

---

---

**Methods for the calibration of vibration  
and shock transducers —**

Part 15:  
**Primary angular vibration calibration by  
laser interferometry**

*Méthodes pour l'étalonnage des transducteurs de vibrations et de  
chocs —*

*Partie 15: Étalonnage angulaire primaire de vibration par interférométrie  
laser*



Reference number  
ISO 16063-15:2006(E)

© ISO 2006

**PDF disclaimer**

This PDF file may contain embedded typefaces. In accordance with Adobe's licensing policy, this file may be printed or viewed but shall not be edited unless the typefaces which are embedded are licensed to and installed on the computer performing the editing. In downloading this file, parties accept therein the responsibility of not infringing Adobe's licensing policy. The ISO Central Secretariat accepts no liability in this area.

Adobe is a trademark of Adobe Systems Incorporated.

Details of the software products used to create this PDF file can be found in the General Info relative to the file; the PDF-creation parameters were optimized for printing. Every care has been taken to ensure that the file is suitable for use by ISO member bodies. In the unlikely event that a problem relating to it is found, please inform the Central Secretariat at the address given below.

© ISO 2006

All rights reserved. Unless otherwise specified, no part of this publication may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying and microfilm, without permission in writing from either ISO at the address below or ISO's member body in the country of the requester.

ISO copyright office  
Case postale 56 • CH-1211 Geneva 20  
Tel. + 41 22 749 01 11  
Fax + 41 22 749 09 47  
E-mail [copyright@iso.org](mailto:copyright@iso.org)  
Web [www.iso.org](http://www.iso.org)

Published in Switzerland

# Contents

Page

Foreword.....	iv
<b>1 Scope .....</b>	<b>1</b>
<b>2 Normative references .....</b>	<b>2</b>
<b>3 Uncertainty of measurement .....</b>	<b>2</b>
<b>4 Requirements for apparatus .....</b>	<b>2</b>
4.1 General.....	2
4.2 Frequency generator and indicator .....	3
4.3 Power amplifier/angular vibration exciter combination.....	3
4.4 Seismic block(s) for vibration exciter and laser interferometer .....	5
4.5 Laser.....	5
4.6 Interferometer.....	5
4.7 Instrumentation for interferometer signal processing.....	8
4.8 Voltage instrumentation, measuring true r.m.s. accelerometer output.....	9
4.9 Distortion-measuring instrumentation .....	9
4.10 Oscilloscope (optional).....	9
4.11 Other requirements.....	9
<b>5 Ambient conditions .....</b>	<b>9</b>
<b>6 Preferred angular accelerations and frequencies .....</b>	<b>10</b>
<b>7 Common procedure for all six methods.....</b>	<b>10</b>
<b>8 Methods using fringe-counting (methods 1A and 1B).....</b>	<b>11</b>
8.1 General.....	11
8.2 Common test procedure for methods 1A and 1B.....	12
8.3 Expression of results .....	12
<b>9 Methods using minimum-point detection (methods 2A and 2B) .....</b>	<b>16</b>
9.1 General.....	16
9.2 Common test procedure for methods 2A and 2B.....	17
9.3 Expression of results .....	17
<b>10 Methods using sine approximation (methods 3A and 3B) .....</b>	<b>21</b>
10.1 General.....	21
10.2 Procedure applied to methods 3A and 3B .....	22
10.3 Data acquisition .....	27
10.4 Data processing .....	27
<b>11 Reporting of calibration results .....</b>	<b>29</b>
<b>Annex A (normative) Uncertainty components in primary angular vibration calibration of vibration and shock transducers by laser interferometry .....</b>	<b>30</b>
<b>Annex B (normative) Equations for the calculation of the angular quantities of rotational angle, <math>\Phi</math>, angular velocity, <math>\Omega</math>, and angular acceleration, <math>\alpha</math>, and of the sensitivities of angular transducers: rotational angle transducers, <math>S_{\Phi}</math>, of angular velocity transducers, <math>S_{\Omega}</math> and angular accelerometers, <math>S_{\alpha}</math>.....</b>	<b>36</b>
<b>Bibliography .....</b>	<b>42</b>

## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 16063-15 was prepared by Technical Committee ISO/TC 108, *Mechanical vibration and shock*, Subcommittee SC 3, *Use and calibration of vibration and shock measuring instruments*.

ISO 16063 consists of the following parts, under the general title *Methods for the calibration of vibration and shock transducers*:

- *Part 1: Basic concepts*
- *Part 11: Primary vibration calibration by laser interferometry*
- *Part 12: Primary vibration calibration by the reciprocity method*
- *Part 13: Primary shock calibration using laser interferometry*
- *Part 15: Primary angular vibration calibration by laser interferometry*
- *Part 21: Vibration calibration by comparison to a reference transducer*
- *Part 22: Shock calibration by comparison to a reference transducer*

The following additional parts are under preparation:

- *Part 23, addressing the angular vibration calibration by comparison to reference transducers*
- *Part 31, addressing the testing of transverse vibration sensitivity*
- *Part 32, addressing the resonance testing*
- *Part 41, addressing the calibration of laser vibrometers*
- *Part 42, addressing the calibration of seismometers*

# Methods for the calibration of vibration and shock transducers —

## Part 15:

## Primary angular vibration calibration by laser interferometry

### 1 Scope

This part of ISO 16063 specifies the instrumentation and procedures used for primary angular vibration calibration of angular transducers, i.e. angular accelerometers, angular velocity transducers and rotational angle transducers (with or without amplifier) to obtain the magnitude and the phase shift of the complex sensitivity by steady-state sinusoidal vibration and laser interferometry. The methods specified in this part of ISO 16063 are applicable to measuring instruments (rotational laser vibrometers in particular) and to angular transducers as defined in ISO 2041 for the quantities of rotational angle, angular velocity and angular acceleration.

It is applicable to a frequency range from 1 Hz to 1,6 kHz and a dynamic range (amplitude) from 0,1 rad/s<sup>2</sup> to 1 000 rad/s<sup>2</sup> (frequency-dependent).

These ranges are covered with the uncertainty of measurement specified in Clause 3. Calibration frequencies lower than 1 Hz (e.g. 0,4 Hz, which is a reference frequency used in other International Standards) and angular acceleration amplitudes smaller than 0,1 rad/s<sup>2</sup> can be achieved using method 3A or method 3B specified in this part of ISO 16063, in conjunction with an appropriate low-frequency angular vibration generator.

Method 1A (cf. Clause 8: fringe-counting, interferometer type A) and method 1B (cf. Clause 8: fringe-counting, interferometer type B) are applicable to the calibration of the magnitude of complex sensitivity in the frequency range of 1 Hz to 800 Hz and under special conditions, at higher frequencies. Method 2A (cf. Clause 9: minimum-point method, interferometer type A) and method 2B (cf. Clause 9: minimum-point method, interferometer type B) can be used for sensitivity magnitude calibration in the frequency range of 800 Hz to 1,6 kHz. Method 3A (cf. Clause 10: sine-approximation method, interferometer type A) and method 3B (cf. Clause 10: sine-approximation method, interferometer type B) can be used for magnitude of sensitivity and phase calibration in the frequency range of 1 Hz to 1,6 kHz. Methods 1A, 1B and 3A, 3B provide for calibrations at fixed angular acceleration amplitudes at various frequencies. Methods 2A and 2B require calibrations at fixed rotational angle amplitudes (angular velocity amplitude and angular acceleration amplitude vary with frequency).

NOTE 1 The numbering 1 to 3 of the methods characterizes the handling of the interferometer output signal(s) analogous to ISO 16063-11: number 1 for fringe counting, number 2 for minimum-point detection and number 3 for sine-approximation. Each of these signal handling procedures can be used together with interferometer types A and B specified in this part of ISO 16063.

Interferometer type A designates a Michelson or Mach-Zehnder interferometer with retro-reflector(s) located at a radius,  $R$ , from the axis of rotation of the angular exciter. This interferometer type is limited to rotational angle amplitudes of 3° maximum. Interferometer type B designates a Michelson or a Mach-Zehnder interferometer using a circular diffraction grating implemented on the lateral surface of the circular measuring table. This interferometer type is not limited as regards the rotational angle amplitude if the diffraction grating covers the whole lateral surface of the disk (i.e. 360°). Usually, the maximum angular vibration is, in this case, limited by the angular vibration exciter.

NOTE 2 Though the calibration methods specified in this part of ISO 16063 are applicable to angular transducers (according to definition in ISO 2041) and, in addition, to measuring instrumentation for angular motion quantities, the specifications are given for transducers as calibration objects, for the sake of simplified description. Some specific information for the calibration of rotational laser vibrometers is given in 4.11 and Figure 11.

## 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 266, *Acoustics — Preferred frequencies*

ISO 2041:1990, *Vibration and shock — Vocabulary*

ISO 16063-1:1998, *Methods for the calibration of vibration and shock transducers — Part 1: Basic concepts*

## 3 Uncertainty of measurement

The limits of the uncertainty of measurement applicable to this part of ISO 16063 shall be as follows:

- a) for the magnitude of sensitivity:
  - 0,5 % of the measured value at reference conditions,
  - $\leq 1$  % of the measured value outside reference conditions;
- b) for the phase shift of sensitivity:
  - 0,5° of the measured value at reference conditions,
  - $\leq 1^\circ$  of the reading outside reference conditions.

Recommended reference conditions are as follows:

- frequency: 160 Hz, 80 Hz, 40 Hz, 16 Hz or 8 Hz (or radian frequency,  $\omega$ : 1 000 rad/s, 500 rad/s, 250 rad/s, 100 rad/s or 50 rad/s);
- angular acceleration: (angular acceleration amplitude or r.m.s. value): 100 rad/s<sup>2</sup>, 50 rad/s<sup>2</sup>, 20 rad/s<sup>2</sup>, 10 rad/s<sup>2</sup>, 5 rad/s<sup>2</sup>, 2 rad/s<sup>2</sup> or 1 rad/s<sup>2</sup>.

Amplifier settings shall be selected for optimum performance with respect to noise, distortion and influence from cut-off frequencies.

The uncertainty of measurement is expressed as the expanded measurement uncertainty in accordance with ISO 16063-1, for the coverage factor  $k = 2$  (referred to, in short, as “uncertainty”).

## 4 Requirements for apparatus

### 4.1 General

Clause 4 gives recommended specifications for the apparatus necessary to comply with the scope of Clause 1 and to obtain the uncertainties of Clause 3.

If desired, systems covering only parts of the ranges may be used, and normally different systems (e.g. exciters) should be used to cover all the frequency and dynamic ranges.

NOTE The apparatus specified in Clause 4 covers all devices and instruments required for any of the six calibration methods described in this part of ISO 16063. The assignment to a particular method is indicated (cf. Figures 2, 3, 4, 5, 6, 7, 8 and 10).

## 4.2 Frequency generator and indicator

A frequency generator and indicator having the following characteristics shall be used:

- a) uncertainty of frequency: maximum 0,05 % of reading;
- b) frequency stability: better than  $\pm 0,05$  % of reading over the measurement time;
- c) amplitude stability: better than  $\pm 0,05$  % of reading over the measurement time.

## 4.3 Power amplifier/angular vibration exciter combination

### 4.3.1 General

A power amplifier/angular vibration exciter combination having the following characteristics shall be used:

- a) total harmonic distortion: 2 % maximum;

NOTE 1 This specification relates to the input quantity for the transducer to be calibrated.

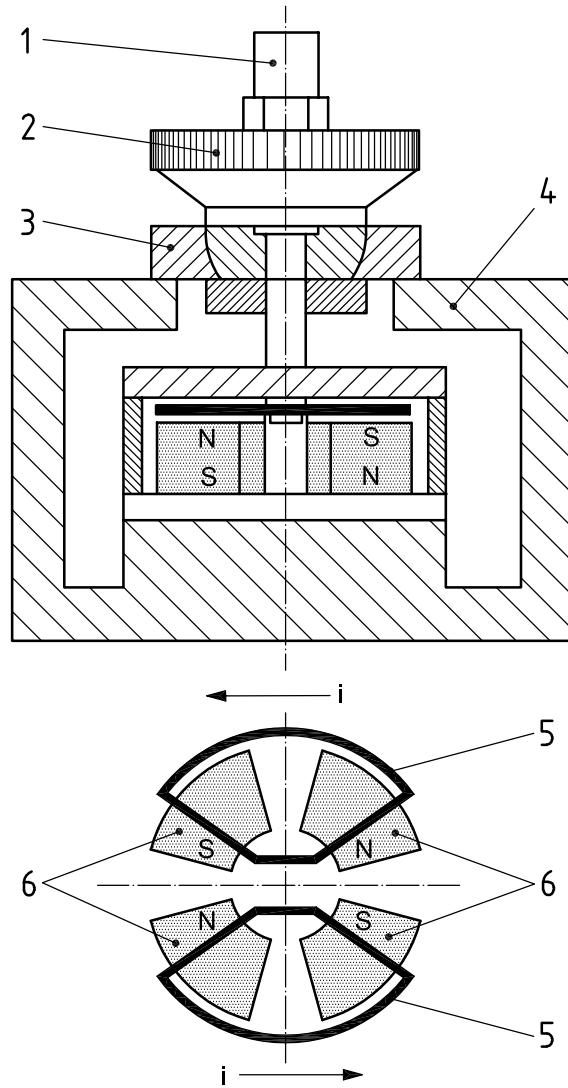
NOTE 2 If method 3A or method 3B is used, greater harmonic distortions can be tolerable.

- b) transverse, and rocking angular acceleration: sufficiently small to prevent excessive effects on the calibration results. For interferometer type A, a transverse motion of less than 1 % of the tangential motion component at the minimum rotational angle displacement can be required. For interferometer type B, a maximum lateral motion (including eccentricity) of 2  $\mu\text{m}$  is tolerated, which can be achieved only if the moving part (measuring table) of the angular exciter is carried in a high-precision rotational air bearing;
- c) hum and noise: 70 dB minimum below full output;
- d) stability of angular acceleration amplitude: better than  $\pm 0,05$  % of reading over the measurement period.

### 4.3.2 Electro-dynamic angular vibration exciter

An electrodynamic vibration exciter is based on the Lorentz force acting on electric charge carriers when these move through a magnetic field.

In analogy to common electrodynamic vibration exciters designed to generate rectilinear vibration, the coil located in the magnetized air gap of a magnetic circuit can be so designed that the Lorentz force generates a dynamic torque exciting the measuring table with the angular transducer to be calibrated to angular vibration. In the working frequency range (i.e. 1 Hz to 1,6 kHz), the amplitude of angular acceleration is proportional to the amplitude of the electric current carried through the coil. An example of an angular vibration exciter is shown in Figure 1. The maximum rotational amplitude is in this case limited to 30° (i.e. double amplitude: 1 rad). Another example of an angular acceleration exciter (amplitude of 60°, i.e. 1 rad) is described in Reference [14].



- Key**
- 1 angular accelerometer
  - 2 diffraction grating
  - 3 air bearing
  - 4 housing
  - 5 coil
  - 6 magnet

**Figure 1 — Example of an angular exciter (mode of function)**

#### 4.3.3 Angular vibration exciter based on a brushless electric motor

Special angular exciters have been designed and manufactured for angular transducer calibration using commercial electric motors.

For the testing of inertial navigation sensors, so-called “rate tables” have been developed for many years. These are often equipped with brushless, three-phase, hollow-shaft motors that are electronically commutated and servo-controlled, in particular for the angular velocity, i.e. angular rate operating mode. Normally, a constant angular velocity is generated. Often, sinusoidal angular velocities with low distortion are achieved.



The progress in control made over the last few years allows this exciter type to be used even to generate angular acceleration. A basic requirement is the use of an air bearing as in the flat-coil exciter (cf. 4.3.2).

As the distortion increases after differentiation, the calibration of angular accelerometers can require a frequency-selective measurement of the transducer output signal, which is ensured by the use of method 3A or 3B (i.e. sine-approximation).

#### 4.4 Seismic block(s) for vibration exciter and laser interferometer

The angular vibration exciter and the interferometer shall be mounted on the same heavy block or on two different heavy blocks so as to prevent relative motion due to ground motion, or to prevent the reaction of the vibration exciter's support structure from excessively influencing the calibration results.

When a common seismic block is used, this should have a moment of inertia at least 2 000 times that of the moving mass. This causes less than 0,05 % reactive angular vibration of angular transducer and interferometer. If the moment of inertia of the seismic block is smaller, its motion generated by the vibrator shall be taken into account.

To suppress disturbing effects of ground motion, the seismic block(s) used in the frequency range of 1 Hz to 1,6 kHz should be suspended on damped springs designed to reduce the uncertainty component due to these effects to less than 0,1 %.

#### 4.5 Laser

A laser of the red helium-neon type or a single-frequency laser with another wavelength of known value shall be used. Under laboratory conditions (i.e. at an atmospheric pressure of 100 kPa, a temperature to 23 °C and a relative humidity of 50 %), the wavelength of a red helium-neon laser is 0,632 81 µm.

If the laser is provided with a manual or automatic atmospheric compensation device, this shall be set to zero or switched off.

#### 4.6 Interferometer

##### 4.6.1 General

The interferometer may be used to transform

- the rotational angle,  $\Phi(t)$ , into a proportional phase shift,  $\varphi_M(t)$ , of the interferometer output signal,
- the angular velocity,  $\Omega(t)$ , into a proportional frequency shift,  $f_D(t)$  (Doppler frequency), of the interferometer output signal.

For both transformations, a homodyne or a heterodyne interferometer (cf. Figures 3 to 8 and 10) and a one-channel or two-channel arrangement (cf. Figures 3 to 8 and 10) may be used.

The first transformation of  $\Phi(t)$  into  $\varphi_M(t)$  is specified in this part of ISO 16063 as a standard procedure whereas the latter transformation of  $\Omega(t)$  into  $f_D(t)$  is given as an option with reference to detailed descriptions in the literature.

The interferometer types A and B basically have in common that the measuring beam senses a translational displacement motion component so that an interferometer arrangement designed for rectilinear vibration measurements can be used. To make the application of such conventional interferometers possible, the quantity of rotational motion to be measured is converted into a representative translational displacement motion component using retro-reflector(s) as measuring reflector(s) for interferometer type A, and a diffraction grating arranged on the rotary measuring table for interferometer type B. In the latter case, an optically reflecting diffraction grating is to be arranged on the lateral surface of an air-borne rotary table to meet the requirement of the tolerable eccentricity of 2 µm.

For methods 1A, 1B (see Figures 3 and 4) and Methods 2A, 2B (see Figures 5 and 6), a common Michelson interferometer with a single light detector is sufficient.

The Michelson interferometer can be realized with a single measuring beam or with two measuring beams.

For methods 3A, 3B, (see Figures 7 and 8), a modified Michelson interferometer with quadrature signal outputs, with two light detectors for sensing the interferometer signal beams, shall be used. The modified Michelson interferometer may be designed according to Figure 9. A quarter wavelength retarder converts the incident, linearly polarized light into two measuring beams with perpendicular polarization states and a phase shift of 90°. After interfering with the linearly polarized reference beam, the two components with perpendicular polarization shall be separated in space using appropriate optics (e.g. a Wollaston prism or a polarizing beam splitter), and detected by two photodiodes.

The two outputs of the modified Michelson interferometer shall have offsets of less than ± 5 % in relation to the amplitude, relative amplitude deviations of less than ± 5 % and deviations of less than ± 5° from the nominal angle of 90°. To comply with these tolerances, appropriate means shall be provided to adjust the offset, the signal level and the angle between the two interferometer signals.

At large rotational angles, it can be difficult to maintain the tolerances stated above for the deviations of the two outputs of the modified Michelson interferometer. To comply with the uncertainty of measurement of Clause 3, the above tolerances shall be complied with at least for small rotational angles of up to  $2 \times 10^{-2}$  rad. For greater amplitudes, greater tolerances are permitted.

**EXAMPLE** For a rotational angle of  $2,5 \times 10^{-2}$  rad (i.e. angular acceleration amplitude of 1 rad/s<sup>2</sup> at a frequency of 1 Hz), the tolerances can be extended to ± 10 % for the offsets and for the relative amplitude deviations, and to ± 20° for the deviation from the nominal angle of 90° (see also NOTE 1 of 10.2).

The tolerances stated above are valid without correction of quadrature fringe measurement errors in interferometer. If the correction procedure after Heydemann<sup>[6]</sup> is applied, greater tolerances are permitted.

For methods 1A, 1B, 2A, 2B, 3A or 3B, another suitable interferometer, e.g. a (modified) Mach-Zehnder heterodyne interferometer (cf. Figure 10) may be used in the place of the (modified) Michelson interferometer.

An interferometer of type A (cf. 4.6.2) or B (cf. 4.6.3) shall be used with a light detector to sense the interferometer signal bands and with a frequency response covering the bandwidth necessary. The maximum bandwidth (frequency  $f_{\max}$ ) needed can be calculated from the maximum angular velocity amplitude,  $\Omega_{\max}$  using Equation (1):

$$f_{\max} = \frac{\Omega_{\max} R}{\Delta s} \quad (1)$$

where

$R$  is the effective radius (cf. 4.6.2 for the definition for interferometer of type A and 4.6.3, for interferometer of type B);

$\Delta s$  is the displacement quantization interval of the interferometer.

For interferometer type A,  $\Delta s = \lambda/2$  in the single measuring beam arrangement and  $\Delta s = \lambda/4$  in the two-beam arrangement with the laser wavelength,  $\lambda$ . For interferometer type B,  $\Delta s = g$  in the single measuring beam arrangement and  $\Delta s = g/2$  in the two-beam arrangement with the grating constant,  $g$ .

#### 4.6.2 Interferometer type A (retro-reflector interferometer)

For methods 1A and 2A, an interferometer of the Michelson type with retro-reflector(s) as measuring reflector(s) shall be used with a light detector for sensing the interferometer signal bands and a frequency response covering the necessary bandwidth (cf. 4.6.1). To compensate the influence of the disturbing motion, a two-beam arrangement (for an example, cf. Figures 3 and 5) shall be used with two retro-reflectors mounted symmetrically (i.e. shifted by 180°) at a distance,  $R$ , from the axis of rotation.

The laser beam emitted by the laser passes to a beam splitter which splits up the beam into two components that are fed in parallel to the retro-reflectors. The reflected beams are superimposed on each other and the relevant part of the resulting light intensity is transformed by the photodetector into an electrical signal (briefly referred to as interferometer signal).

**NOTE** The two-beam arrangement leads not only to compensation of the disturbing motion (e.g. from ground vibration) but also to doubling of the sensitivity (quantization interval of  $\lambda/4$  instead of  $\lambda/2$ ). The retro-reflectors (instead of plane mirrors) compensate (in a certain range, cf. Appendix B) for the tilting effect of the rotational motion. Moreover, the interferometer accommodates (in a certain range) disturbing motion in the transverse direction without the uncertainty of measurement being affected.

For method 3A, a quadrature interferometer with retro-reflectors, measuring and reference reflectors shall be used. In the homodyne interferometer version shown in Figures 7 and 9, the light source is a stabilized single-frequency laser. The diameter of the laser beam is expanded by lenses to reduce the divergence of the beam. The polarized laser beam is split by the beam splitter into a measuring beam and a reference beam. The reference beam is reflected and shifted in parallel by a retro-reflector (reference reflector). As the  $\lambda/8$  retardation waveplate is traversed twice, a path difference of  $\lambda/4$  is obtained. At the same time, the reflected laser beam is split into two beams, each with a direction of polarization orthogonal to the other, that show a phase shift of  $90^\circ$  (i.e. circular polarization). The measuring beam is also shifted in parallel when reflected by the retro-reflector mounted on the measuring table, retaining its linear polarization. The linearly polarized reflected measuring beam and the circularly polarized reference beam are superimposed. When passing the Wollaston prism, which is inclined by  $45^\circ$  with reference to the direction of polarization of the reflected measuring beam, two linearly polarized beam components are obtained whose directions of polarization are perpendicular to each other. After separation of the two components in space, two different interference systems are derived having a phase shift of  $90^\circ$  with respect to each other. The two photodetectors transform the relevant parts of the light intensities into electrical signals that show a sinusoidal and a cosinusoidal dependence on the displacement of the measuring reflector.

#### 4.6.3 Interferometer type B (diffraction-grating interferometer)

An interferometer with a diffraction grating as measuring reflector shall be used (e.g. a Michelson interferometer) with a light detector for sensing the interferometer signal bands and with a frequency response covering the necessary bandwidth (cf. 4.6.1).

For methods 1B and 2B, a modified Michelson interferometer with diffraction grating is used (cf. Figures 2, 4 and 6).

The angular acceleration, the angular velocity or the rotational angle are measured by a special diffraction grating interferometer developed on the basis of a high-resolution grating (e.g. a sine-phase grating of 2 400 grooves/mm or 3 000 grooves/mm, manufactured by holography) (examples are described in References [12] and [13]). An optical reflection grating is located on the air-borne measuring table of the angular vibration exciter, concentrically to the axis of rotation (cf. Figure 2). The light beam emitted by a frequency-stabilized single-frequency He-Ne laser is split into two parallel beams striking the grating symmetrical to the axis of rotation at the angle at which the first-order beams diffracted by reflection (according to the diffraction formula for oblique incidence) return into the direction of the incident beam. The first-order diffracted light beams are superposed in the optical arrangement. When the moving part is rotated, these light beams undergo a frequency change opposite in sign and of the same amount that is proportional to the tangential velocity and, thus, also to the angular velocity. The interfering beams give rise to a light intensity whose significant component shows a periodic dependence on the rotational angle.

For method 3B, a homodyne quadrature interferometer with diffraction grating is used (cf. Figure 8).

In the quadrature diffraction-grating interferometer in the single measuring beam arrangement, the light beam is split into the reference beam and the measuring beam. The measuring beam strikes the grating at the angle at which the first-order beam diffracted by reflection returns into the direction of the incident beam. The first-order beam diffracted in accordance with the diffraction equation for oblique incidence and the reference beam are superimposed in the optical arrangement. The interfering light beams yield a light intensity whose significant component depends sinusoidally on the rotational angle.

For high-accuracy requirements (relative uncertainty of calibration smaller than 0,5 %), a grating interferometer calibration procedure shall be carried out once for a measuring table with an individual diffraction grating disk, to accurately determine the quantization intervals of the displacement,  $\Delta s$ , and of the rotational angle,  $\Delta\phi$  (cf. procedure in Clause 7).

## 4.7 Instrumentation for interferometer signal processing

### 4.7.1 General

The instrumentation used has in common that the phase-modulated electric current or voltage at the output(s) of the photodetector(s) is demodulated to extract the vibration parameter(s) of interest (e.g. amplitude and initial phase of the sinusoidal rotational angle). Different techniques are to be used for methods 1A, 1B (cf. 4.7.2), methods 2A, 2B (cf. 4.7.3) and methods 3A, 3B (cf. 4.7.4).

### 4.7.2 Instrumentation for fringe counting (for methods 1A and 1B)

The counting instrumentation shall have the following characteristics:

- a) frequency range: 1 Hz to the maximum frequency needed (20 MHz is typically used);
- b) maximum uncertainty: 0,01 % of reading.

The counter may be replaced by a ratio counter offering the same uncertainty.

### 4.7.3 Instrumentation for zero-point detection (for methods 2A and 2B)

A tunable bandpass filter or spectrum analyser with the following characteristics shall be used:

- a) frequency range:  $\leq 800$  Hz to  $\geq 1,6$  kHz;
- b) bandwidth:  $< 12$  % of centre frequency;
- c) filter slopes: equal to or greater than 24 dB per octave;
- d) signal-to-noise ratio: greater than 70 dB below maximum signal;
- e) dynamic range: greater than 60 dB.

Instrumentation for zero detection (not needed with spectrum analyser), with a frequency range from 800 Hz to 1,6 kHz shall be used. The range shall be sufficient for detecting output noise from the bandpass filter.

### 4.7.4 Instrumentation for sine-approximation (for methods 3A and 3B)

A waveform recorder with a computer interface capable of analog-to-digital conversion and storage of the two interferometer quadrature outputs and the accelerometer output shall be used. The amplitude resolution, the sampling rate and the memory shall be sufficient for calibration in the intended amplitude range with the uncertainty specified in Clause 3. Typically, an amplitude resolution of  $\geq 10$  bits is used for the accelerometer output. For the quadrature signal outputs of the interferometer, a resolution of  $\geq 8$  bits is sufficient. A two-channel waveform recorder may be used for the interferometer output signals, and another waveform recorder (with higher resolution and lower sampling rate) for the angular transducer output signal. In each case, conversion of the data from the interferometer and the angular transducer output signals shall begin and end at the same time, with an uncertainty that meets the uncertainty requirements of Clause 3.

A sufficient number of samples (cf. 10.3) are required of the shortest period of the interferometer output signal occurring at maximum velocity. For a particular angular acceleration amplitude, at decreasing frequencies, larger displacement amplitudes occur that require higher sampling rates and larger memories. If such capabilities are not available, the angular acceleration amplitude shall be reduced.

To calibrate an angular accelerometer at a vibration frequency of 10 Hz and an angular acceleration amplitude of  $1\,000\text{ rad/s}^2$ , a memory of  $\geq 4$  Mbytes should be used if a sampling frequency of  $\geq 20$  kHz is applied.

A computer with data-processing program (for methods 3A and 3B) in accordance with the procedure for the calculations stated in 10.4 shall be used.

#### 4.8 Voltage instrumentation, measuring true r.m.s. accelerometer output

Voltage instrumentation, measuring true r.m.s. accelerometer output, having the following characteristics shall be used:

- a) frequency range:  $\leq 1$  Hz to  $\geq 1,6$  kHz;
- b) maximum uncertainty: 0,1 % of reading.

The r.m.s. value shall be multiplied by a factor of  $\sqrt{2}$  to obtain the (single) amplitude.

For methods 1A, 1B, 2A and 2B, an r.m.s. voltmeter shall be used. For methods 3A and 3B, special voltage-measuring instrumentation in accordance with 4.7.4 shall be used; an r.m.s. voltmeter may be applied in addition (optional).

#### 4.9 Distortion-measuring instrumentation

Distortion-measuring instrumentation capable of measuring the total harmonic distortion of  $< 1$  % to 5 % and with the following characteristics shall be used.

- a) frequency range:  $\leq 1$  Hz to  $\geq 1,6$  kHz, with the capability of measuring up to the fifth harmonic;
- b) maximum uncertainty: 10 % of reading in the distortion range of 0,5 % to 5 %.

#### 4.10 Oscilloscope (optional)

An oscilloscope for optimizing the interferometer and for checking the waveform of the interferometer and accelerometer signals, with a frequency range from 1 Hz to 2 MHz minimum, may be used.

#### 4.11 Other requirements

The transducer to be calibrated shall be structurally rigid. The base strain sensitivity, the transverse sensitivity and the stability of the angular accelerometer/amplifier combination (if calibrated as a single unit) shall be taken into account in the calculation of the uncertainty of measurement (cf. Annex A).

All effects influencing the measurement result shall be included in the uncertainty calculation.

Methods 1B, 2B and 3B can be applied to calibrate rotational laser vibrometers if the motion parameter is sensed simultaneously by the standard device (cf. 4.6 and 4.7) and by the laser interferometer being calibrated. If the motion sensing periods of both measurement systems are different, the rotational vibration amplitude shall be sufficiently stable to meet the uncertainty requirement of Clause 3. An example of an arrangement for the calibration of rotational laser interferometers is shown in Figure 11.

### 5 Ambient conditions

The calibration shall be carried out under the following ambient conditions:

- a) room temperature:  $(23 \pm 3)$  °C;
- b) relative humidity: 75 % max.

Care should be taken that external vibration and noise do not affect the quality of the measurements.

## 6 Preferred angular accelerations and frequencies

The angular accelerations (amplitude or r.m.s. value) and frequencies equally covering the angular accelerometer range should preferably be chosen from the following series;

a) angular acceleration (methods 1A, 1B, 3A and 3B):

— 0,1 rad/s<sup>2</sup>, 0,2 rad/s<sup>2</sup>, 0,5 rad/s<sup>2</sup>, 1 rad/s<sup>2</sup>, 2 rad/s<sup>2</sup>, 5 rad/s<sup>2</sup>, 10 rad/s<sup>2</sup>, 20 rad/s<sup>2</sup>, 50 rad/s<sup>2</sup>, 100 rad/s<sup>2</sup>, 200 rad/s<sup>2</sup>, 500 rad/s<sup>2</sup>, 1 000 rad/s<sup>2</sup> (1 000 rad/s<sup>2</sup> is valid for amplitude only);

b) frequency:

— selected from the standardized one-third-octave frequency series (in accordance with ISO 266) between 1 Hz and 1,6 kHz or the series of radian(s) frequencies evolving from  $\omega = 1\,000$  rad/s.

## 7 Common procedure for all six methods

Methods A1, B1, A2, B2, A3 and B3 have in common that the interferometer (type A or B) senses a displacement at a point situated at a distance,  $R$ , the “effective radius”, from the axis of rotation of the circular measuring table of the angular vibration exciter. From the displacement amplitude,  $\hat{s}$  sensed by the interferometer, the amplitude of the rotational angle,  $\hat{\phi}$ , is obtained using Equation (2):

$$\hat{\phi} = \frac{\hat{s}}{R} \quad (2)$$

where  $R$  is the effective radius whose value shall be determined from a special interferometer calibration carried out once before the interferometer can be used for transducer calibrations. In all cases, the measurement of  $\hat{s}$  is based on the comparison with an accurately known value of a very small length in the sub-micrometer range. For interferometer type A, this is the wavelength  $\lambda = 0,632\,81\ \mu\text{m}$  of the laser of the red helium-neon type, which is known *a priori*. In interferometer type B, this is the grating constant,  $g$  (groove length, i.e. grating constant of  $0,333\,33\ \mu\text{m}$  of a sine-phase diffraction grating having 3 000 grooves/mm). The measure,  $g$ , shall be accurately known (from length measurements carried out by the manufacturer of the diffraction grating).

NOTE If the grating constant is not known with sufficient accuracy, the angular quantization interval  $\Delta\phi$ , which corresponds to one interferometer signal period, can be determined by a special diffraction grating interferometer calibration<sup>[13]</sup>. Then, the expression  $\Delta s = R \Delta\phi$  with the displacement quantization interval  $\Delta s$  (e.g.  $\Delta s = g$  in a single-beam arrangement of interferometer type B, cf. Figure 8) can be used to eliminate the radius,  $R$ ; cf. Equation (2), which is no longer used to calculate  $\hat{\phi}$ .

All six methods apply the result for the rotational angle amplitude,  $\hat{\phi}$ , obtained from Equation (2) to calculate the following:

a) sensitivity (magnitude),  $S_{\phi}$ , of rotational angle transducers, using Equation (3):

$$S_{\phi} = \frac{\hat{u}}{\hat{\phi}} \quad (3)$$

b) sensitivity (magnitude),  $S_{\Omega}$ , of angular velocity transducers, using Equations (4) and (5):

$$S_{\Omega} = \frac{\hat{u}}{\hat{\Omega}} \quad (4)$$

where

$$\hat{\Omega} = 2\pi \times f \hat{\phi} \quad (5)$$

c) sensitivity (magnitude),  $S_\alpha$ , of angular accelerometers, using Equations (6) and (7):

$$S_\alpha = \frac{\hat{u}}{\hat{\alpha}} \quad (6)$$

where

$$\hat{\alpha} = 4\pi^2 \times f^2 \hat{\phi} \quad (7)$$

where

$\hat{u}$  is the amplitude of the angular transducer output,  $u$ , (e.g. output voltage of an angular accelerometer);

$\hat{\Omega}$  is the amplitude of the angular velocity,  $\Omega$ ;

$\hat{\alpha}$  is the amplitude of the angular acceleration,  $\alpha$ .

As the phase shift of the complex sensitivity of angular transducers can be measured only by methods 3A and 3B, the common procedures used for phase shift calibrations are specified in Clause 10.

## 8 Methods using fringe-counting (methods 1A and 1B)

### 8.1 General

This method is applicable to sensitivity magnitude calibration the frequency range from 1 Hz to 800 Hz.

**NOTE** At the frequency of 800 Hz and an angular acceleration amplitude of 1 000 rad/s<sup>2</sup>, the rotational angle amplitude is  $4 \times 10^{-5}$  rad. This corresponds to a displacement amplitude of 2  $\mu$ m if a retro-reflector or a diffraction grating is arranged in a distance of 50 mm from the axis of rotation (i.e. a diffraction grating at the lateral surface of a disk 100 mm in diameter). Using the fringe-counting method without special means to suppress the quantization error (see References [1] and [11]), displacement amplitudes down to 2  $\mu$ m can be measured with an uncertainty specified in Clause 3. Methods 1A and 1B can also be applied at smaller amplitudes if the quantization error is suppressed [1], [11]. This allows calibration at a specified angular acceleration amplitude (e.g. 1 000 rad/s<sup>2</sup>) to be performed at higher frequencies.

In both interferometer types A and B (i.e. in methods 1A and 1B), the number of signal periods (e.g. intensity maxima),  $N$ , is given by Equation (7):

$$N = 4\hat{s} / \Delta s \quad (8)$$

resulting in Equation (9)

$$\hat{s} = \frac{\Delta s}{4} \times \frac{f_f}{f} \quad (9)$$

where

$\hat{s}$  is the displacement amplitude sensed by the laser interferometer, required to apply Equations (2) to (7);

$\Delta s$  is the quantization interval, equal to  $\lambda/2$ , specified by Equations (10) and (11) for the two versions of interferometer type A and by Equations (12) and (13) for the two versions of interferometer type B;

$f$  is the frequency of the angular vibration exciter;

$f_f$  is the (mean) fringe frequency.

Inserting the relevant expression for  $\Delta s$ , i.e. Equations (10) or (11) for the type A interferometer and Equations (12) or (13) for the type B interferometer and using Equation (2) to transform the displacement into a rotational angle, the rotational angle amplitude,  $\hat{\phi}$ , is obtained. The angular velocity amplitude,  $\hat{\Omega}$ , and the angular acceleration amplitude,  $\hat{\alpha}$ , are calculated from Equations (5) and (7), respectively. To calculate the sensitivity of an angular transducer, Equations (3), (4) and (6) are applied.

## 8.2 Common test procedure for methods 1A and 1B

After optimizing the interferometer (see 4.6), determine the sensitivity of the transducer to be calibrated at the angular vibration frequencies and angular acceleration amplitudes demanded (see Clause 6) by measuring either the fringe frequency with the counter (see 4.7.2) (fringe counting in accordance with Figure 3 and Figure 4 may be used) or the ratio between the angular vibration frequency and the fringe frequency with a ratio counter (cf. 4.7.2).

## 8.3 Expression of results

See also B.2.

### 8.3.1 Method 1A (retro-reflector interferometer)

Calculate the displacement amplitude from the fringe frequency readings calculated in Equation (9) using Equation (10) for the two-beam arrangement shown in Figure 3:

$$\Delta s = \lambda/4 \tag{10}$$

or Equation (11) for the single measuring beam arrangement shown in Figure 7:

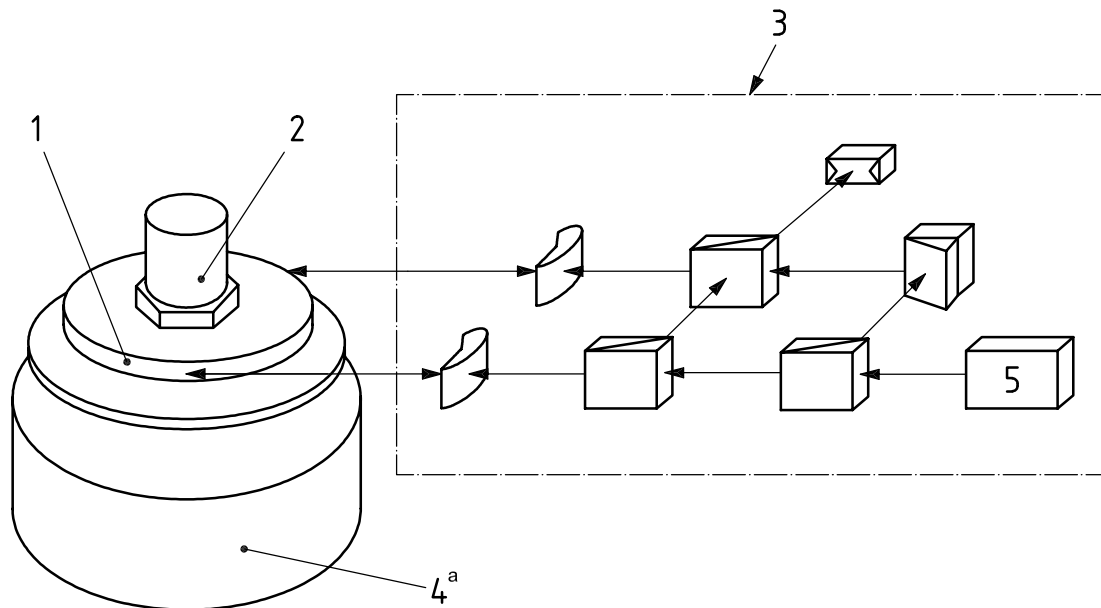
$$\Delta s = \lambda/2 \tag{11}$$

where  $\lambda$  is the wavelength  $\lambda = 0,632\ 81\ \mu\text{m}$  of the laser of the red helium-neon type.

Calculate the amplitude,  $\hat{\phi}$ , of the rotational angle,  $\phi$ , using Equation (2); calculate the sensitivity,  $S_\phi$  (magnitude), of the rotational angle transducer from Equation (3); the sensitivity,  $S_\Omega$  (magnitude), of the angular velocity transducer from Equations (4) and (5); and the sensitivity,  $S_\alpha$  (magnitude), of the angular accelerometer using Equations (6) and (7).



### 8.3.2 Method 1B (diffraction-grating interferometer)



#### Key

- 1 diffraction grating
- 2 calibration object
- 3 interferometer
- 4 angular acceleration exciter

<sup>a</sup> See also Figure 1.

**Figure 2 — Angular acceleration exciter in conjunction with a diffraction-grating interferometer (homodyne version)**

Calculate the displacement amplitude from the fringe frequency readings calculated in Equation (9) using Equation (12) for the two-beam arrangement shown in Figure 4:

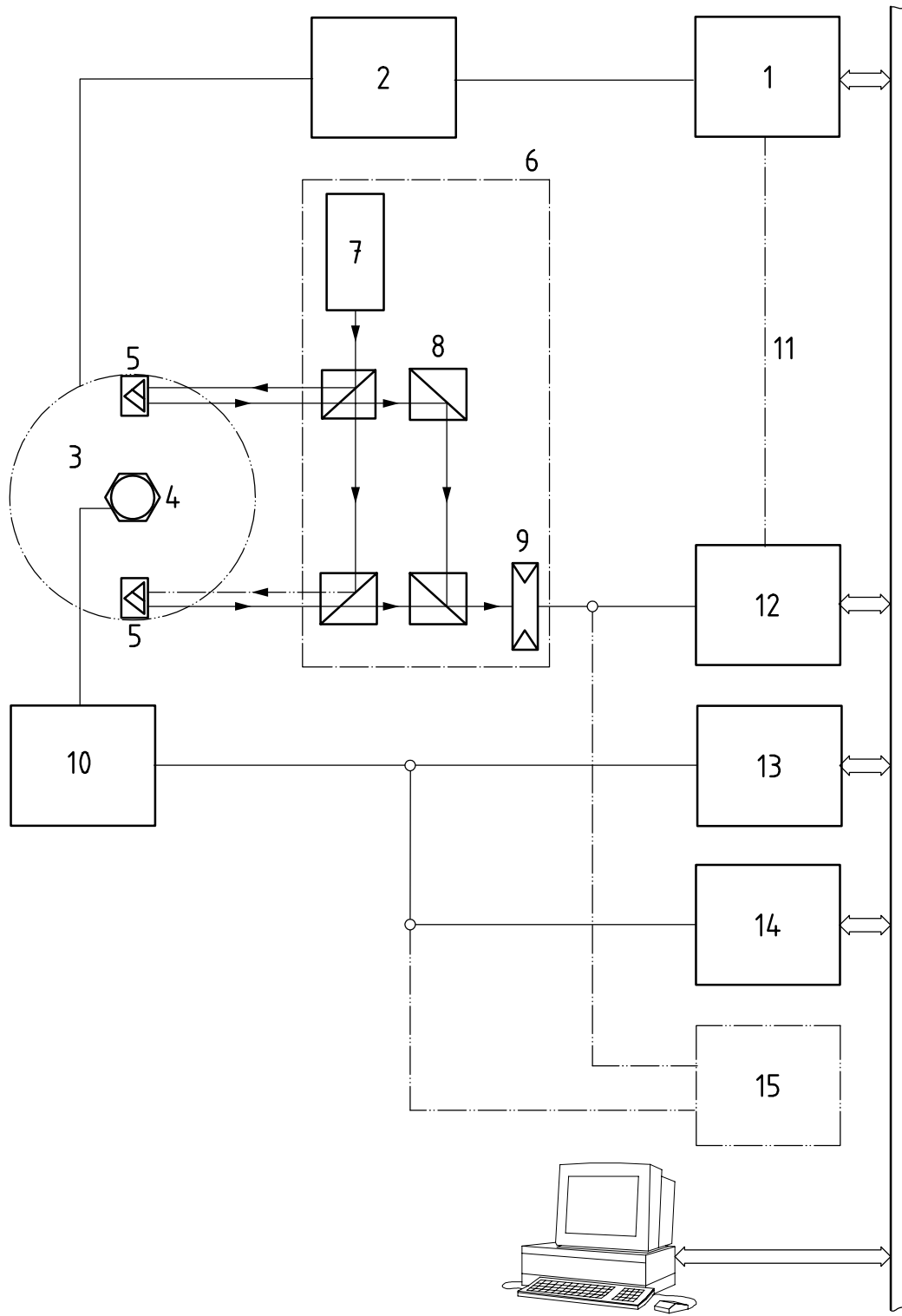
$$\Delta s = g/2 \quad (12)$$

or Equation (13) for the single-beam arrangement shown in Figure 8:

$$\Delta s = g \quad (13)$$

where  $g$  is the grating constant of the diffraction grating (groove length, i.e. grating constant of 0,333 33  $\mu\text{m}$  of a sine-phase diffraction grating having 3 000 grooves/mm).

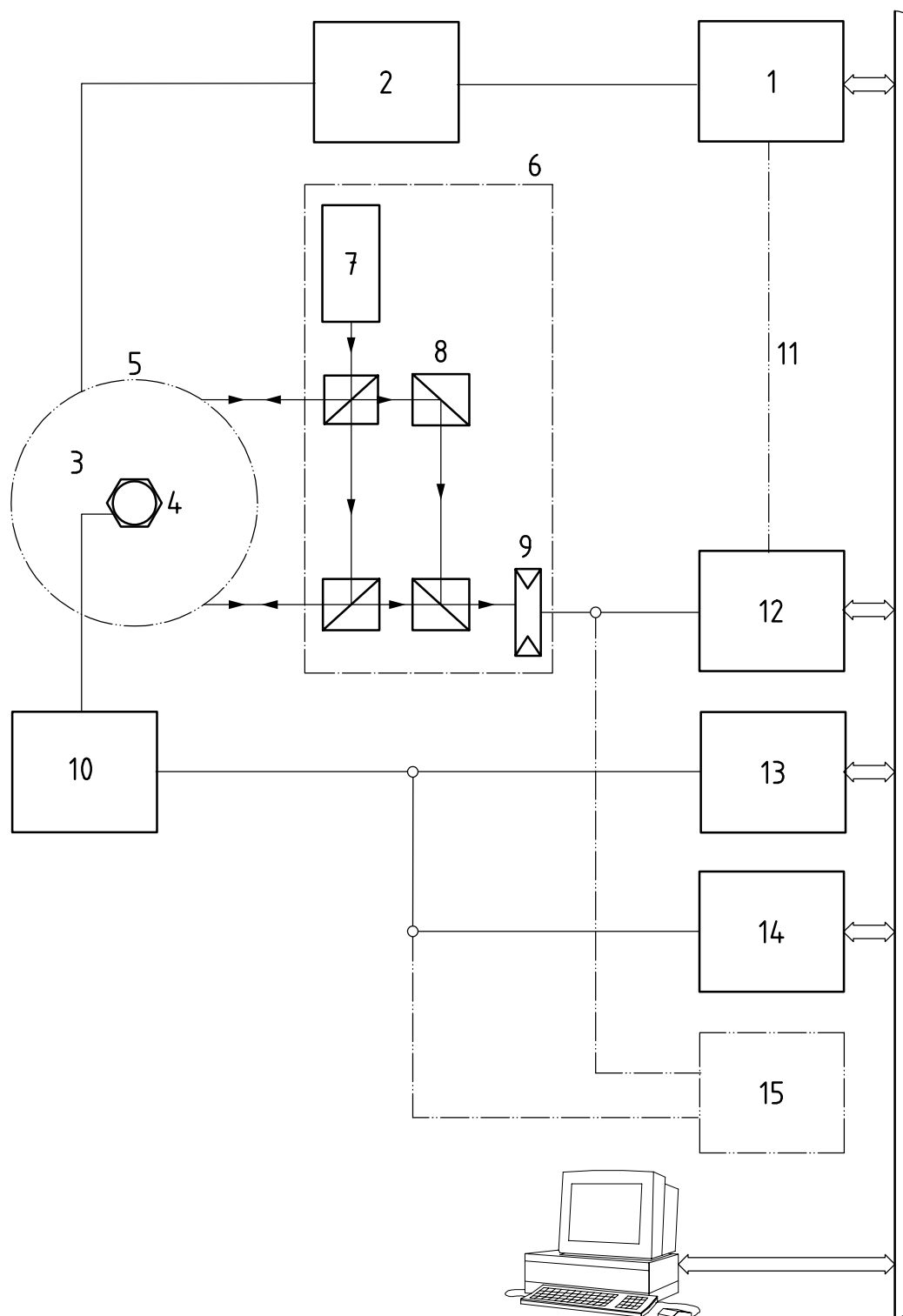
Calculate the amplitude,  $\hat{\phi}$ , of the rotational angle,  $\phi$ , using Equation (2); calculate the sensitivity,  $S_{\phi}$  (magnitude), of the rotational angle transducer from Equation (3); the sensitivity,  $S_{\Omega}$  (magnitude), of the angular velocity transducer from Equations (4) and (5); and the sensitivity,  $S_{\alpha}$  (magnitude), of the angular accelerometer using Equations (6) with (7).



**Key**

- |                             |                        |                                       |
|-----------------------------|------------------------|---------------------------------------|
| 1 frequency generator (4.2) | 6 interferometer (4.6) | 11 with ratio counter (4.7.2)         |
| 2 power amplifier (4.3)     | 7 laser (4.5)          | 12 counter (or ratio counter) (4.7.2) |
| 3 angular exciter (4.3)     | 8 beam splitter        | 13 voltmeter (4.8)                    |
| 4 angular transducer        | 9 light detector       | 14 distortion meter (4.9)             |
| 5 retro-reflector           | 10 amplifier           | 15 oscilloscope (4.10)                |

**Figure 3 — Example of a measuring system for method 1A (retro-reflector interferometer, fringe counting)**



**Key**

- |                             |                        |                                       |
|-----------------------------|------------------------|---------------------------------------|
| 1 frequency generator (4.2) | 6 interferometer (4.6) | 11 with ratio counter (4.7.2)         |
| 2 power amplifier (4.3)     | 7 laser (4.5)          | 12 counter (or ratio counter) (4.7.2) |
| 3 angular exciter (4.3)     | 8 beam splitter        | 13 voltmeter (4.8)                    |
| 4 angular transducer        | 9 light detector       | 14 distortion meter (4.9)             |
| 5 retro-reflector           | 10 amplifier           | 15 oscilloscope (4.10)                |

**Figure 4 — Example of a measuring system for method 1B (diffraction-grating interferometer, fringe counting)**

## 9 Methods using minimum-point detection (methods 2A and 2B)

### 9.1 General

This method is used for the calibration of the magnitude of the sensitivity in the frequency range from 800 Hz to 1,6 kHz.

The method illustrated in this section is based on a determination of the displacement using the arguments corresponding to the zero crossings of the Bessel function of first kind and first order.

NOTE An equally valid approach is to determine the displacement using the arguments corresponding to the zero crossings of the Bessel function of first kind and zero order. However, this technique requires modulating of the position of the reference mirror to be implemented (see Reference [2]).

Considering the frequency spectrum of the intensity and adjusting the angular vibration amplitude to such a level that the component of the same frequency as the vibration frequency is zero, it is possible to calculate the displacement amplitude sensed by the interferometer using Equation (14):

$$\hat{s} = \hat{s}_n = x_n \times \frac{\Delta s}{2\pi} \tag{14}$$

where

$\hat{s}$  is the displacement amplitude sensed by the laser interferometer;

$x_n$  are the arguments corresponding to the zero points of the Bessel function, as given in Table 1;

$\Delta s$  is the quantization interval;

as specified by Equations (10) and (11) for the two versions of interferometer type A and by Equations (12) and (13) for the two versions of interferometer type B. To obtain the relevant values of the rotational angle amplitude, Equation (2) is to be used. The angular velocity amplitude,  $\hat{\Omega}$ , and the angular acceleration amplitude,  $\hat{\alpha}$ , are calculated from Equations (5) and (7), respectively. The calculation of the sensitivity of the angular transducer is made using Equations (3), (4) and (6).

**Table 1 — Values for the arguments,  $x_n$ , corresponding to the zero points of the Bessel function of first kind and first order**

Zero point No. <i>n</i>	$x_n$	Zero point No. <i>n</i>	$x_n$
0	0	16	51,043 53
1	3,831 70	17	54,185 56
2	7,015 59	18	57,327 53
3	10,173 46	19	60,469 45
4	13,323 69	20	63,611 36
5	16,470 63	21	66,753 23
6	19,615 86	22	69,895 07
7	22,760 09	23	73,036 90
8	25,903 68	24	76,178 70
9	29,046 83	25	79,320 49
10	32,189 68	26	82,462 27
11	35,332 30	27	85,604 02
12	38,474 77	28	88,745 77
13	41,617 09	29	91,887 52
14	44,759 32	30	95,029 24
15	47,901 46		

## 9.2 Common test procedure for methods 2A and 2B

After optimizing the interferometer (see 4.6), determine the sensitivity of the transducer being calibrated at the angular vibration frequencies and angular acceleration amplitudes demanded (see Clause 6). Filter the signal from the light detector (4.6) through a bandpass filter (4.7.3) with the centre frequency equal to the frequency of the angular vibrator. This filtered signal has a number of minimum points at special vibration amplitudes (i.e. displacement amplitudes,  $\hat{s}_n$ , and rotational angle amplitudes,  $\hat{\phi}_n$ ) corresponding to the arguments,  $x_n$ , specified in Table 1.

Set the calibration frequency and adjust the vibrator amplitude from zero to a value at which the filtered light detector signal after reaching a maximum value, returns to a minimum value. This minimum value is minimum point No. 1 at which the displacement amplitude has a defined value depending on the type and configuration of the interferometer (cf. 9.3).

The measuring system for the minimum-point method is shown in Figures 5 and 6.

NOTE 1 The sensitivity of an accelerometer can also be determined using the Bessel function of first kind and zero order by modulating the position of the reference mirror at a frequency that is small relative to the frequency of calibration, and setting the centre frequency of the bandpass filter, or frequency analyser, to that of the modulation frequency of the mirror (see Reference [3]).

NOTE 2 Modulation of the position of the reference mirror can also be used to improve the efficiency of the minimum-point method using the Bessel function of first kind and first order (see Reference [4]).

## 9.3 Expression of results

See also B.3.

### 9.3.1 Method 2A (retro-reflector interferometer)

Calculate the displacement amplitude  $\hat{s} = \hat{s}_n$  from the associated argument,  $x_n$ , using Equation (14)

where

$\Delta s = \lambda/4$  [cf. Equation (10) for the two-beam arrangement shown in Figure 3]; or

$\Delta s = \lambda/2$  [cf. Equation (11) for the single-beam arrangement shown in Figure 7 where the wavelength of the laser of the red helium-neon type  $\lambda = 0,632\ 81\ \mu\text{m}$ ].

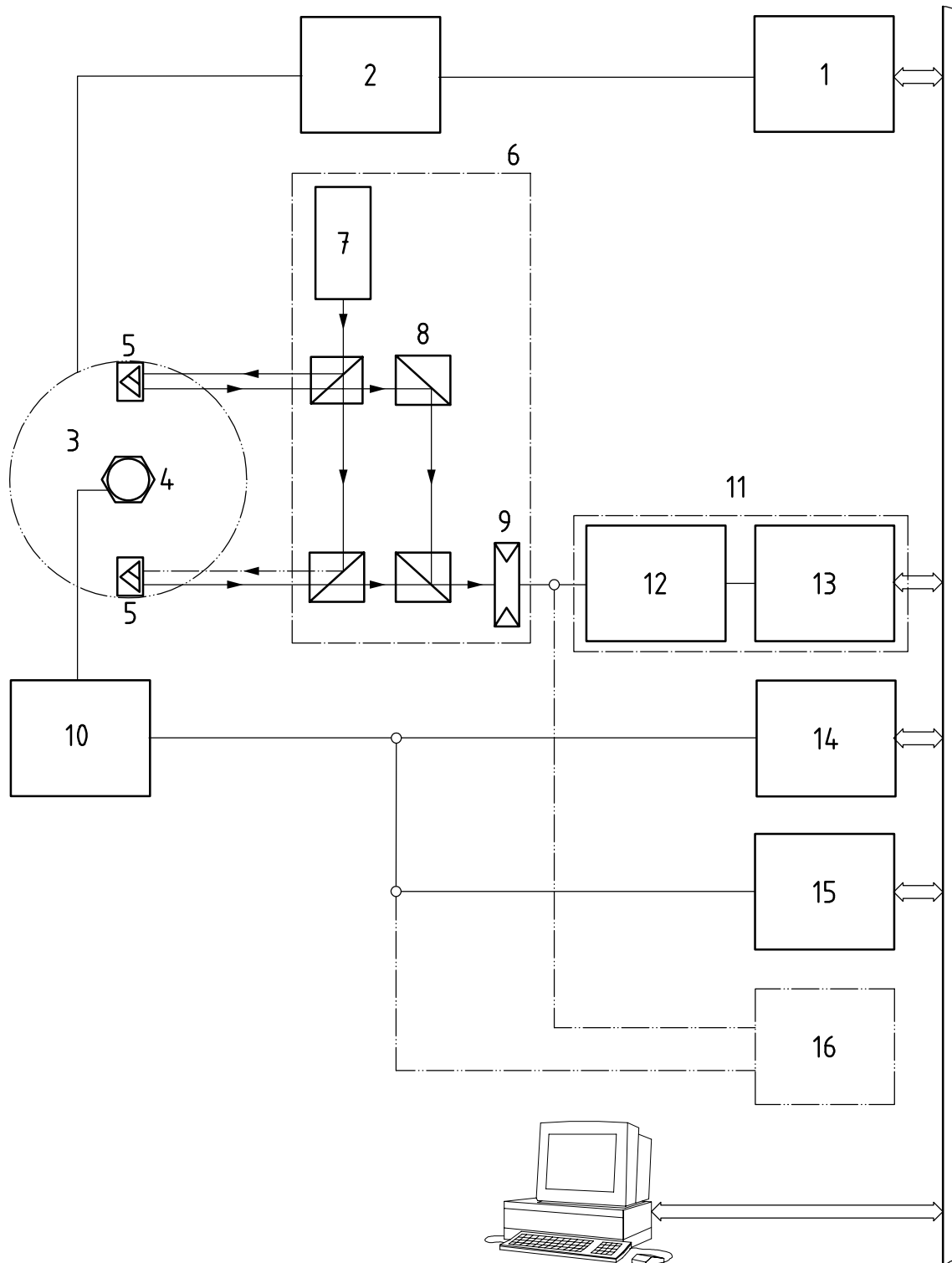
NOTE Application of Equation (14) can be dispensed with if the displacement amplitude,  $\hat{s}_n$ , associated with the  $n$ th minimum point from Table 2 is taken.

For the single-beam arrangement shown in Figure 7, the amplitude,  $\hat{s}$ , converted into the amplitude of the rotational angle using Equation (2) is the same as,  $\hat{s}_n$ , taken from Table 2, as shown in Equation (15):

$$\hat{s} = \hat{s}_n \quad (15)$$

For the two-beam arrangement shown in Figure 3, the amplitude,  $\hat{s}$ , converted into the amplitude of the rotational angle using Equation (2) is half the value of  $\hat{s}_n$  taken from Table 2, as shown in Equation (16):

$$\hat{s} = \frac{\hat{s}_n}{2} \quad (16)$$



**Key**

- |                             |                              |   |
|-----------------------------|------------------------------|---|
| 1 frequency generator (4.2) | 7 laser (4.5)                | 12 bandpass filter tuned to vibration frequency (4.7.3) |
| 2 power amplifier (4.3)     | 8 beam splitter              | 13 voltmeter  |
| 3 angular exciter (4.3)     | 9 light detector             | 14 voltmeter (4.8)                                      |
| 4 angular transducer        | 10 amplifier                 | 15 distortion meter (4.9)                               |
| 5 retro-reflector           | 11 spectrum analyser (4.7.3) | 16 oscilloscope (4.10)                                  |
| 6 interferometer (4.6)      |                              |   |

**Figure 5 — Example of a measuring system for method 2A  
(homodyne retro-reflector interferometer, minimum-point detection)**

### 9.3.2 Method 2B (diffraction-grating interferometer)

Calculate the displacement amplitude  $\hat{s} = \hat{s}_n$  from the associated argument,  $x_n$ , using Equation (14)

where

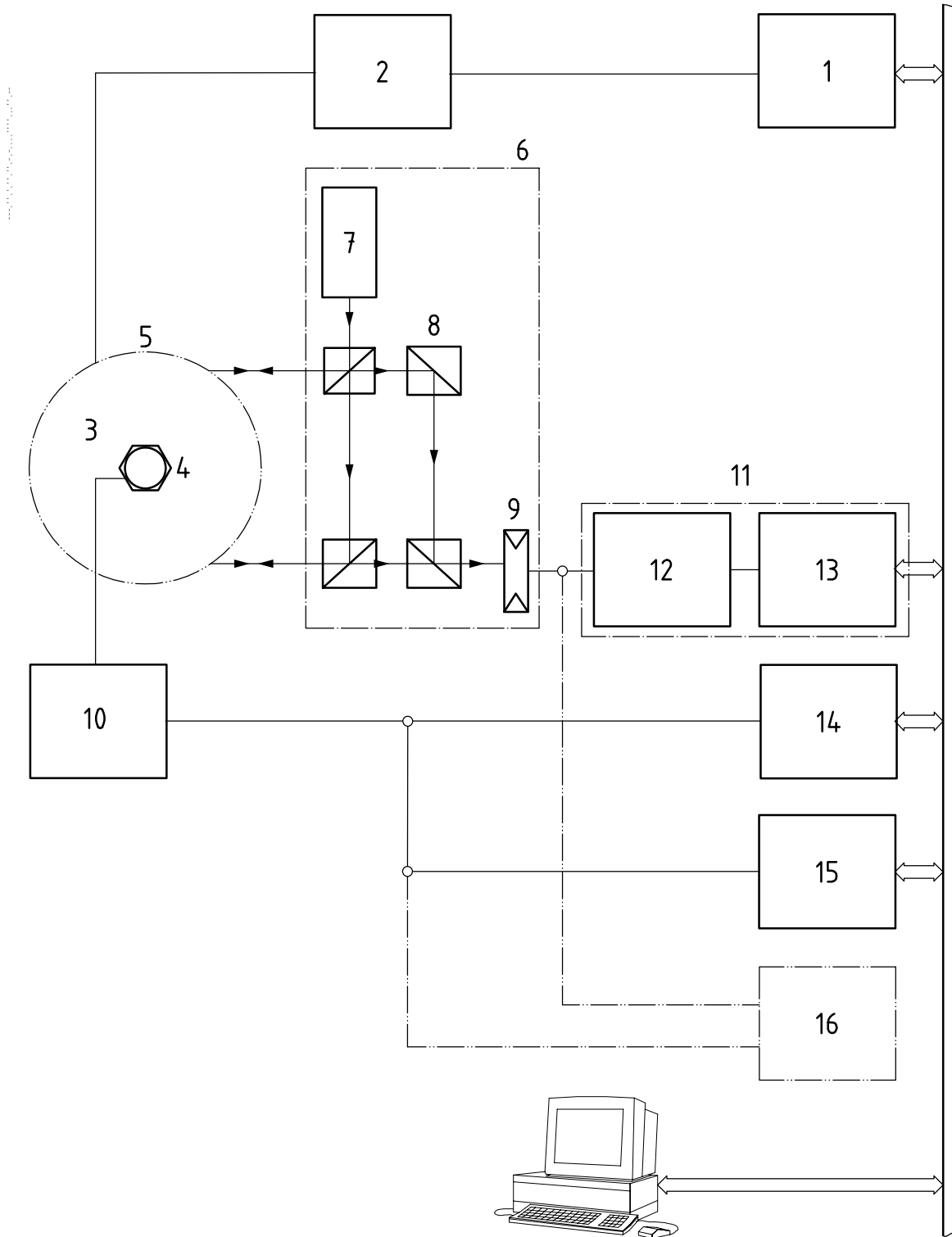
$\Delta s = g/2$  [cf. Equation (12) for the two-beam arrangement shown in Figure 6]; or

$\Delta s = g$  [cf. Equation (13) for the single-beam arrangement shown in Figure 8];

where  $g$  is the grating constant of the diffraction grating (groove length, i.e. grating constant of 0,333 33  $\mu\text{m}$  of a sine-phase diffraction grating having 3 000 grooves/mm).

NOTE Application of Equation (14) can be dispensed with if the displacement amplitude,  $\hat{s}_n$ , associated with the  $n$ th minimum point from Table 2 is taken.

For the single-beam arrangement shown in Figure 8, the amplitude,  $\hat{s}$ , converted into the amplitude of the rotational angle using Equation (2) is the same as  $\hat{s}_n$  taken from Table 2 [cf. Equation (15)]. For the two-beam arrangement shown in Figure 6, the amplitude,  $\hat{s}$ , converted into the amplitude of the rotational angle using Equation (2) is half the value of  $\hat{s}_n$  taken from Table 2 [cf. Equation (16)].



**Key**

- |                             |                              |   |
|-----------------------------|------------------------------|---|
| 1 frequency generator (4.2) | 7 laser (4.5)                | 12 bandpass filter tuned to vibration frequency (4.7.3) |
| 2 power amplifier (4.3)     | 8 beam splitter              | 13 voltmeter  |
| 3 angular exciter (4.3)     | 9 light detector             | 14 voltmeter (4.8)                                      |
| 4 angular transducer        | 10 amplifier                 | 15 distortion meter (4.9)                               |
| 5 diffraction grating       | 11 spectrum analyser (4.7.3) | 16 oscilloscope (4.10)                                  |
| 6 interferometer (4.6)      |                              |   |

**Figure 6 — Example of a measuring system for method 2B  
(homodyne diffraction-grating interferometer, minimum-point detection)**



**Table 2 — Displacement amplitudes for minimum points ( $\lambda = 0,623\ 81\ \mu\text{m}$ ) to calculate the amplitudes of the rotational angle,  $F^a$ , of the angular velocity,  $W^b$ , and of the angular acceleration,  $\alpha^c$**

Minimum point No. $n$	Displacement amplitude $\hat{s}_n$ $\mu\text{m}$	Minimum point No. $n$	Displacement amplitude $\hat{s}_n$ $\mu\text{m}$
0	0	17	2,728 6
1	0,193 0	18	2,886 8
2	0,353 3	19	3,045 0
3	0,512 3	20	3,203 3
4	0,670 9	21	3,361 5
5	0,829 4	22	3,519 7
6	0,987 8	23	3,677 9
7	1,146 1	24	3,836 1
8	1,304 4	25	3,994 3
9	1,462 7	26	4,152 5
10	1,621 0	27	4,310 7
11	1,779 2	28	4,468 9
12	1,937 5	29	4,627 1
13	2,095 7	30	4,785 3
14	2,253 9		
15	2,412 2		
16	2,570 4		

<sup>a</sup> See Equation (2).  
<sup>b</sup> See Equation (5).  
<sup>c</sup> See Equation (7).

Calculate the amplitude,  $\hat{\phi}$ , of the rotational angle,  $\phi$ , using Equation (2); calculate the sensitivity,  $S_\phi$  (magnitude), of the rotational angle transducer from Equation (3); the sensitivity,  $S_\Omega$  (magnitude), of the angular velocity transducer from Equations (4) and (5); and the sensitivity,  $S_\alpha$  (magnitude), of the angular accelerometer using Equations (6) and (7).

When the calibration results are reported, the expanded uncertainty of measurement in calibration shall be calculated and reported in accordance with Annex A.

## 10 Methods using sine approximation (methods 3A and 3B)

### 10.1 General

This method is applicable to sensitivity magnitude and/or phase calibration in the frequency range from 1 Hz to 1,6 kHz.

NOTE The sine-approximation measurement method is also applicable to lower frequencies than 1 Hz and higher frequencies than 1,6 kHz in conjunction with an angular exciter appropriate for the extended frequency range.

In addition to the calibration results obtained by the equations valid for all six methods (cf. Clause 7), methods 3A and 3B also allow the phase shift of the complex sensitivity of angular transducers to be measured.

The initial phase,  $\varphi_{\Phi}$ , of the rotational angle is represented by the initial phase of the displacement,  $\hat{s}$ , as given in Equation (17):

$$\varphi_{\Phi} = \varphi_s \quad (17)$$

Using the value,  $\varphi_{\Phi}$ , obtained by the laser interferometer, the amplitude initial phase,  $\varphi_{\Omega}$ , of the angular velocity,  $\Omega$ , is calculated from Equation (18):

$$\varphi_{\Omega} = \varphi_{\Phi} - \pi/2 \quad (18)$$

and the initial phase,  $\varphi_{\alpha}$ , of the angular acceleration,  $\alpha$ , is calculated from Equation (19):

$$\varphi_{\alpha} = \varphi_{\Phi} - \pi \quad (19)$$

The phase shift,  $\Delta\varphi_{\Phi}$ , of rotational angle transducers is calculated from Equation (20):

$$\Delta\varphi_{\Phi} = \varphi_u - \varphi_{\Phi} \quad (20)$$

The phase shift,  $\Delta\varphi_{\Omega}$ , of angular velocity transducers is calculated from Equation (21):

$$\Delta\varphi_{\Omega} = \varphi_u - \varphi_{\Omega} \quad (21)$$

The phase shift,  $\Delta\varphi_{\alpha}$ , of angular accelerometers is calculated from Equation (22):

$$\Delta\varphi_{\alpha} = \varphi_u - \varphi_{\alpha} \quad (22)$$

In Equations (20), (21) and (22),  $\varphi_u$  is the initial phase of the output of the transducer being calibrated.

## 10.2 Procedure applied to methods 3A and 3B

See also B.4.

Install the equipment in accordance with Figures 7, 8 and 9.

The laser interferometer (for an example, see Figures 7 and 8) shall be adjusted to give output signals  $u_1$  and  $u_2$  in phase quadrature within the tolerances stated in 4.6.

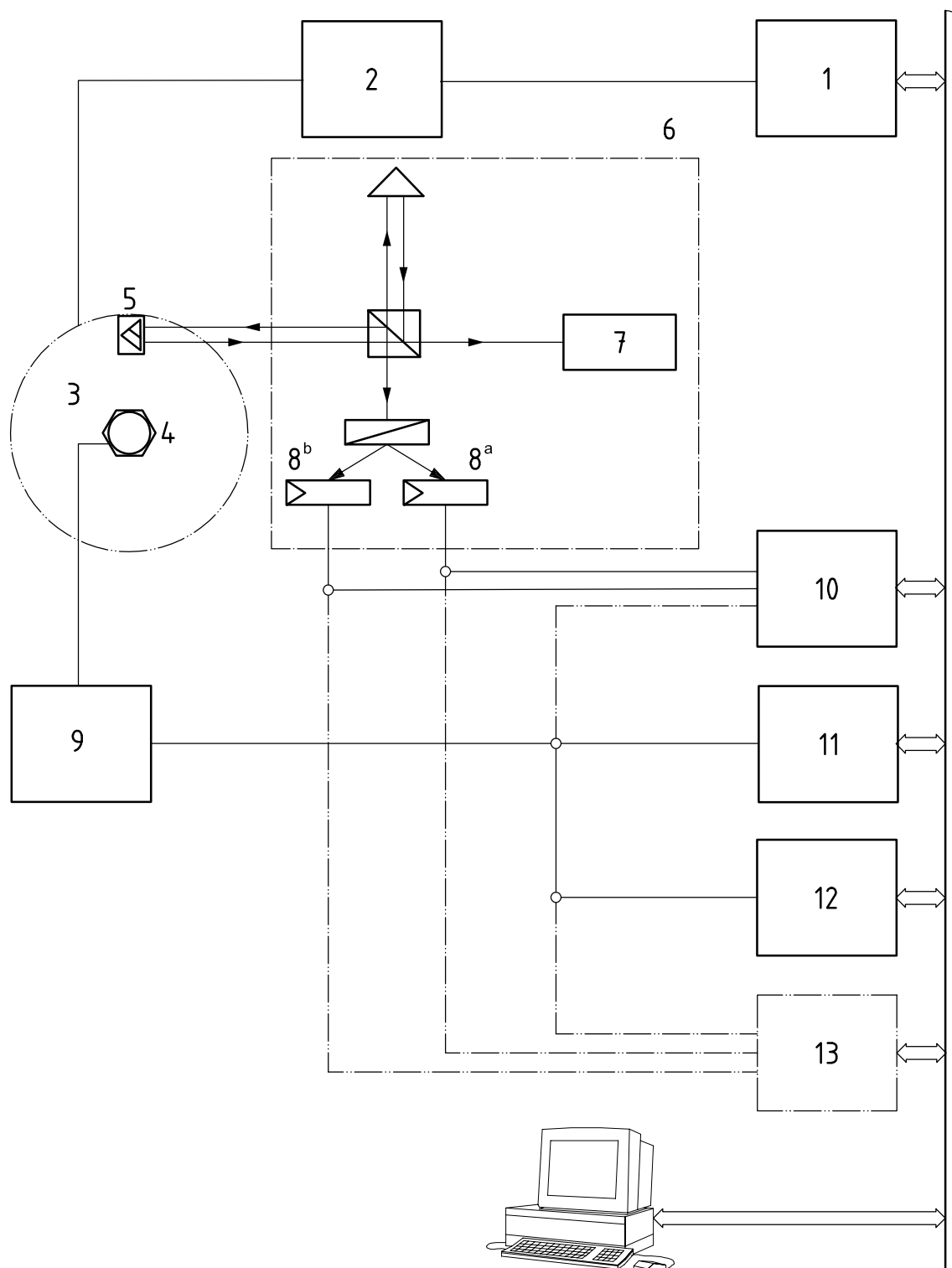
After the interferometer (cf. 4.6) settings have been optimized, measure the magnitude and the phase shift of the angular transducer sensitivity at the specified vibration frequencies and amplitudes (cf. Clause 6) as follows.

The transducer shall be vibrated sinusoidally. The displacement amplitude should be large enough to give at least one full bright/dark cycle of the interferometer output.

NOTE 1 In the arrangement described in the Note of 8.1, at displacement amplitudes  $\leq 0,5 \mu\text{m}$ , a worst-case error component  $\geq 0,3 \%$  in the sensitivity magnitude measurement and  $\geq 0,3^\circ$  in the phase shift measurement can be caused by the combined effects of the disturbing parameters of the quadrature output signals within the tolerances stated in 4.6. Suppression of this error component is possible by more careful adjustment than tolerated in 4.6 (see Reference [5]) or by application of the correction procedure given in Reference [6]).

NOTE 2 To measure the magnitude and the phase shift of the complex sensitivity of angular transducers down to small displacement amplitudes in the nanometer range, the sine-approximation method in conjunction with an appropriate heterodyne technique can be applied as reported in References [7] and [8]. This allows calibration at a preferred moderate angular acceleration amplitude to be performed at high vibration frequencies.

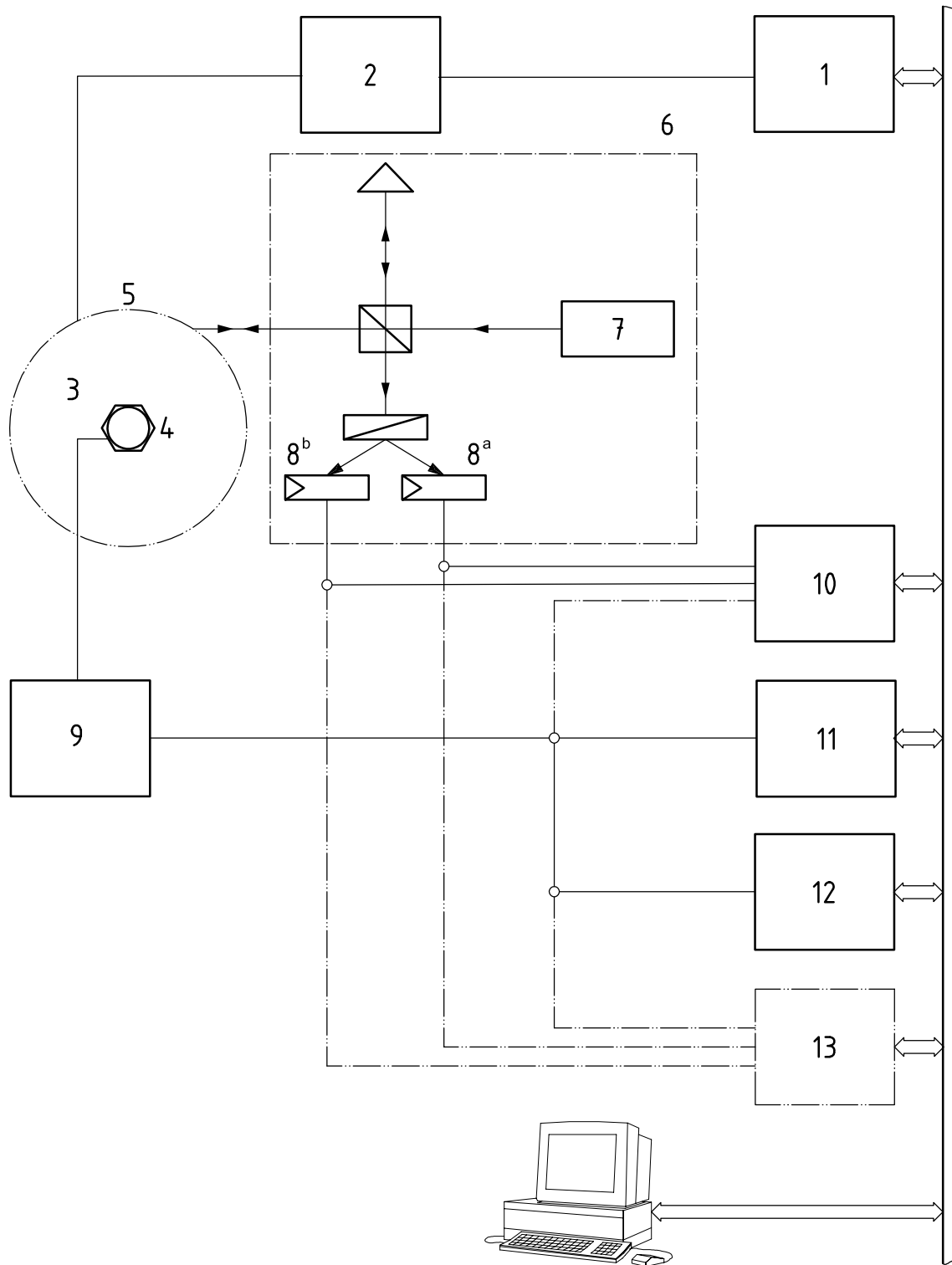
NOTE 3 In order to enhance the rejection efficiency of sine approximation (i.e. attenuation of the influence of disturbing signals), windowing of the displacement values or modulation phase values can be applied<sup>[9]</sup> if the procedure has proved to meet the uncertainty requirements of Clause 3.



**Key**

- |                             |                        |                                      |
|-----------------------------|------------------------|--------------------------------------|
| 1 frequency generator (4.2) | 6 interferometer (4.6) | 10 digital waveform recorder (4.7.4) |
| 2 power amplifier (4.3)     | 7 laser (4.5)          | 11 voltmeter (4.8)                   |
| 3 angular exciter (4.3)     | 8 light detectors      | 12 distortion meter (4.9)            |
| 4 angular transducer        | 9 amplifier            | 13 oscilloscope (4.10)               |
| 5 retro-reflector           |                        |                                      |
| a Phase shift of 0°.        |                        |                                      |
| b Phase shift of 90°.       |                        |                                      |

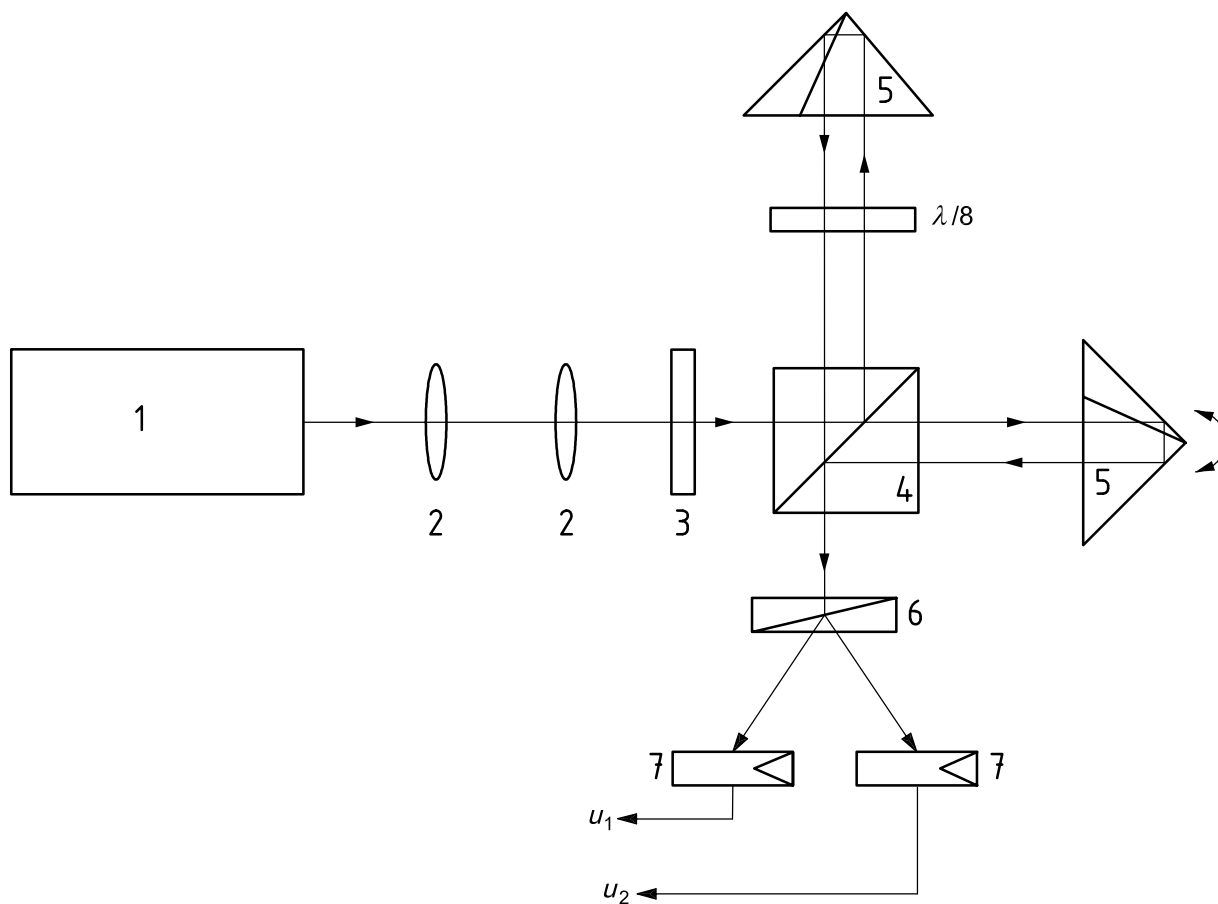
**Figure 7— Example of a measuring system for method 3A (homodyne retro-reflector interferometer, sine approximation)**



**Key**

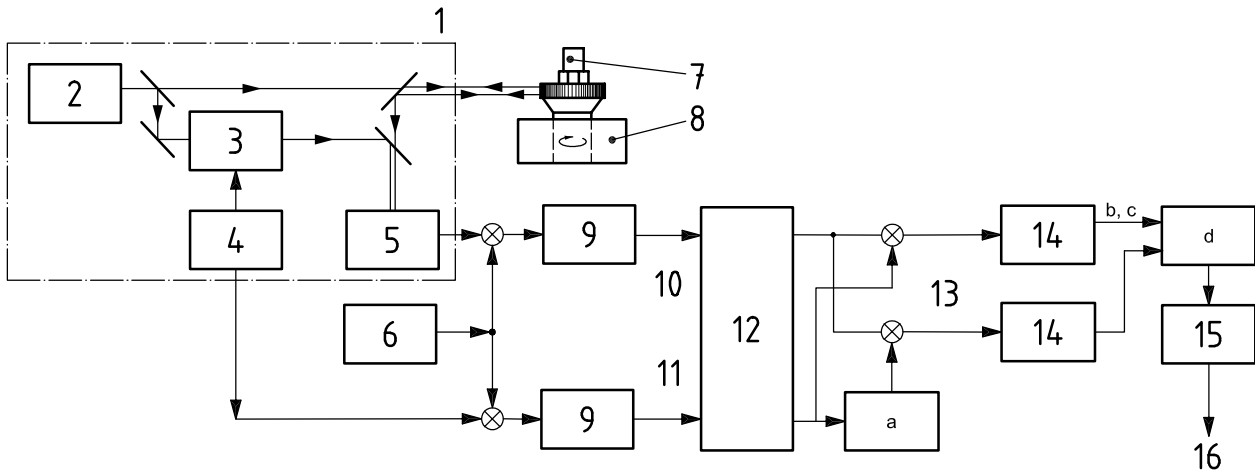
- |                             |                        |                                      |
|-----------------------------|------------------------|--------------------------------------|
| 1 frequency generator (4.2) | 6 interferometer (4.6) | 10 digital waveform recorder (4.7.4) |
| 2 power amplifier (4.3)     | 7 laser (4.5)          | 11 voltmeter (4.8)                   |
| 3 angular exciter (4.3)     | 8 light detectors      | 12 distortion meter (4.9)            |
| 4 angular transducer        | 9 amplifier            | 13 oscilloscope (4.10)               |
| 5 diffraction grating       |                        |                                      |
- a Phase shift of 0°.  
b Phase shift of 90°.

**Figure 8 — Example of a measuring system for method 3B  
(homodyne diffraction-grating interferometer, sine approximation)**

**Key**

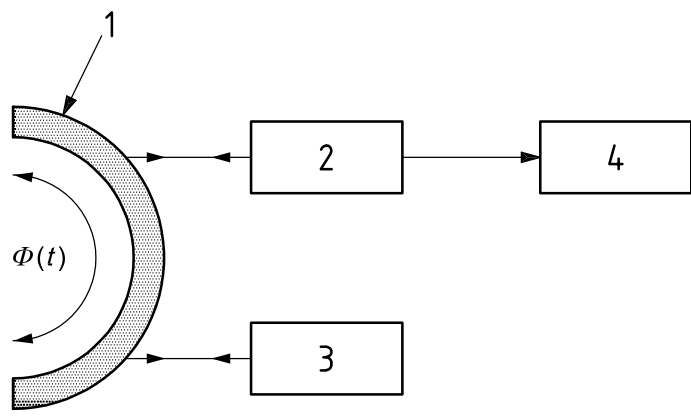
- 1 laser
- 2 telescope
- 3 polarizer
- 4 beam splitter
- 5 retro-reflector
- 6 wollaston prism
- 7 light detectors

**Figure 9 — Modified Michelson interferometer with retro-reflector(s) and quadrature output**



- Key**
- |   |   |
|---|---|
| 1 interferometer                            | 9 low-pass filter                                       |
| 2 laser                                     | 10 measurement signal                                   |
| 3 Bragg cell                                | 11 reference signal                                     |
| 4 quartz generator                          | 12 transient recorder                                   |
| 5 photodetector                             | 13 quadrature signal synthesis                          |
| 6 synthesis generator                       | 14 digital filtering                                    |
| 7 angular accelerometer                     | 15 algorithms   |
| 8 angular accelerometer exciter (air-borne) | 16 measurement results (vibration and shock parameters) |
- a 90°.  
 b  $y(t_i)$ .  
 c  $z(t_i)$ .  
 d  $\arctan y(t_i)/z(t_j)$ .

**Figure 10 — Example of a measuring system using heterodyne interferometry in accordance with method 3B (Mach-Zehnder heterodyne interferometer with diffraction grating, frequency conversion, transient recorder and digital data processing)**



- Key**
- |                             |  |
|-----------------------------|--|
| 1 diffraction grating       | 3 rotational laser vibrometer being calibrated |
| 2 interferometer (standard) | 4 signal-processing system                     |

**Figure 11 — Example of an arrangement for the calibration of rotational laser vibrometers**

Method B is applied here in the version with a diffraction grating interferometer using a sine-phase grating with 2 400 grooves/mm. It is arranged on the lateral surface of a disk forming the measuring table 100 mm in diameter.

### 10.3 Data acquisition

The cut-off frequencies of low-pass and, if any, high-pass filters shall be chosen so that disturbing effects of low- and high-pass filtering on the calibration results are tolerable (cf. Reference [5]). To fulfil Nyquist's theorem, the sampling rate shall be set so that the highest frequency content is smaller than half the sampling rate.

Analog-to-digital conversion of the angular transducer output voltage may be performed at the same or at a lower sampling rate than that for the conversion of the interferometer output signals. The three sampling processes shall start and end at the same points of time and at least for the two interferometer signals be synchronized by a single system clock.

The quadrature signals shall be equidistantly sampled during a measurement time  $t_0 < t < t_0 + T_{\text{Meas}}$ . The series of measurement values,  $\{u_1(t_i)\}$  and  $\{u_2(t_i)\}$ , sampled within  $t_0 < t < t_0 + T_{\text{Meas}}$  shall have a sampling interval,  $\Delta t = t_i - t_{i-1} = \text{const}$ .

The sampled series of accelerometer output values are  $\{u(t_i)\}$ .

Transfer the data to the computer memory.

### 10.4 Data processing

**10.4.1** Obtain the magnitude and the phase shift of the accelerometer sensitivity by the following steps.

**10.4.2** Calculate a series of modulation phase values from the sampled interferometer output values,  $\{u_1(t_i)\}$  and  $\{u_2(t_i)\}$ , using Equation (23):

$$\varphi_{\text{Mod}}(t_i) = \arctan \frac{u_2(t_i)}{u_1(t_i)} + n\pi \quad (23)$$

where  $n = 0, 1, 2, \dots$

Choose an integer number,  $n$ , so that discontinuities of  $\varphi_{\text{Mod}}(t_i)$  are avoided for the values  $n\pi$ .

A procedure for calculating the number,  $n$ , is described in Reference [10].

Calculate a series of displacement values,  $\{s(t_i)\}$ , using Equation (24):

$$s(t_i) = \frac{\lambda}{4\pi} \varphi_{\text{Mod}}(t_i) \quad (24)$$

**10.4.3** Approximate the obtained series of displacement values by solving the following system of  $N + 1$  equations for the three unknown parameters,  $A$ ,  $B$  and  $C$ , using the least-squares method as shown in Equation (25):

$$s(t_i) = A \cos \omega t_i - B \sin \omega t_i + C \quad (25)$$

where

$$i = 0, 1, 2, \dots, N;$$

$$A = \hat{s} \cos \varphi_s;$$

$$B = \hat{s} \sin \varphi_s;$$

$C$  is a constant;

$\omega$  is the vibration radian(s) frequency,  $\omega = 2\pi f$ ;

$\varphi_s$  is the initial phase angle of the displacement;

$N + 1$  denotes the number of samples synchronously taken over the measurement period given in 10.3.

From among the resulting parameter values of this “sine approximation”, the parameter,  $C$ , is not used in this context.

**10.4.4** Calculate the displacement amplitude,  $\hat{s}$ , and the displacement initial phase angle,  $\varphi_s$ , from the parameter values,  $A$  and  $B$ , obtained through sine approximation using Equations (26) and (27):

$$\hat{s} = \sqrt{A^2 + B^2} \quad (26)$$

$$\varphi_s = \arctan \frac{B}{A} \quad (27)$$

**10.4.5** Calculate the amplitude,  $\hat{\phi}$ , and initial phase angle,  $\varphi_\phi$ , from  $\hat{s}$  and  $\varphi_s$  using Equations (2) and (17):

If an angular velocity transducer is being calibrated, calculate the amplitude,  $\hat{\Omega}$ , and the initial phase,  $\varphi_\Omega$ , of the angular velocity from  $\hat{\phi}$  and  $\varphi_\phi$  using Equations (5) and (18).

If an angular accelerometer is being calibrated, calculate amplitude,  $\hat{\alpha}$ , and initial phase,  $\varphi_\alpha$ , of the angular acceleration from  $\hat{\phi}$  and  $\varphi_\phi$  using Equations (7) and (19).

**10.4.6** Approximate the series of sampled angular transducer output values,  $\{u(t_i)\}$ , by the sine-approximation method in accordance with 10.4.3. Rewritten for the transducer output denoted  $u$ , the following system of  $N + 1$  equations derived from Equation (28) is solved:

$$\{u(t_i)\} = A_u \cos \omega t_i - B_u \sin \omega t_i + C_u \quad (28)$$

where

$$A_u = \hat{u} \cos \varphi_u;$$

$$B_u = \hat{u} \sin \varphi_u;$$

$C_u$  is a constant;

$\hat{u}$  is the angular transducer output amplitude;

$\varphi_u$  is the output initial phase angle.

Calculate the angular transducer output amplitude,  $\hat{u}$ , and the initial phase angle,  $\varphi_u$ , from the parameter values,  $A_u$  and  $B_u$ , obtained by sine approximation using Equations (29) and (30):

$$\hat{u} = \sqrt{A_u^2 + B_u^2} \quad (29)$$

$$\varphi_u = \arctan \frac{B_u}{A_u} \quad (30)$$



If a rotational angle transducer is being calibrated, calculate the magnitude,  $S_{\phi}$ , and the phase shift,  $\Delta\phi_{\phi}$ , of its complex sensitivity from the values,  $\hat{u}$  and  $\phi_u$ , obtained in 10.4.6, and the values,  $\hat{\phi}$  and  $\phi_{\phi}$ , obtained in 10.4.5 using Equations (3) and (20).

If an angular velocity transducer is being calibrated, calculate the magnitude,  $S_{\Omega}$ , and the phase shift,  $\Delta\phi_{\Omega}$ , of its complex sensitivity from the values,  $\hat{u}$  and  $\phi_u$ , obtained in 10.4.6, and the values,  $\hat{\phi}$  and  $\phi_{\phi}$ , obtained in 10.4.5 using Equations (4) and (21).

If an angular accelerometer is being calibrated, calculate the magnitude,  $S_{\alpha}$ , and the phase shift,  $\Delta\phi_{\alpha}$ , of its complex sensitivity from the values,  $\hat{u}$  and  $\phi_u$ , obtained in 10.4.6, and the values,  $\hat{\phi}$  and  $\phi_{\phi}$ , obtained in 10.4.5 using Equations (6) and (22).

When the calibration results are reported, the expanded uncertainty of measurement in calibration shall be calculated and reported in accordance with Annex A. In A.1.1, A.1.2, A.1.3.1 and A.2.1, the magnitude of the complex sensitivity is denoted by  $S$ .

## 11 Reporting of calibration results

When the calibration results are reported, in addition to the calibration method, at least the following conditions and characteristics shall be stated:

- a) ambient conditions:
  - temperature of angular transducer,
  - ambient air temperature;
- b) mounting technique:
  - material of mounting surface,
  - mounting torque (if the angular transducer is stud-mounted),
  - oil or grease (if used),
  - cable fixing,
  - orientation (about vertical or horizontal axis),
  - all amplifier settings (if adjustable), for example:
    - gain,
    - cut-off frequencies of filters;
- c) calibration results:
  - values of calibration frequencies and amplitudes,
  - values of sensitivity (magnitude and phase shift, if measured),
  - expanded uncertainty of measurement, coverage factor  $k$  if different from  $k = 2$ .

**Annex A**  
(normative)

**Uncertainty components in primary angular vibration calibration of vibration and shock transducers by laser interferometry**

**A.1 Calculation of the relative expanded uncertainty for the sensitivity (magnitude) and the expanded uncertainty for the phase shift for calibration frequencies, amplitudes, and settings of amplifier gain and cut-off frequencies**

**A.1.1 Calculation of  $U_{rel}(S)$  for methods 1A and 1B**

In accordance with ISO 16063-1, the relative expanded measurement uncertainty of the sensitivity (magnitude),  $U_{rel}(S)$ , for the calibration frequencies, amplitudes, and settings of amplifier gain and cut-off frequencies shall be calculated from Equations (A.1) and (A.2):

$$U_{rel}(S) = k u_{c,rel}(S) \tag{A.1}$$

$$u_{c,rel}(S) = \frac{u_c(S)}{S} = \frac{1}{S} \sqrt{\sum_i u_i^2(S)} \tag{A.2}$$

with the coverage factor  $k = 2$  (see Table A.1).

**Table A.1 — Uncertainty components for methods 1A and 1B**

<i>i</i>	Standard uncertainty component $u(x_i)$	Source of uncertainty <sup>a</sup>	Uncertainty contribution $u_i(S)$
1	$u(\hat{u}_V)$	measurement of output voltage of angular transducer (voltmeter)	$u_1(S)$
2	$u(\hat{u}_D)$	measurement of output voltage of effect of total distortion on angular transducer	$u_2(S)$
3	$u(\hat{u}_T)$	measurement of output voltage of effect of transverse and rocking motion angular transducer (transverse sensitivity)	$u_3(S)$
4	$u(\hat{s}_Q)$	effect of displacement quantization on displacement measurement	$u_4(S)$
5	$u(\hat{s}_H)$	effect of trigger hysteresis on displacement measurement	$u_5(S)$
6	$u(\hat{s}_F)$	filtering effect on displacement measurement (frequency band limitation)	$u_6(S)$
7	$u(\hat{s}_{VD})$	effect of voltage disturbance on displacement measurement (e.g. random noise in the photoelectric measuring chain)	$u_7(S)$
8	$u(\hat{s}_{MD})$	effect of motion disturbance on displacement measurement (e.g. total distortion; relative motion between reference surface of angular transducer and spot sensed by interferometer)	$u_8(S)$
9	$u(\hat{s}_{PD})$	effect of phase disturbance on displacement measurement (e.g. phase noise of interferometer signal)	$u_9(S)$
10	$u(\hat{\phi}_{Int})$	interferometer function (conversion displacement into rotational angle)	$u_{10}(S)$
11	$u(f_{FG})$	vibration frequency measurement (frequency generator and indicator)	$u_{11}(S)$
12	$u(S_{RE})$	residual effects on sensitivity measurement (e.g. random effect in repeat measurements; experimental standard deviation of arithmetic mean)	$u_{12}(S)$

<sup>a</sup> The sources of uncertainties may be subdivided and numbered in a way differing from that used in the above table, provided that each effect significantly influencing the measurement result has been taken into account.

### A.1.2 Calculation of $U_{\text{rel}}(S)$ for methods 2A and 2B

$U_{\text{rel}}(S)$  for methods 2A and 2B shall be calculated as described in A.1.1 but using the uncertainty components in Table A.2.

**Table A.2 — Uncertainty components for methods 2A and 2B**

$i$	Standard uncertainty component $u(x_i)$	Source of uncertainty <sup>a</sup>	Uncertainty contribution $u_i(y)$
1	$u(\hat{u}_V)$	measurement of output voltage of angular transducer (voltmeter)	$u_1(S)$
2	$u(\hat{u}_D)$	measurement of output voltage of effect of total distortion on angular transducer	$u_2(S)$
3	$u(\hat{u}_T)$	measurement of output voltage of effect of transverse and rocking motion on angular transducer (transverse sensitivity)	$u_3(S)$
4	$u(\hat{s}_Z)$	effect of minimum-point resolution on displacement measurement	$u_4(S)$
5	$u(\hat{s}_{VD})$	effect of voltage disturbance on displacement measurement (e.g. hum and noise)	$u_5(S)$
6	$u(\hat{s}_{MD})$	effect of motion disturbance on displacement measurement (e.g. relative motion between angular transducer reference surface and spot sensed by the interferometer)	$u_6(S)$
7	$u(\hat{\phi}_{\text{int}})$	interferometer function (transformation displacement to rotational angle)	$u_7(S)$
8	$u(f_{FG})$	vibration frequency measurement (frequency generator and indicator)	$u_8(S)$
9	$u(S_{RE})$	residual effects on sensitivity measurement (e.g. random effect in repeat measurements; experimental standard deviation of arithmetic mean)	$u_9(S)$

<sup>a</sup> The sources of uncertainties may be subdivided and numbered in a way differing from that used in the above table, provided that each effect significantly influencing the measurement result has been taken into account.

**A.1.3 Calculation of  $U_{rel}(S)$  and  $U(\Delta\phi)$  for methods 3A and 3B**

**A.1.3.1 Calculation of  $U_{rel}(S)$**

$U_{rel}(S)$  for methods 3A and 3B shall be calculated as under A.1.1 but using the uncertainty components in Table A.3.

**Table A.3 — Uncertainty components for methods 3A and 3B**

<i>i</i>	Standard uncertainty component $u(x_i)$	Source of uncertainty <sup>a</sup>	Uncertainty contribution $u_i(y)$
1	$u(\hat{u}_V)$	angular transducer output voltage measurement (waveform recorder; e.g. ADC resolution)	$u_1(S)$
2	$u(\hat{u}_F)$	voltage filtering effect on angular transducer output amplitude measurement (frequency band limitation)	$u_2(S)$
3	$u(\hat{u}_D)$	effect of voltage disturbance on angular transducer output voltage measurement (e.g. hum and noise)	$u_3(S)$
4	$u(\hat{u}_T)$	effect of transverse, rocking and bending angular transducer on accelerometer output voltage measurement (transverse sensitivity)	$u_4(S)$
5	$u(\hat{\phi}_{M,Q})$	effect of interferometer quadrature output signal disturbance on displacement amplitude measurement (e.g. offsets, voltage amplitude deviation, deviation from 90° nominal angle difference)	$u_5(S)$
6	$u(\hat{\phi}_{M,F})$	interferometer signal filtering effect on displacement amplitude measurement (frequency band limitation)	$u_6(S)$
7	$u(\hat{\phi}_{M,VD})$	effect of voltage disturbance on displacement amplitude measurement (e.g. random noise in photoelectric measuring chains)	$u_7(S)$
8	$u(\hat{\phi}_{M,MD})$	effect of motion disturbance on displacement amplitude measurement (e.g. drift; relative motion between accelerometer reference surface and spot sensed by the interferometer)	$u_8(S)$
9	$u(\hat{\phi}_{M,PD})$	effect of phase disturbance on phase amplitude measurement (e.g. phase noise of interferometer signals)	$u_9(S)$
10	$u(\hat{\phi}_{Int})$	interferometer function (transformation displacement to rotational angle)	$u_{10}(S)$
11	$u(f_{FG})$	vibration frequency measurement (frequency generator and indicator)	$u_{11}(S)$
12	$u(S_{RE})$	residual effects on sensitivity measurement (e.g. random effect in repeat measurements; experimental standard deviation of arithmetic mean)	$u_{12}(S)$

<sup>a</sup> The sources of uncertainties may be subdivided and numbered in a way differing from that used in the above table, provided that each effect significantly influencing the measurement result has been taken into account.

### A.1.3.2 Calculation of $U(\Delta\varphi)$

In accordance with ISO 16063-1, the expanded uncertainty of measurement of the phase shift,  $U(\Delta\varphi)$ , for the calibration frequencies, amplitudes, and settings of amplifier gain and cut-off frequencies shall be calculated from Equations (A.3) and (A.4):

$$U(\Delta\varphi) = k u_c(\Delta\varphi) \quad (\text{A.3})$$

$$u_c(\Delta\varphi) = \sqrt{\sum_i u_i^2(\Delta\varphi)} \quad (\text{A.4})$$

with the coverage factor  $k = 2$  (see Table A.4).

**Table A.4 — Uncertainty components for measurement of phase shift,  $U(\Delta\varphi)$**

$i$	Standard uncertainty component $u(x_i)$	Source of uncertainty <sup>a</sup>	Uncertainty contribution $u_i(y)$
1	$u(\varphi_{u,V})$	angular transducer output phase measurement (waveform recorder; e.g. ADC resolution)	$u_1(\Delta\varphi)$
2	$u(\varphi_{u,F})$	voltage filtering effect on angular transducer output phase measurement (frequency band limitation)	$u_2(\Delta\varphi)$
3	$u(\varphi_{u,D})$	effect of voltage disturbance on angular transducer output phase measurement (e.g. hum and noise)	$u_3(\Delta\varphi)$
4	$u(\varphi_{u,T})$	effect of transverse and rocking motion on angular transducer output phase measurement (transverse sensitivity)	$u_4(\Delta\varphi)$
5	$u(\varphi_{s,Q})$	effect of interferometer quadrature output signal disturbance on displacement phase measurement (e.g. offsets, voltage amplitude deviation, deviation from 90° nominal angle difference)	$u_5(\Delta\varphi)$
6	$u(\varphi_{s,F})$	interferometer signal filtering effect on displacement phase measurement (frequency band limitation)	$u_6(\Delta\varphi)$
7	$u(\varphi_{s,VD})$	effect of voltage disturbance on displacement phase measurement (e.g. random noise in photoelectric measuring chains)	$u_7(\Delta\varphi)$
8	$u(\varphi_{s,MD})$	effect of motion disturbance on displacement phase measurement (e.g. drift; relative motion between accelerometer reference surface and spot sensed by the interferometer)	$u_8(\Delta\varphi)$
9	$u(\varphi_{s,PD})$	effect of phase disturbance on displacement phase measurement (e.g. phase noise of interferometer signals)	$u_9(\Delta\varphi)$
10	$u(\varphi_{\phi,Int})$	interferometer function (transformation displacement to rotational angle)	$u_{10}(\Delta\varphi)$
11	$u(\Delta\varphi_{RE})$	residual effects on phase shift measurement (e.g. random effect in repeat measurements; experimental standard deviation of arithmetic mean)	$u_{11}(\Delta\varphi)$

<sup>a</sup> The sources of uncertainties may be subdivided and numbered in a way differing from that used in the above table, provided that each effect significantly influencing the measurement result has been taken into account.

## A.2 Calculation of the relative expanded uncertainty for the sensitivity (magnitude) and the expanded uncertainty for the phase shift over the complete frequency and amplitude range

### A.2.1 Calculation of $U_{rel}(S_t)$

The relative expanded measurement uncertainty of the sensitivity (magnitude),  $U_{rel}(S_t)$ , calculated in accordance with A.1.1, A.1.2 or A.1.3.1, is valid only for the calibration frequencies, amplitudes, and settings of amplifier gain and cut-off frequencies. The relative expanded measurement uncertainty of the sensitivity,  $U_{rel}(S_t)$ , for the complete frequency and amplitude range, at any time during the interval between successive calibrations, shall be calculated from Equations (A.5) and (A.6):

$$U_{rel}(S_t) = k u_{c,rel}(S_t) \tag{A.5}$$

$$u_{c,rel}(S_t) = \frac{u_c(S_t)}{S} = \frac{1}{S} \sqrt{\sum_i u_i^2(S_t)} \tag{A.6}$$

with the coverage factor  $k = 2$  (cf. ISO 16063-1; see Table A.5).

**Table A.5 — Uncertainty components for measurement of sensitivity (magnitude),  $U_{rel}(S_t)$**

<i>i</i>	Standard uncertainty component $u(x_i)$	Source of uncertainty <sup>a</sup>	Uncertainty contribution $u_i(y)$
1	$u(S)$	uncertainty of sensitivity calculated at calibration frequencies, amplitudes and amplifier gain settings in accordance with A.1.1, A.1.2 or A.1.3.1	$u_1(S_t)$
2	$u(e_{T,A})$	reference amplifier tracking (deviations in gain for different amplification settings)	$u_2(S_t)$
3	$u(e_{L,f,A})$	deviation from constant amplitude-frequency characteristic of reference amplifier	$u_3(S_t)$
4	$u(e_{L,f,P})$	deviation from constant amplitude-frequency characteristic of reference angular transducer	$u_4(S_t)$
5	$u(e_{L,a,A})$	amplitude effect on gain of reference amplifier	$u_5(S_t)$
6	$u(e_{L,a,P})$	amplitude effect on sensitivity (magnitude) of reference angular transducer	$u_6(S_t)$
7	$u(e_{I,A})$	instability of reference amplifier gain and effect of source impedance on gain	$u_7(S_t)$
8	$u(e_{I,P})$	instability of sensitivity (magnitude) of reference angular transducer	$u_8(S_t)$
9	$u(e_{E,A})$	environmental effects on gain of reference amplifier	$u_9(S_t)$
10	$u(e_{E,P})$	environmental effects on sensitivity (magnitude) of reference angular transducer	$u_{10}(S_t)$

<sup>a</sup> The sources of uncertainties may be subdivided and numbered in a way differing from that used in the above table, provided that each effect significantly influencing the measurement result has been taken into account.

### A.2.2 Calculation of $U(\Delta\phi_t)$

The expanded uncertainty of measurement of the phase shift of the complex sensitivity,  $U(\Delta\phi_t)$ , calculated in accordance with A.1.3.2, is only valid for the calibration frequencies, amplitudes, and settings of amplifier gain and filter cut-off frequencies. The expanded uncertainty of measurement of the phase shift,  $U(\Delta\phi_t)$ , for the complete frequency and amplitude range, at any time during the interval between successive calibrations, shall be calculated from Equations (A.7) and (A.8):

$$U(\Delta\phi_t) = k u_c(\Delta\phi_t) \tag{A.7}$$

$$u_c(\Delta\varphi_t) = \sqrt{\sum_i u_i^2(\Delta\varphi_t)} \quad (\text{A.8})$$

with the coverage factor  $k = 2$  (cf. ISO 16063-1; see Table A.6).

**Table A.6 — Uncertainty components for measurement of phase shift of the complex sensitivity,  $U(\Delta\varphi)$**

$i$	Standard uncertainty component $u(x_i)$	Source of uncertainty <sup>a</sup>	Uncertainty contribution $u_i(y)$
1	$u(\Delta\varphi)$	uncertainty of phase shift calculated at calibration frequencies, amplitudes and amplifier gain settings in accordance with A.1.3.2	$u_1(\Delta\varphi_t)$
2	$u(e_{T,A}^*)$	reference amplifier tracking (deviations in phase for different amplification settings)	$u_2(\Delta\varphi_t)$
3	$u(e_{L,f,A}^*)$	deviation from linear phase-frequency characteristic of reference amplifier	$u_3(\Delta\varphi_t)$
4	$u(e_{L,f,P}^*)$	deviation from linear phase-frequency characteristic of reference angular transducer	$u_4(\Delta\varphi_t)$
5	$u(e_{L,a,A}^*)$	amplitude effect on phase shift of reference amplifier	$u_5(\Delta\varphi_t)$
6	$u(e_{L,a,P}^*)$	amplitude effect on phase shift of reference angular transducer	$u_6(\Delta\varphi_t)$
7	$u(e_{I,A}^*)$	instability of reference amplifier phase shift, and effect of source impedance on phase shift	$u_7(\Delta\varphi_t)$
8	$u(e_{I,P}^*)$	instability of reference angular transducer phase shift	$u_8(\Delta\varphi_t)$
9	$u(e_{E,A}^*)$	environmental effects on phase shift of reference amplifier	$u_9(\Delta\varphi_t)$
10	$u(e_{E,P}^*)$	environmental effects on phase shift of reference angular transducer	$u_{10}(\Delta\varphi_t)$

<sup>a</sup> The sources of uncertainties may be subdivided and numbered in a way differing from that used in the above table, provided that each effect significantly influencing the measurement result has been taken into account.

## Annex B (normative)

### Equations for the calculation of the angular quantities of rotational angle, $\Phi$ , angular velocity, $\Omega$ , and angular acceleration, $\alpha$ , and of the sensitivities of angular transducers: rotational angle transducers, $S_\Phi$ , of angular velocity transducers, $S_\Omega$ , and angular accelerometers, $S_\alpha$

#### B.1 General

In accordance with ISO 2041 and ISO 16063-1, the complex sensitivity of a vibration and shock transducer is given in Equation (B.1):

$$\underline{\hat{S}} = \hat{S} \exp j(\Delta\varphi) \quad (\text{B.1})$$

where the magnitude,  $\hat{S}$ , is described by Equation (B.2):

$$\hat{S} = \frac{\hat{u}}{\hat{x}} \quad (\text{B.2})$$

and the phase shift,  $\Delta\varphi$ , of the complex sensitivity to a motion quantity,  $x$ , is described by Equation (B.3):

$$\Delta\varphi = \varphi_u - \varphi_x \quad (\text{B.3})$$

where

- $x$  represents the rotational angle,  $\Phi$ , the angular velocity,  $\Omega$ , or the angular acceleration,  $\alpha$ ;
- $\hat{x}$ ,  $\hat{\Phi}$ ,  $\hat{\Omega}$  and  $\hat{\alpha}$  represent the amplitudes of these four quantities, respectively;
- $\hat{u}$  is the amplitude of the angular transducer output,  $u$ , (e.g. output voltage of an angular accelerometer).

Equations (B.2) and (B.3) for magnitude and phase shift are specifically valid for rotational angle transducers, as given in Equations (B.4) and (B.5):

$$\hat{S}_\Phi = \frac{\hat{u}}{\hat{\Phi}} \quad (\text{B.4})$$

$$\Delta\varphi_\Phi = \varphi_u - \varphi_\Phi \quad (\text{B.5})$$

for angular velocity transducers, as given in Equations (B.6) and (B.7):

$$\hat{S}_\Omega = \frac{\hat{u}}{\hat{\Omega}} \quad (\text{B.6})$$

$$\Delta\varphi_\Omega = \varphi_u - \varphi_\Omega \quad (\text{B.7})$$



and for angular accelerometer, as given in Equations (B.8) and (B.9):

$$\hat{s}_\alpha = \frac{\hat{u}}{\hat{\alpha}} \quad (\text{B.8})$$

$$\Delta\varphi_\alpha = \varphi_u - \varphi_\alpha \quad (\text{B.9})$$

The motion quantity measured by laser interferometry specified in this part of ISO 16063 is the rotational angle,  $\Phi$  (i.e. amplitude,  $\hat{\Phi}$ ) in all six methods 1A, 1B, 2A, 2B, 3A and 3B, and initial phase,  $\varphi_\phi$ , in methods 3A and 3B. Using the values  $\hat{\Phi}$  and  $\varphi_\phi$  obtained with the laser interferometer, the amplitude,  $\hat{\Omega}$ , and initial phase,  $\varphi_\Omega$ , of the angular velocity are calculated as given in Equations (B.10) and (B.11):

$$\hat{\Omega} = 2\pi \times f \hat{\Phi} \quad (\text{B.10})$$

$$\varphi_\Omega = \varphi_\phi - \frac{\pi}{2} \quad (\text{B.11})$$

and the amplitude,  $\hat{\alpha}$ , and initial phase,  $\varphi_\alpha$  of the angular acceleration are calculated as given in Equations (B.12) and (B.13):

$$\hat{\alpha} = 4\pi \times f^2 \hat{\Phi} \quad (\text{B.12})$$

$$\varphi_\alpha = \varphi_\phi - \pi \quad (\text{B.13})$$

The two interferometer types specified in this part of ISO 16063 use a laser of the red helium-neon type whose ultimate line in the emission spectrum has a wavelength of 0,632 81  $\mu\text{m}$ . For interferometer type A (i.e. retro-reflector interferometer), the displacement corresponding to the separation of two fringes (intensity maxima or intensity minima) as given in Equation (B.14) for the two-beam arrangement shown in Figures 3 and 5:

$$\Delta s = \lambda/4 \quad (\text{B.14})$$

or as given in Equation (B.15) for the single measuring beam arrangement shown in Figure 7:

$$\Delta s = \lambda/2 \quad (\text{B.15})$$

The measuring table of the angular exciter shall behave as a rigid body in the specified frequency range up to 1,6 kHz, so that the initial phase,  $\varphi_s$ , at the same time represents the initial phase,  $\varphi_\phi$ , as given in Equation (B.16):

$$\varphi_\phi = \varphi_s \quad (\text{B.16})$$

The two interferometer types A and B basically have in common that they sense a displacement at a point situated at a distance,  $R$  ("effective radius"), from the axis of rotation of the circular measuring table of the angular vibration exciter. From the displacement amplitude,  $\hat{s}$ , sensed by the interferometer, the rotational angle amplitude is obtained from Equation (B.17):

$$\hat{\Phi} = \frac{\hat{s}}{R} \quad (\text{B.17})$$

The displacement sensed by the interferometer and the effective radius are explained specifically for each of the two interferometer types.

For interferometer type A (i.e. retro-reflector), three systematic effects, in particular, which affect the measurement result as a function of the rotational angle, are taken into account.

The basic characteristic of the retro-reflector interferometer is the nonlinear function between the displacement amplitude,  $\hat{s}$ , sensed by the interferometer (i.e. the component acting in the direction of the incident beam) and the rotational angle amplitude (measured) as given in Equation (B.18):

$$\hat{s} = R \sin \hat{\Phi} \quad (\text{B.18})$$

For small rotational angle amplitudes,  $\hat{\Phi} \leq 3^\circ$ , Equation (B.18) can be replaced with Equation (B.19), which is approximately valid with a relative deviation  $< 0,05\%$ :

$$\hat{s} = R \cdot \hat{\Phi} \quad (\text{B.19})$$

If needed, a corrected result can be calculated using Equation (B.20):

$$\sin \hat{\Phi} \approx \hat{\Phi} - \frac{\hat{\Phi}^3}{3} \quad (\text{B.20})$$

The above equations are valid if the interferometer is ideally adjusted such that the two incident laser beams are parallel and the edges of the retro-reflectors as well as the axis of the rotary measuring table (i.e. moving part) are perpendicular to the plane formed by the two incident laser beams. A deviation,  $\varphi$ , from these directions causes a systematic error,  $\Delta s$ , because the path difference deviates by the factor,  $\cos \varphi$  (the so-called "cosine error"), from the displacement to be measured as given in Equations (B.21) and (B.22):

$$\Delta s = -s(\cos \varphi - 1) \quad (\text{B.21})$$

$$\Delta \hat{\Phi} = -\hat{\Phi}(\cos \varphi - 1) \quad (\text{B.22})$$

Moreover, when subjected to rotational motion, an angle,  $\delta$ , between the direction of the incident laser beam and the direction of motion of the retro-reflector occurs that causes a transversal shift,  $\Delta y$ , of the laser beam at the photodetector surface as given in Equation (B.23):

$$\Delta y = 2s \cdot \sin \delta \quad (\text{B.23})$$

As the laser light beam travelling from the (fixed) reference retro-reflector to the photodetector is not displaced, the interference pattern is shifted in the transverse direction. The retro-reflector interferometer described in this part of ISO 16063 (cf. Figures 3, 5 and 7) makes use of concentric circular fringes (i.e. divergence between measuring and reference laser light beams). It is advisable to minimize the transverse shift of the fringes by choosing as small a divergence as possible. This can be achieved using a telescope.

An obstacle to achieving greater rotational angle amplitudes, rather, is the interferometer maladjustment due to the parallel shift of the incident and reflected measuring laser light beams when the cube corner is rotated about the axis of the measuring table of the angular vibration exciter.

For interferometer type A, the radius,  $R$ , is not a precisely definable quantity. As a retro-reflector is used to prevent tilting from affecting the measurement results, the incident beam and the reflected beams are displaced with respect to one another. Another source of uncertainty is the determination of  $R$ .  $R$  is the distance between the two retro-reflectors in the two-beam arrangement or the distance between the retro-reflector and the axis of rotation in the single-beam arrangement.

For the interferometer type B (i.e. diffraction-grating interferometer), the displacement corresponding to the separation between two fringes (intensity maxima or intensity minima) for the two-beam arrangement shown in Figures 4 and 6 is given in Equation (B.24):

$$\Delta s = g/2 \quad (\text{B.24})$$

or for the single-beam arrangement shown in Figure 8 as given in Equation (B.25):

$$\Delta s = g \quad (\text{B.25})$$

where  $g$  is the grating constant (manufacturing period), e.g. the groove length of  $g = 1/(2\,400) \text{ mm} = 0,416\,6 \text{ }\mu\text{m}$  for a reflecting sine-phase grating with 2 400 grooves/mm<sup>[13]</sup>. Equations (B.19) and (B.20) are valid for interferences generated by the diffracted laser light beams of first order based on Equation (B.26):

$$k \frac{\lambda}{g} = \sin \alpha_L + \sin \beta_L \quad (\text{B.26})$$

where

- $k$  is the diffraction order ( $k = \pm 1$ );
- $\lambda$  the wavelength of the laser light;
- $\alpha_L$  angle of the incident laser beam;
- $\beta_L$  angle of the diffracted laser beam.

As the diffraction-grating interferometer senses the displacement described at any point of the lateral surface of the circular disk forming the measuring table (i.e. circular displacement), the rotational angle amplitude is limited by the angular range covered by the diffraction grating, unless the whole lateral surface (i.e. 360°) is covered by the diffraction grating, or by any technical limitation except for interferometer type B (e.g. maximum rotational angle of angular vibration exciter). It is another advantage compared to interferometer type A that the radius of the disk on which the diffraction grating is situated (lateral surface) is an accurately defined and measurable quantity. However, interferometer type B requires compliance with much tighter tolerances than does interferometer type A as regards the transverse motion (maximum eccentricity of 2  $\mu\text{m}$ ).

## B.2 Procedures for methods 1A and 1B

For both interferometer types A and B, and, thus, in methods 1A and 1B, the number of signal periods (e.g. intensity maxima) is as given in Equation (B.27):

$$N = 4\hat{s} / \Delta s \quad (\text{B.27})$$

which can be rewritten as Equation (B.28)

$$\hat{s} = \frac{\Delta s}{4} \times \frac{f_f}{f} \quad (\text{B.28})$$

where

- $f$  is the frequency of the angular vibration exciter;
- $f_f$  is the (mean) fringe frequency;
- $\Delta s = \lambda/2$ .

Inserting the expression for  $\Delta s$ , i.e. Equations (B.14) or (B.15) for the type A interferometer and Equations (B.19) or (B.20) for the type B interferometer, and using Equation (B.17) to transform the displacement into a rotational angle, the rotational angle amplitude,  $\hat{\phi}$ , is obtained. The angular velocity amplitude,  $\hat{\Omega}$ , and the angular acceleration amplitude,  $\hat{a}$ , are calculated from Equations (B.10) and (B.12), respectively. To calculate the sensitivity of an angular transducer, Equations (B.4), (B.6) and (B.8) are applied.

### B.3 Procedure for methods 2A and 2B

Considering the frequency spectrum of the intensity and adjusting the angular vibration amplitude to the level at which the component of the same frequency as the vibration frequency is zero, it is possible to calculate the displacement amplitude sensed by the interferometer, as given in Equation (B.29):

$$\hat{s} = x_n \times \frac{\Delta s}{2\pi} \quad (\text{B.29})$$

where  $x_n$  are the arguments corresponding to the zero points of the Bessel function as given in Table 1. To obtain the corresponding values of the rotational angle amplitude, Equation (B.17) is used. The angular velocity amplitude,  $\hat{\Omega}$ , and the angular acceleration amplitude,  $\hat{\alpha}$ , are calculated from Equations (B.10) and (B.12), respectively. To calculate the sensitivity of an angular transducer, Equations (B.4), (B.6) and (B.8) are employed.

### B.4 Procedure for methods 3A and 3B

When the angular vibration exciter generates a sinusoidal displacement at a spot sensed by the interferometer of type A or B at a distance,  $R$  (effective radius), from the axis of the rotary moving part (measurement table) as given in Equation (B.30):

$$s = \hat{s} \cos(\omega t + \varphi_s) \quad (\text{B.30})$$

then the angular transducer output signal follows the relationship given in Equation (B.31):

$$u = \hat{u} \cos(\omega t + \varphi_u). \quad (\text{B.31})$$

The output of the first photodetector can be written as Equation (B.32):

$$u_1(t) = \hat{u}_1 \cos \varphi_{\text{Mod}}(t) = \hat{u}_1 \cos[\varphi_0 + \hat{\varphi}_M \cos(\omega t + \varphi_s)] \quad (\text{B.32})$$

where the modulation phase is composed of the initial phase angle of the photodetector signal,  $\varphi_0$ , and a modulation term,  $\hat{\varphi}_M$ , as given in Equation (B.33):

$$\varphi_{\text{Mod}} = \varphi_0 + \hat{\varphi}_M \cos(\omega t + \varphi_s) \quad (\text{B.33})$$

The amplitude,  $\hat{\varphi}_M$ , of  $\varphi_M$  is proportional to the displacement as given in Equation (B.34):

$$\varphi_M = \hat{\varphi}_M \cos(\omega t + \varphi_s) \quad (\text{B.34})$$

where  $\hat{\varphi}_M$  is given by Equation (B.35):

$$\hat{\varphi}_M = 2\pi \frac{\hat{s}}{\Delta s} \quad (\text{B.35})$$

It is presupposed that there is no phase shift between the displacement,  $s(t)$ , and the sinusoidal phase term,  $\varphi_M(t)$ .

A second photodetector output that is in quadrature is then expressed as given in Equation (B.36):

$$u_2(t) = \hat{u}_2 \cos \varphi_{\text{Mod}}(t) = \hat{u}_2 \cos[\varphi_0 + \hat{\varphi}_M \cos(\omega t + \varphi_s)] \quad (\text{B.36})$$

where  $\hat{u}_2 = \hat{u}_1$ .

The quadrature signals are equidistantly sampled during a measurement time  $t_0 < t < t_0 + T_{\text{Meas}}$ . The series of measurement values,  $\{u_1(t_i)\}$  and  $\{u_2(t_i)\}$ , sampled within  $t_0 < t < t_0 + T_{\text{Meas}}$  have a sampling interval  $\Delta t = t_i - t_{i-1} = \text{const}$ .

From the two quadrature signals, the displacement values,  $s_i = s(t_i)$ , are successively calculated during the measurement time as given in Equation (B.37):

$$s_i = s(t_i) = \frac{\Delta s}{2\pi} \varphi_M(t_i) = \frac{\Delta s}{2\pi} \left( \arctan \frac{u_2(t_i)}{u_1(t_i)} + n\pi \right) \quad (\text{B.37})$$

where  $n = 0, 1, 2, \dots$

The required procedures, especially the calculation of the arctan function with successive “phase unwrapping” (see Reference [10]), are standard procedures in digital signal processing. From the values,  $s(t_i)$ , the parameters  $\hat{s}$  and  $\varphi_s$  of the displacement can be calculated by solving the system of  $N + 1$  equations of the form of Equation (B.38), which is linear with respect to the parameters:

$$s(t_i) = A \cos \omega t_i - B \sin \omega t_i + C \quad (\text{B.38})$$

where

$$n = 0, 1, 2, \dots;$$

$$A = \hat{s} \cos \varphi_s, \quad B = \hat{s} \sin \varphi_s; \quad (\text{B.39})$$

$C$  is a constant.

$N + 1$  denotes the number of samples synchronously taken over the above-mentioned measurement time. Parameters  $A$ ,  $B$  and  $C$  are calculated using the least-squares sum method (sine approximation). From their values, the displacement amplitude,  $\hat{s}$ , and the displacement phase,  $\varphi_s$ , can be obtained as given in Equations (B.40) and (B.41)

$$\hat{s} = \sqrt{A^2 + B^2} \quad (\text{B.40})$$

$$\varphi_s = \arctan \frac{B}{A} \quad (\text{B.41})$$

To obtain the corresponding values of the rotational angle amplitude and its initial phase, Equations (B.16) and (B.17) are used. The angular velocity amplitude,  $\hat{\Omega}$ , and the angular acceleration amplitude,  $\hat{\alpha}$ , are calculated from Equations (B.10) and (B.12), respectively. To calculate the magnitude and the phase shift of the complex sensitivity of an angular transducer, Equations (B.4) to (B.9) are used.

The above relationships, especially Equations (B.32) and (B.36), presuppose ideal conditions that are not fulfilled in reality. Under practical conditions, the signals from the photodetectors deviate with respect to their amplitudes,  $\hat{u}_1, \hat{u}_2$ , and the nominal phase shift,  $\pi/2$ , and different offsets,  $\hat{u}_{0,1}, \hat{u}_{0,2}$ , can occur. The influences from non-ideal quadrature signals within the tolerance ranges stated in 4.6 are briefly characterized in Note 1 of 10.2 (for a detailed investigation, cf. Reference [7]).

## Bibliography

- [1] VON MARTENS, H.-J., Interferometric counting methods for measuring displacements in the range  $10^{-9}$  to 1 m. *Metrologia*, **24**, No. 4, 1987, pp. 163-170
- [2] ROBINSON, D.C., SERBYN, M.R. AND PAYNE, B.F., A description of NBS Calibration Services in mechanical vibration and shock. *NBS Technical Note*, 1232, 1987
- [3] SCHMIDT, V.A., EDELMAN, S., SMITH, E.R. AND PIERCE, E.T., Modulated photoelectric measurement of vibration. *Journal of the Acoustical Society of America*, **34**, No. 4, 1962, pp. 455-458
- [4] CLARK, N.H., An improved method for calibrating reference standard accelerometers. *Metrologia*, **19**, 1983, pp. 103-107
- [5] LINK, A. AND VON MARTENS, H.-J., *Proposed primary calibration method for amplitude and phase response of accelerometers*. ISO/TC 108/SC 3/WG 6 N 59, Sept. 1995
- [6] HEYDEMANN, P.L.M., Determination and correction of quadrature fringe measurement errors in interferometers. *Applied Optics*, **20**, No. 19, 1981, pp. 3382-3384
- [7] LINK, A., GERHARDT, J. AND VON MARTENS, H.-J., Amplitude and phase calibration of accelerometers in the nanometer range by heterodyne interferometry. *SPIE*, **2868**, 1996, pp. 37-48
- [8] WABINSKI, W. AND VON MARTENS, H.-J., Time interval analysis of interferometer signals for measuring amplitude and phase of vibrations. *SPIE*, **2868**, 1996, pp. 166-177
- [9] SILL, R.D., Accelerometer calibration to 50 kHz with a quadrature laser interferometer. *Proceedings NCSL, Workshop & Symposium, Session 7B, Atlanta GA July 1997*, pp. 767-773
- [10] TRIBOLET, J.M., A new phase unwrapping algorithm. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, **ASSP-25**, No. 2 April 1977, pp. 170-177
- [11] VON MARTENS, H.-J., Investigations into the uncertainties of interferometric measurements of linear and torsional vibrations. *Shock and vibration*, **4**, No. 5/6, 1997, pp. 327-340
- [12] VON MARTENS, H.-J., AND TÄUBNER, A., Interferometric low-frequency calibration of translation and rotation quantity transducers. In: *Proceedings of the 7<sup>th</sup> International Meeting on Low Frequency Noise & Vibration*, Edinburgh, Great Britain, 1993, pp. 77-82
- [13] TÄUBNER, A., AND VON MARTENS, H.-J., Measurement of angular acceleration, angular velocities and rotation angles by grating interferometry, In: *Measurement* **24**, 1998, pp. 21-32
- [14] SCHLAAK, H.-J., Excitation of low-frequency rotational vibration. In: *Journal of Low Frequency Noise*, **12**, 1993, No. 4, pp. 123-127
- [15] VON MARTENS, H.-J., Generalization and analysis of the fringe-counting method for interferometric measurement of motion quantities. In: *Measurement* **25**, 1999, pp. 71-87
- [16] MARZOLF, J.G., AND BULL, S.J., Angle Measuring Interferometer. In: *The Review of Scientific Instruments*, Vol. **35**, No. 9, 1964, pp. 1212-1215
- [17] ISO 16063-11:1999, *Methods for the calibration of vibration and shock transducers — Part 11: Primary vibration calibration by laser interferometry*
- [18] ISO 5348, *Mechanical vibration and shock — Mechanical mounting of accelerometers*
- [19] BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML, *Guide to the expression of uncertainty in measurement (GUM)*, 1995



---

---

**ICS 17.160**

Price based on 42 pages