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ISO 17123-7

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Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 7: Optical plumbing instruments

Optique et instruments d'optique — Méthodes d'essai sur site des instruments géodésiques et d'observation —

Partie 7: Instruments de plombage optique

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Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 17123-7 was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 6, *Geodetic and surveying instruments*.

This first edition of ISO 17123-7 cancels and replaces ISO 8322-5:1991, which has been technically revised.

ISO 17123 consists of the following parts, under the general title *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments*:

— *Part 1: Theory*

— *Part 2: Levels*

— *Part 3: Theodolites*

— *Part 4: Electro-optical distance meters (EDM instruments)*

— *Part 5: Electronic tacheometers*

— *Part 6: Rotating lasers*

— *Part 7: Optical plumbing instruments*

Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 7: **Optical plumbing instruments**

1 Scope

This part of ISO 17123 specifies field procedures to be adopted when determining and evaluating the precision (repeatability) of optical plumbing instruments and their ancillary equipment, when used in building and surveying measurements. This part of ISO 17123 is not applicable to optical plummets as a device in tribrachs or in surveying instruments. Primarily, these tests are intended to be field verifications of the suitability of a particular instrument for the immediate task at hand and to satisfy the requirements of other standards. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

This part of ISO 17123 can be thought of as one of the first steps in the process of evaluating the uncertainty of a measurement (more specifically a measurand). The uncertainty of a result of a measurement is dependent on a number of factors. These include among others: repeatability, reproducibility (between-day repeatability) and a thorough assessment of all possible error sources, as prescribed by the ISO *Guide to the expression of uncertainty in measurement (GUM)*.

These field procedures have been developed specifically for *in situ* applications without the need for special ancillary equipment and are purposefully designed to minimize atmospheric influences and effects of imperfect adjustment of the optical plumbing instrument.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*

ISO 4463-1, *Measurement methods for building — Setting-out and measurement — Part 1: Planning and organization, measuring procedures, acceptance criteria*

ISO 7077, *Measuring methods for building — General principles and procedures for the verification of dimensional compliance*

ISO 7078, *Building construction — Procedures for setting out, measurement and surveying — Vocabulary and guidance notes*

ISO 9849, *Optics and optical instruments — Geodetic and surveying instruments — Vocabulary*

ISO 17123-1, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 1: Theory*

Guide to the expression of uncertainty in measurement (GUM), BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 1993, corrected and reprinted in 1995

International vocabulary of basic and general terms in metrology (VIM). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 2nd ed., 1993

3 Terms and definitions

For the purpose of this document, the terms and definitions given in ISO 3534-1, ISO 4463-1, ISO 7077, ISO 7078, ISO 9849, ISO 17123-1, the *GUM* and the *VIM* apply.

4 Requirements

Before commencing a survey, it is important that the operator ensure that the precision in use of the measuring equipment is appropriate for the intended measuring task.

The optical plumbing instrument and its ancillary equipment shall be in known and acceptable states of permanent adjustment according to the methods specified in the manufacturer's handbook, and used with tripods as recommended by the manufacturer.

The results of these tests are influenced by meteorological conditions. An overcast sky and low wind speed guarantee the most favourable weather conditions. The particular conditions to be taken into account may vary depending on where the tasks are to be undertaken. Note shall also be taken of the actual weather conditions at the time of measurement and the surroundings in which the measurements are made. The conditions chosen for the tests shall match those expected when the intended measuring task is actually carried out (see ISO 7077 and ISO 7078). Above all, this is applicable to the range of vertical distances (plumbing heights) over which the plumbing operation shall be carried out.

Tests performed in laboratories would provide results which are almost unaffected by many of the above-mentioned measuring conditions, but the costs for such tests are very high, and therefore they are not practicable for most users. In addition, laboratory tests yield precisions much higher than those that can be obtained under field conditions.

For the test procedure described in this part of ISO 17123, a rectangular x-y grid, used as target plate, is necessary. The spacing t of this grid shall fulfil the following condition:

$$t \geqslant 2{,}9 \times \frac{h}{\Gamma} \tag{1}$$

where

2,9 is a constant factor permitting a good estimation in the grid interval;

h is the plumbing height, expressed in meters;

Γ is the magnifying power of the telescope.

The value of t is obtained in millimetres.

5 Types of optical plumbing instruments

Types of optical plumbing instruments include:

— instruments using spirit levels;

— instruments using a compensator;

— instruments using two compensators.

The spirit level or a single compensator ensures that the line of sight is in a vertical plane only perpendicular to the direction of pointing. The plumb line is the intersection of two vertical planes perpendicular to each other and needs settings and measurements in two perpendicular directions of pointing.

An instrument with two compensators ensures that the line of sight is coincident with the plumb line in any direction. Although the differences of the designs are obvious, only one test procedure shall be used for the different types of optical plumbing instruments.

Optical plumbing instruments are able to sight upwards or downwards or both. The test procedure is the same in all these cases.

The same test procedure is applicable to laser-emitting plumbing instrument, but with an observation means required separately for observing laser spot on the target.

6 Test principle

The test procedure shall be adopted to determine the measure of precision of a particular optical plumbing instrument and its ancillary equipment under field conditions.

The measure of precision of any type of optical plumbing instruments is dependent on the plumbing height. Thus the achievable measure of precision in use is expressed as the relative experimental standard deviation of one component of the point transferred one time over the corresponding plumbing height:

$s_{\text{ISO-plumb}}$

Furthermore, this procedure may be used to determine:

— the measure of precision in the use of optical plumbing instruments by a single survey team with a single instrument and its ancillary equipment at a given time;

— the measure of precision in the use of a single instrument over time and differing environmental conditions;

— the measure of precision in the use of each of several optical plumbing instruments in order to enable a comparison of their respective achievable precisions to be obtained under similar field conditions.

Statistical tests should be applied to determine whether the experimental standard deviation, s, obtained belongs to the population of the instrumentation's theoretical standard deviation, σ, whether two tested samples belong to the same population, whether the standard deviation of the x-component is equal to the standard deviation of the y-component and whether the line of sight is coincident with the plumb line.

These deviations of the line of sight do not affect the standard deviation obtained by the test procedure as described in Clause 7.

The same test procedure is applied to laser-emitting plumbing instrument, but with an observation means required separately for observing laser spot on the target.

7 Test procedure

7.1 Test configuration

A rectangular x-y grid as described in Clause 4, shall be established at the plumbing height, h, similar to that of the intended plumbing operation. Figure 1 shows an example of an x-y grid with an interval of 2 mm and a numbering that avoids exchanges of x-y values. This graduated plate shall be levelled approximately and located vertically above or below the mark over or under which the plumbing instrument is centred.

The orientation of the cross hairs of the telescope shall be parallel to the axes of the target plate within the accuracy of setting.

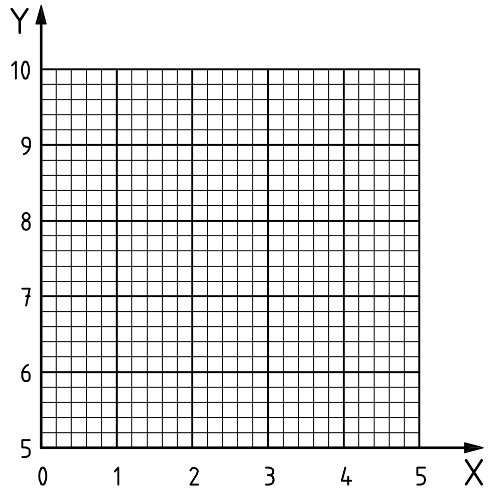


Figure 1 — Example of an x-y grid

7.2 Measurement

Before commencing the measurements, the instrument shall be allowed to acclimatize to the ambient temperature. The time required is about 2 min per degree Celsius temperature difference. Furthermore, the user shall check and adjust the collimation error before the test as specified by the manufacturer.

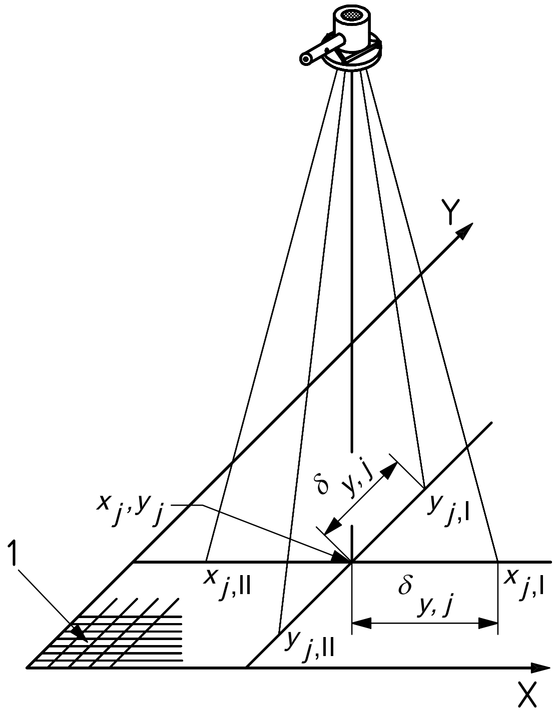
Three series of measurements ($m = 3$, for $i = 1,\ldots, m$) shall be carried out. Each series shall consist of $n = 10$ (for $j = 1,\ldots, n$) sets of measurements. Between the particular series of measurements the instrument has to be lifted and to be set up again. When setting up the instrument, special care shall be taken for centring the instrument above the ground point respectively below the mark above it.

Any set of measurements consists of two observations, $x_{j,\mathrm{I}}$ and $x_{j,\mathrm{II}}$, with the telescope in diametrically opposite positions I and II (e.g. pointing along the $+x$-axis and the $-x$-axis) and of two further observations $y_{j,\mathrm{I}}$ and $y_{j,\mathrm{II}}$, with the telescope again in diametrically opposite positions I and II (e.g. pointing along the $+y$-axis and the $-y$-axis).

7.3 Calculation

The measurements of each series are evaluated separately. The index i for the ith series is added only to the symbols of the final results.

First, the differences of the readings, $x_{j,\mathrm{I}}$ and $x_{j,\mathrm{II}}$, respectively $y_{j,\mathrm{I}}$ and $y_{j,\mathrm{II}}$, in the telescope positions I and II are calculated and divided by 2. These values δx_j, δy_j are the deviations from the plumb line (see Figure 2):



Key

1 grid plate

Figure 2 — Explanatory figure for the calculations

$$\delta x_j = \frac{1}{2} \times \left(x_{j,\text{I}} - x_{j,\text{II}} \right); \quad j = 1, \ldots, 10 \tag{2}$$

$$\delta y_j = \frac{1}{2} \times \left(y_{j,\text{I}} - y_{j,\text{II}} \right); \quad j = 1, \ldots, 10 \tag{3}$$

The following step is the calculation of the quasi-observations, x_j and y_j:

$$x_j = \frac{1}{2} \times \left(x_{j,\text{I}} + x_{j,\text{II}} \right); \quad j = 1, \ldots, 10 \tag{4}$$

$$y_j = \frac{1}{2} \times \left(y_{j,\text{I}} + y_{j,\text{II}} \right); \quad j = 1, \ldots, 10 \tag{5}$$

where

x_j is the mean value of the observations, $x_{j,\text{I}}$ and $x_{j,\text{II}}$;

y_j is the mean value of the observations, $y_{j,\text{I}}$ and $y_{j,\text{II}}$.

The mean values of the quasi-observations over all ten sets are:

$$\bar{x} = \frac{1}{10} \times \sum_{j=1}^{10} x_j \tag{6}$$

$$\overline{y} = \frac{1}{10} \times \sum_{j=1}^{10} y_j \qquad (7)$$

$$\overline{\delta x} = \frac{1}{10} \times \sum_{j=1}^{10} \delta x_j \qquad (8)$$

$$\overline{\delta y} = \frac{1}{10} \times \sum_{j=1}^{10} \delta y_j \qquad (9)$$

With the mean values, $\overline{x}$ and $\overline{y}$, the residuals of the quasi-observations, $r_{x,j}$ and $r_{y,j}$, are calculated:

$$r_{x,j} = \overline{x} - x_j \qquad (10)$$

$$r_{y,j} = \overline{y} - y_j \qquad (11)$$

The final results of the ith series of measurements are:

$$\sum r_{x,\,i}^2 = \sum_{j=1}^{10} r_{x,\,i,\,j}^2 \qquad (12)$$

$$\sum r_{y,\,i}^2 = \sum_{j=1}^{10} r_{y,\,i,\,j}^2 \qquad (13)$$

$$\sum r_i{}^2 = \sum r_{x,\,i}^2 + \sum r_{y,\,i}^2 \qquad (14)$$

$$\nu_{x,i} = \nu_{y,i} = 10 - 1 = 9 \qquad (15)$$

$$\nu_i = 20 - 2 = 18 \qquad (16)$$

$$s_{x,\,i} = \sqrt{\frac{\sum r_{x,\,i}^2}{\nu_{x,\,i}}} = \sqrt{\frac{\sum r_{x,\,i}^2}{9}} \qquad (17)$$

$$s_{y,\,i} = \sqrt{\frac{\sum r_{y,\,i}^2}{\nu_{y,\,i}}} = \sqrt{\frac{\sum r_{y,\,i}^2}{9}} \qquad (18)$$

$$s_i = \sqrt{\frac{\sum r_i{}^2}{\nu_i}} = \sqrt{\frac{\sum r_i{}^2}{18}} \qquad (19)$$

where

$\sum r_{x,\,i}^2$ is the sum of the squares of the residuals in x-direction;

$\sum r_{y,\,i}^2$ is the sum of the squares of the residuals in y-direction;

$\sum r_i^2$ is the total sum of the squares of the residuals;

$\nu_{x,j} = \nu_{y,j}$ is the number of degrees of freedom for the x- and y-components, respectively;

ν_i is the number of degrees of freedom;

$s_{x,i}$ is the standard deviation of the x-component of a transferred point for the plumbing height, h, determined in both telescope positions;

$s_{y,i}$ is the standard deviation of the y-component of a transferred point for the plumbing height, h, determined in both telescope positions;

s_i is the standard deviation of a transferred point for the plumbing height, h, determined in both telescope positions.

In order to obtain more meaningful results of the test procedure, it is suggested to calculate in addition the x- and y-components of the experimental standard deviation separately:

$$\sum_{i=1}^{3} \nu_{x,i} = \sum_{i=1}^{3} \nu_{y,i} = 27 \tag{20}$$

$$s_x = \sqrt{\frac{\sum r_{x,1}^2 + \sum r_{x,2}^2 + \sum r_{x,3}^2}{\nu_{x,1} + \nu_{x,2} + \nu_{x,3}}} \tag{21}$$

$$s_y = \sqrt{\frac{\sum r_{y,1}^2 + \sum r_{y,2}^2 + \sum r_{y,3}^2}{\nu_{y,1} + \nu_{y,2} + \nu_{y,3}}} \tag{22}$$

The total sum of degrees of freedom amounts to:

$$\nu = \sum_{i=1}^{3} \nu_i = 54 \tag{23}$$

and the standard deviation of a transferred point for the plumbing height, h, determined once in both telescope positions, calculated from the measurements of all series:

$$s = \sqrt{\frac{\sum r_1^2 + \sum r_2^2 + \sum r_3^2}{\nu}} \tag{24}$$

The measure of precision reads as follows:

$$s_{\text{ISO-plumb}} = \frac{s}{h} \quad [\text{or } = s \text{ (at } h)] \tag{25}$$

The estimated deviation of the line of sight from the plumb line, δ, can be evaluated by the quasi-observations, $\overline{\delta x}$, $\overline{\delta y}$, of each series i:

$$\delta_x = \frac{\sum_{i=1}^{3} \overline{\delta x_i}}{3} \tag{26}$$

$$\delta_y = \frac{\sum_{i=1}^{3} \overline{\delta y_i}}{3} \tag{27}$$

$$\delta = \sqrt{\delta_x^2 + \delta_y^2} \tag{28}$$

The experimental standard deviation of the deviation, δ, is calculated as follows:

$$s_\delta = s\left(\frac{1}{\sqrt{3} \times \sqrt{10}}\right) \tag{29}$$

8 Statistical tests

8.1 General

Statistical tests are recommended for the test procedure given in this part of ISO 17123.

For the interpretation of the results, statistical tests shall be carried out using:

— the experimental standard deviation, s, of one plumbing operation carried out in both telescope positions,

— the deviation, δ, of the line of sight and its experimental standard deviation, s_δ; for dual-axis-compensator instruments, δx and δy should be separately investigated with their experimental standard deviations $s_{\delta x}$ and $s_{\delta y}$,

in order to answer the following questions (see Table 1).

a) Is the calculated experimental standard deviation, s, smaller than the value, σ, stated by the manufacturer or smaller than another predetermined value, σ?

b) Do two experimental standard deviations, s and $\tilde{s}$, as determined from two different samples of measurements belong to the same population assuming that both samples have the same plumbing height, h, and the same number of degrees of freedom, ν?

The experimental standard deviations, s and $\tilde{s}$, may be obtained from:

1) two samples of measurements by the same instrument but different observers, or

2) two samples of measurements by the same instrument at different times, or

3) two samples of measurements by different instruments.

c) Is the standard deviation, s_x, of the x-component equal to the standard deviation, s_y, of the y-component of the result of the plumbing operation?

d) Is the deviation, δ, of the line of sight equal to zero?

For the following tests, a confidence level of $1 - \alpha = 0{,}95$ is assumed.

Table 1 — Statistical tests

Question	Null hypothesis	Alternative hypothesis
a)	$s \leqslant \sigma$	$s > \sigma$
b)	$\sigma = \tilde{\sigma}$	$\sigma \neq \tilde{\sigma}$
c)	$\sigma_x = \sigma_y$	$\sigma_x \neq \sigma_y$
d)	$\delta = 0$	$\delta \neq 0$

8.2 Response to Question a) in 8.1

The null hypothesis stating that the experimental standard deviation, s, is smaller than or equal to a theoretical or a predetermined value, σ, is not rejected if the following condition is fulfilled:

$$s \leqslant \sigma \sqrt{\frac{\chi^2_{1-\alpha}(\nu)}{\nu}} \quad (30)$$

$$s \leqslant \sigma \sqrt{\frac{\chi^2_{0,95}(54)}{54}} \quad (31)$$

$$\chi^2_{0,95}(54) = 72{,}15 \quad (32)$$

$$s \leqslant \sigma \sqrt{\frac{72{,}15}{54}} \quad (33)$$

$$s \leqslant \sigma \times 1{,}16 \quad (34)$$

Otherwise, the null hypothesis is rejected.

8.3 Response to Question b) in 8.1

In the case of two different samples, a test indicates whether the experimental standard deviations, s and $\tilde{s}$, belong to the same population. The corresponding null hypothesis, $\sigma = \tilde{\sigma}$, is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{1-\alpha/2}(\nu_1, \nu_2)} \leqslant \frac{s^2}{\tilde{s}^2} \leqslant F_{1-\alpha/2}(\nu_1, \nu_2) \quad (35)$$

$$\frac{1}{F_{0,975}(54, 54)} \leqslant \frac{s^2}{\tilde{s}^2} \leqslant F_{0,975}(54, 54) \quad (36)$$

$$F_{0,975}(54, 54) = 1{,}71 \quad (37)$$

$$0{,}58 \leqslant \frac{s^2}{\tilde{s}^2} \leqslant 1{,}71 \quad (38)$$

Otherwise, the null hypothesis is rejected.

8.4 Response to Question c) in 8.1

The null hypothesis stating that the experimental standard deviations, s_x and s_y, belong to the same population is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{1-\alpha/2}(\nu_x, \nu_y)} \leqslant \frac{s_x^2}{s_y^2} \leqslant F_{1-\alpha/2}(\nu_x, \nu_y) \quad (39)$$

$$\nu_x = \nu_y = 27 \quad (40)$$

$$\frac{1}{F_{0,975}(27,27)} \leqslant \frac{s_x^2}{s_y^2} \leqslant F_{0,975}(27,27) \tag{41}$$

$$F_{0,975}(27,27) = 2,16 \tag{42}$$

$$0,46 \leqslant \frac{s_x^2}{s_y^2} \leqslant 2,16 \tag{43}$$

Otherwise, the null hypothesis is rejected.

8.5 Response to Question d) in 8.1

The hypothesis of coincidence of the line of sight with the vertical plane respectively the plumb line (null hypothesis of δ) is not rejected if the following condition is fulfilled:

$$|\delta| \leqslant s_\delta \times t_{1-\alpha/2}(\nu) \tag{44}$$

$$|\delta| \leqslant s_\delta \times t_{0,975}(54) \tag{45}$$

$$s_\delta = \frac{s}{\sqrt{3} \times \sqrt{10}} \tag{46}$$

$$t_{0,975}(54) = 2,00 \tag{47}$$

$$|\delta| \leqslant \frac{s}{\sqrt{3} \times \sqrt{10}} \times 2,00 \tag{48}$$

$$|\delta| \leqslant s \times 0,37 \tag{49}$$

Otherwise, the null hypothesis is rejected.

The number of degrees of freedom and, thus, the corresponding test values $\chi^2_{1-\alpha}(\nu)$, $F_{1-\alpha/2}(\nu,\nu)$ and $t_{1-\alpha/2}(\nu)$ (taken from reference books on statistics) change if a different number of measurements is analysed.

Annex A
(informative)

Example of the test procedure

A.1 Measurements

Table A.1 contains in the Columns 2 to 5 the measured values of ten sets of readings (j) at the x-y grid plate (target plate) in x- and y-direction, each in telescope Positions I and II (measured values $x_{j,\text{I}}$, $y_{j,\text{I}}$, $x_{j,\text{II}}$ and $y_{j,\text{II}}$) of the series of measurements No. 1 (the series of measurements No. 2, and No. 3 were not printed).

Table A.1 — Measurements and residuals

Reading	Measured values in telescope positions I and II				Deviations from the plumb line		Mean value of observations		Residuals			
1	2	3	4	5	6	7	8	9	10	11	12	13
j	$x_{j,\text{I}}$	$y_{j,\text{I}}$	$x_{j,\text{II}}$	$y_{j,\text{II}}$	δx_j	δy_j	x_j	y_j	$r_{x,j}$	$r_{y,j}$	$r_{x,j}^2$	$r_{y,j}^2$
	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm^2	mm^2
1	36,7	71,8	37,5	72,3	–0,40	–0,25	37,10	72,05	0,06	–0,04	0,003 6	0,001 6
2	36,8	71,8	37,3	72,3	–0,25	–0,25	37,05	72,05	0,11	–0,04	0,012 1	0,001 6
3	36,9	71,8	37,4	72,3	–0,25	–0,25	37,15	72,05	0,01	–0,04	0,000 1	0,001 6
4	36,9	71,8	37,4	72,2	–0,25	–0,20	37,15	72,00	0,01	0,01	0,000 1	0,000 1
5	36,9	71,8	37,6	72,2	–0,35	–0,20	37,25	72,00	–0,09	0,01	0,008 1	0,000 1
6	36,8	71,7	37,6	72,2	–0,40	–0,25	37,20	71,95	–0,04	0,06	0,001 6	0,003 6
7	36,8	71,8	37,4	72,2	–0,30	–0,20	37,10	72,00	0,06	0,01	0,003 6	0,000 1
8	36,8	71,7	37,8	72,2	–0,50	–0,25	37,30	71,95	–0,14	0,06	0,019 6	0,003 6
9	36,7	71,9	37,3	72,2	–0,30	–0,15	37,00	72,05	0,16	–0,04	0,025 6	0,001 6
10	36,9	71,8	37,6	72,2	–0,35	–0,20	37,25	72,00	–0,09	0,01	0,008 1	0,000 1
$\sum$	368,2	717,9	374,9	722,3	–3,35	–2,20	371,55	720,10	0,05	0,00	0,082 5	0,014 0

Measurement conditions:	
Set:	i = 1
Observer:	S. Miller
Weather:	cloudy, +10 °C
Instrument type and number:	NN xxx 630401
Magnification of the telescope:	Γ = 31,5
Plumbing height:	10,1 m
Interval of the grid plate:	1 mm
Date:	1999-04-15

A.2 Calculations

First, the differences, δx_j and δy_j, according to Equations (2) and (3) (see Columns 6 and 7 in Table A.1) and the quasi-observations, x_j and y_j, are calculated [according to Equations (4) and (5), see Columns 8 and 9 in Table A.1]. With the sums of each series of 10 observations (see last lines of the Columns 2 to 9), the mean values of the first series of observations are calculated according to Equations (6) to (9):

$$\sum_{j=1}^{10} x_j = 371{,}55 \text{ mm} \qquad \bar{x} = 37{,}16 \text{ mm}$$

$$\sum_{j=1}^{10} y_j = 720{,}10 \text{ mm} \qquad \bar{y} = 72{,}01 \text{ mm}$$

$$\sum_{j=1}^{10} \delta x_j = -3{,}35 \text{ mm} \qquad \overline{\delta x} = -0{,}34 \text{ mm}$$

$$\sum_{j=1}^{10} \delta y_j = -2{,}20 \text{ mm} \qquad \overline{\delta y} = -0{,}22 \text{ mm}$$

Using the mean values, $\bar{x}$ and $\bar{y}$, the residuals, $r_{x,j}$ and $r_{y,j}$, according to Equations (10) and (11) and their squares (see Columns 10 to 13 in Table A.1) are calculated and subsequently the results of the first series of measurements according to Equations (12) to (14):

$$\sum r_{x,1}^2 = 0{,}082\,5 \text{ mm}^2$$

$$\sum r_{y,1}^2 = 0{,}014\,0 \text{ mm}^2$$

$$\sum r_1{}^2 = 0{,}096\,5 \text{ mm}^2$$

The corresponding standard deviations, $s_{x,1}$, $s_{y,1}$ and s_1, of the first series of measurements are calculated according to Equations (17) to (19), with the degrees of freedom (15) and (16):

$$s_{x,1} = \sqrt{\frac{0{,}082\,5 \text{ mm}^2}{9}} = 0{,}10 \text{ mm}$$

$$s_{y,1} = \sqrt{\frac{0{,}014\,0 \text{ mm}^2}{9}} = 0{,}04 \text{ mm}$$

$$s_1 = \sqrt{\frac{0{,}0965 \text{ mm}^2}{18}} = 0{,}07 \text{ mm}$$

The final results for Series 2 and 3 are:

$$\sum {}_{x,2}^2 = 0{,}037\,2 \text{ mm}^2$$

$$\sum {}_{y,2}^2 = 0{,}083\,6 \text{ mm}^2$$

$$\sum {}_{x,3}^2 = 0{,}020\,0 \text{ mm}^2$$

$$\sum {}_{y,3}^2 = 0{,}029\,2 \text{ mm}^2$$

$s_{x,2} = 0{,}06$ mm

$s_{y,2} = 0{,}10$ mm

$s_2 = 0{,}08$ mm

$s_{x,3} = 0{,}05$ mm

$s_{y,3} = 0{,}06$ mm

$s_3 = 0{,}05$ mm

$\overline{\delta x_2} = -0{,}40$ mm

$\overline{\delta y_2} = -0{,}30$ mm

$\overline{\delta x_3} = -0{,}44$ mm

$\overline{\delta y_3} = -0{,}24$ mm

With the total sum of degrees of freedom equal to 54, then according to Equation (23), the experimental standard deviation, s, is calculated according to Equation (24):

$$s = \sqrt{\frac{0{,}096\,5 \text{ mm}^2 + 0{,}120\,8 \text{ mm}^2 + 0{,}049\,2 \text{ mm}^2}{54}} = 0{,}07 \text{ mm}$$

The experimental standard deviation, $s_{\text{ISO-plumb}}$, is obtained according to Equation (25):

$$s_{\text{ISO-plumb}} = \frac{s}{h} = 1 : \frac{h}{s} = 1 : 144\,286 \approx 1 : 140\,000$$

Furthermore, the parameters derived from all series of measurements are calculated according to Equations (26) and (27):

$$\delta_x = \frac{-0{,}34 \text{ mm} - 0{,}40 \text{ mm} - 0{,}44 \text{ mm}}{3} = -0{,}39 \text{ mm}$$

$$\delta_y = \frac{-0{,}22 \text{ mm} - 0{,}30 \text{ mm} - 0{,}24 \text{ mm}}{3} = -0{,}25 \text{ mm}$$

and in some cases, according to Equation (28):

$\delta = 0{,}46$ mm

The experimental standard deviation of δ is calculated according to Equation (29) as

$$s_\delta = s\left(\frac{1}{\sqrt{3} \times \sqrt{10}}\right) = 0{,}01 \text{ mm}$$

A.3 Statistical tests

A.3.1 Statistical test according to Question a) in 8.1

$\sigma = 1{:}100\,000 = 0{,}000\,010\,0$

$s_{\text{ISO-plumb}} = 1{:}140\,000 = 0{,}000\,007\,1$

$\nu = 54$

$0{,}000\,007\,1 \leqslant 0{,}000\,010\,0 \times 1{,}16$

$0{,}000\,007\,1 \leqslant 0{,}000\,011\,6$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviation, $s_{\text{ISO-plumb}}$, is smaller than or equal to the manufacturer's value, σ, is not rejected at the confidence level of 95 %.

A.3.2 Statistical test according to Question b) in 8.1

$s = 0{,}000\,007\,1$

$\tilde{s} = 0{,}000\,006\,0$ (assumption)

$\nu = 54$

$$0{,}58 \leqslant \frac{5{,}0 \times 10^{-11}}{3{,}6 \times 10^{-11}} \leqslant 1{,}71$$

$$0{,}58 \leqslant 1{,}39 \leqslant 1{,}71$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviations, s and $\tilde{s}$, belong to the same population is not rejected at the confidence level of 95 %.

A.3.3 Statistical test according to Question c) in 8.1

$s_x = 0{,}072$ mm

$s_y = 0{,}068$ mm

$\nu_x = \nu_y = 27$

$$0{,}46 \leqslant \frac{0{,}005\,2}{0{,}004\,6} \leqslant 2{,}16$$

$$0{,}46 \leqslant 1{,}12 \leqslant 2{,}16$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviations, s_x and s_y, belong to the same population is not rejected at the confidence level of 95 %.

A.3.4 Statistical test according to Question d) in 8.1

$s = 0{,}07$ mm

$\delta = 0{,}46$ mm

$s_\delta = 0{,}01$ mm

$\nu = 54$

$$0{,}46 \text{ mm} \leqslant 0{,}01 \text{ mm} \times 2{,}00$$

$$0{,}46 \text{ mm} \leqslant 0{,}02 \text{ mm}$$

Since the above condition is not fulfilled, the null hypothesis stating that the deviation, δ, of the line of sight is equal to zero is rejected at the confidence level of 95 %, i.e. the instrument should be adjusted.

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