INTERNATIONAL STANDARD

ISO 16610-21

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Geometrical product specifications (GPS) — Filtration —

Part 21:
Linear profile filters: Gaussian filters

Spécification géométrique des produits (GPS) — Filtrage —

Partie 21: Filtres de profil linéaires: Filtres gaussiens

Reference number
ISO 16610-21:2011(E)

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 16610-21 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO 16610-21 cancels and replaces ISO 11562:1996, which has been technically revised.

ISO 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

— *Part 1: Overview and basic concepts* [Technical Specification]

— *Part 20: Linear profile filters: Basic concepts* [Technical Specification]

— *Part 21: Linear profile filters: Gaussian filters*

— *Part 22: Linear profile filters: Spline filters* [Technical Specification]

— *Part 28: Profile filters: End effects* [Technical Specification]

— *Part 29: Linear profile filters: Spline wavelets* [Technical Specification]

— *Part 30: Robust profile filters: Basic concepts* [Technical Specification]

— *Part 31: Robust profile filters: Gaussian regression filters* [Technical Specification]

— *Part 32: Robust profile filters: Spline filters* [Technical Specification]

— *Part 40: Morphological profile filters: Basic concepts* [Technical Specification]

— *Part 41: Morphological profile filters: Disk and horizontal line-segment filters* [Technical Specification]

— *Part 49: Morphological profile filters: Scale space techniques* [Technical Specification]

The following parts are planned:

— *Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets*

— *Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets*

- *Part 42: Morphological profile filters: Motif filters*
- *Part 60: Linear areal filters: Basic concepts*
- *Part 61: Linear areal filters: Gaussian filters*
- *Part 62: Linear areal filters: Spline filters*
- *Part 69: Linear areal filters: Spline wavelets*
- *Part 70: Robust areal filters: Basic concepts*
- *Part 71: Robust areal filters: Gaussian regression filters*
- *Part 72: Robust areal filters: Spline filters*
- *Part 80: Morphological areal filters: Basic concepts*
- *Part 81: Morphological areal filters: Sphere and horizontal planar segment filters*
- *Part 82: Morphological areal filters: Motif filters*
- *Part 89: Morphological areal filters: Scale space techniques*

Introduction

This part of ISO 16610 is a geometrical product specification (GPS) standard and is to be regarded as a global GPS standard (see ISO/TR 14638). It influences the chain links 3 and 5 of all chains of standards.

The ISO/GPS masterplan given in ISO/TR 14638 gives an overview of the ISO/GPS system of which this document is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this document, unless otherwise indicated.

For more detailed information about the relation of this part of ISO 16610 to the GPS matrix model, see Annex D.

This part of ISO 16610 develops the terminology and a concept of Gaussian filters. It has the transmission for the cut-off wavelength as 50 % since the short wave and long wave portions of surface profile are separated and can be reconstructed without altering the surface profile.

Geometrical product specifications (GPS) — Filtration —

Part 21:
Linear profile filters: Gaussian filters

1 Scope

This part of ISO 16610 specifies the metrological characteristics of the Gaussian filter, for the filtration of profiles. It specifies, in particular, how to separate long and short wave components of a surface profile.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO/TS 16610-1:2006, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

ISO/TS 16610-20:2006, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO/TS 16610-1, ISO/TS 16610-20, ISO/IEC Guide 98-3 and ISO/IEC Guide 99 and the following apply.

3.1
linear profile filters
profile filter to separate profiles into long wave and short wave components

[ISO/TS 16610-20:2006]

3.1.1
open profile
finite length surface profile with two ends

[ISO/TS 16610-1:2006]

3.1.2
closed profile
connected finite length surface profile without ends

[ISO/TS 16610-1:2006]

3.1.3
cut-off wavelength
wavelength of a sinusoidal profile of which 50 % of the amplitude is transmitted by the profile filter

[ISO/TS 16610-20:2006]

4 Characteristics of Gaussian profile filter for an open profile

4.1 Gaussian weighting function for an open profile

The weighting function of an open profile filter (see Figure 1) has the equation of the Gaussian function with the cut-off wavelength λ_c, where c is cut-off. The equation is given by:

$$s(x) = \frac{1}{\alpha \times \lambda_c} \times \exp\left[-\pi\left(\frac{x}{\alpha \times \lambda_c}\right)^2\right] \tag{1}$$

where

x is the distance from the centre (maximum) of the weighting function;

λ_c is the cut-off wavelength;

α is a constant, to provide 50 % transmission characteristic at the cut-off λ_c.

For implementation, the weighting function equation is as follows:

$$s(x) = \begin{cases} \frac{1}{\alpha \times \lambda_c} \times \exp\left[-\pi\left(\frac{x}{\alpha \times \lambda_c}\right)^2\right] & \text{for } -L_c \times \lambda_c \leqslant x \leqslant L_c \times \lambda_c \\ 0 & \text{Otherwise} \end{cases} \tag{2}$$

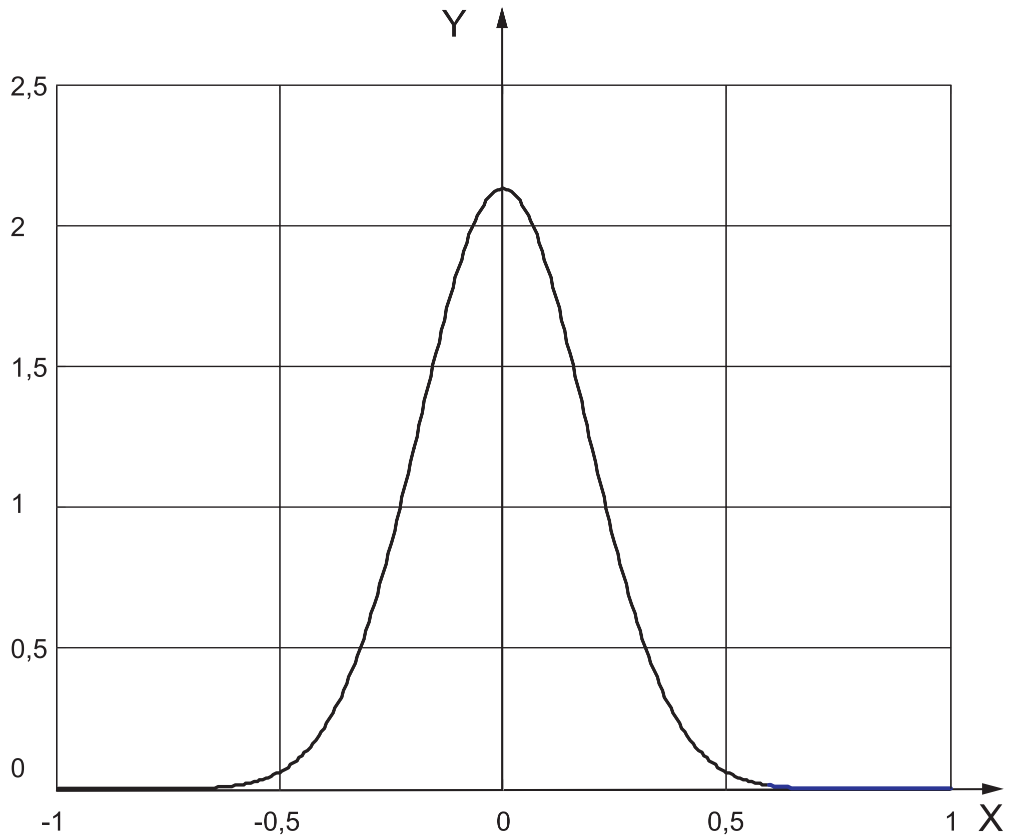
where

L_c is a truncation constant of the weighting function (see Annex A for recommended values);

α is given by:

$$\alpha = \sqrt{\frac{\ln 2}{\pi}} \approx 0,469\,7 \tag{3}$$

The graph of the weighting function is shown in Figure 1.



Key

X x/λ_c

Y $\lambda_c \times s(x)$

Figure 1 — Weighting function of a Gaussian profile filter for an open profile

4.2 Transmission characteristics of an open profile

4.2.1 Transmission characteristic of the long wave component for an open profile

The transmission characteristic (see Figure 2) is determined from the weighting function by means of the Fourier transformation. The transmission characteristic of the long wave component (mean line) is given by:

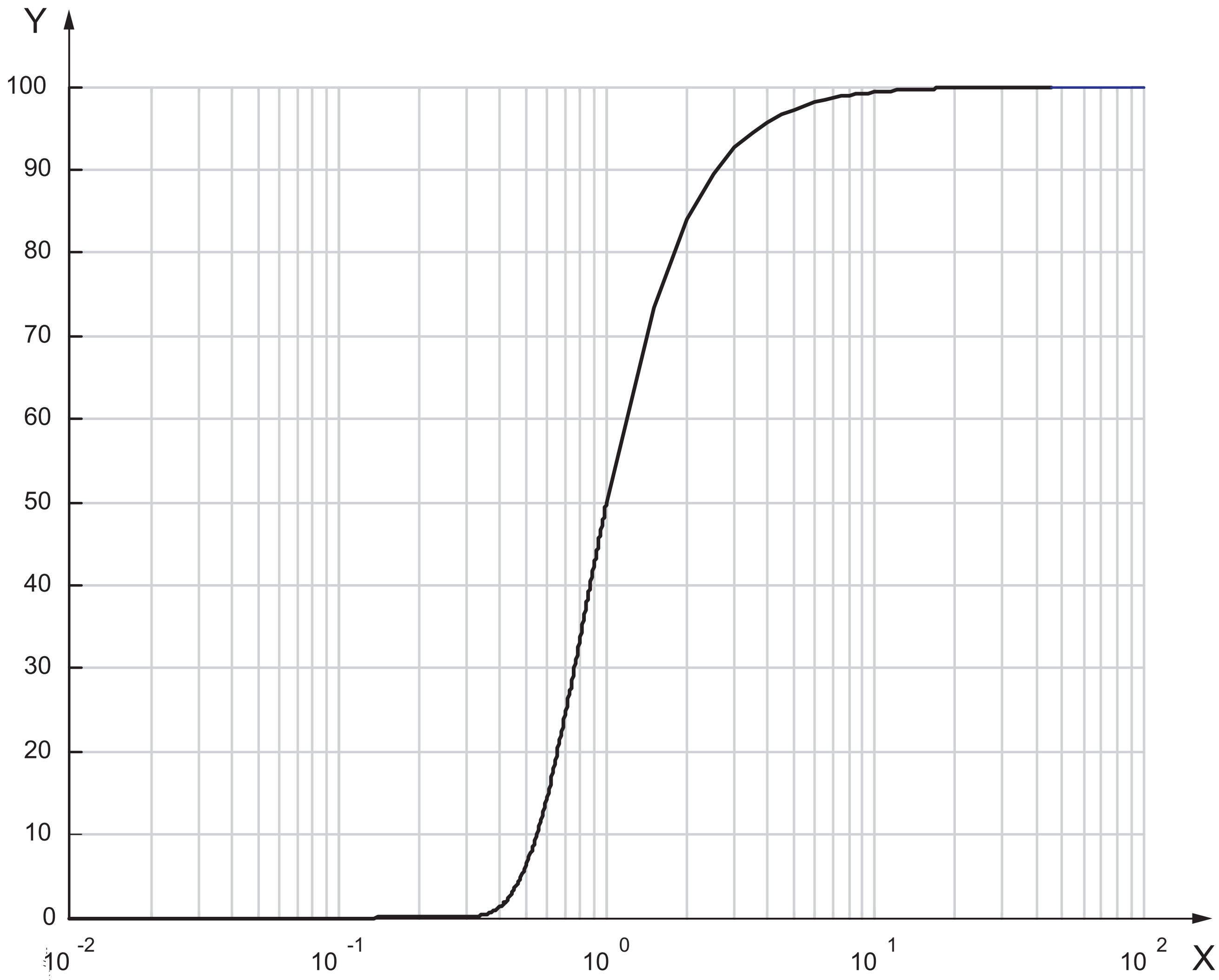
$$\frac{a_1}{a_0} = \exp\left[-\pi\left(\frac{\alpha \times \lambda_c}{\lambda}\right)^2\right] \quad (4)$$

where

a_0 is the amplitude of a sinusoidal wave profile before filtering;

a_1 is the amplitude of this sinusoidal profile in the mean line;

λ is the wavelength of this sinusoidal profile.



Key

X λ/λ_c

Y amplitude transmission a_1/a_0, in percent

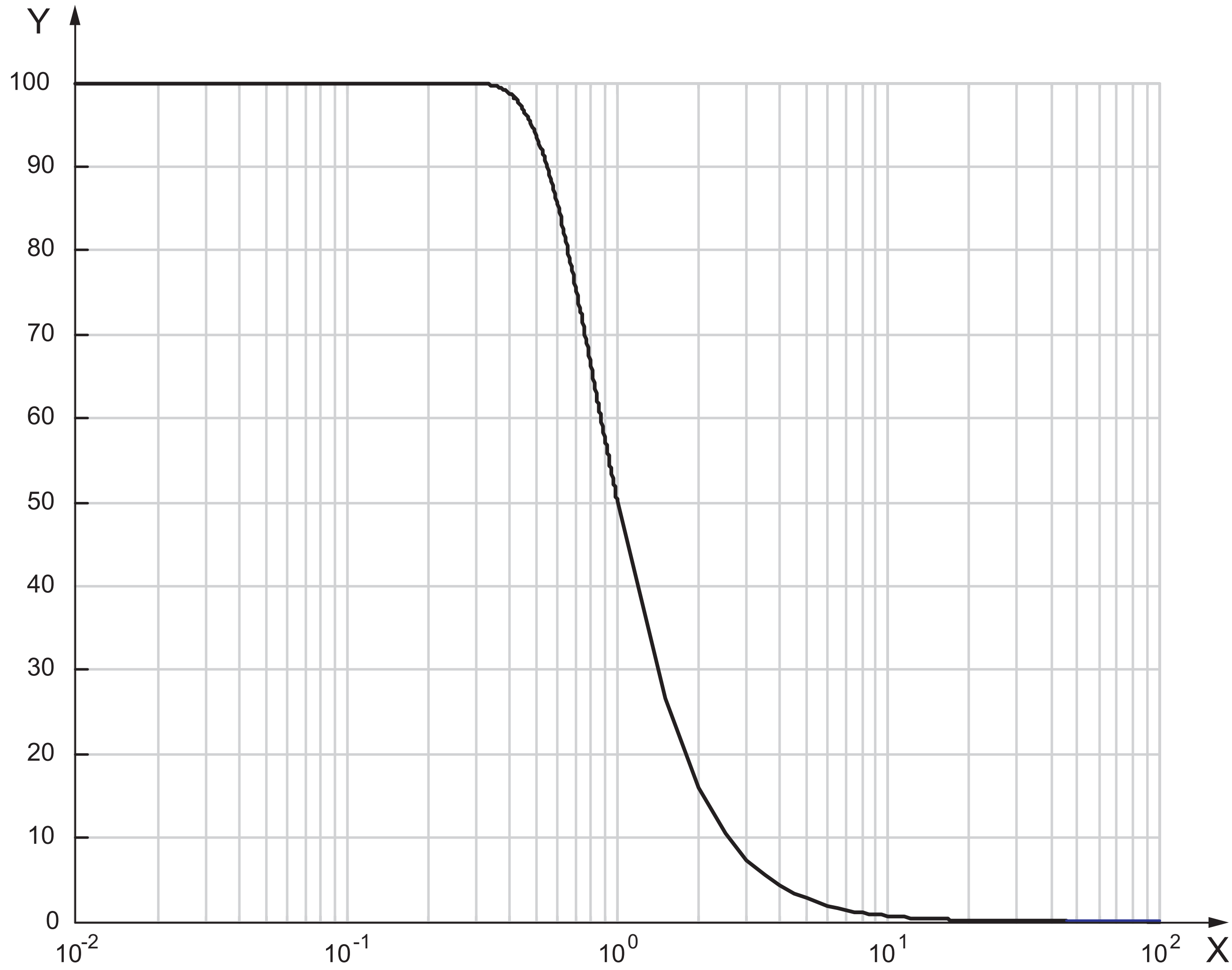
Figure 2 — Long wave transmission function of the Gaussian filter for an open profile

4.2.2 Transmission characteristic of the short wave component for an open profile

The transmission characteristic (see Figure 3) is determined from the weighting function by means of the Fourier transformation and is complementary to the transmission characteristic of the long wave profile component. The transmission characteristic of the short wave component is given by:

$$\frac{a_2}{a_0} = 1 - \exp\left[-\pi\left(\frac{\alpha \times \lambda_c}{\lambda}\right)^2\right] \qquad \frac{a_2}{a_0} = 1 - \frac{a_1}{a_0} \tag{5}$$

where a_2 is the amplitude of the short wave component of a sinusoidal wave profile.



Key

X λ/λ_c

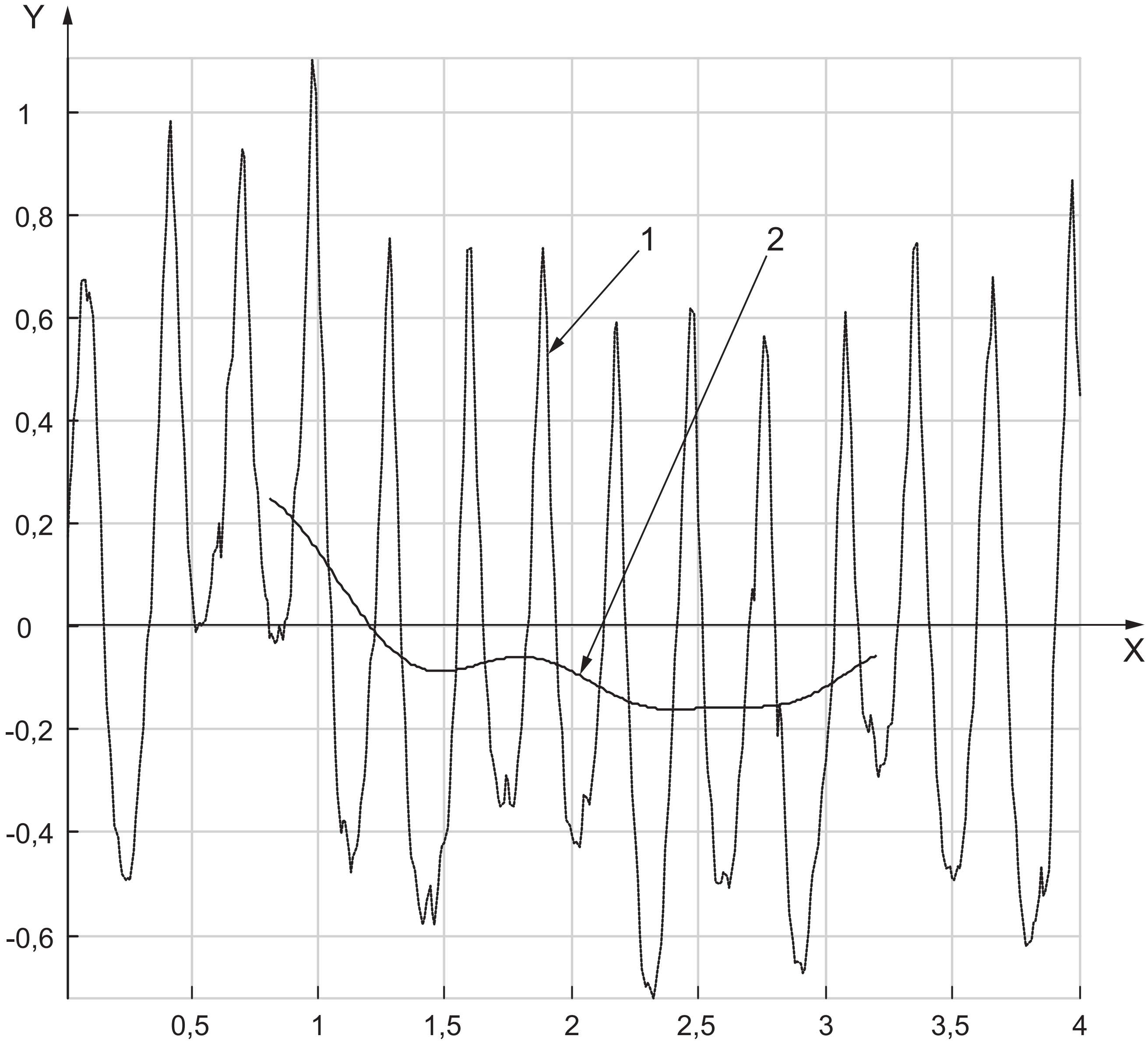
Y amplitude transmission a_1/a_0, in percent

Figure 3 — Short wave transmission function of the Gaussian filter for an open profile

4.3 End effects

Since open profiles are only defined for a finite length; the convolution of the Gaussian filter with the open profile will cause unintentional changes in the filtration response in the end portions of the profile. The end portion of an open profile where end effects are significant are called the *end effect regions*.

One strategy to reduce these end effects is to take a longer profile and remove the end effect regions to leave a filtration response with insignificant end effects (see Figure 4).



Key

1 unfiltered profile

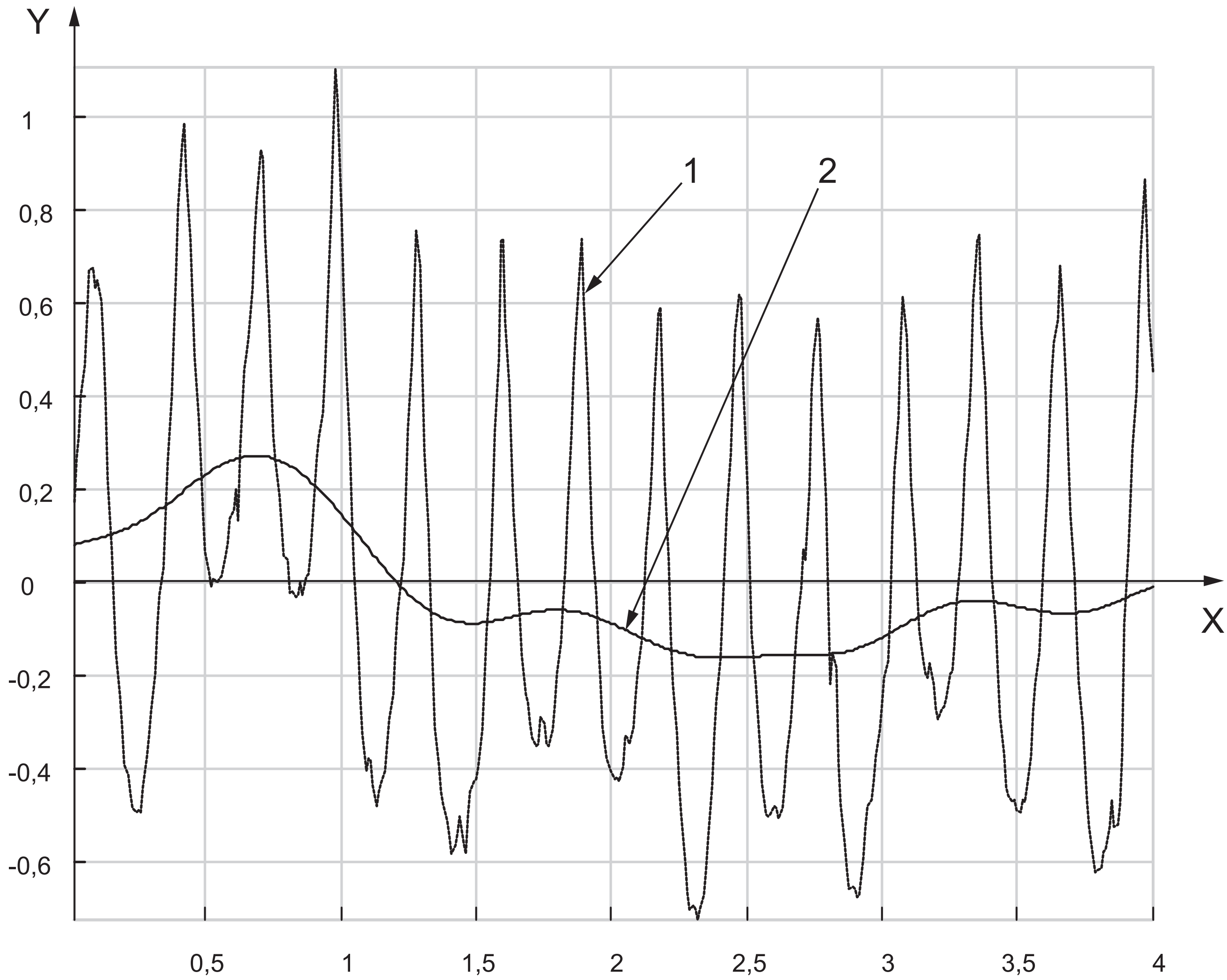
2 filtered profile

X length, mm

Y height, µm

Figure 4 — Example of Gaussian filtration ($\lambda_c = 0{,}8$ mm) **with the removal of the end effect regions**

Alternative strategies to reduce end effects are to apply the techniques used in ISO/TS 16610-28. For an illustrative example, see Figure 5.



Key

1 unfiltered profile

2 filtered profile

X length, mm

Y height, µm

Figure 5 — Example of Gaussian filtration (λ_c = 0,8 mm) using the moment retainment criterion from ISO/TS 16610-28

5 Characteristic of Gaussian profile filter for a closed profile

5.1 Introduction

A closed profile filter is not recommended if the length (circumference) is less than $2\lambda_c$.

5.2 Gaussian weighting function for a closed profile

The weighting function of a closed profile filter (see Figure 6) has the equation of the Gaussian density function wrapped around the closed profile of length L. With the cut-off frequency $f_c = L/\lambda_c$, the equation is as follows:

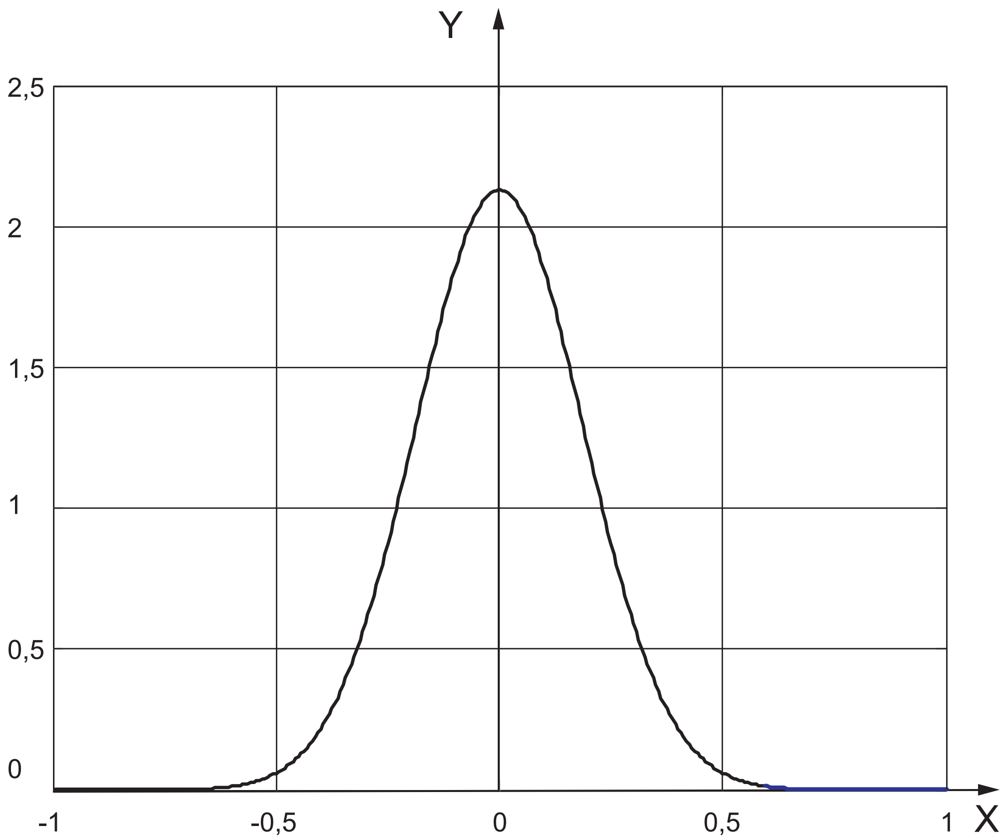
$$s(x)=\begin{cases}\dfrac{f_c}{\alpha\times L}\times\exp\left[-\pi\left(\dfrac{x\times f_c}{\alpha\times L}\right)^2\right] & -\dfrac{L_c\times L}{f_c}\leqslant x\leqslant\dfrac{L_c\times L}{f_c}\\[2ex] 0 & \text{Otherwise}\end{cases}\qquad(6)$$

where

- x is the distance along the closed profile from the centre (maximum) of the weighting function;
- f_c is the cut-off frequency in undulations per revolution;
- L is the length of the closed profile;
- L_c is a truncation constant (see Annex A);
- α is a constant given by

$$\alpha=\sqrt{\frac{\ln 2}{\pi}}\approx 0{,}469\,7\qquad(7)$$

The graph of the weighting function for a closed profile filter is shown in Figure 4.



Key

X $(f_c/L)\times x$

Y $(L/f_c)\times s(x)$

Figure 6 — Weighting function of a closed Gaussian profile filter

5.3 Transmission characteristic of a closed profile filter

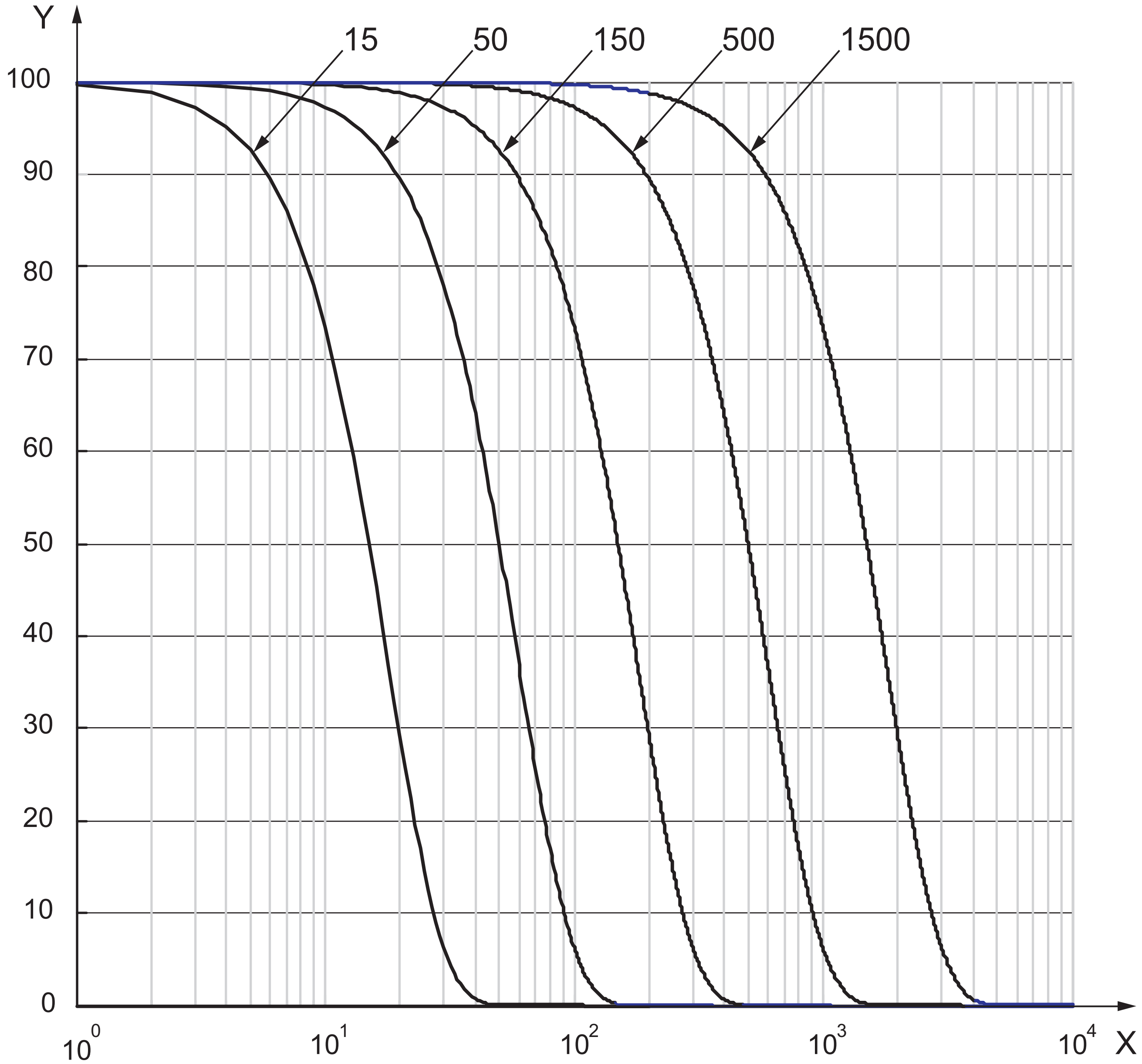
5.3.1 Transmission characteristic of the long wave component for a closed profile

The filter characteristic (see Figure 7) is determined from the weighting function by means of the Fourier transformation. The filter characteristic for the mean line when $\lambda_c \ll L$ can be approximated by the following equation:

$$\frac{a_1}{a_0} = \exp\left[-\pi\left(\frac{\alpha \times f}{f_c}\right)^2\right] \tag{8}$$

where

- a_0 is the amplitude of sinusoidal wave roughness profile before filtering;
- a_1 is the amplitude of this sinusoidal profile in the mean line;
- f is the frequency of the sinusoidal profile in undulations per revolution.



Key

X undulations per revolution (UPR)

Y amplitude transmission a_1/a_0, in percent

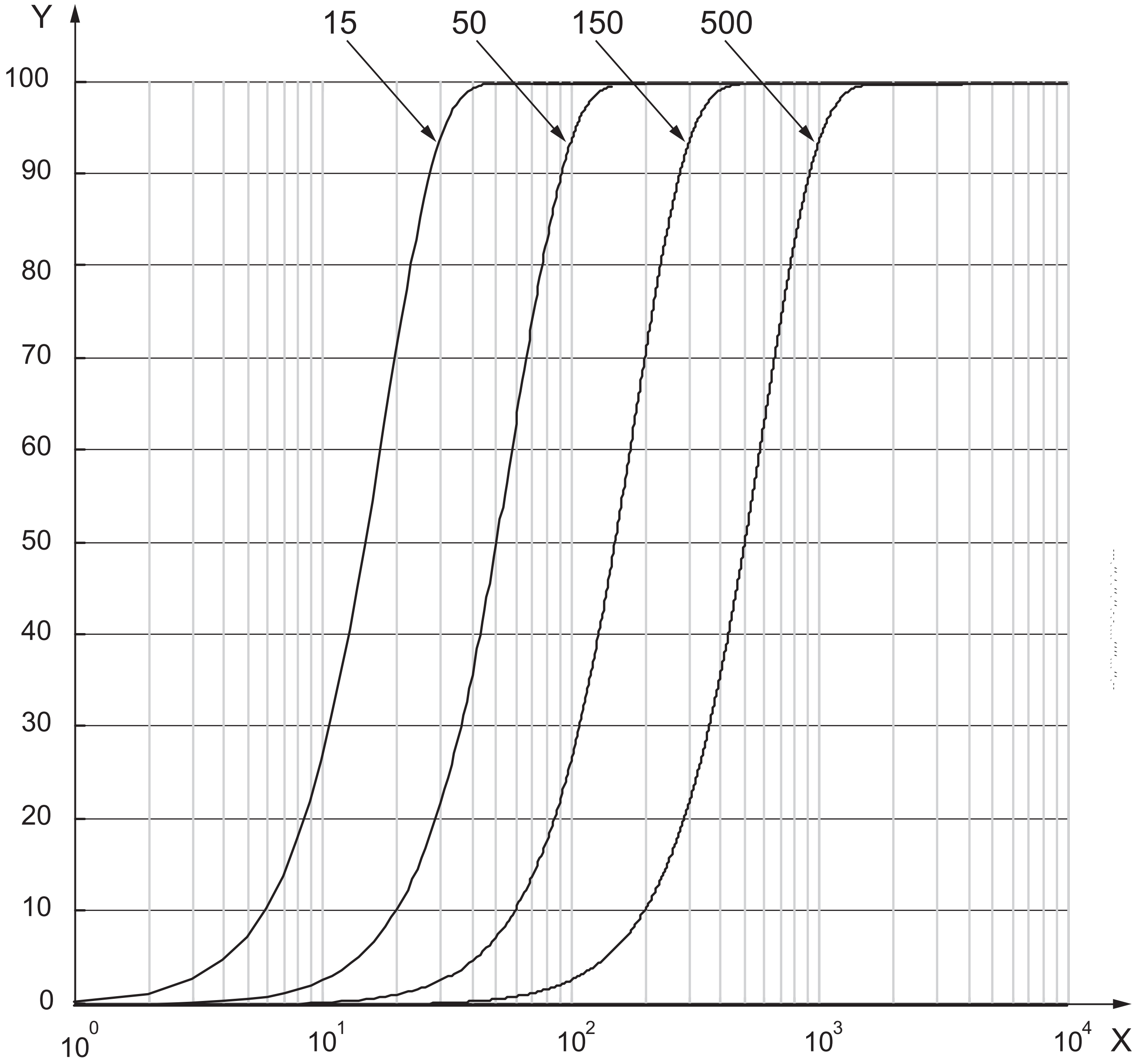
Figure 7 — Transmission characteristic of the long wave profile component for closed profile filters of different UPR values

5.3.2 Transmission characteristic of the short wave profile component

The transmission characteristic (Figure 8) of the short wave profile component is complementary to the transmission characteristic of the long wave profile component. The short wave profile component is the difference between the surface profile and the long wave profile component. The equation as a function of the limiting wavelength λ_c when $\lambda_c << L$ can be approximated by:

$$\frac{a_2}{a_0} = 1 - \exp\left[-\pi\left(\frac{\alpha \times f}{f_c}\right)^2\right] \qquad \frac{a_2}{a_0} = 1 - \frac{a_1}{a_0} \tag{9}$$

where a_2 is the amplitude of the sine wave roughness profile.



Key

X undulations per revolution (UPR)

Y amplitude transmission a_2/a_0, in percent

Figure 8 — Transmission characteristic of the short wave profile component for a closed profile filter in undulations per revolution

Annex A
(informative)

Implementation errors for open and closed profiles (convolution algorithm)

A.1 Open profiles

In theory, the Gaussian weighting function for an open profile takes a positive value from minus infinity to plus infinity, that is to say it has an infinite support. But the Gaussian weighting function approches zero very rapidly away from the centre, so that, in regions sufficiently far away from the centre, it is effectively zero for any practical implementation. In other words, the Gaussian weighting function has finite support in any practicable implementation. This is equivalent to implementating a truncated Gaussian weighting function where the truncated Gaussian weighting function for an open profile is defined as:

$$\tilde{s}(x)=\begin{cases} 0 & \text{for} \quad x < -L_\text{c}\times\lambda_\text{c} \\ s(x) & \text{for} \quad -L_\text{c}\times\lambda_\text{c} \leqslant x \leqslant L_\text{c}\times\lambda_\text{c} \\ 0 & \text{for} \quad x > L_\text{c}\times\lambda_\text{c} \end{cases} \tag{A.1}$$

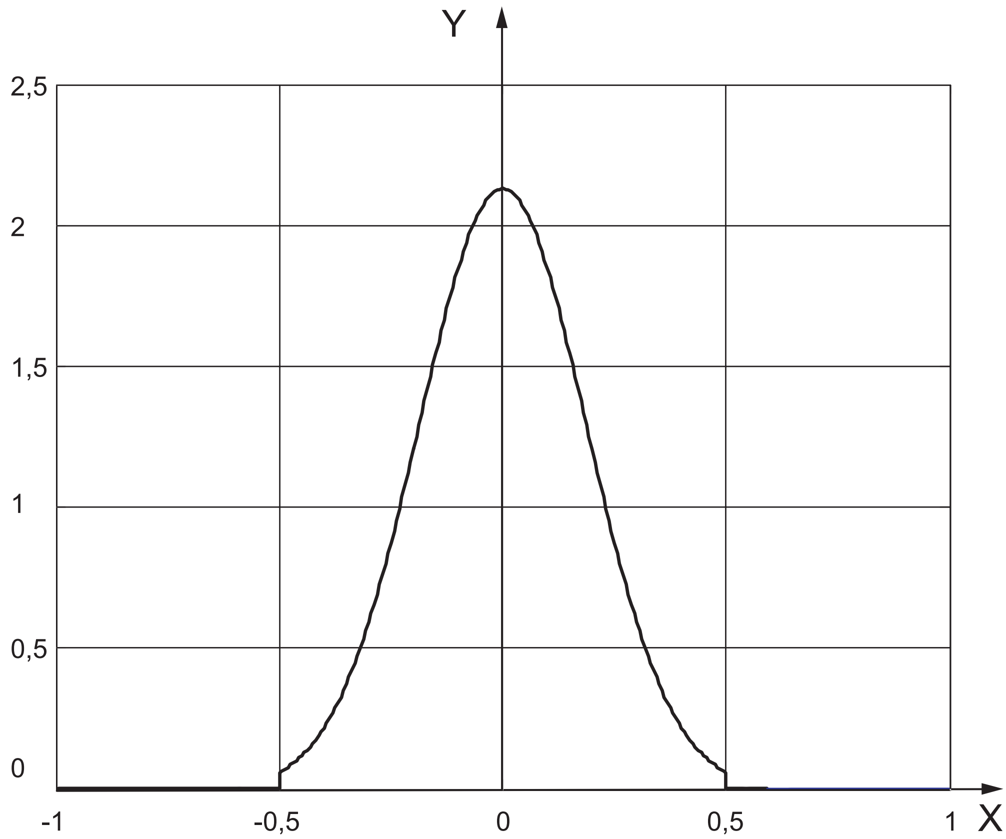
where

x is the distance along the closed profile from the centre (maximum) of the weighting function;

λ_c is the cut-off wavelength;

L_c is the truncation constant.

An example of the truncated Gaussian weighting function for an open profile is found in Figure A.1.



Key

X x/λ_c

Y $\lambda_c \times \tilde{s}(x)$

Figure A.1 — Example of a $L_c = 0{,}5$ truncated Gaussian weighting function for an open profile

The truncated weighting function is an approximation to the true Gaussian weighting function and as such when implementing a convolution with the truncated weighting function, there will always be an error compared to implementing a convolution with the true Gaussian weighting function. The truncation constant, L_c, should be chosen so that this implementation error is at an acceptable level for the application.

NOTE The implementation error is different from the error caused by end effects (see ISO/TS 16610-28) and also does not include errors due to the digital implementation of the Gaussian filter.

Clause A.2 provides the mathematics needed to calculate the implementation error for a given truncation constant L_c.

A.2 Mathematics for an open profile

Mathematically, the tructation of a weighting function to a finite interval[1] can be described by multiplication with a rectangular function, thus for the weighting function truncated to the finite interval is given by:

$$\tilde{s}(x) = s(x) \times r(L_c, x) \tag{A.2}$$

1) For details, see Reference [6].

where

$s(x)$ is the Gaussian weighting function;

$\tilde{s}(x)$ is the truncated Gaussian weighting function;

$r(L_c,x)$ is the rectangular function, defined by:

$$r(L_c, x) = \begin{cases} 1, & \text{for} \quad -L_c \times \lambda_c \leqslant x \leqslant L_c \times \lambda_c \\ 0, & \text{otherwise} \end{cases} \tag{A.3}$$

The implementation error is defined to be the maximum deviation of the transfer function of the truncated weighting function from the transfer function of the Gaussian weighting function.

The Fourier transform of the truncated Gaussian weighting function $\tilde{S}(\omega)$ is:

$$\tilde{S}(\omega) = (S * R)(\omega) = \int_{-\infty}^{\infty} S(v) \times R(L_c, \omega - v) \mathrm{d}v \tag{A.4}$$

where

$S(v)$ is the Fourier transform of the Gaussian weighting function;

$R(.,.)$ is the Fourier transform of the rectangular function which can be shown to be:

$$R(L_c, \omega) = \int_{-L_c \times \lambda_c}^{L_c \times \lambda_c} \mathrm{e}^{-i\omega x} \mathrm{d}x = 2 \times L_c \times \lambda_c \times \mathrm{sinc}\left(\frac{L_c \times \lambda_c}{\pi} \omega \right) \tag{A.5}$$

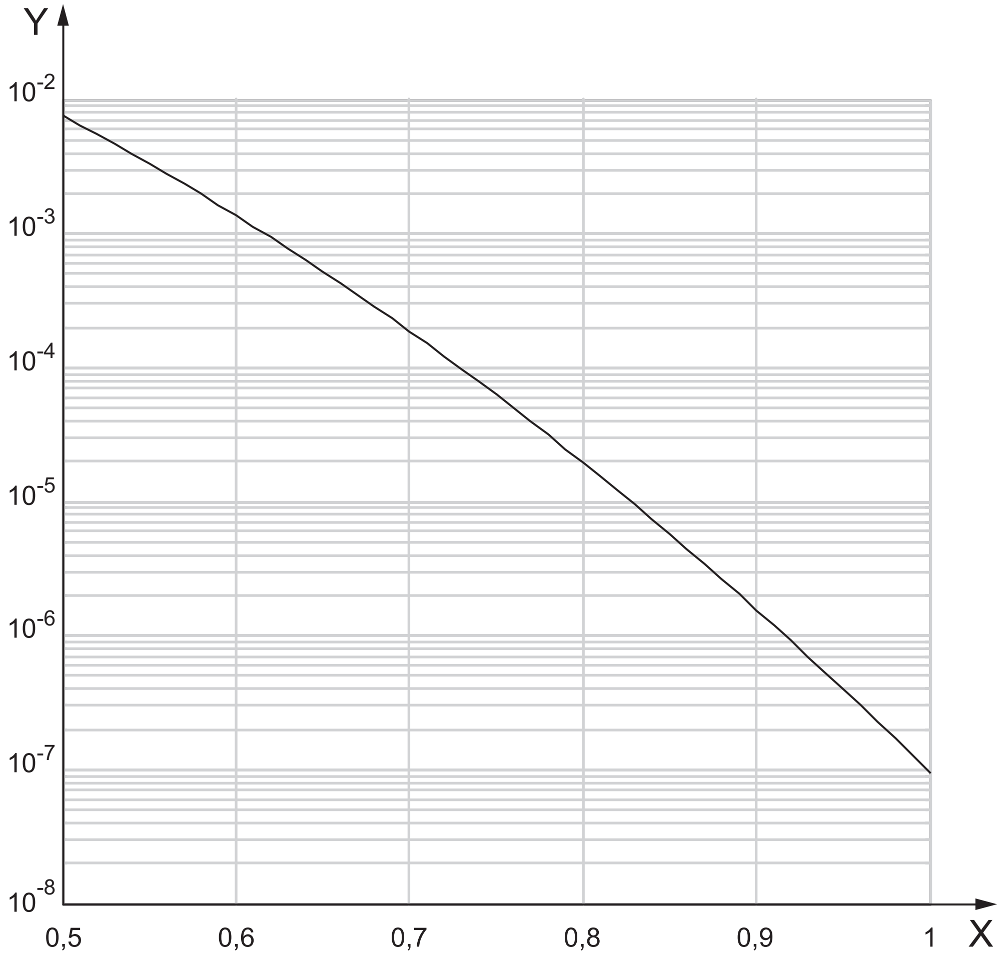
where $\mathrm{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$ is the sinc function.

Using Equation (A.3), the maximum deviation of the transfer function of the truncated weighting function from the transfer function of the Gaussian weighting function is at $\omega = 0$. This results in a maximum error[2)] of

$$\mathrm{erfc}\left(\frac{L_c \times \sqrt{\pi}}{\alpha \times \lambda_c} \right)$$

displayed as a graph (Figure A.2) and as a table (Table A.1).

2) For details, see Reference [6].



Key

X truncation constant, L_c

Y implementation error

Figure A.2 — Implementation errors for different truncation constants, L_c

Table A.1 — Implementation errors for different truncation constants, L_c

Truncation constants L_c	**Implementation error** %
0,5	0,76
0,6	0,14
0,8	1,96 e–003
1,0	9,47 e–006

On a profile $w(x)$, an estimate of the maximum error can be obtained[3)] as:

$$\Delta w(x) \leqslant p_{\text{max}} \times \text{erfc}\left(\frac{L_{\text{c}} \times \sqrt{\pi}}{\alpha \times \lambda_{\text{c}}}\right) \tag{A.6}$$

where p_{max} is the largest absolute value of the profile.

A.3 Closed profiles

The weighting function of a closed profile filter has the equation of the Gaussian density function wrapped around the closed profile of length L. With the cut-off frequency $f_{\text{c}} = L/\lambda_{\text{c}}$, the equation is as follows:

$$s(x) = \begin{cases} \dfrac{f_{\text{c}}}{\alpha \times L} \times \exp\left[-\pi\left(\dfrac{x \times f_{\text{c}}}{\alpha \times L}\right)^2\right] & -\dfrac{L_{\text{c}} \times L}{f_{\text{c}}} \leqslant x \leqslant \dfrac{L_{\text{c}} \times L}{f_{\text{c}}} \\ 0 & \text{Otherwise} \end{cases} \tag{A.7}$$

where

x is the distance along the closed profile from the centre (maximum) of the weighting function;

f_{c} is the cut-off frequency in undulations per revolution;

L is the length of the closed profile;

L_{c} is a truncation constant;

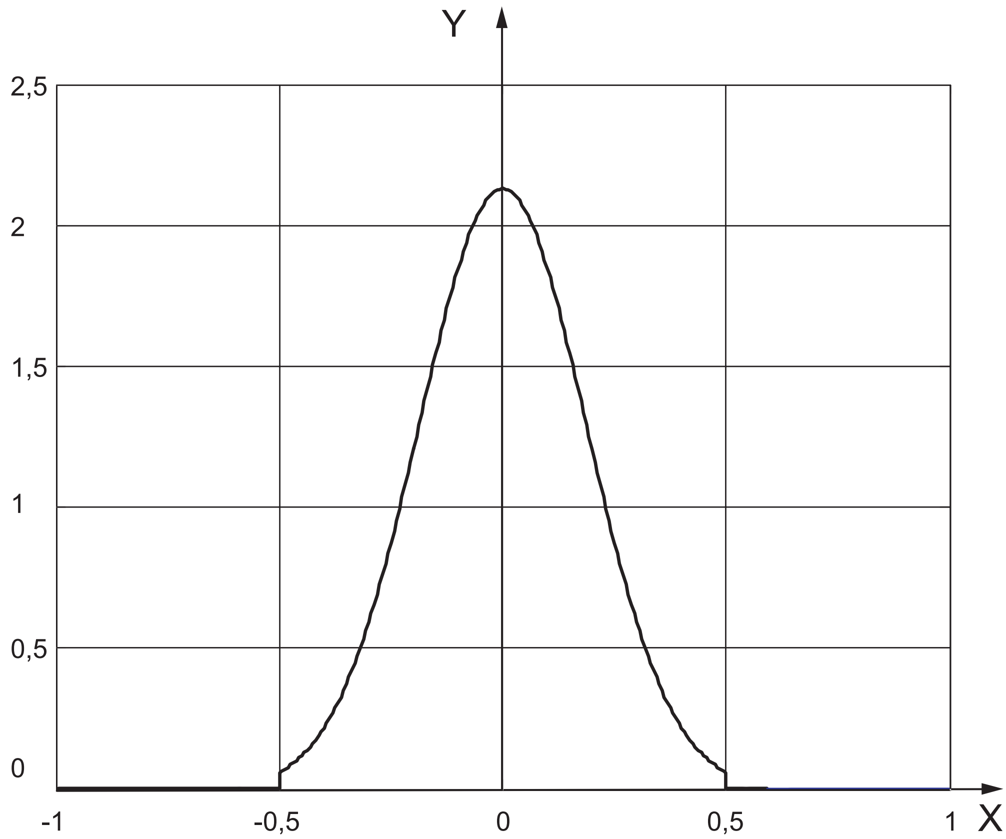
α is a constant given by:

$$\alpha = \sqrt{\frac{\ln 2}{\pi}} \approx 0{,}469\,7 \tag{A.8}$$

This is already in the form of a truncated Gaussian weighting function.

An example of the truncated Gaussian weighting function for an closed profile is found in Figure A.3.

3) For details, see Reference [6].



Key

X $f_c/L \times x$

Y $L/f_c \times \tilde{s}(x)$

Figure A.3 — Example of a $L_c = 0{,}5$ truncated Gaussian weighting function for a closed profile

The truncated weighting function is an approximation to the true Gaussian weighting function and as such when implementing a convolution with the truncated weighting function there will always be an error compared to implementing a convolution with the true Gaussian weighting function. The truncation constant, L_c, should be chosen so that this implementation error is at an acceptable level for the application.

A.4 Mathematics for a closed profile

The mathematics for the truncated Gaussian weighting function for a closed profile is identical to that of the open profile except with λ_c replaced by L/f_c, i.e. $\lambda_c = L/f_c$. Figure A.2 and Table A.1 are also identical for the closed profile case as the open profile case.

A.5 Recommendations

It is recommended that:

1) for general use, the truncation value be $L_c = 0{,}5$;

2) for use with precision surfaces where it is desirable to reduce the implementation error to 0,14 %[4], a truncation value of $L_c = 0,6$ be used;

3) for reference software, a truncation value of $L_c = 1,0$ be used[5] so that the implementation error is insignificant.

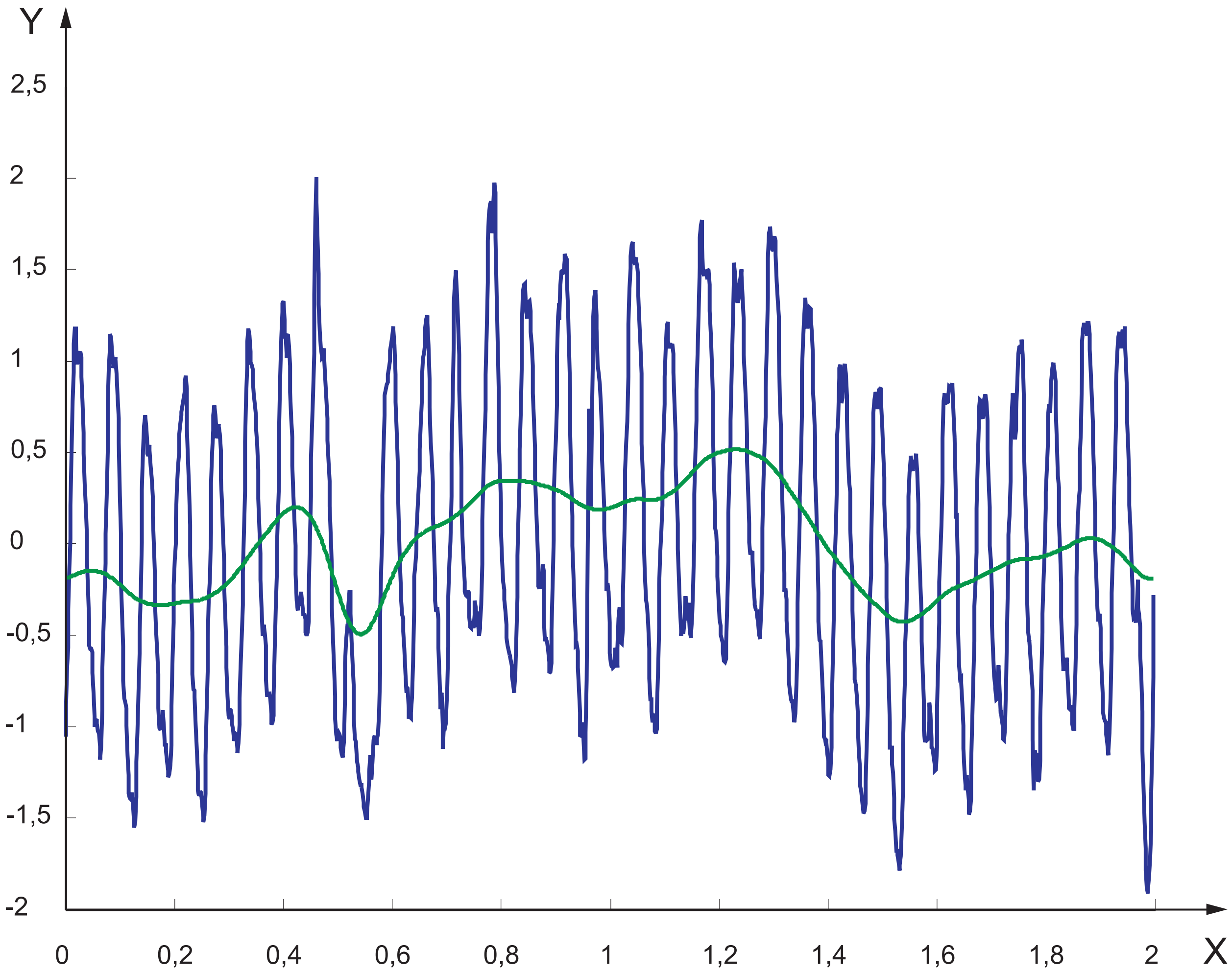
4) See References [4] and [5].

5) See Reference [6].

Annex B
(informative)

Examples

Examples for the application of the Gaussian profile filter into open and closed profile are given for information purposes.



Key

X distance *x*, mm

Y height *y*, µm

Figure B.1 — Open Gaussian filter with (λ_c = 0,8 mm), using the moment retainment criterion for end effect correction, applied on a milled surface

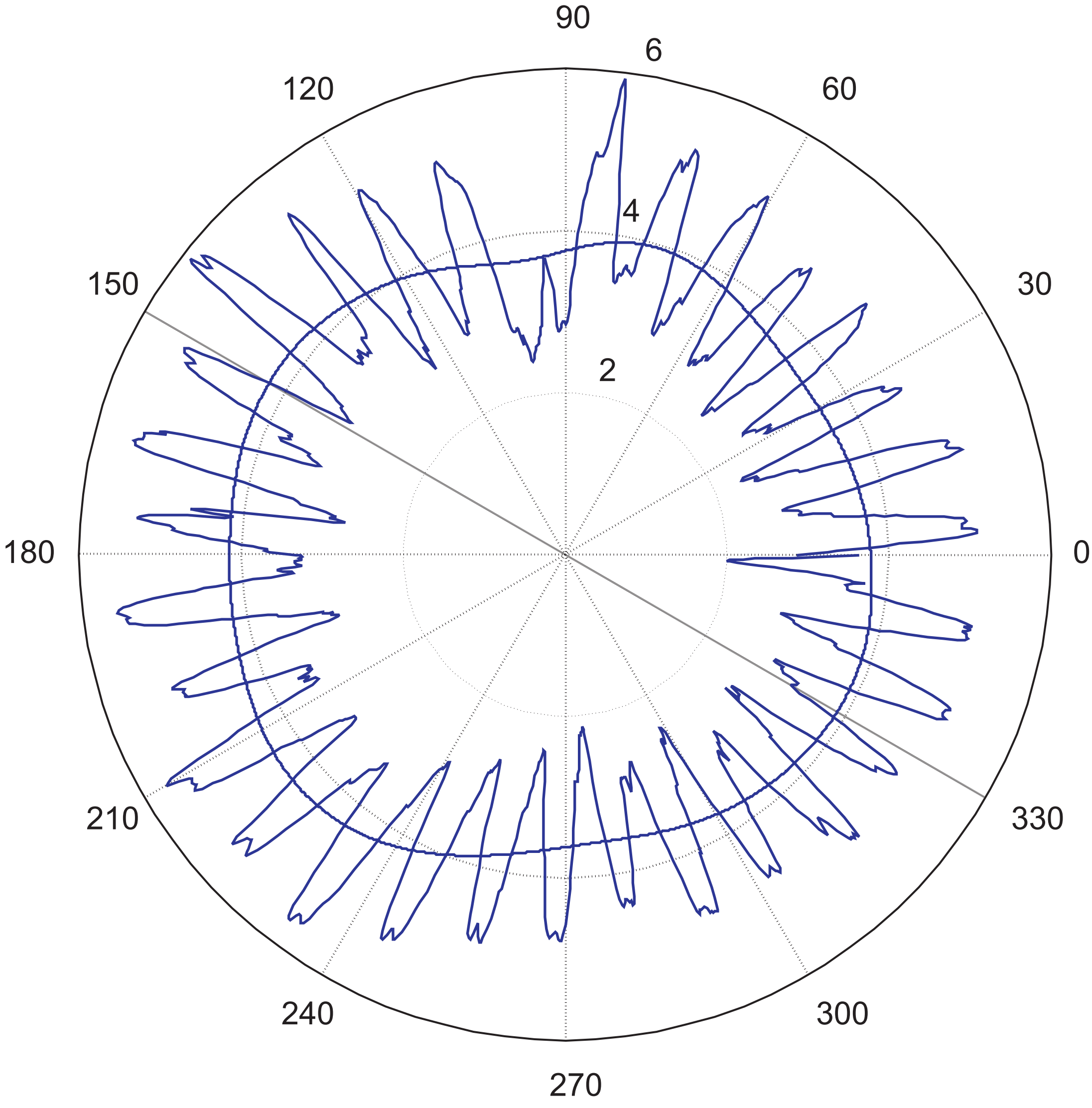


Figure B.2 — Closed Gaussian filter with f_c = 5 UPR, **applied on a milled/stamped surface**

Annex C
(informative)

Relationship to the filtration matrix model

C.1 General

For full details about the filtration matrix model, see ISO/TS 16610-1.

C.2 Position in the filtration matrix model

This part of ISO 16610 is a particular filter document in the column "Profile filters, Linear" (see Figure C.1).

	Filters: ISO 16610 series					
General	Part 1					
	Profile filters			Areal filters		
Fundamental	Part 11			Part 12		
	Linear	**Robust**	**Morphological**	**Linear**	**Robust**	**Morphological**
Basic Concepts	Part 20	Part 30	Part 40	Part 60	Part 70	Part 80
Particular Filters	Parts 21-25	Parts 31-35	Parts 41-45	Parts 61-65	Parts 71-75	Parts 81-85
How to filter	Parts 26-28	Parts 36-38	Parts 46-48	Parts 66-68	Parts 76-78	Parts 86-88
Multiresolution	Part 29	Part 39	Part 49	Part 69	Part 79	Part 89

Figure C.1 — Relationship to the filtration matrix model

Annex D
(informative)

Relation to the GPS matrix model

D.1 General

For full details about the GPS matrix model, see ISO/TR 14638.

The ISO/GPS masterplan given in ISO/TR 14638 gives an overview of the ISO/GPS system of which this document is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this document, unless otherwise indicated.

D.2 Information about this part of ISO 16610 and its use

This part of ISO 16610 specifies the metrological characteristics of the linear Gaussian filter.

D.3 Position in the GPS matrix model

This part of ISO 16610 is a global GPS document that influences the chain links 3 and 5 of all chains of standards, as graphically illustrated in Figure D.1.

Fundamental GPS Standards

Global GPS standards

General GPS standards						
Chain link number	**1**	**2**	**3**	**4**	**5**	**6**
Size			X		X	
Distance			X		X	
Radius			X		X	
Angle			X		X	
Form of a line independent of datum			X		X	
Form of a line dependent on datum			X		X	
Form of a surface independent of datum			X		X	
Form of a surface dependent on datum			X		X	
Orientation			X		X	
Location			X		X	
Circular run-out			X		X	
Total run-out			X		X	
Datums			X		X	
Roughness profile			X		X	
Waviness profile			X		X	
Primary profile			X		X	
Surface imperfections			X		X	
Edges			X		X	

Figure D.1 — Position in the GPS matrix model

D.4 Related International Standards

The related International Standards are those of the chains of standards indicated in Figure D.1.

Bibliography

[1] ISO 8015, *Geometrical product specifications (GPS) — Fundamentals — Concepts, principles and rules*

[2] ISO 11562:1996, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Metrological characteristics of phase correct filters*

[3] ISO 14253-1:1998, *Geometrical Product Specifications (GPS) — Inspection by measurement of workpieces and measuring equipment — Part 1: Decision rules for proving conformance or non-conformance with specifications*

[4] ISO/TR 14638, *Geometrical product specification (GPS) — Masterplan*

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