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Mechanical vibration and shock — Signal processing —

Part 2: Time domain windows for Fourier Transform analysis

Vibrations et chocs mécaniques — Traitement du signal —

Partie 2: Fenêtres des domaines temporels pour analyse par transformation de Fourier

Reference number
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Foreword

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 18431-2 was prepared by Technical Committee ISO/TC 108, *Mechanical vibration and shock*.

ISO 18431 consists of the following parts, under the general title *Mechanical vibration and shock — Signal processing*:

— *Part 1: General introduction*

— *Part 2: Time domain windows for Fourier Transform analysis*

The following parts are under preparation:

— *Part 3: Bilinear methods for joint time-frequency analysis*

— *Part 4: Shock response spectrum analysis*

— *Part 5: Methods for time-scale analysis*

Introduction

Vibration and shock data can consist of displacement, velocity or acceleration measurements, which can be either stationary or non-stationary with respect to time. For both classes of signals, spectral decomposition with Fourier Transformation is one of the analysis tools. In digital signal processing, there are N uniformly spaced (in time) samples of the observed signal. The application of the Discrete Fourier Transform to these N samples produces a series of simple periodic functions of sines and cosines, whose amplitudes and harmonic balance are determined by the time domain window applied to the N samples.

This part of ISO 18431 specifies the three most common windows used.

Mechanical vibration and shock — Signal processing —

Part 2:
Time domain windows for Fourier Transform analysis

1 Scope

This part of ISO 18431 specifies the algebraic functions which describe a selected set of time domain windows used for pre-processing digitally sampled vibration and shock data as a precursor to Discrete Fourier Transform spectral analysis. This selected set consists of Hanning, flat-top and rectangular time windows.

This part of ISO 18431 is one of a series of documents that details the tools available for time domain, frequency domain, and joint time and frequency domain signal processing.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2041:1990, *Vibration and shock — Vocabulary*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2041 and the following apply.

3.1
Discrete Fourier Transform
DFT
discrete transform in time and frequency, based on the Fourier integral transform, used to obtain a spectral estimation of N uniformly time-spaced samples of a signal observed over a finite duration

$$X(m) = \frac{1}{f_s} \sum_{n=0}^{N-1} x(n) e^{-i2\pi nm/N}$$

where the symbols are as defined in Clause 4

3.2
Fast Fourier Transform
FFT
algorithm for computing the Discrete Fourier Transform (DFT) with optimized computational efficiency

NOTE This algorithm is typically either the Cooley-Tukey (see Reference [1]) or Sande-Tukey algorithm.

3.3
time windows
weighting function applied to an ensemble of sampled data to reduce the amount of energy which flows into adjacent frequencies (spectral leakage) caused by sampling a signal that is not periodic within the finite time record of the observation interval, i.e. that has truncated sinusoidal components

4 Symbols

$a(i)$ constants for flat-top window

B_e equivalent noise bandwidth

f_s sampling frequency

i index for flat-top window constants

m frequency sample

n time sample

N block size of sampled data; the number of sampled points that are transformed

$w(n)$ window function in the time domain

$W(m)$ window function in the frequency domain

$x(n)$ sampled physical quantity in the time domain

$X(n)$ Digital Fourier Tranform of $x(n\ \Delta t)$

5 Common time domain windows

5.1 General

There are three common time domain windows in use with Fourier analysis: Hanning, flat top and rectangular.

NOTE The latter is not really an algebraically applied window, but is included in this document for completeness.

Table 1 — Window properties

Window type	**Highest sidelobe** dB	**Sidelobe rolloff** dB/decade	**Noise bandwidth** No. of lines [a]	**Maximum amplitude error** dB
Hanning	–32	–60	1,50	1,4
Flat-top	–93	~0	3,77	<0,01
Rectangular	–13	–20	1,00	3,9

[a] Relative to line spacing.

The noise bandwidth and maximum amplitude error imply that, where knowledge of amplitude is paramount (e.g. during calibration), either the flat top or Hanning window is appropriate and, where frequency resolution is paramount (e.g. for identifying sidebands), either the rectangular or Hanning window is appropriate.

The equivalent noise bandwidth is

$$B_{\mathrm{e}} = \frac{\frac{1}{N}\sum_{n=0}^{N-1} w^2(n)}{\left(\frac{1}{N}\sum_{n=0}^{N-1} w(n)\right)^2} \cdot \frac{f_{\mathrm{s}}}{N} \quad (1)$$

NOTE More information on the use of time domain windows can be found in References [2], [3] and [4].

5.2 Hanning window

For the purposes of this part of ISO 18431, the Hanning window is defined as

$$\omega(\nu) = 1 - \cos(2\pi\nu/N) \quad (2)$$

where

$\nu = 0, 1, \ldots\ldots N - 1$

N is the number of samples in the time record.

Figure 1 shows an example of a 1 024-point Hanning window sampled (f_s) at 1 024 samples per second.

5.3 Flat-top window

For the purposes of this part of ISO 18431, the flat top window is defined as

$$w(n) = 1 + a(1)\cos(2\pi n/N) + a(2)\cos(4\pi n/N) + a(3)\cos(6\pi n/N) + a(4)\cos(8\pi n/N) \quad (3)$$

where

$n = 0, 1, \ldots N - 1$

$a(1) = -1{,}933$

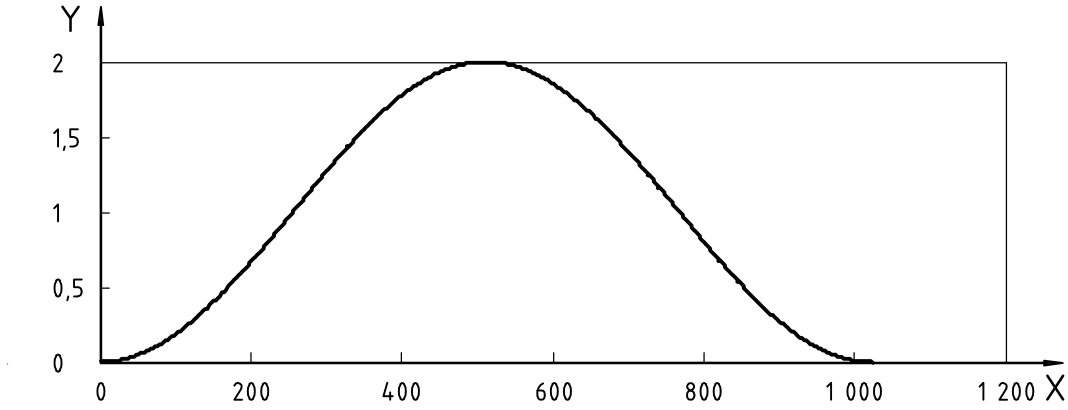
$a(2) = +1{,}286$

$a(3) = -0{,}388$

$a(4) = +0{,}0322$

N is the number of samples in the time record.

Figure 2 shows an example of a 1 024 point flat-top window sampled (f_s) at 1 024 samples per second.

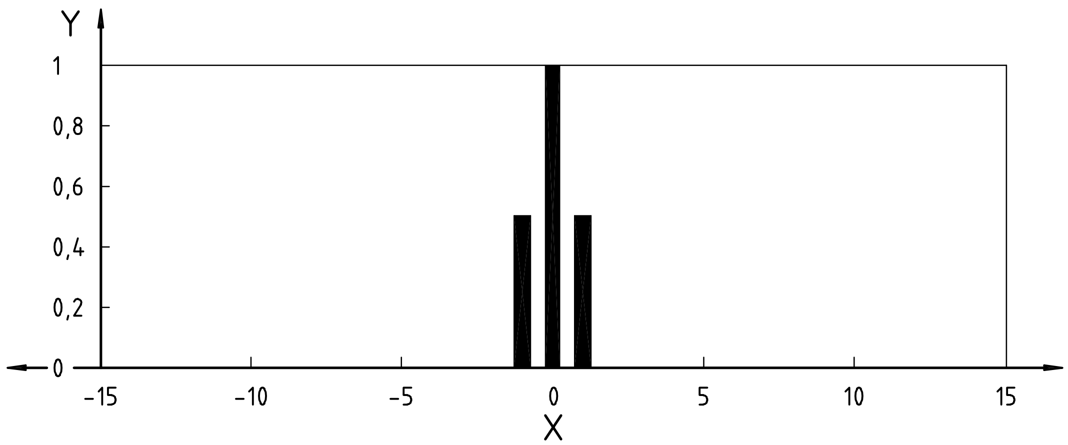


a)

Key

X sample

Y amplitude, $w(n)$



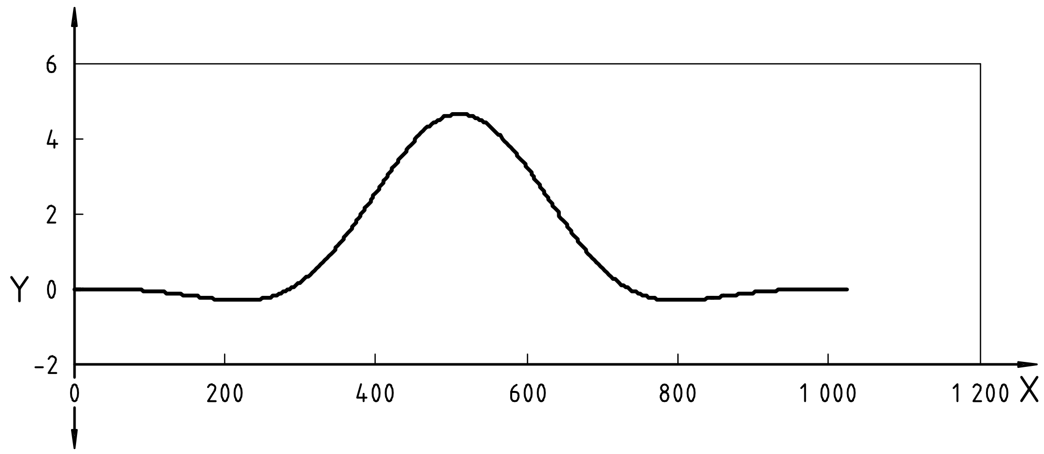
b)

Key

X frequency, in hertz

Y amplitude, $W(m)$

Figure 1 — Hanning window for 1 024 samples

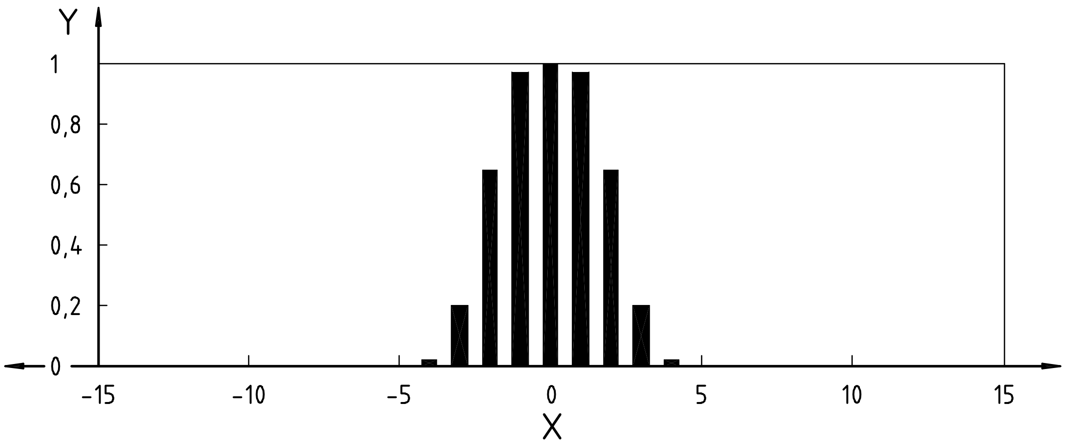


a)

Key

X sample

Y amplitude, $w(n)$



b)

Key

X frequency, in hertz

Y amplitude, $W(m)$

Figure 2 — Flat-top window for 1 024 samples

5.4 Rectangular window

For the purposes of this part of ISO 18431, the rectangular window is defined as

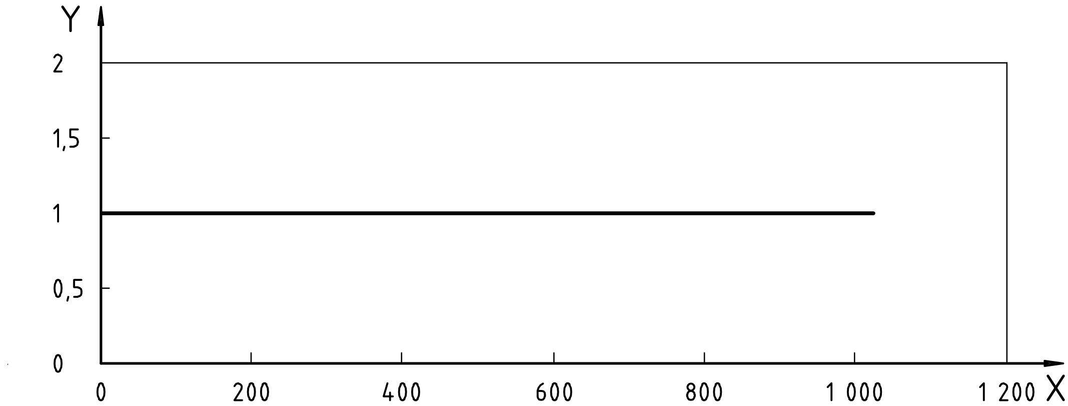
$$w(n) = 1 \quad (4)$$

where

n = 0, 1, ...N – 1

N is the number of samples in the time record.

Figure 3 shows an example of a 1 024 point rectangular window sampled (f_S) at 1 024 samples per second.

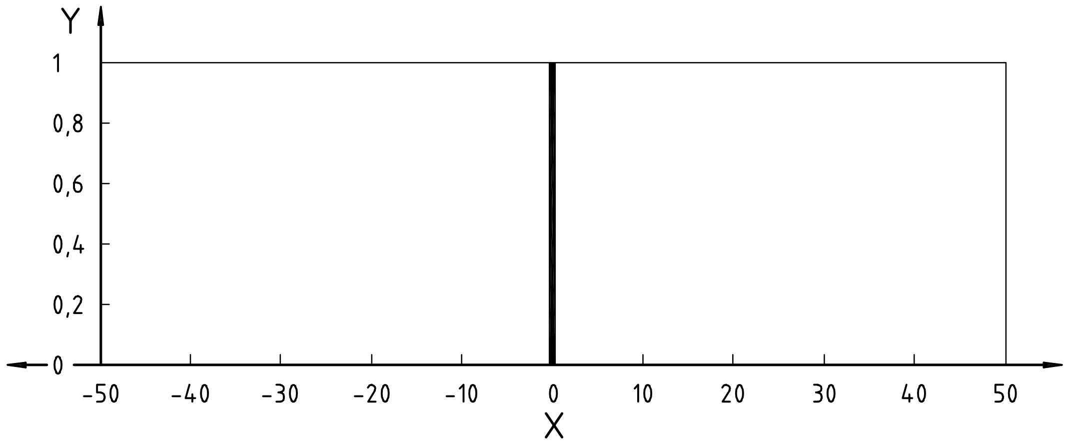


a)

Key

X sample

Y amplitude, $w(n)$



b)

Key

X frequency, in hertz

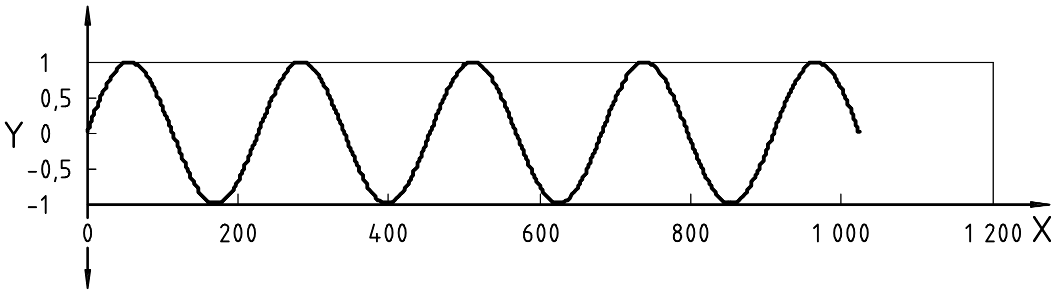
Y amplitude, $W(m)$

Figure 3 — Rectangular window for 1 024 samples

6 Examples

6.1 Common windows applied to a truncated sinusoidal signal

Figure 4 and Table 2 show an example of a sine wave of 4 1/2 cycles sampled at 1 024 samples per second (f_s); the results are independent of a phase shift. It shows the extent of the noise bandwidth and amplitude error.

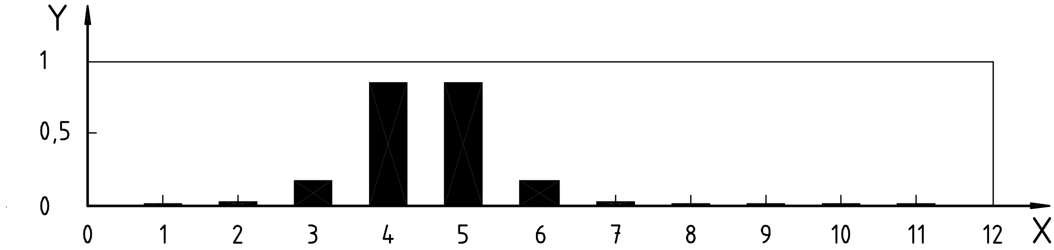


a) Sine wave

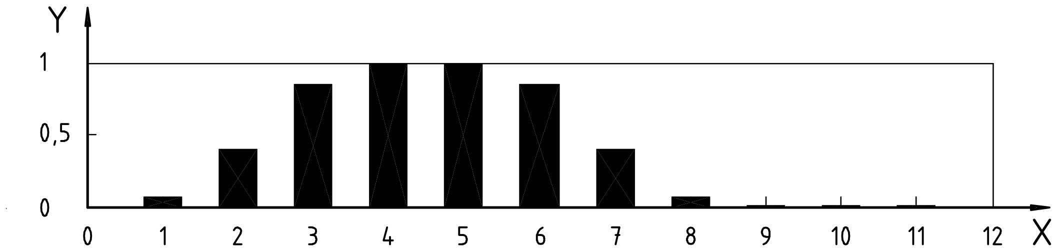
Key

X sample

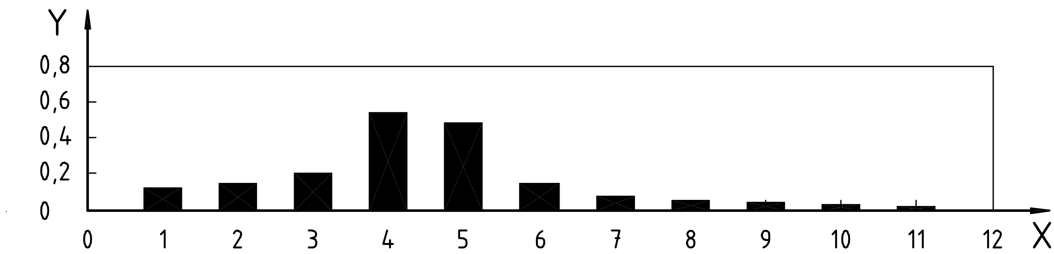
Y signal, $x(n)$



b) Hanning window



c) Flat-top window



d) Rectangular window

Key

X frequency, in hertz

Y $X(m)$

Figure 4 — Example of the common windows applied to a truncated sinusoid of 4 1/2 cycles

Table 2 — Common windows applied to a truncated sinusoidal signal

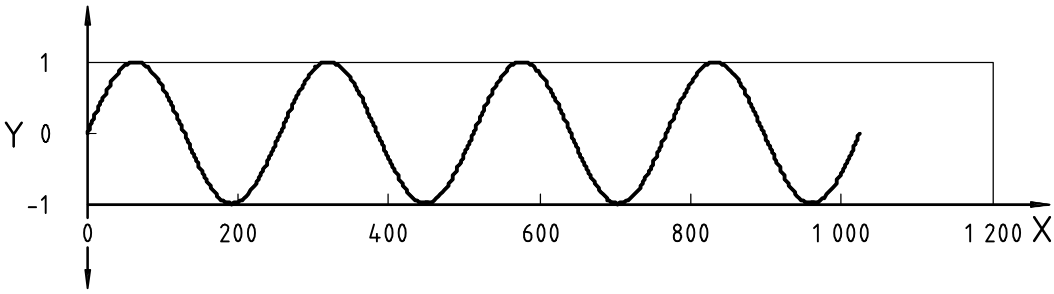
Frequency	Hanning	Flat-top	Rectangular
0	0,0073	0,0033	0,1415
1	0,0101	0,0694	0,1488
2	0,0254	0,3988	0,1763
3	0,1705	0,8507	0,2546
4	0,8483	0,9989	0,6741
5	0,8492	0,9990	0,6031
6	0,1695	0,8506	0,1819
7	0,0240	0,3989	0,0997
8	0,0079	0,0693	0,0655
9	0,0035	0,0017	0,0472
10	0,0019	0,0000	0,0359

6.2 Common windows applied to a non-truncated sinusoidal signal

Figure 5 and Table 3 show an example of a sine wave of 4 cycles sampled at 1 024 samples per second (f_S); the results are independent of a phase shift. It shows the extent of the noise bandwidth with zero amplitude error.

Table 3 — Common windows applied to a non-truncated sinusoidal signal

Frequency	Hanning	Flat-top	Rectangular
0	0,0000	0,0000	0,0000
1	0,0000	0,1940	0,0000
2	0,0000	0,6430	0,0000
3	0,5000	0,9665	0,0000
4	1,0000	1,0000	1,0000
5	0,5000	0,9665	0,0000
6	0,0000	0,6430	0,0000
7	0,0000	0,1940	0,0000
8	0,0000	0,0160	0,0000
9	0,0000	0,0000	0,0000
10	0,0000	0,0000	0,0000

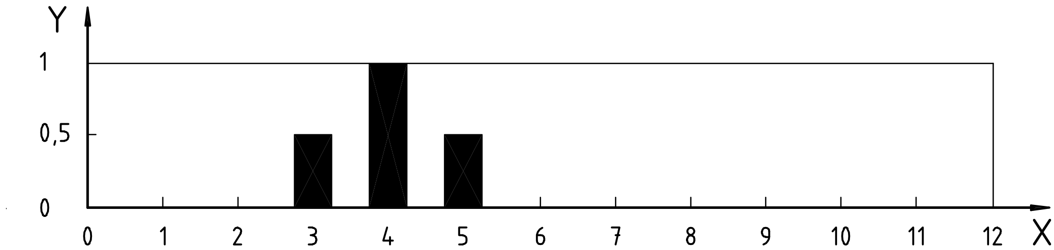


a) Sine wave

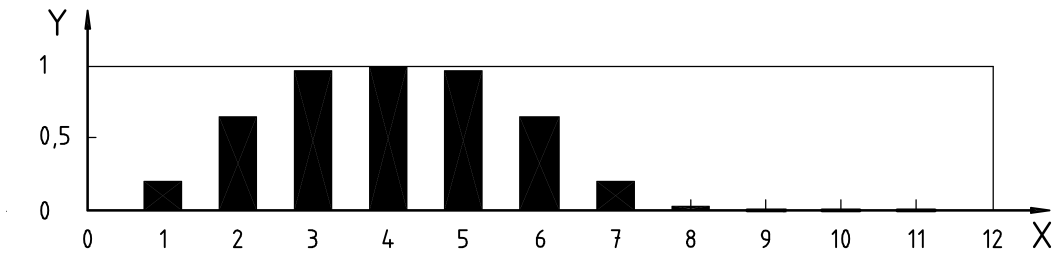
Key

X sample

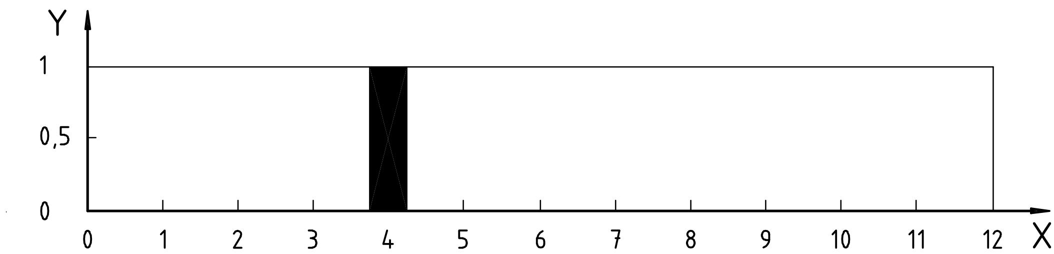
Y signal, $x(n)$



b) Hanning window



c) Flat-top window



d) Rectangular window

Key

X frequency, in hertz

Y $X(m)$

Figure 5 — Example of the common windows applied to a non-truncated sinusoid of 4 cycles

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