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Methods for the calibration of vibration and shock transducers —

Part 21:

Vibration calibration by comparison to a reference transducer

Méthodes pour l'étalonnage des transducteurs de vibrations et de chocs —

Partie 21: Étalonnage de vibrations par comparaison à un transducteur de référence

Reference number ISO 16063-21:2003(E)

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 16063-21 was prepared by Technical Committee ISO/TC 108, *Mechanical vibration and shock*, Subcommittee SC 3, *Use and calibration of vibration and shock measuring instruments*.

This first edition of ISO 16063-21 cancels and replaces ISO 5347-3:1993, which has been technically revised.

ISO 16063 consists of the following parts, under the general title *Methods for the calibration of vibration and shock transducers*:

- *Part 1: Basic concepts*
- *Part 11: Primary vibration calibration by laser interferometry*
- *Part 12: Primary vibration calibration by the reciprocity method*
- *Part 13: Primary shock calibration using laser interferometry*
- *Part 21: Vibration calibration by comparison to a reference transducer*
- *Part 22: Shock calibration by comparison to an accelerometer, velocity or force transducer*

Introduction

The ISO 16063 series of standards is concerned with methods for the calibration of vibration and shock transducers under both standard laboratory conditions and in the field.

As such, the intended user group of this part of ISO 16063 is wide, ranging from metrologists in mechanical vibration to technicians evaluating the vibration characteristics of a machine or structure, or human exposure to vibration. The key to the application of this part of ISO 16063 is in the careful detailed specification and evaluation of measurement uncertainty, i.e. the error budget and computation of expanded uncertainty associated with the measurement of vibration.

This part of ISO 16063 is particularly intended for those engaged in vibration measurements requiring traceability to primary national or international standards through a secondary, reference, working or check standard (portable calibrator intended for field use) as defined in the *International vocabulary of basic and general terms in metrology* (VIM). The specifications for the instrumentation and the procedures given are intended to be used for calibration of rectilinear vibration transducers (with or without signal conditioning) to obtain the magnitude and (optionally) phase shift of the complex sensitivity at frequencies in the range of 0,4 Hz to 10 kHz.

Methods for the calibration of vibration and shock transducers —

Part 21: **Vibration calibration by comparison to a reference transducer**

1 Scope

This part of ISO 16063 describes the calibration of rectilinear vibration transducers by comparison. Although it mainly describes calibration using direct comparison to a standard calibrated by primary methods, the methods described can be applied between other levels in the calibration hierarchy.

This part of ISO 16063 specifies procedures for performing calibrations of rectilinear vibration transducers by comparison in the frequency range from 0,4 Hz to 10 kHz. It is primarily intended for those who are required to meet ISO standardized methods for the measurement of vibration under laboratory conditions, where the uncertainty of measurement is relatively small. It can also be used under field conditions, where the uncertainty of measurement may be relatively large.

From knowledge of all significant sources of uncertainty affecting the calibration, the expanded uncertainty can be evaluated using the methods given in this part of ISO 16063. It also covers the assessment of uncertainties for calibrations performed using a check standard.

Comparison calibrations made in accordance with this part of ISO 16063 need to allow for the environmental conditions of the reference transducer calibration.

NOTE Transducer calibrations made under extreme environmental conditions are covered by other International Standards.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 266, *Acoustics — Preferred frequencies*

ISO 2041:1990, *Vibration and shock — Vocabulary*

ISO 16063-1:1998, *Methods for the calibration of vibration and shock transducers — Part 1: Basic concepts*

ISO 16063-11:1999, *Methods for the calibration of vibration and shock transducers — Part 11: Primary vibration calibration by laser interferometry*

Guide to the expression of uncertainty in measurement (GUM). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 19931)

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¹⁾ Corrected and reprinted in 1995.

3 Uncertainty of measurement

3.1 All users of this part of ISO 16063 are expected to make uncertainty budgets according to Annex A to document their level of uncertainty (see example in Annex D).

To help set up systems fulfilling different requirements two examples are given. System requirements for each are set up and the attainable uncertainty is given. Example 1 is typical for calibrations under well-controlled laboratory conditions with the requirement to obtain a high accuracy. Example 2 is typical for calibrations where less than the highest accuracy can be accepted or where calibration conditions are such that only less narrow tolerances can be maintained. These two examples will be used throughout this part of ISO 16063.

a) **Example 1**

The reference transducer is calibrated by primary means and documented uncertainty. The calibration may be transferred to a working standard for practical reasons. The temperature and other conditions are kept within narrow limits during the comparison calibration as indicated in the appropriate clauses.

b) **Example 2**

The reference transducer is not calibrated by primary means, but has a traceable calibration, as defined in VIM (see [2]), with the corresponding uncertainty documented. The calibration may be transferred to a working standard for practical reasons. The requirements on other parameters and instruments are indicated in the appropriate clauses.

3.2 For both examples, the minimum calibration requirement for the reference transducer is calibration under suitable reference conditions (i.e. frequency, amplitude and temperature). Normally the conditions will be chosen as indicated in ISO 16063-11.

It is applicable for the following parameters:

Frequency range: 20 Hz to 5 000 Hz, optionally 0,4 Hz to 10 000 Hz (see Note 1) Dynamic range: 10 m/s^2 to 1 000 m/s² r.m.s., optionally 0,1 m/s² to 1 000 m/s² (frequency dependent)

NOTE The indicated frequency ranges are not mandatory and single-point calibrations are also acceptable.

At any given frequency and amplitude of acceleration, velocity or displacement, the dynamic range will be limited by the noise floor and the amount of distortion produced by the excitation apparatus (if no filtering is used) or its maximum power. (Techniques are also used to counteract the inherent distortion at large displacements for spring-controlled exciters by changing the waveform of the input voltage.) Typical maximum values for electrodynamic vibration exciters designed for the frequency range from 10 Hz to 10 kHz are 200 m/s² to 1000 m/s² r.m.s. acceleration, 0,5 m/s to 1 m/s r.m.s. velocity and 5 mm peak displacement. The lower limits will be set by the noise in the two measurement channels, and by the bandwidth used. Typical values used for measurement are 50 m/s2 to 100 m/s2 r.m.s. acceleration or 0,1 m/s r.m.s. velocity. For calibrators, values between 1 m/s² and 10 m/s² r.m.s. are normally used. A graph similar to the one shown in Annex C is useful when considering the ranges covered.

When measurements are performed at the lowest frequencies, the limiting factor is normally displacement. At 1 Hz, typical values for long-stroke vibrators are 1 m/s2 to 2 m/s2 r.m.s. acceleration or 0,1 m/s to 0,3 m/s r.m.s. velocity.

3.3 The attainable uncertainties (expanded uncertainties calculated using a coverage factor of 2 in accordance with ISO 16063-1) for the two examples are given in Table 1. In practice, these limits may be exceeded depending on the uncertainty with which the reference transducer has been calibrated, the response characteristics of the reference transducer and the transducer to be calibrated, the vibratory characteristics of the exciter and the instrumentation used in the measurement apparatus. It is the responsibility of the laboratory or end user to make sure that the reported values of expanded uncertainty are credible.

Table 1 — Attainable uncertainties of magnitude and phase shift of the complex sensitivity

4 Requirements for apparatus and environmental conditions

4.1 General

The examples referred to in this clause are those described in Clause 3.

If the recommended specifications listed below are met for each item, the uncertainties given in Clause 3 should be obtainable over the applicable frequency range depending on the uncertainty with which the reference transducer has been calibrated, and the response characteristics of the reference transducer and transducer to be calibrated. Other combinations of requirements can, however, lead to the same uncertainty. Special instrumentation may be required in order to meet the expanded uncertainties given in Clause 3 at frequencies less than 1 Hz. It is mandatory to document the expanded uncertainty using the methods of Annex A.

4.2 Environmental conditions

These shall be the following.

4.3 Reference transducer

This should preferably be calibrated together with the amplifier.

a) **Example 1**

The transducer shall be calibrated in accordance with suitable primary methods or by comparison against a transducer calibrated in accordance with suitable primary methods (see ISO 16063-11 or other parts) with an expanded uncertainty of 0,5 % (magnitude) and 0,5 $^{\circ}$ (phase shift) at selected reference frequency and acceleration (the uncertainties are those obtained when calculating expanded uncertainties using a coverage factor of 2). Higher uncertainty values are accepted at high and low frequencies.

b) **Example 2**

The transducer shall be calibrated by suitable and known methods with traceability to a primary reference transducer and an uncertainty of less than 2 % (magnitude) and 2° (phase shift) at selected reference frequency and acceleration (the uncertainties are those obtained when calculating expanded uncertainties using a coverage factor of 2). Higher uncertainty values are accepted at high and low frequencies.

The reference transducer may be of the so-called back-to-back type meant for direct mounting of the transducer to be calibrated on top of it in a so-called back-to-back configuration (see Figure 1). It may also be a transducer with normal mounting provisions used underneath a fixture in line with the transducer to be calibrated. It is not recommended to mount the two transducers side by side as rocking motions will often be present, causing large errors in many circumstances. For calibrators, the reference transducer may be an integral part of a moving element.

Subclauses 4.4 to 4.8 specify characteristics of apparatus that contribute to the uncertainty of measurement.

4.4 Vibration generation equipment

This shall fulfil the requirements given in Table 2.

Table 2 — Vibration generation equipment

The hum and noise will only be important when inside the measurement bandwidth used. For every combination of frequency, acceleration and load that is used during calibration, the magnitude of the transverse, bending and rocking accelerations, hum and noise shall be consistent with the uncertainties given in Clause 3.

Static or dynamic base strain introduced to the transducer from the attachment surfaces shall not unduly influence the calibration result.

All mounting surfaces between any two transducers compared shall have flatness and roughness specifications suitable for the purpose. If the highest frequency range is used, strict tolerances are necessary. The surface on which the transducer is to be mounted shall have a roughness value, expressed as the arithmetical mean deviation *Ra* of < 1 µm. The flatness shall be such that the surface is contained between two parallel planes 5 µm apart, over the area corresponding to the maximum mounting surface of any

transducer to be calibrated. The drilled and tapped hole for connecting the transducer shall have a perpendicularly tolerance to the surface of $<$ 10 µm, i.e. the centreline of the hole shall be contained in a cylindrical zone with 10 um diameter and a height equal to the hole depth.

The mounting surface of the vibration exciter should be perpendicular to the direction of motion. Any deviation from perpendicularity should be taken into account in the uncertainty budget, see Annex A.

Key

- 1 exciter
- 2 amplifiers
- 5 reference transducer
- 6 transducer to be calibrated 7 voltmeter
- 3 power amplifier
- 4 frequency generator and indicator
- 8 distortion meter for occasional checks
- 9 oscilloscope for visual inspection (optional)
- 10 phase meter (optional)

Figure 1 — Example of a measuring system for vibration calibration by comparison to a reference transducer

NOTE 1 Multisine, sine or random generators can be used in conjunction with frequency analysers. Typically Fast Fourier Transform (FFT) analysers are used for random and multisine signals and Single-Sine Correlation or Frequency Response Analysers (FRA) are used for single-sine signals. The distortion is then normally of no importance. Therefore analysers are normally preferred instead of broadband r.m.s. voltmeters which, although fundamentally more accurate, are sensitive to distortion and other signals at frequencies differing from the measurement frequency. Measurement of coherence can be used to estimate whether or not the signal-to-noise ratio and the linearity of the transducers are within well-defined limits when spectral averaging is used. With random excitation and 64 averages a minimum coherence limit of 0,98 will ensure that the errors due to signal-to-noise ratio and linearity are less than 0,9 % for a dual channel measurement. In rare cases, broadband excitation can, however, create unwanted (transverse) vibration or output signals at a measuring frequency due to non-linear behaviour of shaker or transducer at other frequencies.

NOTE 2 The items in 4.3 and 4.4 may be integrated into a calibrator.

4.5 Voltage measuring instrumentation

Two alternative set-ups are considered.

a) A single voltmeter measuring true r.m.s. at transducer amplifier output is used. The outputs from the reference transducer and the transducer to be calibrated are measured consecutively and the reference transducer output at least twice. This equipment shall fulfil the requirements given in Table 3.

b) An instrument measuring voltage ratio between transducer amplifier outputs is used. This equipment shall have the characteristics specified in Table 4.

Table 4 — Voltage measuring instrumentations

4.6 Distortion measuring instrumentation

Distortion measuring instrumentation (limited use, see Note) capable of measuring total harmonic distortion of 1 % to 10 % shall have the characteristics specified in Table 5.

4.7 Oscilloscope

An oscilloscope or similar display may be used for examining the waveforms of the transducer signals.

Its use is strongly recommended but not mandatory.

4.8 Phase shift measuring instrumentation

This equipment shall have the characteristics specified in Table 6.

Table 6 — Phase shift measuring

5 Calibration

5.1 Preferred amplitudes and frequencies

Six frequencies, each with associated acceleration (amplitude or r.m.s. value) and equally covering the transducer range, should preferably be chosen from the following series.

- a) Acceleration $(m/s²)$:
	- $-$ 1, 2, 5, 10 or their multiples of ten.

If broadband signals are used, these values are the total r.m.s. values.

- b) Frequency:
	- selected from standardized one-third-octave frequency series (see ISO 266).

If broadband signals are used, the desired range should be covered in one or more calibrations.

Values chosen should preferably be the same as those used in the reference transducer calibration. If the transducer is to be calibrated at frequencies and accelerations other than those at which the reference has been calibrated, the characteristics of the reference transducer should be assessed at those frequencies and accelerations. The resulting uncertainty component shall be taken into account in the uncertainty budget (see Annex A).

5.2 Measurement requirements

When a calibration is to be performed using a new set-up or a new transducer, it is good practice to carry out the calibration more than once to ensure sufficient repeatability.

It is important to ensure that cable motion and base strain do not appreciably affect the measurement results, particularly at low frequencies. Altering the attachment of the cable, the mounting of the transducer, or both, and noting changes in the measurement results or harmonic distortion may be used to evaluate effects due to these causes. If the measured sensitivity or distortion does not change significantly when compared to the uncertainty in the calibration, then these influences may be neglected. The mounting conditions of the transducer should also be repeatable. This can be verified by remounting the transducer several times and measuring the sensitivity after each successive attachment of the transducer.

If the transducer under test is not being calibrated in combination with an associated signal conditioner or amplifier, then the gain and frequency response (i.e. magnitude and, if needed, phase shift) of the complex sensitivity of the signal conditioner or amplifier used with the transducer under test should be determined in a traceable fashion at all measurement frequencies. The sensitivity and frequency response of the reference (transducer plus amplifier) shall also be determined in a traceable fashion at all measurement frequencies.

If any variations, significant compared to the desired uncertainty, are found in the above tests, these should be quantified by making a sufficiently large number of repeated measurements to get a good estimate of the variance. This shall then be included in the final uncertainty statement. This is especially important if the measurement is not made at the frequencies and amplitudes at which the reference transducer was calibrated.

5.3 Calibration procedure

The surfaces of the reference transducer (or fixture) and the transducer to be calibrated shall be examined to verify that they are free from burrs, etc. and that they comply with the manufacturer's flatness specifications and the specifications of Clause 4.

Mount the reference transducer (see 4.4) and the transducer to be calibrated back-to-back or in-line on a fixture on the exciter or on the exciter with integral working reference transducer using the recommended torque. Below approximately 5 kHz, good fixtures with known characteristics may be used between the transducers. At higher frequencies, the direct back-to-back configuration or integral working reference transducer shall be used. An example of a block diagram of a typical laboratory calibration apparatus is shown in Figure 1. The voltmeter, selector, generator and phase meter are often substituted by a two-channel instrumentation (e.g. dual-channel analyser with internal generator or voltage ratio meter) with sufficient accuracy.

Measure the ratio of the two outputs and the relative phase shift, if needed.

Determine the sensitivity at the reference frequency, for accelerometers preferably at 160 Hz (second choice: 80 Hz), and at the reference acceleration, for accelerometers preferably at 100 m/s² (other choices: 10 m/s², 20 m/s² or 50 m/s²), then determine the sensitivity at other calibration frequencies and accelerations. The results shall be given in absolute terms and/or as a relative deviation (percentage or decibels) and degrees deviation from the sensitivity at the reference point.

In the case of stud-mounted transducers, a thin film of light oil, wax or grease should be used between the mounting surfaces of the transducer(s) and exciter, particularly in the case of calibrations performed at high frequencies (see ISO 5348 for details).

6 Expression of results

If the reference transducer and the transducer under test respond to the same vibration quantity, calculate the magnitude S_2 and the phase shift φ_2 of the complex sensitivity of the transducer to be calibrated using the following formulae:

Magnitude: Phase shift:

$$
S_2 = \frac{X_2}{X_1} S_1 \qquad \varphi_2 = \varphi_{2,1} + \varphi_1
$$

where

- S_1 , φ_1 are the magnitude and phase shift of the complex sensitivity of the reference transducer;
- X_1 is the output from the reference transducer;
- X_2 is the output from the transducer to be calibrated;
- $\varphi_{2,1}$ is the phase shift between the outputs from the transducer to be calibrated and the reference transducer.

If the two transducers measure different vibration quantities, calculate the sensitivity of the transducer to be calibrated, using the following formulae.

a) If the magnitude and phase shift of the complex acceleration sensitivity S_a , φ_a were measured:

Magnitude: Phase:

 $S_v = 2\pi f S_a$ $\varphi_v = \varphi_a - 90^\circ$

$$
S_s = 4\pi^2 f^2 S_a \qquad \varphi_s = \varphi_a - 180^\circ
$$

b) If the magnitude and phase shift of the complex velocity sensitivity S_v , φ_v were measured:

 $S_s = 2\pi f S_v$ $\varphi_s = \varphi_v - 90^\circ$

where

 S_a , φ_a are the magnitude and phase shift of the complex acceleration sensitivity;

 S_v , φ , are the magnitude and phase shift of the complex velocity sensitivity;

 S_s , φ_s are the magnitude and phase shift of the complex displacement sensitivity;

f is the frequency of the vibration, in hertz.

The phase shift is undetermined within modulus 180° until definitions of directions for the transducers, motions and electrical signals are specified (see ISO 16063-1:1998, 3.3, and Annex B for guidance).

For calibrators designed to provide one or more fixed amplitude(s) at one or more frequencies, the sensitivity of the transducer to be calibrated is obtained by using the specified amplitude *A* and the measured output *X* in the formula

$$
S = \frac{X}{A}
$$

where *A* and *S* must refer to the same vibration quantity.

7 Reporting the calibration results

In addition to the calibration method and instrumentation with calibration due dates, at least the following conditions and characteristics shall be stated when the calibration results are reported.

- a) Ambient conditions, Example 1:
	- $-$ temperature of the transducer if measured, or
	- ambient air temperature and estimated difference from transducer temperature if this is not measured.
- b) Ambient conditions, Example 2:
	- $-$ estimated transducer temperature.
- c) Mounting technique:
	- material of mounting surface,
	- mounting torque (if stud mounted and optional for Example 2) or adhesive used,
	- characteristics of mounting components or adapters (if used),
	- $\overline{}$ oil or grease or wax (if used),
	- cable fixing,
	- orientation (vertical or horizontal).
- d) All amplifier settings (if adjustable) when the transducer is calibrated in combination with a signal conditioner or amplifier:
	- $-$ gain,
	- cut-off frequencies and slope of filters.
- e) Calibration results:
	- values of calibration frequencies and vibration amplitudes,
	- values of sensitivity (magnitude and phase shift, if measured),
	- coherence of the measurement if measured,
	- expanded uncertainty of the calibration (calculated in accordance with Annex A).
- f) Coverage factor, if different from *k* = 2 (corresponding to a confidence level of 95 % for a normal distribution).

Annex A

(normative)

Expression of uncertainty of measurement in calibration

A.1 Calculation of expanded uncertainty of measurement, *U*

A.1.1 Purpose of *U*

The uncertainty of measurement in calibration shall be expressed by the "expanded uncertainty" *U* in accordance with GUM, based on the approach recommended by the International Committee for Weights and Measures (CIPM). The purpose of *U* is to provide an interval $y - U$ to $y + U$ within which the value of *Y*, the specific quantity subject to calibration and estimated by y , can be expected to lie with high probability. To confidently assert that $y - U \leq Y \leq y + U$, the expanded uncertainty *U* shall be determined as follows.

A.1.2 Corrections

Every effort has to be made to identify each effect that significantly influences the measurement result and to compensate for such effects by applying the estimated corrections or correction factors.

If an effect influencing the measurement result is appropriately described by a probability distribution (preferably probability density, see A.1.3) having a significant expected value (in particular for an asymmetrical distribution), the latter shall be treated as systematic error and compensated by correction.

A.1.3 Standard uncertainty estimation

Each component of uncertainty that contributes to the uncertainty of the measurement shall be represented by a standard deviation u_i , termed "standard uncertainty", equal to the positive square root of the variance u_i^2 .

Some standard uncertainties can be obtained as statistically estimated standard deviations by the statistical analysis of series of observations (type A evaluation of standard uncertainty). Other standard uncertainties shall be evaluated as the standard deviation of a probability distribution describing the scientific judgement of all possible values of the respective quantity (type B evaluation of standard uncertainty). The judgement is based on all information available about the quantity. In particular, if there is no specific information about the possible values of a quantity responsible for systematic effects except that these values are within the bounds *b*_− and *b*₊, a uniform distribution over the interval [*b*_−; *b*₊] may be used to represent this information. It has a standard uncertainty $b/\sqrt{3}$ where $b = (b_+ - b_-)/2$. The expected value is $(b_+ + b_-)/2$ to be used for correction in this case.

If an influence quantity can be considered uniformly distributed (rectangular probability density) but is known to be transformed into the measurement result with a specific non-linear function (e.g. sinusoidal; polynomial of second or third order), this information shall be taken into account by choosing the associated distribution model.

EXAMPLE The sensitivity *S* of an accelerometer to sinusoidal accelerations in the nominal measurement direction is calculated from the output, voltage or charge amplitude \hat{x} , stimulated by a vibration, acceleration amplitude \hat{a} , using the formula $S = \hat{x}/\hat{a}$. Among the various disturbing effects influencing the measurement result in calibration, there may be a significant transverse motion component from the vibration exciter, acceleration amplitude \hat{a}_T , transformed into an error component $e_{\hat{x}T}$ in the output, in conjunction with the accelerometer's transverse sensitivity, S_T . It is assumed for this example that the acceleration to be measured and the transverse acceleration have the same frequency and that there is no phase angle difference. As the transverse sensitivity is usually sinusoidally dependent on the angle β between the direction of maximum transverse sensitivity $(S_{T,max})$ and the direction of a transverse excitation, the error component can be expressed by

 $e_{\hat{\tau}} = S_{\tau} \hat{a}_{\tau} = S_{\tau} \hat{a}_{\tau} \hat{a}_{\tau}$ \hat{a}_{τ} \hat{a}_{τ} $\cos \beta$

If the values of the maximum transverse sensitivity $(S_{\tau,max})$ and the maximum transverse acceleration $(\hat{a}_{\tau,max})$ are known while the angle β is not, it is reasonable to assume a rectangular distribution of β within the interval $[-\pi, \pi]$. Thus, the influence quantity, i.e. transverse acceleration, with rectangularly distributed angle β leads to a measurement error component $e_{\hat{\mathbf{x}}\mathsf{T}}$ whose probability density is described by

$$
w(e_{\hat{x}}) = \frac{1}{b\pi\sqrt{1 - \left(\frac{e_{\hat{x}}}{b}\right)^2}}
$$

 $-b < e_{\hat{x}}$ _T $< b$

 $b=S_{\textsf{T},\textsf{max}}\hat{a}_{\textsf{T},\textsf{max}}$

(often referred to as arcsin distribution). The associated standard uncertainty is

$$
u\left(e_{\hat{x}\mathsf{T}}\right)=b/\sqrt{2}
$$

The expected value $E\{e_{\hat{x}}\}$ is zero in this case. This is the best estimate of the error $e_{\hat{x}}\tau$.

A.1.4 Combined standard uncertainty

The "combined standard uncertainty" u_c , as the standard uncertainty of the measurement of *Y*, shall be determined by combination of the individual standard uncertainties (and covariances as appropriate) using the law of propagation of uncertainty. Accordingly, the combined standard uncertainty is obtained from

$$
u_{\mathbf{c}}(y) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_{i}}\right)^{2} u^{2}(x_{i}) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_{i}} \frac{\partial f}{\partial x_{j}} u(x_{i}, x_{j})}
$$
(A.1)

This equation is based on a first-order Taylor series approximation of

$$
Y = f(X_1, X_2, \dots, X_N) \tag{A.2}
$$

where *Y* is the measurand determined from *N* input quantities $X_1, X_2, ..., X_N$ through a functional relationship *f*.

An estimate of the measurand *Y*, denoted by y , is obtained from Equation (A.1) using input estimates $x_1, x_2, ...,$ x_N for the values of the input quantities. Thus the output estimate, which is the result of measurement, is given by

$$
y = f(x_1, x_2, \dots, x_N) \tag{A.3}
$$

In Equation (A.1), the symbols $\partial f/\partial x_i$ are often referred to as sensitivity coefficients c_i . They are equal to the partial derivatives $\partial f/\partial X_i$ evaluated at $X_i = x_i$. The symbol $u(x_i,x_j)$ designates the estimated covariance associated with x_i and x_j .

For the case where no significant correlations are present, Equation (A.1) is reduced to

$$
u_{\mathbf{c}}(y) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)}
$$
(A.4)

NOTE The first-order Taylor series approximation of Equation (A.2) resulting in Equation (A.1) is only applicable if the model function f is sufficiently linear with respect to the variation of the input estimates x_i within the ranges characterized by the uncertainties $u(x_i)$. This is not the case in the example given in A.1.3 if the angle β is considered to be an input quantity X_i . To overcome this obstacle, which similarly exists with other influence quantities acting in measurements within calibrations of vibration and shock transducers, an adequate model has been introduced (see reference [4]). To briefly

specify this model for the example above, a factor $(1 - e_{xT}/\hat{x})$ with $e_{xT}/\hat{x} \ll 1$ is introduced, as an input quantity X_i , into the functional relationship used for calculating the measurand. Equation (A.2), specially tailored to the example, is reduced to three input quantities

$$
Y = f(X_1, X_2, X_3)
$$

where

- *Y* is the measurand (sensitivity *S*);
- X_1 is the accelerometer output (voltage or charge amplitude \hat{x});
- X_2 is the acceleration amplitude;

 $X_3 = (1 - e_{\hat{x}} / \hat{x})$.

Thus, the relationship

$$
Y = \frac{X_1}{X_2} X_3
$$

can be established. The first Taylor series approximation can be used now, leading to the relative combined standard uncertainty

$$
\frac{u_{c}(y)}{y} = \sqrt{\left(\frac{u(x_{1})}{x_{1}}\right)^{2} + \left(\frac{u(x_{2})}{x_{2}}\right)^{2} + \left(\frac{u(x_{3})}{x_{3}}\right)^{2}}
$$

if there are no significant correlations.

Using the symbols introduced in the Example, the latter relationship can be written as follows:

$$
\frac{u_{\mathsf{C}}(S)}{S} = \sqrt{\left(\frac{u(\hat{x})}{\hat{x}}\right)^2 + \left(\frac{u(\hat{a})}{\hat{a}}\right)^2 + \left(\frac{u(e_{\hat{x}T}/\hat{x})}{1}\right)^2}
$$

where

- $u(\hat{x})/\hat{x}$ is the relative standard uncertainty of the output voltage amplitude measurement;
- $u(\hat{a})/\hat{a}$ the relative standard uncertainty in acceleration amplitude measurement;

$$
u(e_{\hat{x}T}/\hat{x}) = u(e_{\hat{x}T})/\hat{x}
$$
, with $u(e_{\hat{x}T}) = b/\sqrt{2}$ as explained in the Example.

Accordingly, further factors whose deviations from the value 1 are similarly expressed by the relative error component of the respective quantity (e.g. voltage, acceleration or the sensitivity as a whole) might be introduced as input quantities (*X*4, *X*5, ...), allowing the variety of uncertainty sources to be taken into account separately.

A.1.5 Expanded uncertainty

The "expanded uncertainty" *U* shall be determined by multiplying *u*c by a coverage factor *k*:

 $U = k u_c$

where a value of $k = 2$ should preferably be used. If it can be assumed that the possible values of the calibration result are approximately normally distributed with approximate standard deviation u_c , the unknown value can be asserted to lie in the interval defined by *U* with a level of confidence, or probability, of approximately 95 %.

A.1.6 Reporting the result

When reporting the result of the measurement *y*, the expanded uncertainty and the value of the coverage factor k used, if different from $k = 2$, shall be stated. In addition, the approximate coverage probability or level of confidence of the interval may be stated.

A.2 Calculation of expanded uncertainties at reference conditions

A.2.1 Calculation of the relative expanded uncertainty *U*rel(*S*) **for the sensitivity magnitude**

The relative expanded uncertainty of measurement of the magnitude of the complex sensitivity, *U*rel(*S*)*,* for each of the applied frequencies, accelerations and amplifier gain settings (if an amplifier is part of the calibrated transducer) is calculated from the following formulae:

$$
U_{\text{rel}}(S) = ku_{\text{c,rel}}(S)
$$

with the coverage factor $k = 2$ (see A.1.5);

$$
u_{\text{c,rel}}(S) = \frac{u_{\text{c}}(S)}{S} = \frac{1}{S} \sqrt{\sum_{i} \left(\frac{\partial f}{\partial x_{i}}\right)^{2} u_{i}^{2}(S) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_{i}} \frac{\partial f}{\partial x_{j}} u(x_{i}, x_{j})}
$$

$$
= \sqrt{\sum_{i} \left(\frac{\partial f}{\partial x_{i}}\right)^{2} u_{\text{rel},i}^{2}(S) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_{i}} \frac{\partial f}{\partial x_{j}} u_{\text{rel}}(x_{i}, x_{j})}
$$

where *S* is the sensitivity and $f(x_1, x_2,...,x_N)$ is the estimated sensitivity and x_i are the estimated input values. $u(x_i,x_j)$ is the estimated covariance of x_i,x_j (zero if uncorrelated). The last part uses u_{rel} to indicate relative values.

NOTE 1 The correlation terms in the above relationship can often be omitted by assigning correlated terms, if any, to the same number *i*. This approach may lead to significant simplifications (see reference [4]).

NOTE 2 It is assumed that the effective number of degrees of freedom is large enough (say greater than 10) to assume that the combined uncertainty approximates the normal distribution (see GUM:1993, G.6.6).

Table A.1 lists a number of uncertainty sources. Although it is believed to contain all the important sources it cannot be guaranteed to be complete because this part of ISO 16063 covers a wide range of different systems and methods, and new ones might be implemented.

A.2.2 Calculation of the expanded uncertainty $U(\Delta\varphi)$ for the phase shift

The expanded uncertainty of measurement of the phase shift ∆ φ of the complex sensitivity *S*, at each of the applied frequencies, accelerations and amplifier gain settings (if an amplifier is part of the calibrated transducer) is calculated from the following formulae:

$$
U(\Delta \varphi) = k u_{\rm c}(\Delta \varphi)
$$

with the coverage factor $k = 2$ (see A.1.5);

$$
u_{\textbf{c}}(\Delta \varphi) = \sqrt{\sum_i \left(\frac{\partial g}{\partial x_i}\right)^2 u_i^2(\Delta \varphi) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j} u(x_i, x_j)}
$$

where $\Delta\varphi$ is the phase shift of the complex sensitivity and $g(x_1,x_2,...,x_N)$ is the estimated phase shift and x_i are the estimated input values; $u(x_i, x_j)$ is the estimated covariance of x_i, x_j (zero if uncorrelated).

NOTE 1 The correlation terms in the above relationship can often be omitted by assigning correlated terms, if any, to the same number *i*. This approach may lead to significant simplifications (see reference [4]).

NOTE 2 It is assumed that the effective number of degrees of freedom is large enough (say greater than 10) to assume that the combined uncertainty approximates the normal distribution (see GUM:1993, G.6.6).

Table A.2 lists a number of uncertainty sources. Although it is believed to contain all the important sources it cannot be guaranteed to be complete because this part of ISO 16063 covers a wide range of different systems and methods, and new ones might be implemented.

\dot{i}	Standard uncertainty component $u(x_i)$	Source of uncertainty	Uncertainty contribution $u_i(y)$
$\mathbf 1$	$u(\Delta \varphi_1)$	The combined standard uncertainty on phase shift for the reference transducer and amplifier combination	$u_1(\Delta \varphi)$
$\overline{2}$	$u(\Delta \varphi_{A})$	Conditioning amplifier phase shift uncertainty	$u_2(\Delta \varphi)$
3	$u(\varphi)$	Phase measurements	$u_3(\Delta \varphi)$
4	$u(\varphi_d)$	Effect of the total harmonic distortion on the phase measurement	$u_4(\Delta \varphi)$
5	$u(\varphi_H)$	Effect of hum and noise on the phase measurement	
6	$u(\varphi_{\mathsf{T}})$	Effect of transverse, rocking and bending vibration on the output phase	
$\overline{7}$	$u(\varphi_{\mathsf{e}})$	Effect of base strain on the output phase	
8	$u(\varphi_{\mathsf{M}})$	Effect of mounting parameters (torque, cable fixing, dummy mass, etc.) on output phase	$u_8(\Delta \varphi)$
9	$u(\varphi_R)$	Effect of relative motion on output phase	
10	Reference transducer phase stability over time $u(\Delta \varphi_{s})$		$u_{10}(\Delta \varphi)$
11	$u(\varphi_{\mathsf{T}})$	Effect of temperature on output phase	$u_{11}(\Delta \varphi)$
12	$u(\varphi_1)$	Effect of non-linearity of transducers on output phase	$u_{12}(\Delta \varphi)$
13	$u(\varphi_1)$	Effect of non-linearity of amplifiers on output phase	$u_{13}(\Delta \varphi)$
14	$u(\varphi_{\rm G})$	Effect of gravitation on transducers on output phase	$u_{14}(\Delta\varphi)$
15	$u(\varphi_{\rm B})$	Effect of magnetic field from exciter on transducers on output phase	$u_{15}(\Delta \varphi)$
16	$u(\varphi_{\rm F})$	Effect of other environmental parameters on output phase	$u_{16}(\Delta \varphi)$
17	$u(\Delta \varphi_1, \Delta \varphi_{2,RE})$	Residual effects on phase shift measurement and on the output phase of the transducer to be calibrated (e.g. random effect in repeated measurements; experimental standard deviation of arithmetic mean)	$u_{17}(\Delta \varphi)$

Table A.2 — Uncertainty components for determination of ∆^ϕ

NOTE 1 If the method chosen uses a working standard transducer, which at certain intervals is calibrated by the reference transducer, then the above calculation needs to be made twice, and the result of the first calculation is used as input to the final calculation.

NOTE 2 The sources of uncertainties may be subdivided and numbered in a way differing from that used in this table, provided that each effect significantly influencing the measurement result has been taken into account (see A.1.3).

A.3 Expanded uncertainties over the complete frequency and amplitude range

A.3.1 Calculation of the relative expanded uncertainty $U_{\sf rel}(S_{\sf t})$ for the magnitude

The relative expanded uncertainty of measurement of the magnitude of the complex sensitivity, $U_{\text{rel}}(S_t)$, calculated in accordance with A.2.1 is only valid for the calibration frequencies, accelerations and amplifier settings (if an amplifier is part of the calibrated transducer). The relative expanded uncertainty of measurement of the magnitude of the complex sensitivity, $U_{\text{rel}}(S_t)$, for the complete frequency and amplitude range, is calculated from the following formulae:

 $U_{\text{rel}}(S_t) = k u_{\text{c,rel}}(S_t)$

with the coverage factor $k = 2$ (see A.1.5);

$$
u_{\text{c,rel}}(S_t) = \frac{u_{\text{c}}(S_t)}{S_t} = \frac{1}{S_t} \sqrt{\sum_i \left(\frac{\partial f}{\partial x_i}\right)^2 u_i^2(S_t) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)}
$$

where

 S_t is the sensitivity determined for the complete frequency and amplitude range;

 $f(x_1, x_2, \ldots, x_N)$ is the estimated sensitivity;

xi is the estimated input values;

 $u(x_i, x_j)$) is the estimated covariance of $x_i x_j$ (zero if uncorrelated).

NOTE 1 The correlation terms in the above relationship can often be omitted by assigning correlated terms, if any, to the same number *i.* This approach may lead to significant simplifications (see reference [4]).

NOTE 2 It is assumed that the effective number of degrees of freedom is large enough (say greater than 10) to assume that the combined uncertainty approximates the normal distribution (see GUM:1993, G.6.6).

Table A.3 lists a number of uncertainty sources. Although it is believed to contain all the important sources, it cannot be guaranteed to be complete because this part of ISO 16063 covers a wide range of different systems and methods, and new ones might be implemented.

A.3.2 Calculation of the expanded uncertainty $U(\Delta \varphi_{\!\! \bm{\mathfrak{t}}})$ for the phase

The expanded uncertainty of measurement of the phase shift ∆_{$\varphi_{\rm t}$} of the complex sensitivity $S_{\rm t}$, calculated in accordance with A.2.2, is only valid for the calibration frequencies, accelerations and amplifier settings (if an amplifier is part of the calibrated transducer). The expanded uncertainty of measurement of the phase shift of the complex sensitivity, $U(S_t)$, for the complete frequency and amplitude range is calculated from the following formulae:

 $U(\Delta \varphi_t) = k u_c(\Delta \varphi_t)$

with the coverage factor $k = 2$ (see A.1.5);

$$
u_{\mathbf{c}}(\Delta \varphi_t) = \sqrt{\sum_i \left(\frac{\partial g}{\partial x_i}\right)^2 u_i^2(\Delta \varphi_t) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j} u(x_i, x_j)}
$$

where

 $\Delta\varphi$ is the phase shift of the complex sensitivity determined for the complete frequency and amplitude range;

 $g(x_1, x_2,...,x_N)$ is the estimated phase shift;

xi is the estimated input values. $u(x_i, x_j)$ is the estimated covariance of x_i, x_j (zero if uncorrelated).

NOTE 1 The correlation terms in the above relationship can often be omitted by assigning correlated terms, if any, to the same number *i*. This approach may lead to significant simplifications (see reference [4]).

NOTE 2 It is assumed that the effective number of degrees of freedom is large enough (say greater than 10) to assume that the combined uncertainty approximates the normal distribution (see GUM:1993, G.6.6).

Table A.4 lists a number of uncertainty sources. Although it is believed to contain all the important sources, it cannot be guaranteed to be complete because this part of ISO 16063 covers a wide range of different systems and methods, and new ones might be implemented.

i	Standard uncertainty component $u(x_i)$	Source of uncertainty	Uncertainty contribution $u_i(y)$
1	$u(\Delta \varphi)$	Uncertainty of the phase shift of the sensitivity calculated at the calibration frequencies, amplitudes and amplifier settings in accordance with A.2.2	$u_1(\Delta \varphi_t)$
$\overline{2}$	$u(e^{\star}_{\mathsf{T.A}})$	Conditioning amplifier tracking (phase deviations for different amplifications)	$u_2(\Delta \varphi_t)$
3	$u(e^*_{L, f, A})$	Conditioning amplifier frequency response (phase deviations for different frequencies)	$u_3(\Delta \varphi_t)$
4	$u(e^{\star}_{L, f, T})$	Transducer frequency response deviations from theoretical curve (phase deviations from assumed curve at different frequencies)	$u_4(\Delta \varphi_t)$
5	$u(e^*_{L, a, A})$	Amplitude effect on amplifier phase	$u_5(\Delta \varphi_t)$
6	$u(e^*_{L, a, T})$	Amplitude effect on sensitivity (phase) of transducer	$u_6(\Delta \varphi_t)$
$\overline{7}$	$u(e^*_{i, A})$	Instability of amplifier phase and effect of source impedance	$u_7(\Delta \varphi_t)$
8	$u(e^*_{i,T})$	Instability of transducer sensitivity (phase)	$u_8(\Delta \varphi_t)$
9	$u(e^{\star}_{E, A})$	Environmental effects on amplifier phase	$u_{9}(\Delta \varphi_{t})$
10	$u(e^{\star}_{E,T})$	Environmental effects on transducer sensitivity (phase)	$u_{10}(\Delta \varphi_t)$
11	$u(e^{\star}_{M, T})$	Additional effects of mounting parameters (torque, cable fixing, dummy mass, etc.) on transducer sensitivity (phase)	$u_{11}(\Delta \varphi_t)$

Table A.4 — Uncertainty components for determination of ∆_{ϕ_t}

Annex B

(normative)

Definitions of amplitude sign and phase shift between mechanical motion and vibration transducer electrical output

B.1 Motion

An object has normally 6 degrees of freedom: 3 degrees for linear motion and 3 degrees for rotational motion.

B.2 Coordinate system

All motions are measured in a coordinate system, and for linear motions a Cartesian coordinate system with *x*, *y* and *z* coordinates is recommended.

Angular motion can be measured in the same coordinate system with the addition of a rotation angle. The angular motion is specified in a polar coordinate system with the coordinate axes as rotational axes and the sign for the rotation is defined by using the right-hand grip rule (with the thumb in the direction of the axes, the fingers will point in the positive rotational direction).

B.3 Transducer types

Several different vibration transducer types are available. Accelerometers, velocity transducers, displacement (proximity) probes are typical transducers.

B.4 Units

Amplitudes for vibration signals are specified in either linear or rotational units. The units for the quantities of acceleration, velocity and displacement (length) are metres per second squared, metres per second and metres, respectively, and the unit for phase (angle) is either degrees or radians. The units for the quantities of angular acceleration, angular velocity and angular displacement (plane angle) are radians per second squared, radians per second and radians, respectively, and the unit for rotational phase (angle) is either degrees or radians.

B.5 Definition of coordinate system

The coordinate system is here defined relative to the transducer. One of the axes is parallel to the transducer main axis, and the positive direction is from the measuring surface and through the transducer.

B.6 Definition of polarity or sign for motion

A linear motion (acceleration, velocity or displacement) has a positive sign when the motion is in the positive direction of the axis parallel to the transducer main axis.

A rotational or angular motion has a positive sign when the motion is in the same direction as the positive angle.

This definition is independent of the mounting of the transducer (contact or non-contact transducer).

B.7 Definition of sensitivity (magnitude and phase shift) for transducers

The transducer converts the physical parameter to another parameter, usually an electrical output signal. The output signal could either be a voltage, current or charge value.

The sensitivity is defined as the ratio between the electrical output and a specified motion. The sensitivity includes a value with sign and a phase shift specification based on the previous definition of coordinate system, motion and the conversion in the transducer.

Annex C

(informative)

Nomogram for conversion between acceleration, velocity and displacement

Figure C.1 shows a nomogram for conversion between acceleration, velocity and displacement of vibration magnitude(s) (r.m.s. or peak or peak-to-peak values) at discrete frequencies. If different parameters are used, the factor $\sqrt{2}$ (r.m.s. to peak) or 2 $\sqrt{2}$ (r.m.s. to peak-to-peak) should be used. With two known parameters, the other two can be found.

For example, if the frequency and acceleration level are known, use the nomogram shown to read the corresponding velocity and displacement magnitude(s) off the respective scales.

Key

- 1 displacement
- 2 acceleration
- 3 velocity
- 4 frequency

Annex D

(informative)

Example of uncertainty calculation

D.1 General

To facilitate the use of the previously given principles, an example based on a set-up as shown in Figure 1 is given in this annex.

The measurement process may be represented as shown in Figure D.1.

Key

- 1 mechanical vibrator
- 2 reference accelerometer set
- 3 voltmeter
- 4 transducer to be calibrated
- 5 amplifier

Figure D.1 — Scheme of the measurement process

D.2 Symbols

The following symbols are used in this annex:

- *a* Acceleration in main direction
- *a*_v Acceleration transverse to main direction
- a_{d} Acceleration distortion in main direction
- *S*₁ Sensitivity magnitude of reference transducer set
- *u*1 Voltage at output of reference transducer set
- V_1 Voltmeter reading (reference voltages and accelerations are r.m.s. values)
- *S*₂ Sensitivity magnitude of transducer to be calibrated
- *S*_A Sensitivity magnitude of conditioning amplifier
- u_2 Voltage at amplifier output (transducer to be calibrated)
- V_2 Voltmeter reading of transducer to be calibrated

D.3 Sensitivity function

An acceleration excitation, *a*, is delivered to the transducers.

The transducer to be calibrated and its associated conditioning amplifier form a calibration set. The output from this set is compared to the output from the calibrated reference transducer set consisting of a reference transducer and its associated conditioning amplifier.

The ratio V_R between the measured output voltage V_2 from the calibration set and the measured output voltage V_1 from the reference transducer set is determined by consecutive measurements. To take drift into account, V_1 is measured before and after the measurement of V_2 . Distortion in the acceleration, $a_{\rm d}$, has an influence on the calculation of the sensitivity of the transducer to be calibrated only if the slopes for the frequency responses are different, as it will give the same influence on both readings (V_1 and V_2) if the slopes are identical. In Table D.1 the reference transducer is assumed to be a quartz-based transducer, which has no slope, and the transducer to be calibrated is assumed to be a PZT (lead titanate/zirconate ceramic) based transducer, which typically has a decade slope of −2 % per frequency, which can be described as

$$
S_2(f) = S_2(f_{\text{ref}}) \left(1 - 0.02 \lg \frac{f}{f_{\text{ref}}}\right)
$$

The output voltage from the conditioning amplifier is given by

 $u_2 = a S_2 S_A$

The acceleration level is calculated from the output voltage from the reference transducer set by

$$
a = \frac{u_1}{S_1}
$$

Inserting a in the formula for u_2 , the sensitivity of the transducer to be calibrated is found as

$$
S_2 = \frac{S_1 u_2}{u_1 S_A}
$$

 V_R is defined by

$$
V_R \equiv \frac{V_2}{V_1}
$$

and, assuming constant acceleration, this gives

$$
\frac{V_2}{V_1} = \frac{u_2}{u_1}
$$

Therefore the model function for S_2 is

$$
S_2 = \frac{S_1}{S_{\mathsf{A}}} V_R
$$

D.4 Calculation of uncertainty

Following reference [4], the influence from quantities not directly included in the above model function is expanded to

$$
S_2 = \frac{S_1}{S_A} V_R \times I_1 \times \dots \times I_M
$$

where the factors I_1 to I_M describe the errors due to influence quantities.

The mean value equals 1 (and with different distributions, normal, rectangular, etc.) and is given as

$$
I_i = \frac{1 - e_{2,i}}{1 - e_{1,i}}
$$

where e_i denotes the *i*th error component due to an influence quantity characterized by an appropriate distribution model. The error component e_i can, only under certain conditions, be estimated by a simple distribution model such as rectangular distribution and mean value equal to 0.

Each influence factor contains the influence from one quantity on V_R from both V_1 and V_2 .

For each influence factor, I_i , the possible correlation between errors is taken into account. Preferably the influence factors are chosen so that correlated influences are contained in one factor rendering all influence factors uncorrelated.

Under the assumption that the errors e_i are small, the second (and higher) order terms in the Taylor series may be neglected.

According to reference [4], the correlation coefficient may be assumed to be zero if there is no indication of a strong correlation.

If the correlation factor between the influences on V_R is unknown, the influences are treated in accordance with Equation D.10 in reference [3]:

$$
u^{2}(y) \leq (| u_{1}(y) | + | u_{2}(y) |)^{2} + u_{r}^{2}(y)
$$

where $u_1(y)$ and $u_2(y)$ are the uncertainties for the correlated elements (standard deviation for approximated normal distributions), and $u_r(y)$ is the combined uncertainty for the uncorrelated elements.

The budget of uncertainty is based upon calculation of relative uncertainties. For uncorrelated elements, the relative combined uncertainty $(k = 1)$ for S_2 , $u_{c,rel}(S_2)$ is given by:

$$
u_{\rm c,rel}^2(S_2) = \frac{u_{\rm c}^2(S_2)}{S_2^2} = \sum_{i=1}^{N} \left(\frac{u_{\rm rel,i} \ c_i}{x_i}\right)^2
$$

where c_i are the sensitivity coefficients, which are found as the partial derivatives of the model function with respect to the different quantities. As the model function only contains factors in the nominator and one denominator, all becomes 1 or −1, which means they are basically without any importance as all are squared.

An example with numbers is given in Table D.1. The expanded relative uncertainty (coverage factor *k* = 2) is:

$$
U_{\text{rel, }95}(S_2) = \frac{U_{95}(S_2)}{|S_2|} = 0,84\%
$$

The expanded uncertainty calculated with the coverage factor $k = 2$ corresponds to the confidence limit for the coverage probability $P = 95$ % (assuming normal distribution).

Table D.1 — Example of budget of uncertainty for a piezoelectric accelerometer at 160 Hz and 100 m/s2s

Quantity	Description	Relative expanded uncertainty or bounds of estimated error components $\%$	Probability distribution model	Factor	Sensitivity coefficient c_i	Relative contribution $u_{\text{rel},i}(y)$
				x_i		%
$I(V_{R, r})$	Influence on V_R measurement from relative motion. Estimated to be less than	0,05	Rectangular	$1/\sqrt{3}$	1	0,029
$I(V_{R, L})$	Influence on V_R measurement from non-linearity of transducers. Estimated to be less than	0.03	Rectangular	$1/\sqrt{3}$	1	0,017
$I(V_{R,1})$	Influence on V_R measurement from non-linearity of amplifiers. Estimated to be less than	0.03	Rectangular	$1/\sqrt{3}$	1	0.017
$I(V_{R, G})$	Influence on V_R measurement from gravity. Estimated to be less than	0,00	Rectangular	$1/\sqrt{3}$	1	0,00
$I(V_{R, B})$	Influence on V_R measurement from magnetic field from exciter. Estimated to be less than	0.03	Rectangular	$1/\sqrt{3}$	1	0,017
$I(V_{R, E})$	Influence on V_R measurement from other environmental effects. Estimated to be less than	0,03	Rectangular	$1/\sqrt{3}$	1	0,017
$I(V_{R,RE})$	Influence on V_R measurement from residual effects (e.g. random effect in repeated measurements; experimental standard deviation of arithmetic mean). Estimated to be less than	0.03	Rectangular	$1/\sqrt{3}$	1	0,017
$u_{rel}(S_2)$	Uncertainty for accelerometer sensitivity S_2		Standard uncertainty $(k = 1)$			0,42

Table D.1 *(continued)*

NOTE 1 The uncertainties involved in the measurements by comparison are listed with their estimated magnitudes. The uncertainties are converted to a standard deviation in the last column using the appropriate distribution model, e.g. rectangular. All the calculated values $u_{rel,i}(y)$ are standard uncertainties $(k = 1$ values).

NOTE 2 The output from the two channels can be described by $u_1^2 = S_1^2 (f_{ref})(a_{ref}^2 + a_3^2)$ and $u_2^2 = S_A^2 (S_2^2 (f_{ref}) a_{ref}^2 + S_2^2 (3f_{ref}) a_3^2)$ following the comments in the text with no dependence on frequency for the re the reference frequency is obtained. Using a_3 = 0,05, this gives an error as calculated above on the ratio of the square roots of the squared outputs. The amplifier is assumed to have a flat response $(S_A \text{ constant})$.

NOTE 3 If the transverse sensitivity directions of the two transducers and the exciter transverse direction are known, but their
relative directions are unknown, an integration over 360° gives the resulting variance σ^2

If all three are assumed to be unknown but rectangularly distributed with the bounds as the values used as known above, it is necessary to normalize by 1/ $\sqrt{3}$, yielding $\sigma^2 = \frac{1}{2}(\frac{1}{3}S_{\nu,2}^2 + \frac{1}{3}S_{\nu,1}^2) \times \frac{1}{3}a_1^2 = \frac{1}{18}(S_{\nu,2}^2 + S_{\nu,1}^2)a_1^2$.

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