
**Geometrical product specifications
(GPS) — Filtration —**

Part 31:
**Robust profile filters: Gaussian
regression filters**

Spécification géométrique des produits (GPS) — Filtrage —

Partie 31: Filtres de profil robustes: Filtres de régression gaussiens



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ISO copyright office
Ch. de Blandonnet 8 • CP 401
CH-1214 Vernier, Geneva, Switzerland
Tel. +41 22 749 01 11
Fax +41 22 749 09 47
copyright@iso.org
www.iso.org

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: www.iso.org/iso/foreword.html.

The committee responsible for this document is ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

This first edition of ISO 16610-31 cancels and replaces ISO/TS 16610-31, which has been technically revised.

A list of all parts in the ISO 16610 series can be found on the ISO website.

Introduction

This document is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO 14638). It influences the chain link C of all chains of standards.

For more detailed information of the relation of this document to the GPS matrix model, see [Annex C](#).

The ISO/GPS matrix model given in ISO 14638 gives an overview of the ISO/GPS system of which this document is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this document, unless otherwise indicated.

This document develops the concept of the discrete robust Gaussian regression filter. The robust process reduces the influence of the deep valleys and high peaks. The subject of this document is the robust Gaussian regression filter of degree $p = 2$, which has very good robust behaviour and form approximation for functional stratified engineering surfaces.

Geometrical product specifications (GPS) — Filtration —

Part 31:

Robust profile filters: Gaussian regression filters

1 Scope

This document specifies the characteristics of the discrete robust Gaussian regression filter for the evaluation of surface profiles with spike discontinuities such as deep valleys and high peaks.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1:2015, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO/IEC Guide 99, ISO 16610-1, ISO 16610-20, ISO 16610-30 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- IEC Electropedia: available at <http://www.electropedia.org/>
- ISO Online browsing platform: available at <http://www.iso.org/obp>

3.1

robust filter

filter that is insensitive to output data against specific phenomena in the input data

3.2

regression filter

M-estimator based on the local polynomial modelling of the profile

3.3

robust Gaussian regression filter

regression filter (3.2) based on the Gaussian weighting function and a *biweight influence function* (3.4)

3.4

biweight influence function

asymmetric function which is scale-invariant, expressed by

$$\psi(x) = \begin{cases} x \times \left(1 - \left(\frac{x}{c} \right)^2 \right)^2 & \text{for } |x| \leq c \\ 0 & \text{for } |x| > c \end{cases}$$

where c is the scale parameter.

4 Robust Gaussian regression filter

4.1 Weighting function

The weighting function of the robust Gaussian regression filter depends on the profile values (distance to the reference line) and the location of the weighting function along the profile.

4.2 Filter equation

4.2.1 General

The robust Gaussian regression filter is derived from the general discrete regression filter (see [Annex A](#)) by setting the degree to $p = 2$, using the biweight influence function and the Gaussian weighting function according to ISO 16610-21. In the case of $p = 2$, the robust Gaussian regression filter follows form components up to the second degree.

4.2.2 Filter equation for the robust Gaussian regression filter for open profiles

For open profiles, the filter equation for the robust Gaussian regression filter is given by [Formula \(1\)](#):

$$w_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \left(\mathbf{X}_k^T \times \mathbf{S}_k \times \mathbf{X}_k \right)^{-1} \times \mathbf{X}_k^T \times \mathbf{S}_k \times \mathbf{z} \quad (1)$$

The regression function is spanned by the matrix

$$\mathbf{X}_k = \begin{bmatrix} 1 & x_{1,k} & x_{1,k}^2 \\ \vdots & \vdots & \vdots \\ 1 & x_{n,k} & x_{n,k}^2 \end{bmatrix} \quad (2)$$

where

$$x_{l,k} = (l - k) \times \Delta x, \quad l = 1, \dots, n \quad (3)$$

The space variant weighting function, S_k , is given by [Formula \(4\)](#):

$$S_k = \begin{bmatrix} s_{1,k} \times \delta_1 & 0 & \dots & 0 \\ 0 & s_{2,k} \times \delta_2 & \dots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \dots & 0 & s_{n,k} \times \delta_n \end{bmatrix} \quad (4)$$

with the Gaussian function

$$s_{l,k} = \frac{1}{\gamma \times \lambda_c} \times \exp \left(-\pi \left(\frac{x_{l,k}}{\gamma \times \lambda_c} \right)^2 \right), \quad l = 1, \dots, n \quad (5)$$

and the parameter

$$\gamma = \sqrt{\frac{-1 - W \left(-\frac{1}{2 \times \exp(1)} \right)}{\pi}} \approx 0,7309 \quad (6)$$

The additional weights

$$\delta_l = \begin{cases} \left(1 - \left(\frac{z_l - w_l}{c} \right)^2 \right)^2 & \text{for } |z_l - w_l| \leq c, \\ 0 & \text{for } |z_l - w_l| > c \end{cases}, \quad l = 1, \dots, n \quad (7)$$

are derived from the biweight influence function with the parameter

$$c = \frac{3}{\sqrt{2} \times \operatorname{erf}^{-1}(0,5)} \times \operatorname{median} |z - \mathbf{w}| \approx 4,4478 \times \operatorname{median} |z - \mathbf{w}| \quad (8)$$

The definition for c is equivalent to three times Rq of the surface roughness for Gaussian distributed profiles and is the default case.

where

- $W(X)$ is the “Lambert W” function;
- $\text{erf}^{-1}(x)$ is the inverse error function;
- n is the number of values in the profile;
- k is the index of the profile ordinate $k = 1, \dots, n$;
- \mathbf{z} is the vector of dimension n of the profile values before filtering;
- \mathbf{w} is the vector of dimension n of the profile values of the filter reference line;
- w_k is the value of the filter mean line at position k ;
- λ_c is the cut-off wavelength of the profile filter;
- Δx is the sampling interval.

NOTE 1 Vector \mathbf{w} gives the profile values of the long-wave component (reference line). The short-wave component, \mathbf{r} , can be obtained by the difference vector, $\mathbf{r} = \mathbf{z} - \mathbf{w}$.

NOTE 2 For surfaces with big pores or peaks at the profile boundaries, the robustness can be increased by setting $P = 0$. In this case, the nominal form is eliminated by using a pre-filtering technique. The filter equation for $P = 0$ results in

$$w_k = \left(\mathbf{X}_k^T \times \mathbf{S}_k \times \mathbf{X}_k \right)^{-1} \times \mathbf{X}_k^T \times \mathbf{S}_k \times \mathbf{z} = \left(\sum_{l=1}^n s_{l,k} \times \delta_l \right)^{-1} \times \sum_{l=1}^n \left(s_{l,k} \times \delta_l \times z_l \right)$$

where

$$\mathbf{X}_k = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \text{ and } \gamma = \sqrt{\frac{\ln 2}{\pi}}$$

4.2.3 Filter equation for robust Gaussian regression filter for closed profiles

For closed profiles, the filter equation for the robust Gaussian regression filter is given by [Formula \(9\)](#):

$$\tilde{\mathbf{w}}_k = \left(\mathbf{1} \quad \mathbf{0} \quad \mathbf{0} \right) \times \left(\tilde{\mathbf{X}}_k^T \times \tilde{\mathbf{S}}_k \times \tilde{\mathbf{X}}_k \right)^{-1} \times \tilde{\mathbf{X}}_k^T \times \tilde{\mathbf{S}}_k \times \tilde{\mathbf{z}} \quad (9)$$

The regression function is spanned by the matrix

$$\tilde{\mathbf{X}}_k = \begin{bmatrix} 1 & \tilde{x}_{1,k} & \tilde{x}_{1,k}^2 \\ \vdots & \vdots & \vdots \\ 1 & \tilde{x}_{n,k} & \tilde{x}_{n,k}^2 \end{bmatrix} \quad (10)$$

with

$$\tilde{x}_{l,k} = \left(\left(l - k + \frac{n}{2} \right) \bmod n - \frac{n}{2} \right) \times \Delta x, \quad l = 1, \dots, n \quad (11)$$

The space variant weighting function, $\tilde{\mathbf{S}}_k$, is given by

$$\tilde{\mathbf{S}}_k = \begin{bmatrix} \tilde{s}_{1,k} \times \tilde{\delta}_1 & 0 & \dots & 0 \\ 0 & \tilde{s}_{2,k} \times \tilde{\delta}_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \dots & 0 & \tilde{s}_{n,k} \times \tilde{\delta}_n \end{bmatrix} \quad (12)$$

with the Gaussian function

$$\tilde{s}_{l,k} = \frac{1}{\gamma \times \lambda_c} \times \exp \left(-\pi \left(\frac{\tilde{x}_{l,k}}{\gamma \times \lambda_c} \right)^2 \right), \quad l = 1, \dots, n \quad (13)$$

and the parameter

$$\gamma = \sqrt{\frac{-1 - W \left(-\frac{1}{2 \times \exp(1)} \right)}{\pi}} \approx 0,7309 \quad (14)$$

The additional weights

$$\tilde{\delta}_l = \begin{cases} \left(1 - \left(\frac{\tilde{z}_l - \tilde{w}_l}{\tilde{c}} \right)^2 \right)^2 & \text{for } |\tilde{z}_l - \tilde{w}_l| \leq \tilde{c} \\ 0 & \text{for } |\tilde{z}_l - \tilde{w}_l| > \tilde{c} \end{cases}, \quad l = 1, \dots, n \quad (15)$$

are derived from the biweight influence function with the parameter

$$\tilde{c} = \frac{3}{\sqrt{2} \times \operatorname{erf}^{-1}(0,5)} \times \operatorname{median} |\tilde{\mathbf{z}} - \tilde{\mathbf{w}}| \approx 4,4478 \times \operatorname{median} |\tilde{\mathbf{z}} - \tilde{\mathbf{w}}| \quad (16)$$

The definition for c is equivalent to three times Rq of the surface roughness for Gaussian distributed profiles and is the default case.

where

- $W(X)$ is the “Lambert W” function;
- $\text{erf}^{-1}(x)$ is the inverse error function;
- n is the number of values in the profile;
- k is the index of the profile ordinate $k = 1, \dots, n$;
- $\tilde{\mathbf{z}}$ is the vector of dimension n of the profile values before filtering;
- $\tilde{\mathbf{w}}$ is the vector of dimension n of the profile values of the filter reference line;
- \tilde{w}_k is the value of the filter mean line at position k ;
- λ_c is the cut-off wavelength of the profile filter;
- Δx is the sampling interval.

Vector $\tilde{\mathbf{w}}$ gives the profile values of the long-wave component (reference line). The short-wave component, $\tilde{\mathbf{r}}$, may be obtained by the difference vector, $\tilde{\mathbf{r}} = \tilde{\mathbf{z}} - \tilde{\mathbf{w}}$.

4.2.4 Transmission characteristics

The weighting function of the robust Gaussian regression filter depends on the profile values and the location along the profile. Therefore, no transmission characteristic can be given.

5 Recommendations for nesting index (cutoff values λ_c)

It is recommended that a nesting index be chosen equivalent to three times the feature width in the profile data set. Otherwise, the nesting index should be chosen from the following series of values:

... 2,5 μm ; 8 μm ; 25 μm ; 80 μm ; 250 μm ; 0,8 mm; 2,5 mm; 8 mm; 25 mm; ...

6 Filter designation

Robust Gaussian regression filters according to this document are designated

FPRG

See also ISO 16610-1:2015, Clause 5.

Annex A (informative)

Examples

The examples for the application of the robust Gaussian regression filter ($p = 2$) are for information only. In [Figures A.1](#) to [A.5](#), profiles of different machined surfaces and the calculated reference lines are shown.

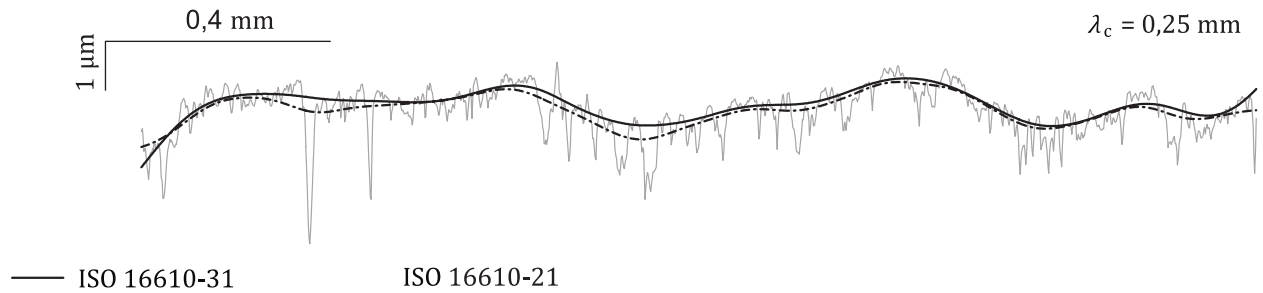


Figure A.1 — Profile of ceramic surface

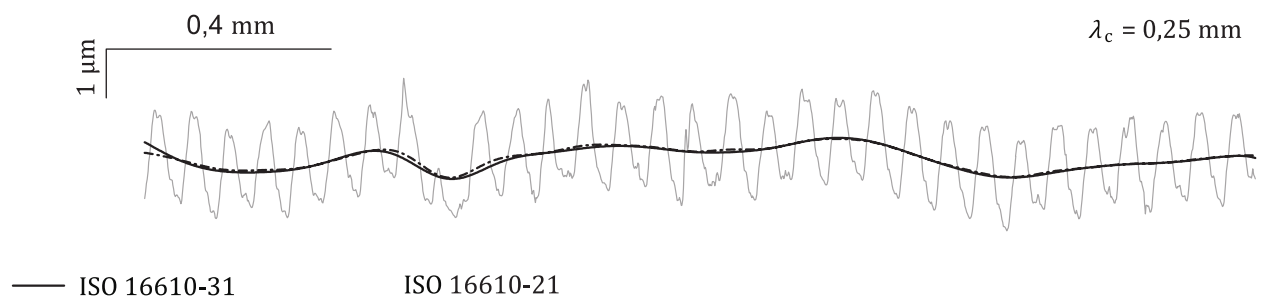


Figure A.2 — Profile of milled surface

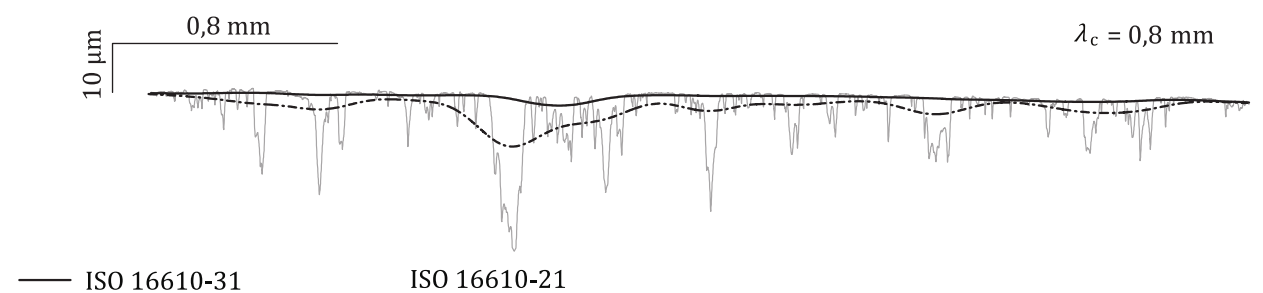


Figure A.3 — Profile of sintered surface with pores

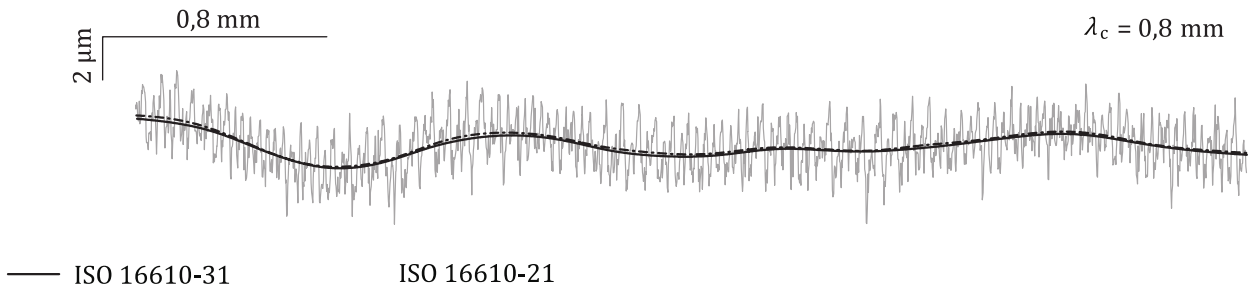


Figure A.4 — Profile of a ground surface

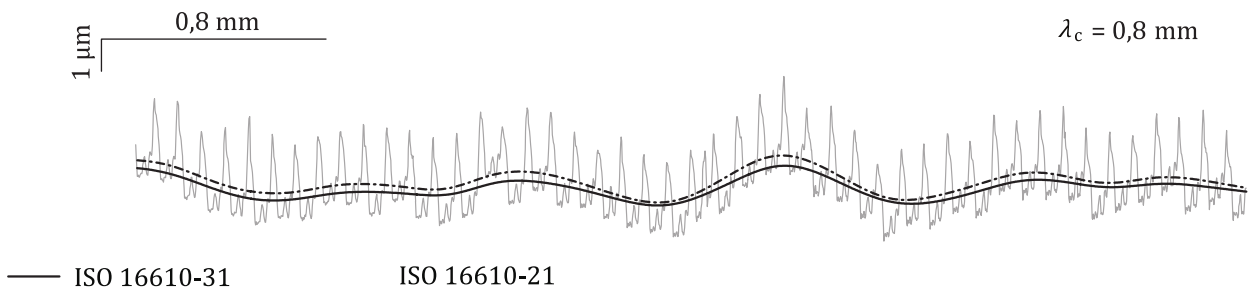


Figure A.5 — Profile of a turned surface

In [Figures A.6 to A.8](#), synthetic profiles with discontinuities and the calculated reference lines are shown.

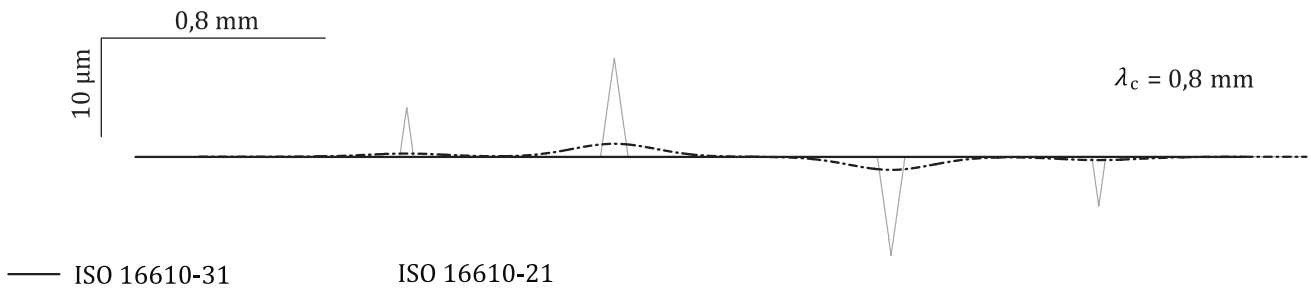


Figure A.6 — Profile with spikes

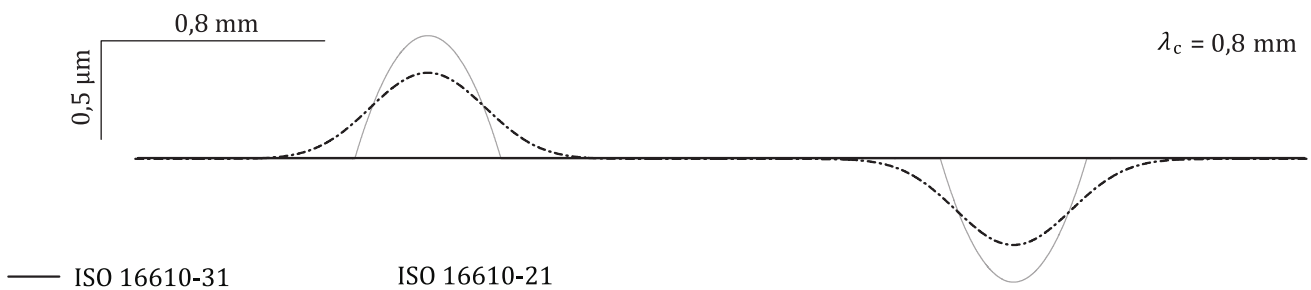


Figure A.7 — Profile with concave and convex structures

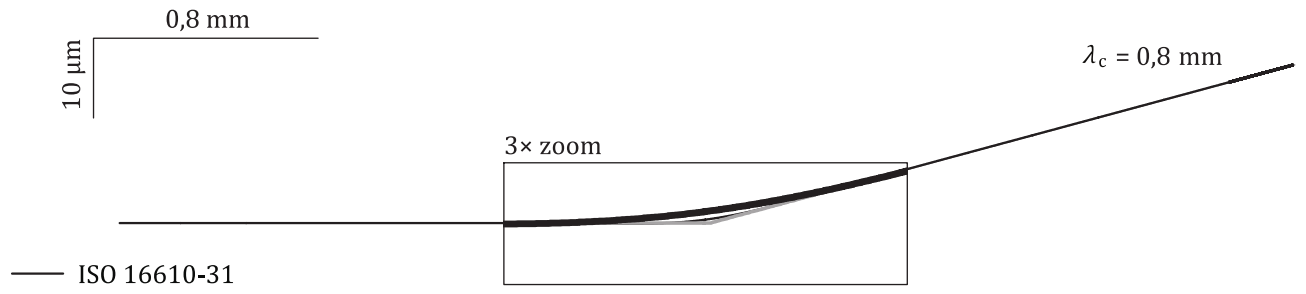


Figure A.8 — Slope up profile

Annex B (informative)

Relationship to the filtration matrix model

B.1 General

For full details about the filtration matrix model, see ISO 16610-1.

B.2 Position in the filtration matrix model

This document is a particular filter document in the column “Profile filters, Robust” (see [Table B.1](#)).

Table B.1 — Relationship to the filtration matrix model

General	Filters: ISO 16610 series					
	ISO 16610-1					
Fundamental	Profile filters			Areal filters		
	ISO 16610-11 ^a			ISO 16610-12 ^a		
	Linear	Robust	Morphological	Linear	Robust	Morphological
Basic concepts	Part 20	Part 30	Part 40	Part 60	Part 70	Part 80
Particular filters	Parts 21–25	Parts 31–35	Parts 41–45	Parts 61–65	Parts 71–75	Parts 81–85
How to filter	Parts 26–28	Parts 36–38	Parts 46–48	Parts 66–68	Parts 76–78	Parts 86–88
Multiresolution	Part 29	Part 39	Part 49	Part 69	Part 79	Part 89

^a At present included in ISO 16610-1.

Annex C (informative)

Relationship to the GPS matrix model

C.1 General

For full details about the GPS matrix model, see ISO 14638.

C.2 Information on this document and its application

This document defines the characteristics of the robust Gaussian regression filter. In particular, the robust separation of long- and short-wave components of surface profiles is defined.

C.3 Position in the GPS matrix model

This document is a general GPS standard that influences the chain link C of all chains of standards, as graphically illustrated in [Table C.1](#).

Table C.1 — Position in the GPS matrix model

	Chain links						
	A	B	C	D	E	F	G
	Symbols and indications	Feature requirements	Feature properties	Conformance and non-conformance	Measurement	Measurement equipment	Calibrations
Size			•				
Distance			•				
Form			•				
Orientation			•				
Location			•				
Run-out			•				
Profile surface texture			•				
Areal surface texture			•				
Surface imperfections			•				

C.4 Related International Standards

The related International Standards are those of the chain of standards indicated in [Table C.1](#).

Bibliography

- [1] ISO 3274, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Nominal characteristics of contact (stylus) instruments*
- [2] ISO 8015, *Geometrical product specifications (GPS) — Fundamentals — Concepts, principles and rules*
- [3] ISO 14253-1, *Geometrical product specifications (GPS) — Inspection by measurement of workpieces and measuring equipment — Part 1: Decision rules for verifying conformity or nonconformity with specifications*
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- [5] ISO 16610-20, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*
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