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Nuclear fuel technology — Tank calibration and volume determination for nuclear materials accountancy —

Part 6:

Accurate in-tank determination of liquid density in accountancy tanks equipped with dip tubes

Technologie du combustible nucléaire — Étalonnage et détermination du volume de cuve pour la comptabilité des matières nucléaires —

Partie 6: Détermination précise de la masse volumique d'un liquide dans une cuve bilan équipée de cannes de bullage

Reference number
ISO 18213-6:2008(E)

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 18213-6 was prepared by Technical Committee ISO/TC 85, *Nuclear energy*, Subcommittee SC 5, *Nuclear fuel technology*.

ISO 18213 consists of the following parts, under the general title *Nuclear fuel technology — Tank calibration and volume determination for nuclear materials accountancy*:

- *Part 1: Procedural overview*
- *Part 2: Data standardization for tank calibration*
- *Part 3: Statistical methods*
- *Part 4: Accurate determination of liquid height in accountancy tanks equipped with dip tubes, slow bubbling rate*
- *Part 5: Accurate determination of liquid height in accountancy tanks equipped with dip tubes, fast bubbling rate*
- *Part 6: Accurate in-tank determination of liquid density in accountancy tanks equipped with dip tubes*

Introduction

ISO 18213 deals with the acquisition, standardization, analysis, and use of calibration data to determine liquid volumes in process tanks for accountability purposes. This part of ISO 18213 is complementary to the other parts, ISO 18213-1 (procedural overview), ISO 18213-2 (data standardization), ISO 18213-3 (statistical methods), ISO 18213-4 (slow bubbling rate), ISO 18213-5 (fast bubbling rate).

The procedure described in this part of ISO 18213 is a two-step procedure. First, a liquid of known density is used to determine the vertical distance between the tips of the two probes (i.e. to calibrate their separation). The calibration step requires synchronous (or as nearly synchronous as possible) measurements of the pressure exerted at the tips of two probes by the calibration liquid in which they are submerged. The measurements obtained are used to make an accurate determination of probe separation. Second, the unknown density of the process liquid is determined with the aid of the probe separation calibration. The density-determination step also requires (nearly) synchronous measurements of the pressure exerted at the tips of two probes by the process liquid of unknown density.

With careful technique, it is possible to make determinations of liquid density with in-tank measurements that approach the accuracy and precision of those made in the laboratory. Moreover, density determinations made with in-tank measurements are automatically made at the observed temperature of the tank liquid. Thus, no additional information about the liquid is required to infer its density at its tank temperature from determinations of its density at some other temperature.

Except that the density of the process liquid is generally not well characterized, the steps involved in determining the height of process liquid in the tank are the same as those for determining the height of calibration liquid. Thus, the method of density determination given in this part of ISO 18213 is very closely related to the procedures given in ISO 18213-4 and ISO 18213-5 for determining liquid height.

Nuclear fuel technology — Tank calibration and volume determination for nuclear materials accountancy —

Part 6:
Accurate in-tank determination of liquid density in accountancy tanks equipped with dip tubes

1 Scope

This part of ISO 18213 specifies a procedure for making accurate determinations of the densities of process liquids from in-tank measurements of the liquid content. This procedure is applicable to tanks equipped with pneumatic systems for determining the liquid content that have two or more bubbler probes of differing lengths. It is necessary that the probes be fixed relative to each other and to the tank in which they are installed.

The methods presented in this part of ISO 18213 yield acceptable results only for clear (i.e. free of suspended solids) liquids that are both homogeneous in concentration and at thermal equilibrium. The accuracy of the method is limited by

- the accuracy of the density determinations for the calibration liquid, and
- the number and accuracy of the liquid height determinations used in the calculations.

With state-of-the-art measurement technology and careful technique, and with water as a calibration liquid, it is possible to determine the density of homogeneous clear liquids with relative accuracy[1] on the order of 2×10^{-4} to 3×10^{-4}, or approximately 0,2 kg/m^3 or 0,3 kg/m^3.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 18213-1:2007, *Nuclear fuel technology — Tank calibration and volume determination for nuclear materials accountancy — Part 1: Procedural overview*

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[1] In ISO 18213, all statements of accuracy are given in terms of the half-width of two standard deviation (95 %) confidence intervals. Thus, the assertion here is that relative standard deviations for density determinations in the range of 0,01 % are possible.

3 Physical principles involved

The methodology in this part of ISO 18213 is based on the basic hydrodynamic principle which states that the pressure, P, exerted by a column of liquid of temperature T_m at the tip of a dip tube inserted therein is given by Equation (1):

$$P = gH_\text{M}\rho_\text{M} \tag{1}$$

where

H_M is the height of the liquid column (at temperature T_m), in m;

ρ_M is the average density of the liquid in the column (at temperature T_m), in kg/m^3;

g is the local acceleration due to gravity, in m/s^2.

If pressure exerted by some quantity of liquid is measured simultaneously at the tips of two dip tubes of differing lengths, then their separation at temperature T_m is given by Equation (2):

$$S_\text{M} = (H_{1,\text{M}} - H_{2,\text{M}}) = (P_1 - P_2)/(g\rho_\text{M}) \tag{2}$$

where

P_1 is the pressure exerted at the tip of the major (long) probe;

P_2 is the pressure exerted at the tip of the minor (short) probe;

$H_{1,\text{M}}$ is the height of the column of liquid above the tip of the major probe;

$H_{2,\text{M}}$ is the height of the column of liquid above the tip of the minor probe.

From Equation (2), the density of the liquid, ρ_M, can be expressed in terms of the probe separation, S_M:

$$\rho_\text{M} = (P_1 - P_2)/[g(H_{1,\text{M}} - H_{2,\text{M}})] = (P_1 - P_2)/(gS_\text{M}) \tag{3}$$

If Equation (2) is used to determine S_M from pressure measurements for a calibration liquid whose density is accurately known, then Equation (3) can be used to determine the density of some unknown liquid from measures of the pressures P_1 and P_2 that the liquid exerts at the tips of the respective probes.

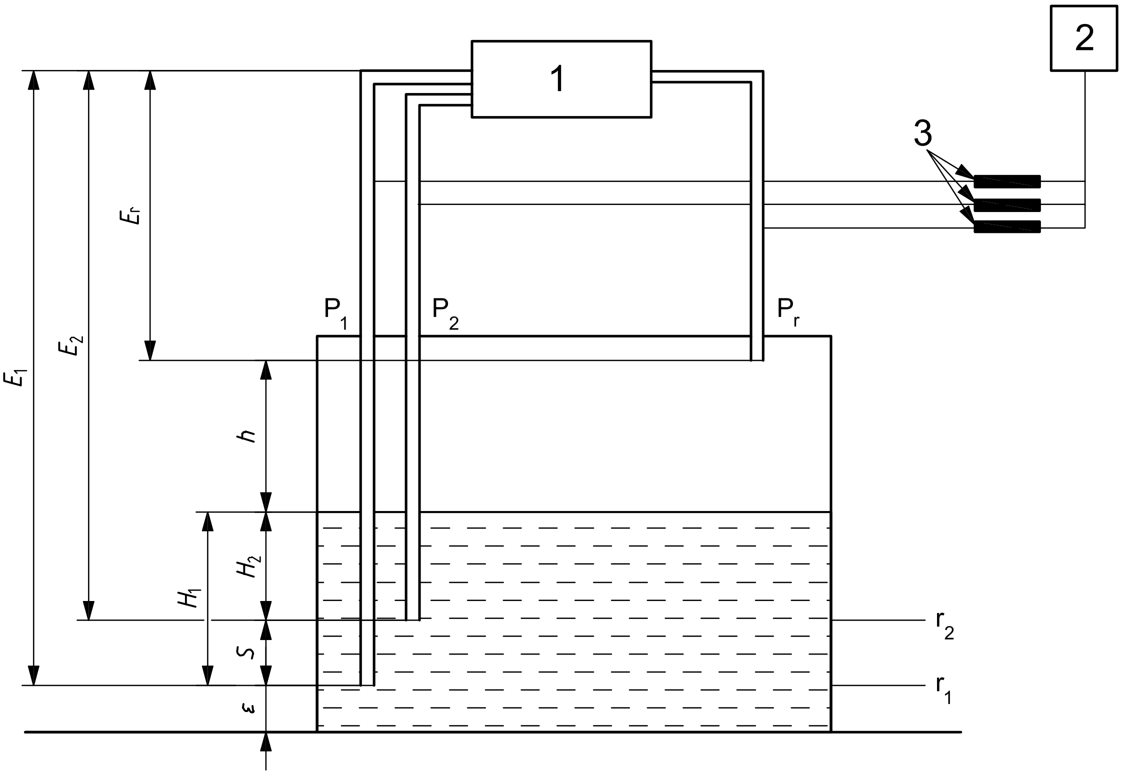
In practice, it is not possible to make direct measurements of the pressure exerted by a column of liquid at the tip of a probe in a process tank. Consequently, tanks are typically equipped with bubbler probe systems for making indirect measurements of pressure. With a bubbler probe system, gas is forced through a probe whose tip is submerged in the tank liquid. When bubbling occurs, the pressure exerted at the tip of the probe by the column of liquid in the tank is equal to that exerted by the gas in the probe line. This pressure is measured with a gauge located at some distance from the tip of the probe. Figure 1 shows the elements of a typical pressure measurement system for determining the liquid content; see ISO 18213-1 for a detailed description of this system.

Factors that can result in differences between actual pressures at the tips of the probes and the observed pressures at the measurement gauge include the buoyancy effect of air, the mass of gas in the probe lines, flow resistance, and the effects of bubble formation and release at the tip of the probe. Adjustments that compensate for these effects are discussed in detail in ISO 18213-4 or ISO 18213-5, respectively, for slow and fast bubbling rates; the discussion in this part of ISO 18213 is limited to those factors that affect the probe separation calibration.

Other factors can also affect the reliability of the calculations indicated by Equations (2) and (3). These include

- variations in the conditions under which required the measurements are made, and
- the accuracy of the density determinations of the calibration liquid.

Temperature variations are particularly significant because of their effect on the density of the liquids involved. Adjustments that compensate for temperature variations are included in the standardization steps of Clause 7. A formula that characterizes the density of demineralized water (a preferred calibration liquid), as a function of temperature with sufficient accuracy for accountability purposes, is given in Clause 4.



Key

1 manometer

2 gas supply (N_2 or air)

3 flowmeters

Probe	Major probe	Minor probe	Reference probe
Probe designation	P_1	P_2	P_r
Reference point	r_1 (primary)	r_2 (secondary)	—
Height of liquid above reference point	H_1	H_2	—
Elevation of pressure gauge (manometer) above reference point	E_1	E_2	E_r
Elevation of reference probe above liquid surface	$h = E_1 - E_r - H_1$	$h = E_2 - E_r - H_2$	—
Elevation of reference point above bottom of tank	ε	$\varepsilon + S$ [a]	—

[a] Vertical distance (probe separation): $S = H_1 - H_2$.

Figure 1 — Elements of a typical pressure measurement system for determining liquid content

4 Required data

The determination of probe separation by means of Equation (2) requires measurements of the pressures exerted by some calibration liquid (a liquid of known density) at the tips of two probes of differing (but fixed) lengths. Required data are obtained by filling the tank with enough calibration liquid to cover the tips of both dip tubes and then measuring the corresponding pressure in each probe line. It is desirable to make replicate readings for each measurement and to repeat this process for several liquid heights which span some height range of interest in the tank[2]. Moreover, these measurements shall be standardized to a predetermined set of reference conditions (see Clause 7) in order to minimize the effect of variations in ambient conditions, especially temperature, during the period of data collection.

The probe separation calibration step requires a (calibration) liquid whose density has been accurately determined at all temperatures of interest. Subject to this constraint, any liquid that is compatible with the process may be used for calibration. Demineralized water is a preferred calibration liquid because its density has been very accurately determined at all operating temperatures and results are readily available. Equation (4) gives very accurate determinations of the density, ρ_M, in kilograms per cubic metre, of air-free (freshly distilled) water for temperatures $T = T_\mathrm{m}$ between 4 °C and 40 °C:

$$\rho_\mathrm{M} = \mathrm{A} + \mathrm{B}T + \mathrm{C}T^2 + \mathrm{D}T^3 + \mathrm{E}T^4 + \mathrm{F}T^5 \tag{4}$$

where

$\mathrm{A} = 999{,}843\ 22$

$\mathrm{B} = 6{,}684\ 416 \times 10^{-2}$

$\mathrm{C} = -8{,}903\ 070 \times 10^{-3}$

$\mathrm{D} = 8{,}797\ 523 \times 10^{-5}$

$\mathrm{E} = -8{,}030\ 701 \times 10^{-7}$

$\mathrm{F} = 3{,}596\ 363 \times 10^{-10}$

For temperatures between 3 °C and 30 °C, the estimated residual standard deviation for this fit is less than 0,001 kg/m^3. For other temperatures between 1 °C and 40 °C, the reported standard deviation does not exceed 0,001 4 kg/m^3.

Equation (4) is based on a recent redetermination of the density of water[5] and is reproduced in ISO 18213-1:2007, Annex A, ISO 18213-4:2008, Annex A and ISO 18213-5:2008, Annex A. A correction to Equation (4) that is applicable to air-saturated water is also given in Reference [5].

Equation (4) can be used to compute the density of water with sufficient accuracy to satisfactorily carry out the probe separation calibration procedure presented in this part of ISO 18213. If some liquid other than water is used for calibration, then its density must be determined with suitable accuracy at all measurement temperatures before the procedure presented herein can be successfully applied.

5 Equipment required

As noted in Clause 4, it is convenient to obtain data for determining probe separation when the tank is being calibrated. In any event, the equipment required to obtain probe separation data is the same as that required to determine liquid height during the calibration exercise. Required equipment is specified in ISO 18213-1:2007, Clause 4.

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2) It is convenient to obtain suitable data for determining probe separation when the tank is being calibrated (see ISO 18213-1).

6 Operating procedures

Operating procedures required to obtain probe separation data are the same as those required to determine liquid height during the tank calibration exercise. Suitable operating procedures are specified in ISO 18213-1:2007, Clause 6. Operating procedures specific to the slow bubbling case are specified in ISO 18213-4:2008, Clause 3.

7 Calculation of probe separation and liquid density

7.1 General

To minimize the effect of changes in ambient conditions, pressure measurements made to determine the probe separation shall be standardized to a fixed set of reference conditions before they are used to calculate the probe separation. However, standardization steps depend on the physical configuration and the bubbling rate of the tank's manometer system. In the configuration shown in Figure 1, the high side of the manometer is connected to the submerged (major or minor) probe and the low side is connected to the reference probe. Under this configuration, pressures in the two submerged probes are measured independently relative to the pressure in the reference probe. Appropriate standardization steps and computing procedures for determining probe separation and liquid density under the illustrated configuration are given for fast and slow bubbling rates, respectively, in 7.2 and 7.3. Estimates of uncertainty for the resulting density estimates are given respectively in 8.1.1 and 8.1.2. The user of this part of ISO 18213 should consult whichever sections pertain to the situation in his or her facility.

An alternative configuration, in which the two submerged probes are connected directly to each other, is possible. However, this configuration is not recommanded because it is not possible to obtain independant measures of the height of the liquid from the two submerged probes.

7.2 Case 1: Fast bubbling rate

7.2.1 Data standardization

The data standardization steps for this case (high side of manometer connected to submerged probe, low side connected to the reference probe fast bubbling rate) follow from Equation (8) of ISO 18213-5:2007, which is reproduced here as Equation (5)[3), 4)]. This equation gives the height of the column of liquid in the tank, $H_{1,\mathrm{M}}$, above the tip of the bubbling (major) probe at the measurement temperature, T_m, in terms of the observed pressure difference, ΔP_1:

$$H_{1,\mathrm{M}} = [\Delta P_1 + gE_1(\rho_{\mathrm{g},1} - \rho_{\mathrm{a,s}}) - gE_\mathrm{r}(\rho_{\mathrm{g,r}} - \rho_{\mathrm{a,s}}) + (\delta_\mathrm{r} - \delta_1) - g\lambda(\rho_\mathrm{M} - \rho_{\mathrm{g},1}) - 2\,\sigma/r_\mathrm{b}]/[g(\rho_\mathrm{M} - \rho_{\mathrm{a,s}})] \qquad (5)$$

where

$\Delta P_1 = P_1(E_1) - P_\mathrm{r}(E_1)$ is the difference in pressure between the bubbling probe and the reference probe, as measured at a gauge located at elevation E_1 above the tip of the bubbling probe;

g is the local acceleration due to gravity;

ρ_M is the density of the liquid in the tank at temperature T_m;

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3) The subscript “1” is used to indicate quantities that refers to the major (long) probe and the subscript “*2*” is used to indicate quantities that refer to the minor (short) probe (see Figure 1).

4) For quantities other than temperature, the letter m is used as a subscript to denote temperature dependence. A lower case m (m) refers to the temperature t_m of liquid in the prover and an upper case m (M) refers to the temperature T_m of liquid in the tank.

$\rho_{a,s}$	is the average density of air in the tank above the liquid surface at the prevailing pressure;
$\rho_{g,1}$	is the average density of gas in the major probe line at the prevailing pressure;
$\rho_{g,r}$	is the average density of gas in the reference probe line at the prevailing pressure;
E_1	is the elevation of the pressure gauge above the primary reference point (the tip of the major probe);
E_r	is the elevation of the pressure gauge above the tip of the reference probe;
δ_1	is the pressure drop in the major probe line due to the gas flow resistance;
δ_r	is the pressure drop in the reference probe line due to flow resistance;
λ	is the distance of the lowest point of the bubble below the tip of the major probe;
σ	is the surface tension for the liquid and gas;
r_b	is the radius of curvature of the bubble at its lowest point.

With appropriate notational changes, a similar expression can be written, in accordance with Equation (6), for the height of liquid in the tank above the tip of the minor probe:

$$H_{2,M} = [\Delta P_2 + g\,E_2(\rho_{g,2} - \rho_{a,s}) - g\,E_r(\rho_{g,r} - \rho_{a,s}) + (\delta_r - \delta_2) - g\,\lambda(\rho_M - \rho_{g,2}) - 2\,\sigma/r_b]/[g(\rho_M - \rho_{a,s})] \tag{6}$$

If it is assumed that the quantities λ, σ, and r_b associated with bubble formation at the tip of the probe are the same for both probes, then

$$[g\,\lambda(\rho_M - \rho_{g,1}) + 2\,\sigma/r_b] - [g\,\lambda(\rho_M - \rho_{g,2}) + 2\,\sigma/r_b] = -\,g\,\lambda(\rho_{g,1} - \rho_{g,2}) \tag{7}$$

in which case the difference between Equations (5) and (6) yields Equation (8) for $S_M = (H_{1,M} - H_{2,M})$:

$$\begin{aligned} S_M &= (H_{1,M} - H_{2,M}) \\ &= [(\Delta P_1 - \Delta P_2) + g\,E_1(\rho_{g,1} - \rho_{a,s}) - g\,E_2(\rho_{g,2} - \rho_{a,s}) + (\delta_2 - \delta_1) + g\,\lambda(\rho_{g,1} - \rho_{g,2})]/\,[g(\rho_M - \rho_{a,s})] \end{aligned} \tag{8}$$

The simplifying assumptions made here are generally acceptable under the operating conditions encountered in practice. However, they should be verified for each application. If any assumption seems questionable or cannot be verified in a particular situation, then the affected terms should be retained in subsequent calculations.

The difference in pressure drop $(\delta_2 - \delta_1)$ in the two lines is generally very small unless the probe separation is very large and the air flow rate is very high. If this quantity is ignored, then Equation (8) becomes Equation (9):

$$S_M = (H_{1,M} - H_{2,M}) = [(\Delta P_1 - \Delta P_2) + g\,E_1(\rho_{g,1} - \rho_{a,s}) - g\,E_2(\rho_{g,2} - \rho_{a,s}) + g\,\lambda(\rho_{g,1} - \rho_{g,2})]/[g(\rho_M - \rho_{a,s})] \tag{9}$$

Equation (9) is the observational analogue of Equation (2). In Equation (9), the pressure differences ΔP_1 and ΔP_2 are observed. For calibration of the probe separation (determination of S_M), the density, ρ_M, can be determined from Equation (4) when the tank liquid is water. If the bubbling gas is air, the densities $\rho_{g,1}$ and $\rho_{g,2}$ can be estimated as indicated in ISO 18213-5:2008, A.3.2. The air density, $\rho_{a,s}$, can be determined as indicated in ISO 18213-5:2008, A.3.4, and the elevations E_1 and E_2 can be determined from engineering drawings for the tank (see Figure 1).

7.2.2 Determination of probe separation

Let D_M denote the (corrected) pressure difference on the right-hand side of Equation (9) observed when the temperature of the tank liquid is T_m:

$$D_\mathrm{M} = [(\Delta P_1 - \Delta P_2) + g\,E_1(\rho_{\mathrm{g},1} - \rho_{\mathrm{a,s}}) - g\,E_2(\rho_{\mathrm{g},2} - \rho_{\mathrm{a,s}}) + g\,\lambda(\rho_{\mathrm{g},1} - \rho_{\mathrm{g},2})] \tag{10}$$

Then, the estimated probe separation, S_M, at temperature T_m can be written as

$$S_\mathrm{M} = (H_{1,\mathrm{M}} - H_{2,\mathrm{M}}) = D_\mathrm{M}/[g(\rho_\mathrm{M} - \rho_{\mathrm{a,s}})] \tag{11}$$

Moreover, the probe separation at some pre-established reference temperature, T_r, is related to S_M by Equation (12):

$$S_\mathrm{r} = S_\mathrm{M}/(1 + \alpha\,\Delta T_\mathrm{M}) = D_\mathrm{M}/[g(\rho_\mathrm{M} - \rho_{\mathrm{a,s}})(1 + \alpha\,\Delta T_\mathrm{M})] \tag{12}$$

where $\Delta T_\mathrm{M} = T_\mathrm{M} - T_\mathrm{r}$ and α is the linear (thermal) coefficient of expansion for the probes.

In practice, it is desirable to average multiple observations, $S_{\mathrm{r,i}}$, of the probe separation, S_r, in order to increase the precision (reduce the variability) of the resulting probe separation estimate. For multiple observations, the probe separation at temperature T_r is estimated from Equation (13):

$$\bar{S}_\mathrm{r} = n^{-1}\Sigma_i\, S_{\mathrm{r},i} = n^{-1}\Sigma_i\, \{D_{\mathrm{M},i}/[g(\rho_{\mathrm{M},i} - \rho_{\mathrm{a,s}})(1 + \alpha\,\Delta T_{\mathrm{M},i})]\} \tag{13}$$

where

n is the number of observations;

$T_{\mathrm{m},i}$ is the temperature of the liquid for the ith observation;

$\rho_{\mathrm{M},i}$ is the density of the liquid for the ith observation;

$\Delta T_{\mathrm{M},i} = T_{\mathrm{m},i} - T_\mathrm{r}$.

7.2.3 Standard error of estimated probe separation

When multiple determinations of S_r are averaged to estimate the probe separation as indicated by Equation (13), the standard error of the estimate, $\bar{S}_\mathrm{r}$, is given by Equation (14):

$$s(\bar{S}_\mathrm{r}) = \{\Sigma_i(S_{\mathrm{r},i} - \bar{S}_r)^2/[n(n-1)]\}^{0,5} \tag{14}$$

where n is the number of observations used to calculate $\bar{S}_\mathrm{r}$. The number of replicates can be chosen to ensure that the variability of $\bar{S}_\mathrm{r}$ does not exceed some pre-established limit.

7.2.4 Density estimation

With the aid of the probe separation estimate, $\bar{S}_\mathrm{r}$, obtained from Equation (13), the unknown density of some liquid can be estimated by the means of the following procedure:

- the temperature of the liquid, $T_{\mathrm{m,u}}$, is observed and recorded;
- the differential pressures, ΔP_1 and ΔP_2, induced by the liquid in the major and minor probe lines, respectively, are observed and recorded;
- the pressure difference $(\Delta P_1 - \Delta P_2)$ is corrected as indicated in Equation (10); the result $D_{\mathrm{M,u}}$ is noted;

— the following estimate of the unknown density, $\rho_{M,u}$, is computed with the aid of Equation (12):

$$\hat{\rho}_{M,u} = D_{M,u}/[(g\bar{S}_r)(1 + \alpha \Delta T_{M,u})] + \rho_{a,s} \tag{15}$$

where $\Delta T_{M,u} = T_{m,u} - T_r$ and α is the linear coefficient of expansion for the probes. Alternatively, one may write

$$\hat{\rho}_{M,u} = \left[\left(\Delta P_1 - \Delta P_2\right) + gE_1\left(\rho_{g,1} - \rho_{a,s}\right) - gE_2\left(\rho_{g,2} - \rho_{a,s}\right) + g\lambda\left(\rho_{g,1} - \rho_{g,2}\right)\right] / \left[\left(g\bar{S}_r\right)\left(1 + \alpha\Delta T_{M,u}\right)\right] + \rho_{a,s} \tag{16}$$

The density estimate given by Equation (15) or Equation (16) is valid only at the temperature $T_{m,u}$. It is not possible to infer the density of the liquid at other temperatures from this estimate without additional information about how the density of the unknown liquid changes with temperature.

7.3 Case 2: Slow bubbling rate

7.3.1 Data standardization

The data standardization steps for this case (high side of manometer connected to submerged probe, low side connected to the reference probe, slow bubbling rate) follow from Equation (5) of ISO 18213-4:2008, which is reproduced here as Equation (17)[5), 6)]. This equation gives the height of the column of liquid in the tank above the tip of the bubbling (major) probe at the measurement temperature, T_m, in terms of the observed pressure difference, ΔP_1:

$$H_{1,M} = [\Delta P_1 + g\,E_1(\rho_{g,1} - \rho_{a,s}) - g\,E_r(\rho_{g,r} - \rho_{a,s}) + (\delta p)_{max}]/[g\,(\rho_M - \rho_{a,s})] \tag{17}$$

where

$\Delta P_1 = P_1(E_1) - P_r(E_1)$	is the difference in pressure between the bubbling probe and the reference probe, as measured at a gauge located at elevation E_1 above the tip of the bubbling probe;
g	is the local acceleration due to gravity;
ρ_M	is the density of the liquid in the tank at temperature T_m;
$\rho_{a,s}$	is the average density of air in the tank above the liquid surface at the prevailing pressure;
$\rho_{g,1}$	is the average density of gas in the major probe line at the prevailing pressure;
$\rho_{g,r}$	is the average density of gas in the reference probe line at the prevailing pressure;
E_1	is the elevation of the pressure gauge above the primary reference point (the tip of the major probe);
E_r	is the elevation of the pressure gauge above the tip of the reference probe;
$(\delta p)_{max}$	is the maximum overpressure observed during the bubbling process, relative to the pressure at the tip of the probe.

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5) The subscript "1" is used to indicate quantities that refers to the major (long) probe and the subscript "2" is used to indicate quantities that refer to the minor probe (see Figure 1).

6) For quantities other than temperature, the letter m is used as a subscript to denote temperature dependence. A lower case m (m) refers to the temperature t_m of liquid in the prover and an upper case m (M) refers to the temperature T_m of liquid in the tank.

With appropriate notational changes, a similar expression can be written, in accordance with Equation (17), for the height of liquid in the tank above the tip of the minor probe:

$$H_{2,\mathrm{M}} = [\Delta P_2 + g\,E_2(\rho_{\mathrm{g},2} - \rho_{\mathrm{a,s}}) - g\,E_\mathrm{r}(\rho_{\mathrm{g,r}} - \rho_{\mathrm{a,s}}) + (\delta p)_{\mathrm{max}}]/[g(\rho_\mathrm{M} - \rho_{\mathrm{a,s}})] \tag{18}$$

For a given liquid, the overpressure is essentially independent of the height of liquid above the tip of the probe. If the overpressure is assumed to be the same for both probes, then the difference between Equation (17) and Equation (18) yields Equation (19) for $S_\mathrm{M} = (H_1 - H_2)$:

$$S_\mathrm{M} = (H_1 - H_2) = [(\Delta P_1 - \Delta P_2) + g\,E_1(\rho_{\mathrm{g},1} - \rho_{\mathrm{a,s}}) - g\,E_2(\rho_{\mathrm{g},2} - \rho_{\mathrm{a,s}})]/[g(\rho_\mathrm{M} - \rho_{\mathrm{a,s}})] \tag{19}$$

The simplifying assumptions made here are generally acceptable under the operating conditions encountered in practice. However, they should be verified for each application. If any assumption seems questionable or cannot be verified in a particular situation, then the affected terms should be retained in subsequent calculations.

Equation (19) is the observational analogue to Equation (2). In Equation (19), the pressure differences ΔP_1 and ΔP_2 are observed. For calibration of the probe separation (determination of S_M), the density ρ_M can be determined from Equation (4) when the tank liquid is water. If the bubbling gas is air, the densities $\rho_{\mathrm{g},1}$ and $\rho_{\mathrm{g},2}$ can be estimated as indicated in ISO 18213-4:2008, A.3.2. The air density, $\rho_{\mathrm{a,s}}$, can be determined as indicated in ISO 18213-4:2008, A.3.4, and the elevations E_1 and E_2 can be determined from engineering drawings for the tank (see Figure 1).

7.3.2 Determination of probe separation

Let D_M denote the (corrected) pressure difference on the right-hand side of Equation (19) observed when the temperature of the tank liquid is T_m:

$$D_\mathrm{M} = [(\Delta P_1 - \Delta P_2) + g\,E_1(\rho_{\mathrm{g},1} - \rho_{\mathrm{a,s}}) - g\,E_2(\rho_{\mathrm{g},2} - \rho_{\mathrm{a,s}})] \tag{20}$$

Then, the estimated probe separation, S_M, at temperature T_m can be written as

$$S_\mathrm{M} = (H_{1,\mathrm{M}} - H_{2,\mathrm{M}}) = D_\mathrm{M}/[g(\rho_\mathrm{M} - \rho_{\mathrm{a,s}})] \tag{21}$$

Moreover, the probe separation at some pre-established reference temperature, T_r, is related to S_M by Equation (22):

$$S_\mathrm{r} = S_\mathrm{M}/(1 + \alpha\,\Delta T_\mathrm{M}) = D_\mathrm{M}/[g(\rho_\mathrm{M} - \rho_{\mathrm{a,s}})(1 + \alpha\,\Delta T_\mathrm{M})] \tag{22}$$

where $\Delta T_\mathrm{M} = T_\mathrm{m} - T_\mathrm{r}$ and α is the linear (thermal) coefficient of expansion for the probes.

In practice, it is desirable to average multiple observations, $S_{\mathrm{r},i}$, of the probe separation, S_r, in order to increase the precision (reduce the variability) of the resulting probe separation estimate. For multiple observations, the probe separation at temperature T_r is estimated from Equation (23):

$$\bar{S}_\mathrm{r} = n^{-1}\Sigma_i\, S_{\mathrm{r},i} = n^{-1}\Sigma_i\{D_{\mathrm{M},i}/[g(\rho_{\mathrm{M},i} - \rho_{\mathrm{a,s}})(1 + \alpha\,\Delta T_{\mathrm{M},i})]\} \tag{23}$$

where

n is the number of observations;

$T_{\mathrm{m},i}$ is the temperature of the liquid for the ith observation;

$\rho_{\mathrm{M},i}$ is the density of the liquid for the ith observation;

$\Delta T_{\mathrm{M},i} = T_{\mathrm{m},i} - T_\mathrm{r}$.

7.3.3 Standard error of estimated probe separation

When multiple determinations of S_r are averaged to estimate the probe separation as indicated by Equation (23), the standard error of the estimate, $\bar{S}_r$, is given by Equation (24):

$$s(\bar{S}_r) = \{\textstyle\sum_i (S_{r,i} - \bar{S}_r)^2/[n(n-1)]\}^{0,5} \tag{24}$$

where n is the number of observations used to calculate $\bar{S}_r$. The number of replicates can be chosen to ensure that the variability of $\bar{S}_r$ does not exceed some pre-established limit.

7.3.4 Density estimation

With the aid of the probe separation estimate, $\bar{S}_r$, obtained from Equation (23), the unknown density of some liquid can be estimated by the means of the following procedure:

- the temperature of the liquid, $T_{m,u}$, is observed and recorded;
- the differential pressures, ΔP_1 and ΔP_2, induced by the liquid in the major and minor probe lines, respectively, are observed and recorded;
- the pressure difference ($\Delta P_1 - \Delta P_2$) is corrected as indicated in Equation (20); the result $D_{M,u}$ is noted;
- the following estimate of the unknown density, $\rho_{M,u}$, is then computed with the aid of Equation (22):

$$\hat{\rho}_{M,u} = D_{M,u}/[(g\bar{S}_r)(1 + \alpha\,\Delta T_{M,u})] + \rho_{a,s} \tag{25}$$

where $\Delta T_{M,u} = T_{m,u} - T_r$ and α is the linear coefficient of expansion for the probes. Alternatively, one may write

$$\hat{\rho}_{M,u} = \left[(\Delta P_1 - \Delta P_2) + gE_1(\rho_{g,1} - \rho_{a,s}) - gE_2(\rho_{g,2} - \rho_{a,s})\right] / \left[(g\bar{S}_r)(1 + \alpha\Delta T_{M,u})\right] + \rho_{a,s} \tag{26}$$

The density estimate given by Equation (25) or Equation (26) is valid only at the temperature $T_{m,u}$. It is not possible to infer the density of the liquid at other temperatures from this estimate without additional information about how the density of the unknown liquid changes with temperature.

8 Uncertainty estimation

8.1 Density

8.1.1 Case 1: Fast bubbling rate

The variance of the density estimate derived in 7.2.4 may be estimated with the aid of variance propagation methods. In terms of the variances of $D_{M,u}$ and $\bar{S}_r$, the variance of the density estimate $\hat{\rho}_{M,u}$ given by Equation (15) may be estimated from Equation (27):

$$s^2(\hat{\rho}_{M,u}) = (1 + \alpha\,\Delta T_{M,u})^{-2}\,[D_{M,u}/(g\bar{S}_r)]^2\,[s^2(D_{M,u})/(D_{M,u})^2 + s^2(g\bar{S}_r)/(g\bar{S}_r)^2)] + s^2(\rho_{a,s}) \tag{27}$$

In Equation (27), the quantity $\bar{S}_r$ is determined from Equation (13) and its estimated variance, $s^2(\bar{S}_r)$, is obtained from Equation (14). The variance of a particular determination, $D_{M,u}$, can be estimated by applying variance propagation methods to variance estimates for ΔP_1 and ΔP_2 in accordance with Equation (10). The contribution of the correction terms to the total variance is usually quite small. If the variance contribution of these terms is ignored, and it is further assumed that ΔP_1 and ΔP_2 are independent, then

$$s^2(D_{M,u}) = s^2(\Delta P_1) + s^2(\Delta P_2) \tag{28}$$

The variance estimates on the right-hand side of Equation (28) may be obtained from repeated measurements of ΔP_1 and ΔP_2.

In Equation (27), the contribution of the term $s^2(\rho_{a,s})$ to the total variability $s^2(\hat{\rho}_{M,u})$ is quite small and can usually be ignored. If it proves necessary to include this term, it can be estimated by applying variance propagation methods to the formula for estimating $\rho_{a,s}$ that is given in ISO 18213-5:2008, A.3.4.

As emphasized in 7.2, any simplifying assumptions should be verified for each application. Any term whose variability contributes (or is suspected of contributing) significantly to the total variability of the density estimate should be retained in subsequent calculations.

8.1.2 Case 2: Slow bubbling rate

The variance of the density estimate derived in 7.3.4 may be estimated with the aid of variance propagation methods. In terms of the variances of $D_{M,u}$ and $\bar{S}_r$, the variance of the density estimate, $\hat{\rho}_{M,u}$, given by Equation (25) may be estimated from Equation (29):

$$s^2(\hat{\rho}_{M,u}) = (1 + \alpha\,\Delta T_{M,u})^{-2}\,[D_{M,u}/(g\bar{S}_r)]^2\,[s^2(D_{M,u})/(D_{M,u})^2 + s^2(g\bar{S}_r)/(g\bar{S}_r)^2)] + s^2(\rho_{a,s}) \tag{29}$$

In Equation (29), the quantity $\bar{S}_r$ is determined from Equation (23) and its estimated variance, $s^2(\bar{S}_r)$, is obtained from Equation (24). The variance of a particular determination, $D_{M,u}$, can be estimated by applying variance propagation methods to variance estimates for ΔP_1 and ΔP_2 in accordance with Equation (20). The contribution of the correction terms to the total variance is usually quite small. If the variance contribution of these terms is ignored, and it is further assumed that ΔP_1 and ΔP_2 are independent, then

$$s^2(D_{M,u}) = s^2(\Delta P_1) + s^2(\Delta P_2) \tag{30}$$

The variance estimates on the right-hand side of Equation (30) may be obtained from repeated measurements of ΔP_1 and ΔP_2.

In Equation (29), the contribution of the term $s^2(\rho_{a,s})$ to the total variability $s^2(\hat{\rho}_{M,u})$ is quite small and can usually be ignored. If it proves necessary to include this term, it can be estimated by applying variance propagation methods to the formula for estimating $\rho_{a,s}$ that is given in ISO 18213-4:2008, A.3.4.

As emphasized in 7.3, any simplifying assumptions should be verified for each application. Any term whose variability contributes (or is suspected of contributing) significantly to the total variability of the density estimate should be retained in subsequent calculations.

8.2 Local acceleration due to gravity

A determination of the local acceleration due to gravity, g, is required to make the corrections indicated by Equation (9) or Equation (19). The local acceleration due to gravity depends on elevation and latitude, and should be determined at the measurement site. In lieu of an on-site determination, the local acceleration due to gravity may be determined with suitable accuracy from charts given in Appendix 7 of Reference [6]. An erroneous value results in a bias in the indicated correction. However, the correction itself is generally quite small, so a small error in the estimate of g will have a negligible effect.

8.3 Changes in reference conditions

The calculations to determine probe separation and liquid density are very sensitive to changes in measurement conditions, especially to variations in temperature. Therefore, it is important that all measurements be made under conditions that are as thermally stable as possible. In any event, the standardization steps indicated in Clause 7 should be carried out for each measurement to ensure that results are not degraded by changes in ambient conditions during the measurement period.

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