INTERNATIONAL STANDARD

ISO 16063-1

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Methods for the calibration of vibration and shock transducers —

Part 1:
Basic concepts

Méthodes pour l'étalonnage des transducteurs de vibrations et de chocs —

Partie 1: Concepts de base

Reference number
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Contents

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 16063-1 was prepared by Technical Committee ISO/TC 108, *Mechanical vibration and shock*, Subcommittee SC 3, *Use and calibration of vibration and shock measuring instruments*.

This first edition of ISO 16063-1 cancels and replaces ISO 5347-0:1987, of which it constitutes a minor revision. A new clause 6, new annex A, and an enlarged bibliography have been included.

ISO 16063 will consist of the following parts, under the general title *Methods for the calibration of vibration and shock transducers:*

— *Part 1: Basic concepts*

— *Part 2: Primary calibrations*

— *Part 3: Secondary calibrations*

— *Part 4: Environmental calibrations*

Parts 2 to 4 are under preparation and will consist of a revision of parts 1 to 23 of ISO 5347.

Annex A of this part of ISO 16063 is for information only.

Introduction

The calibration of vibration and shock transducers has become increasingly important as the need has grown for accurate measurements of the shocks and vibrations to which man and a wide variety of equipment are subjected in service. Several methods have been used or proposed for these calibrations and some of them are described in this part of ISO 16063. Clause 5 describes methods which have proved to be reliable means for the primary calibration of vibration and shock transducers.

Methods of calibration for both vibration and shock transducers are included in this International Standard because it has proved to be impracticable to make a distinction between transducers used in measurements of vibrations and those used in measurements of shocks.

This International Standard is limited to the calibration of acceleration, velocity and displacement transducers. It does not deal with transducers used for measurements of force, pressure or strain, even though some of these may be calibrated using similar methods. Furthermore, transducers used to measure rotational vibratory motion are also excluded because, at present, they are few in number and the calibration hardware and methods are somewhat different from those for the rectilinear transducers covered by this International Standard.

This part of ISO 16063 contains definitions and describes basic primary calibration. In addition, it describes, in general terms, various methods for the calibration of vibration and shock transducers as well as methods for measuring characteristics other than sensitivity. In order to be able to carry out a calibration with known accuracy, detailed specifications for instruments and procedures have to be laid down. Information of this kind for each method of calibration will be specified in subsequent parts of ISO 16063 (i.e. revisions of parts 1 to 23 of the ISO 5347 series).

The transducer may be calibrated as a unit by itself; it may include a cable connection and/or a conditioning device. The calibration system shall always be properly described.

A bibliography is included and the references are referred to in the text by numbers in square brackets.

Methods for the calibration of vibration and shock transducers —

Part 1
Basic concepts

1 Scope

This part of ISO 16063 describes methods for the calibration of vibration and shock transducers. It also includes methods for the measurement of characteristics in addition to the sensitivity.

One primary calibration method has been selected as the preferred method (see 5.2.1). Comparison calibration methods for vibration and shock are also described (see 5.3). More detailed descriptions are given in parts 1 to 23 of ISO 5347 (see references [1] to [22]).

This part of ISO 16063 is applicable to continuous-reading rectilinear acceleration, velocity and displacement transducers and recommends a preferred method which has proved to give reliable and reproducible results.

It is not applicable to methods for the calibration of rotational transducers.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this part of ISO 16063. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this part of ISO 16063 are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 1101:1983, *Technical drawings — Geometrical tolerancing — Tolerances of form, orientation, location and run-out — Generalities, definitions, symbols, indications on drawings.*

ISO 2041:1990, *Vibration and shock — Vocabulary.*

ISO 2954:1975, *Mechanical vibration of rotating and reciprocating machinery — Requirements for instruments for measuring vibration severity.*

GUM: *Guide to the Expression of Uncertainty in Measurement.* BIPM/IEC/IFCC/ISO/OIML/IUPAC,1995.

3 Terms and definitions

For the purposes of this part of ISO 16063, the terms and definitions given in ISO 2041, together with the following, apply.

3.1
transducer
device for converting the mechanical motion to be measured, for example acceleration in a given direction, into a quantity which may be conveniently measured or recorded

NOTE A transducer may include auxiliary equipment for amplifying, supplying necessary operating power, providing necessary circuit elements, indicating or recording its output, etc.

3.1.1
operating range
range of frequency and of amplitude for which the transducer behaves as a linear transducer within specified limits of tolerance

3.1.2
reciprocal transducer
bilateral electromechanical transducer for which the ratio of the applied current to force produced (when the transducer is restrained so the velocity is zero) equals the ratio of the applied velocity to the voltage produced (when the transducer is open-circuited so the current is zero)

EXAMPLES: Electromagnetic and piezo-electric transducers.

3.1.3
unilateral transducer
transducer employing strain gauges as sensing elements for which an electrical excitation does not cause a perceptible mechanical effect in the transducer

3.2
input signal
signal applied to the input of the transducer

EXAMPLE: The acceleration applied to the mounting surface.

3.3
output signal
signal generated by the transducer in response to a given input signal

NOTE 1 For single-ended transducers, the acceleration vector is considered positive when directed into the mounting surface of the transducer. For back-to-back reference accelerometers, the acceleration vector is considered positive when directed from the top surface into the accelerometer to be calibrated by comparison.

NOTE 2 The phase of the output quantity (e.g. voltage, charge, current, resistance, etc.) should be specified with reference to the defined positive acceleration vector or the derived quantities (velocity or displacement).

3.4
sensitivity
for a linear transducer, the ratio of the output to input during sinusoidal excitation parallel to a specified axis of sensitivity at the mounting surface

NOTE 1 In general, the sensitivity includes both amplitude and phase information and is, consequently, a complex quantity which varies with frequency.

The sinusoidal input motion may be represented by the following equations:

$$s=\hat{s}\exp\left[\mathrm{j}(\omega t+\varphi_1)\right]=\hat{s}\left[\cos(\omega t+\varphi_1)+\mathrm{j}\sin(\omega t+\varphi_1)\right] \quad (1)$$

$$v=\mathrm{j}\omega s=\hat{v}\exp\left[\mathrm{j}(\omega t+\varphi_1+\pi/2)\right]=\hat{v}\left[\cos(\omega t+\varphi_1+\pi/2)+\mathrm{j}\sin(\omega t+\varphi_1+\pi/2)\right] \quad (2)$$

$$a=\mathrm{j}\omega v=\hat{a}\exp\left[\mathrm{j}(\omega t+\varphi_1+\pi)\right]=\hat{a}\left[\cos(\omega t+\varphi_1+\pi)+\mathrm{j}\sin(\omega t+\varphi_1+\pi)\right] \quad (3)$$

$$u=\hat{u}\exp\left[\mathrm{j}(\omega t+\varphi_2)\right]=\hat{u}\left[\cos(\omega t+\varphi_2)+\mathrm{j}\sin(\omega t+\varphi_2)\right] \quad (4)$$

where

s is the complex quantity of the displacement;

v is the complex quantity of the velocity;

a is the complex quantity of the acceleration;

u is the complex quantity of the output;

$\hat{s}$ is the peak amplitude of sinusoidal displacement;

$\hat{v}$ is the peak amplitude of sinusoidal velocity;

$\hat{a}$ is the peak amplitude of sinusoidal acceleration;

ω is the angular frequency;

φ_1 and φ_2 are the phase angles;

t is the time;

j is the imaginary unit.

The displacement sensitivity, S_s, expressed in the units of the output signal per metre, is

$$S_s = \frac{u}{s} = \hat{S}_s \exp\left[-\mathrm{j}\left(\varphi_1 - \varphi_2\right)\right] \tag{5}$$

where

$\hat{S}_s = \frac{\hat{u}}{\hat{s}}$ is the magnitude of the displacement sensitivity;

$(\varphi_1 - \varphi_2)$ is the phase lag.

The velocity sensitivity, S_v, expressed in the units of the output signal per metre per second, is

$$S_v = \frac{u}{v} = \hat{S}_v \exp\left[-\mathrm{j}\left(\varphi_1 + \pi/2 - \varphi_2\right)\right] \tag{6}$$

where

$\hat{S}_v = \frac{\hat{u}}{\hat{v}}$ is the magnitude of the velocity sensitivity;

$(\varphi_1 + \pi/2 - \varphi_2)$ is the phase lag.

The acceleration sensitivity, S_a, expressed in the units of the output signal per metre per second squared, is

$$S_a = \frac{u}{a} = \hat{S}_a \exp\left[-\mathrm{j}\left(\varphi_1 + \pi - \varphi_2\right)\right] \tag{7}$$

where

$\hat{S}_a = \frac{\hat{u}}{\hat{a}}$ is the magnitude of the acceleration sensitivity;

$(\varphi_1 + \pi - \varphi_2)$ is the phase lag.

Usually, the displacement sensitivity is determined for a displacement transducer, the velocity sensitivity for a velocity transducer, and the acceleration sensitivity for an acceleration transducer. In general, the sensitivity magnitudes and the phase angles are functions of the frequency, $f = \omega/2\pi$.

NOTE 2 A displacement, velocity or acceleration transducer in which the corresponding sensitivity does not become zero as the frequency approaches zero is said to have a zero-frequency response (direct-current response). Sensitivity under constant acceleration corresponds to $\omega = 0$ and the phase lag is zero. Examples of transducers with zero-frequency response are acceleration transducers employing strain gauges, potentiometers, differential transformers, force-balance (servo) or variable reluctance circuits as sensing elements. Seismic self-generating transducers, such as piezo-electric and electrodynamic transducers, are examples of transducers without zero-frequency response.

3.5
transverse sensitivity ratio (TSR)
ratio of the output of a transducer, when oriented with its axis of sensitivity transverse to the direction of the input, to the output when the axis of sensitivity is aligned in the direction of the same input

3.6
vibration generator
any device for applying a controlled motion to the mounting surface of a transducer

NOTE Vibration generators are sometimes referred to as exciters or shakers.

4 Characteristics to be measured

4.1 General

The primary object of the calibration of a transducer is to determine its calibration factor (sensitivity) over the amplitude and frequency range for the degree of freedom for which the transducer is to be used. In addition, it may be important to know its response to motions in the other five degrees of freedom; for example, for a rectilinear acceleration transducer, its response should be known to motions at right angles to the sensitive direction and to rotations. Other important factors include damping, phase lag, non-linearity or variation in response with amplitude of motion, effect of temperature and pressure changes, and other extraneous conditions such as motion of the connection cable.

4.2 Direct response

4.2.1 Frequency response and phase response

The sensitivity of a transducer is obtained by placing the transducer with its sensitivity axis parallel to the direction of motion of the vibration generator, measuring the motion or input applied by the vibration generator, and measuring the output of the transducer. Both continuous-reading and peak-reading transducers can also be calibrated with a controlled transient excitation whose amplitude and frequency components are within the working range of the transducer. To detect any resonances, the output of the transducer should be observed while varying the vibration generator frequency slowly and continuously over the frequency range. In general, only information concerning magnitude sensitivity calibration is given as a function of frequency. However, for the use of a vibration transducer close to its upper or lower frequency limits, or for special applications, the phase response may be required. This is determined by measuring the phase lag between the output signal and the mechanical excitation over the frequency range of interest.

4.2.2 Non-linearity

Deviations from linearity of the output of a transducer (amplitude distortions) are determined by measuring its output magnitude as the magnitude of the input is increased from the smallest value to the largest value for which the transducer is designed. When a sinusoidal vibration generator is used, the measurement should be repeated for several frequencies.

Non-linearity may take several forms. The sensitivity of the transducer may change progressively with increasing amplitude, there may be a permanent change leading to a displacement of the zero after subjecting the transducer to vibration or shock, or there may be stops that limit the range of motion suddenly.

The type and magnitude of the non-linearity of a transducer may be indicated by its amplitude distortion and by comparing its resonance curve, its phase lag, and its decrement with the corresponding characteristics for the idealized linear transducer. The permissible deviations from linearity will depend on the measurements to be made. Non-linearity should be expected at the upper limit of the useful dynamic range of the transducer.

4.3 Spurious response

4.3.1 Temperature dependency

The sensitivity, damping ratio and resonance frequency of many transducers change as a function of temperature. Temperature response calibrations are usually performed using a comparison method. The standard transducer is mounted axially in line with the test transducer. The test transducer is placed inside a temperature chamber and the standard transducer is located outside the chamber or otherwise protected from changes in temperature in such a way that its sensitivity remains constant to within 2 % for the ambient temperatures present during the entire calibration. The vibration generator is used only at frequencies where it is known that the transverse motion is less than 25 % of the axial motion. The vibration generator is selected and a fixture designed so that there is negligible relative motion between the test and standard transducers at frequencies at which the calibration is to be performed.

An alternative procedure for performing temperature response calibrations is to mount the standard and test transducers on a suitable fixture inside the temperature chamber. This method is limited to temperature ranges over which the response of the standard transducer is known.

For transducers which respond to static acceleration, the zero unbalance is measured at the maximum and minimum temperatures.

Transducers with internal damping greater than 10 % of the critical damping should be calibrated at a minimum of four frequencies at a single vibration amplitude and at each of four temperatures in addition to room temperature. This method is equally applicable to transducers, such as the electrodynamic types, which utilize a coil of wire in their operation. The frequencies are selected throughout the frequency range of intended use.

The internal capacitance and resistance of piezo-electric transducers shall be measured after stabilization at the maximum calibration temperature.

If the measured resistance of a piezo-electric accelerometer at the maximum calibration temperature is so low that it affects the low-frequency response of the type of amplifier to be used, a low-frequency response calibration should be performed at that temperature. A number of frequencies shall be selected to describe adequately the frequency response. The calibration should be performed on the complete system, using the amplifier that is used with the accelerometer.

NOTE High temperature may affect the low-frequency response of the accelerometer as well as the noise and stability of the accelerometer-amplifier combination. Temperature response deviations are computed as the change in calibration factor determined at the test temperature referred to the room temperature (20 °C) calibration factor (measured at a frequency in the range of frequencies over which the transducer response is uniform). This change is expressed as a percentage of the room temperature calibration factor. It is usually desirable to select transducers which have temperature response deviations not exceeding +15 % throughout the temperature range of intended use.

4.3.2 Transient temperature sensitivity in piezo-electric transducers

Pyroelectric outputs are generated in all piezo-electric transducers subjected to transient temperatures. This is especially true for ferroelectric materials. The magnitude of the pyroelectric outputs depends upon the material constituting the crystal and the design of the transducer. Usually, the predominant frequency of the pyroelectric output is considerably less than 1 Hz. Also, most of the pyroelectric output from the transducer is filtered owing to the low-frequency characteristics of most amplifiers.

Accordingly, the pyroelectric output is dependent on the rate of change in temperature and on the characteristics of the amplifier, together with the characteristics of the transducer. The pyroelectric test is performed using the type of amplifier normally used with the transducer. The transducer is attached to an aluminium block by the usual means of attachment. Both are quickly immersed in an ice-water bath or a bath of other suitable liquid at a temperature which differs by approximately 20 °C from room temperature. The liquid in the bath should be described. The mass of the block should be approximately 10 times the mass of the transducer. Precautions are required to ensure that the liquid does not penetrate the transducer or that electrical leakage resistance is not lowered by the liquid at the connector, etc. The maximum amplifier output and the time from the start of the transient at which this maximum output is reached are measured on a direct-current oscilloscope or recorder. If the output reverses within the first 2 s and reaches a peak of opposite polarity, the magnitude and time of this peak are also recorded. For an accelerometer, the transient temperature sensitivity is expressed in equivalent metres per second squared per degree Celsius [(m/s^2)/°C] by dividing the maximum transducer output by the product of the difference between the bath temperature and room temperature and the accelerometer sensitivity.

For special applications using amplifiers having significantly different low-frequency characteristics, the pyroelectric test is performed with the specific amplifier to be used. Also, for applications in which the transient temperature rate differs greatly from that described by the above conditions, the test may be performed by simulating the particular temperature environment.

4.3.3 Transverse sensitivity ratio

The transverse sensitivity ratio (TSR) is usually determined at a single frequency below 500 Hz. The frequency used shall be reported. Sinusoidal motion is applied at a frequency at which it is known that the motion in a plane perpendicular to the sensing axis is at least 100 times the motion in the direction of the sensing axis. For transverse sensitivity ratios less than 1 %, the requirements for motion are more severe and extreme care and skill are required to obtain the value of the transverse sensitivity ratio.

The transducer is mounted and rotated about its sensing axis through 360°, in increments of 45° or less, to determine the maximum transverse response.

NOTE Experimental transverse sensitivity measurements on accelerometers indicate no detectable frequency dependence up to about 2 000 Hz. Only limited data are presently available regarding the transverse response within the frequency range from 2 000 Hz to 10 000 Hz. Several experimenters have stated that their measurement results usually indicate the high-frequency transverse response (that is, 2 000 Hz to 10 000 Hz) to be of the same order of magnitude as in a low-frequency determination (that is, less than 500 Hz). Generally, it is considered that for accelerometers whose axial resonance frequency is greater than 30 kHz, major transverse resonances will be greater than 10 kHz and, thus, beyond a transducer's normal operating range. For vibration transducers of other types, even less information is currently available. If possible, the lowest frequency of transverse resonance should be determined.

4.3.4 Sensitivity to rotational motion

Certain rectilinear vibration transducers are susceptible to rotational inputs. Examples of these include flexion-type piezo-electric and piezo-resistive accelerometers, and pendulum force-balance (servo) accelerometers. Attention is drawn to the existence of rotational sensitivity, and precautions may have to be taken to preclude a measurement error due to this effect. The rotational sensitivity of rectilinear vibration transducers can be determined by special methods developed for sensitivity calibrations of rotational vibration transducers (see reference [36]).

4.3.5 Strain sensitivity

The technique described below is the preferred method to determine the error produced in a transducer output due to bending of its base.

The transducer is mounted on a simple cantilever beam which produces a radius of curvature of 25 m and a strain of 250×10^{-6}.

A steel cantilever beam is clamped to a rigid support. The beam is 76 mm wide and 12,5 mm thick with a free length of 1 450 mm.

The natural frequency is very close to 5 Hz. The strain is measured by strain gauges bonded to the beam near the pickup mounting location about 40 mm from the clamped end. The motion at the mounting location can be checked by means of a transducer attached using extra isolation against base bending. A transducer with a calibration factor more than 10 times higher than the units under test is normally adequate. The outputs from the strain gauges and the transducer under test are recorded. The system is excited by manually deflecting the free end of the beam. The output of the transducer is recorded at a point where the strain in the surface of the beam is 250×10^{-6}. (This is equivalent to a radius of curvature of 25 m.) The error is the difference between the motion of the beam at the mounting location and the motion indicated by the transducer. The strain sensitivity, for a strain of 10^{-6}, is determined by dividing the above difference by 250.

The strain sensitivity should be tested at various strain amplitudes, in various directions. The maximum strain sensitivity of some transducers can produce significant errors in certain applications and mounting conditions. For example, some piezo-electric accelerometers produce error signals of several per cent at certain frequencies where strains are produced in vibration generators used for calibration purposes.

4.3.6 Magnetic sensitivity

The transducer is placed in a known magnetic field at 50 Hz or 60 Hz, and rotation of the transducer is started. The maximum electrical output of the transducer is recorded. For accelerometers, metres per second squared per tesla is recorded as the equivalent based on the sensitivity. For velocity transducers, metres per second per tesla over the useful frequency range is recorded as the equivalent. Induced mechanical vibrations and spurious electrical noise shall be eliminated from the test assembly.

4.3.7 Mounting torque sensitivity

The change in calibration factor due to transducer mounting torque is determined by applying torques of one-half the specified mounting torque, the specified, and twice the maximum specified. This test applies only to transducers that are mounted by screws, bolts, or other threaded fasteners. If more than one fastener is used in the normal mounting, the torques should be applied to each fastener.

Care should be taken to ensure that the transducer mounting surface is free from burrs or other surface defects which would prevent a flat mounting. The test surface to which the transducer is to be mounted should be flat and smooth and made from steel. The recommended values of flatness and roughness are a curvature less than 5 µm and an r.m.s. ground finish of 2 µm or better.

The test surface on which the transducer is to be mounted should be drilled and tapped square to the mounting surface with a perpendicularity of 0,05 mm or better (see ISO 1101). The interface lubrication normally recommended should be used and stated. The torque should always be applied from an unmounted condition, that is from zero torque for each of the three test torques. The torque sensitivity is recorded as the change in transducer calibration factor for one-half and twice the specified torque in relation to the specified torque. The uncertainty in the applied torque should not exceed ± 15 %.

4.3.8 Special environments

The operation of some transducers may be adversely affected in certain special environments, such as strong electrostatic, variable magnetic or radio-frequency fields, acoustic fields, in the case of cable effects, and nuclear irradiation. At present, there are no generally accepted techniques for measuring the effect of such special environments on a transducer, although special tests have been developed in instances where adverse effects could be expected (see ISO 2954).

5 Calibration methods

5.1 General

In order to perform a direct calibration of a transducer, it is necessary to use a vibration generator which applies a controllable and measurable input to the transducer and to provide a means for recording or measuring the output of the transducer. The transducer shall be attached to the vibration generator (or placed near it in the case of transducers whose output depends on the relative motion between the transducer and the vibrating object).

The attachment shall be sufficiently rigid to transmit the motion of the vibration generator to the transducer over the frequency range of the transducer. This requires that the natural frequency of the system, consisting of the transducer regarded as the mass and the attachment as the spring of a single-degree-of-freedom system, be high compared with the highest frequency component of the motion of the vibration generator. The vibration generator may be a support for tilting the transducer relative to the pull of gravity, a centrifuge, an electrodynamic vibration generator, or the anvil of a ballistic pendulum. The tilting support and centrifuge are used for calibration at zero frequency. Rotational calibration is used for low-frequency calibration for the Earth's gravitational field. The electrodynamic vibration generator is normally used for steady-state sinusoidal calibrations. Ballistic pendulums, which apply transient excitation, may be used as a complementary method to the electrodynamic vibration generator, to bring out natural frequency response and to permit calibration a high accelerations and velocities. In addition, shock excitation may be used to verify transducer performance for high accelerations and velocity changes and to check that auxiliary instrumentation connected to the transducer functions properly under transient conditions.

A number of calibration methods are described in this part of ISO 16063 and they may be used for special purposes. However, the use of a laser interferometer is recommended for primary calibration. Whenever possible, it is recommended that standard transducers be calibrated by this method, and if only one frequency is used, this should preferably be 160 Hz, 80 Hz, 16 Hz or 8 Hz depending on the application. Frequency response may be obtained by calibration at discrete frequencies over the frequency range of interest or as the frequency response relative to the sensitivity at the reference frequency with less accuracy. Most other calibration needs can be covered by comparison against a standard transducer having primary calibration. The calibration is always referred to the moving base of the transducer and, for "back-to-back" calibration standards, to the mounting base for the unknown transducer.

5.2 Primary calibration methods

5.2.1 Calibration by measuring displacement amplitude and frequency

5.2.1.1 General

Many dynamic calibration methods depend on the accurate measurement of the displacement amplitude of the vibration to which the transducer is subjected. This method is generally used for continuous-reading transducers. The sinusoidal motion applied by the vibration generator should be along a well-defined straight line; lateral motions should be negligible.

The measured displacements can be used to calculate velocities, v, and accelerations, a, using the formulae $\hat{v}=2\pi f\hat{s}$ and $\hat{a}=(2\pi f)^2\hat{s}$ which are derived by single and double differentation, respectively, for the sinusoidal displacement, s, and frequency, f. These formulae assume that the harmonic and noise content of the motion remains negligible even after the differentation. They emphasize the need for minimizing the distortion due to the electrical power sources or due to other causes such as mechanical resonance. Harmonics are also objectionable since they may excite resonant response in a transducer.

Once the displacement amplitude is known, the transducer sensitivity may be calculated as the ratio of the measured transducer output to the velocity or the acceleration amplitude. The displacement shall be measured by laser interferometry. The method is well described in references [23] to [28], [37] and [38].

The methods of sensitivity calculation based on displacement amplitude measurement by laser interferometry generally give good accuracy from 0,1 Hz to 10 kHz (corresponding to displacement amplitudes of 0,5 m to 20 nm). Special methods based on interferometric displacement measurement allow primary phase calibration to be performed in addition to the sensitivity calibration. As an alternative to laser interferometry based on displacement measurement, good accuracy in absolute sensitivity and phase calibration of vibration transducers may also be achieved by current-state laser doppler velocimetry [39]. Considerable errors in the measurement of displacement will occur if the reference mirror is perturbed at the frequency (or a harmonically related frequency) at which the accelerometer is vibrated. Error may also result from perturbation of the beam splitter. It is advisable to monitor for such pertubation using a very sensitive accelerometer.

5.2.1.2 Theory for the ideal interferometer

The principle of operation is shown in Figure 1, where E_0, E_1 and E_2 represent the electric field vectors, and l_1 and l_2 represent the actual path lengths the beams have to travel after the beamsplitter. The displacement to be measured is represented by s (mirror 2).

The electric field vectors E_1 and E_2 can be represented by the formulae

$$E_1 = A_1 \exp\left[\mathrm{j}\left(\omega t + \frac{4\pi}{\lambda} l_1\right)\right]$$

$$E_2 = A_2 \exp\left\{\mathrm{j}\left[\omega t + \frac{4\pi}{\lambda}(l_2 + s)\right]\right\}$$

where λ is the wavelength of the laser light.

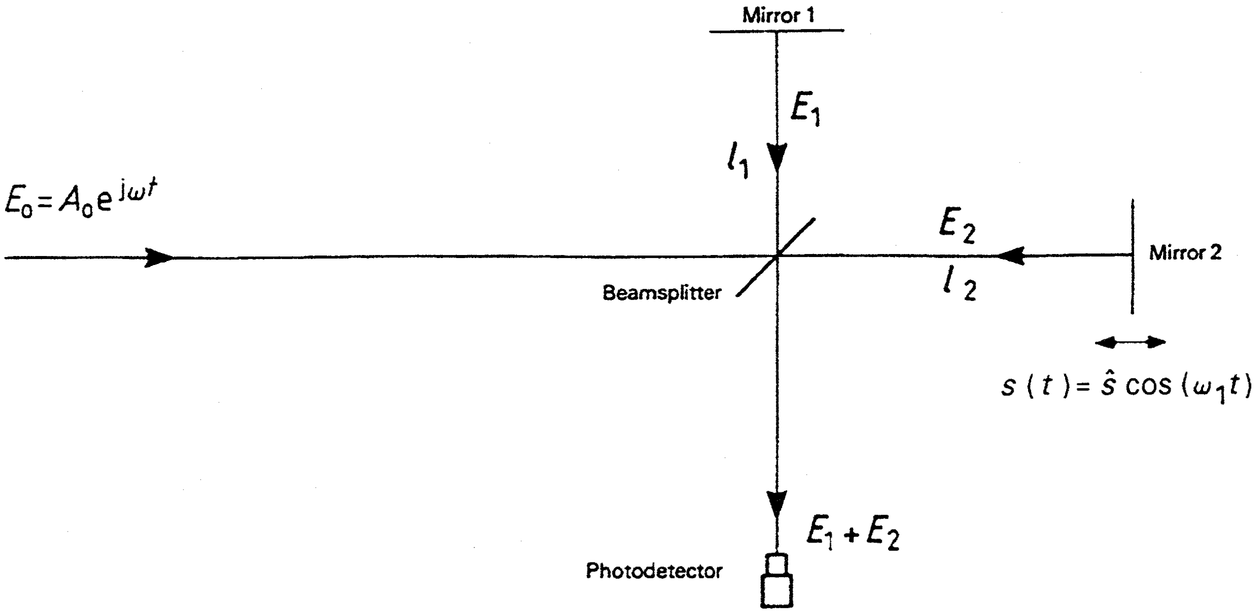

Figure 1 — Principle of the ideal interferometer

The intensity of the photodetector $I(t)$ is given by the formula

$$I(t)\approx|E_1+E_2|^2=A+B\cos\left[\frac{4\pi}{\lambda}(L+s)\right]$$

where

A and B are constants of the system;

$L=l_2-l_1$

From the intensity expression, it can be seen that the maxima will occur when

$$\frac{4\pi}{\lambda}(l_2-l_1+s)=2n\pi$$

and, therefore, the displacement corresponding to the distance between two intensity maxima is given by $\Delta s=\lambda/2$. The number of maxima, R_f, for one vibration cycle is then

$$R_f=4\hat{s}/(\lambda/2)=8\hat{s}/\lambda$$

which is commonly referred to as the "frequency ratio" because it can be calculated by dividing the number of fringes counted during 1 s by the vibration frequency.

The displacement amplitude, $\hat{s}$, is thus given by the formula

$$\hat{s}=R_f\cdot\lambda/8$$

If, in addition to the frequency ratio, the vibration frequency is measured, one can also compute the velocity and acceleration.

The same system can be used to measure displacement amplitude at frequencies outside the range recommended earlier for the fringe-counting method. Several other methods can be devised by considering the frequency spectrum of the intensity $I(t)$. As given in reference [23], the expansion gives

$$I(t)=A+B\cos\frac{4\pi L}{\lambda}\left[J_0\left(\frac{4\pi\hat{s}}{\lambda}\right)-2J_2\left(\frac{4\pi\hat{s}}{\lambda}\right)\cos(2\omega_1 t)+2J_4\left(\frac{4\pi\hat{s}}{\lambda}\right)\cos(4\omega_1 t)-\ldots\right]-$$
$$B\sin\frac{4\pi\hat{s}}{\lambda}\left[2J_1\left(\frac{4\pi\hat{s}}{\lambda}\right)\cos(\omega_1 t)-2J_3\left(\frac{4\pi\hat{s}}{\lambda}\right)\cos(3\omega_1 t)+\ldots\right]$$

The following two examples adequately illustrate the type of signal processing that is required here.

a) Adjusting the vibration amplitude to a level which makes the n^{th} harmonic component zero, one can solve the equation $J_n\left(\frac{4\pi}{\lambda}\hat{s}\right)=0$, to obtain $\hat{s}$.

b) In cases where it is not possible or practical to calibrate at amplitude levels required by the $J_n\left(\frac{4\pi}{\lambda}\hat{s}\right)=0$ method, one can extract the value of $\hat{s}$ from the ratio of two harmonic components, for example, by solving for $\hat{s}$ from

$$\frac{J_1\left(\frac{4\pi}{\lambda}\hat{s}\right)}{J_3\left(\frac{4\pi}{\lambda}\hat{s}\right)}=\frac{\hat{u}_1}{\hat{u}_3}$$

where $\hat{u}_1$ and $\hat{u}_3$ are the measured magnitudes of the first and third harmonics.

5.2.1.3 Measuring system

An example of a measuring system is shown in Figure 2. The transducer is a so-called reference transducer and the sensitivity shall be determined for the upper surface (reference mounting surface). The laser has an output power of 1 mW, and the detector is a normal silicon phototransistor. The pulse generator is used to obtain a well-defined signal for the counter input instead of the internal crystal oscillator. The frequency analyser is used to select the appropriate frequency when the zero-point method is used. The laser and the interferometer system and the vibrator system should be mounted on independent heavy vibration isolation blocks (for example each of mass more than 400 kg) to avoid perturbation of the reference mirror or the beam splitter by the reaction of the vibrator support structure.

5.2.2 Calibration by reciprocity method (see references [28] to [30])

Primary calibrations may also be carried out using the technique of reciprocity calibration. The reciprocity theory is applicable to the calibration of vibration standards in the amplitude range where the transducer's electrical output is linearly proportional to the motion of the vibration generator on which it is calibrated. The theory shows a reciprocity relationship for the driver coil of the vibration generator and equates the ratios of force/current and potential difference/velocity.

When the calibrator is energized with current in the driving coil at a specified frequency, the sensitivity, S_{uc} is defined as the ratio of the potential difference, u_{13} in volts, generated by the accelerometer, to the acceleration, $\hat{a}$ in metres per second squared, at the mounting surface, that is

$$S_{\text{uc}}=\frac{u_{13}}{\hat{a}} \qquad (8)$$

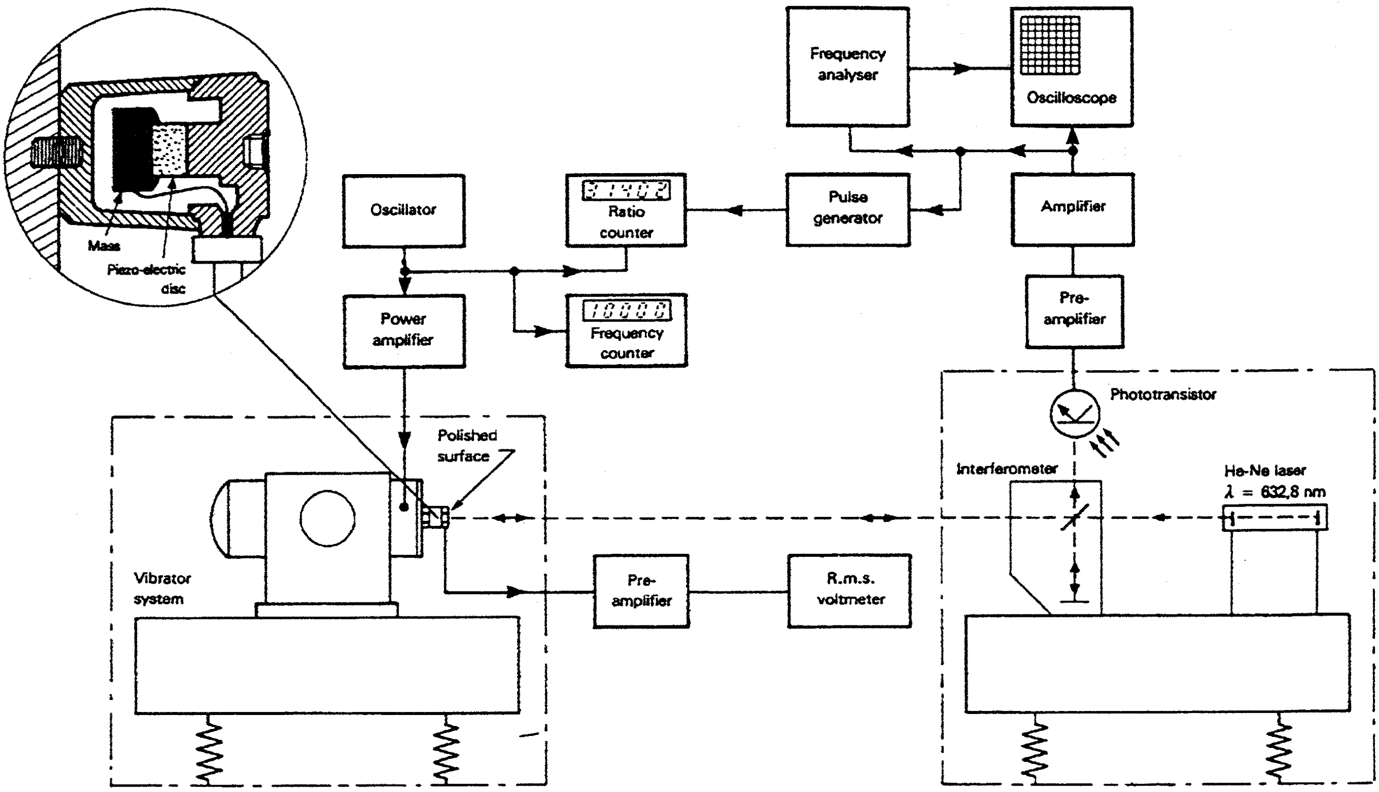

Figure 2 — Example of a measuring system using an interferometer

The object of the reciprocity method is to determine the sensitivity S_{uc} that when the potential difference u_{13} is measured, the acceleration $\hat{a}$ may be computed by use of equation (8). S_{uc} is determined from the following equation:

$$S_{uc} = S_0 + s_Z Z_m \tag{9}$$

where Z_m is the mechanical impedance of the transducer, in kilograms per second.

The quantities S_0 and s_Z are determined by the following two experiments and computational procedure.

Experiment 1

Several weights are attached to the mounting table. For each weight, and with no weight attached, the transfer admittance Y_e in amperes per volt, is measured between the driving coil and the accelerometer, and is given by the formula

$$Y_e = \frac{i}{u_{13}} \tag{10}$$

where

i is the current, in amperes, in the driving coil;

u_{13} is the potential difference, in volts, generated by the accelerometer.

Experiment 2

The moving parts of the calibrator are set into sinusoidal vibration by connecting the mounting table of the calibrator to a vibration generator and then energizing the vibration generator[1]. The ratio, u_{13}/u_{15}, of the potential, u_{13}, generated in the accelerometer to the open-circuit potential, u_{15}, generated in the driving coil is measured.

Computational procedure

Determine the ordinate intercept, J, and the slope, Q, of the function $W/(Y_{eW} - Y_{e0})$ when plotted against the mass W, of the weight attached to the mounting table in experiment 1, and where Y_{eW} is the value of Y_e with a weight of mass W attached, and Y_{e0}, is the value for $W = 0$. This plot is made by separating $W/(Y_{eW} - Y_{e0})$ into its real and imaginary parts from which the real and imaginary parts of J and Q are determined. The quantities S_0 and s_Z, in equation (9) are then given by

$$S_0 = \sqrt{\mathrm{j}\omega J\, u_{13} / u_{15}} \tag{11}$$

$$s_Z = \sqrt{(u_{13} / u_{15}) / \mathrm{j}\omega J} \tag{12}$$

where

ω is the angular frequency, in radians per second;

j is the imaginary unit.

5.2.3 Calibration by centrifuge

5.2.3.1 Single centrifuge

A centrifuge consists of a balanced table or arm which can be made to rotate about a vertical axis at a uniform angular velocity. With this device, an accurately known constant acceleration can be applied to an acceleration transducer for as long a time as is desired. Centrifuges capable of subjecting transducers to masses of several kilograms and to accelerations up to 6×10^5 m/s^2 have been built and centrifuges rated at lower accelerations are commercially available.

Only rectilinear acceleration transducers having zero-frequency response can be calibrated on a centrifuge.

To make a calibration, the acceleration transducer shall be mounted on the table or arm of the centrifuge with its axis of sensitivity carefully aligned on a radius of the circle of rotation. The acceleration acting on the transducer is

$$a = \omega^2 r \tag{13}$$

where

ω is the angular frequency, in radians per second, of the centrifuge;

r is the distance from the axis of rotation to the centre of gravity of the mass element of the transducer.

It is necessary to mount the transducer at such a distance from the axis of rotation that the deflection of the mass element of the transducer can be neglected in the determination of r.

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[1] Some electrodynamic vibration generators have been constructed having two drive coils mechanically connected to the armature and mounting table of the vibration generator. In this instance, it is not necessary to use a separate vibration generator.

Most transducers are so constructed that it is not easy to measure r directly. The value of r may be determined from readings taken for the transducer mounted at two positions separated by a known distance, Δr, while it is rotated on the centrifuge. It is good practice to adjust the speed so that approximately the same value of acceleration is applied to the transducer in both positions. The value of r in the second position, designated r_2, is

$$r_2 = \frac{\Delta r}{1 - \omega_2^2 u_1 / \omega_1^2 u_2} \tag{14}$$

where

ω_1 is the angular frequency in the first position, for which $r = r_1 = r_2 - \Delta r$;

ω_2 is the angular frequency in the second position, for which $r = r_2$;

u_1 is the transducer output due to the angular frequency ω_1;

u_2 is the transducer output due to the angular frequency ω_2.

With this value of r_2 and the angular frequency ω_2, the acceleration can be determined from equation (13).

The determination of r may be eliminated if the transducer has a linear range which extends down to the acceleration due to gravity, g. If this is the case, the transducer is first calibrated at $\pm g$ by the tilting support method. The transducer is then placed on the centrifuge and the angular frequency ω_1 at which the output is that corresponding to g is determined. The applied acceleration at another angular frequency, ω, is then

$$a = g\omega^2 / \omega_1^2 \tag{15}$$

The angular frequency, ω, has to be determined more accurately than the radial distance, r, because the applied acceleration varies as the square of this quantity. Most centrifuges that are designed for calibration purposes are equipped with a tachometer which gives a direct indication of the rate of rotation to within about 2 %. A much more accurate determination of the rate of rotation is possible either by stroboscopic means or by any one of several devices (for example a device using a photoelectric cell or a magnet) which produces pulses at a rate proportional to the speed. The pulse rate may be determined with an electronic counter.

In the calibration of electromechanical transducers on a centrifuge, leads are brought through slip rings and brushes. Since acceleration transducers with zero-frequency response are relatively low-impedance devices, shielding from external fields and cable noise presents no particular problem. The electrical noise from a worn-in slip-ring assembly of good design is negligible under normal circumstances. However, certain acceleration transducers using strain gauges as the sensing element contain only one or two active elements, the other resistances of the Wheatstone bridge circuit being added externally. With such a transducer, the entire bridge should be mounted on the rotating table to avoid false signals which would otherwise occur as a result of small changes in resistance of the slip-ring assembly. Alternatively, a Kelvin bridge circuit may be used.

In the calibration of transducers at small accelerations, gravity may have a significant effect if the transducer is sensitive to transverse accelerations. If possible, the transducer should be placed on the centrifuge with the axis of maximum transverse sensitivity of the transducer in a horizontal plane.

The sensitivity of an acceleration transducer at zero frequency can be determined with an expanded uncertainty of measurement of 1 % or better on a good centrifuge. Calibration on a centrifuge will, of course, give no indication of the usable frequency range.

5.2.3.2 Tilted centrifuge

When the axis of rotation of a centrifuge is not parallel with the Earth's gravitational vector, the acceleration applied to a transducer mounted on the centrifuge will have a gravitational component, $g \sin\theta \sin(\omega t)$, which will be added to the centripetal acceleration, $\omega^2 r$, where θ is the angle between the rotational axis and the vertical axis. The effect of the gravitational component on a transducer approximated by a spring-mass system is discussed in reference [31]. Sinusoidal acceleration can be applied to the transducer when $r = 0$ and $\theta = 90°$. In this instance, the acceleration due to gravity can be determined with minimal uncertainty. The limit is, of course, $\pm g$ at whatever frequency is produced by the rotation. The maximum frequency is generally less than several hundred hertz as limited by the centrifuge and balancing ability.

5.2.3.3 **Dual centrifuge** (see reference [32])

The dual centrifuge consists essentially of a small centrifuge eccentrically mounted on a large centrifuge made to rotate about this vertical axis in accordance with 5.2.3.1 (see Figure 3). The vibration transducer, A, is attached to the small centrifuge, which is driven independently. When the two centrifuges are driven at constant angular frequencies, the sensitivity axis of the transducer is alternatively changing its direction relative to the centre of the large centrifuge at a frequency, in hertz, equal to the angular frequency, in revolutions per second, of the small centrifuge relative to the large centrifuge. The component of acceleration, a, applied along the sensitive axis of the transducer at any time t is

$$a = r_{\mathrm{c}}\omega^2 \cos(\omega_{\mathrm{p}}t) + r(\omega \pm \omega_{\mathrm{p}})^2 \tag{16}$$

where

ω is the angular frequency, in radians per second, of the large centrifuge;

ω_{p} is the angular frequency, in radians per second, of the small centrifuge relative to the large centrifuge;

r_{c} is the distance between the centres of the two centrifuges;

r is the distance from the centre of gravity of the mass element of the transducer to the centre of the small centrifuge.

The sign in the last term of equation (16) is plus when ω and ω_{p} are either both clockwise or both counterclockwise, and minus when they are in opposite directions.

When the term $r(\omega \pm \omega_{\mathrm{p}})^2$ can be neglected, the applied acceleration along the sensitive axis of the transducer reduces to the sinusoidal term

$$a = r_{\mathrm{c}}\omega^2 \cos(\omega_{\mathrm{p}}t) \tag{17}$$

Thus, the component of acceleration applied along the sensitive axis of the transducer is sinusoidal. There is also a component of acceleration applied transverse to the sensitive axis, which renders this method non-applicable to transducers with high transverse sensitivity ratios. The term $r(\omega \pm \omega_{\mathrm{p}})^2$ is zero if ω and ω_{p} are equal but in opposite directions.

When a dual centrifuge of this type is built in the following manner, equation (17) is exact and the component of acceleration along the sensitive axis of the transducer should be purely sinusoidal. A pulley is fixed in space with its centre coincident with the centre of the large centrifuge. A pulley of the same size is fixed concentrically to the small centrifuge and connected to the other pulley by a belt. The large centrifuge is driven by a motor and the angular frequency of both centrifuges about their respective centres will always be equal and opposite in direction.

The dual centrifuge is useful for applying sinusoidal accelerations up to 500 m/s^2 in the approximate frequency range from 0,7 Hz to 10 Hz.

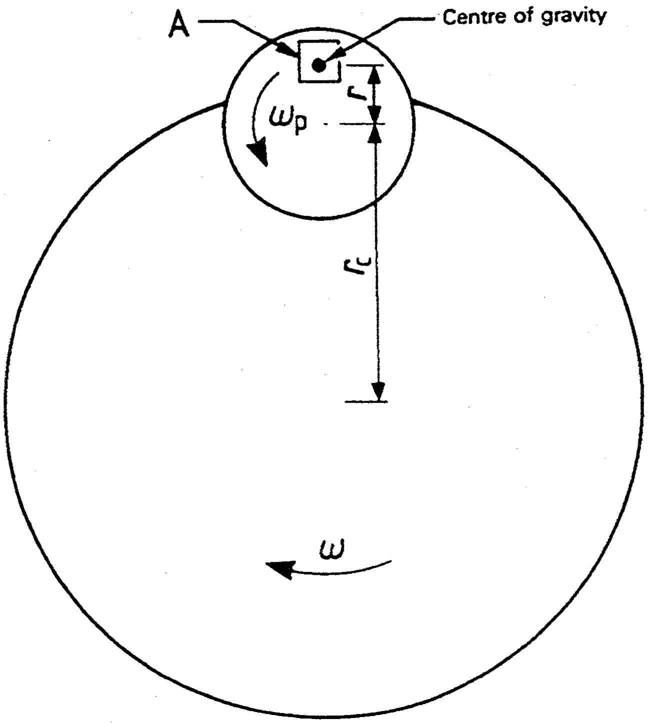

Figure 3 — Dual centrifuge

5.2.3.4 Tilting support calibrator

The tilting support calibrator utilizes the Earth's gravitational field for the calibration of rectilinear acceleration transducers with zero-frequency response and with negligible transverse sensitivity. It is useful over the range from $-g$ to $+g$. The transducer to be calibrated is fastened to a platform at the end of an arm which indicates the component of acceleration along the arm. The arm may be set at an angle, φ, relative to the vertical between 0 and 180°. It is furnished with a pointer to read off the angle, φ, from a divided circle. Care should be taken to level the base to which the transducer is attached in the position $\varphi = 0$. Positioning of the arm to ± 0,1° or better is possible with an accurately divided circle.

The component of acceleration along the arm is given by

$$a = g\cos\varphi \tag{18}$$

The change in acceleration corresponding to an angular displacement, $\Delta\varphi$, is therefore

$$\Delta a = g\sin\varphi\Delta\varphi \tag{19}$$

The acceleration transducer is subjected to a component of acceleration at right angles, a_t, to its sensitive direction equal to

$$a_\mathrm{t} = g\sin\varphi \tag{20}$$

Usually, this does not affect the results of the calibration of transducers with negligible transverse sensitivity.

NOTE Vibration isolation of the test apparatus may be required to achieve satisfactory results. Electronic filtering and averaging in direct-current voltage-sensing instrumentation may help to reduce the effects of bench-top vibration, etc.

5.2.4 Shock calibration method

To measure the time-varying acceleration, laser interferometry based on fringe counting and time interval measurement may be applied [40]. From the measured displacement values and the conjugate times, the acceleration is obtained through an interpolation polynomial and double differentation. A small measurement uncertainty of the acceleration peak value or of shock sensitivity of high quality accelerometers has been demonstrated by using a specially developed shock acceleration exciter whose hammer and anvil are airborne [41]. Most primary shock calibrations are based on the principle of change in velocity (see references [28] and [33]). This is because velocity is a physical parameter which can be measured practically. The usual configuration is to mount the transducer to be calibrated on an anvil suspended by some means in a resting position (see reference [34]). A hammer of some sort is then allowed to strike the anvil, thus generating a transient motion of the anvil. The impact shall be controlled so that the velocity change is not so rapid, or so slow, as to excite important frequency components outside of the response range of the instruments and rigid-body dynamics is a suitably accurate approximation. The velocity or acceleration transducer to be calibrated shall have a mass which is small compared with the anvil mass to which it is attached and should have its sensitivity direction carefully aligned with the direction of the impact force during collision. During impact, the accelerometer output versus time is recorded. Immediately following impact, the anvil velocity, Δv, is measured.

The velocity may be measured by timing the anvil over a known distance. Photoelectric or magnetic transducers can be used to trigger an electronic timer. The velocity is a direct result of the acceleration applied during impact:

$$\Delta v = \int_{t_1}^{t_2} a(t)\,\mathrm{d}t \tag{21}$$

where

Δv is the velocity increment, in metres per second;

$a(t)$ is the time-varying acceleration, in metres per second squared.

The accelerometer output, $u_r(t)$, is then

$$u_r(t) = S_r\, a(t) \tag{22}$$

where S_r is the sensitivity of the reference standard, in units of the output signal per metre per second squared.

Combining equations (21) and (22), and solving for S_r, gives

$$S_r = \left[\int_{t_1}^{t_2} u_r(t)\,\mathrm{d}t\right] / \Delta v \tag{23}$$

Equation (23) makes possible the calibration of a linear acceleration transducer from its recorded output during the ballistic impact. If the impact is against a linear spring, it will have the shape of a half-sine pulse of area $A = 0{,}637\ hb$ where h and b are the height and the width respectively of the pulse. The pulse shapes and durations are usually adjusted by varying mass, impacting mediums and some initial conditions such as drop height, air pressure, or other physical parameters depending on the nature of the shock generator.

Both techniques (impact against an anvil and impact against a linear spring) are practical to obtain the value of Δv as required for equation (23). The output of the acceleration transducer may be recorded during impact as a function of time by a high-speed oscillograph or from a storage-type oscilloscope and photographed. Potential difference scales and times scales can be verified by superimposing a known potential difference signal, u_c and a known time area, y_c and x_c, respectively. The potential difference, K_1, and time scale, K_2, factors are given by

$$K_1 = u_c / y_c \qquad K_2 = t_c / x_c \tag{24}$$

The area under the record of accelerometer output versus time is defined by

$$A = \int_{x_1}^{x_2} y\,\mathrm{d}x \tag{25}$$

where x_1 and x_2 denote the beginning and the end of the impact.

Substitution of these quantities into equation (23) results in

$$S_r = \frac{K_1 K_2 K A}{\Delta v} \tag{26}$$

The area, A, can be obtained by graphic integration of the recorded acceleration time history. A planimeter is useful in measuring the area under the acceleration-time record. Care has to be taken in the determination of area with regard to zero offsets, overshoot and ringing.

The integral shown in equation (23) may also be determined by electronic integration or by digital recording and summation techniques. This accelerates the calibration process and reduces the subjective errors and operator fatigue.

Based on the principle of change in velocity, primary shock calibrations of quality accelerometers can be performed with uncertainties of less than 5 % over most reasonable ranges of shock amplitudes and durations. An important assumption made is that the transducer being calibrated has linear frequency response in the frequency range of interest. If it does not, errors will result which are very difficult to evaluate. Furthermore, a single value of sensitivity is determined and does not in any way yield any practical information regarding frequency or phase response.

5.3 Comparison calibration methods

A vibration transducer calibrated by a primary calibration method may be used as a reference standard for the calibration of other transducers. The method described in 5.2.1 is the preferred method.

In making such a calibration, both the calibrated reference transducer 1 and the transducer 2 to be calibrated are subjected to the same input motion by suitably mounting the transducers, and their outputs, u_1 and u_2, or the ratio of the two outputs, are measured. If the two transducers sense the same vibration parameter, i.e. both sense velocity or both sense acceleration, and if the responses of both transducers 1 and 2 are linear, the amplitude sensitivity, S_2, of transducer 2 is related to the corresponding amplitude sensitivity, S_1, of the calibrated transducer 1 by

$$S_2 = \frac{u_2}{u_1} S_1 \tag{27}$$

Calibration by this comparison method is limited to the range of frequencies, time duration of pulses, and amplitudes for which transducer 1 has been calibrated. The complex sensitivity S_2 may also be determined if the phase response of transducer 1 is known and if the phase relationship between u_2 and u_1 is measured. If the two transducers sense different vibration parameters, for example if a velocity transducer is compared to an accelerometer, the sensitivity S_2 will be related by some power of $\mathrm{j}\omega$. Best results are generally obtained when the two transducers are rigidly mounted in a back-to-back configuration with the sensing axes parallel with the direction of motion. Care has to be taken to ensure that transducers 1 and 2 experience the same motion. If both transducers are rectilinear and are placed on the table of a vibration generator, the rocking motion of the table has to be negligible. Wave-form distortion is generally not as critical for comparison calibrations as for primary calibrations, especially if both transducers sense the same vibration parameter. However, the presence of harmonic components in the motion may increase the output voltages of the two transducers by different amounts, depending on the ratio of the sensitivities at the frequencies of the harmonics.

Practical calibration is normally carried out using an electrodynamic vibration generator. A back-to-back reference accelerometer is mounted on the vibration generator with the calibrated surface opposite the table. The unknown accelerometer is placed on the back-to-back reference accelerometer. For comparison at high frequencies, the reference accelerometer shall be calibrated with a mass load of the same order of magnitude as that of the unknown accelerometer.

The electrical output of the reference accelerometer preamplifier may be conveniently and precisely compared against the output of the unknown accelerometer using a precision amplifier with attenuator and comparing the signals by adjusting to the zero indication using a balance comparator meter.

It shall be noted that the sensitivity of the reference transducer may depend on the mass load at the surface where the unknown transducer is mounted. Therefore the sensitivity values for the reference transducer shall be known for the mass of the unknown transducer [35].

6 Expression of uncertainty of measurement

The uncertainty of measurement in calibration shall be expressed by the *expanded uncertainty* U in accordance with the *Guide to the Expression of Uncertainty in Measurement* (GUM), based on the approach recommended by the International Committee for Weights and Measures (CIPM).

Annex A
(informative)

Expression of uncertainty of measurement in calibration

A.1 General

See clause 6.

The purpose of U is to provide an interval $y - U$ to $y + U$ within which the value of Y, the specific quantity subject to calibration and estimated by y, can be expected to lie with high probability. To assert confidently that $y - U \leqslant Y \leqslant y + U$, determine the expanded uncertainty U as follows.

A.2 Calculation of expanded uncertainty of measurement

A.2.1 Every effort has to be made to identify each effect that significantly influences the measurement result and to compensate for such effects by applying the estimated corrections or correction factors.

If an effect influencing the measurement result is appropriately described by a probability distribution (preferably probability density, cf. A.2.2) having a significant expected value (in particular for an asymmetrical distribution), treat the latter as systematic error and compensate by correction.

A.2.2 Represent each component of uncertainty that contributes to the uncertainty of the measurement by a standard deviation u_i, termed *standard uncertainty*, equal to the positive square root of the variance u_i^2.

Some standard uncertainties may be obtained as statistically estimated standard deviations by the statistical analysis of series of observations (referred to as type A evaluation of standard uncertainty in GUM). Evaluate other standard uncertainties as standard deviation of a probability distribution describing the scientific judgement of all possible values of the respective quantity (type B evaluation of standard uncertainty). The judgement is based on all information available about the quantity. In particular, if there is no specific information about the possible values of a quantity responsible for systematic effects except that these values are within the bounds b_- and b_+, a uniform distribution over the interval $[b_-;\ b_+]$ may be used to represent this information. It has a standard uncertainty $b/\sqrt{3}$ where $b = (b_+ - b_-)/2$. The expected value $(b_+ + b_-)/2$ is to be used for correction in this case.

If an influence quantity can be considered uniformly distributed (rectangular probability density) but is known to be transformed into the measurement result with a specific nonlinear function (e.g. sinusoidal; polynomial of second or third order), take this information into account by choosing the associated distribution model.

EXAMPLE

The sensitivity S of an accelerometer to sinusoidal accelerations in the nominal measurement direction is calculated from the output, voltage or charge amplitude $\hat{x}$, stimulated by a vibration, acceleration amplitude $\hat{a}$, using the formula $S = \hat{x}/\hat{a}$. Among the various disturbing effects influencing the measurement result in calibration, there may be a significant transverse motion component from the vibration exciter, acceleration amplitude $\hat{a}_\mathrm{T}$, transformed into an error component $e_{\hat{x}\mathrm{T}}$ in the output, in conjunction with the accelerometer's transverse sensitivity, S_T. It is assumed for this example that the acceleration to be measured and the transverse acceleration have the same frequency and that there is no phase angle difference. As the transverse sensitivity is usually sinusoidally dependent on the angle β between the direction of maximum transverse sensitivity $(S_{\mathrm{T,max}})$ and the direction of a transverse excitation, the error component can be expressed by

$$e_{\hat{x}\mathrm{T}} = S_{\mathrm{T}}\,\hat{a}_{\mathrm{T}} = S_{\mathrm{T,max}}\,\hat{a}_{\mathrm{T,max}} \cos\beta \,.$$

If the values of the maximum transverse sensitivity $(S_{\mathrm{T,max}})$ and the maximum transverse acceleration $(\hat{a}_{\mathrm{T,max}})$ are known while the angle β is not, it is reasonable to assume a rectangular distribution of β within the interval $\lfloor -\pi;\pi \rfloor$. Thus, the influence quantity, i.e. transverse acceleration, with rectangularly distributed angle β leads to a measurement error component $e_{\hat{x}\mathrm{T}}$ whose probability density is described by

$$w(e_{\hat{x}\mathrm{T}}) = \frac{1}{b\pi\sqrt{1-\left(\frac{e_{\hat{x}\mathrm{T}}}{b}\right)^2}}, \qquad -b < e_{\hat{x}\mathrm{T}} < b, \qquad b = S_{\mathrm{T,max}}\,\hat{a}_{\mathrm{T,max}}$$

(often referred to as arcsin distribution). The associated standard uncertainty is

$$u(e_{\hat{x}\mathrm{T}}) = b/\sqrt{2}$$

The expected value $\mathrm{E}\{e_{\hat{x}\mathrm{T}}\}$ is zero in this case. This is the best estimate of the error $e_{\hat{x}\mathrm{T}}$.

A.2.3 Determine the *combined standard uncertainty* u_{c}, as the standard uncertainty of the measurement of Y, by combination of the individual standard uncertainties (and covariances as appropriate) using the law of propagation of uncertainty. Accordingly, the combined standard uncertainty is obtained from

$$u_{\mathrm{c}}(y) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i,x_j)} \tag{A.1}$$

This equation is based on a first-order Taylor series approximation of

$$Y = f(X_1, X_2, \ldots X_N) \tag{A.2}$$

where Y is the measurand determined from N input quantities X_1, X_2, ..., X_N through a functional relationship f.

An estimate of the measurand Y, denoted by y, is obtained from equation (A.1) using input estimates x_1, x_2, ..., x_N for the values of the input quantities. Thus the output estimate which is the result of measurement is given by

$$y = f(x_1, x_2, \ldots, x_N)\,. \tag{A.3}$$

In equation (A.1), the symbols $\partial f/\partial x_i$ are often referred to as sensitivity coefficients c_i. They are equal to the partial derivatives $\partial f/\partial X_i$ evaluated at $X_i = x_i$. Symbol $u(x_i,x_j)$ designates the estimated covariance associated with x_i and x_j.

For the case where no significant correlations are present, equation (A.1) is reduced to

$$u_{\mathrm{c}}(y) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)} \tag*{(A4)}$$

The first-order Taylor series approximation of equation (A.2) resulting in equation (A.1) is only applicable if the model function f is sufficiently linear with respect to the variation of the input estimates x_i within the ranges characterized by the uncertainties $u(x_i)$. This is not the case in the example given in A.2.2 if the angle β is considered to be an input quantity X_i. To overcome this obstacle which similarly exists with other influence quantities acting in measurements within calibrations of vibration and shock transducers, an adequate model has been introduced (cf. reference [42]). To briefly specify this model for the example above, a factor $(1 - e_{\hat{x}\mathrm{T}}/\hat{x})$ with

$e_{\hat{x}\mathrm{T}}/\hat{x} << 1$ is introduced, as an input quantity X_i, into the functional relationship used for calculating the measurand. Equation (A.2) specially tailored to the example, is reduced to three input quantities,

$$Y = f(X_1, X_2, X_3)$$

where Y designates the measurand (sensitivity S), X_1 the accelerometer output (voltage or charge amplitude $\hat{x}$), X_2 the acceleration amplitude and where $X_3 = (1 - e_{\hat{x}\mathrm{T}}/\hat{x})$. Thus, the relationship

$$Y = \frac{X_1}{X_2} X_3$$

can be established. The first Taylor series approximation can be used now, leading to the relative combined standard uncertainty

$$\frac{u_\mathrm{c}(y)}{y} = \sqrt{\left(\frac{u(x_1)}{x_1}\right)^2 + \left(\frac{u(x_2)}{x_2}\right)^2 + \left(\frac{u(x_3)}{x_3}\right)^2}$$

if there are no significant correlations.

Using the symbols introduced in the example, the latter relationship can be written as follows:

$$\frac{u_\mathrm{c}(S)}{S} = \sqrt{\left(\frac{u(\hat{x})}{\hat{x}}\right)^2 + \left(\frac{u(\hat{a})}{\hat{a}}\right)^2 + \left(\frac{u(e_{\hat{x}\mathrm{T}}/\hat{x})}{1}\right)^2}$$

where $u(\hat{x})/\hat{x}$ is the relative uncertainty of the output voltage amplitude measurement, $u(\hat{a})/\hat{a}$ the relative uncertainty in acceleration amplitude measurement and $u(e_{\hat{x}\mathrm{T}}/\hat{x}) = u(e_{\hat{x}\mathrm{T}})/\hat{x}$, with $u(e_{\hat{x}\mathrm{T}}) = b/\sqrt{2}$ as explained in the example. Accordingly, further factors whose deviations from the value 1 are similarly expressed by the relative error component of the respective quantity (e.g. voltage, acceleration, or the sensitivity as a whole) might be introduced as input quantities $(X_4, X_5, \ldots)$, allowing the variety of uncertainty sources to be taken into account separately.

A.2.4 Determine the *expanded uncertainty* U by multiplying u_c by a coverage factor k:

$$U = k\,u_\mathrm{c}$$

where a value of $k = 2$ is preferably to be used. If it can be assumed that the possible values of the calibration result are approximately normally distributed with approximate standard deviation u_c, the unknown value can be asserted to lie in the interval defined by U with a level of confidence, or probability, of approximately 95 %.

A.2.5 When reporting the result of the measurement y, the expanded uncertainty and the value of the coverage factor k used, if different from $k = 2$, shall be stated. In addition, the approximate coverage probability or level of confidence of the interval may be stated.

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