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**Passenger cars — Vehicle dynamic  
simulation and validation — Steady-  
state circular driving behaviour**

*Voitures particulières — Simulation et validation dynamique des  
véhicules — Tenue de route en régime permanent sur trajectoire  
circulaire*



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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: [www.iso.org/iso/foreword.html](http://www.iso.org/iso/foreword.html).

The committee responsible for this document is ISO/TC 22, *Road vehicles*, Subcommittee SC 33, *Vehicle dynamics and chassis components*.

## Introduction

The main purpose of this document is to provide a repeatable and discriminatory method for comparing simulation results to measured test data from a physical vehicle for a specific type of test.

The dynamic behaviour of a road vehicle is a very important aspect of active vehicle safety. Any given vehicle, together with its driver and the prevailing environment, constitutes a closed-loop system that is unique. The task of evaluating the dynamic behaviour is therefore very difficult since the significant interactions of these driver-vehicle-environment elements are each complex in themselves. A complete and accurate description of the behaviour of the road vehicle should include information obtained from a number of different tests.

Since this test method quantifies only one small part of the complete vehicle handling characteristics, the validation method associated with this test can only be considered significant for a correspondingly small part of the overall dynamic behaviour.



# Passenger cars — Vehicle dynamic simulation and validation — Steady-state circular driving behaviour

## 1 Scope

This document specifies a method for comparing computer simulation results from a vehicle mathematical model to measured test data for an existing vehicle according to steady-state circular driving tests as specified in ISO 4138 or the Slowly Increasing Steer Test that is an alternative to ISO 4138. The comparison is made for the purpose of validating the simulation tool for this type of test when applied to variants of the tested vehicle.

It is applicable to passenger cars as defined in ISO 3833.

NOTE The Slowly Increasing Steer method is described in regulations such as USA FMVSS 126 “Federal Register Vol 72, No. 66, April 6, 2007” and UN/ECE Regulation No. 13-H, “Uniform provisions concerning the approval of passenger cars with regard to braking”.

## 2 Normative references

The following documents are referred to in text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 4138, *Passenger cars — Steady-state circular driving behaviour — Open-loop test methods*

ISO 15037-1, *Road vehicles — Vehicle dynamics test methods — Part 1: General conditions for passenger cars*

## 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 1176, ISO 2416, ISO 3833, ISO 8855 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— IEC Electropedia: available at <http://www.electropedia.org/>

— ISO Online browsing platform: available at <http://www.iso.org/obp>

### 3.1

#### **simulation**

calculation of motion variables of a vehicle from equations in a mathematical model of the vehicle system

### 3.2

#### **simulation tool**

simulation environment including software, models, input data, and hardware case of hardware-in-the-loop simulation

### 3.3

#### **cross plot**

plot where the horizontal axis shows values for a variable other than time (e.g. lateral acceleration)

## 4 Principle

Open-loop test methods defined in ISO 4138 are used to determine the steady-state circular driving behaviour of passenger cars as defined in ISO 3833.

The test characterizes vehicle-handling behaviour in steady-state conditions covering a range of cornering conditions from straight-line up to limit conditions for steering control. Results are typically reported by cross plotting steady-state measures of variables of interest against steady-state levels of lateral acceleration, and possibly calculating characteristic values based on gradients of the plotted data.

Within this document, the purpose of the test is to demonstrate that a vehicle simulation tool can predict the vehicle behaviour within specified tolerances. The vehicle simulation tool is used to simulate a specific existing vehicle running through a steady-state test as specified in ISO 4138, or, alternatively, a Slowly Increasing Steer Test used in stability control evaluation. Simulation results are used to define graphical boundaries for overlaid cross-plots, and the data from physical testing are overlaid to see if the measurements fall within the acceptable ranges.

NOTE This document may be used for several purposes. Depending on the purpose of the validation, only parts of the validation requirements may be met.

The existing vehicle is physically tested at least three times to allow the test data to be compared with the simulation results.

## 5 Variables

The following variables shall be compared:

- lateral acceleration;
- steering-wheel angle;
- sideslip angle;
- roll angle.

The steering-wheel torque shall also be compared if this document is used to validate a simulation tool for the purpose of predicting steering torque during steady-state circular driving as defined in ISO 4138.

Measurement requirements shall be taken from ISO 4138 and ISO 15037-1, unless noted otherwise.

For the purpose of this document, lateral acceleration should be measured directly by an inertial measurement unit, rather than using the alternative calculation methods provided in ISO 4138.

## 6 Simulation tool requirements

### 6.1 General

The simulation tool used to predict behaviour of a vehicle of interest shall include a mathematical model capable of calculating variables of interest for the test procedures being simulated. In this document, the mathematical model is used to simulate a steady-state cornering manoeuvres (see [7.2](#)) and provide calculated values of the variables of interest from [Clause 5](#).

The procedure for obtaining input data from experiments may differ for simulation tools, however, the input data shall not be manipulated for better correlation. However, adaptation of input data to actual testing conditions such as road friction should be allowed.

NOTE Active controllers and active intervention systems that prevent a steady-state condition from being established are not relevant for the tests covered in this document.



## 6.2 Mass and inertia

The mathematical model should include all masses, such as the chassis, engine, payloads, unsprung masses, etc. The value of the mass and the location of the centre of mass are essential properties of the vehicle for the tests covered in this document. On the other hand, moments and products of inertia have no effect under steady-state conditions, when angular accelerations are negligible.

Vehicles with significant torsional frame compliance require a more detailed representation that includes frame-twist effects that occur in extreme manoeuvres.

## 6.3 Tires

The vertical, lateral, and longitudinal forces and moments where each tire contacts the ground provide the main actions on the vehicle. The fidelity of the prediction of vehicle movement depends on the fidelity of the calculated tire forces and moments. Differences between the tire force and moment measurements used for the model and those of used in vehicle testing can be expected due to different wear and aging histories. Although difficult to account for these differences, it is important to acknowledge and understand them.

Large lateral slip angles and inclination can occur under the conditions covered in this document. Longitudinal slip ratios are usually limited to the amounts needed to generate longitudinal forces to maintain a target speed in the test. The tire model shall cover the entire ranges of slip (lateral and longitudinal), inclination angle relative to the ground, and load that occur in the tests being simulated.

The surface friction coefficient between the tire and ground is an important property for the limit friction conditions that can be encountered in steady-state circular driving tests.

The simulated tests take place on a flat homogenous surface; detailed tire models that handle uneven surfaces are not needed. If the test surface has inclination for water drainage, this should be included in the simulation.

The simulated tests involve conditions that are intended to be steady-state; therefore, transient effects in tire response (e.g. relaxation length) are not needed.

## 6.4 Suspensions

The properties of the suspensions that determine how the tire is geometrically located, oriented, and loaded against the ground shall be represented properly in the model in order for the tire model to generate the correct tire forces and moments. The suspension properties also determine how active and reactive forces and moments from the tires are transferred to the sprung mass.

The suspension properties should include change of location and orientation of the wheel due to suspension deflection and applied load as would be measured in a physical system in kinematics and compliance (K&C) tests.

The model shall cover the full nonlinear range encountered in the steady-state steering tests for springs, jounce and rebound bumpers and auxiliary roll moments due to anti-roll bars and other sources of roll stiffness.

Rate-dependent forces such as shock absorbers are not significant in steady-state testing.

The model shall include the effects of active intervention systems, if applicable during the steady-state conditions covered in this document.

## 6.5 Steering system

The steering system interacts with the suspensions to determine how the tire is oriented on the ground.

The model shall include kinematical and compliance relationships between the steering-wheel angle and road wheel angle.

The model shall include the effects of active intervention systems, if applicable during the steady-state conditions covered in this document.

### 6.6 Aerodynamics

The model should include aerodynamic effects that influence tire load and overall vehicle drag for the speeds covered in the testing.

### 6.7 Brake system

If the brakes are not engaged during the testing, then the brake system is not needed. However, if an active controller engages that uses the brakes to control the vehicle during the steady-state conditions covered in this document (see [6.9](#)), then the vehicle brake model shall include the actuators and response properties that affect the controlled vehicle response.

### 6.8 Powertrain

In the steady-state steering manoeuvre covered in this document, the powertrain is applied as needed to achieve the vehicle target lateral acceleration. The transfer of drive torque to the wheels should be included in the model, with the proper drivetrain configuration (front-wheel drive, rear-wheel drive, all-wheel drive, electric motors, differentials, etc.).

Aspects of powertrain behaviour that are important for other kinds of tests (engine power, dynamic responses to throttle, shifting and clutch behaviour) might not be needed for the steady-state steering manoeuvres; however, if a chassis control system engages, then any aspects of the powertrain that influence the controller behaviour shall be included in the powertrain model.

### 6.9 Active controllers

If any electronic controller is engaged in the physical vehicle for the steady-state steering manoeuvres covered in this document, its model shall be included in the simulated version.

Physical controllers and/or mechanical components may be linked to the simulated vehicle by hardware in the loop.

The controller model should include actuators that are not already part of the vehicle brake model (see [6.7](#)) and control logic.

### 6.10 Data acquisition

Signals extracted from the simulation should mimic the signals extracted from the physical vehicle. For example, sensor location, orientation, data processing (filtering, etc.; see [7.4](#)) in the simulation should match the physical test setup.

### 6.11 Driver controls

The test methods described in [7.2](#) require control of steering and speed. The simulation tool shall be capable of providing the driver controls (steering, throttle, gear selection) required for the selected test method.

## 7 Physical testing

### 7.1 General

An existing vehicle of interest shall be tested using a constant-radius test or constant-speed test as defined in ISO 4138, or alternatively, the Slowly Increasing Steer Test variant of the constant-speed test used in stability control evaluation (e.g. UN/ECE Regulation No. 13-H, "Uniform provisions concerning

the approval of passenger cars with regard to braking.”). Unless noted otherwise in this document, the tests should be conducted and results should be reported as specified in ISO 4138 and ISO 15037-1.

## 7.2 Test methods

### 7.2.1 Constant-radius

The constant-radius method requires driving the vehicle at several speeds over a circular path of known radius. The standard radius of the path specified in ISO 4138 is 100 m, but larger and smaller radii may be used, with 40 m the smallest recommended radius and 30 m the minimum permitted radius.

Results depend on the turn radius; therefore, the radius used shall be reported along with the plots.

ISO 4138 defines two variations of this method, one with a set of tests each done with a constant speed and the other with a single test in which speed is slowly increased. The variation used in physical testing shall also be used in the simulation and shall be reported along with the method.

### 7.2.2 Constant speed

#### 7.2.2.1 General

The constant-speed method requires driving the vehicle at a constant speed over a range of lateral acceleration generated with changes in steering. The plotted curves can vary with speed; therefore, the speed shall be reported along with the plots.

#### 7.2.2.2 Multiple tests with discrete steer or radius

ISO 4138 defines two variations of this method: one involving discrete changes in steering-wheel angle, and the second involving discrete changes in path radius (marked with painted circles on the surface of the test space). The variation used in physical testing shall also be used in the simulation, and shall be reported along with the method.

#### 7.2.2.3 Slowly Increasing Steer Test

In a third variation, an open-loop test is performed at a constant speed with steering control provided by a robot. In this test, the steering pattern increases the steering-wheel angle at a constant rate to run through the desired range of lateral acceleration, or until limits of test space, or vehicle stability are reached.

The rate of steering-wheel increase shall be 13,5°/s or less.

The same rate of steering-wheel increase shall be used for all physical tests and simulations, and shall be reported along with the plots.

## 7.3 Documentation of limit condition

The steady-state circular turning procedures described in ISO 4138 involve testing until steady-state conditions can no longer be achieved. Depending on the method and variation used, limits might be reached due to the capability of the driver, lack of space, intervention of active systems that cause dynamic motions that prevent steady conditions from occurring, or other reasons.

In each test series, the factor causing the limit condition shall be reported.

## 7.4 Low-pass filtering of measured data

The vehicle behaviour of interest in this document involves steady conditions or slowly changing conditions. The low-pass filtering of the variables measured in these test may use a cut-off frequency

lower than those mentioned in ISO 15037-1; the cut-off frequency for the tests used in this document may be as low as 1,0 Hz.

## 8 Simulation

### 8.1 General

The simulation tool shall be configured to simulate the method from [7.2](#) that was chosen to physically test the existing vehicle. Because the computer simulations might not be sensitive to some factors of interest in physical testing, not all of the details specified in ISO 4138 and ISO 15037-1 apply. Requirements from ISO 4138 and ISO 15037-1 for repeated tests, warm-up, transducer properties, filtering, etc. might be irrelevant. Factors that do not affect the simulations may be neglected to reduce the number of simulations or simplify the data processing.

### 8.2 Simulation procedure

#### 8.2.1 Direction of steer

The simulation shall be run for both directions (steering clockwise and steering counter-clockwise).

#### 8.2.2 Tests with steady-state conditions

This document supports four test procedures defined in ISO 4138 (see [7.2.1](#) and [7.2.2.2](#)). Three of these procedures involve driving the vehicle with a constant speed and constant steer or constant turn radius until a steady-state is reached. If one of these procedures is chosen, then the conditions for steady-state lateral acceleration shall be defined by values separated by intervals no less than 0,1 m/s<sup>2</sup> (0,01 g) and no greater than 0,25 m/s<sup>2</sup> (0,025 g).

#### 8.2.3 Tests with slowly changing conditions

One of the constant-radius methods from ISO 4138 allows tests to be done under the condition of slowly increasing speed (see [7.2.1](#)). Another option is the Slowly Increasing Steer Test ([7.2.2.3](#)). If one of these procedures is used, then a single simulation is made for each steering direction.

## 8.3 Data recording

### 8.3.1 General

The purpose of each test method (see [7.2](#)) is to obtain values of the variables of interest (see [Clause 5](#)) for corresponding values of lateral acceleration. These values shall be extracted from the simulation tool and written to file such that they can be used to calculate auxiliary variables that are used as coordinates for boundaries (see [9.2](#)), and possibly for characteristic gradient metrics defined in ISO 4138.

### 8.3.2 Tests with steady-state conditions

This document supports three test procedures defined in ISO 4138 that involve driving the vehicle at constant speed and constant steer or constant turn radius until a steady-state is reached. If one of these procedures is chosen, then the variables of interest (see [Clause 5](#)) shall be recorded when steady-state is achieved in the simulation.

**NOTE** It is acceptable to record the variables of interest with a shorter time step during the simulation, and then extract the steady-state values for each condition through post-processing.

### 8.3.3 Tests with slowly changing conditions

If a test procedure is chosen that involves slowly changing conditions (see 8.2.3), then the variables of interest (see Clause 5) shall be recorded for intervals of lateral acceleration no less than 0,01 g (0,1 m/s<sup>2</sup>) and no greater than 0,25 m/s<sup>2</sup> (0,025 g).

NOTE The coordinates for boundaries in the cross plots of variables of interest (see 9.2) involve the use of gradient variables calculated using a finite-difference method. The intervals of lateral acceleration are specified in this section to limit numerical artefacts resulting from the finite difference calculations. It is acceptable to record the variables of interest with a fixed time step during the simulation, and then extract the values for acceptable intervals of lateral acceleration through post-processing.

In tests where speed is slowly increasing, transient responses can occur due to gear shifting. If possible, data should not be recorded until these transient responses have settled.

## 9 Comparison between simulation and physical test results

### 9.1 General

Data points from the simulation and testing are reported in the form of cross plots in which the steady-state values of steering-wheel angle, sideslip angle, and roll angle are plotted on the ordinate; values of lateral acceleration are plotted on the abscissa.

Values from simulation are considered to be fully repeatable and reproducible, and are used to calculate upper and lower boundaries for the purpose of determining whether the simulation is valid when compared to data points from physical testing.

### 9.2 Calculation of boundary points

Given a set of lateral acceleration values associated with the X-axis and a set of values of another variable associated with the Y-axis, the X and Y coordinates for the top boundary are:

$$X_T = X - \Delta Y \varepsilon_X^2 / D \quad (1)$$

$$Y_T = Y + \Delta X \varepsilon_Y^2 / D \quad (2)$$

The X and Y coordinates for the bottom boundary are:

$$X_B = X + \Delta Y \varepsilon_X^2 / D \quad (3)$$

$$Y_B = Y - \Delta X \varepsilon_Y^2 / D \quad (4)$$

where

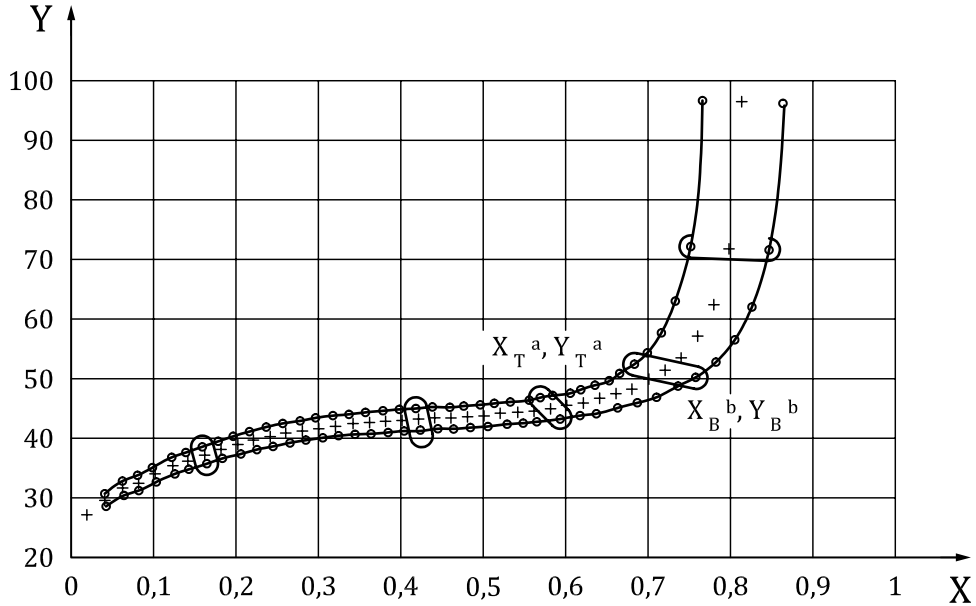
$$D = [(\Delta X \varepsilon_Y)^2 + (\Delta Y \varepsilon_X)^2]^{1/2} \quad (5)$$

and  $\Delta X$  is the difference between the X-axis variable ( $A_y$ ) for the current value and preceding value (with original units such as g or m/s<sup>2</sup>),  $\Delta Y$  is the difference between the Y-axis variable (e.g. steering-wheel angle) for the current value and preceding value,  $\varepsilon_X$  is the tolerance for the lateral acceleration, and  $\varepsilon_Y$  is the tolerance for the Y-axis variable (see 9.3).

NOTE See Annex A for the derivation of Formulae (1) to (5).

For example, Figure 1 shows top and bottom boundaries for a vehicle with limit understeer behaviour (indicated by greatly increasing steering-wheel angle as limit lateral acceleration is reached). In the figure, several ovals are used to group top and bottom boundary points, indicated with circular markers, with the corresponding point from the simulation, indicated with a "+" marker.

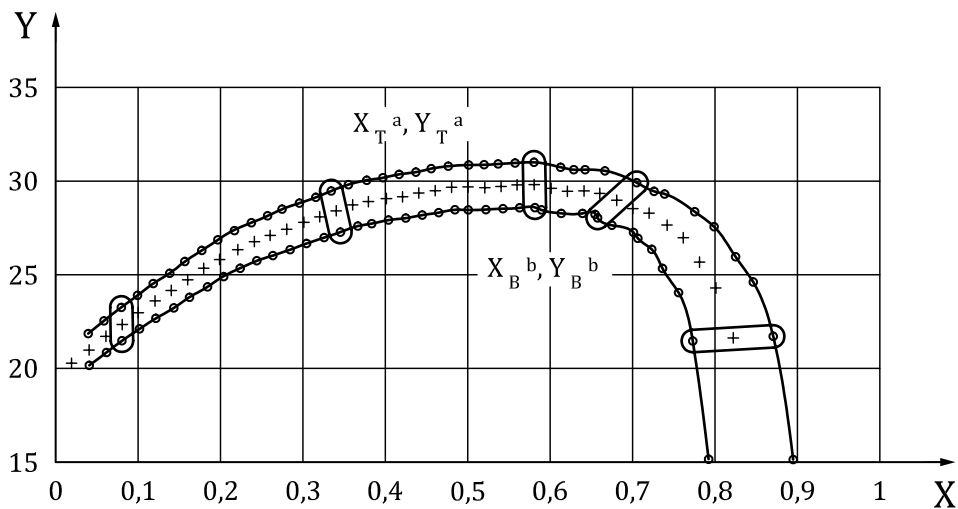
Figure 2 shows example plots for a vehicle with limit oversteer.



**Key**

- X lateral acceleration
- Y boundaries for Steering Wheel Angle
- $X_T^a, Y_T^a$  top boundary points
- $X_B^b, Y_B^b$  bottom boundary points

**Figure 1 — Steer vs. lateral acceleration boundaries for an example simulation with limit understeer**



**Key**

- X lateral acceleration
- Y boundaries for Steering Wheel Angle
- $X_T^a, Y_T^a$  top boundary points
- $X_B^b, Y_B^b$  bottom boundary points

**Figure 2 — Steer vs. lateral acceleration boundaries for an example simulation with limit oversteer**



### 9.3 Tolerances for cross-plot boundary points

Tolerances  $\varepsilon_X$  and  $\varepsilon_Y$  for each cross plot are calculated using an offset and gain with the form:

$$\varepsilon_X = [\text{X offset}] + [\text{X gain}] \times |X| \quad (6)$$

$$\varepsilon_Y = [\text{Y offset}] + [\text{Y gain}] \times |Y| \quad (7)$$

The offsets and gains for the three cross plots are listed in [Table 1](#) for the constant-radius test methods (see [7.2.1](#)) and [Table 2](#) for the constant-speed methods (see [7.2.2](#)).

**Table 1 — Offsets and gains used to define tolerances  $\varepsilon_X$  and  $\varepsilon_Y$  for constant-radius tests**

Variable on Y-axis	X offset (m/s <sup>2</sup> )	X gain	Y offset (deg)	Y gain
Steering wheel angle (deg)	0,1	0,06	1,0	0,03
Sideslip angle (deg)	0,1	0,06	0,3	0,04
Roll angle (deg)	0,1	0,06	0,2	0,2

**Table 2 — Offsets and gains used to define tolerances  $\varepsilon_X$  and  $\varepsilon_Y$  for constant-speed (variable steer) tests**

Variable on Y-axis	X offset (m/s <sup>2</sup> )	X gain	Y offset (deg)	Y gain
Steering wheel angle (deg)	0,1	0,06	5,0	0,03
Sideslip angle (deg)	0,1	0,06	0,3	0,04
Roll angle (deg)	0,1	0,06	0,2	0,2

NOTE Separate tolerances are needed for the two categories of testing because the levels of steering for the constant speed tests start at zero, whereas the steering for the constant radius tests starts with Ackermann steering.

### 9.4 Comparison of simulation cross plots with measured test data

Values of lateral acceleration, steering-wheel angle, sideslip angle, and roll angle from physical testing shall be obtained as specified in ISO 4138 or the Slowly Increasing Steer Test and plotted along with the boundaries obtained from the simulation.

Results should be shown for three or more repeat tests, each made in both directions (left and right turning).

If the test results all lie within the boundaries, then the simulation tool is considered valid for determining steady-turning behaviour up to the limits in lateral acceleration covered in the testing.

## 10 Documentation

The simulation shall be documented to the extent needed to reproduce the simulated tests. This should include names of software tools, including version numbers, and internal model names. A list of files used to run the simulation shall be provided, and copies of the files shall be archived.

This document includes various options for characterizing steady-turning behaviour. The method that is used shall be documented, along with specification for options within the selected method (e.g. radius of turn for a constant-radius test, speed for a constant speed test, etc.). Details of the methods used to extract steady-state values from the measured test data and the simulated tests shall be included in the documentation.

If this document is used to validate a simulation tool solely for the purpose of reporting results defined in ISO 4138, then characteristic gradients values that are normally calculated from the measured test data should also be calculated from the simulated test data. Both values should be reported.



## Annex A (informative)

### Combined tolerance for normalized geometric profile

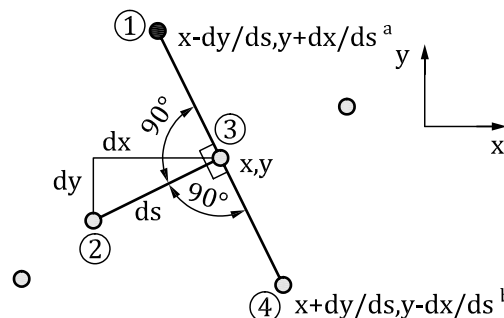
Results from steady-state turning tests are reported by cross plotting steady-state values of a variable of interest on the Y-axis against steady-state values of lateral acceleration on the X-axis.

When the two variables being plotted have the same dimensional units (e.g. m), geometric dimensioning and tolerancing (GD&T) standards (published by ISO, ASME, ANSI, etc.) can be used for evaluating the match between profiles. The 2D curved line is called a profile, and the tolerance is applied at each point in a direction normal to the profile.

A complication in the case of the steady-state turning plots is that the measures on the horizontal and vertical axes have different units (e.g. acceleration, angle) so it is not possible to combine tolerances directly. Instead, they must be normalized. A combined tolerance is defined using variables that have been normalized locally using the tolerances for the two variables  $\epsilon_x$  and  $\epsilon_y$ , and then applied using a profile control method.

In the following equations, the original scaled variables are designated  $X$  and  $Y$  and the normalized variables are lower-case italic  $x$  and  $y$ . With both variables being normalized to their respective tolerances, the variables  $x$  and  $y$  are dimensionless and scaled such that a value of 1 corresponds to the tolerance in any direction in the  $x$ - $y$  plane.

[Figure A.1](#) shows several points in the  $x$ - $y$  plane. Points ② and ③ are two consecutive vehicle dynamic variables (e.g. lateral acceleration and steering-wheel angle), normalized by the tolerances for the point shown ③. The distance from the reference point ③ to a corresponding point on the top boundary ① is unity by definition of the normalization; the distance from the reference point to a corresponding point on the bottom ④ boundary is also unity by definition.



#### Key

- <sup>a</sup> top boundary
- <sup>b</sup> bottom boundary points

**Figure A.1 — Top and bottom boundary points based on normalized slopes**

The normalized variables and differences are

$$x = X/\epsilon_x \quad (\text{A.1})$$

$$y = Y/\epsilon_y \quad (\text{A.2})$$

$$dx = \Delta X/\epsilon_x \quad (\text{A.3})$$

$$dy = \Delta Y / \varepsilon_y \quad (\text{A.4})$$

where the difference between the X-axis variable (lateral acceleration) for the current test (3) and preceding test (2) is  $\Delta X$  with original units such as g or m/s<sup>2</sup>, and the difference between the Y-axis variable (e.g. steering-wheel angle) for the two tests ((2) and (3)) is  $\Delta Y$  with original units such as degrees.

The distance in normalized space between the two points ((2) and (3)) is

$$ds = [dx^2 + dy^2]^{1/2} = [(\Delta X / \varepsilon_x)^2 + (\Delta Y / \varepsilon_y)^2]^{1/2} \quad (\text{A.5})$$

A profile control method specifies the tolerance boundaries by points that are the specified distance from the line (profile) and normal to it. With this definition, the coordinates for the point in the top boundary are

$$x_T = x - dy/ds \quad (\text{A.6})$$

$$y_T = y + dx/ds \quad (\text{A.7})$$

For the bottom boundary, they are

$$x_B = x + dy/ds \quad (\text{A.8})$$

$$y_B = y - dx/ds \quad (\text{A.9})$$

To simplify the following equations, [Formula A.5](#) is reorganized as

$$ds \varepsilon_x \varepsilon_y = [(\Delta X \varepsilon_y)^2 + (\Delta Y \varepsilon_x)^2]^{1/2}$$

$$ds = D / (\varepsilon_x \varepsilon_y) \quad (\text{A.10})$$

where

$$D = [(\Delta X \varepsilon_y)^2 + (\Delta Y \varepsilon_x)^2]^{1/2} \quad (\text{A.11})$$

Going back to the scaled variables, the X and Y coordinates for the top boundary are

$$X_T = X - \Delta Y \varepsilon_x^2 / D \quad (\text{A.12})$$

$$Y_T = Y + \Delta X \varepsilon_y^2 / D \quad (\text{A.13})$$

and the X and Y coordinates for the bottom boundary are

$$X_B = X + \Delta Y \varepsilon_x^2 / D \quad (\text{A.14})$$

$$Y_B = Y - \Delta X \varepsilon_y^2 / D \quad (\text{A.15})$$

## Bibliography

- [1] ISO 1176, *Road vehicles — Masses — Vocabulary and codes*
- [2] ISO 2416, *Passenger cars — Mass distribution*
- [3] ISO 3833, *Road vehicles — Types — Terms and definitions*
- [4] ISO 8855, *Road vehicles — Vehicle dynamics and road-holding ability — Vocabulary*

