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Measurement of apparent thermal conductivity of wet porous building materials by a periodic method

Détermination de la conductivité thermique apparente des matériaux de construction poreux et mouillés par une méthode périodique

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

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The committee responsible for this document is ISO/TC 163, *Thermal performance and energy use in the built environment*, Subcommittee SC 1, *Test and measurement methods*.

Introduction

Most building materials, with the exception of glass and metals, are porous and, thus, absorb moisture due to condensation, rain and water uptake from the ground. The absorbed moisture may damage the materials through, e.g. rotting or frost damage and, thus, may cause their performance to deteriorate. In particular, an increase in the moisture content of insulation material causes a reduction of its thermal resistance, which must be avoided as much as possible to preserve its performance. However, infiltration of rain water into a brick wall or joints of tiles and uptake of ground water into the foundation (footing) are very difficult to avoid. Therefore, it is important to understand the changes in the thermal properties (thermal conductivity and heat capacity) of porous materials due to changes in their moisture content.

ISO 10051 specifies a steady-state method for measuring the thermal conductivity of a moist building material. In the steady-state method, a nonuniform distribution of moisture content in the test piece is inevitable, since the imposed temperature gradient causes moisture transfer. The nonuniform moisture distribution makes it difficult to define which moisture content the measured thermal conductivity corresponds to. ISO 10051 categorizes the moisture distribution in the test piece into several types and estimates the thermal conductivity corresponding to each type.

Since theoretical and experimental research has recently been performed concerning heat and moisture transfer in porous materials (see References [5], [7], [8], [9] and [10]), along with measurements and the construction of a database of hygrothermal properties (see Reference [6]), hygrothermal behaviour can now be predicted with reasonable accuracy.

This International Standard describes a transient method for measuring the thermal conductivity of a wet porous building material and a method of evaluating the measurement uncertainty, on the basis of both theoretical developments for heat and mass transfer and the constructed database of hygrothermal properties. The evaluation of the measurement uncertainty makes possible a simple and, thus, practical method for measuring thermal conductivity.

NOTE Thermal conductivity is one of the necessary hygrothermal properties. Since heat transfer and mass transfer in porous material interact with each other, an exact value of the thermal conductivity must be given in order to examine the validity of the theoretical models. Thus, precisely speaking, the above-mentioned theoretical models have not been validated, and the construction of the model and the measurements of the hygrothermal properties must be carried out in parallel. Nonetheless, it seems reasonable to expect that measurement of the thermal conductivity with an allowable accuracy is possible using a suitable measuring method. This is the basis for the present document.

Measurement of apparent thermal conductivity of wet porous building materials by a periodic method

1 Scope

This International Standard describes a method of measuring the thermal conductivity (diffusivity) of a wet porous building material and a method of evaluating the measurement uncertainty.

While ISO 10051 is the current International Standard, based on a steady-state method, this International Standard proposes a method that makes use of a non-steady-state method which uses a small temperature change with a short period as an input. Along with the measurement, an evaluation of the measurement uncertainty is described, which makes possible a simple and practical measuring method.

This International Standard intends to measure the apparent (effective) thermal conductivity, including latent heat transfer caused by vapour movement. The situation in which moisture and/or air movement occur due to convection or gravity is excluded. The application of this International Standard to high moisture content is excluded so that the gravity effect can be neglected. This International Standard can be applied to a porous material heavier than about 100 kg/m^3, in which radiative heat transfer can be neglected.

This International Standard specifies the following:

a) a non-steady-state method of measuring thermal conductivity;

b) an approximation formula for the measurement uncertainty caused by moisture movement and nonuniform moisture distribution (and, thus, a determination of the measuring conditions that satisfy the upper limit of measurement uncertainty);

c) an estimate of the heat transfer caused by moisture (vapour) movement.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

There are no normative references in this document.

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 9346, ISO 10051 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— IEC Electropedia: available at http://www.electropedia.org/

— ISO Online browsing platform: available at http://www.iso.org/obp

3.1
apparent thermal conductivity of a wet material
λ^*
intrinsic material property dependent upon moisture content and temperature, but not on testing conditions

Note 1 to entry: Since this value includes the influence of heat transfer due to phase change (condensation and evaporation), it is called apparent thermal conductivity.

4 Symbols and units

Symbol	Quantity	Units
a	thermal diffusivity of the material	m^2/s
A, B, C, D	defined in Formula (B.14)	—
c	porosity	m^3/m^3
c'	specific heat of the material	J/(kg·K)
c_s	specific heat of the dry material	J/(kg·K)
D_T	moisture (vapour and liquid water) conductivity related to temperature gradient	kg/(m·s·K)
D_θ	Moisture (vapour and liquid water) conductivity related to water content gradient	m^2/s
D_{Tl}	liquid water conductivity related to temperature gradient	kg/(m·s·K)
$D_{\theta l}$	liquid water conductivity related to water content gradient	m^2/s
D_{Tv}	vapour conductivity related to temperature gradient	kg/(m·s·K)
$D_{\theta v}$	vapour conductivity related to water content gradient	m^2/s
E, E_1, E_2	defined in Formula (B.12)	—
I_0	amplitude of the input surface temperature	K
k_v	vapour diffusivity	kg/[m·s·(kg/kg′)]
R	latent heat of vaporization	J/kg
S	specific surface area inside the material, i.e. ratio of pore surface area to the material volume	m^2/m^3
t	time	s
T	temperature	K
T_0	initial temperature	K
$T_1, T_2, \theta_1, \theta_2$	the first and second terms of the perturbation solution of the temperature and water content, respectively	K, kg/m^3
x	coordinate	m
X	humidity ratio of moist air in the pores of the specimen	kg/kg′
X_i	equilibrium humidity ratio with liquid or capillary water at the interface in the material	kg/kg′
$U_3, U_7, Q_1 - Q_6, R_1 - R_6, V_{18}$	coefficients appearing solutions θ_2 and T_2	—
α_1, α_2	defined in Formula (B.13)	—
α'	effective vapour transfer coefficient at the interface	kg/(m·s·kg)/kg′
γ	specific weight of dry air	kg/m^3
γ'	density of the material	kg/m^3
γ_s	density of the dry material	kg/m^3
$\eta_1, \zeta_1, \beta_1, \xi_1, \kappa_1, \gamma_1$	water content coefficients of $D_{\theta l}$, $D_{\theta v}$, D_{Tl}, D_{Tv}, c' γ', λ	—
$\eta_2, \zeta_2, \beta_2, \xi_2, \gamma_2$	temperature coefficients of $D_{\theta l}$, $D_{\theta v}$, D_{Tl}, D_{Tv}, λ	—
λ	thermal conductivity without moisture movement	W/(m·K)

Symbol	Quantity	Units
λ^*	thermal conductivity defined as $\lambda^* = \lambda + RD_{Tv}$	W/(m·K)
θ	water content of material	kg/m^3
θ_0	initial water content of material	kg/m^3
Θ_0	water content by weight	%
ω	angular velocity of the input surface temperature	rad/s

5 Determination of thermal conductivity of a wet porous material by non-steady-state method (periodic method)

When measuring the thermal conductivity of a porous material in which moisture transfer may occur, the nonuniformity of the water content distribution must be kept as low as possible under the temperature gradient. In order to minimize the nonuniformity, a periodic method is adopted as a transient method of measuring thermal conductivity (see example in Annex C). Since positive and negative temperature gradients are generated in turn in this method, the (time-averaged) water content distribution can be expected to remain uniform.

By measuring the temperatures at two points in the sample (usually one of them is at the sample surface), the thermal diffusivity (not conductivity) can be determined based on the amplitude ratio or phase difference of these two temperatures. If moisture movement does occur, a similar method can be used to determine the thermal diffusivity if the input cyclic temperature fluctuation is kept small enough that the change in transport properties is also small enough that the system can be regarded as linear (see Annex B).

6 Measurement by periodic method

6.1 Test procedure

A schematic diagram of the apparatus for the periodic method is given in Figure 1. The whole system is installed in a climate chamber whose temperature is kept at the mean temperature of the sample under measurement. The sample is preconditioned at a certain water content, and then the whole surface is made impermeable to moisture movement. A periodic temperature variation is imposed on the sample, and the temperatures at (at least) two points in the sample are measured by thermocouples.

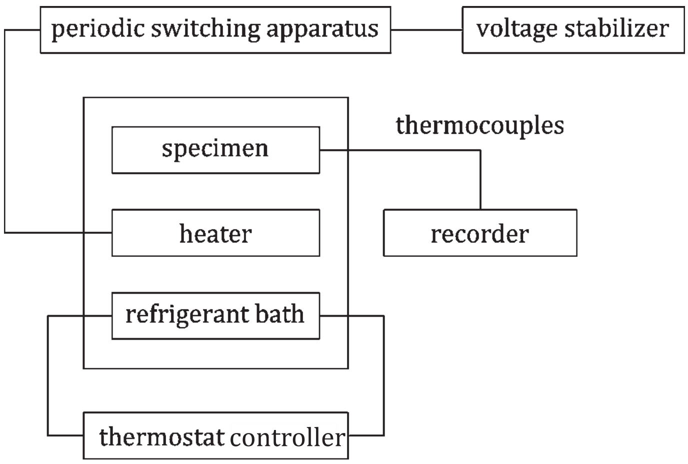


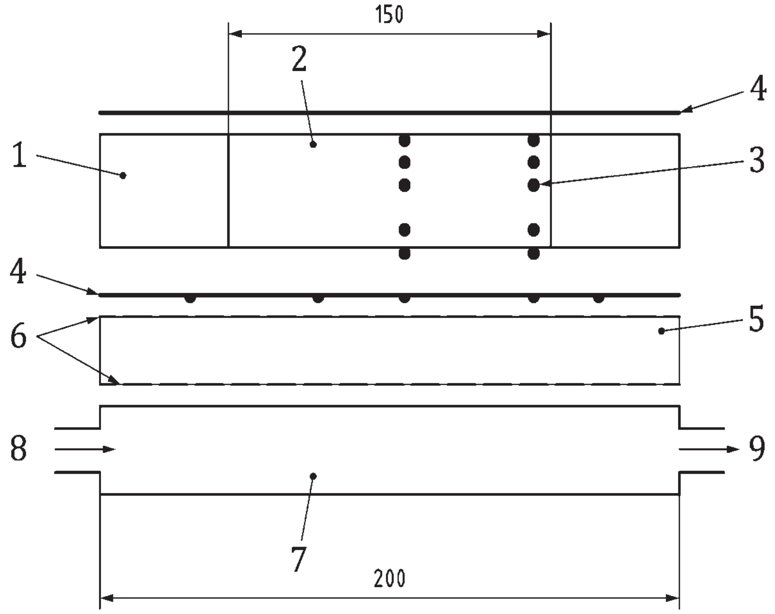
Figure 1 — Schematic diagram for measuring thermal diffusivity

6.2 Measuring apparatus

6.2.1 Overall design

A detailed schematic of the measuring apparatus is shown in Figure 2. Refrigerant kept at a low constant temperature is circulated in a metal refrigerant bath [200 mm (length) × 200 mm (wide) × 50 mm (height)] in order to avoid a temperature increase due to heating by the heater. A heater, a damping layer and a sample are placed on the refrigerant bath in order, and another damping layer is placed on the sample to reduce the influence of the temperature fluctuations of the climate chamber surrounding the measuring apparatus. Either a sinusoidal or stepwise electric current is generated in the heater. The stepwise wave becomes almost sinusoidal as it flows through the damping layers before arriving at the sample surface. Thermocouples are inserted into the sample and connected to the recorders. The output from the thermocouples is recorded by both an analogue recorder and a digital recorder. The digital recorder is used for a long-term record for temperatures at multiple points, while the analogue recorder is used for recording the temperature wave in a short-term measurement.

Dimensions in millimetres



Key

1	thermal insulation material
2	specimen
3	thermocouple
4	rubber film
5	heater
6	copper plates
7	refrigerant bath
8	refrigerant in
9	refrigerant out

Figure 2 — Vertical cross section of apparatus

6.2.2 Generator of the sinusoidal or stepwise electric wave

A sinusoidal electric wave is generated by an arbitrary wave generator and is sent to the heater. When such an apparatus is not available, a cyclic stepwise electric wave is generated and input to the heater by switching a constant electric current on and off using a relay. A cyclic on-off switching of the relay can be realized with a combination of two timers.

6.2.3 Heater

The lower surface of the sample is uniformly heated (with a cyclic change) by a film heater. The refrigerant bath under the sample works to reduce any increase in the average sample temperature due to the heating and to realize the necessary average temperature.

6.2.4 Specimen

The sample size should follow the requirement in ISO 10456 for thermal conductivity measurement, that is, it should be 150 mm × 150 mm. However, the sample should be thick enough that the amplitude of the temperature variation arriving at the upper surface of the sample becomes almost 0. The sample surfaces (boundary) should be made impermeable to moisture flow by a suitable water barrier and should also be made adiabatic by a thermal insulation material.

6.3 Specimen preparation and preconditioning

6.3.1 Initial uniform moisture content and adiabatic and impermeable boundaries

Before the measurement, a predetermined amount of water is added to the sample in order to obtain the required average moisture content. The sample is placed in a climate chamber with a constant temperature and kept for a time long enough to produce a uniform distribution of moisture content in the sample.

6.3.2 Embedding and the position of the thermocouples

Several thermocouples are inserted into the sample from its side surfaces. At least two thermocouples on the lower and upper surfaces and one at one depth in the vertical direction are necessary. In addition, the temperature distribution on the lower and upper surfaces should be measured in order to check the uniformity of the temperature distribution in the horizontal direction.

The exact positions of the thermocouples are decided, for example, by making use of the linear temperature distribution under steady-state conditions when the sample is dry with a temperature difference between the upper and lower surfaces.

6.4 Derivation of thermal diffusivity from measured temperatures (see Annex B)

6.4.1 Solution for heat flow without moisture

When a sample is dry and, thus, the thermal properties are constant, the temperature at time t and position x under a sinusoidal surface temperature input (at $x = 0$) is given in Formula (1):

$$T\left(t,x\right) = I_0 \exp\left(-\sqrt{\frac{\omega}{2a}}x\right) \times \sin\left(\omega t - \sqrt{\frac{\omega}{2a}}x\right) + T_\mathrm{m} \qquad (1)$$

where

T_m is the (constant) average temperature;

a is the thermal diffusivity of the sample material;

I_0 is the amplitude of the surface temperature variation;

ω is the angular velocity of the input temperature.

By making use of this solution, the thermal diffusivity, a, for example, is given as Formula (2) and Figure 3:

$$a = \frac{1}{\varphi_0^2}\left(\frac{\omega}{2}\right)\left(x_1 - x_2\right) \tag{2}$$

where

x_1 and x_2 are any two points in the material (usually one of them is the surface);

φ_0 is the phase difference between the temperatures at x_1 and x_2.

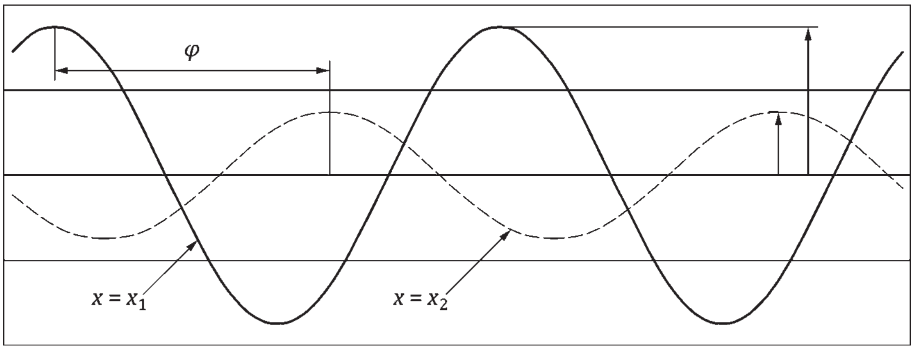


Figure 3 — Temperature waves at two points (amplitude ratio and phase difference, φ**)**

6.4.2 Solution for heat flow with moisture

By using the fact that generally, $\alpha_2\left(\cong \frac{1}{\sqrt{A}}\right)$ is larger than $\alpha_1\left(\cong \frac{1}{\sqrt{C}}\right)$, Formula (B.11) approximately becomes

$$T_1\left(t, x\right) = \alpha_1 \frac{EE_1}{B} \exp\left(-\alpha_1 \sqrt{\frac{\omega}{2}} x\right) \times \sin\left(\omega t - \alpha_1 \sqrt{\frac{\omega}{2}} x\right) \tag{3}$$

because of the rapid exponential decay of the second term. In Formula (3), α_1, B, E and E_1 are given by Formulae (B.12) and (B.13).

Based on this solution, the apparent thermal diffusivity, $1/\alpha_1^2$, when there is moisture can be as given in Formula (4):

$$\frac{1}{\alpha_1^2} \cong C\left(1 + \frac{BD}{C^2}\right) = \frac{1}{\varphi^2}\left(\frac{\omega}{2}\right)\left(x_1 - x_2\right)^2 \tag{4}$$

where

φ is the phase difference of the temperatures at points x_1 and x_2.

6.5 Estimation of measuring uncertainty due to moisture (vapour) movement

The uncertainty in the thermal diffusivity calculated using the phase difference relative to that of Formula (2) is

$$\frac{\varphi_0^2}{\varphi^2} - 1 = \frac{1}{C}\frac{1}{\alpha_1^2} = 1 - \frac{BD}{C^2} \tag{5}$$

where an approximation, Formula (B.17), was used. As expected, Formula (5) represents the influence of the term $R\frac{\partial}{\partial x}\left(D_{\theta\mathrm{v}}\frac{\partial\theta}{\partial x}\right)$.

The perturbation solution, T_2, can be used to determine the amplitude of the input temperature variation, because this temperature expresses the influence of the nonlinear change of the hygrothermal properties, i.e. the degree of nonuniform distribution of the moisture content.

6.6 Thermal conductivity

Thermal conductivity can be obtained by multiplying the heat capacity and the density of the material with the obtained thermal diffusivity. This product, along with its change due to moisture content, can be found in the references.

7 Test report

The test report shall include the following:

a) a reference to this International Standard, i.e. ISO 16957;

b) product identification;

- — product name, factory, manufacturer or supplier;
- — type of product;
- — production code number;
- — the form in which the product arrived at the laboratory;
- — other information if necessary, e.g. thickness, dry density;

c) measurement apparatus;

- — cyclic heating device (unit) and cooling unit;
- — measurement of temperature (locations and time interval);
- — insulation of sample;

d) preparation of test piece (sample);

- — insulation to sides of sample;
- — method of ensuring no moisture flow through sample surfaces;
- — preconditioning of sample (temperature and moisture content);
- — temperature measurement inside sample (method, locations and time interval);

e) test procedure;

— date of the start and duration of the test;

— the method of sampling;

— heat input (mean temperature, amplitude, etc.);

— moisture content of sample;

— any factors which may have influenced the results;

f) results;

— table of the measured values (mean and amplitude of temperatures at two depths from sample surface, phase difference between temperatures at two depths, average moisture content);

— graph showing time profiles of temperatures at two depths from sample surface.

Annex A
(informative)

Theoretical background

A.1 Fundamental formulae of vapour, liquid water and heat transfer

Recently, the mechanism of the simultaneous heat and moisture transfer in porous material has been investigated and clarified (see References [5], [7], [8], [9] and [10]), and measurements of the transport properties have been made.[6]

Formulae that describe the simultaneous flow of heat and moisture in a moist porous material are as follows:[9]

vapour balance,

$$c\gamma \frac{\partial X}{\partial t} = \frac{\partial}{\partial x}\left[k_{\mathrm{v}} \frac{\partial X}{\partial x}\right] + \alpha'_{\mathrm{i}} S\left(X_{\mathrm{i}} - X\right) \tag{A.1}$$

liquid water balance,

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[D_{\theta\mathrm{l}} \frac{\partial \theta}{\partial x}\right] + \frac{\partial}{\partial x}\left[D_{\mathrm{Tl}} \frac{\partial T}{\partial x}\right] + \alpha'_{\mathrm{i}} S\left(X - X_{\mathrm{i}}\right) \tag{A.2}$$

heat balance, and

$$c'\gamma' \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\lambda \frac{\partial T}{\partial x}\right] + R\alpha'_{\mathrm{i}} S\left(X - X_{\mathrm{i}}\right) \tag{A.3}$$

absorption isotherm

$$X_{\mathrm{i}} = g(\theta, T) \tag{A.4}$$

where the sensible heat transported by vapour and liquid is neglected.

Assuming α'_{i} is infinite (local equilibrium), using Formula (A.4), Formulae (A.1) to (A.3) are transformed as follows:

moisture balance,

$$\left(1 + c\gamma \frac{\partial g}{\partial \theta}\right)\frac{\partial \theta}{\partial t} + \left(c\gamma \frac{\partial g}{\partial T}\right)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[D_{\theta} \frac{\partial \theta}{\partial x}\right] + \frac{\partial}{\partial x}\left[D_{\mathrm{T}} \frac{\partial T}{\partial x}\right] \tag{A.5}$$

heat balance

$$\left(Rc\gamma \frac{\partial g}{\partial \theta}\right)\frac{\partial \theta}{\partial t} + \left(Rc\gamma \frac{\partial g}{\partial T} + c'\gamma'\right)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\left(\lambda + RD_{\mathrm{Tv}}\right)\frac{\partial \theta}{\partial x}\right] + R\frac{\partial}{\partial x}\left(D_{\theta\mathrm{v}} \frac{\partial \theta}{\partial x}\right) \tag{A.6}$$

where

$$D_{\theta} = D_{\theta\mathrm{v}} + D_{\theta\mathrm{l}} = k_{\mathrm{v}} \frac{\partial g}{\partial \theta} + D_{\theta\mathrm{l}}, D_{\mathrm{T}} = D_{\mathrm{Tv}} + D_{\mathrm{Tl}} = k_{\mathrm{v}} \frac{\partial g}{\partial T} + D_{\mathrm{Tl}} \tag{A.7}$$

A.2 Implication of the thermal conductivity of a wet porous material

In Formula (A.3), λ represents the thermal conductivity, which is mainly determined by conduction through the constituents (solid skeleton, water and air) in the case of no moisture movement, and of course it varies with the water content. On the other hand, as seen in Formula (A.6), λ and RD_{Tv} always appear in the conjunct form $\lambda^* = \lambda + RD_{\mathrm{Tv}}$, and λ^* is the coefficient to the temperature gradient.

As mentioned above, λ and RD_{Tv} never appear separately in the fundamental Formulae (A.5) and (A.6) or in the boundary conditions. Consequently, D_{Θ}, D_{T}, $\lambda^* = \lambda + RD_{\mathrm{Tv}}$ and $D_{\Theta\mathrm{v}}$ are necessary to calculate the heat and mass transfer, while the values of λ and D_{Tv} are not necessary. Because of these properties of the fundamental formulae, only λ^* (not λ) is given by any method of measuring thermal diffusivity. Thus, methods that minimize the effects of moisture movement can mostly minimize the effect of the term $R\dfrac{\partial}{\partial x}\left(D_{\theta\mathrm{v}}\dfrac{\partial\theta}{\partial x}\right)$ and measure λ^* more accurately. The periodic method, for example, satisfies this criterion, which is made clear in Annex B. It is concluded from the above discussion that in order to solve the formulae of heat and mass transfer, one must measure and use the values of λ^*, not λ.

On the other hand, as λ is defined under the assumption that there is no moisture movement under the temperature gradient, it is a hypothetical quantity. As mentioned above, only λ^* can be measured in this system, and λ cannot be measured, except in special cases, but may be estimated by assuming an appropriate model.

Annex B
(informative)

Derivation of thermal conductivity from measured temperatures

B.1 Measurement of thermal diffusivity by periodic method

B.1.1 Objective

It is estimated from the result of Annex A that the thermal diffusivity corresponding to λ^* can be measured in a non-steady-state method minimizing the liquid movement and, thus, non-uniform moisture distribution. As a representative of many transient methods, the periodic method is examined here.

The objective of the following analysis is to estimate quantitatively the measurement uncertainty caused by moisture movement, and to derive a formula to give the measurement uncertainty that is composed of the material properties and is applicable to any material.

This problem will be solved by a perturbation method assuming that the transfer coefficients are linear functions of water content and temperature and that the solution is a power series of the input surface temperature variation amplitude.

The influences of moisture are those caused by

a) the existence of the moisture movement, and

b) variation of the transfer coefficients with water content and temperature.

Approximately, the first term of the solution can be considered to represent a), and the second term, b).

B.2 Formulation

B.2.1 Perturbation from initial conditions

Suppose that the material is semi-infinite and that its surface temperature variations are sinusoidal. The basic formulae are Formulae (A.5) and (A.6) (neglecting smaller order terms on the left-hand side).

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left\{ \begin{aligned} & D_{\theta \mathrm{l}}^{0}\left[1+\eta_1\left(\theta-\theta_0\right)\right]\left[1+\eta_2\left(T-T_0\right)\right]\frac{\partial \theta}{\partial x}+ \\ & D_{\theta \mathrm{v}}^{0}\left[1+\varsigma_1\left(\theta-\theta_0\right)\right]\left[1+\varsigma_2\left(T-T_0\right)\right]\frac{\partial \theta}{\partial x} \end{aligned} \right\}+ \frac{\partial}{\partial x}\left\{ \begin{aligned} & D_{\mathrm{Tl}}^{0}\left[1+\beta_1\left(\theta-\theta_0\right)\right]\left[1+\beta_2\left(T-T_0\right)\right]\frac{\partial T}{\partial x}+ \\ & D_{\mathrm{Tv}}^{0}\left[1+\xi_1\left(\theta-\theta_0\right)\right]\left[1+\xi_2\left(T-T_0\right)\right]\frac{\partial T}{\partial x} \end{aligned} \right\} \tag{B.1}$$

$$c'\gamma'\left[1+\kappa_1\left(\theta-\theta_0\right)\right]\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left\{\begin{matrix}\lambda^0\left[1+\gamma_1\left(\theta-\theta_0\right)\right]\left[1+\gamma_2\left(T-T_0\right)\right]\frac{\partial T}{\partial x}+\\ RD^0_{\mathrm{Tv}}\left[1+\xi_1\left(\theta-\theta_0\right)\right]\left[1+\xi_2\left(T-T_0\right)\right]\frac{\partial T}{\partial x}\end{matrix}\right\}+ R\frac{\partial}{\partial x}\left\{D^0_{\theta\mathrm{v}}\left[1+\varsigma_1\left(\theta-\theta_0\right)\right]\left[1+\varsigma_2\left(T-T_0\right)\right]\frac{\partial\theta}{\partial x}\right\} \tag{B.2}$$

with initial conditions

$$T=T_0,\ \theta=\theta_0 \tag{B.3}$$

and boundary conditions

$$\left\{\begin{matrix}D^0_{\theta\mathrm{l}}\left[1+\eta_1\left(\theta-\theta_0\right)\right]\left[1+\eta_2\left(T-T_0\right)\right]+\\ D^0_{\theta\mathrm{v}}\left[1+\varsigma_1\left(\theta-\theta_0\right)\right]\left[1+\varsigma_2\left(T-T_0\right)\right]\end{matrix}\right\}\frac{\partial\theta}{\partial x}+ \left\{\begin{matrix}D^0_{\mathrm{Tl}}\left[1+\beta_1\left(\theta-\theta_0\right)\right]\left[1+\beta_2\left(T-T_0\right)\right]+\\ D^0_{\theta\mathrm{v}}\left[1+\xi_1\left(\theta-\theta_0\right)\right]\left[1+\xi_2\left(T-T_0\right)\right]\end{matrix}\right\}\frac{\partial T}{\partial x}=0\left(x=0\right) \tag{B.4}$$

$$\theta=Finite\left(x\to+\infty\right) \tag{B.5}$$

$$T=T_0+I_0\sin\omega t\left(x=0\right) \tag{B.6}$$

$$T=Finite\left(x\to+\infty\right) \tag{B.7}$$

where zero superscripts represent the values at Θ_0 and T_0.

Transforming Θ and T to Θ' and T' by the relation

$$T'=T-T_0,\theta'=\theta-\theta_0 \tag{B.8}$$

The transformed formulae have the same form as Formulae (B.1) to (B.7), except that Θ_0 and T_0 are 0. Hereafter, we rewrite Θ' and T' as Θ and T.

B.2.2 Moisture content and temperature as functions of small input

We assume a power series solution as

$$\begin{aligned}\theta&=I_0\theta_1+I_0{}^2\theta_2+I_0{}^3\theta_3+\cdots\\ T&=I_0T_1+I_0{}^2T_2+I_0{}^3T_3+\cdots\end{aligned} \tag{B.9}$$

Substituting these formulae into Formulae (B.1) to (B.7) and equating the coefficients of the same order of I_0, we obtain the formulae for Θ_1, T_1, Θ_2, T_2, etc.

B.3 Solution

B.3.1 Solution: First order (periodic solution)

$$\theta_1\left(t,x\right)=\alpha_1E_1\exp\left(-\alpha_1\sqrt{\frac{\omega}{2}}x\right)\times\sin\left(\omega t-\alpha_1\sqrt{\frac{\omega}{2}}x\right)- \\ \alpha_2E_2\exp\left(-\alpha_2\sqrt{\frac{\omega}{2}}x\right)\times\sin\left(\omega t-\alpha_2\sqrt{\frac{\omega}{2}}x\right) \tag{B.10}$$

$$T_1\left(t,x\right)=\alpha_1\frac{EE_1}{B}\exp\left(-\alpha_1\sqrt{\frac{\omega}{2}}x\right)\times\sin\left(\omega t-\alpha_1\sqrt{\frac{\omega}{2}}x\right)- \\ \alpha_2\frac{EE_2}{B}\exp\left(-\alpha_2\sqrt{\frac{\omega}{2}}x\right)\times\sin\left(\omega t-\alpha_2\sqrt{\frac{\omega}{2}}x\right) \tag{B.11}$$

where

$$E=\frac{B}{\left(\alpha_1-\alpha_2\right)\left[C-\left(CA-BD\right)\left(\alpha_1{}^2+\alpha_1\alpha_1+\alpha_1{}^2\right)\right]} \\ E_1=C-\left(CA-BD\right)\alpha_1{}^2, E_2=C-\left(CA-BD\right)\alpha_2{}^2 \tag{B.12}$$

α_1 and α_2 are positive roots of the quadratic equation ($\alpha_1 < \alpha_2$)

$$\left(AC-BD\right)\alpha^4-\left(A+C\right)\alpha^2+1=0 \tag{B.13}$$

$$A=D_\theta^0, B=D_\mathrm{T}^0, C=\frac{\lambda^0+RD_\mathrm{Tv}^0}{\left(c'\gamma'\right)^0}, D=\frac{RD_{\theta\mathrm{v}}^0}{\left(c'\gamma'\right)^0} \tag{B.14}$$

B.3.2 Solution: Second order

The periodic solutions are

$$\theta_2\left(t,x\right)=2U_3\exp\left(-\alpha_1\sqrt{\omega}x\right)\times\cos\left(2\omega t-\alpha_1\sqrt{\omega}x\right)+2U_7\exp\left(-\alpha_2\sqrt{\omega}x\right)\times\cos\left(2\omega t-\alpha_2\sqrt{\omega}x\right)+ \\ 2Q_1\exp\left(-2\sqrt{\frac{\omega}{2}}\alpha_1x\right)\times\cos\left(2\omega t-2\sqrt{\frac{\omega}{2}}\alpha_1x\right)+2Q_2\exp\left(-2\sqrt{\frac{\omega}{2}}\alpha_2x\right)\times\cos\left(2\omega t-2\sqrt{\frac{\omega}{2}}\alpha_2x\right)+ \\ 2Q_3\exp\left[\sqrt{\frac{\omega}{2}}\left(\alpha_1+\alpha_2\right)x\right]\times\cos\left[2\omega t\sqrt{\frac{\omega}{2}}\left(\alpha_1+\alpha_2\right)x\right]+Q_4\exp\left(-2\sqrt{\frac{\omega}{2}}\alpha_1x\right)+ \\ Q_5\exp\left(-2\sqrt{\frac{\omega}{2}}\alpha_2x\right)+2Q_6'\exp\left[-\sqrt{\frac{\omega}{2}}\left(\alpha_1+\alpha_2\right)x\right]\times\cos\left[\sqrt{\frac{\omega}{2}}\left(\alpha_1-\alpha_2\right)x\right]+ \\ 2Q_6''\exp\left[-\sqrt{\frac{\omega}{2}}\left(\alpha_1+\alpha_2\right)x\right]\times\sin\left[\sqrt{\frac{\omega}{2}}\left(\alpha_1-\alpha_2\right)x\right] \tag{B.15}$$

$$
\begin{aligned}
T_2\left(t,x\right) = {} & 2\frac{1-A\alpha_1^2}{B\alpha_1^2}U_3\exp\left(-\alpha_1\sqrt{\omega}x\right)\times\cos\left(2\omega t-\alpha_1\sqrt{\omega}x\right)+ \\
& 2\frac{1-A\alpha_2^2}{B\alpha_1^2}U_7\exp\left(-\alpha_2\sqrt{\omega}x\right)\times\cos\left(2\omega t-\alpha_2\sqrt{\omega}x\right)+ \\
& 2R_1\exp\left(-2\sqrt{\frac{\omega}{2}}\alpha_1 x\right)\times\cos\left(2\omega t-\sqrt{\frac{\omega}{2}}\alpha_1 x\right)+ \\
& 2R_2\exp\left(-2\sqrt{\frac{\omega}{2}}\alpha_2 x\right)\times\cos\left(2\omega t-\sqrt{\frac{\omega}{2}}\alpha_2 x\right)+ \\
& 2R_3\exp\left[-\sqrt{\frac{\omega}{2}}\left(\alpha_1+\alpha_2\right)x\right]\times\cos\left[2\omega t-\sqrt{\frac{\omega}{2}}\left(\alpha_1+\alpha_2\right)x\right]+ \\
& R_4\exp\left(-2\sqrt{\frac{\omega}{2}}\alpha_1 x\right)+R_5\exp\left(-2\sqrt{\frac{\omega}{2}}\alpha_2 x\right)+ \\
& 2R_6'\exp\left[-\sqrt{\frac{\omega}{2}}\left(\alpha_1+\alpha_2\right)x\right]\times\cos\left[\sqrt{\frac{\omega}{2}}\left(\alpha_1-\alpha_2\right)x\right]+ \\
& 2R_6''\exp\left[-\sqrt{\frac{\omega}{2}}\left(\alpha_1+\alpha_2\right)x\right]\times\sin\left[\sqrt{\frac{\omega}{2}}\left(\alpha_1-\alpha_2\right)x\right]+V_{18}
\end{aligned}
\tag{B.16}
$$

B.4 Approximation of the coefficients appearing in the solution

B.4.1 Coefficients in terms of heat and moisture properties

By assuming $\frac{A}{C} << 1, \frac{BD}{AC} < 1$, etc. (which should be derived from the hygrothermal properties), the following approximate formulae are obtained:

$$
\begin{aligned}
& \alpha_1 \cong \frac{1}{\sqrt{C}}\left(1-\frac{BD}{2C^2}\right), \alpha_2 \cong \frac{1}{\sqrt{A}}\left(1+\frac{BD}{2AC}\right), E=\frac{B}{\sqrt{C}}\left(1-\frac{\sqrt{A}}{\sqrt{C}}\cdot\frac{BD}{AC}\right) \\
& E_1 \cong C\left(1-\frac{A}{C}\right), E_2 \cong \frac{BD}{C}\left(1+\frac{A}{C}-\frac{BD}{C^2}\right), \alpha_1 E_1\frac{E}{B} \cong 1-\frac{\sqrt{A}}{\sqrt{C}}\cdot\frac{BD}{AC} \\
& \alpha_2 E_2\frac{E}{B} \cong 1-\frac{\sqrt{A}}{\sqrt{C}}\cdot\frac{BD}{AC}\left(1+\frac{BD}{2AC}\right), 2\frac{1-A\alpha_1^2}{B\alpha_1^2}U_3 \cong -2\left(R_1+R_2\right)
\end{aligned}
\tag{B.17}
$$

B.5 Determination of thermal diffusivity

B.5.1 Solution for heat flow without moisture

The solution is

$$T\left(t,x\right)=I_0\exp\left(-\sqrt{\frac{\omega}{2a}}x\right)\times\sin\left(\omega t-\sqrt{\frac{\omega}{2a}}x\right)+T_\mathrm{m} \tag{B.18}$$

and, for example, the method using the phase difference gives the thermal diffusivity, a, as

$$a=\frac{1}{\varphi_0^2}\left(\frac{\omega}{2}\right)\left(x_1-x_2\right)^2 \tag{B.19}$$

where x_1 and x_2 are any two points in the material, and φ_0 is the phase difference between the temperatures at x_1 and x_2.

B.5.2 Solution for heat flow with moisture

By using the fact that generally, $\alpha_2\left(\cong\frac{1}{\sqrt{A}}\right)$ is larger than $\alpha_1\left(\cong\frac{1}{\sqrt{C}}\right)$, Formula (B.11) approximately becomes

$$T_1\left(t,x\right)=\alpha_1\frac{EE_1}{B}\exp\left(-\alpha_1\sqrt{\frac{\omega}{2}}x\right)\times\sin\left(\omega t-\alpha_1\sqrt{\frac{\omega}{2}}x\right) \tag{B.20}$$

because of the rapid exponential decay of the second term.

Based on this solution, when there is moisture, the apparent thermal diffusivity $\frac{1}{\alpha_1^2}$ can be given by

$$\frac{1}{\alpha_1^2}\cong C\left(1+\frac{BD}{C^2}\right)=\frac{1}{\varphi^2}\left(\frac{\omega}{2}\right)\left(x_1-x_2\right)^2 \tag{B.21}$$

where φ is the phase difference of the temperatures at x_1 and x_2, and

$$A=D_\theta^0, B=D_\mathrm{T}^0, C=\frac{\lambda^0+RD_\mathrm{Tv}^0}{\left(c'\gamma'\right)^0}, D=\frac{RD_{\theta\mathrm{v}}^0}{\left(c'\gamma'\right)^0} \tag{B.22}$$

Thus, it is clear that $\frac{BD}{C^2}$ expresses the measurement uncertainty.

B.6 Observational uncertainty

The uncertainty in the thermal diffusivity calculated using the phase difference, φ, measured in this case relative to that of Formula (B.19) is

$$\frac{\varphi_0^2}{\varphi^2}-1=\frac{1}{C}\cdot\frac{1}{\alpha_1^2}=1-\frac{BD}{C^2} \tag{B.23}$$

where the approximation $\frac{1}{\alpha_1^2}\cong C\left(1+\frac{BD}{C^2}\right)$ [Formula (B.17)] was used.

As expected, Formula (B.23) represents the influence of the term $R\frac{\partial}{\partial x}\left(D_{\theta\mathrm{v}}\frac{\partial\theta}{\partial x}\right)$.

Similarly, the perturbation solution, T_2, can be used to determine the amplitude of the input temperature variation.

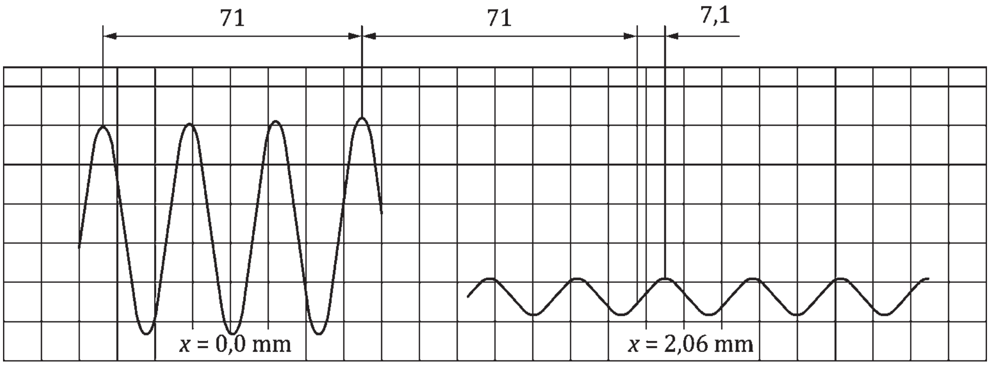
Annex C
(informative)

Example of measurement by periodic method

C.1 Thermal diffusivity and thermal conductivity of wood fibre board

C.1.1 Under dry conditions (see Figure C.1)

ω = 0,265 rad/s (period 24 s), T_0 = 25,2 °C, I_0 = 0,338 K



NOTE Under dry conditions, moisture content by weight Θ = 6 %.

Figure C.1 — Measured temperature profiles at two positions

Minimum testing time is about 1 h. It depends on angular velocity of input temperature and temperature diffusivity of the sample.

C.1.2 Under wet conditions (see Figure C.2)

ω = 0,265 rad/s (period 24 s), Θ ($\theta/c_s\gamma_s \times 100$) = 170 %, T_0 = 26,9 °C, I_0 = 0,338 K

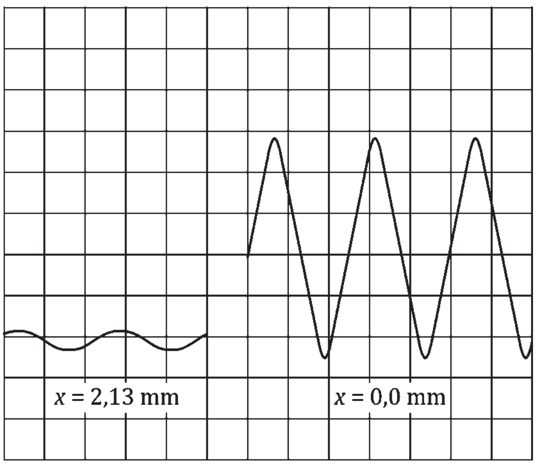
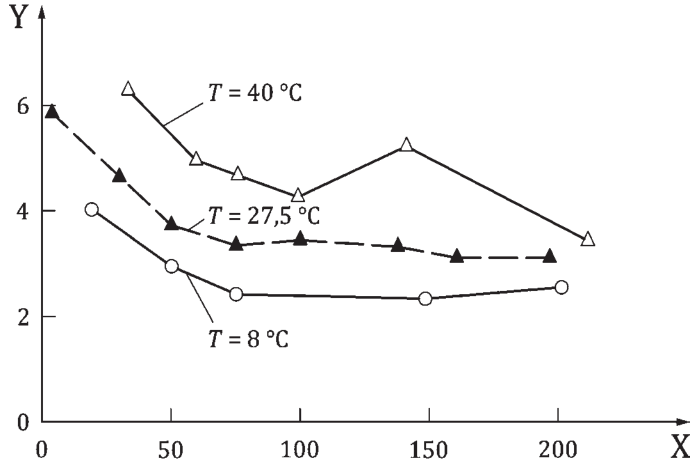


Figure C.2 — Measured temperature profiles at two positions (under wet conditions, moisture content by weight Θ =170 %)

Minimum testing time is about 1 h. It depends on angular velocity of input temperature and temperature diffusivity of the sample.

C.1.3 Thermal diffusivity (see Figure C.3)



Key

X moisture content (%)

Y thermal diffusivity (m^2/h)

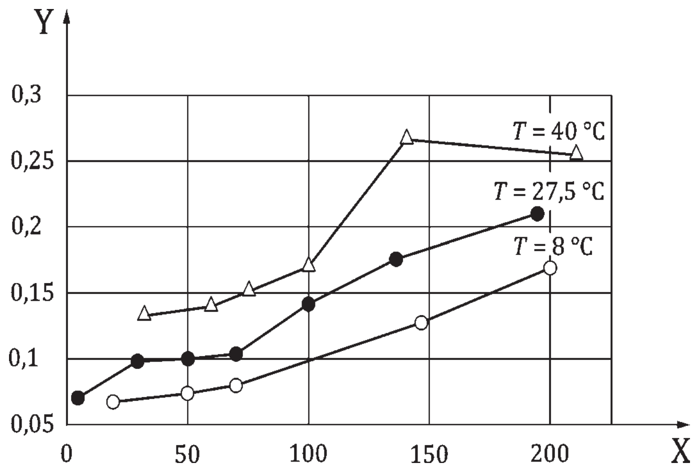
Figure C.3 — Measured thermal diffusivity of wood fibre board

C.2 Measured thermal conductivity

The thermal conductivities obtained are shown in Figure C.4. In calculating the thermal conductivity, the volumetric heat capacity of the material was calculated with Formula (C.1):

$$c'\gamma' = \left(c_s + \theta\right)\gamma_s \tag{C.1}$$

where c_s and γ_s are specific heat and density of the solid skeleton, respectively. The mean temperatures of the sample were 8,0 °C; 27,5 °C and 40,0 °C.



Key

X moisture content (%)

Y thermal conductivity (W/m·K)

Figure C.4 — Thermal conductivity of wood fibre board

C.3 Theoretical uncertainty

In these measurements, the maximum theoretical uncertainty of the periodic method calculated by Formula (B.23) is 0,1 % at Θ = 10 %, which is sufficiently small compared with the uncertainty caused by experimental processes.

C.4 Discussion

The results obtained for a wood fibre board show that the thermal conductivity, λ^*, increases rapidly with water content, from 0,06 (W/m·K) when it is air dry to 0,21 (W/m·K) at 200 % of water content at 25 °C. This fact suggests that the variation of the thermal conductivity with water content must be considered.

Thermal conductivity increases with the mean temperature, and its change is rather large. This increase is because of the variation of the values of λ and D_{Tv}, which are components of λ^*, with temperature. Making use of this fact, the values of D_{Tv} and λ can be determined separately by the results obtained.

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