
**Geometrical product specifications
(GPS) — Filtration —**

**Part 22:
Linear profile filters: Spline filters**

*Spécification géométrique des produits (GPS) — Filtrage —
Partie 22: Filtres de profil linéaires: Filtres splines*



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ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.org
Web www.iso.org

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In other circumstances, particularly when there is an urgent market requirement for such documents, a technical committee may decide to publish other types of normative document:

- an ISO Publicly Available Specification (ISO/PAS) represents an agreement between technical experts in an ISO working group and is accepted for publication if it is approved by more than 50 % of the members of the parent committee casting a vote;
- an ISO Technical Specification (ISO/TS) represents an agreement between the members of a technical committee and is accepted for publication if it is approved by 2/3 of the members of the committee casting a vote.

An ISO/PAS or ISO/TS is reviewed after three years in order to decide whether it will be confirmed for a further three years, revised to become an International Standard, or withdrawn. If the ISO/PAS or ISO/TS is confirmed, it is reviewed again after a further three years, at which time it must either be transformed into an International Standard or be withdrawn.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TS 16610-22 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO/TS 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

- *Part 1: Overview and basic concepts*
- *Part 20: Linear profile filters: Basic concepts*
- *Part 22: Linear profile filters: Spline filters*
- *Part 29: Linear profile filters: Spline wavelets*
- *Part 31: Robust profile filters: Gaussian regression filters*
- *Part 32: Robust profile filters: Spline filters*
- *Part 40: Morphological profile filters: Basic concepts*

- *Part 41: Morphological profile filters: Disk and horizontal line-segment filters*
- *Part 49: Morphological profile filters: Scale space techniques*

The following parts are under preparation:

- *Part 21: Linear profile filters: Gaussian filters*
- *Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets*
- *Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets*
- *Part 30: Robust profile filters: Basic concepts*
- *Part 42: Morphological profile filters: Motif filters*
- *Part 60: Linear areal filters: Basic concepts*
- *Part 61: Linear areal filters: Gaussian filters*
- *Part 62: Linear areal filters: Spline filters*
- *Part 69: Linear areal filters: Spline wavelets*
- *Part 70: Robust areal filters: Basic concepts*
- *Part 71: Robust areal filters: Gaussian regression filters*
- *Part 72: Robust areal filters: Spline filters*
- *Part 80: Morphological areal filters: Basic concepts*
- *Part 81: Morphological areal filters: Sphere and horizontal planar segment filters*
- *Part 82: Morphological areal filters: Motif filters*
- *Part 89: Morphological areal filters: Scale space techniques*

Introduction

This part of ISO/TS 16610 is a geometrical product specification (GPS) Technical Specification and is to be regarded as a global GPS Technical Report (see ISO/TR 14638). It influences the chain links 3 and 5 of all chains of standards.

For more detailed information about the relation of this part of ISO/TS 16610 to the GPS matrix model, see Annex F.

This part of ISO/TS 16610 develops the terminology and concepts of spline filters.

The spline filter has an advantage over a conventional phase correct filter in that for open profiles, the ends of the measured profile are still usable.

.....

Geometrical product specifications (GPS) — Filtration —

Part 22:

Linear profile filters: Spline filters

1 Scope

This part of ISO/TS 16610 specifies spline filters for the filtration of profiles. It specifies in particular how to separate the long and short wave component of a profile.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO/TS 16610-1:2006, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic terminology*

ISO/TS 16610-20:2006, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

International vocabulary of basic and general terms in metrology (VIM). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 2nd ed., 1993

3 Terms and definitions

For the purposes of this document, the terms and definitions given in VIM, ISO/TS 16610-1 and ISO/TS 16610-20, and the following apply.

3.1

spline

linear combination of piecewise polynomials, with a smooth fit between the pieces

NOTE The degree of the spline is equal to the degree of the polynomial of the highest degree used, e.g. a cubic spline is made of cubic polynomials.

3.2

cardinal spline

basic function of the space of **splines** (3.1) with infinite support

3.3

natural spline

spline (3.1) which is a straight line beyond the end points

3.4 spline profile filter

linear profile filter based on **splines** (3.1)

NOTE For the purposes of this part of ISO/TS 16610, the result of the low-pass filtering is a spline.

4 Spline profile filters

4.1 General

For a spline filter to conform with this part of ISO/TS 16610, it shall satisfy the filter equations given in 4.3.2, for open profiles, and 4.3.3, for closed profiles.

NOTE A comparison with Gaussian filters is given in Annex B, and illustrative examples are given in Annex C. A concept diagram for the concepts of spline filters is given in Annex D, and the relationship to the filtration matrix model is given in Annex E.

4.2 Weighting function

The weighting function of a spline profile filter cannot be given by a simple closed formula. Filter equations are therefore used instead of weighting functions to describe the spline profile filters. However, where necessary, a numerical calculation of the weighting function for a spline profile filter is always possible. If the sampling interval Δx is small enough, and the spline profile filter is based on cardinal cubic splines, the weighting function $s(x)$, with a default value of $\beta = 0$, can be approximated by the continuous function

$$s(x) = \frac{\pi}{\lambda_c} \sin\left(\sqrt{2} \frac{\pi}{\lambda_c} |x| + \frac{\pi}{4}\right) \exp\left(-\sqrt{2} \frac{\pi}{\lambda_c} |x|\right) \tag{1}$$

NOTE This is the weighting function of the ideal spline profile filter, based on cardinal cubic splines. A graph of this function is shown in Figure 1.

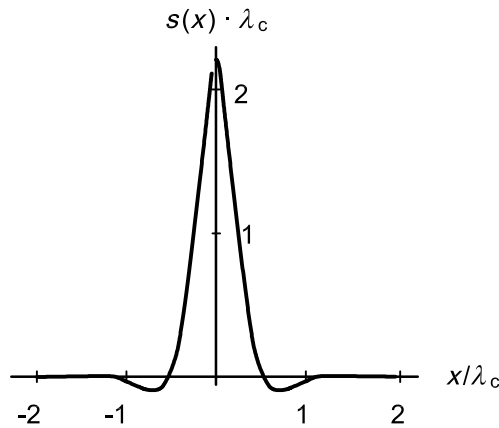


Figure 1 — Weighting function of the ideal spline profile filter, based on cardinal cubic splines

4.3 Filter equations

4.3.1 General

The following filter equations for spline profile filters are based on cardinal cubic splines (for more details, see [1] and [2]).

4.3.2 Filter equation for the non-periodic spline profile filter

Non-periodic spline profile filters should be used in cases of filtering open profiles. The filter equation for the non-periodic spline profile filter is given by

$$\left[1 + \beta\alpha^2 P + (1 - \beta)\alpha^4 Q\right] w = z \quad (2)$$

with the matrix

$$P = \begin{pmatrix} 1 & -1 & & & & & & & \\ -1 & 2 & -1 & & & & & & \\ & -1 & 2 & -1 & & & & & \\ & & \ddots & \ddots & \ddots & & & & \\ & & & -1 & 2 & -1 & & & \\ & & & & -1 & 2 & -1 & & \\ & & & & & -1 & 1 & & \end{pmatrix} \quad Q = \begin{pmatrix} 1 & -2 & 1 & & & & & & & & & \\ -2 & 5 & -4 & 1 & & & & & & & & \\ 1 & -4 & 6 & -4 & 1 & & & & & & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & & & & & & \\ & & & 1 & -4 & 6 & -4 & 1 & & & & \\ & & & & 1 & -4 & 5 & -2 & & & & \\ & & & & & 1 & -2 & 1 & & & & \end{pmatrix} \quad (3)$$

with n rows, n columns and the parameters

$$\alpha = \frac{1}{2 \sin \frac{\pi \Delta x}{\lambda_c}} \quad \text{and} \quad 0 \leq \beta \leq 1 \quad (4)$$

where

- n is the number of extracted data values of the profile;
- z is the vector of dimension n of the profile values before filtering;
- w is the vector of dimension n of the profile values in the filtered profile;
- λ_c is the cut-off wavelength of the profile filter;
- Δx is the sampling interval.

NOTE 1 The vector w gives the profile values of the long wave component. The short wave component r can be obtained by the difference vector $r = z - w$, i.e. by subtracting the long wave component values obtained by the filtering process from the extracted data values of the profile.

NOTE 2 β is called the tension parameter, and controls how tightly the spline curve fits through the data points.

NOTE 3 Algorithms to solve matrix equations can be found in [6].

4.3.3 Filter equation for the periodic spline profile filter

Periodic spline profile filters should be used in cases of filtering closed profiles. The filter equation for the periodic spline profile filter is given by

$$\left[1 + \beta \alpha^2 \tilde{P} + (1 - \beta) \alpha^4 \tilde{Q} \right] \tilde{w} = \tilde{z} \tag{5}$$

with the matrix

$$\tilde{P} = \begin{pmatrix} 2 & -1 & & & -1 \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & \ddots & \ddots & \ddots & \ddots \\ & & & -1 & 2 & -1 \\ -1 & & & & -1 & 2 & -1 \end{pmatrix} \quad \tilde{Q} = \begin{pmatrix} 6 & -4 & 1 & & & 1 & -4 \\ -4 & 6 & -4 & 1 & & & 1 \\ 1 & -4 & 6 & -4 & 1 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & 1 & -4 & 6 & -4 & 1 \\ 1 & & & 1 & -4 & 6 & -4 \\ -4 & 1 & & & 1 & -4 & 6 \end{pmatrix} \tag{6}$$

with n rows, n columns and the parameters

$$\alpha = \frac{1}{2 \sin \frac{\pi \Delta x}{\lambda_c}} \text{ and } 0 \leq \beta \leq 1 \tag{7}$$

where

- n is the number of extracted data values of the profile;
- \tilde{z} is the vector of dimension n of the profile values before filtering;
- \tilde{w} is the vector of dimension n of the profile values in the filtered profile;
- λ_c is the cut-off wavelength of the profile filter;
- Δx is the sampling interval.

NOTE 1 The vector \tilde{w} gives the profile values of the long wave component. The short wave component \tilde{r} can be obtained by the difference vector $\tilde{r} = \tilde{z} - \tilde{w}$, i.e. by subtracting the long wave component values obtained by the filtering process from the extracted data values of the profile.

NOTE 2 β is called the tension parameter, and controls how tightly the spline curve fits through the data points.

NOTE 3 Algorithms to solve matrix equations can be found in [6].

4.4 Transmission characteristics

4.4.1 General

The following transmission characteristics for spline profile filters are based on cardinal cubic splines. For more details, see [3]. For the influence of the sampling interval, see Annex A.

4.4.2 Transmission characteristic of the long wave profile component

The filter characteristic (see Figure 2) is determined from the filter equations of the spline profile filter by means of the Fourier transformation. The filter characteristic for the long wave component is approximated (for very small Δx) by the following equation:

$$\frac{a_1}{a_0} = \left[1 + \beta \alpha^2 \sin^2 \frac{\pi \Delta x}{\lambda} + 16(1 - \beta) \alpha^4 \sin^4 \frac{\pi \Delta x}{\lambda} \right]^{-1} \quad (8)$$

where

a_0 is the amplitude of the sinusoidal profile before filtering;

a_1 is the amplitude of the sinusoidal profile in the long wave component;

λ is the wavelength of the sine profile ($\lambda \geq 2\Delta x$);

Δx is the sampling interval.

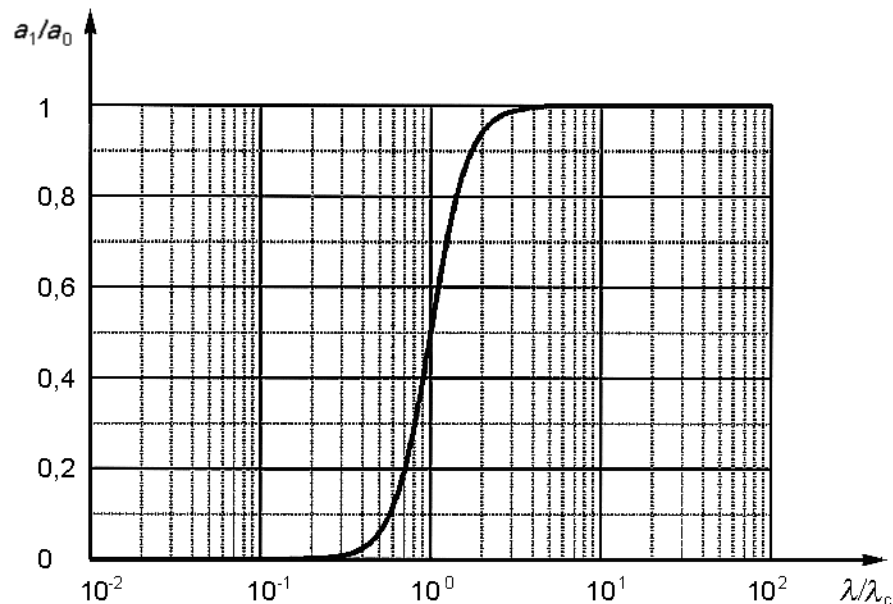


Figure 2 — Transmission characteristic for the long wave profile component ($\beta = 0$; $\Delta x = \lambda_c / 200$)

4.4.3 Transmission characteristic of the short wave profile component

The transmission characteristic of the short wave profile component is complementary to the transmission characteristic of the long wave profile component. The short wave profile component is the difference between the surface profile and the long wave profile component. The filter characteristic for the short wave profile component (see Figure 3) has the following equation:

$$\frac{a_2}{a_0} = 1 - \frac{a_1}{a_0} = \left[4\beta \alpha^2 \sin^2 \frac{\pi \Delta x}{\lambda} + 16(1 - \beta) \alpha^4 \sin^4 \frac{\pi \Delta x}{\lambda} \right] \left[1 + \beta \alpha^2 \sin^2 \frac{\pi \Delta x}{\lambda} + 16(1 - \beta) \alpha^4 \sin^4 \frac{\pi \Delta x}{\lambda} \right]^{-1} \quad (9)$$

where

a_2 is the amplitude of the sinusoidal profile short wave component.

NOTE The sum of the transmission characteristics of the short wave profile component and the long wave profile component equals one.

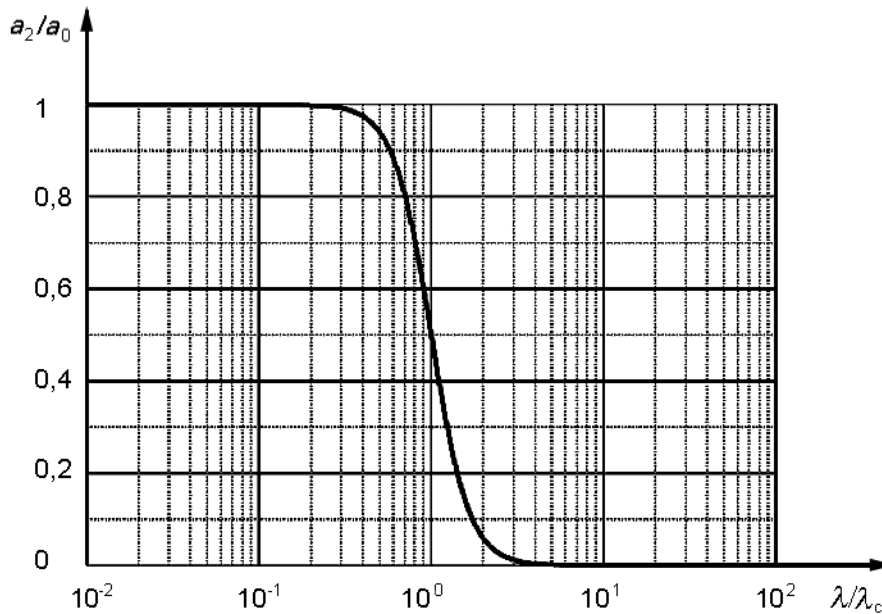


Figure 3 — Transmission characteristic for the short wave profile component ($\beta = 0; \Delta x = \lambda_c / 200$)

5 Recommendations

5.1 Nesting Index (cut-off values)

It is recommended that the nesting index (the cut-off value λ_c) be chosen from a logarithmic series (constant ratio) of values. Experience has shown that a constant ratio of approximately the square root of ten between successive scale values is optimum. The nesting index should be chosen from the following series of values:

... 2,5 μm ; 8 μm ; 25 μm ; 80 μm ; 250 μm ; 0,8 mm; 2,5 mm; 8 mm; 25 mm;

5.2 Tension parameter (β)

If not otherwise specified, the tension parameter β takes the value zero.

5.3 Implementation

It is strongly recommended that the matrix equations given in 4.3.2 and 4.3.3 be used to implement the spline filter.

6 Filter designation

Spline filters in accordance with this part of ISO/TS 16610 are designated

FPLS

See also ISO/TS 16610-1:2006, Clause 5.

Annex A (informative)

Influence of the sampling interval

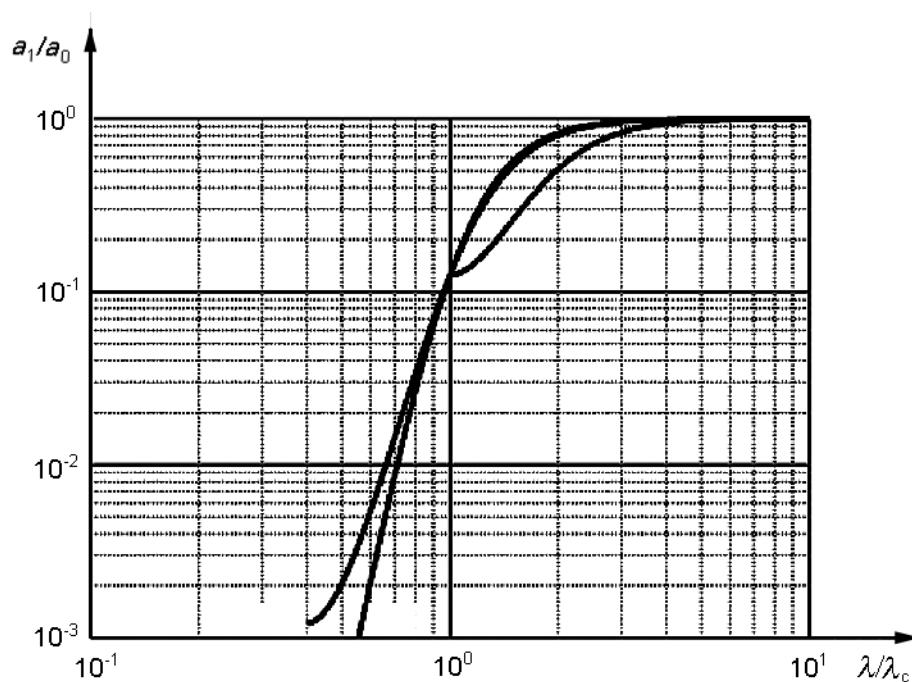
The transfer characteristic of the spline profile filter, as for every digital filter, depends on the sampling interval Δx (see Figure A.1). If Δx is small enough, the sine in all equations may be replaced by its argument.

EXAMPLE The error for $\Delta x = 0,01\lambda_c$ is $\varepsilon < 5,2 \times 10^{-6}$

Consequently, the equation for the parameter α in 4.2.3 simplifies to

$$\frac{a_1}{a_0} = \left[1 + \beta \left(\frac{\lambda_c}{\lambda} \right)^2 + (1 - \beta) \left(\frac{\lambda_c}{\lambda} \right)^4 \right]^{-1} \quad (\text{A.1})$$

i.e. it no longer depends on Δx . However, according to the sampling theorem (see ISO/TS 14406), this is only valid for $\lambda \geq 2\Delta x$.



NOTE According to the sampling theorem, $\lambda / \lambda_c \geq 2\Delta x / \lambda_c$ is valid.

Figure A.1 — Influence of the sampling interval Δx on the transfer function of the spline profile filter for the parameters $\beta = 0$; $\Delta x / \lambda_c$: 0,1; 0,2 and 0,5

Annex B (informative)

Comparison of spline profile filter and Gaussian filter

In order to compare the transmission characteristics of the spline profiler filter and the Gaussian filter, a graphical representation of the deviation, in accordance with ISO 11562:1996, is given (see Figure B.1).

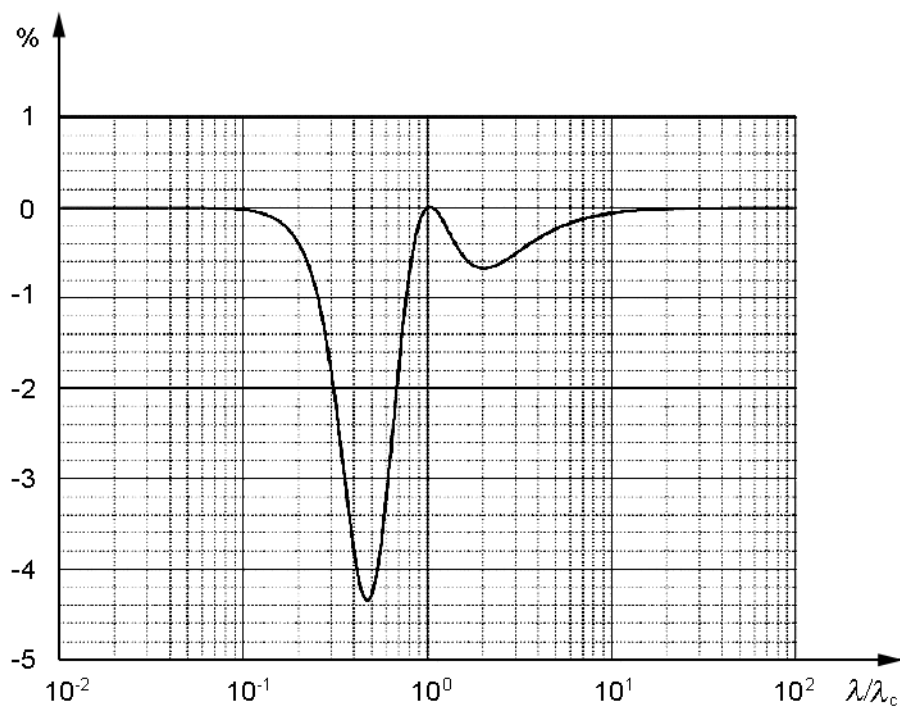


Figure B.1 — Deviation of the transmission characteristic of the spline profile filter from the Gaussian filter ($\beta = 0,625242$; $\Delta x = \lambda_c / 200$)

Annex C (informative)

Illustrative examples

The following examples for the application of the spline profile filter and the Gaussian filter (currently standardized profile filter) are given for information purposes.

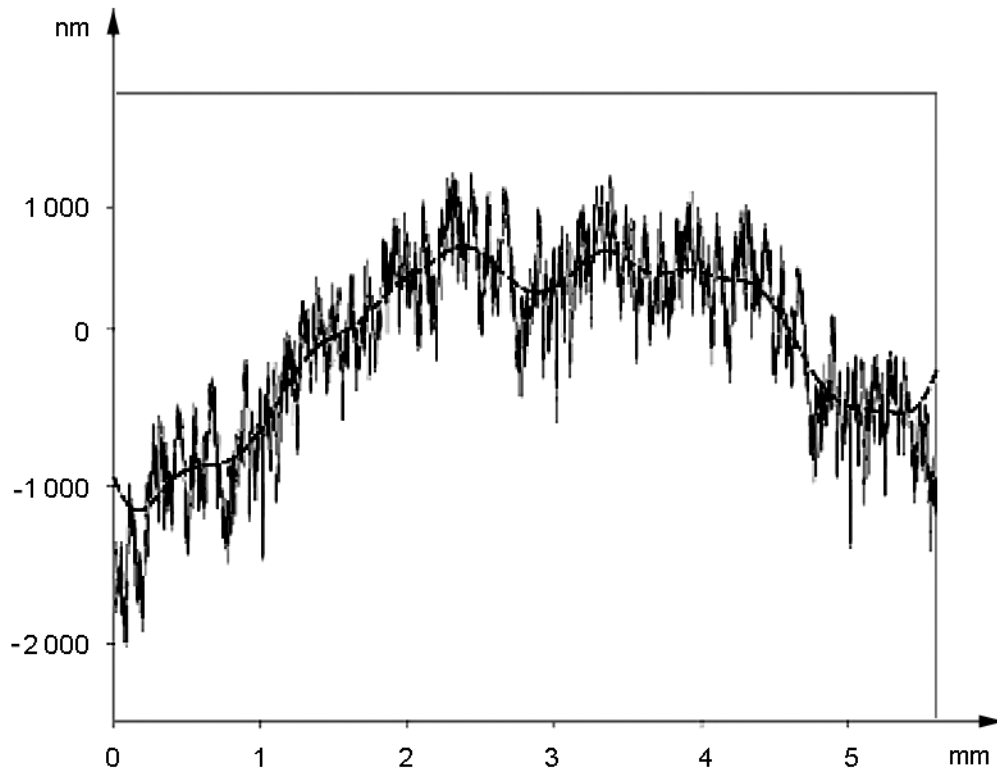


Figure C.1 — Gaussian filter with $\lambda_c = 0,8$ mm, in accordance with ISO 11562:1996

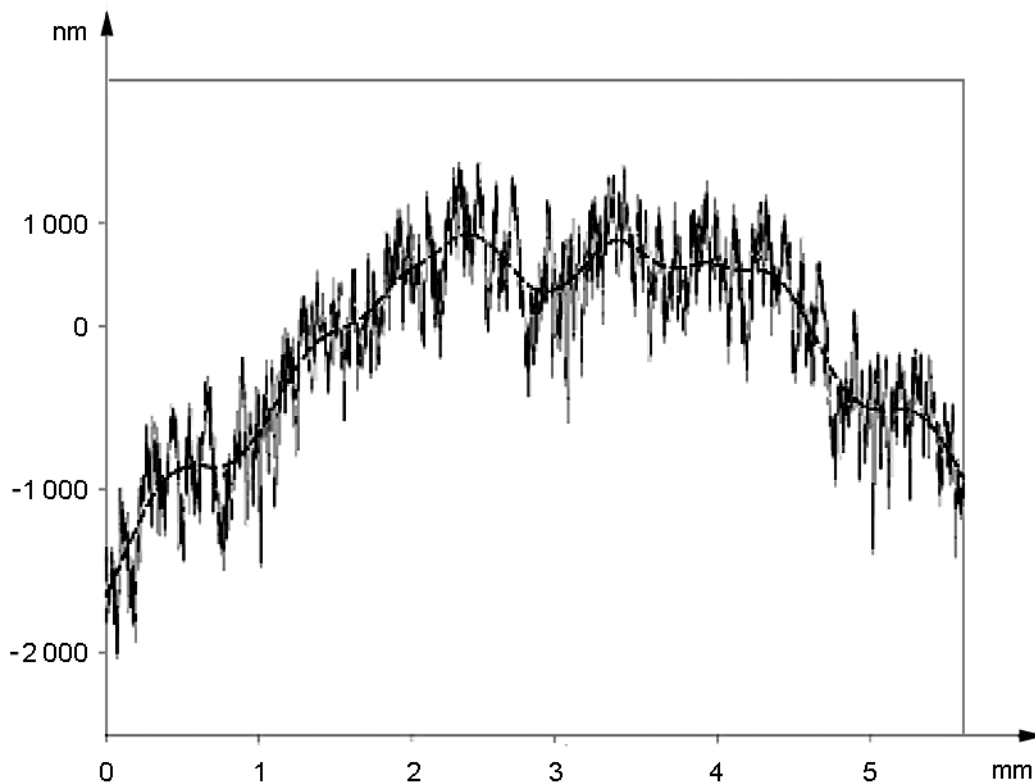


Figure C.2 — Spline profile filter with $\lambda_c = 0,8$ mm and $\beta = 0$

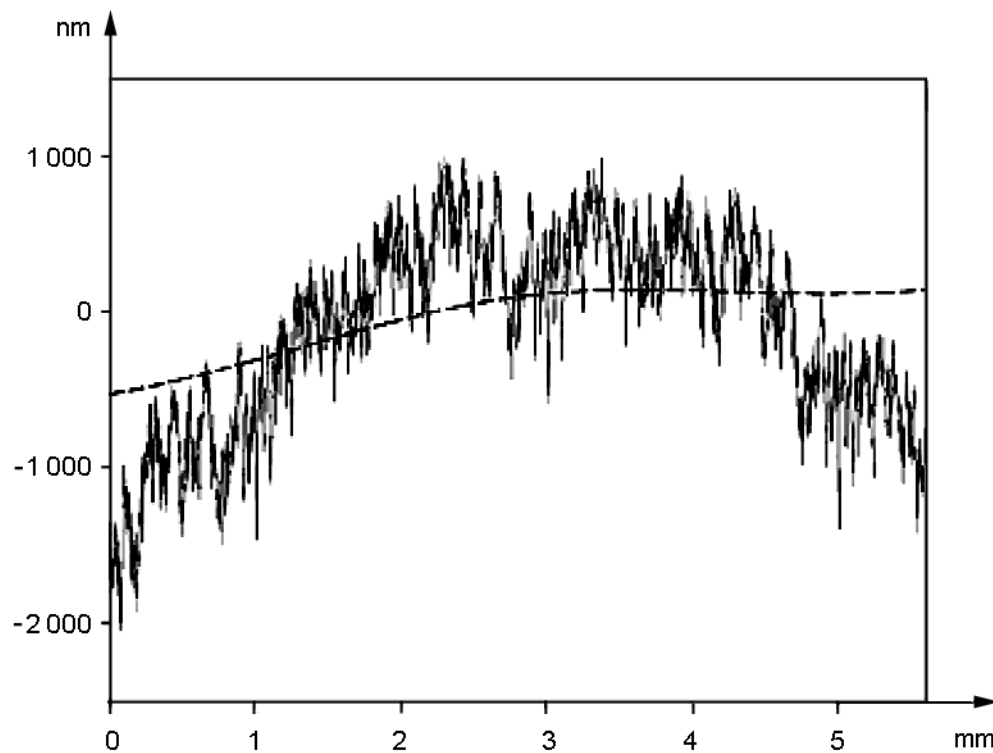


Figure C.3 — Gaussian filter with $\lambda_c = 8$ mm, in accordance with ISO 11562:1996

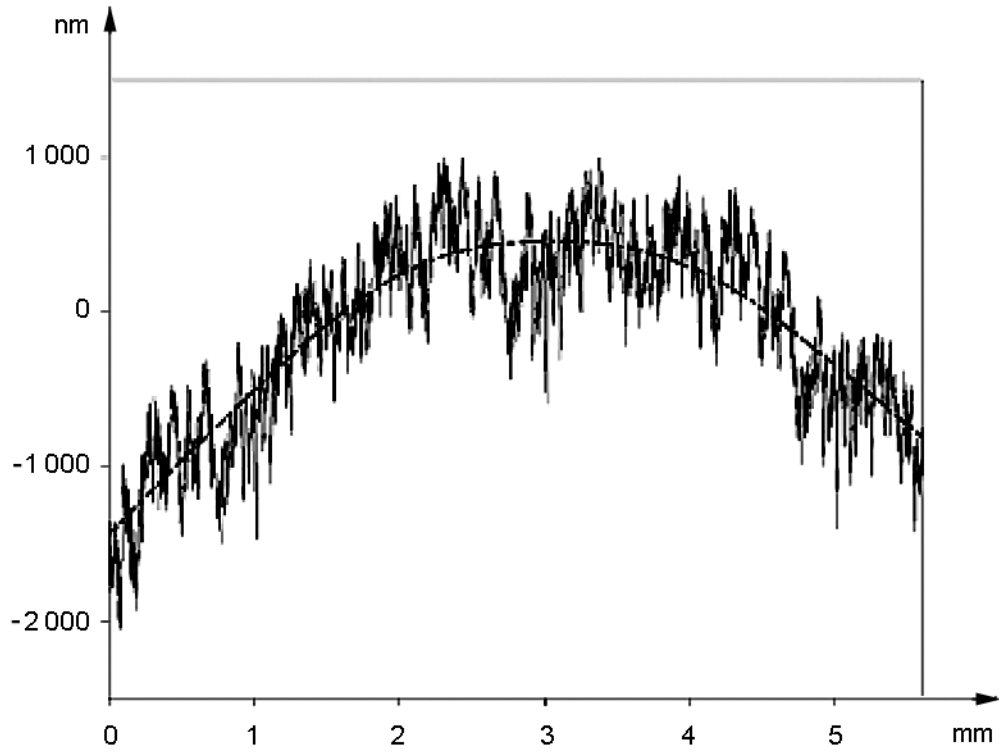
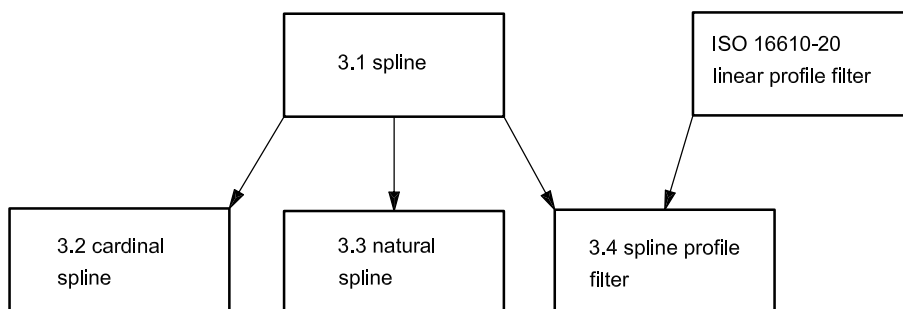


Figure C.4 — Spline profile filter with $\lambda_c = 8$ mm and $\beta = 0$

Annex D (informative)

Concept diagram

The following is a concept diagram for this part of ISO/TS 16610.



Annex E (informative)

Relationship to the filtration matrix model

For full details about the filtration matrix model see ISO/TS 16610-1.

E.1 Position in the filtration matrix model

This part of ISO/TS 16610 is a basic concept document that influences all filtration standards in the column: Profile filters, Linear (see Figure E.1).

General	Filters: ISO/TS 16610 series					
	Part 1					
Fundamental	Profile filters			Areal filters		
	Part 11 ^a			Part 12 ^a		
	Linear	Robust	Morphological	Linear	Robust	Morphological
Basic concepts	Part 20	Part 30	Part 40	Part 60	Part 70	Part 80
Particular filters	Parts 21-25	Parts 31-35	Parts 41-45	Parts 61-65	Parts 71-75	Parts 81-85
How to Filter	Parts 26-28	Parts 36-38	Parts 46-48	Parts 66-68	Parts 76-78	Parts 86-88
Multiresolution	Part 29	Part 39	Part 49	Part 69	Part 79	Part 89

^a At present included in Part 1.

Figure E.1 — Relationship to the filtration matrix model

Annex F (informative) Relationship to the GPS matrix model

For full details about the GPS matrix model, see ISO/TR 14638.

F.1 Information about this Technical Specification and its use

This part of ISO/TS 16610 specifies the characteristics of cardinal cubic spline filters for the filtration of profiles. It specifies in particular how to separate the long and short wave component of a profile.

F.2 Position in the GPS matrix model

This part of ISO/TS 16610 is a global GPS Technical Specification, which influences chain links 3 and 5 of all chains of standards in the GPS matrix structure, as graphically illustrated in Figure F.1.

Global GPS standards	
General GPS standards	
Chain link number	1 2 3 4 5 6
Size	
Distance	
Radius	
Angle	
Form of line independent of datum	
Form of line dependent of datum	
Form of surface independent of datum	
Form of surface dependent of datum	
Orientation	
Location	
Circular run-out	
Total run-out	
Datums	
Roughness profile	
Waviness profile	
Primary profile	
Surface imperfections	
Edges	

Figure F.1 — Position in the GPS matrix model

F.3 Related International Standards

The related International Standards are those of the chains of standards indicated in Figure F.1.

Bibliography

- [1] ISO 11562:1996, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Metrological characteristics of phase correct filters*
- [2] ISO/TS 14406¹⁾, *Geometrical product specifications (GPS) — State-of-the-art of dimensional and associated tolerancing*
- [3] ISO 14638:1995, *Geometrical Product Specification (GPS) — Masterplan*
- [4] KRYSZEK, M., *Form filtering by splines*, Measurement 18, 9-15 (1996)
- [5] KRYSZEK, M., *Discrete L-spline filtering in roundness measurements*, Measurement 18, 129-138 (1996)
- [6] KRYSZEK, M., *Transfer Function of Discrete Spline Filters*, in: P. CIARLINI et. al. (Eds.), *Advanced Mathematical Tools in Metrology III*, S. 203-210, World Scientific Publishing, Singapore (1997)
- [7] GOLUB GENE H. & VAN LOAN CHARLES F., *Matrix Computations* (John Hopkins series in Mathematical Sciences), The Johns Hopkins University Press, ISBN: 0801854148 (1996)

1) Under preparation.

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