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Mechanical vibration and shock — Signal processing

Part 1: General introduction

Vibrations et chocs mécaniques — Traitement du signal

Partie 1: Introduction générale

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Foreword

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ISO 18431-1 was prepared by Technical Committee ISO/TC 108, *Mechanical vibration and shock*.

ISO 18431 consists of the following parts, under the general title *Mechanical vibration and shock — Signal processing*:

— *Part 1: General introduction*

— *Part 2: Time domain windows for Fourier Transform analysis*

— *Part 4: Shock response spectrum analysis*

The following parts are under preparation:

— *Part 3: Bilinear methods for joint time-frequency analysis*

— *Part 5: Methods for time-scale analysis*

Introduction

In the recent past, nearly all data analysis has been accomplished through mathematical operations on digitized data. This state of affairs has been accomplished through the widespread use of digital signal acquisition systems and computerized data-processing equipment. The analysis of data is therefore primarily a digital signal-processing task.

The analysis of experimental vibration and shock data should be thought of as a part of the process of experimental mechanics that includes all steps from experimental design through data evaluation and understanding.

This part of ISO 18431 assumes that the data have been sufficiently reduced so that the effects of instrument sensitivity have been included. The data considered in this part of ISO 18431 are considered to be a sequence of time samples of a physical quantity, such as a component of velocity, acceleration, displacement or force. Experimental methods for obtaining these data are outside the scope of this part of ISO 18431.

Mechanical vibration and shock — Signal processing

Part 1:
General introduction

1 Scope

This part of ISO 18431 defines the mathematical transformations, including the physical units, that convert each category of vibration and shock data into a form that is suitable for quantitative comparison between experiments and for quantitative specifications. It is applicable to the analysis of vibration that is deterministic or random, and transient or continuous signals. The categories of signals are defined in Clause 6.

Extreme care is to be exercised to identify correctly the type of signal being analysed in order to use the correct transformation and units, especially with the frequency domain analysis.

The data may be obtained experimentally from measurements of a mechanical structure or obtained from numerical simulation of a mechanical structure. This category of data is very broad because there is a wide variety of mechanical structures, for example, microscopic instruments, musical instruments, automobiles, manufacturing machines, buildings and civil structures. The data can determine the response of machines or of humans to mechanical vibration and shock.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2041:1990, *Vibration and shock — Vocabulary*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2041 and the following apply.

3.1
aliasing
false representation of spectral energy caused by mixing of spectral components above the Nyquist frequency with those spectral components below the Nyquist frequency

3.2
confidence interval
range within which the true value of a statistical quantity will lie, given a value of the probability

3.3
data
sampled measurements of a physical quantity

3.4
statistical degrees of freedom
number of independent variables in a statistical estimate of a probability

3.5
frequency resolution
difference between two adjacent spectral lines

3.6
number of lines
number of spectral lines that are displayed

3.7
Nyquist frequency
maximum usable frequency available in data taken at a given sampling rate

$$f_{\mathrm{N}} = f_{\mathrm{s}}/2$$

where

f_{N} is the Nyquist frequency;

f_{s} is the sampling frequency

3.8
record length
number of data points comprising a contiguous set of sampled data points

3.9
sampling
measurement of a varying physical quantity at a sequence of values of time, angle, revolutions or other mechanical, independent variable

3.10
sampling frequency
number of samples per unit of time for uniformly sampled data

3.11
sampling interval
number of units (e.g. time, angle, revolutions) between two successive samples

3.12
sampling period
duration of time between two successive samples

3.13
sampling rate
number of samples per unit of time, angle, revolutions or other mechanical, independent variable for uniformly sampled data

3.14
side-lobes
sequence of peaks in the frequency domain caused by the use of a finite time window with the Fourier Transform

3.15
signal bandwidth
interval over frequency between the upper and lower frequencies of interest

3.16
spectral leakage
width of the peak in the power spectrum due to a single spectral component caused by using a finite window with the Fourier Transform

4 Symbols and abbreviated terms

ADC	analog-to-digital converter
B	signal bandwidth
B_e	equivalent noise bandwidth
C_a	amplitude scaling factor
DFT	Discrete Fourier Transform
$E\{\ \}$	expectation operator that computes the statistical mean value or average value
$F(n)$	time-dependent force
$H_1(m)$	frequency response function of the first type
$H_2(m)$	frequency response function of the second type
K	summation limit of time delay k or length of window $w(k)$
I	number of data blocks
L	record length
L_i	level in units U_x for amplitude histogram of signal $x(n)$
N	data block length: the number of sampled points that are transformed
$O_x(k,m)$	wavelet transform of $x(n)$
$P_{xx}(m)$	power spectral density of signal $x(n)$
$P_{xx,\mathrm{low}}(m)$	low frequency part of the power spectral density of signal $x(n)$
$P_{x2,\mathrm{low}}(m)$	low frequency part of the power spectral density of signal $x^2(n)$
$P_{xy}(m)$	cross power spectral density of signal $x(n)$ with $y(n)$
Q	quality factor of a single degree-of-freedom system
$R_{xx}(m)$	r.m.s. spectrum of signal $x(n)$
$S_x(m,n)$	short-time Fourier Transform of $x(n)$
T	total time of a block of digital data = $N\Delta t$
$V(k,m)$	Cohen class filter for smoothing the Wigner distribution
$X(m)$	Discrete Fourier transform of $x(n)$
$Y(m)$	Discrete Fourier transform of $y(n)$
b	number of increments, also known as bits, in an ADC
$c_{xx}(k,n)$	auto-covariance of $x(n)$
$c_{xy}(k,n)$	cross-covariance of $x(n)$ with $y(n)$
$e_{xx}(m)$	energy spectral density of signal $x(n)$
$e_{xy}(m)$	cross energy spectral density of signals $x(n)$ and $y(n)$
f	frequency = $m\Delta f$
f_N	Nyquist frequency, the highest frequency present in a sampled signal
f_n	natural frequency of a single degree-of-freedom system
f_s	sampling frequency = $1/\Delta t$
i	index of data block
k	index of time shift
l	index of summation
m	index of frequency or scale
$\overline{x(n)}$	mean of non-stationary signal $x(n)$

$\bar{x}$	mean of stationary signal $x(n)$
n	index of time
p	lower limit of summation
q	upper limit of summation
r	upper limit of summation
$r_{xx}(k,n)$	auto-correlation of non-stationary data $x(n)$
$r_{xx}(k)$	auto-correlation of stationary data $x(n)$
$r_{xy}(k,n)$	cross-correlation of non-stationary data $x(n)$ with $y(n)$
$r_{xy}(k)$	cross-correlation of stationary data $x(n)$ with $y(n)$
t	time = $n\Delta t$
$v_x(n)$	variance of the non-stationary data $x(n)$
v_x	variance of the data $x(n)$
$w(n)$	window function
$x(n)$	physical data in the time domain
$y(n)$	physical data in the time domain
Δt	sample period
Δf	frequency resolution
ε_r	relative random error
$\gamma_{xy}{}^2(m)$	coherence function
$\nu(n)$	noise component of measured signal
$\psi(n)$	mother wavelet
$\sigma_x{}^2$	statistical variance of x
$\Xi_x(m,n)$	Cohen class Wigner distribution using Cohen class filter $V(n,m)$
$\Omega_x(m,n)$	Wigner distribution of signal $x(n)$

5 Signal conditioning

5.1 Cautionary overview

The electrical signal from a transducer shall be properly conditioned for digitization by an analog-to-digital converter (ADC). This signal conditioning requires the determination of several parameters associated with amplification, filtering and digitization. The selection of these parameters is very important for the acquisition of data that is appropriate for signal processing.

5.2 Filtering

Before the signal can be successfully digitized by the ADC, the signal shall be low-pass filtered to prevent aliasing. Aliasing occurs when there are components of the signal at a frequency that is too high. The highest frequency in the signal is limited by the sampling frequency, f_S, of the ADC. The range of settings of f_S are found in the specifications of the ADC. The highest frequency component of the signal may be no greater than $f_\mathrm{N} = f_\mathrm{S}/2$, which is known as the Nyquist frequency. The upper frequency of the low-pass filter depends on the roll-off characteristics of the filter and the spectral properties of the signal. If the phase of the data is important, attention shall also be paid to the phase characteristics of the filter.

The following test should be performed to check the adequacy of the low-pass filter. A signal should be digitized and recorded. Then a Fourier transform should be performed on the data. The amplitude of the Fourier-transformed signal at the Nyquist frequency should be less than or equal to the expected noise level of the Fourier-transformed signal at the frequency of interest. If this is not the case, then the sampling rate should be increased or the upper frequency of the low-pass filter should be lowered.

In addition to the low-pass filter, a high-pass filter may also be required because a non-negligible d.c. component of the signal may reduce the useful range of the ADC. Reducing or eliminating this offset prior to digitizing is preferable unless the d.c. component or low-frequency components are important.

The external analog anti-aliasing filtering considerations for a sigma delta ADC are different. The analog signal shall meet the Nyquist criterion for the high frequency 1-bit digitizer, not the frequency for the end result. With sigma delta digitizers, the manufacturer usually includes the low-pass filter needed for the analog input and an internal digital low-pass filter to match the output sample rate.

5.3 Sampling

The ADC converts an analog signal into a sequence of integers. The output integers are proportional to the input over a range of voltage. This range of voltage is given in the specifications of the ADC and determines the proper gain setting discussed in 5.1.

NOTE The number of increments b in the largest output number determines the dynamic range of the ADC, which is specified in terms of decibels, $6b + 1{,}8$ dB.

The sequence of numbers is sampled at a rate called the sampling frequency, f_s, discussed in 5.1. A signal may be resampled to order track the signal into samples that are equal increments of units other than time, for example angular displacement or degrees.

Another parameter to be selected is the number of samples, the record length. The record length shall be large enough to capture the whole signal if the signal is transient or limited in time.

The sampling frequency, f_s, fixes the following parameters:

— the maximum (Nyquist) frequency $f_N = f_s/2$

— the sampling interval $\Delta t = 1/f_s$

6 Determination of signal type

6.1 Signal taxonomy

The signals that make up the data are considered to be approximate members of idealized categories. In this part of ISO 18431, the signals are categorized by the taxonomy shown in Table 1.The category of signal often determines the methods of analysis. If an inappropriate analysis is used, then the results may be misleading or inconclusive. Usually data contain a mixture of two types of signals. For example, a signal may be the sum of a deterministic, non-periodic transient signal and a random, stationary, continuous signal. The triggering, filtering and processing to determine the characteristics of the two signals are very different. The type of signal to be described determines the signal conditioning digitization and data analysis. For example, specifying a mechanical environment for equipment requires random, non-stationary transients to be sufficiently described so that the mechanical conditions can be experimentally modelled with deterministic, non-periodic transients.

The signal taxonomy shown in Table 1 implies the decision tree required to determine the nature of the signal of interest and also the subsequent data analysis.

Table 1 — Signal taxonomy

Decision tree	Deterministic				Random			
	Periodic		Non-periodic		Stationary		Non-stationary	
	Sinusoidal	Harmonic	Non-harmonic sinusoidal	Transient	Ergodic	Non-ergodic	Transient	Continuous
Example signals	Vibration from imbalanced rotor	Vibration from gears	Vibration from rolling bearings	Vibration from modal impact	Fluid dynamic noise	Vibration from jet engine with several operating states	Vehicle vibration from pothole	Vibration during rocket take-off
Appropriate spectral transform	r.m.s.	r.m.s.	r.m.s.	Energy	Power	Power	Energy	Power

6.2 Deterministic signals

6.2.1 Description of deterministic signals

Deterministic signals do not contain components that average to zero when subsequently captured signals are averaged together.

NOTE A deterministic signal can reoccur over time or it can occur only once. Deterministic signals are caused by an event. If the event recurs over time, a deterministic signal has the same behaviour each time. The event can be accessible for use or not. The event can be periodic, continuous or impulsive in time.

6.2.2 Periodic signals

6.2.2.1 Description of periodic signals

These signals are generated from a periodic excitation and appear steady over subsequent signal captures. Also they do not decay significantly with time over the duration of the signal capture.

6.2.2.2 Sinusoidal signals

Sinusoidal signals consist of single frequency components. This single frequency component is described by its amplitude and phase relative to a reference.

NOTE Sinusoidal signals are usually caused by a sinusoidal event in a linear system.

6.2.2.3 Harmonic signals

Complicated periodic signals contain harmonically related components, the phase of which is related to a reference excitation signal. The Fourier series may be applied to this case with the lowest frequency being the inverse of the period of the signal.

A time history that is periodic may be represented by a superposition of sinusoids whose frequencies are integral multiples of the fundamental frequency. The Fourier series includes a constant, which represents the mean value and the sinusoidal terms and frequencies f, $2f$, …, nf, where f is the fundamental frequency and n is an integer limited in magnitude by the frequency range of interest or the nature of the physical phenomenon being analysed. The average values and the amplitudes of the sinusoidal components are obtained from the integrals defined in ISO 2041:1990, A.18, under the definition of Fourier coefficients.

6.2.3 Non-periodic signals

6.2.3.1 Description of non-periodic signals

These signals are of two types. One type contains frequency components that are not harmonically related. The other type decays over time.

6.2.3.2 Non-harmonic sinusoidal signals

These signals contain a non-harmonic sequence of spectral components that are related by the internal components of the machine, e.g. gear ratios that have rational but not integer values.

These signals are composed of sinusoids whose frequencies are not all integral multiples of a common fundamental frequency. Such a vibration can be viewed as a superposition of a number of periodic vibrations where each periodic vibration includes those of the sinusoids that are harmonically related. Fourier series cannot be used since periodicity is not present. The discrete frequency components of either periodic or multi-sinusoidal vibration data may be isolated individually by narrowband pass filtering provided the bandwidth of the filter is smaller than the difference of adjacent discrete frequencies (see 5.2 and 5.3).

6.2.3.3 Transient signals

Transient signals decay rapidly and therefore contain a continuous distribution of spectral components. The power spectral density is an inappropriate description of these signals.

6.3 Random signals

6.3.1 Description of random signals

Random signals can originate with or without any reference excitation signal. They differ from deterministic signals by varying either over time or over subsequent measurements.

For the purpose of analysis, a set of statistically independent blocks of data is required.

6.3.2 Stationary signals

6.3.2.1 Description of stationary signals

A stationary signal possesses statistical characteristics that do not change over time. Thus they do not increase or decrease over time.

6.3.2.2 Ergodic signals

An ergodic signal has statistical properties that permit averages over time to replace averages over ensemble.

6.3.2.3 Non-ergodic signals

A non-ergodic signal has statistical properties that must be generated by the performance of several experiments.

EXAMPLE There may be statistical variations between machines that cause the signal of interest to be characterized through an average of measurements taken with a sequence of machines. Averages over time of a signal taken from a single machine do not produce a meaningful statistical average over a set of machines.

6.3.3 Non-stationary signals

6.3.3.1 Description of non-stationary signals

A non-stationary signal has time-dependent statistical properties.

6.3.3.2 Continuous signals

A continuous, non-stationary random signal must be described by statistical characteristics that are functions of time. Because the signal is continuous, the signal must be decomposed into the power spectrum.

6.3.3.3 Transient signals

A transient, non-stationary random signal must be described by statistical characteristics that are functions of time. This time-limited signal should be decomposed into an energy spectrum.

This may be characterized by a Fourier spectrum, which is a continuous function of frequency, namely all frequencies are present and the spectrum defines the distribution of magnitude and phase with frequency. It is analogous to the Fourier series used to describe periodic data with the assumption that the periodicity is arbitrarily long. Thus, the frequency terms are spaced arbitrarily closely.

7 Analysis of signals

7.1 Preprocessing of signals

7.1.1 Purpose of preprocessing

The signal is considered to be a set of samples $x(n)$, $0 < n < (N - 1)$ that represent measurements of a physical quantity of a mechanical system at time samples $n\Delta t$. The random signal $x_i(n)$, $0 < n < (N - 1)$, $0 < i < (I - 1)$, consists of I statistically independent blocks of data. The recorded signal may be analysed in the time domain as a function of the time sample n or as a function of the frequency sample m, or as a function of both time and frequency. The purpose of signal analysis, for random or deterministic signals, is to determine underlying behaviour or causes of the mechanical process being measured.

Before processing the signal to generate time domain products, the digitized signal is often preprocessed with linear filters to remove unwanted narrow band, or out-of-band, noise. This preprocessing may be performed using the Fourier Transform.

7.1.2 Fourier Transform

The sequence of data samples with physical units U in the time domain, $x(n)$, is transformed into the frequency domain by the discrete Fourier Transform equation defined in this document as:

$$X(m) = \frac{1}{f_\mathrm{s}} \sum_{n=0}^{N-1} x(n) \mathrm{e}^{-i\frac{2\pi}{N}nm} \qquad (1)$$

This equation approximates the definition of the continuous Fourier Transform in ISO 2041. Note that the units of X are U/Hz or U s for transient signals.

The inverse Discrete Fourier Transform also approximates the continuous inverse Fourier Transform and is defined in this part of ISO 18431 as

$$x(n) = \frac{f_\mathrm{s}}{N} \sum_{m=0}^{N-1} X(m) \mathrm{e}^{i\frac{2\pi}{N}nm} \qquad (2)$$

Often the data are conditioned with a window function that decreases the effects of discontinuities at $n = 0$ and $N - 1$. These window functions are defined in the time domain as $w(n)$ and are used in the forward transform as

$$Y(m) = \frac{1}{f_s} \sum_{n=0}^{N-1} w(n) x(n) e^{-i\frac{2\pi}{N}nm} \qquad (3)$$

NOTE The above definitions of the Fourier Transform differ from some mathematical descriptions that fail to incorporate the units of the quantities being transformed.

After the blocksize N and the sampling frequency f_s are chosen, the following relevant parameters are fixed:

— total time $T = N/f_s$

— frequency resolution $\Delta f = f_s/N$

If the frequency resolution Δf is required to be less than some value, then the N must be increased or f_s decreased to achieve the necessary resolution.

7.1.3 Signal filtering

A time domain signal is filtered through the use of a low-pass, high-pass, band-pass or band-reject filter.

7.2 Time domain analysis

7.2.1 Appropriate data for analysis

Time domain analysis may be performed on a vibration time-history as recorded or after being filtered by an appropriate bandwidth filter to improve the signal-to-noise ratio.

7.2.2 Magnitude analysis

The objective of this analysis is to determine the variation of the vibration magnitude as a function of time. The linear signal of a deterministic non-periodic transient signal can be plotted in terms of physical units U as a function of time. Alternatively the magnitude of the signal $|x(n)|$ or the squared magnitude of the signal $|x(n)|^2$ may be plotted in terms of physical units U or U^2 respectively as a function of time.

7.2.3 Statistical analysis

7.2.3.1 Purpose of statistical analysis

Statistical analysis of random data is carried out in order to determine the statistics of various functions of the signal as a function of time.

7.2.3.2 Distribution function analysis

The objective of instantaneous value analysis is to determine the cumulative distribution function or the probability density function of the instantaneous values of the vibration data. The cumulative distribution function at any level L_i is obtained by measuring the percentage of time samples that does not exceed that level. The probability density function between the two levels L_i and L_{i+1} is obtained by measuring the percentage of time that the instantaneous value lies between L_i and L_{i+1}.

7.2.3.3 Moment analysis

A random, time domain signal may be described in terms of its moments. The first statistical moment is commonly called the mean value. The definition of the mean value is

$$\overline{x(n)} = E\{x(n)\} \tag{4}$$

where the expectation is performed as an average over S statistically independent blocks of data.

If the signal is stationary, then the mean does not vary as a function of time. If the signal is stationary and ergodic then the expectation operator E is an average over $N \geqslant S$ time samples

$$\bar{x} = \frac{1}{N}\sum_{n=0}^{N-1} x(n) \tag{5}$$

If a non-stationary signal is considered to be wide-sense stationary over a time interval of K samples and ergodic, then the expectation operator E is an average over $K \geqslant S$ samples

$$\overline{x(n)} = \frac{1}{K}\sum_{k=0}^{K-1} x(n-k) \tag{6}$$

If the number of samples over which the signal is considered to be wide-sense stationary and K is insufficiently large, the average shall be taken over a number of statistically independent measurements I, $IK \geqslant S$ samples

$$\overline{x(n)} = \frac{1}{I}\sum_{i=0}^{I-1} \frac{1}{K}\sum_{k=0}^{K-1} x_i(n-k) \tag{7}$$

The second moment about the mean of a random signal is commonly called the variance and is defined as

$$v_x(n) = E\left\{\left|x(n) - \overline{x(n)}\right|^2\right\} \tag{8}$$

If the signal is stationary, then the mean and variance does not vary as a function of time. If the signal is stationary and ergodic, then the expectation operator E is an average over $N \geqslant S$ time samples

$$v_x(n) = \frac{1}{N}\sum_{n=0}^{N-1}\left|x(n) - \overline{x(n)}\right|^2 \tag{9}$$

If a non-stationary signal is considered to be wide-sense stationary and ergodic over an interval of K samples, then the expectation operator E is an average over $K \geqslant S$ samples

$$v_x(n) = \frac{1}{K}\sum_{k=0}^{K-1}\left|x(n-k) - \overline{x(n-k)}\right|^2 \tag{10}$$

If the number of samples over which the signal is considered to be wide-sense stationary and K is insufficiently large, the average shall be taken over a number I of statistically independent blocks of data, $IK \geqslant S$ samples

$$\overline{x(n)} = \frac{1}{I}\sum_{i=0}^{I-1} \frac{1}{K}\sum_{k=0}^{K-1}\left|x_i(n-k) - \overline{x(n-k)}\right|^2 \tag{11}$$

7.2.4 Covariance analysis

The similarity between time-delayed subcomponents of a signal are shown in the two-time covariance function. For a deterministic non-periodic transient signal, the auto-covariance is

$$c_{xx}(k,n) = x(n)x(k) \tag{12}$$

For two deterministic non-periodic transient signals, the cross-covariance is

$$c_{xy}(k,n) = x(n)y(k) \tag{13}$$

For a random non-stationary transient signal, the auto-covariance is

$$c_{xx}(k,n) = E\left\{\left(x(n) - \overline{x(n)}\right)\left(x(k) - \overline{x(k)}\right)\right\} \tag{14}$$

For two random non-stationary transient signals, the cross-covariance is

$$c_{xy}(k,n) = E\left\{\left(x(n) - \overline{x(n)}\right)\left(y(k) - \overline{y(k)}\right)\right\} \tag{15}$$

7.2.5 Correlation analysis

7.2.5.1 Purpose of correlation analysis

Correlation analysis is a means of determining certain characteristics of a single stationary vibration time-history, or the relation between multiple different stationary time-histories, by comparing the vibration value at one time with the vibration value at another time. If the signal is random and stationary, then the correlation depends only on the time differences. If the signal is random and non-stationary, then the correlation depends on each time.

7.2.5.2 Auto-correlation analysis

Auto-correlation analysis is employed to examine the magnitude and time delay characteristics of record of data and thus serves as a substitute for or supplement to spectral analysis and statistical analysis. The units of the autocorrelation are U^2.

For non-stationary signals, the correlation function is the average of the product of the signal at time n and the delay k

$$r_{xx}(k,n) = E\left\{x(n)x(n-k)\right\} \tag{16}$$

where the average is taken over independent data blocks. The number of data blocks in the average must be sufficiently large.

If the signal is stationary, the correlation function is a function of the time delay k

$$r_{xx}(k) = E\left\{x(n)x(n-k)\right\} \tag{17}$$

If the signal is non-ergodic, the average is taken over independent data blocks.

If the signal is ergodic, the correlation function is found by averages over the data record:

$$r_{xx}(k) = \frac{1}{(q-p+1)} \sum_{n=p}^{q} x(n)x(n-k)$$
$$\text{for } -L+1 \leqslant k \leqslant L-1\text{: } p = \max(0,k),\ q = \min(L-1,\ L-1+k) \tag{18}$$

The record length L must be sufficiently long so that the average over ($p - q$ + 1) is statistically sufficient.

7.2.5.3 Cross-correlation analysis

Cross-correlation analysis is employed to determine shared magnitude and time delay characteristics for two vibrational time-histories representing distinct measurements. The units of the autocorrelation are $U_x U_y$.

For non-stationary signals, the correlation function is the average of the product of the signal at time n and delay k

$$r_{xy}(k,n) = E\{x(n)y(n-k)\} \tag{19}$$

where the average is taken over independent data blocks. The number of data blocks in the average must be sufficiently large.

If the signals are stationary, the correlation function is a function of the time delay k:

$$r_{xy}(k) = E\{x(n)y(n-k)\} \tag{20}$$

If the signal is non-ergodic, the average is taken over independent data blocks.

If the signal is ergodic, the correlation function is found by averages over in the data record:

$$r_{xy}(k) = \frac{1}{(q-p+1)} \sum_{n=p}^{q} x(n)y(n-k)$$
$$\text{for } -L+1 \leqslant k \leqslant L-1\text{: } p = \max(0,k),\ q = \min(L-1,\ L-1+k) \tag{21}$$

The record length L must be sufficiently long so that the average over ($p - q$ + 1) is statistically sufficient.

7.2.6 Convolution product

The convolution product is the sum of the product of two time histories at a sequence of time lags. The convolution is performed on transient signals. The linear convolution of $x_1(t)$ with $x_2(t)$ is defined as

$$y(n) = \frac{1}{f_s} \sum_{n=p}^{q} x_1(n-k)x_2(k)$$
$$\text{for } -N+1 \leqslant n \leqslant N-1\text{: } p = \max(-N+1+n,0),\ q = \min(N-1,n) \tag{22}$$

The units of y are $U_{x1}U_{x2}$ s. The circular convolution is defined in terms of the Fourier transform

$$y(n) = \frac{1}{Nf_s} \sum_{m=0}^{N-1} \left(\sum_{l=0}^{N-1} x_1(l) e^{-i\frac{2\pi}{N}lm} \right) \left(\sum_{l=0}^{N-1} x_2(l) e^{-i\frac{2\pi}{N}lm} \right) e^{i\frac{2\pi}{N}mn} \tag{23}$$

The duration of the transient signal should not exceed N/2 time samples to avoid wrap-around errors. This is usually accomplished by increasing the length of the data by including a sequence of zeros at the end of the record.

7.2.7 Shock response spectrum

A transient vibration or shock may be defined in terms of the response of an idealized mechanical system that is excited by the vibration. Conceptually, an array of single-degree-of-freedom systems is subjected to the vibration or shock to be analysed. The single-degree-of-freedom systems have different natural frequencies (f_n) but the same Q factor. The time-history may result from applying base acceleration or as a force $F(t)$ directly to the masses. Each simple system responds in a manner dependent solely on its natural frequency and Q factor. In its simplest and most common form, the shock response spectrum is a plot of the maximum response experienced by each simple system in response to the applied time-history as a function of the natural frequency of the systems. In principle, the maximum response is described by a number of interconnected points, obtained by analysing a sufficient number of simple systems to define adequately the variation of the spectrum with natural frequency. The maximum response is usually expressed in terms of absolute acceleration, but relative displacement, pseudo-velocity and equivalent static acceleration are also quite common.

7.2.8 Impulse response function

The impulse response function is computed to be the inverse Fourier Transform of the frequency response function. The inverse Fourier Transform must be limited to the interval where the signal-to-noise ratio is sufficiently large.

7.3 Frequency domain analysis of signals

7.3.1 Objective of frequency-domain analysis

The objective of spectral analysis of vibration is to determine the characteristics of the vibration as a function of frequency. Appropriate methods of spectral analysis for vibration are described in this subclause.

7.3.2 Windowing

If the frequency components of a periodic signal coincide with the spectral lines $m\Delta f$ (an integral number of periods in the total time), the Fourier transform can be simply applied. If the total time does not comprise an integral number of periods, leakage lobes in the frequency domain will occur. Suppression of the lobes can be obtained by use of a suitable window function $w(n)$ in the time domain.

The application of a smoothing window function in the time domain will moderate the magnitude of the side-lobes of the spectral peaks. However the application of a smoothing window also introduces spreading of each spectral peak into adjacent frequency samples. Thus a correction for the levels of the spectral peaks must be introduced.

The amplitude scaling constant is

$$C_a = \frac{1}{N}\sum_{n=0}^{N-1} w(n) \qquad (24)$$

and the equivalent noise bandwidth

$$B_e = \frac{\frac{1}{N}\sum_{n=0}^{N-1} w^2(n)}{\left(\frac{1}{N}\sum_{n=0}^{N-1} w(n)\right)^2} \cdot \frac{f_s}{N} \qquad (25)$$

7.3.3 R.m.s. spectrum

The amplitude spectrum is used to quantify the components of sinusoidal, harmonic and non-harmonic signals and is defined as

$$R_{xx}(0) = \frac{f_s}{NC_a}\left|X(0)\right|$$

$$R_{xx}(m) = \frac{\sqrt{2}f_s}{NC_a}\left|X(m)\right| \quad \left(\text{for } 1 \leqslant m \leqslant \frac{N}{2}-1\right) \tag{26}$$

Windowing of the time domain signals is assumed. The units of this quantity are U r.m.s.

7.3.4 Energy spectral density

The energy spectral density is suitable for transient signals. For a deterministic non-stationary, transient signal the energy spectral density is

$$e_{xx}(m) = 2\left|X(m)\right|^2 \quad \left(\text{for } 1 \leqslant m \leqslant \frac{N}{2}-1\right) \tag{27}$$

Time windowing of transient signals is rarely done and no amplitude scaling is necessary. The units of this quantity are U^2s/Hz, U^2/Hz^2 or U^2s^2.

If the transient signal is random, then the energy spectral density is defined to be

$$e_{xx}(m) = E\left\{\left|X(m)\right|^2\right\} \quad \left(\text{for } 1 \leqslant m \leqslant \frac{N}{2}-1\right) \tag{28}$$

The average is taken over a sufficient number of independent data blocks and is described in 7.5. The variance defined in Equation (11) shall be performed with the energy spectral density of a random signal.

7.3.5 Power spectral density

This is also known as the auto spectral density.

Random, continuous signals are described by the power spectral density.

The power spectral density is computed as the average

$$P_{xx}(m) = E\left\{\frac{2f_s}{TC_a^2NB_e}\left|X(m)\right|^2\right\} \quad \left(\text{for } 1 \leqslant m \leqslant \frac{N}{2}-1\right) \tag{29}$$

where the average is taken over various times in a long time record. The units of this quantity are U^2/Hz. If the signal is non-ergodic, then the expectation shall be taken over an ensemble of independent data blocks.

Random stationary ergodic and non-ergodic signals are described by the power spectral density, where the expectation is accomplished by an average over a sufficient number of statistically independent data blocks.

NOTE A random non-stationary continuous signal is described by the Fourier transform of the auto-correlation function

$$P_{xx}(m,n) = \frac{1}{Nf_s}\sum_{k=0}^{K-1} r_{xx}(k,n)\,\mathrm{e}^{-i\frac{2\pi}{N}km}$$

where the auto-correlation function is determined with a sufficiently large value of K to ensure that the complete response is estimated.

7.3.6 Cross-spectral density

The frequency domain relationship between two signals may be displayed as the cross-spectral density. For transient signals described by the energy spectral density the cross energy spectral density is used:

$$e_{xy}(m) = 2X^*(m)Y(m) \quad (\text{for } 0 \leqslant m \leqslant N/2-1) \tag{30}$$

The units of this quantity are U^2s/Hz, U^2/Hz^2 or U^2s^2.

For signals described by the power spectral density the cross power spectral density is used:

$$P_{xy}(m) = \frac{2}{T}X^*(m)Y(m) \quad (\text{for } 0 \leqslant m \leqslant N/2-1) \tag{31}$$

The cross-spectral densities are complex quantities. The magnitude and phase of this quantity are displayed as a function of frequency $m\Delta f$.

7.3.7 Coherence function

The coherence function γ_{xy}^2 is a dimensionless measure of the relationship between two signals in the frequency domain. The coherence function is plotted as a function of $m\Delta f$. For two deterministic signals, the complex coherence function is the following ratio of either energy

$$\gamma_{xy}^2(m) = \frac{\left|e_{xy}\right|^2}{e_{xx}e_{yy}} \quad \left(\text{for } 0 \leqslant m \leqslant N/2-1\right) \tag{32}$$

or power spectra

$$\gamma_{xy}^2(m) = \frac{\left|P_{xy}\right|^2}{P_{xx}P_{yy}} \quad \left(\text{for } 0 \leqslant m \leqslant N/2-1\right) \tag{33}$$

7.3.8 Frequency response function

The vibration time-history of a deterministic response may be related to the input by finding the ratio of the two signals in the frequency domain.

If the input and output signals are deterministic, periodic, sinusoidal signals, the complex values of the input and response signals must be determined.

If the input and output signals are deterministic, periodic, complicated signals, at least one complete period of the input and response signal must be captured in the time window.

If the input and output signals are deterministic, non-periodic, almost-periodic signals, then the input and response signal must be captured in a sufficiently long time window so that the frequencies are well resolved. In general, a suitable window function must be used to well resolve the spectral content of the signals. In addition, the windows must be long enough to include the time-delay of the response in a large mechanical system.

If the input and output signals are deterministic, non-periodic transient signals, the complete input and response signals must be captured in the time window.

The frequency response function from signals defined with energy spectral densities is defined either as the ratio of the cross-energy spectral density of the input and response to the energy spectral density of the input signal

$$H_1(m) = \frac{e_{yx}(m)}{e_{xx}(m)} \tag{34}$$

or as the ratio of the energy spectral density of the response and the cross energy spectral density of the input and response

$$H_2(m) = \frac{e_{yy}(m)}{e_{yx}(m)} \tag{35}$$

For periodic or non-harmonic signals, the values of m in Equations (38) and (39) must be limited to the frequencies where a signal is present. Often most frequencies contain only noise.

The frequency response function derived from signals defined with power spectral densities is defined as the ratio of the cross-power spectral density of the input and response to the power spectral density of the input signal

$$H_1(m) = \frac{P_{yx}(m)}{P_{xx}(m)} \tag{36}$$

or as the ratio of the power spectral density of the response and the cross power spectral density of the input and response

$$H_2(m) = \frac{P_{yy}(m)}{P_{yx}(m)} \tag{37}$$

In each case, the units of the frequency response function are $\mathrm{U}_y / \mathrm{U}_x$ regardless of whether the inputs and outputs were described by energy or by power spectra.

The frequency response function is only valid at frequencies where the correlation coefficient and input signal are high.

The coherence function γ_{xy}^2 may be used to estimate the measurement noise. To estimate the energy or power spectrum of the noise $\nu(n)$ present in the input $x(n)$

$$e_{\nu\nu}(m) = \left[1 - \gamma^2(m)\right] e_{xx}(m)$$

or

$$P_{\nu\nu}(m) = \left[1 - \gamma^2(m)\right] P_{xx}(m)$$

Likewise, the energy or power spectrum of the measurement noise in the output $y(n)$

$$e_{\nu\nu}(m) = \left[1 - \gamma^2(m)\right] e_y(m)$$

or

$$P_{\nu\nu}(m) = \left[1 - \gamma^2(m)\right] P_{yy}(m)$$

7.4 Time-frequency distributions

7.4.1 Short-time Fourier Transform

The short-time Fourier Transform of a signal is given by

$$S_x(m,n) = \frac{1}{f_s} \sum_{k=0}^{K-1} x(n+k)\, w(k) \mathrm{e}^{-i\frac{2\pi}{N}km} \quad \left(\text{for } K < N,\ 0 < k < N-K\right) \tag{38}$$

where the window function w containing K samples shall be specified. The increment of the time index n may be increased from one; common choices are either the window length K or one-half of the window length $K/2$.

The units of this quantity are U/Hz.

7.4.2 Wigner distribution

The Wigner time-frequency distribution of a signal is a quadratic signal distribution defined by

$$\begin{aligned} &\Omega_x(m,n) = \frac{f_s}{N} \sum_{k=0}^{r} X(m+k)\, X^*(m-k) \mathrm{e}^{+i\frac{2\pi}{N}2nk} \\ &\text{for } 0 \leqslant m \leqslant \frac{N}{2} - 1,\ 0 \leqslant n \leqslant N,\ \ r = \min\left(\frac{N}{2} - 1 - m, m\right) \end{aligned} \tag{39}$$

This distribution can be smoothed and filtered by a Cohen class of filter $V(n,m)$:

$$\begin{aligned} &\Xi_x(m,n) = \frac{1}{f_s} \sum_{k=0}^{N-1} \mathrm{e}^{-i\frac{2\pi}{N}nk} \sum_{l=q}^{r} V(k, m-1)\, x(l+k)\, x(l-k) \\ &\text{for } q = \max(N-1-m, k),\ r = \min(N-1-k, m) \end{aligned} \tag{40}$$

The filter shall be specified.

The frequency increment of the time-frequency distribution is twice that of the Fourier transform.

The units of this quantity are $\mathrm{U^2s}$.

7.4.3 Continuous wavelet distribution

The wavelet distribution of a signal is a linear signal distribution defined by

$$O_x(k,m) = \frac{1}{f_s} 2^{m/2} \sum_{n=0}^{r} x(n)\, \psi\left(2^m n - k\right) \tag{41}$$

where $\psi(n)$ is the mother wavelet. Common mother wavelets and their use are given in ISO 18341-3.

The units of this quantity are U/Hz.

7.5 Averages of random stationary, ergodic signals

7.5.1 Estimates and errors

Two types of errors arise during the process of estimating averaged quantities from random data. The mean value of the error is called bias error, while the variation around the mean is called random error. The random error may be made arbitrarily small by averaging over a long time, while the bias error is not affected by averaging.

An estimate of the mean value $\bar{x}$ of a random signal $x(t)$ can be found by averaging $x(t)$ over time T. The variance of the random error depends on the power spectral density P_{xx} of the signal $x(t)$ and the averaging time T. Only the low frequency part of P_{xx}, $P_{xx,\text{low}}$, comes into play and the major contribution comes from frequencies $f < \frac{5}{T}$.

The variance of the estimate of the mean is

$$\text{Var}(\bar{x}) = \frac{1}{2T} \cdot \frac{1}{5} \sum_{m=1}^{5} P_{xx}(m) \tag{42}$$

For the case of $x(t)$ being low-pass noise with constant spectral density over the bandwidth B, and with r.m.s. value for the dynamic part σ_x, the power spectral density is $\frac{\sigma_x^2}{B}$ (single-sided) and the variance of the estimate of the mean

$$\text{Var}(\bar{x}) = \frac{\sigma_x^2}{2BT} \tag{43}$$

The relative random error, ε_r is

$$\varepsilon_\text{r} = \frac{\sigma_x}{\bar{x}} \cdot \sqrt{\frac{1}{2BT}} \tag{44}$$

For a band-pass signal with bandwidth B, the low frequency part of the power spectral density may be small, leading to a small random error in the estimate of the mean.

If the mean of $x^2(t)$ [where the mean of $x(t)$ is zero] is estimated, the power spectral density of $x^2(t)$ comes into play. For low-pass noise with constant spectral density over the bandwidth B, the low frequency part of the power spectral density of $x^2(t)$ is

$$P_{x^2x^2,\text{low}} = \frac{2\sigma_x^4}{B} \tag{45}$$

which gives

$$\text{Var}(\sigma_x^2) = \frac{\sigma_x^4}{BT} \tag{46}$$

and the relative random error

$$\varepsilon_\text{r} = \sqrt{\frac{\sigma_x^4}{BT\sigma_x^4}} = \sqrt{\frac{1}{BT}} \tag{47}$$

For a band-pass signal with bandwidth B, the low frequency part of the power spectral density for $x^2(t)$ has the same expression as for the low-pass case

$$S_{x^2,\mathrm{low}} = \frac{2\sigma_x^4}{B} \tag{48}$$

leading to the same expression for the relative random error as for the low-pass case.

7.5.2 Random error for power spectral density (PSD) calculations

The relative random error for one average of a PSD calculation is unity and is independent of frequency. If the averaging is performed over I non-overlapping data blocks, the relative random error is

$$\varepsilon_\mathrm{r} = \frac{1}{\sqrt{I}} \tag{49}$$

Overlap averaging may be used. This means that some of the data are used more than once. This means that the results used in the averaging process are somewhat correlated, resulting in somewhat higher random error. It is common practice to use time windows to calculate power spectral density and the amount of correlation depends on the window used. The Hanning window with up to 50 % overlap may be used and the equation for the relative error still is approximately valid.

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