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Statistical interpretation of data —

Part 7:
Median — Estimation and confidence intervals

Interprétation statistique des données —

Partie 7: Médiane — Estimation et intervalles de confiance

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ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.ch
Web www.iso.ch

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Contents

Page

Forms

Table

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

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International Standard ISO 16269-7 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 3, *Application of statistical methods in standaridization*.

ISO 16269 consists of the following parts, under the general title *Statistical interpretation of data*:

— *Part 7: Median — Estimation and confidence intervals*

The following will be the subjects of future parts to ISO 16269:

— *Part 1: Guide to statistical interpretation of data*

— *Part 2: Presentation of statistical data*

— *Part 3: Tests for departure from normality*

— *Part 4: Detection and treatment of outliers*

— *Part 5: Estimation and tests of means and variances for the normal distribution, with power functions for tests*

— *Part 6: Determination of statistical tolerance intervals*

Annexes A and B of this part of ISO 16269 are for information only.

Statistical interpretation of data —

Part 7:
Median — Estimation and confidence intervals

1 Scope

This part of ISO 16269 specifies the procedures for establishing a point estimate and confidence intervals for the median of any continuous probability distribution of a population, based on a random sample size from the population. These procedures are distribution-free, i.e. they do not require knowledge of the family of distributions to which the population distribution belongs. Similar procedures can be applied to estimate quartiles and percentiles.

NOTE The median is the second quartile and the fiftieth percentile. Similar procedures for other quartiles or percentiles are not described in this part of ISO 16269.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this part of ISO 16269. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this part of ISO 16269 are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 2602, *Statistical interpretation of test results — Estimation of the mean — Confidence interval.*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.*

3 Terms, definitions and symbols

3.1 Terms and definitions

For the purposes of this part of ISO 16269, the terms and definitions given in ISO 2602 and ISO 3534-1 and the following apply.

3.1.1
***k*th order statistic of a sample**
value of the *k*th element in a sample when the elements are arranged in non-decreasing order of their values

NOTE For a sample of n elements arranged in non-decreasing order, the *k*th order statistics is $x_{[k]}$ where

$$x_{[1]} \leqslant x_{[2]} \leqslant \dots \leqslant x_{[n]}$$

3.1.2
median of a continuous probability distribution
value such that the proportions of the distribution lying on either side of it are both equal to one half

NOTE In this part of ISO 16269, the median of a continuous probability distribution is called the population median and is denoted by M.

3.2 Symbols

a lower bound to the values of the variable in the population

b upper bound to the values of the variable in the population

C confidence level

c constant used for determining the value of k in equation (1)

k number of the order statistic used for the lower confidence limit

M population median

n sample size

T_1 lower confidence limit derived from a sample

T_2 upper confidence limit derived from a sample

u fractile of the standardized normal distribution

$x_{[i]}$ ith smallest element in a sample when the elements are arranged in a non-decreasing order of their values

$\tilde{x}$ sample median

y intermediate value calculated to determine k using equation (1)

4 Applicability

The method described in this part of ISO 16269 is valid for any continuous population, provided that the sample is drawn at random.

NOTE If the distribution of the population can be assumed to be approximately normal, the population median is approximately equal to the population mean and the confidence limits should be calculated in accordance with ISO 2602.

5 Point estimation

A point estimate of the population median is given by the sample median, $\tilde{x}$. The sample median is obtained by numbering the sample elements in non-decreasing order of their values and taking the value of

— the $[(n + 1)/2]$th order statistic, if n is odd, or

— the arithmetic mean of the $(n/2)$th and $[(n/2) + 1]$th order statistics, if n is even.

NOTE This estimator is in general biased for asymmetrical distributions, but an estimator that is unbiased for any population does not exist.

6 Confidence interval

6.1 General

A *two-sided confidence interval* for the population median is a closed interval of the form $[T_1, T_2]$, where $T_1 < T_2$; T_1 and T_2 are called the *lower* and *upper confidence limits*, respectively.

If a and b are respectively the lower and upper bounds of the variable in the population, a *one-sided confidence interval* will be of the form $[T_1, b)$ or of the form $(a, T_2]$.

NOTE For practical purposes, a is often taken to be zero for variables that cannot be negative, and b is often taken to be infinity for variables with no natural upper bound.

The practical meaning of a confidence interval is that the experimenter claims that the unknown M lies within the interval, while admitting a small nominal probability that this assertion may be wrong. The probability that intervals calculated in such a way cover the population median is called the confidence level.

6.2 Classical method

The classical method is described in annex A. It involves solving a pair of inequalities. Alternatives to solving these inequalities are given below for a range of confidence levels.

6.3 Small samples ($5 \leqslant n \leqslant 100$)

The values of k satisfying the equations in annex A for eight of the most commonly used confidence levels for sample sizes varying from 5 to 100 sampling units are given in Table 1 for the one-sided case and in Table 2 for the two-sided case. The values of k are given such that the lower confidence limit is

$$T_1 = x_{[k]}$$

and the upper confidence limit is

$$T_2 = x_{[n-k+1]}$$

where $x_{[1]}, x_{[2]}, ..., x_{[n]}$ are the ordered observed values in the sample.

For small values of n, it can happen that confidence limits based on order statistics are unavailable at certain confidence levels.

An example of the calculation of the confidence limits for small samples is given in B.1 and shown in Form A of annex B.

Table 1 — Exact values of k for sample sizes varying from 5 to 100: one-sided case

Sample size n	k								Sample size n	k							
	Confidence level %									Confidence level %							
	80	90	95	98	99	99,5	99,8	99,9		80	90	95	98	99	99,5	99,8	99,9
5	2	1	1	[a]	[a]	[a]	[a]	[a]	55	24	23	21	20	19	18	17	16
6	2	1	1	1	[a]	[a]	[a]	[a]	56	25	23	22	20	19	18	17	17
7	2	2	1	1	1	[a]	[a]	[a]	57	25	24	22	21	20	19	18	17
8	3	2	2	1	1	1	[a]	[a]	58	26	24	23	21	20	19	18	17
9	3	3	2	2	1	1	1	[a]	59	26	25	23	22	21	20	19	18
10	4	3	2	2	1	1	1	1	60	27	25	24	22	21	20	19	18
11	4	3	3	2	2	1	1	1	61	27	25	24	23	21	21	19	19
12	5	4	3	3	2	2	1	1	62	28	26	25	23	22	21	20	19
13	5	4	4	3	2	2	2	1	63	28	26	25	23	22	21	20	19
14	5	5	4	3	3	2	2	2	64	29	27	25	24	23	22	21	20
15	6	5	4	4	3	3	2	2	65	29	27	26	24	23	22	21	20
16	6	5	5	4	3	3	2	2	66	30	28	26	25	24	23	21	21
17	7	6	5	4	4	3	3	2	67	30	28	27	25	24	23	22	21
18	7	6	6	5	4	4	3	3	68	31	29	27	26	24	23	22	21
19	8	7	6	5	5	4	3	3	69	31	29	28	26	25	24	23	22
20	8	7	6	5	5	4	4	3	70	31	30	28	26	25	24	23	22
21	9	8	7	6	5	5	4	4	71	32	30	29	27	26	25	23	23
22	9	8	7	6	6	5	4	4	72	32	31	29	27	26	25	24	23
23	9	8	8	7	6	5	5	4	73	33	31	29	28	27	26	24	23
24	10	9	8	7	6	6	5	5	74	33	31	30	28	27	26	25	24
25	10	9	8	7	7	6	5	5	75	34	32	30	29	27	26	25	24
26	11	10	9	8	7	7	6	5	76	34	32	31	29	28	27	26	25
27	11	10	9	8	8	7	6	6	77	35	33	31	30	28	27	26	25
28	12	11	10	9	8	7	7	6	78	35	33	32	30	29	28	26	25
29	12	11	10	9	8	8	7	6	79	36	34	32	30	29	28	27	26
30	13	11	11	9	9	8	7	7	80	36	34	33	31	30	29	27	26
31	13	12	11	10	9	8	8	7	81	37	35	33	31	30	29	28	27
32	14	12	11	10	9	9	8	7	82	37	35	34	32	31	29	28	27
33	14	13	12	11	10	9	8	8	83	38	36	34	32	31	30	28	28
34	15	13	12	11	10	10	9	8	84	38	36	34	33	31	30	29	28
35	15	14	13	11	11	10	9	9	85	39	37	35	33	32	31	29	28
36	15	14	13	12	11	10	10	9	86	39	37	35	34	32	31	30	29
37	16	15	14	12	11	11	10	9	87	40	38	36	34	33	32	30	29
38	16	15	14	13	12	11	10	10	88	40	38	36	34	33	32	31	30
39	17	16	14	13	12	12	11	10	89	41	38	37	35	34	32	31	30
40	17	16	15	14	13	12	11	10	90	41	39	37	35	34	33	31	30
41	18	16	15	14	13	12	11	11	91	41	39	38	36	34	33	32	31
42	18	17	16	14	14	13	12	11	92	42	40	38	36	35	34	32	31
43	19	17	16	15	14	13	12	12	93	42	40	39	37	35	34	33	32
44	19	18	17	15	14	14	13	12	94	43	41	39	37	36	35	33	32
45	20	18	17	16	15	14	13	12	95	43	41	39	38	36	35	34	33
46	20	19	17	16	15	14	13	13	96	44	42	40	38	37	35	34	33
47	21	19	18	17	16	15	14	13	97	44	42	40	38	37	36	34	33
48	21	20	18	17	16	15	14	13	98	45	43	41	39	38	36	35	34
49	22	20	19	17	16	16	15	14	99	45	43	41	39	38	37	35	34
50	22	20	19	18	17	16	15	14	100	46	44	42	40	38	37	36	35
51	22	21	20	18	17	16	15	15									
52	23	21	20	19	18	17	16	15									
53	23	22	21	19	18	17	16	15									
54	24	22	21	19	19	18	17	16									

[a] A confidence interval and confidence limit cannot be determined for this sample size at this confidence level.

Table 2 — Exact values of k for sample sizes varying from 5 to 100: two-sided case

Sample size n	k								Sample size n	k							
	Confidence level %									Confidence level %							
	80	90	95	98	99	99,5	99,8	99,9		80	90	95	98	99	99,5	99,8	99,9
5	1	1	[a]	[a]	[a]	[a]	[a]	[a]	55	23	21	20	19	18	17	16	15
6	1	1	1	[a]	[a]	[a]	[a]	[a]	56	23	22	21	19	18	18	17	16
7	2	1	1	1	[a]	[a]	[a]	[a]	57	24	22	21	20	19	18	17	16
8	2	2	1	1	1	[a]	[a]	[a]	58	24	23	22	20	19	18	17	17
9	3	2	2	1	1	1	[a]	[a]	59	25	23	22	21	20	19	18	17
10	3	2	2	1	1	1	1	[a]	60	25	24	22	21	20	19	18	17
11	3	3	2	2	1	1	1	1	61	25	24	23	21	21	20	19	18
12	4	3	3	2	2	1	1	1	62	26	25	23	22	21	20	19	18
13	4	4	3	2	2	2	1	1	63	26	25	24	22	21	20	19	19
14	5	4	3	3	2	2	2	1	64	27	25	24	23	22	21	20	19
15	5	4	4	3	3	2	2	2	65	27	26	25	23	22	21	20	19
16	5	5	4	3	3	3	2	2	66	28	26	25	24	23	22	21	20
17	6	5	5	4	3	3	2	2	67	28	27	26	24	23	22	21	20
18	6	6	5	4	4	3	3	2	68	29	27	26	24	23	23	21	21
19	7	6	5	5	4	4	3	3	69	29	28	26	25	24	23	22	21
20	7	6	6	5	4	4	3	3	70	30	28	27	25	24	23	22	21
21	8	7	6	5	5	4	4	3	71	30	29	27	26	25	24	23	22
22	8	7	6	6	5	5	4	4	72	31	29	28	26	25	24	23	22
23	8	8	7	6	5	5	4	4	73	31	29	28	27	26	25	23	23
24	9	8	7	6	6	5	5	4	74	31	30	29	27	26	25	24	23
25	9	8	8	7	6	6	5	5	75	32	30	29	27	26	25	24	23
26	10	9	8	7	7	6	5	5	76	32	31	29	28	27	26	25	24
27	10	9	8	8	7	6	6	5	77	33	31	30	28	27	26	25	24
28	11	10	9	8	7	7	6	6	78	33	32	30	29	28	27	25	25
29	11	10	9	8	8	7	6	6	79	34	32	31	29	28	27	26	25
30	11	11	10	9	8	7	7	6	80	34	33	31	30	29	28	26	25
31	12	11	10	9	8	8	7	7	81	35	33	32	30	29	28	27	26
32	12	11	10	9	9	8	7	7	82	35	34	32	31	29	28	27	26
33	13	12	11	10	9	9	8	7	83	36	34	33	31	30	29	28	27
34	13	12	11	10	10	9	8	8	84	36	34	33	31	30	29	28	27
35	14	13	12	11	10	9	9	8	85	37	35	33	32	31	30	28	27
36	14	13	12	11	10	10	9	8	86	37	35	34	32	31	30	29	28
37	15	14	13	11	11	10	9	9	87	38	36	34	33	32	30	29	28
38	15	14	13	12	11	10	10	9	88	38	36	35	33	32	31	30	29
39	16	14	13	12	12	11	10	9	89	38	37	35	34	32	31	30	29
40	16	15	14	13	12	11	10	10	90	39	37	36	34	33	32	30	30
41	16	15	14	13	12	12	11	10	91	39	38	36	34	33	32	31	30
42	17	16	15	14	13	12	11	11	92	40	38	37	35	34	33	31	30
43	17	16	15	14	13	12	12	11	93	40	39	37	35	34	33	32	31
44	18	17	16	14	14	13	12	11	94	41	39	38	36	35	33	32	31
45	18	17	16	15	14	13	12	12	95	41	39	38	36	35	34	33	32
46	19	17	16	15	14	14	13	12	96	42	40	38	37	35	34	33	32
47	19	18	17	16	15	14	13	12	97	42	40	39	37	36	35	33	32
48	20	18	17	16	15	14	13	13	98	43	41	39	38	36	35	34	33
49	20	19	18	16	16	15	14	13	99	43	41	40	38	37	36	34	33
50	20	19	18	17	16	15	14	14	100	44	42	40	38	37	36	35	34
51	21	20	19	17	16	16	15	14									
52	21	20	19	18	17	16	15	14									
53	22	21	19	18	17	16	15	15									
54	22	21	20	19	18	17	16	15									

[a] A confidence interval and confidence limits cannot be determined for this sample size at this confidence level.

6.4 Large samples ($n > 100$)

For sample sizes in excess of 100 sampling units, an approximation of k for the confidence level (1 – α) may be determined as the integer part of the value obtained from the following equation:

$$y = \frac{1}{2}\left[n + 1 - u\left(1 + \frac{0{,}4}{n}\right)\sqrt{n - c}\right] \qquad (1)$$

where

u is a fractile of the standardized normal distribution; values of u are given in Table 3 for a one-sided confidence interval and in Table 4 for a two-sided interval;

c is given in Table 3 for a one-sided confidence interval and in Table 4 for a two-sided interval.

The values of k obtained by means of the empirical equation (1) are in complete agreement with the correct values given in Tables 1 and 2. Provided all 8 decimal places of u are retained, this approximation is extremely accurate and gives the correct values for k for all eight confidence levels at all sample sizes from 5 up to over 280 000, for both one- and two-sided confidence intervals.

An example of the calculation of the confidence limits for large samples is given in B.2 and shown in Form B of annex B.

NOTE For ease of use, the values of c in Tables 3 and 4 are given to the minimum number of decimal places necessary to guarantee the fullest possible accuracy of equation (1).

Table 3 — Values of u and c for the one-sided case

Confidence level %	u	c
80,0	0,841 621 22	0,75
90,0	1,281 551 56	0,903
95,0	1,644 853 64	1,087
98,0	2,053 748 92	1,3375
99,0	2,326 347 88	1,536
99,5	2,575 829 30	1,74
99,8	2,878 161 73	2,014
99,9	3,090 232 29	2,222

Table 4 — Values of u and c for the two-sided case

Confidence level %	u	c
80,0	1,281 551 56	0,903
90,0	1,644 853 64	1,087
95,0	1,959 964 00	1,274
98,0	2,326 347 88	1,536
99,0	2,575 829 30	1,74
99,5	2,807 033 76	1,945
99,8	3,090 232 29	2,222
99,9	3,290 526 72	2,437

Annex A
(informative)

Classical method of determining confidence limits for the median

Assume that a sample of size n is to be drawn at random from a continuous population. Under these conditions, the probability that precisely k of the sample values will be less than the population median is described by the binomial distribution:

$$P\left(k;\ n, \frac{1}{2}\right) = \binom{n}{k}\left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{n-k} = \binom{n}{k}\frac{1}{2^n}$$

This is also the probability that precisely k of the sample values will be *greater than* the population median.

The lower and upper limits of a two-sided confidence interval of confidence level (1– α) are given by the pair of order statistics $(x_{[k]}, x_{[n-k+1]})$ where the integer k is determined in such a way that

$$\sum_{i=0}^{k-1}\binom{n}{i}\frac{1}{2^n} \leqslant \frac{\alpha}{2} \tag{A.1}$$

and

$$\sum_{i=0}^{k}\binom{n}{i}\frac{1}{2^n} > \frac{\alpha}{2}\ ; \tag{A.2}$$

i.e.

$$\sum_{i=0}^{k-1}\binom{n}{i} \leqslant 2^n \cdot \frac{\alpha}{2} \tag{A.3}$$

and

$$\sum_{i=0}^{k}\binom{n}{i} > 2^n \cdot \frac{\alpha}{2}. \tag{A.4}$$

In the one-sided case, $\alpha/2$ in equations (A.1) to (A.4) is replaced by α.

Annex B
(informative)

Examples

B.1 Example 1

Electric cords for a small appliance are flexed by a test machine until failure. The test simulates actual use, under highly accelerated conditions. The 24 times of failure, in hours, are given below; seven of them are censored times and are marked with an asterisk [1)]:

57,5	77,8	88,0	96,9	98,4	100,3
100,8	102,1	103,3	103,4	105,3	105,4
122,6	139,3	143,9	148,0	151,3	161,1*
161,2*	161,2*	162,4*	162,7*	163,1*	176,8*

An estimate of the median and a lower confidence limit on the median at 95 % confidence are required.

A point estimate of the median lifetime is

$$\tilde{x} = (x_{[12]} + x_{[13]})/2$$

$$= (105{,}4 + 122{,}6)/2$$

$$= 114{,}0 \text{ h}$$

The lower one-sided confidence limit for the median with confidence level 95 % is obtained by reading from Table 1 the value of k for n = 24 and confidence level 95 % for the one-sided case, and then looking for the kth failure time in the above list.

The value from Table 1 is k = 8 and $x_{[8]}$ = 102,1, so it may be asserted with 95 % confidence that the population median is no lower than 102,1 h.

NOTE It is possible to estimate a median and lower bounded confidence interval without observing the largest values in the sample.

The calculation of the median is presented in table form in Form A overleaf. The calculations themselves are shown in italics.

1) When an item is removed from a test without having failed, the time for this test is referred to as a "censored time".

Form A — Calculation of an estimate of a median

Blank form	Completed form
Data identification Data and observation procedure: Units: Remarks:	**Data identification** Data and observation procedure: *Time to failure of 24 electric cords*, *flexed by a test machine. The test simulates actual use, but highly accelerated.* Units: *Hours* Remarks: *The seven longest times to failure were censored. As this is fewer than half of the times, the median can still be calculated.*
Preliminary operation Arrange the observed values into ascending order, i.e. $x_{[1]}, x_{[2]}, \cdots, x_{[n]}$	**Preliminary operation** Arrange the observed values into ascending order, i.e $x_{[1]}, x_{[2]}, \cdots, x_{[n]}$
Information required Sample size, *n:* $n =$ a) Sample size is odd: ☐ b) Sample size is even: ☐	**Information required** Sample size, *n* $n = 24$ a) Sample size is odd: ☐ b) Sample size is even: ☒
Initial calculation required For a) $m = (n+1)/2$: $m =$ For b) $m = n/2$: $m =$	**Initial calculation required** For a) $m = (n+1)/2$: $m =$ For b) $m = n/2$: $m = 12$
Calculation of the sample median, $\tilde{x}$ For a), $\tilde{x}$ is equal to the *m*th smallest (or largest) observed values, i.e. $\tilde{x} = x_{[m]}$: $\tilde{x} =$ For b), $\tilde{x}$ is equal to the arithmetic mean of the *m*th and (*m*+1)th smallest (or largest) observed values, i.e. $\tilde{x} = (x_{[m]} + x_{[m+1]})/2$: $x_{[m]} =$ $x_{[m+1]} =$ $\tilde{x} = (\; + \;)/2 =$	**Calculation of the sample median,** $\tilde{x}$ For a), $\tilde{x}$ is equal to the *m*th smallest (or largest) observed values, i.e. $\tilde{x} = x_{[m]}$: $\tilde{x} =$ For b), $\tilde{x}$ is equal to the arithmetic mean of the *m*th and (*m*+1)th smallest (or largest) observed values, i.e. $\tilde{x} = (x_{[m]} + x_{[m+1]})/2$: $x_{[m]} = 105{,}4$ $x_{[m+1]} = 122{,}6$ $\tilde{x} = (105{,}4 + 122{,}6)/2 = 114{,}0$
Result The sample median (estimate of the population median) is $\tilde{x} =$	**Result** The sample median (estimate of the population median) is $\tilde{x} = 114{,}0$.

B.2 Example 2

The breaking strengths of 120 lengths of nylon yarn are given below in newtons (N), arranged in ascending order along rows:

31,3	33,3	33,5	35,6	36,0	36,2	36,5	37,5	37,8	37,9	38,8	39,1	40,3	40,4	40,8
41,0	41,8	42,4	42,9	43,1	43,2	43,5	43,9	43,9	44,0	44,2	44,2	44,5	44,7	44,7
45,0	45,6	46,0	46,0	46,1	46,1	46,3	46,3	46,3	46,4	46,5	46,7	47,1	47,1	47,1
47,2	47,3	47,4	47,5	47,5	47,8	47,8	47,9	47,9	48,0	48,0	48,2	48,2	48,3	48,3
48,3	48,5	48,6	48,6	48,6	48,6	48,8	48,9	48,9	48,9	49,0	49,0	49,1	49,1	49,1
49,1	49,2	49,2	49,3	49,4	49,4	49,4	49,4	49,5	49,5	49,6	49,7	49,9	49,9	50,0
50,1	50,2	50,2	50,3	50,3	50,3	50,5	50,7	50,8	50,9	50,9	51,0	51,0	51,2	51,4
51,4	51,4	51,6	51,6	51,8	52,0	52,2	52,2	52,4	52,5	52,6	52,8	52,9	53,2	53,3

A point estimate of the median breaking strength is required, together with a two-sided confidence interval at 99 % confidence.

A point estimate of the median breaking strength is

$$\tilde{x} = (x_{[60]} + x_{[61]})/2 = (48{,}3 + 48{,}3)/2 = 48{,}3 \text{ N}$$

For $n > 100$, Tables 1 and 2 do not provide the appropriate value of k for confidence limits. As two-sided confidence limits are required, equation (1) is to be used in conjunction with Table 4. The values of u and c for 99 % confidence are found from Table 4 to be u = 2,575 829 30 and c = 1,74. Inserting these into equation (1) with n = 120 gives y = 46,448. Taking the integer part of 46,448 gives k = 46.

A 99 % two-sided confidence interval on the population median repair time is therefore

$$(x_{[k]},\ x_{[n-k+1]}) = (x_{[46]}, x_{[75]}) = (47{,}2,\ 49{,}1) \text{ N}$$

It may therefore be asserted with at least 99 % confidence that the population median breaking strength lies in the interval (47,2, 49,1) N.

The calculation of the confidence interval is presented in table form in Form B with the calculations shown in italics.

Form B — Calculation of a confidence interval for a median

Blank form

Data identification

Data and observation procedure:

Units:

Remarks:

Preliminary operation

Arrange the observed values into ascending order, *i.e.*

$x_{[1]}, x_{[2]}, \ldots, x_{[n]}$

Information required

Sample size, n $n =$

Confidence level C: $C =$ %

a) $n \leqslant 100$ one-sided interval ☐

b) $n \leqslant 100$ two-sided interval ☐

c) $n > 100$ one-sided interval ☐

d) $n > 100$ two-sided interval ☐

For a) or c) with an upper confidence limit, the lower bound to x in the population is required: $a =$

For a) or c) with a lower confidence limit, the upper bound to x in the population is required: $b =$

Determination of k

For a), find k from Table 1: $k =$

For b), find k from Table 2: $k =$

For c), find u and c from Table 3: $u =$ $c =$

For d), find u and c from Table 4: $u =$ $c =$

For c) or d), calculate y from equation (1): $y =$

then calculate k as the integer part of y: $k =$

Determination of the confidence limits T_1 and/or T_2

For a) or c) with a lower limit,

and for b) or d), set $T_1 = x_{[k]}$ $T_1 =$

For a) or c) with an upper limit,

and for b) or d), calculate $m = n - k + 1$: $m =$

then set $T_2 = x_{[m]}$: $T_2 =$

Result

For a single lower confidence limit, the $C =$ % confidence interval for the population median is $[T_1, b) = [\ ,\ ]$.

For a single upper confidence limit, the $C =$ % confidence interval for the population median is $[a, T_2) = [\ ,\ ]$.

For two-sided confidence limits, the $C =$ % symmetric confidence interval for the population median is $[T_1, T_2) = [\ ,\ ]$.

Completed form

Data identification

Data and observation procedure: *Breaking strengths of 120 lengths of nylon yarn.*

Units: *Newtons*.

Remarks: *Two-sided confidence interval required at 99 % confidence*.

Preliminary operation

Arrange the observed values into ascending order, *i.e.*

$x_{[1]}, x_{[2]}, \ldots, x_{[n]}$

Information required

Sample size, n: *n = 120*

Confidence level C: *C = 99 %*

a) $n \leqslant 100$ one-sided interval ☐

b) $n \leqslant 100$ two-sided interval ☐

c) $n > 100$ one-sided interval ☐

d) $n > 100$ two-sided interval ☒

For a) or c) with an upper confidence limit, the lower bound to x in the population is required: $a =$

For a) or c) with a lower confidence limit, the upper bound to x in the population is required: $b =$

Determination of k

For a), find k from Table 1: $k =$

For b), find k from Table 2: $k =$

For c), find u and c from Table 3: $u =$ $c =$

For d), find u and c from Table 4: *u = 2,575 829 30* *c = 1,74*

For c) or d), calculate y from equation (1): *y = 46,448*

then calculate k as the integer part of y: *k = 46*

Determination of the confidence limits T_1 and/or T_2

For a) or c) with a lower limit,

and for b) or d), set $T_1 = x_{[k]}$ $T_1 = 47{,}2$

For a) or c) with an upper limit,

and for b) or d), calculate $m = n - k + 1$: *m = 75*

then set $T_2 = x_{[m]}$: $T_2 = 49{,}1$

Result

For a single lower confidence limit, the $C =$ % confidence interval for the population median is $[T_1, b) = [\ ,\ ]$.

For a single upper confidence limit, the $C =$ % confidence interval for the population median is $[a, T_2) = [\ ,\ ]$.

For two-sided confidence limits, the $C =$ *99* % symmetric confidence interval for the population median is $[T_1, T_2)$ = [*47,2, 49,1*].

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