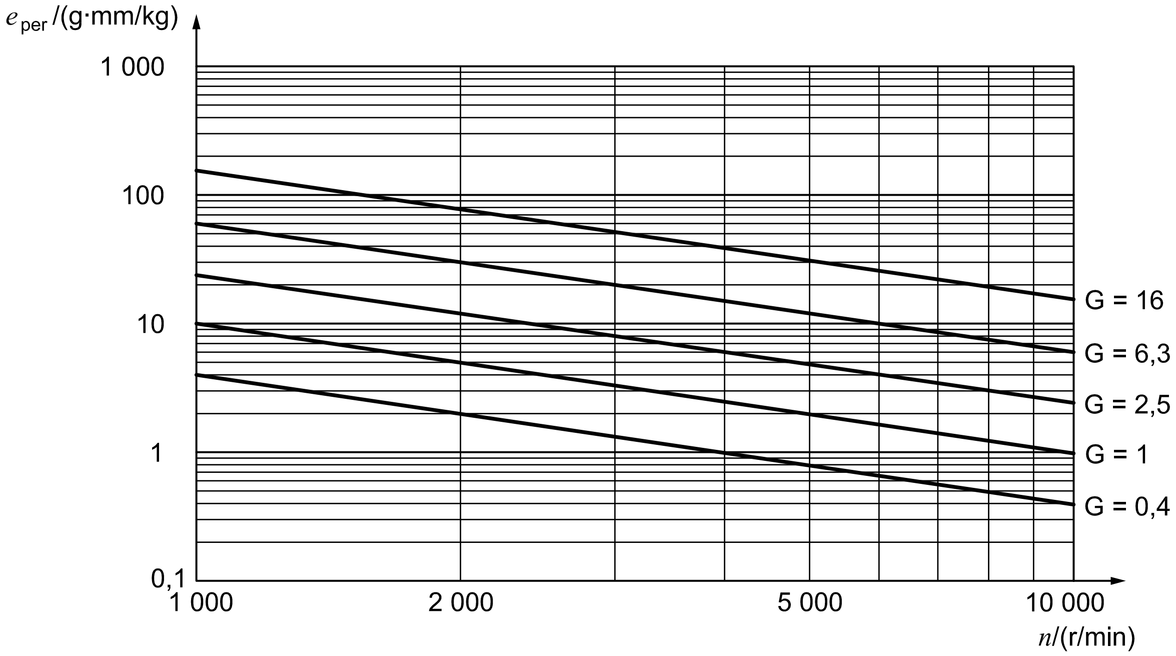
The balance quality grade, G, is constant for a given machine type (e.g. $e_{\mathrm{per}}\Omega$ = 2,5 mm/s = G 2,5).

NOTE The operating speed Ω is the numerical value of the angular velocity of the rotational speed n in r/min (revolutions per minute), expressed in rad/s (radians per second), with $\Omega = 2\pi n/60 \approx n/10$.

Reorganizing Formula (2) yields an expression for permissible eccentricity based on the machine operating speed and balance quality grade:

$$e_{\mathrm{per}} = \frac{G}{\Omega} \quad (3)$$

Permissible eccentricity as a function of rotational speed is shown in Figure 1 for various balance quality grades. For machines with balance quality grade G 2,5 and a rotational speed of n = 3 000 r/min, the permissible eccentricity is 8,0 µm and for 3 600 r/min machines, the permissible eccentricity is 6,6 µm.



Key

e_{per} permissible eccentricity

n rotational speed

NOTE Units g·mm/kg are equivalent to µm.

Figure 1 — Permissible eccentricity according to ISO 1940-1[1]

5.4.3 Allowable vibration magnitude

ISO 7919-2[2] and ISO 7919-4[3] define evaluation zones for steady-state shaft vibration. These zones are:

— zone A: the vibration of newly commissioned machines;

— zone B: acceptable for unrestricted long-term operation;

— zone C: unsatisfactory for long-term continuous operation, machine to be operated for limited period until suitable opportunity arises for remedial action;

— zone D: vibration sufficient to cause damage to machine.

These peak-to-peak vibration zone boundaries are inversely proportional to the square root of the maximum normal operating speed n, in revolutions per minute, as shown in Formulae (4) to (6):

Zone boundary A/B

$$S_{(\mathrm{p-p})} = \frac{4\,800}{\sqrt{n}}\ \mu\mathrm{m} \tag{4}$$

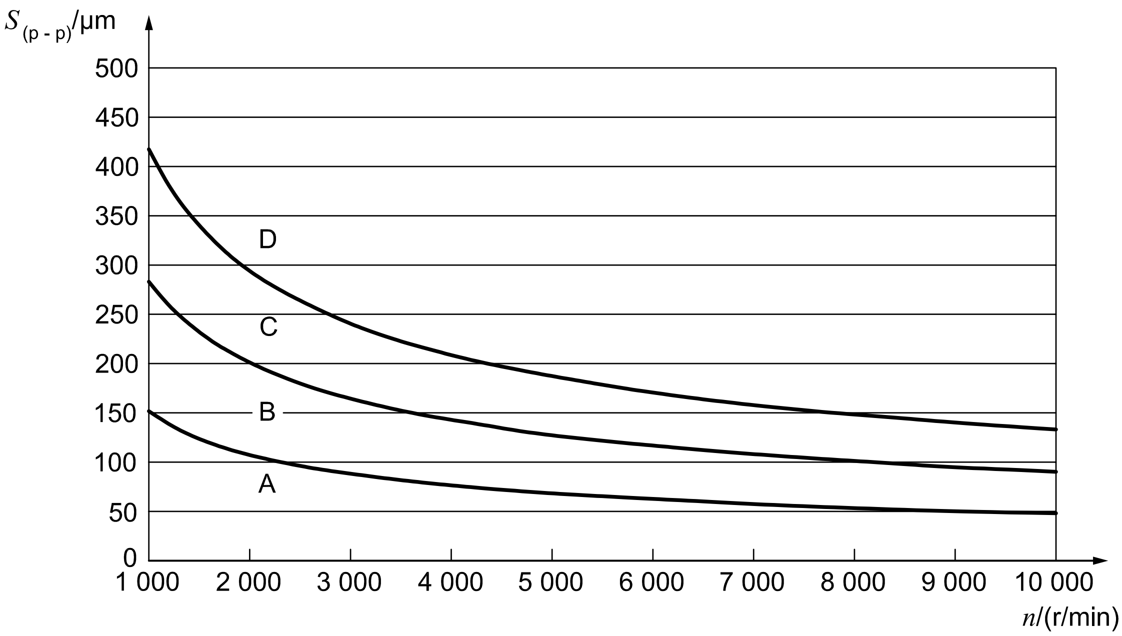
Zone boundary B/C

$$S_{(\mathrm{p-p})} = \frac{9\,000}{\sqrt{n}}\ \mu\mathrm{m} \tag{5}$$

Zone boundary C/D

$$S_{(\mathrm{p-p})} = \frac{13\,200}{\sqrt{n}}\ \mu\mathrm{m} \tag{6}$$

Figure 2 shows how Formulae (4) to (6) vary with rotational speed and Table 3 shows vibration limits for common type II machine operating speeds.



Key

$S_{(\mathrm{p-p})}$ peak-to-peak vibration magnitude

n machine operating speed

NOTE A to D are the zones given in 5.4.3.

Figure 2 — Zone boundary curves according to ISO 7919-2[2] and ISO 7919-4[3]

Table 3 — Zone boundary values for common machine operating speeds

Rotational speed	$S_{(p-p)}$ **at zone boundary** A/B	$S_{(p-p)}$ **at zone boundary** B/C	$S_{(p-p)}$ **at zone boundary** C/D
r/min	µm	µm	µm
1 500	123,9	232,4	340,8
1 800	113,1	212,1	311,1
3 000	87,6	164,3	241,0
3 600	80,0	150,0	220,0
NOTE A to D are the zones given in 5.4.3.			

5.4.4 Steady-state derivation

Using Formulae (3) to (6) and those for modal sensitivity (see Annex A), a speed-dependent expression for modal amplification factors can be established for each of the evaluation zones based on the machine balance quality grade. Formula (7) shows the general form of the derived modal amplification factor:

$$M_n = \frac{S_{(p-p)\,\mathrm{ISO}\,7919}}{2\,e_{\mathrm{ISO}\,1940-1}} \tag{7}$$

NOTE 1 The modal amplification factor defining the modal sensitivity (ratio of the modal displacement to the modal eccentricity) is a non-dimensional quantity. The peak-to-peak vibration zone boundary values given in the relevant parts of ISO 7919 are divided by 2 so that both numerator and denominator in Formula (7) are amplitudes.

More specifically, for zone boundary A/B:

$$M_n = \frac{2{,}4\pi\sqrt{n}}{30G} \tag{8}$$

for zone boundary B/C:

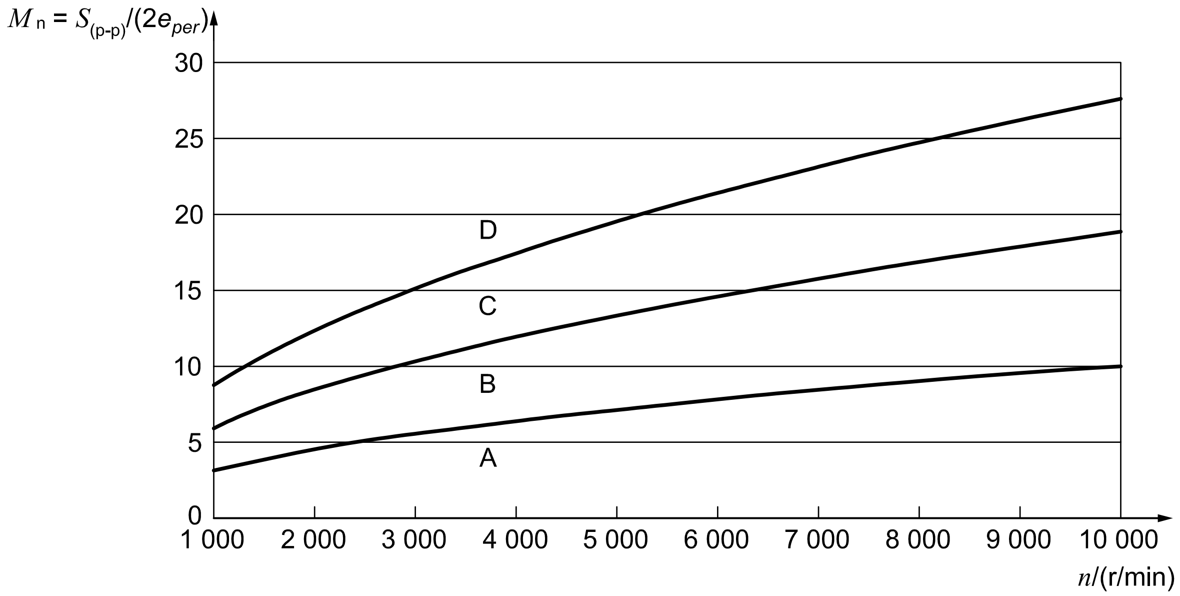
$$M_n = \frac{4{,}5\pi\sqrt{n}}{30G} \tag{9}$$

and for zone boundary C/D:

$$M_n = \frac{6{,}6\pi\sqrt{n}}{30G} \tag{10}$$

NOTE 2 A to D are the zones given in 5.4.3.

As can be seen from Formulae (8) to (10) where the maximum normal operating speed n is given in revolutions per minute, the modal amplification factor can be derived as a speed-dependent term. Formulae (8) to (10) are shown graphically in Figure 3.



Key

M_n modal amplification factor

n machine operating speed

NOTE A to D are the zones given in 5.4.3.

Figure 3 — Speed-dependent modal amplification factors

Calculated modal amplification factors for common machine operating speeds are shown in Table 4.

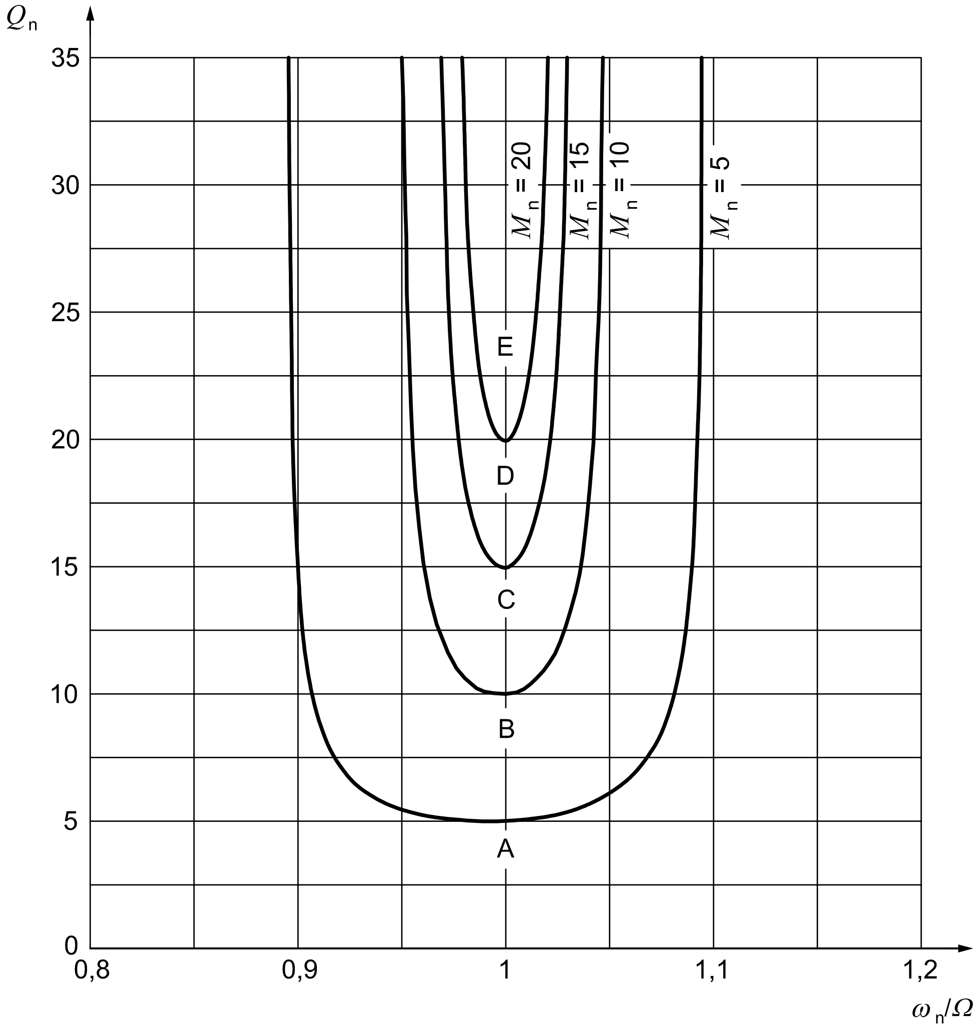
Table 4 — Modal amplification factor M_n for common machine operating speeds

Rotational speed r/min	M_n at zone boundary		
	A/B	B/C	C/D
1 500	3,9	7,3	10,7
1 800	4,3	8,0	11,7
3 000	5,5	10,3	15,1
3 600	6,0	11,3	16,6
NOTE A to D are the zones given in 5.4.3.			

To simplify modal sensitivity evaluation, it is desirable to eliminate operating speed variation from consideration. Since many type II machines operate at 3 000 r/min or 3 600 r/min, a conservative approach is to use M_n constants of 5, 10, 15, and 20 for modal sensitivity range boundaries A/B, B/C, C/D, and D/E (see 5.2 for range descriptions). Figures 4 and 5 apply to all machines independent of their nominal operating speed.

5.5 Operating speed

Using Formula (A.2), curves that define modal sensitivity ranges are easily developed for rotational speeds in the nominal operating region. Figure 4 shows such curves for type II machines. Two examples of how to use Figure 4 can be found in Annex C.



Key

Q_n amplification factor at resonance speed

ω_n/Ω resonance speed to operating speed ratio

NOTE A to E are the modal sensitivity ranges given in 5.2.

Figure 4 — Type II modal sensitivity near operating speed

The range values need to be multiplied by correction factors for the machine susceptibility type as specified in 4.2 and shown in Table 5.

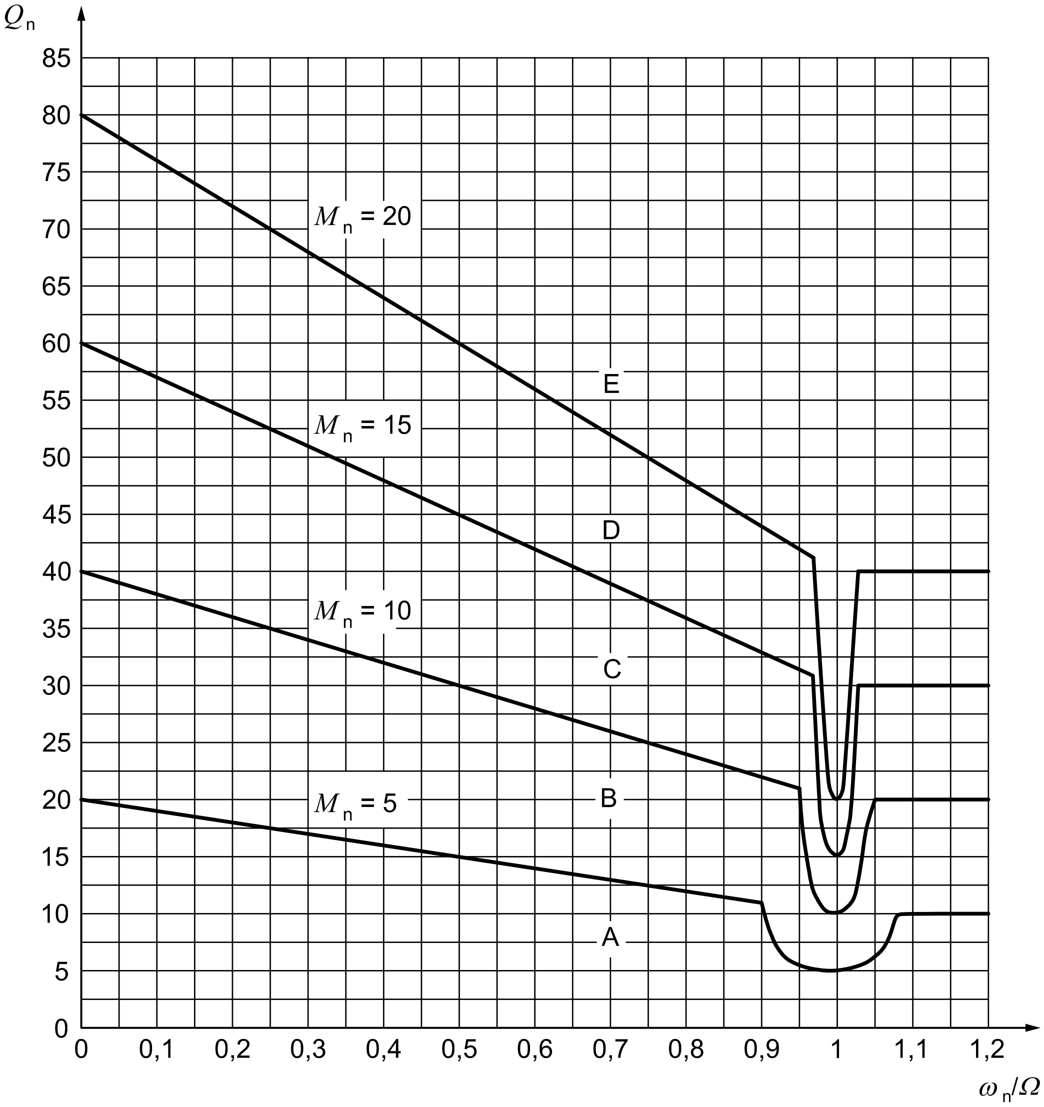
Table 5 — Modal amplification factors M_n for different machine types

Range boundary	M_n		
	Type I	Type II	Type III
A/B	6,7	5,0	3,3
B/C	13,3	10,0	6,7
C/D	20,0	15,0	10,0
D/E	26,7	20,0	13,3
NOTE A to E are the modal sensitivity ranges given in 5.2.			

5.6 Transient speed

Higher amplification factors are permissible in the transient operating speed range, or while passing through resonance rotational speeds.

The transient speed regions of operation have criteria based on the resonance speed to operating speed ratio. For speed ratios below the operating range, the modal sensitivity curves can be characterized at straight sloping lines from Q_n equal to $4M_n$ at a speed ratio of zero with a slope of $-2M_n$. These sloped lines continue until intersecting the operating range shown in Figure 4. For speed ratios above the operating range, the sensitivity curves are straight lines with a Q_n value of $2M_n$ and a slope of zero. Figure 5 shows the modal sensitivity ranges for a type II machine.



Key

Q_n amplification factor at resonance speed

ω_n/Ω resonance speed to operating speed ratio

NOTE A to E are the modal sensitivity ranges given in 5.2.

Figure 5 — Type II modal sensitivity for range of operation

6 Experimental determination of modal sensitivity near resonance speed under operational conditions

6.1 General

Once-per-revolution vibration is normally measured in amplitude and phase so that the Nyquist diagram procedure given in 6.2 can be used. If only amplitude measurements are available, the Bode diagram procedure given in 6.3 should be used.

6.2 Nyquist diagram procedure

At resonance rotational speeds, the sensitivity to unbalance is dependent on the damping that is present in the system. As the damping itself can depend on many parameters, it is recommended that sensitivity tests be performed with the machine under operating conditions as close to normal as possible (e.g. at normal operating temperatures).

In many cases, the response of the system close to a given resonance speed occurs predominantly in the corresponding mode only, so that its behaviour can be modelled by an equivalent single degree of-freedom system. Under these circumstances, the damping and the flexural resonance speed can be found from measurements during slow run-up or coast-down, where the rate of change of speed is small.

Such a single degree-of-freedom system describes the vibration in the nth mode, and the following relationship for the maximum modal sensitivity Q_n is applicable:

$$Q_n \approx \left| \frac{\omega_n \Omega_{45}}{\omega_n^2 - \Omega_{45}^2} \right| \tag{11}$$

where

ω_n is the nth resonance rotational speed;

Ω_{45} are the speeds where the phase has shifted by ±45° from that at the resonance speed.

The maximum modal sensitivity Q_n equals the value of M_n when the rotor rotational speed Ω equals the nth resonance speed ω_n, i.e. $\Omega = \omega_n$.

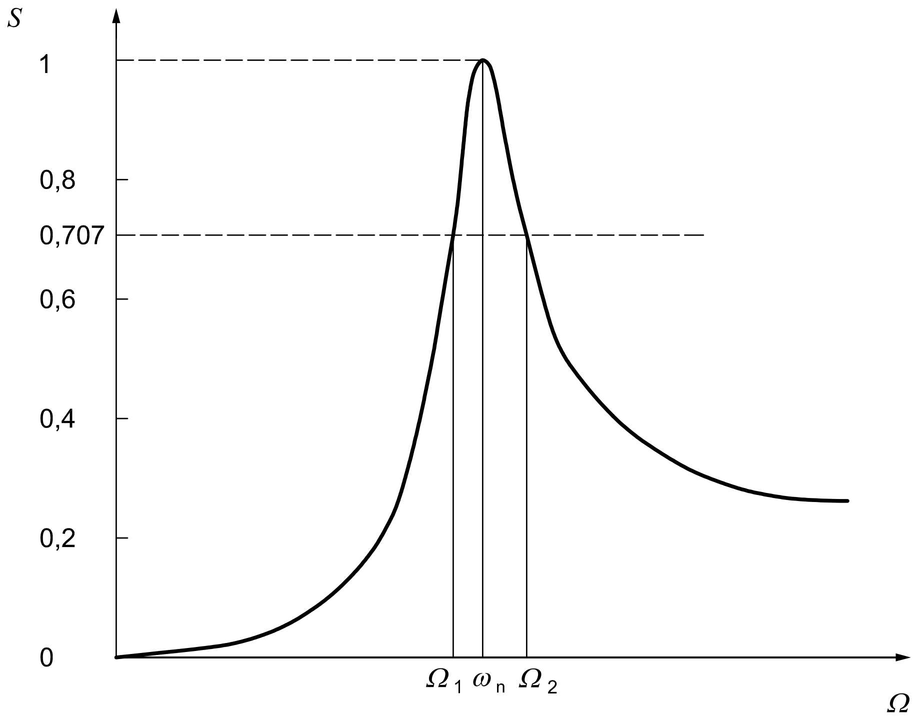
An example of the procedure is shown in Annex B.

Under certain circumstances, the magnitude and phase pattern on the Nyquist plot may be irregular because there are several modes in proximity to each other. In such cases, an evaluation of the modal sensitivity can be made if a trial mass set is added to the rotor that preferentially excites the mode of interest. The procedure is then applied to the difference vectors, which describe the response to the trial mass set.

With the exception of transducers located close to the nodes of the flexural principal modes, any once-per-revolution vibration data can be used for the procedure, but data from transducers installed at the location of relatively high amplitudes give more accurate results.

6.3 Bode diagram procedure

If only a plot of the once-per-revolution displacement amplitude versus rotational speed is available, it may still be possible to find the vibration amplification factor Q_n by the procedure shown in Figure 6.



Key

S displacement amplitude as multiple of maximum value S_{ω_n}

Ω rotational speed

Figure 6 — Bode diagram procedure for estimating modal sensitivity

If ω_n is the rotor speed corresponding to the maximum vibration amplitude S_{ω_n} and Ω_1 and Ω_2 are the rotational speeds where the displacement is 0,707 of the maximum amplitude, then the amplification factor Q_n is:

$$Q_n = \frac{\omega_n}{\Omega_2 - \Omega_1} \tag{12}$$

This approach has accuracy limitations, if the damping is low or the shape of the resonance curve is significantly influenced by adjacent modes or other factors. In such cases, the use of a trial mass set as explained in 6.2 may be applicable.

The speed of response of the measuring apparatus response rate and an insufficient number of sampling points can also be a problem.

7 Numerical values for the local sensitivity

In many cases (e.g. on rotors with an overhung portion, rotors with limited clearance on certain rotor parts or on rotors which in service run close to resonance speeds) the local sensitivity (influence coefficient) may be of interest at a variety of rotational speeds in the operational speed range including the resonance speeds.

The magnitude of the local sensitivity that is measured on a machine is, among other things, a function of the location of the measurement plane and the axial location of the test unbalance. It therefore differs from the modal sensitivity which has a single value at a given rotor speed.

It is generally only necessary to measure local sensitivity at measuring points where at rotational speeds of interest the vibration amplitudes need to be limited. Here, the measured sensitivity M_n at a location and speed of interest is evaluated as follows (see Annex A, Note 2):

$$M_n = \frac{S_{\text{per}}}{e_{\text{per}}} \qquad (13)$$

where

S_{per} is the permissible amplitude;

e_{per} is the permissible residual eccentricity during operation.

NOTE Permissible amplitude can be obtained from the relevant parts of ISO 7919, and permissible residual eccentricity can be obtained from ISO 11342.[4] Note that the relevant parts of ISO 7919 specify peak-to-peak values rather than amplitudes (0-peak values).

Alternatively, values mutually agreed upon for this purpose between the machine manufacturer and the user may be substituted.

Depending on the operational parameters of a machine (e.g. rotational speed, resonance speed to operating speed ratio), it might be advisable to limit the maximum permissible local sensitivity by applying a factor (≤1,0) to Formula (13).

It is common practice to accept higher vibration limits during run-up and run-down than at operating speed. It is therefore possible to accept higher local sensitivity at other than operating speeds, if the vibration does not exceed the agreed limits.

8 Experimental determination of the local sensitivity

8.1 General

For measuring local sensitivity values, it is recommended that where possible rotor planes be used to produce the maximum vibration response for the modes and rotational speeds of interest, and where it is easily possible to add test masses.

8.2 Procedure

a) Prepare the machine for normal operation.

b) Run the machine to the desired data collection speed Ω. This rotational speed is often chosen as that speed in the operating speed range that is closest to a resonance speed. Wait until vibration and other relevant parameters are steady and measure the once-per-revolution vibration in the agreed planes k. During measurement, the rotational speed, load and other parameters that could influence the state of vibration should be held as constant as possible.

c) Attach a single trial mass to the rotor in the agreed plane r, producing the unbalance $\boldsymbol{U}_r$. The trial mass should be big enough to produce a clearly measurable change in the state of vibration from that measured in b), but not so large that dangerous vibrations are developed at any rotational speed that the machine runs through or operates at. Sometimes it may be necessary to attach a trial mass set.

d) Measure the vibration under the same conditions as in b).

e) Calculate for each measuring plane the vectorial difference between the measured values and those found in b) and d). This is the value $\boldsymbol{S}_k$. The magnitude of this value, S_k, divided by the magnitude of the trial mass unbalance, U_r, is the local sensitivity to unbalance (see Annex A). This value, $S_{k,r}$, is the value for the chosen data collection speed.

The linearity of the system and repeatability of the measured values shall be taken into account.

9 Damped unbalance sensitivity analysis

If the relevant experimental data are not available, a numerical analysis for machines that pass through or approach resonance speeds during run-up or service can be used.

Such analyses may include the following in a mathematical model:

a) stiffness, mass and damping characteristics of the rotor and support system;

b) bearing and seal stiffness and damping as a function of the rotor rotational speed.

These models should:

— identify natural frequencies and corresponding mode shapes;

— calculate modal damping;

— calculate local sensitivities at specified rotor axial locations and for specified unbalance planes.

Amount of unbalance applied to the mathematical model can be calculated from the machine balance quality grade, operational speed and rotor mass using Formulae (1) and (2); see Annex D for an example. Response to this calculated unbalance should be compared to acceptable vibration amplitudes defined in Formulae (4) to (6). Additionally, the amplification factor Q_n can be plotted against the speed ratio for modal sensitivity classification using Figure 5.

In calculation, unbalance should be placed in appropriate locations for modes of interest. For example, unbalance should be placed near mid-span for the first mode and in anti-phase near the ends for the second mode.

Annex A
(informative)

Explanations of terms

A.1 General

The sensitivity to unbalance is usually numerically expressed in two ways, see A.2 and A.3.

A.2 Local sensitivity

The local sensitivity $S_{k,r}$ can be expressed as follows:

$$S_{k,r} = \frac{S_k}{U_r} \quad \text{(A.1)}$$

where

- S_k is the magnitude of change in once-per-revolution vibration in plane k;
- U_r is the magnitude of change in trial mass unbalance, attached to plane r, in the rotor (or the change in the trial unbalance set).

NOTE The local sensitivity is frequently referred to as the “influence coefficient”. It is a dimensional quantity.

A.3 Modal sensitivity

The modal sensitivity in terms of the modal amplification factor M_n for excitation of the machine by unbalance for mode n is given by:

$$M_n = \frac{(\Omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\Omega}{\omega_n}\right)^2\right]^2 + 4\zeta_n^2\left(\frac{\Omega}{\omega_n}\right)^2}} \quad \text{(A.2)}$$

where

- Ω is the rotational speed;
- ω_n is the nth undamped resonance speed;
- ζ_n is the damping ratio of the nth mode.

NOTE 1 Formula (A.2) is derived from the Jeffcott rotor model:

$$S = \frac{me\Omega^2}{\sqrt{\left(K - m\Omega^2\right)^2 + \left(C\Omega\right)^2}} \quad \text{(A.3)}$$

where

S is the vibration amplitude;

m is the rotor mass;

e is the eccentricity of the rotor mass;

K is the stiffness;

C is the damping.

After dividing both the numerator and denominator of Formula (A.3) by K

$$S = \frac{\left(me\Omega^2 / K\right)}{\sqrt{\left(1 - \frac{m\Omega^2}{K}\right)^2 + \left(\frac{C\Omega}{K}\right)^2}} \tag{A.4}$$

Formula (A.4) can be simplified by substituting ω_n for $\sqrt{K / m}$, which is the nth undamped resonance speed for a single degree of freedom, and $2\zeta_n/\omega_n$ for C/K, which is a simplification using damping factor ζ_n:

$$S = \frac{e\left(\Omega/\omega_n\right)^2}{\sqrt{\left[1 - \left(\frac{\Omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta_n \frac{\Omega}{\omega_n}\right)^2}} \tag{A.5}$$

By definition, S/e is M_n, thus Formula (A.5) is equivalent to Formula (A.2).

Under conditions where the rotational speed equals a resonance speed, M_n approximates $1/(2\zeta_n)$. This is the maximum amplification at resonance denoted by Q_n, it is influenced only by the amount of damping in the system.

NOTE 2 Modal sensitivity in terms of the modal amplification factor for mode n is a non-dimensional quantity.

Annex B
(informative)

Example of polar plot diagram procedure

Readings taken at different rotational speeds are illustrated in Figure B.1. An analysis of the readings indicates that the first resonance speed is n_1 = 3 000 r/min and the speeds N_{45} at which the phase angle has been displaced by 45° are 2 710 r/min and 3 320 r/min. The amplification factor Q_1 can then be calculated using Formula (11), it is tabulated in Table B.1.

If the first and second resonance speeds are close together or if secondary resonances are superimposed, more sophisticated techniques are required to evaluate the modal properties.

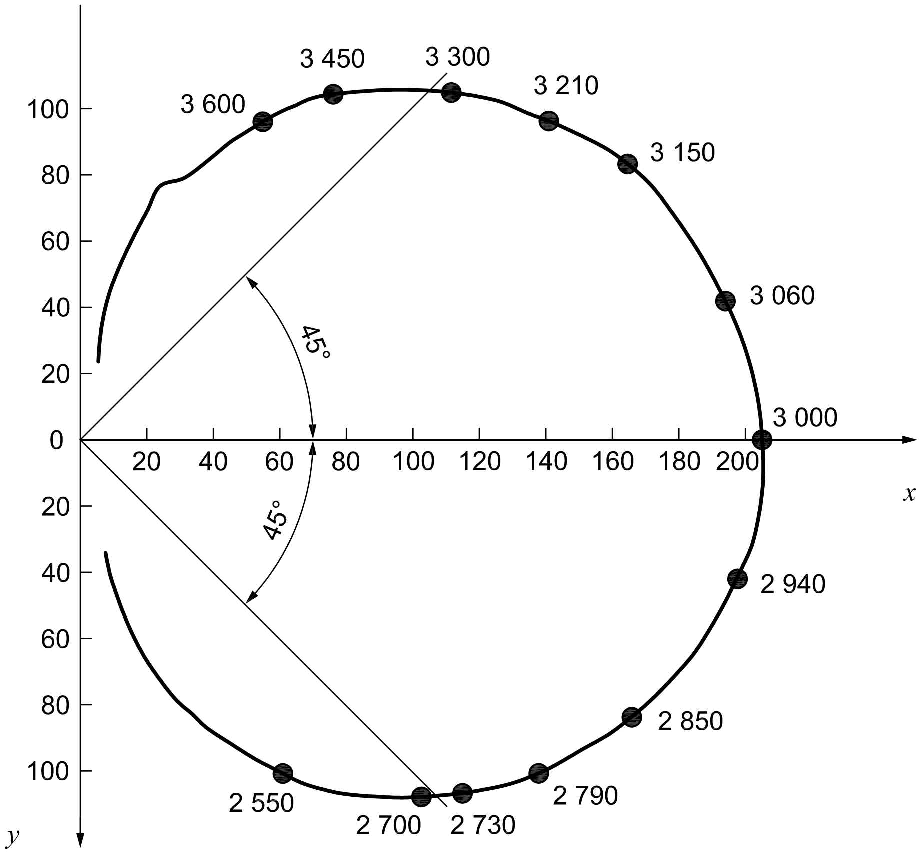


Figure B.1 — Nyquist diagram of vibration amplitudes in *x*- and *y*-directions and phase angle while passing through a resonance speed at 3 000 r/min

Table B.1 — Measurement results for n_1 = 3 000 r/min

N_{45}	Q_1
2 710 r/min	4,91
3 320 r/min	4,92

Annex C
(informative)

Examples of classification according to modal sensitivity

C.1 Example 1

Machine:	Gas turbine (type II)
Constant operating speed:	3 000 r/min
First resonance speed:	2 850 r/min
Damping ratio from experiment *in situ*:	$\zeta_1 = 0{,}04$

Since the resonance speed is very close to the operating speed, Q_1 can be estimated using the damping ratio:

$$Q_1 = \frac{1}{2\zeta_1} = \frac{1}{2 \times 0{,}04} = 12{,}5$$

The resonance speed to operating speed ratio is

$$\frac{\omega_1}{\Omega} = 0{,}95$$

From Figures 4 and 5, it can be seen that the machine is in range B (low sensitivity).

C.2 Example 2

Same machine as in Example 1, but with the resonance speed equal to the operating speed.

In this case, the resonance speed to operating speed ratio is

$$\frac{\omega_1}{\Omega} = 1{,}0$$

From Figure 4, it can be seen that the machine is in range C (moderate sensitivity).

Annex D
(informative)

Example of mathematical model applied unbalance

Machine:	Gas turbine (type II)
Constant operating speed n:	3 000 r/min
Rotor mass m:	80 000 kg

According to ISO 1940-1,[1] the balance quality grade G for gas turbines is 2,5 mm/s. From Formula (2) the permissible eccentricity is:

$$e_{\text{per}} = \frac{G}{\Omega} = \frac{2{,}5}{3\,000\left(2\pi/60\right)} = 0{,}008 \text{ mm}$$

Thus, the permissible residual unbalance of the machine is

$$U_{\text{per}} = e_{\text{per}}m = 0{,}008 \text{ mm} \times 80\,000 \text{ kg} = 640 \text{ kg·mm}$$

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12) To become ISO 21940-11 when revised.

13) To become ISO 21940-12 when revised.

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