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# Mechanical vibration and shock — Signal processing —

## Part 3: Methods of time-frequency analysis

*Vibrations et chocs mécaniques — Traitement du signal —*

*Partie 3: Méthodes d'analyses en temps et fréquence et par échelle de temps*

Reference number
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# Foreword

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The committee responsible for this document is ISO/TC 108, *Mechanical vibration, shock and condition monitoring*.

ISO 18431 consists of the following parts, under the general title *Mechanical vibration and shock — Signal processing*:

— *Part 1: General introduction*

— *Part 2: Time domain windows for Fourier Transform analysis*

— *Part 3: Methods of time-frequency analysis*

— *Part 4: Shock-response spectrum analysis*

# Introduction

Time-frequency analysis is used to quantitatively display a vibration or shock in terms of time and frequency. This is useful for analysing vibrations in a machine at varying speeds, e.g. in an automobile at varying engine rotational frequencies. Time-frequency analysis is also used to quantitatively display impulsive responses from machinery, e.g. the response of an impact. The duration of the impact as well as the frequency response is displayed. The frequency response can be displayed in terms of frequency, RPM, or octaves. The four methods included in this part of ISO 18431 are the short-time Fourier transform, the Wigner-Ville transform, the Choi-Williams transform, and the wavelet transform. When any of these methods is used with the correctly specified parameters, time and frequency components of shocks and vibrations are displayed quantitatively. Quantitative display enables quantitative specification of the machinery.

# Mechanical vibration and shock — Signal processing —

# Part 3:
# Methods of time-frequency analysis

## 1 Scope

This part of ISO 18431 specifies methods for the digital calculation of a time-frequency analysis of a given sampled measurement of a physical or engineering quantity, such as acceleration, force, or displacement, over an interval of time. Several mathematical formulations of time-frequency transformations are given with requirements for recording of parameters and recommendations.

The data can be obtained experimentally from measurements of a mechanical structure or obtained from numerical simulation of a mechanical structure. This category of data is very broad because there is a wide variety of mechanical structures, e.g. microscopic instruments, musical instruments, automobiles, manufacturing machines, buildings, and civil structures. The data can determine the response of machines or of humans to mechanical vibration and shock.

## 2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2041, *Mechanical vibration, shock and condition monitoring — Vocabulary*

ISO 18431-1, *Mechanical vibration and shock — Signal processing — Part 1: General introduction*

## 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2041, ISO 18431-1, and the following apply.

**3.1**
**short-time Fourier transform**
Fourier-related transform used to determine the time dependence of the sinusoidal frequency and phase content of a time-varying vibration

**3.2**
**Wigner-Ville distribution**
quadratic time-frequency description of a vibration based on the autocorrelation of a signal

**3.3**
**generalized Wigner-Ville distribution**
time-frequency description of a vibration based on a filtered autocorrelation of a signal

**3.4**
**Choi-Williams transform**
generalized Wigner-Ville distribution of a vibration using a specific kernel

**3.5**
**wavelet transform**
time-frequency description of a vibration based on the scaled frequency transformation of a signal

## 4 Symbols

| | |
|---|---|
| $f_s$ | = $1/\Delta t$, sampling frequency; the inverse of the sampling period |
| $K$ | number of samples in window $w(k)$ |
| $m$ | index of scaled frequency |
| $N$ | data block length; the number of sampled points that are transformed |
| $n$ | index of time |
| $S_x(m,n)$ | short-time Fourier transform of sampled data $x(n)$ |
| $T$ | = $N\Delta t$, total time of a block of sampled data |
| $t$ | = $n\Delta t$, time |
| u | units of the sampled signal, e.g. displacement or velocity |
| $V(n,m)$ | Cohen class filter for smoothing the Wigner distribution |
| $X(m)$ | discrete Fourier transform of $x(n)$ |
| $\tilde{X}(m)$ | namely $\tilde{X}(m)=0$ for $N/2 \le m$ |
| $x(n)$ | sampled physical data in the time domain |
| $\tilde{x}(n)$ | analytic sampled signal |
| $\Delta f$ | frequency resolution |
| $\Delta n$ | increment of index of time |
| $\Delta t$ | sample period |
| $\Xi_x(m,n)$ | Choi-Williams distribution |
| $\sigma$ | filter parameter of Choi-Williams distribution |
| $\Omega_x(m,n)$ | Wigner distribution of signal $x(n)$ |
| $O_x(k,m)$ | wavelet transform of $x(n)$ |

## 5 Time-frequency transforms

### 5.1 Short-time Fourier transform

The short-time Fourier transform of a signal is the Fourier transform of segments shorter than a given data block. It shows how the spectrum changes with time. It is defined by Formula (1) and detailed in Reference [3].

$$S_x(m,n)=\frac{1}{f_s}\sum_{k=0}^{K-1}x(n+k)w(k)\exp\left(-i2\pi\frac{mk}{K}\right) \quad (1)$$

for

$0 \le m < K/2$ and $0 \le n < N-K$

The following parameters shall be specified: the short-time integration window, $K$, window function $w(k)$, $0 \le k \le K-1$, and the increment of the index of time, $\Delta n$.

The quantity $|S_x(m,n)|^2$ is plotted for quantitative analysis. The units are $u^2/Hz^2$.

NOTE 1 The increment of the index of time is often 1, $K/2$, or $K$.

NOTE 2 The value of $K$ depends on the time and frequency dependence of the signal to be analysed. Larger values of $K$ show finer frequency details and smaller values of $K$ show finer time details.

## 5.2 Generalized Wigner-Ville transform

### 5.2.1 Wigner-Ville transform

The Wigner-Ville time-frequency transform of a signal is defined by Formula (2).

$$\Omega_x(m,n) = \frac{f_s}{N} \sum_{k=0}^{r} \tilde{X}(m+k)\tilde{X}^*(m-k)\exp\left(+i\frac{2\pi}{N}2nk\right) \quad (2)$$

for

$0 \le m \le (N/2)-1$, $0 \le n \le N$, $r = \min[(N/2)-1-m, n]$

The absolute value of $\Omega_x(m,n)$ is plotted for quantitative analysis. The units are $u^2$ s.

NOTE 1 The Wigner-Ville transform is also expressed in terms of the autocorrelation with Formula (3).

$$\Omega_x(m,n) = \begin{cases} \frac{2}{f_s}\mathrm{Re}\left[\sum_{k=0}^{2n} x^*\left(n-\frac{k}{2}\right)x\left(n+\frac{k}{2}\right)\exp\left(-i2\pi\frac{mk}{n}\right)\right] & \text{for} \quad m < n, 0 \le n < \frac{N}{2} \\ \frac{2}{f_s}\mathrm{Re}\left[\sum_{k=0}^{2(N-n)} x^*\left(n-\frac{k}{2}\right)x\left(n+\frac{k}{2}\right)\exp\left(-i2\pi\frac{mk}{N-n}\right)\right] & \text{for} \quad m < (N-n), \frac{N}{2} \le n < N \end{cases} \quad (3)$$

NOTE 2 The Wigner distribution is derived from the autocorrelation of the complete data block. This allows greater accuracy in the resolution of time and frequency components, but also generates spurious components in the distribution.

NOTE 3 The sample period of time in the Wigner distribution is twice that of the sample period of the original data $x(n)$ because the time increments of the autocorrelation have twice the period. Therefore, the time corresponding to sample $n$ is $t = n2\Delta t$.

NOTE 4 An example of the resolution and spurious components are shown in Annex A.

### 5.2.2 Choi-Williams transform

The Choi-Williams transform is defined by Formulae (4) and (5).

$$\Xi_x(m,n) = \frac{2}{f_s}\sum_{k=0}^{N-1}\exp\left(-i4\pi\frac{mk}{N}\right)\sum_{l=k}^{N-k-1} V(k,l,n;\sigma)\tilde{x}(l+k)\tilde{x}^*(l-k) \quad (4)$$

where

$$V(k,l,n;\sigma) = \frac{1}{\sqrt{4\pi k^2/\sigma}}\exp\left[-\frac{(l-n)^2}{4k^2/\sigma}\right] \quad \text{and} \quad V(0,l,n;\sigma) = 0 \quad (5)$$

The value of parameter $\sigma$ is specified. This parameter is dimensionless.

The absolute values of $\Xi_x(m,n)$ are plotted for quantitative analysis. The units of $|\Xi_x(m,n)|$ are $u^2$ s.

NOTE This transform decreases the specific types of spurious components.

## 5.3 Wavelet transform

### 5.3.1 Definition of continuous wavelet transform

The continuous wavelet transform is defined by Formula (6).

$$O_x\left(k,m\right)=\frac{1}{f_s}2^{m/2}\sum_{n=0}^{r}\tilde{x}\left(n\right)\Psi\left(2^m n-k\right) \tag{6}$$

for

$r \leq 2^{-m}(N-1)$

where

$\Psi(n)$ is the mother wavelet used in the analysis.

The specific mother wavelet shall be stated. The absolute value squared $|O_x(k,m)|^2$ is plotted. The units are $u^2/Hz^2$.

NOTE The wavelet transform is based on performing analysis over scaled frequencies. Therefore, the frequency resolution is coarser at lower frequencies and finer at higher frequencies. The time resolution is less at lower frequencies and greater at higher frequencies.

### 5.3.2 Definition of mother wavelet

The mother wavelet, $\Psi(n)$, determines the appearance of the transform. The choice of the proper mother wavelet is based on the time and frequency characteristics of the signal to be analysed. The definition of the Gaussian enveloped sinusoid is defined by Formula (7).

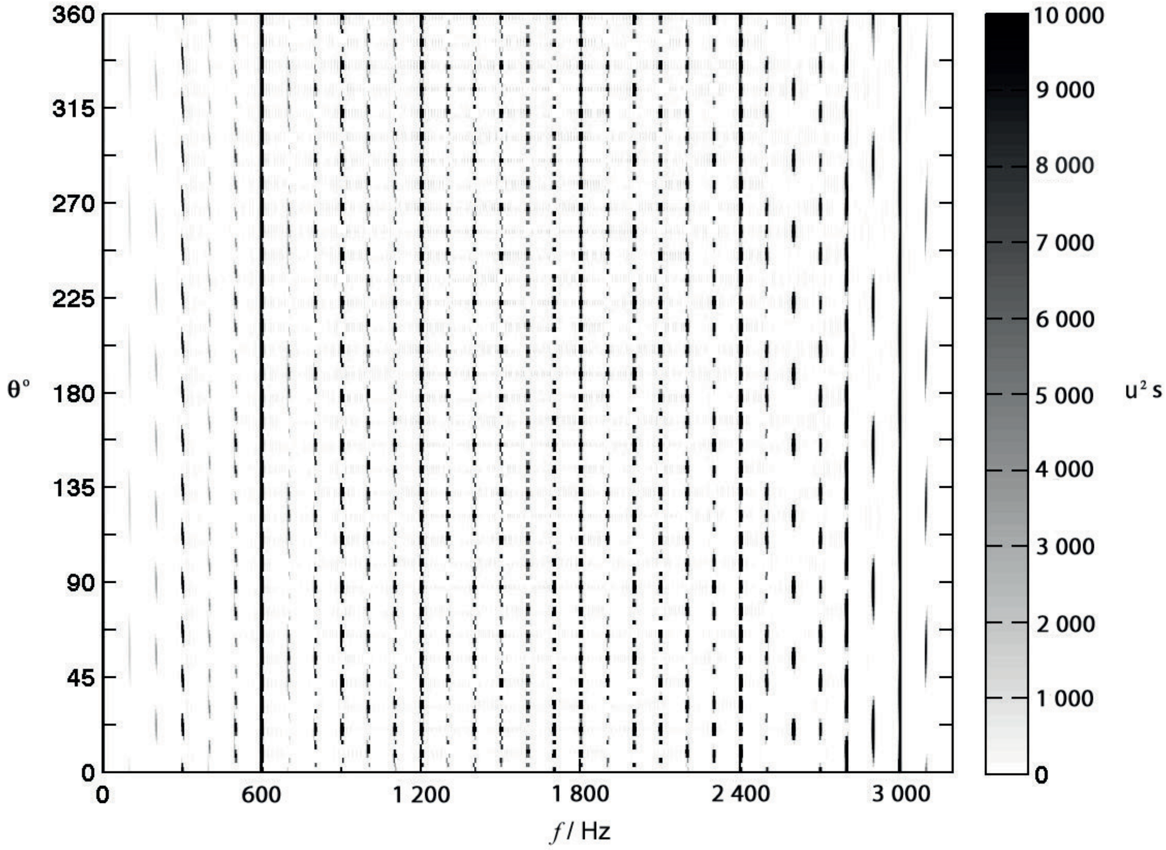
$$\Psi\left(n\right)=c\exp\left(-\sigma^2 n^2\right)\exp\left(-i2\pi\frac{f_0}{f_s}n\right) \tag{7}$$

where the constant $c$ is imaginary so that the real part of $\Psi$ is real with a magnitude that normalizes $\Psi$ over the range of the summation.

# Annex A
(informative)

# Analysis of gear tooth fault using Wigner distribution

For the example of a pinion gear in Reference [6], we consider an input gear with 24 teeth and a pinion gear with 16 teeth. The rotation frequency of the pinion gear is 37,5 Hz, resulting in a mesh frequency of 600 Hz. A gear signal is numerically simulated with 2 048 samples with a sampling frequency of 6 600 Hz. The upper time half of the signal is set to zero to prevent aliasing in the time domain. The analytic signal representation of the signal is used in Formula (2) to prevent aliasing the frequency domain. The case of perfect gear teeth is modeled as a periodic series of unit impulses with period 1/600 s or 11 samples. Therefore, $x(n)$ = [1000000000010...]$^{\tau}$. The absolute value of the Wigner-Ville transform of the perfect case is shown in Figure A.1. The horizontal axis of the plot is frequency and the vertical axis is pinion rotation angle. The harmonics of the mesh frequency are seen at multiples of 600 Hz. The positions of the gear meshes occur at multiples of 22,5°. The greyscale is chosen so that large values are darker and the differences between the perfect and defective pinion gears are apparent.



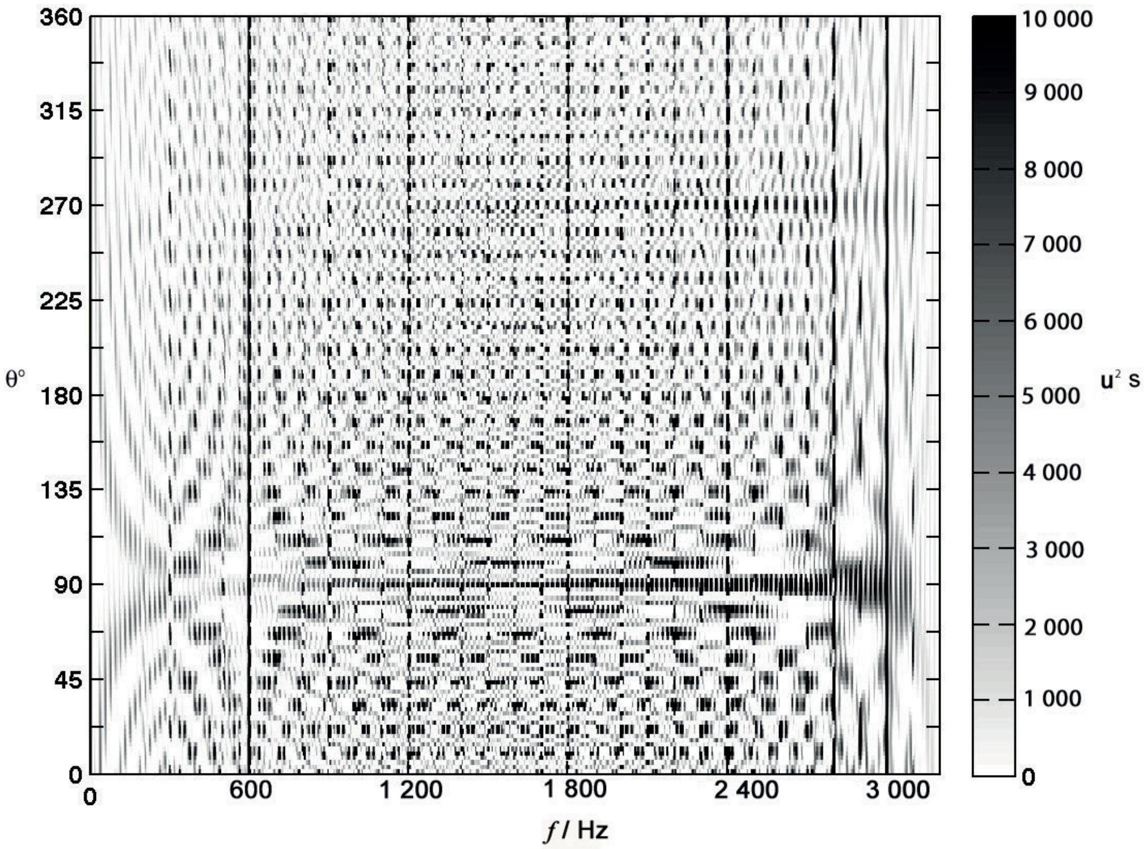

**Key**

θ pinion angle

*f* frequency

**Figure A.1 — The Wigner transform of an impulse sequence simulating the vibration of gears meshing — Perfect pinion**

The fourth tooth of the pinion gear is modeled to be defective with a vibration by replacing the fourth impulse and two adjoining zeros in $x(n)$ = [... 010 ...]$^T$ with three data samples so that $x_{defective}(n)$ = [...0..5 – 2,0 0,5 ...]$^T$. The absolute value of the Wigner-Ville transform is shown in Figure A.2 with the same axes and greyscale. Note that there is a modification in the image at the angular position of the fourth tooth, namely 90°. The detection of the defective tooth can then be measured using image processing techniques.




**Key**

θ pinion angle

*f* frequency

**Figure A.2 — The absolute value of the Wigner transform of a pinion gear with a defective tooth**

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