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**Optics and optical instruments —  
Field procedures for testing geodetic  
and surveying instruments —**

**Part 5:  
Total stations**

*Optique et instruments d'optique — Méthodes d'essai sur site des  
instruments géodésiques et d'observation —*

*Partie 5: Stations totales*



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Published in Switzerland

# Contents

|   | Page      |
|---|-----------|
| <b>Foreword</b> .....   | <b>iv</b> |
| <b>Introduction</b> .....   | <b>v</b>  |
| <b>1 Scope</b> .....  | <b>1</b>  |
| <b>2 Normative references</b> .....   | <b>1</b>  |
| <b>3 Terms and definitions</b> .....  | <b>2</b>  |
| <b>4 General</b> .....  | <b>2</b>  |
| 4.1 Requirement.....  | 2         |
| 4.2 Procedure 1: Simplified test procedure.....   | 2         |
| 4.3 Procedure 2: Full test procedure.....   | 2         |
| <b>5 Simplified test procedure</b> .....  | <b>3</b>  |
| 5.1 Configuration of the test field.....  | 3         |
| 5.2 Measurement.....  | 3         |
| 5.3 Calculation.....  | 4         |
| <b>6 Full test procedure</b> .....  | <b>5</b>  |
| 6.1 Configuration of the test field.....  | 5         |
| 6.2 Measurement.....  | 6         |
| 6.3 Calculation.....  | 7         |
| 6.4 Statistical tests.....  | 11        |
| 6.5 Combined standard uncertainty evaluation (Type A and Type B).....   | 13        |
| <b>Annex A (informative) Example of the simplified test procedure</b> .....   | <b>15</b> |
| <b>Annex B (informative) Example of the full test procedure</b> .....   | <b>17</b> |
| <b>Annex C (informative) Example for the calculation of a combined uncertainty budget (Type A and Type B)</b> ..... | <b>23</b> |
| <b>Annex D (informative) Sources which are not included in uncertainty evaluation</b> .....                         | <b>26</b> |
| <b>Bibliography</b> .....   | <b>27</b> |

## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 17123-5 was prepared by Technical Committee ISO/TC 172, *Optics and optical instruments*, Subcommittee SC 6, *Geodetic and surveying instruments*.

This second edition cancels and replaces the first edition (ISO 17123-5:2005), which has been technically revised.

ISO 17123 consists of the following parts, under the general title *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments*:

- *Part 1: Theory*
- *Part 2: Levels*
- *Part 3: Theodolites*
- *Part 4: Electro-optical distance meters (EDM measurements to reflectors)*
- *Part 5: Total stations*
- *Part 6: Rotating lasers*
- *Part 7: Optical plumbing instruments*
- *Part 8: GNSS field measurement systems in real-time kinematic (RTK)*

[Annexes A](#), [B](#) and [C](#) of this part of ISO 17123 are for information only.

## Introduction

This part of ISO 17123 specifies field procedures for adoption when determining and evaluating the uncertainty of measurement results obtained by geodetic instruments and their ancillary equipment, when used in building and surveying measuring tasks. Primarily, these tests are intended to be field verifications of suitability of a particular instrument for the immediate task. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

The definition and concept of uncertainty as a quantitative attribute to the final result of measurement was developed mainly in the last two decades, even though error analysis has already long been a part of all measurement sciences. After several stages, the CIPM (Comité Internationale des Poids et Mesures) referred the task of developing a detailed guide to ISO. Under the responsibility of the ISO Technical Advisory Group on Metrology (TAG 4), and in conjunction with six worldwide metrology organizations, a guidance document on the expression of measurement uncertainty was compiled with the objective of providing rules for use within standardization, calibration, laboratory, accreditation and metrology services. ISO/IEC Guide 98-3 was first published in 1995.

With the introduction of uncertainty in measurement in ISO 17123 (all parts), it is intended to finally provide a uniform, quantitative expression of measurement uncertainty in geodetic metrology with the aim of meeting the requirements of customers.

ISO 17123 (all parts) provides not only a means of evaluating the precision (experimental standard deviation) of an instrument, but also a tool for defining an uncertainty budget, which allows for the summation of all uncertainty components, whether they are random or systematic, to a representative measure of accuracy, i.e. the combined standard uncertainty.

ISO 17123 (all parts) therefore provides, for defining for each instrument investigated by the procedures, a proposal for additional, typical influence quantities, which can be expected during practical use. The customer can estimate, for a specific application, the relevant standard uncertainty components in order to derive and state the uncertainty of the measuring result.



# Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

## Part 5: Total stations

### 1 Scope

This part of ISO 17123 specifies field procedures to be adopted when determining and evaluating the precision (repeatability) of coordinate measurement of total stations and their ancillary equipment when used in building and surveying measurements. Primarily, these tests are intended to be field verifications of the suitability of a particular instrument for the immediate task at hand and to satisfy the requirements of other standards. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

These field procedures have been developed specifically for *in situ* applications without the need for special ancillary equipment and are purposely designed to minimize atmospheric influences.

### 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 4463-1, *Measurement methods for building — Setting-out and measurement — Part 1: Planning and organization, measuring procedures, acceptance criteria*

ISO 7077, *Measuring methods for building — General principles and procedures for the verification of dimensional compliance*

ISO 7078, *Building construction — Procedures for setting out, measurement and surveying — Vocabulary and guidance notes*

ISO 9849, *Optics and optical instruments — Geodetic and surveying instruments — Vocabulary*

ISO 12858-2, *Optics and optical instruments — Ancillary devices for geodetic instruments — Part 2: Tripods*

ISO 17123-1, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 1: Theory*

ISO 17123-3, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 3: Theodolites*

ISO 17123-4, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 4: Electro-optical distance meters (EDM measurements to reflectors)*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM: 1995)*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

### 3 Terms and definitions

For the purpose of this document, the terms and definitions given in ISO 3534-1, ISO 4463-1, ISO 7077, ISO 7078, ISO 9849, ISO 17123-1, the GUM and the VIM apply.

### 4 General

#### 4.1 Requirement

Before commencing the measurements, it is important that the operator ensures that the precision in use of the measuring equipment is appropriate for the intended measuring task.

The total station and its ancillary equipment shall be in known and acceptable states of permanent adjustment according to the methods specified in the manufacturer's reference manual, and used tripods with reflectors as recommended by the manufacturer.

The coordinates are considered as observables because on modern total stations they are selectable as output quantities.

All coordinates shall be measured on the same day. The instrument should always be levelled carefully. The correct zero-point correction of the reflector prism shall be used.

The results of these tests are influenced by meteorological conditions, especially by the gradient of temperature. An overcast sky and low wind speed guarantee the most favourable weather conditions. Actual meteorological data shall be measured in order to derive atmospheric corrections, which shall be added to the raw distances. The particular conditions to be taken into account may vary depending on where the tasks are to be undertaken. These conditions shall include variations in air temperature, wind speed, cloud cover and visibility. Note should also be taken of the actual weather conditions at the time of measurement and the type of surface above which the measurements are made. The conditions chosen for the tests should match those expected when the intended measuring task is actually carried out (see ISO 7077 and ISO 7078).

Tests performed in laboratories would provide results which are almost unaffected by atmospheric influences, but the costs for such tests are very high, and therefore they are not practicable for most users. In addition, laboratory tests yield precisions much higher than those that can be obtained under field conditions.

This part of ISO 17123 describes two different field procedures as given in Clauses 5 and 6. The operator shall choose the procedure which is most relevant to the project's particular requirements.

To evaluate angle measurement and distance measurement separately, see ISO 17123-3 and ISO 17123-4.

#### 4.2 Procedure 1: Simplified test procedure

The simplified test procedure provides an estimate as to whether the precision of a given total station is within the specified permitted deviation in accordance with ISO 4463-1.

The simplified test procedure is based on a limited number of measurements. This test procedure relies on measurements of x-, y- and z-coordinates in a test field without nominal values. The maximum difference from mean value is calculated as an indicator for the precision.

A significant standard deviation cannot be obtained. If a more precise assessment of the total station under field conditions is required, it is recommended to adopt the more rigorous full test procedure as given in Clause 6.

#### 4.3 Procedure 2: Full test procedure

The full test procedure shall be adopted to determine the best achievable measure of precision of a total station and its ancillary equipment under field conditions.



This procedure is based on measurements of coordinates in a test field without nominal values. The experimental standard deviation of the coordinate measurement of a single point is determined from least squares adjustments.

The full test procedure given in Clause 6 of this part of ISO 17123 is intended for determining the measure of precision in use of a particular total station. This measure of precision in use is expressed in terms of the experimental standard deviations of a coordinate measured once in both face positions of the telescope;

$$s_{\text{ISO-TS-XY}}, s_{\text{ISO-TS-Z}}$$

Furthermore, this procedure may be used to determine:

the measure of precision in use of total stations by a single survey team with a single instrument and its ancillary equipment at a given time;

the measure of precision in use of a single instrument over time;

the measure of precision in use of each of several total stations in order to enable a comparison of their respective achievable precisions to be obtained under similar field conditions.

Statistical tests should be applied to determine whether the experimental standard deviations obtained belong to the population of the instrumentation's theoretical standard deviations and whether two tested samples belong to the same population.

## 5 Simplified test procedure

### 5.1 Configuration of the test field

Two target points ( $T_1$ ,  $T_2$ ) shall be set out as indicated in Figure 1. The targets should be firmly fixed on to the ground. The distance between two target points should be set longer than the average distance (e.g. 60 m) according to the intended measuring task. Their heights should be as different as the surface of the ground allows.

Two instrument stations ( $S_1$ ,  $S_2$ ) shall be set out approximately in line with two target points.  $S_1$  shall be set 5 m to 10 m away from  $T_1$  and in the opposite direction to  $T_2$ .  $S_2$  shall be set between two target points and 5 m to 10 m away from  $T_2$ .

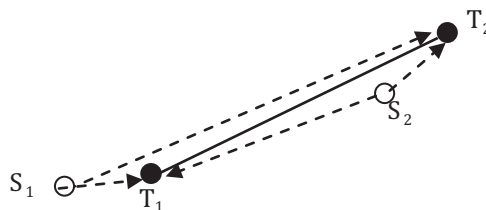


Figure 1 — Configuration of the test field

### 5.2 Measurement

One set consists of two measurements to each target point in one telescope face at one of the instrument stations.

The coordinates of the two target points shall be measured by 4 sets (telescope face: I – II – I – II) at the instrument station  $S_1$ . The instrument is shifted to station  $S_2$  and the same sequence of measurements is carried out. Station coordinates and the reference orientation of the station are discretionary in each set.

On-board or stand-alone software shall be used for the observations. It is preferable to use the same software which will be used for the practical work.

The sequence of the measurements is shown in Table 1.

**Table 1 — Sequence of the measurements for one series**

| Seq. No | Instrument station<br>i | Target point<br>j | Set<br>k | Telescope face | x                  | y                  | z                  |
|---------|-------------------------|-------------------|----------|----------------|--------------------|--------------------|--------------------|
| 1       | 1                       | 1                 | 1        | I              | x <sub>1,1,1</sub> | y <sub>1,1,1</sub> | z <sub>1,1,1</sub> |
| 2       |                         | 2                 |          |                | x <sub>1,2,1</sub> | y <sub>1,2,1</sub> | z <sub>1,2,1</sub> |
| 3       |                         | 1                 | 2        | II             | x <sub>1,1,2</sub> | y <sub>1,1,2</sub> | z <sub>1,1,2</sub> |
| 4       |                         | 2                 |          |                | x <sub>1,2,2</sub> | y <sub>1,2,2</sub> | z <sub>1,2,2</sub> |
| 5       |                         | 1                 | 3        | I              | x <sub>1,1,3</sub> | y <sub>1,1,3</sub> | z <sub>1,1,3</sub> |
| 6       |                         | 2                 |          |                | x <sub>1,2,3</sub> | y <sub>1,2,3</sub> | z <sub>1,2,3</sub> |
| 7       |                         | 1                 | 4        | II             | x <sub>1,1,4</sub> | y <sub>1,1,4</sub> | z <sub>1,1,4</sub> |
| 8       |                         | 2                 |          |                | x <sub>1,2,4</sub> | y <sub>1,2,4</sub> | z <sub>1,2,4</sub> |
| 9       | 2                       | 1                 | 1        | I              | x <sub>2,1,1</sub> | y <sub>2,1,1</sub> | z <sub>2,1,1</sub> |
| ⋮       | ⋮                       | ⋮                 |          | ;              |                    |                    |                    |
| 15      | 2                       | 1                 | 4        | II             | x <sub>2,1,4</sub> | y <sub>2,1,4</sub> | z <sub>2,1,4</sub> |
| 16      |                         | 2                 |          |                | x <sub>2,2,4</sub> | y <sub>2,2,4</sub> | z <sub>2,2,4</sub> |

### 5.3 Calculation

#### 5.3.1 x-, y-coordinates

The evaluation of the test results is given by the deviation of the horizontal distance of each set from the mean value of all measured horizontal distances.

Each horizontal distance between two target points  $l_{i,k}$  is calculated as

$$l_{i,k} = \sqrt{(x_{i,2,k} - x_{i,1,k})^2 + (y_{i,2,k} - y_{i,1,k})^2} \quad i = 1,2 \quad k = 1,2,3,4 \quad (1)$$

Their mean value  $L$  is calculated as

$$L = \frac{1}{8} \sum_{i=1}^2 \sum_{k=1}^4 l_{i,k} \quad (2)$$

The half values of the deviation of each distance from its mean value,  $r_{j,k}$  are calculated

$$r_{i,k} = \frac{l_{i,k} - L}{2} \quad i = 1,2 \quad k = 1,2,3,4 \quad (3)$$

The maximum value  $d_{xy}$  of the  $r_{i,k}$  is defined as

$$d_{xy} = \max |r_{i,k}| \quad i = 1,2 \quad k = 1,2,3,4 \quad (4)$$

### 5.3.2 z-coordinate

The height differences  $d_{z,i,k}$  between target points are calculated using measured z-coordinate values in each set.

$$d_{z,i,k} = z_{i,2,k} - z_{i,1,k} \quad i = 1, 2 \quad k = 1, 2, 3, 4 \quad (5)$$

The mean value  $a_z$  of height difference in all sets is

$$a_z = \frac{1}{8} \sum_{i=1}^2 \sum_{k=1}^4 d_{z,i,k} \quad (6)$$

The differences  $r_{z,i,k}$  between height differences of two target points and the mean value  $a_z$  are

$$r_{z,i,k} = d_{z,i,k} - a_z \quad i = 1, 2 \quad k = 1, 2, 3, 4 \quad (7)$$

Half of the maximum difference value  $d_z$  is calculated as

$$d_z = \frac{1}{2} \max |r_{z,i,k}| \quad (8)$$

### 5.3.3 Evaluation

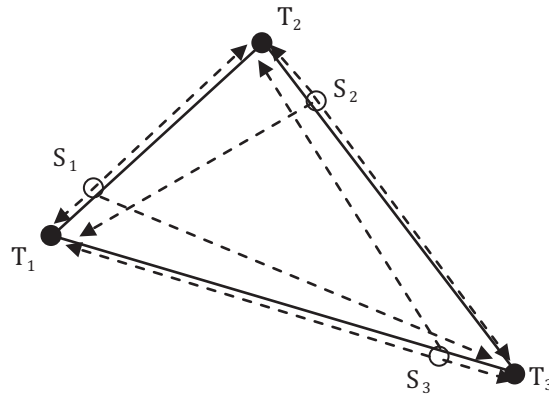
The differences  $d_{xy}$  and  $d_z$  shall be within the specified permitted deviation,  $p_{xy}$  and  $p_z$  respectively, (in accordance with ISO 4463-1 for the intended measuring task). If  $p_{xy}$  and  $p_z$  are not given, they shall be  $d_{xy} \leq 2,5 \times \sqrt{2} \times s_{\text{ISO-TS-XY}}$  and  $d_z \leq 2,5 \times \sqrt{2} \times s_{\text{ISO-TS-Z}}$  respectively, where  $s_{\text{ISO-TS-XY}}$  and  $s_{\text{ISO-TS-Z}}$  are the experimental standard deviations of the  $x,y$  and  $z$  measurements respectively, determined according to the full test procedure with the same instrument.

## 6 Full test procedure

### 6.1 Configuration of the test field

Three target points ( $T_1, T_2, T_3$ ) shall be set out at the corner of the triangle (see Figure 2). The targets should be firmly fixed on to the ground. The distances of target points should be different and at least one distance should be longer than the average distance (e.g. 60 m) according to the intended measuring task. Their heights should be as different as the surface of the ground allows.

Three instrument stations ( $S_1, S_2, S_3$ ) shall be set out close to each triangular side approximately 5 m to 10 m away from each target point.



**Figure 2 — Example of field configuration for full test**

## 6.2 Measurement

One set consists of three measurements to each target point with a single telescope face at each instrument station.

From the instrument stations  $S_1$ ,  $S_2$ ,  $S_3$ , the coordinates of the three target points shall be measured by four sets of observation sequences (telescope face: I – II – I – II).

The station coordinates and the orientation are discretionary for each station set up. These configurations should not be changed while measuring four sets of observations from the same station point.

On-board or stand-alone software shall be used for the observations. It is preferable to use the same software which will be used for the practical work.

The sequence of the measurements is shown in Table 2.

Table 2 — Sequence of the measurements for one series

| Seq. No | Instrument station<br>i | Target point<br>j | Set<br>k | Telescope face | x                  | y                  | z                  |
|---------|-------------------------|-------------------|----------|----------------|--------------------|--------------------|--------------------|
| 1       | 1                       | 1                 | 1        | I              | x <sub>1,1,1</sub> | y <sub>1,1,1</sub> | z <sub>1,1,1</sub> |
| 2       |                         | 2                 |          |                | x <sub>1,2,1</sub> | y <sub>1,2,1</sub> | z <sub>1,2,1</sub> |
| 3       |                         | 3                 |          |                | x <sub>1,3,1</sub> | y <sub>1,3,1</sub> | z <sub>1,3,1</sub> |
| 4       |                         | 1                 | 2        | II             | x <sub>1,1,2</sub> | y <sub>1,1,2</sub> | z <sub>1,1,2</sub> |
| 5       |                         | 2                 |          |                | x <sub>1,2,2</sub> | y <sub>1,2,2</sub> | z <sub>1,2,2</sub> |
| 6       |                         | 3                 |          |                | x <sub>1,3,2</sub> | y <sub>1,3,2</sub> | z <sub>1,3,2</sub> |
| 7       |                         | 1                 | 3        | I              | x <sub>1,1,3</sub> | y <sub>1,1,3</sub> | z <sub>1,1,3</sub> |
| 8       |                         | 2                 |          |                | x <sub>1,2,3</sub> | y <sub>1,2,3</sub> | z <sub>1,2,3</sub> |
| 9       |                         | 3                 |          |                | x <sub>1,3,3</sub> | y <sub>1,3,3</sub> | z <sub>1,3,3</sub> |
| 10      |                         | 1                 | 4        | II             | x <sub>1,1,4</sub> | y <sub>1,1,4</sub> | z <sub>1,1,4</sub> |
| 11      |                         | 2                 |          |                | x <sub>1,2,4</sub> | y <sub>1,2,4</sub> | z <sub>1,2,4</sub> |
| 12      |                         | 3                 |          |                | x <sub>1,3,4</sub> | y <sub>1,3,4</sub> | z <sub>1,3,4</sub> |
| 13      | 2                       | 1                 | 1        | I              | x <sub>2,1,1</sub> | y <sub>2,1,1</sub> | z <sub>2,1,1</sub> |
| ⋮       | ⋮                       | ⋮                 |          |                |                    |                    |                    |
| 34      | 3                       | 1                 | 4        | II             | x <sub>3,1,4</sub> | y <sub>3,1,4</sub> | z <sub>3,1,4</sub> |
| 35      |                         | 2                 |          |                | x <sub>3,2,4</sub> | y <sub>3,2,4</sub> | z <sub>3,2,4</sub> |
| 36      |                         | 3                 |          |                | x <sub>3,3,4</sub> | y <sub>3,3,4</sub> | z <sub>3,3,4</sub> |

### 6.3 Calculation

#### 6.3.1 x-, y-coordinates

Construction of the mathematical model of the triangle is carried out as follows.

Calculate the horizontal distances  $l_{i,3,k}$  between T<sub>1</sub> and T<sub>2</sub>;  $l_{i,1,k}$  between T<sub>2</sub> and T<sub>3</sub>;  $l_{i,2,k}$  between T<sub>3</sub> and T<sub>1</sub> respectively by measured coordinates  $(x_{i,j,k}, y_{i,j,k})$ .

$$l_{i,j,k} = \sqrt{(x_{i,j-1,k} - x_{i,j+1,k})^2 + (y_{i,j-1,k} - y_{i,j+1,k})^2} \quad (9)$$

$i = 1, 2, 3; j = 1, 2, 3$  (if  $j - 1$  is 0 or  $j + 1$  is 4, then replace it by 3 or 1 respectively);  $k = 1, 2, 3, 4$ .

The mean length of each side  $L_j$ :

$$L_j = \frac{1}{12} \sum_{i=1}^3 \sum_{k=1}^4 l_{i,j,k} \quad j = 1, 2, 3 \quad (10)$$

The coordinates of the mathematical model of the triangle M<sub>i</sub> ( $i = 1, 2, 3$ ) is defined based on M<sub>1</sub> = (0,0) and the line from M<sub>1</sub> to M<sub>2</sub> as the x-axis.

Coordinates of  $M_1$ :

$$M_1 (X_1, Y_1) = (0, 0) \tag{11}$$

Coordinates of  $M_2$ :

$$M_2 (X_2, Y_2) = (L_3, 0) \tag{12}$$

Coordinates of  $M_3$ :

$$M_3 (X_3, Y_3) = \left[ \frac{-(L_1^2) + L_2^2 + L_3^2}{2L_3}, \sqrt{L_2^2 - \left[ \frac{-(L_1^2) + L_2^2 + L_3^2}{2L_3} \right]^2} \right] \tag{13}$$

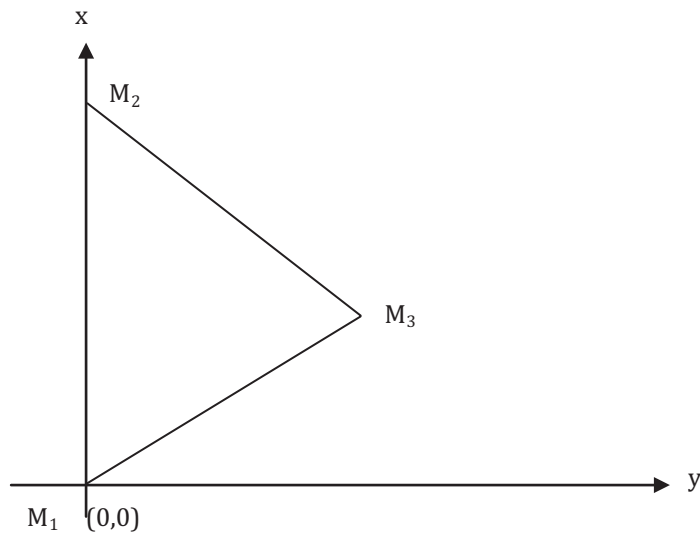


Figure 3 — Mathematical model of the triangle

The coordinates of the centre of gravity of the mathematical model,  $(X_g, Y_g)$ :

$$(X_g, Y_g) = \left\{ \frac{\sum_{j=1}^3 X_j}{3}, \frac{\sum_{j=1}^3 Y_j}{3} \right\} \quad (14)$$

The coordinates of the centre of gravity of the triangle obtained at each instrument station,  $(x_{g,i}, y_{g,i})$ :

$$(x_{g,i}, y_{g,i}) = \left( \frac{\sum_{j=1}^3 \sum_{k=1}^4 x_{i,j,k}}{12}, \frac{\sum_{j=1}^3 \sum_{k=1}^4 y_{i,j,k}}{12} \right) \quad i=1,2,3 \quad (15)$$

Shift the coordinates to coincide the centre of gravity of the mathematical model on the centre of gravity of the measured triangle.

The coordinates of the centre of gravity of the mathematical model  $(X_{t,i,j,k}, Y_{t,i,j,k})$  after the shift are calculated as

$$X_{t,i,j,k} = X_j + (x_{g,i} - X_{g,i}), \quad Y_{t,i,j,k} = Y_j + (y_{g,i} - Y_{g,i}) \quad i=1,2,3, \quad j=1,2,3, \quad k=1,2,3,4 \quad (16)$$

Rotate the mathematical model around the centre of gravity to minimize residuals of the apex coordinates between the mathematical model and respective measured triangles.

Rotation angle  $\theta_{i,k}$  is

$$\theta_{i,k} = \tan^{-1} \left( \frac{q_{i,k}}{p_{i,k}} \right) \quad i=1,2,3, \quad k=1,2,3,4 \quad (17)$$

$$q_{i,k} = \frac{\sum_{j=1}^3 \left( (X_{t,i,j,k} - x_{g,i}) \times (y_{i,j,k} - y_{g,i}) - (Y_{t,i,j,k} - y_{g,i}) \times (x_{i,j,k} - x_{g,i}) \right)}{\sum_{j=1}^3 \left( (X_{t,i,j,k} - x_{g,i})^2 + (Y_{t,i,j,k} - y_{g,i})^2 \right)} \quad (18)$$

$$p_{i,k} = \frac{\sum_{j=1}^3 \left( (X_{t,i,j,k} - x_{g,i}) \times (x_{i,j,k} - x_{g,i}) + (Y_{t,i,j,k} - y_{g,i}) \times (y_{i,j,k} - y_{g,i}) \right)}{\sum_{j=1}^3 \left( (X_{t,i,j,k} - x_{g,i})^2 + (Y_{t,i,j,k} - y_{g,i})^2 \right)} \quad (19)$$

Apex coordinates of mathematical model  $(X_{m,i,j,k}, Y_{m,i,j,k})$  after the rotation:

$$\begin{aligned} X_{m,i,j,k} &= x_{g,i} + \cos_{i,k} \times (X_{t,i,j,k} - x_{g,i}) - \sin_{i,k} \times (Y_{t,i,j,k} - y_{g,i}) \\ Y_{m,i,j,k} &= y_{g,i} + \sin_{i,k} \times (X_{t,i,j,k} - x_{g,i}) + \cos_{i,k} \times (Y_{t,i,j,k} - y_{g,i}) \quad i=1,2,3, \quad j=1,2,3, \quad k=1,2,3,4 \end{aligned} \quad (20)$$

Residuals ( $r_{x,i,j,k}, r_{y,i,j,k}$ ) of the coordinates of the measured triangles from those of the rotated mathematical model are

$$r_{x,i,j,k} = x_{i,j,k} - X_{m,i,j,k} \quad i = 1,2,3 \quad j = 1,2,3 \quad k = 1,2,3,4 \quad (21)$$

$$r_{y,i,j,k} = y_{i,j,k} - Y_{m,i,j,k} \quad i = 1,2,3 \quad j = 1,2,3 \quad k = 1,2,3,4 \quad (22)$$

The sum of squares of residuals is

$$\sum r_{xy}^2 = \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^4 (r_{x,i,j,k}^2 + r_{y,i,j,k}^2) \quad (23)$$

Since there are 3 sides of the mathematical model, 6 [= 2 (components) × 3 (instrument stations)] centre of gravity points of the measured triangle and 12 [= 4 (sets) × 3 (instrument stations)] rotation parameters, the number of unknown parameters  $\nu = 3 + 6 + 12 = 21$ . Thus the number of degrees of freedom is

$$\nu_{XY} = 72 - 21 = 51 \quad (24)$$

The experimental standard deviation is

$$s_{XY} = \sqrt{\frac{\sum r_{xy}^2}{51}} \quad (25)$$

Finally, the standard uncertainty of x-, y-coordinates is:

$$u_{\text{ISO-TS-XY}} = s_{XY} \quad (26)$$



### 6.3.2 z-coordinate

The height difference between  $T_1$  and  $T_2$  (and  $T_3$ ) is calculated using measured z-values for each set.

$$d_{z,i,j,k} = z_{i,j,k} - z_{i,1,k} \quad (27)$$

$$i = 1, 2, 3 \quad j = 2, 3 \quad k = 1, 2, 3, 4$$

The mean values of  $d_{z,i,2,k}$  and  $d_{z,i,3,k}$  are

$$a_{zj} = \frac{1}{12} \sum_{i=1}^3 \sum_{k=1}^4 d_{z,i,j,k} \quad j = 2, 3 \quad (28)$$

The residuals  $r_{z,i,j,k}$  of the height differences  $d_{z,i,2,k}$ ,  $d_{z,i,3,k}$  from obtained mean values for each set of measurements are calculated as

$$r_{z,i,j,k} = d_{z,i,j,k} - a_{zj} \quad i = 1, 2, 3, \quad j = 2, 3, \quad k = 1, 2, 3, 4 \quad (29)$$

The sum of the squares of the residuals is obtained by

$$\sum r_z^2 = \sum_{i=1}^3 \sum_{j=2}^3 \sum_{k=1}^4 r_{z,i,j,k}^2 \quad (30)$$

The number of degrees of freedom is

$$v_z = 24 - 2 = 22 \quad (31)$$

Finally, the standard deviation of z-coordinate is

$$s_z = \sqrt{\frac{\sum r_z^2}{22}} \quad (32)$$

Its standard uncertainty is

$$u_{\text{ISO-TS-Z}} = s_z$$

## 6.4 Statistical tests

### 6.4.1 General

Statistical tests are applicable for the full test procedure only.

For the interpretation of the results, statistical tests shall be carried out using the experimental standard deviation of a coordinate measured on the test triangle in order to answer the following questions (see Table 3).

- Is the calculated experimental standard deviation,  $s$ , smaller than or equal to a corresponding value,  $\sigma$ , stated by the manufacturer or smaller than another predetermined value,  $\sigma$ ?
- Do two experimental standard deviations,  $s$  and  $\tilde{s}$ , as determined from two different samples of measurements, belong to the same population, assuming that both samples have the same number of degrees of freedom,  $\nu$ ?

The experimental standard deviations,  $s$  and  $\tilde{s}$ , may be obtained from

two samples of measurements by the same instrument but different observers;

two samples of measurements by the same instrument at different times; or

two samples of measurements by different instruments.

For the following tests, a confidence level of  $1 - \alpha = 0,95$  and, according to the design of measurements, a number of degrees of freedom of  $\nu_{XY} = 51$  for the x- and y-coordinates and  $\nu_Z = 22$  for the z-coordinate are assumed.

**Table 3 — Statistical tests**

| Question | Null hypothesis           | Alternate hypothesis         |
|----------|---------------------------|------------------------------|
| a)       | $s \leq \sigma$           | $s > \sigma$                 |
| b)       | $\sigma = \tilde{\sigma}$ | $\sigma \neq \tilde{\sigma}$ |

**6.4.2 Response to Question a)**

The null hypothesis stating that the experimental standard deviation,  $s$ , is smaller than or equal to a theoretical or a predetermined value,  $\sigma$ , is not rejected if the following condition is fulfilled:

for x and y

$$s \leq \sigma \times \sqrt{\frac{\chi^2_{1-\alpha}(\nu_{XY})}{\nu_{XY}}}$$

$$s \leq \sigma \times \sqrt{\frac{\chi^2_{0,95}(51)}{51}}$$

$$\chi^2_{0,95}(51) = 68,67$$

$$s \leq \sigma \times \sqrt{\frac{68,67}{51}}$$

$$s \leq \sigma \times 1,16$$

for z

$$s \leq \sigma \times \sqrt{\frac{\chi^2_{1-\alpha}(\nu_Z)}{\nu_Z}}$$

$$s \leq \sigma \times \sqrt{\frac{\chi^2_{0,95}(22)}{22}}$$

$$\chi^2_{0,95}(22) = 33,92$$

$$s \leq \sigma \times \sqrt{\frac{33,92}{22}}$$

$$s \leq \sigma \times 1,24$$

(33)

(34)

(35)

(36)

(37)

Otherwise, the null hypothesis is rejected.

**6.4.3 Response to Question b)**

In the case of two different samples, a test indicates whether the experimental standard deviations,  $s$  and  $\tilde{s}$  belong to the same population. The corresponding null hypothesis,  $\sigma = \tilde{\sigma}$  is not rejected if the following condition is fulfilled:

for x and y

$$\frac{1}{F_{1-\alpha/2}(\nu_{XY}, \nu_{XY})} \leq \frac{s^2}{\tilde{s}^2} \leq F_{1-\alpha/2}(\nu_{XY}, \nu_{XY})$$

$$\frac{1}{F_{0,975}(51, 51)} \leq \frac{s^2}{\tilde{s}^2} \leq F_{0,975}(51, 51)$$

$$F_{0,975}(51, 51) = 1,74$$

$$0,57 \leq \frac{s^2}{\tilde{s}^2} \leq 1,74$$

for z

$$\frac{1}{F_{1-\alpha/2}(\nu_Z, \nu_Z)} \leq \frac{s^2}{\tilde{s}^2} \leq F_{1-\alpha/2}(\nu_Z, \nu_Z)$$

$$\frac{1}{F_{0,975}(22, 22)} \leq \frac{s^2}{\tilde{s}^2} \leq F_{0,975}(22, 22)$$

$$F_{0,975}(22, 22) = 2,36$$

$$0,42 \leq \frac{s^2}{\tilde{s}^2} \leq 2,36$$

(38)

(39)

(40)

(41)

Otherwise, the null hypothesis is rejected.

The number of degrees of freedom and, thus, the corresponding test values  $\chi^2_{1-\alpha/2}$  and  $F_{1-\alpha/2}(v,v)$  (taken from reference books on statistics) change if a different number of measurements is analysed.

### 6.5 Combined standard uncertainty evaluation (Type A and Type B)

The sources of uncertainty (influence quantities) are described in Table 4 as an uncertainty budget.

**Table 4 — Typical influence quantities of the total station**

| Sources of uncertainty                               | Symbol                 | Evaluation | Distribution                             |
|--|------------------------|------------|--|
| <b>I. Result of measurement</b>                      |                        |            |  |
| Standard deviation of x-, y- and z-coordinates       | $u_{\text{ISO-TS}}$    | Type A     | normal                                   |
| <b>II. Relevant sources of the total station</b>     |                        |            |  |
| Distance uncertainty on the specification            | $u_{r\text{-ts}}$      | Type B     | normal, or specified by the manufacturer |
| Horizontal angle uncertainty on the specification    | $u_{\phi\text{-ts}}$   | Type B     | normal, or specified by the manufacturer |
| Vertical angle uncertainty on the specification      | $u_{\theta\text{-ts}}$ | Type B     | normal, or specified by the manufacturer |
| Minimum display digit                                | $u_{\text{disp}}$      | Type B     | rectangular                              |
| <b>III. Error patterns from the mechanical setup</b> |                        |            |  |
| Torsion of a tripod (ISO 12858-2)                    | $u_{\text{trd}}$       | Type B     | rectangular                              |
| Stability of a tripod height (ISO 12858-2)           | $u_{\text{hs}}$        | Type B     | rectangular                              |
| <b>IV. Error sources of the atmospheres</b>          |                        |            |  |
| Temperature  | $u_{\text{temp}}$      | Type B     | normal                                   |
| Pressure   | $u_{\text{prs}}$       | Type B     | normal                                   |
| Relative humidity                                    | $u_{\text{rh}}$        | Type B     | normal                                   |

Uncertainty on the polar coordinates system is described as

$$u_r = \sqrt{u_{r\text{-ts}}^2 + u_{\text{temp}}^2 + u_{\text{prs}}^2 + u_{\text{rh}}^2} \tag{42}$$

$$u_\phi = \sqrt{u_{\phi\text{-ts}}^2 + u_{\text{trd}}^2} \tag{43}$$

$$u_\theta = \sqrt{u_{\theta\text{-ts}}^2 + u_{\text{hs}}^2} \tag{44}$$

The transfer formula to the rectangular coordinate from the polar coordinate is

$$ux^2 + uy^2 = (\cos\theta \cdot u_r)^2 + (r \cdot \sin\theta \cdot u_\theta)^2 + (r \cdot \cos\theta \cdot u_\phi)^2 \tag{45}$$

$$u_z^2 = (\sin\theta \cdot u_r)^2 + (r \cdot \cos\theta \cdot u_\theta)^2 \quad (46)$$

Combined uncertainty is

$$u_{x,y} = \sqrt{u_{\text{ISO-TS-XY}}^2 + (u_x^2 + u_y^2) + u_{\text{disp}}^2} \quad (47)$$

$$u_z = \sqrt{u_{\text{ISO-TS-Z}}^2 + u_z^2 + u_{\text{disp}}^2} \quad (48)$$

Expanded uncertainty is, with coverage factor  $k = 2$

$$U_{x,y} = 2 \times u_{x,y} \quad (49)$$

$$U_z = 2 \times u_z \quad (50)$$

## Annex A (informative)

### Example of the simplified test procedure

#### A.1 Measurements

In Table A.1 all measurements are compiled according to the observation scheme given in Table 1.

**Table A.1 — Measurements**

| Seq. No | Instrument station<br>i | Target point<br>j | Set<br>k | Telescope face | x      | y       | z      |
|---------|-------------------------|-------------------|----------|----------------|--------|---------|--------|
| 1       | 1                       | 1                 | 1        | I              | 6,979  | 4,886   | 9,934  |
| 2       |                         | 2                 |          |                | 59,617 | 25,117  | 6,763  |
| 3       |                         | 1                 | 2        | II             | 6,979  | 4,886   | 9,933  |
| 4       |                         | 2                 |          |                | 59,619 | 25,117  | 6,762  |
| 5       |                         | 1                 | 3        | I              | 6,978  | 4,885   | 9,934  |
| 6       |                         | 2                 |          |                | 59,618 | 25,116  | 6,764  |
| 7       |                         | 1                 | 4        | II             | 6,979  | 4,885   | 9,934  |
| 8       |                         | 2                 |          |                | 59,620 | 25,116  | 6,762  |
| 9       | 2                       | 1                 | 1        | I              | 8,344  | -47,323 | 12,767 |
| 10      |                         | 2                 |          |                | 1,214  | 8,619   | 9,596  |
| 11      |                         | 1                 | 2        | II             | 8,346  | -47,322 | 12,764 |
| 12      |                         | 2                 |          |                | 1,213  | 8,619   | 9,596  |
| 13      |                         | 1                 | 3        | I              | 8,344  | -47,323 | 12,767 |
| 14      |                         | 2                 |          |                | 1,213  | 8,619   | 9,596  |
| 15      |                         | 1                 | 4        | II             | 8,345  | -47,324 | 12,766 |
| 16      |                         | 2                 |          |                | 1,213  | 8,619   | 9,596  |

Observer: Y. Ohshima  
Weather: sunny  
Temperature: 29 °C  
Air pressure: 1006 hPa  
Instrument type and number: NT xxx 309090  
Date: 2010-07-08

## A.2 Calculation

### A.2.1 x-, y-coordinates

According to Formula (1):

$$\begin{aligned} l_{1,1} &= 56,3920 & l_{2,1} &= 56,3945 \\ l_{1,2} &= 56,3938 & l_{2,2} &= 56,3939 \\ l_{1,3} &= 56,3938 & l_{2,3} &= 56,3947 \\ l_{1,4} &= 56,3948 & l_{2,4} &= 56,3958 \end{aligned}$$

and according to Formula (2):

$$L = 56,3942$$

and according to Formula (3):

$$\begin{aligned} r_{1,1} &= -0,0011 & r_{2,1} &= 0,0002 \\ r_{1,2} &= -0,0002 & r_{2,2} &= -0,0001 \\ r_{1,3} &= -0,0002 & r_{2,3} &= 0,0003 \\ r_{1,4} &= 0,0003 & r_{2,4} &= 0,0008 \end{aligned}$$

and according to Formula (4):

$$d_{x,y} = 0,0011$$

### A.2.2 z-coordinate

According to Formula (5):

$$\begin{aligned} d_{z,1,1} &= -3,171 & d_{z,2,1} &= -3,171 \\ d_{z,1,2} &= -3,171 & d_{z,2,2} &= -3,168 \\ d_{z,1,3} &= -3,170 & d_{z,2,3} &= -3,171 \\ d_{z,1,4} &= -3,172 & d_{z,2,4} &= -3,170 \end{aligned}$$

and according to Formula (6):

$$a_z = -3,1705$$

and according to Formula (7):

$$\begin{aligned} r_{Z,1,1} &= -0,0005 & r_{Z,2,1} &= -0,0005 \\ r_{Z,1,2} &= -0,0005 & r_{Z,2,2} &= 0,0025 \\ r_{Z,1,3} &= 0,0005 & r_{Z,2,3} &= -0,0005 \\ r_{Z,1,4} &= -0,0015 & r_{Z,2,4} &= 0,0005 \end{aligned}$$

and according to Formula (8):

$$d_z = 0,0012$$

## Annex B (informative)

### Example of the full test procedure

#### B.1 Measurements of x- and y-coordinates

Table B.1 contains an example of observed data taken in accordance with the full test procedure.

**Table B.1 — Measurements**

| Seq. No   | Instrument station<br>i | Target point<br>j | Set<br>k | Telescope face | x      | y      | z      |
|---|-------------------------|-------------------|----------|----------------|--------|--------|--------|
| 1   | 1                       | 1                 | 1        | I              | 57,053 | 50,000 | 10,902 |
| 2   |                         | 2                 |          |                | 1,469  | 39,157 | 13,120 |
| 3   |                         | 3                 |          |                | 39,429 | -2,997 | 10,641 |
| 4   |                         | 1                 | 2        | II             | 57,053 | 50,001 | 10,902 |
| 5   |                         | 2                 |          |                | 1,470  | 39,159 | 13,121 |
| 6   |                         | 3                 |          |                | 39,426 | -2,998 | 10,640 |
| 7   |                         | 1                 | 3        | I              | 57,054 | 50,001 | 10,902 |
| 8   |                         | 2                 |          |                | 1,468  | 39,156 | 13,120 |
| 9   |                         | 3                 |          |                | 39,427 | -2,997 | 10,640 |
| 10  |                         | 1                 | 4        | II             | 57,054 | 50,000 | 10,902 |
| 11  |                         | 2                 |          |                | 1,470  | 39,158 | 13,121 |
| 12  |                         | 3                 |          |                | 39,428 | -2,998 | 10,640 |
| Observer: Y. Ohshima<br>Weather: sunny<br>Temperature: 29 °C<br>Air pressure: 1006 hPa<br>Instrument type and number: NT xxx 309090<br>Date: 2010-07-08 |                         |                   |          |                |        |        |        |

Table B.1 (continued)

| Seq. No | Instrument station<br>i | Target point<br>j | Set<br>k | Telescope face | x      | y      | z      |
|---------|-------------------------|-------------------|----------|----------------|--------|--------|--------|
| 13      | 2                       | 1                 | 1        | I              | 23,040 | 96,697 | 8,837  |
| 14      |                         | 2                 |          |                | 45,141 | 44,555 | 11,056 |
| 15      |                         | 3                 |          |                | 78,535 | 90,411 | 8,576  |
| 16      |                         | 1                 | 2        | II             | 23,043 | 96,698 | 8,834  |
| 17      |                         | 2                 |          |                | 45,139 | 44,555 | 11,056 |
| 18      |                         | 3                 |          |                | 78,535 | 90,412 | 8,576  |
| 19      |                         | 1                 | 3        | I              | 23,042 | 96,697 | 8,835  |
| 20      |                         | 2                 |          |                | 45,142 | 44,555 | 11,056 |
| 21      |                         | 3                 |          |                | 78,534 | 90,412 | 8,574  |
| 22      |                         | 1                 | 4        | II             | 23,040 | 96,696 | 8,834  |
| 23      |                         | 2                 |          |                | 45,140 | 44,555 | 11,056 |
| 24      |                         | 3                 |          |                | 78,534 | 90,412 | 8,574  |
| 25      | 3                       | 1                 | 1        | I              | 74,685 | 92,755 | 11,703 |
| 26      |                         | 2                 |          |                | 18,066 | 93,974 | 13,922 |
| 27      |                         | 3                 |          |                | 46,198 | 44,716 | 11,442 |
| 28      |                         | 1                 | 2        | II             | 74,686 | 92,752 | 11,703 |
| 29      |                         | 2                 |          |                | 18,068 | 93,975 | 13,922 |
| 30      |                         | 3                 |          |                | 46,198 | 44,715 | 11,442 |
| 31      |                         | 1                 | 3        | I              | 74,687 | 92,752 | 11,703 |
| 32      |                         | 2                 |          |                | 18,068 | 93,976 | 13,922 |
| 33      |                         | 3                 |          |                | 46,199 | 44,715 | 11,442 |
| 34      |                         | 1                 | 4        | II             | 74,689 | 92,751 | 11,701 |
| 35      |                         | 2                 |          |                | 18,068 | 93,975 | 13,923 |
| 36      |                         | 3                 |          |                | 46,199 | 44,715 | 11,442 |

Observer: Y. Ohshima  
 Weather: sunny  
 Temperature: 29 °C  
 Air pressure: 1006 hPa  
 Instrument type and number: NT xxx 309090  
 Date: 2010-07-08



## B.2 Calculation

### B.2.1 x-, y-coordinates

According to Formula (10):

$$L_1 = 56,7267$$

$$L_2 = 55,8499$$

$$L_3 = 56,6321$$

According to Formula (15):

$$(x_{g,1}, y_{g,1}) = (32,6501, 28,7202)$$

$$(x_{g,2}, y_{g,2}) = (48,9054, 77,2213)$$

$$(x_{g,3}, y_{g,3}) = (46,3176, 77,1476)$$

According to Formula (16):

| <i>i</i> | <i>k</i> | $X_{t,i,1,k}$ | $Y_{t,i,1,k}$ | $X_{t,i,2,k}$ | $Y_{t,i,2,k}$ | $X_{t,i,3,k}$ | $Y_{t,i,3,k}$ |
|----------|----------|---------------|---------------|---------------|---------------|---------------|---------------|
| 1        | 1        | 4,6245        | 12,5063       | 61,2566       | 12,5063       | 32,0691       | 61,1479       |
| 1        | 2        | 4,6245        | 12,5063       | 61,2566       | 12,5063       | 32,0691       | 61,1479       |
| 1        | 3        | 4,6245        | 12,5063       | 61,2566       | 12,5063       | 32,0691       | 61,1479       |
| 1        | 4        | 4,6245        | 12,5063       | 61,2566       | 12,5063       | 32,0691       | 61,1479       |
| 2        | 1        | 20,8799       | 61,0074       | 77,5120       | 61,0074       | 48,3244       | 109,6490      |
| 2        | 2        | 20,8799       | 61,0074       | 77,5120       | 61,0074       | 48,3244       | 109,6490      |
| 2        | 3        | 20,8799       | 61,0074       | 77,5120       | 61,0074       | 48,3244       | 109,6490      |
| 2        | 4        | 20,8799       | 61,0074       | 77,5120       | 61,0074       | 48,3244       | 109,6490      |
| 3        | 1        | 18,2920       | 60,9337       | 74,9241       | 60,9337       | 45,7366       | 109,5753      |
| 3        | 2        | 18,2920       | 60,9337       | 74,9241       | 60,9337       | 45,7366       | 109,5753      |
| 3        | 3        | 18,2920       | 60,9337       | 74,9241       | 60,9337       | 45,7366       | 109,5753      |
| 3        | 4        | 18,2920       | 60,9337       | 74,9241       | 60,9337       | 45,7366       | 109,5753      |

According to Formula (20):

| <i>i</i> | <i>k</i> | $X_{m,i,1,k}$ | $Y_{m,i,1,k}$ | $X_{m,i,2,k}$ | $Y_{m,i,2,k}$ | $X_{m,i,3,k}$ | $Y_{m,i,3,k}$ |
|----------|----------|---------------|---------------|---------------|---------------|---------------|---------------|
| 1        | 1        | 57,0529       | 49,9998       | 1,4685        | 39,1571       | 39,4289       | -2,9964       |
| 1        | 2        | 57,0539       | 49,9987       | 1,4690        | 39,1586       | 39,4274       | -2,9968       |
| 1        | 3        | 57,0530       | 49,9997       | 1,4685        | 39,1573       | 39,4287       | -2,9965       |
| 1        | 4        | 57,0536       | 49,9990       | 1,4688        | 39,1581       | 39,4279       | -2,9967       |
| 2        | 1        | 23,0401       | 96,6970       | 45,1410       | 44,5555       | 78,5351       | 90,4112       |
| 2        | 2        | 23,0408       | 96,6980       | 45,1398       | 44,5556       | 78,5356       | 90,4101       |
| 2        | 3        | 23,0399       | 96,6968       | 45,1414       | 44,5554       | 78,5350       | 90,4116       |
| 2        | 4        | 23,0399       | 96,6968       | 45,1414       | 44,5554       | 78,5350       | 90,4116       |
| 3        | 1        | 74,6859       | 92,7539       | 18,0670       | 93,9740       | 46,1998       | 44,7149       |
| 3        | 2        | 74,6867       | 92,7525       | 18,0678       | 93,9754       | 46,1982       | 44,7149       |
| 3        | 3        | 74,6868       | 92,7524       | 18,0679       | 93,9755       | 46,1981       | 44,7149       |
| 3        | 4        | 74,6869       | 92,7521       | 18,0681       | 93,9758       | 46,1978       | 44,7149       |

According to Formula (23):

$$\sum r_{xy}^2 = 0,0000616$$

According to Formula (25) and (26):

$$S_{XY} = 0,00110$$

$$s_{\text{ISO-TS-XY}} = 0,00110$$

### B.2.2 z-coordinate

According to Formula (27), (28), (29), (30):

| Instrument station <i>i</i> | Set <i>k</i> | $d_{z,i,2,k}$ | $d_{z,i,3,k}$ | $r_{z,i,2,k}$ | $r_{z,i,3,k}$  | $r_z^2_{i,2,k}$ | $r_z^2_{i,3,k}$ |
|-----------------------------|--------------|---------------|---------------|---------------|----------------|-----------------|-----------------|
| 1                           | 1            | 2,218         | -0,261        | -0,00175      | -0,00025       | 0,0000031       | 0,0000001       |
|                             | 2            | 2,219         | -0,262        | -0,00075      | -0,00125       | 0,0000006       | 0,0000016       |
|                             | 3            | 2,218         | -0,262        | -0,00175      | -0,00125       | 0,0000031       | 0,0000016       |
|                             | 4            | 2,219         | -0,262        | -0,00075      | -0,00125       | 0,0000006       | 0,0000016       |
| 2                           | 1            | 2,219         | -0,261        | -0,00075      | -0,00025       | 0,0000006       | 0,0000001       |
|                             | 2            | 2,222         | -0,258        | 0,00225       | 0,00275        | 0,0000051       | 0,0000076       |
|                             | 3            | 2,221         | -0,261        | 0,00125       | -0,00025       | 0,0000016       | 0,0000001       |
|                             | 4            | 2,222         | -0,260        | 0,00225       | 0,00075        | 0,0000051       | 0,0000006       |
| 3                           | 1            | 2,219         | -0,261        | -0,00075      | -0,00025       | 0,0000006       | 0,0000001       |
|                             | 2            | 2,219         | -0,261        | -0,00075      | -0,00025       | 0,0000006       | 0,0000001       |
|                             | 3            | 2,219         | -0,261        | -0,00075      | -0,00025       | 0,0000006       | 0,0000001       |
|                             | 4            | 2,222         | -0,259        | 0,00225       | 0,00175        | 0,0000051       | 0,0000031       |
| $\Sigma d_{z,i,j,k}$        |              | 26,637        | -3,129        |               |                |                 |                 |
|                             |              | $a_{z,2}$     | $a_{z,3}$     |               |                |                 |                 |
|                             |              | 2,2198        | -0,2607       |               | $\Sigma r_z^2$ | 0,0000425       |                 |

According to Formula (32):

$$s_{\text{ISO-TS-Z}} = 0,00139$$

### B.3 Statistical tests

#### B.3.1 Statistical test according to Question a)

Test for  $x$  and  $y$ ;

$$\begin{aligned}\sigma &= 5,0 \text{ mm} \\ s_{\text{ISO-TS-XY}} &= 1,10 \text{ mm} \\ v_{\text{XY}} &= 51 \\ 1,10 \text{ mm} &\leq 5,0 \text{ mm} \times 1,16 \\ 1,10 \text{ mm} &\leq 5,8 \text{ mm}\end{aligned}$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviation

$$s_{\text{ISO-TS-XY}} = 1,10 \text{ mm}$$

$$\sigma = 5,0 \text{ mm}$$

is smaller than or equal to the manufacturer's value is not rejected at the confidence level of 95 %.

Test for  $z$  :

$$\begin{aligned}\sigma &= 5,0 \text{ mm} \\ s_{\text{ISO-TS-Z}} &= 1,39 \text{ mm} \\ v_z &= 15 \\ 1,39 \text{ mm} &\leq 5,0 \text{ mm} \times 1,24 \\ 1,39 \text{ mm} &\leq 6,2 \text{ mm}\end{aligned}$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviation

$$s_{\text{ISO-TS-Z}} = 1,39 \text{ mm}$$

is smaller than or equal to the manufacturer's value  $\sigma = 5,0 \text{ mm}$  is not rejected at the confidence level of 95 %.

#### B.3.2 Statistical test according to Question b)

Test for  $x$  and  $y$ :

$$\begin{aligned}s &= 1,10 \text{ mm} \\ \tilde{s} &= 1,15 \text{ mm} \\ v_{\text{XY}} &= 51 \\ 0,57 &\leq \frac{1,12 \text{ mm}^2}{1,32 \text{ mm}^2} \leq 1,74 \\ 0,57 &\leq 0,85 \leq 1,74\end{aligned}$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviations  $s = 1,10 \text{ mm}$  and  $\tilde{s} = 1,15 \text{ mm}$  belong to the same population is not rejected at the confidence level of 95 %.

**Test for z:**

$$s = 1,39 \text{ mm}$$

$$\tilde{s} = 1,55 \text{ mm}$$

$$v_z = 22$$

$$0,42 \leq \frac{1,93 \text{ mm}^2}{2,40 \text{ mm}^2} \leq 2,36$$

$$0,42 \leq 0,80 \leq 2,36$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviations  $s=1,39 \text{ mm}$  and  $\tilde{s}=1,55 \text{ mm}$  belong to the same population is not rejected at the confidence level of 95 %.

## Annex C (informative)

### Example for the calculation of a combined uncertainty budget (Type A and Type B)

#### C.1 Uncertainty budget example

##### C.1.1 Sources of uncertainty

The analysis of measurements:  $u_{\text{ISO-TS}}$

$$s_{\text{ISO-TS}}$$

are obtained from [Annex B](#)

$$s_{\text{ISO-TS-XY}} = 0,00110 \text{ m}$$

$$s_{\text{ISO-TS-Z}} = 0,00139 \text{ m}$$

Total station:

According to the specification by the manufacturer, the uncertainty of distance  $u_{\text{r-ts}}$  is obtained by applying the manufacturer's specification  $\pm (3 + 2\text{ppm} \times D)$  and maximum measured distance = 57 m.

$$u_{\text{r-ts}} = 3 + 2 \times 57000 \times 10^{-6} = 3,1 \text{ mm}$$

The uncertainty of horizontal angle measurement  $u_{\phi\text{-ts}}$  is obtained by applying the manufacturer's specification 5" (according to ISO 17123-3) as

$$u_{\phi\text{-ts}} = 5''$$

The uncertainty of vertical angle measurement  $u_{\theta\text{-ts}}$  is obtained by applying the manufacturer's specification 5" (according to ISO 17123-3) as

$$u_{\theta\text{-ts}} = \frac{5}{\sqrt{3}} = 2,89''$$

The uncertainty of minimum display digit  $u_{\text{disp}}$

$$u_{\text{disp}x} = u_{\text{disp}y} = u_{\text{disp}z} = \frac{0,5}{\sqrt{3}} = 0,29 \text{ mm}$$

when minimum digit is 1 mm.

Tripod:

The influenced quantity of the tripod  $u_{\text{trd}}$

$$u_{\text{trd}} = \frac{3}{\sqrt{3}} = 1,73''$$

with the estimated torsion according to ISO 12858-2 and rectangular distribution.

The stability of the tripod height  $u_{\text{hs}}$  is estimated within 0,5 mm according to ISO 12858-2, which can be omitted from the budget.

Atmospheric condition

The uncertainty of temperature  $u_{\text{temp}}$  :

$$u_{\text{temp}} = 1 \times 57000 \times 10^{-6} = 0,057 \text{ mm} , \text{ with } \pm 1 \text{ }^\circ\text{C from experience}$$

The uncertainty of pressure  $u_{\text{prs}}$  :

$$u_{\text{prs}} = 0,3 \times 5 \times 57000 \times 10^{-6} = 0,086 \text{ mm} , \text{ with } 5 \text{ hPa on experience}$$

The uncertainty of humidity  $u_{\text{rh}}$  can be omitted from the budget, as its influence is so small for the maximum distance of 100 m in the test.

### C.1.2 Uncertainty calculation

The uncertainty on polar coordinate is calculated according Formula (42), (43), (44):

$$u_r = \sqrt{u_{r-ts}^2 + u_{\text{temp}}^2 + u_{\text{prs}}^2} = \sqrt{3,114^2 + 0,057^2 + 0,086^2} = 3,116 \text{ mm}$$

$$u_\phi = \sqrt{u_{\phi-ts}^2 + u_{\text{trd}}^2} = \sqrt{5^2 + 1,73^2} = 5,29''$$

$$u_\theta = \sqrt{u_{\theta-ts}^2} = 2,89''$$

The uncertainty on rectangular coordinate is calculated according to Formula (45), (46):

$$u_x^2 + u_y^2 = 11,85 \text{ mm}$$

$$u_z^2 = 3,12 \text{ mm}$$

Combined uncertainty is calculated according to Formula (47), (48):

$$u_{xy} = \sqrt{1,10^2 + 11,85 + 0,29^2} = 3,63$$

$$u_z = \sqrt{1,39^2 + 3,12 + 0,29^2} = 2,27$$

Table C.1 — Uncertainty budget on rectangular coordinate

| Input quantity         | Input estimates | Standard uncertainty $u(x_i)/\text{mm}$ | Distribution | Sensitivity coefficient | $u_i(c_{xy}) \equiv  c_i  \times u(x_i) / \text{mm}$ | Evaluation | Remark   |
|------------------------|-----------------|---|--------------|-------------------------|--|------------|----------|
| $u_{\text{ISO-TS-XY}}$ | -               | 1,06                                    | normal       | 1                       | 1,06   | Type A     | eq. (25) |
| $u_{\text{ISO-TS-Z}}$  | -               | 1,39                                    | normal       | 1                       | 1,39   | Type A     | eq.(32)  |
| $(ux^2+uy^2)^{0,5}$    | Dmax=57m, Va=1° | 3,44                                    | normal       | 1                       | 3,44   | Type B     |          |
| $u_z$                  | Dmax=57m, Va=1° | 1,77                                    | normal       | 1                       | 1,77   | Type B     |          |
| $u_{\text{disp}}$      | 0               | 0,29                                    | rectangle    | 1                       | 0,29   | Type B     |          |
| Va=Elevation Angle     |                 | Final results                           |              | $u_{xy}$                | 3,63   |            |          |
|                        |                 |   |              | $u_z$                   | 2,27   |            |          |

## C.2 Expanded uncertainty

$$U_{xy} = 2 \times 3,63 \approx 7 \text{ mm}$$

$$U_z = 2 \times 2,27 \approx 5 \text{ mm}$$

## Annex D (informative)

### Sources which are not included in uncertainty evaluation

The sources of uncertainty shown in Table D.1 are not to be evaluated individually, since those are already considered in the corresponding influence quantities listed in Table 4 or not relevant.

**Table D.1 — Sources of uncertainty not to be evaluated individually**

| Source of uncertainty               | Distance | Vertical angle | Horizontal angle |
|-------------------------------------|----------|----------------|------------------|
| Resolving power of telescope        | •        | •              | •                |
| Cross hair error                    |          | •              | •                |
| Centring of total station           | •        | •              | •                |
| Sighting axis and vertical axis     |          |                | •                |
| Vertical-axis tilt of total station | •        | •              | •                |
| Line-of-sight error                 |          |                | •                |
| Tilting-axis error                  |          | •              | •                |
| Graduation error of H circle        |          |                | •                |
| Eccentric error of H circle         |          |                | •                |
| Vertical compensate error           |          | •              |                  |
| Horizontal compensate error         |          |                | •                |
| Additional constant                 | •        |                |                  |
| Prism constant error                | •        |                |                  |
| Parameter of atmospheric factor     | •        |                |                  |
| Centring of prism                   | •        |                |                  |
| Direction of prism face             | •        | •              | •                |



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