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**Centrifugal pumps handling viscous
liquids — Performance corrections**

*Pompes centrifuges pour la manutention de liquides visqueux —
Corrections des caractéristiques de fonctionnement*



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Foreword

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 17766 was prepared by Technical Committee ISO/TC 115, *Pumps*, Subcommittee SC 3, *Installation and special application*.

Centrifugal pumps handling viscous liquids — Performance corrections

1 Scope

This Technical Report gives performance corrections for all worldwide designs of centrifugal and vertical pumps of conventional design, in the normal operating range, with open or closed impellers, single or double suction, pumping Newtonian fluids are included.

2 Symbols and abbreviated terms

A complete list of symbols and definitions used in this document is given below¹⁾.

A	= Suction geometry variable used in the calculation to correct net positive suction head required
B	= Parameter used in the viscosity correction procedures; the B parameter is used as a normalizing pump Reynolds number and to adjust the corrections for the pump specific speed
BEP	= Best efficiency point (the rate of flow and head at which pump efficiency is a maximum at a given speed)
C_{η}	= Efficiency correction factor
$C_{\eta-RR}$	= Efficiency correction factor due to disc friction only
C_H	= Head correction factor
C_{BEP-H}	= Head correction factor that is applied to the flow at maximum pump efficiency for water
C_{NPSH}	= Net positive suction head correction factor
C_Q	= Rate of flow correction factor
d_2	= Impeller outlet diameter in m (ft)
g	= Acceleration due to gravity in m/s^2 (ft/s^2)
H	= Head per stage in m (ft)
$H_{BEP-vis}$	= Viscous head in m (ft); the head per stage at the rate of flow at which maximum pump efficiency is obtained when pumping a viscous liquid
H_{BEP-W}	= Water head in m (ft); the head per stage at the rate of flow at which maximum pump efficiency is obtained when pumping water
H_L	= Hydraulic losses in m (ft)
H_{th}	= Theoretical head (flow without losses) in m (ft)

1) A derogation has been granted to ISO/TC 115/SC 3 for this document to use the industry abbreviation NPSHR in the mathematical symbols $NPSHR_{BEP-W}$, and $NPSHR_W$.

- H_{vis} = Viscous head in m (ft); the head per stage when pumping a viscous liquid
- $H_{vis-tot}$ = Viscous head in m (ft); the total head of the pump when pumping a viscous liquid
- H_W = Water head in m (ft); the head per stage when pumping water
- N = Pump-shaft rotational speed in rpm
- N_S = Specific speed
 (USCS units) = $\frac{NQ_{BEP-W}^{0.5}}{H_{BEP-W}^{0.75}}$
- n_s = Specific speed
 (metric units) = $\frac{NQ_{BEP-W}^{0.5}}{H_{BEP-W}^{0.75}}$
- The specific speed of an impeller is defined as the speed in revolutions per minute at which a geometrically similar impeller would run if it were of such a size as to discharge one cubic meter per second (m³/s) against one meter of head (metric units) or one US gallon per minute against one foot of head (USCS units). These units shall be used to calculate specific speed.
- NOTE The rate of flow for the pump is used in this definition, not the rate of flow at the impeller eye.
- NPSHA = Net positive suction head in m (ft) available to the pump
- NPSHR = Net positive suction head in m (ft) required by the pump based on the standard 3 % head drop criterion
- NPSHR_{BEP-W} = Net positive suction head in m (ft) required for water at the maximum efficiency rate of flow, based on the standard 3 % head drop criterion
- NPSHR_{vis} = Viscous net positive suction head in m (ft) required in a viscous liquid
- NPSHR_W = Net positive suction head in m (ft) required on water, based on the standard 3 % head drop criterion
- P = Power; without subscript: power at coupling in kW (hp)
- P_m = Mechanical power losses in kW (hp)
- P_u = Useful power transferred to liquid; $P_u = \rho g H Q$ in kW (hp)
- P_{RR} = Disc friction power loss in kW (hp)
- P_{vis} = Viscous power in kW (hp); the shaft input power required by the pump for the viscous conditions
- P_W = Pump-shaft input power required for water in kW (hp)
- Q = Rate of flow in m³/h (gpm)
- Q_{BEP-W} = Water rate of flow in m³/h (gpm) at which maximum pump efficiency is obtained
- Q_{vis} = Viscous rate of flow in m³/h (gpm); the rate of flow when pumping a viscous liquid
- Q_W = Water rate of flow in m³/h (gpm); the rate of flow when pumping water
- q^* = Ratio of rate of flow to rate of flow at best efficiency point: $q^* = Q/Q_{BEP}$
- Re = Reynolds-number: $Re = \omega r_2^2/\nu$
- r_2 = Impeller outer radius in m (ft)
- s = Specific gravity of pumped liquid in relation to water at 20 °C (68 °F)

V_{vis}	=	Kinematic viscosity in centistokes (cSt) of the pumped liquid
V_{W}	=	Kinematic viscosity in centistokes (cSt) of water reference test liquid
η	=	Overall efficiency (at coupling)
$\eta_{\text{BEP-W}}$	=	Water best efficiency
η_{h}	=	Hydraulic efficiency
η_{vis}	=	Viscous efficiency; the efficiency when pumping a viscous liquid
η_{vol}	=	Volumetric efficiency
η_{W}	=	Water pump efficiency; the pump efficiency when pumping water
μ	=	Dynamic (absolute) viscosity in N·s/m ² (lb·s/ft ²)
ν	=	Kinematic viscosity in m ² /s (ft ² /s)
ρ	=	Density in kg/m ³ (slugs/ft ³)
ψ	=	Head coefficient
ω	=	Angular velocity of shaft or impeller in rad/s

3 Summary

The performance of a rotodynamic (centrifugal or vertical) pump on a viscous liquid differs from the performance on water, which is the basis for most published curves. Head (H) and rate of flow (Q) will normally decrease as viscosity increases. Power (P) will increase, as will net positive suction head required (NPSHR) in most circumstances. Starting torque may also be affected.

The Hydraulic Institute (HI) has developed a generalized method for predicting performance of rotodynamic pumps on Newtonian liquids of viscosity greater than that of water. This is an empirical method based on the test data available from sources throughout the world. The HI method enables pump users and designers to estimate performance of a particular rotodynamic pump on liquids of known viscosity, given the performance on water. The procedure may also result in a suitable pump being selected for a required duty on viscous liquids.

Performance estimates using the HI method are only approximate. There are many factors for particular pump geometries and flow conditions that the method does not take into account. It is nevertheless a dependable approximation when only limited data on the pump are available and the estimate is needed.

Theoretical methods based on loss analysis may provide more accurate predictions of the effects of liquid viscosity on pump performance when the geometry of a particular pump is known in more detail. This document explains the basis of such theoretical methods. Pump users should consult pump manufacturers to determine whether or not more accurate predictions of performance for a particular pump and viscous liquid are available.

This document also includes technical considerations and recommendations for pump applications on viscous liquids.

Calculations based on the Hydraulic Institute's Viscosity Correction method (VCM) have been mathematically modeled in a web-based HIVCM™ tool.

Available at www.pumps.org, the HIVCM™ tool allows pump users, manufacturers, and third-party software providers access to rapid analysis of a pump's hydraulic performance on water vs. specified viscous liquids. Use of the HIVCM™ tool in pump selection will provide reliable and consistent calculations based on the methodology outlined in this Technical Report.

HIVCM™ is a trademark owned by the Hydraulic Institute. Please visit www.pumps.org for more information.

4 Introduction

The performance (head, flow, efficiency $[\eta]$, and power) of a rotodynamic pump is obtained from the pump's characteristic curves, which are generated from test data using water. When a more viscous liquid is pumped, the performance of the pump is reduced. Absorbed power will increase and head, rate of flow, and efficiency will decrease.

It is important for the user to understand a number of facts that underlie any attempt to quantify the effects of viscosity on rotodynamic pump operation. First, the test data available are specific to the individual pumps tested and are thus not of a generic nature. Second, what data are available are relatively limited in the range of both pump size and viscosity of the liquid. Third, all existing methods of predicting the effects of viscosity on pump performance show discrepancies with the limited test data available. Fourth, the empirical method presented in this document was chosen based on a statistical comparison of various possible correction procedures. The chosen method was found to produce the least amount of variance between calculated and actual data. Considering all of the above, it must be recognized that this method cannot be used as a theoretically rigorous calculation that will predict the performance correction factors with great precision. It is rather meant to allow a general comparison of the effect of pumping higher viscosity liquids and to help the user avoid misapplication without being excessively conservative. See Clause 6 for types of pumps for which the method is applicable.

As a footnote to the preceding paragraph, it should be recognized that there are methods developed by individuals and companies that deal with the actual internal hydraulic losses of the pump. By quantifying these losses the effect of liquid viscosity can, in theory, be calculated. These procedures take into account the specific pump internal geometry, which is generally unavailable to the pump user. Furthermore, such methods still require some empirical coefficients that can only be derived correctly when sufficient information on the pumps tested in viscous liquids is available. The test data collected by HI from sources around the world did not include sufficiently detailed information about the pumps tested to validate loss analysis methods. It is nevertheless recognized that a loss analysis method will probably be more accurate than the empirical method in this document, especially for pumps with special features and particular geometry.

In addition to the correction procedures, the document provides a qualitative description of the various hydraulic losses within the pump that underlie the performance reduction. Procedures for determining the effect of viscosity on starting torque and NPSHR are also provided.

The previous HI Standard for viscosity correction in Reference [24] was based on data supplied up to 1960. This new document is based on an expanded data set up to 1999 which has modified the correction factors for rate of flow, head, and power. Updated correction factors are influenced by the pump size, speed, and specific speed. In general, the head and flow have an increased correction while the power (efficiency) correction is less. The most significant changes in the correction factors occur at flows less than 25 m³/h (100 gpm) and $n_s < 15$ ($N_s < 770$).

5 Fundamental considerations

5.1 Viscous correction factors

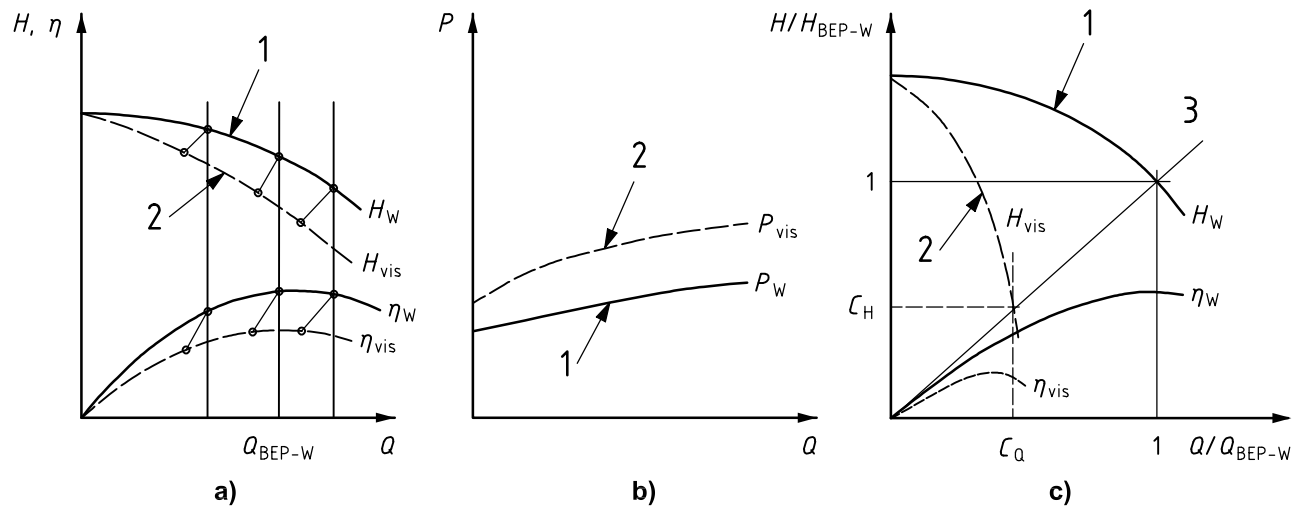
When a liquid of high viscosity, such as heavy oil, is pumped by a rotodynamic pump, the performance is changed in comparison to performance with water, due to increased losses. The reduction in performance on viscous liquids may be estimated by applying correction factors for head, rate of flow, and efficiency to the performance with water.

Thus the curves of head, flow and efficiency for viscous liquids (subscript vis) are estimated from the head, flow, and efficiency measured with water (subscript W) by applying the correction factors C_H , C_Q , and C_η , respectively. These factors are defined in Equation (1):

$$C_H = \frac{H_{vis}}{H_W} \quad ; \quad C_Q = \frac{Q_{vis}}{Q_W} \quad ; \quad C_\eta = \frac{\eta_{vis}}{\eta_W} \quad (1)$$

Figure 1 (a) and (b) shows schematically how the head, efficiency, and power characteristics typically change from operation with water to pumping a highly viscous liquid.

If measured data are normalized to the best efficiency point (BEP) when pumping water (BEP-W), the factors C_H and C_Q can be read directly on Figure 1 (c). A straight line between BEP-W and the origin of the H - Q curve ($H = 0; Q = 0$) is called the diffuser or volute characteristic. Test data reported in References [10] and [14] in the Bibliography show that BEPs for viscous liquids follow this diffuser or volute characteristic. Analysis of test data on viscous pumping collected by HI from sources around the world also confirms this observation. It is consequently a good approximation to assume C_H is equal to C_Q at the BEPs for viscous liquids.



Key

- 1 water
- 2 viscous liquid
- 3 volute or diffuser characteristic

Figure 1 — Modification of pump characteristics when pumping viscous liquids

5.2 Methods for determining correction factors

Correction factors can be either defined empirically from a data bank containing measurements on various pumps with water and liquids of different viscosities or from a physical model based on the analysis of the energy losses in the pump. Examples of such loss analysis methods are given in References [7], [8], [9], [10] and [18] of the Bibliography.

Analysis of the limited data available shows that empirical and loss analysis methods predict head correction functions with approximately the same accuracy. Loss analysis methods are, however, more precise in predicting power requirements for pumping viscous liquids. It is also possible to investigate the influence of various design parameters on viscous performance and to optimize pump selection or design features for operation with highly viscous liquids by applying the loss analysis procedures.

Further theoretical explanations of the principles of loss analysis methods are given in Clause 7 of this document. Use of such methods may require more information about pump dimensions than is generally available to the user. A loss analysis procedure may be expected to provide more accurate predictions of pump performance with viscous liquids when such detailed information is available.

The HI method explained in Clause 6 of this document is based on empirical data. It provides a way of predicting the effects of liquid viscosity on pump performance with adequate accuracy for most practical purposes. The method in this document gives correction factors similar to the previous HI method. The new method matches the experimental data better than the old HI method that has been widely used throughout the

world for many years. The standard deviation for the head correction factor, C_H , is 0,1. Estimates of viscous power, P_{vis} , are subject to a standard deviation of 0,15.

6 Synopsis of Hydraulic Institute method

6.1 Generalized method based on empirical data

The performance of rotodynamic pumps is affected when handling viscous liquids. A marked increase in power, a reduction in head, and some reduction in the rate of flow occur with moderate and high viscosities. Starting torque and NPSHR may also be affected.

The HI correction method provides a means of determining the performance of a rotodynamic pump handling a viscous liquid when its performance on water is known. The equations are based on a pump performance Reynolds number adjusted for specific speed (parameter B), which has been statistically curve-fitted to a body of test data. These tests of conventional single-stage and multi-stage pumps cover the following range of parameters: closed and semi-open impellers; kinematic viscosity 1 to 3 000 cSt; rate of flow at BEP with water $Q_{BEP-W} = 3 \text{ m}^3/\text{h}$ to $260 \text{ m}^3/\text{h}$ (13 gpm to 1 140 gpm); head per stage at BEP with water $H_{BEP-W} = 6 \text{ m}$ to 130 m (20 ft to 430 ft).

The correction equations are, therefore, a generalized method based on empirical data, but are not exact for any particular pump. The generalized method may be applied to pump performance outside the range of test data indicated above, as outlined in Clause 6 and with the specific instructions and examples in 6.5 and 6.6. There will be increased uncertainty of performance prediction outside the range of test results.

When accurate information is essential, pump performance tests should be conducted with the particular viscous liquid to be handled. Prediction methods based on an analysis of hydraulic losses for a particular pump design may also be more accurate than this generalized method.

6.2 Viscous liquid performance correction limitations

Because the equations provided in 6.5 and 6.6 are based on empirical rather than theoretical considerations, extrapolation beyond the limits shown in 6.5 and 6.6 would go outside the experience range that the equations cover and is not recommended.

The correction factors are applicable to pumps of hydraulic design with essentially radial impeller discharge ($n_s \leq 60$, $N_s \leq 3\,000$), in the normal operating range, with fully open, semi-open, or closed impellers. Do not use these correction factors for axial flow type pumps or for pumps of special hydraulic design. See Clause 8 for additional guidance.

Use correction factors only where an adequate margin of NPSH available (NPSHA) over NPSHR is present in order to cope with an increase in NPSHR caused by the increase in viscosity. See 7.3 to estimate the increase in NPSHR.

The data used to develop the correction factors are based on tests of Newtonian liquids. Gels, slurries, paper stock, and other non-Newtonian liquids may produce widely varying results, depending on the particular characteristics of the media.

6.3 Viscous liquid symbols and definitions used for determining correction factors

A	=	Suction geometry variable used in the calculation to correct net positive suction head required
B	=	Parameter used in the viscosity correction procedures; the B parameter is used as a normalizing pump Reynolds number and to adjust the corrections for the pump specific speed
BEP	=	Best efficiency point (the rate of flow and head at which pump efficiency is a maximum at a given speed)
C_{η}	=	Efficiency correction factor
C_H	=	Head correction factor
$C_{\text{BEP-H}}$	=	Head correction factor that is applied to the flow at maximum pump efficiency for water
C_{NPSH}	=	Net positive suction head correction factor
C_Q	=	Rate of flow correction factor
$H_{\text{BEP-vis}}$	=	Viscous head in m (ft); the head per stage at the rate of flow at which maximum pump efficiency is obtained when pumping a viscous liquid
$H_{\text{BEP-W}}$	=	Water head in m (ft); the head per stage at the rate of flow at which maximum pump efficiency is obtained when pumping water
H_{vis}	=	Viscous head in m (ft); the head per stage when pumping a viscous liquid
$H_{\text{vis-tot}}$	=	Viscous head in m (ft); the total head of the pump when pumping a viscous liquid
H_W	=	Water head in m (ft); the head per stage when pumping water
N	=	Pump-shaft rotational speed in rpm
N_S	=	Specific speed
		(USCS units) = $\frac{NQ_{\text{BEP-W}}^{0.5}}{H_{\text{BEP-W}}^{0.75}}$
n_s	=	Specific speed
		(metric units) = $\frac{NQ_{\text{BEP-W}}^{0.5}}{H_{\text{BEP-W}}^{0.75}}$
		The specific speed of an impeller is defined as the speed in revolutions per minute at which a geometrically similar impeller would run if it were of such a size as to discharge one cubic meter per second (m^3/s) against one meter of head (metric units) or one US gallon per minute against one foot of head (USCS units). These units shall be used to calculate specific speed.
		NOTE The rate of flow for the pump is used in this definition, not the rate of flow through the impeller eye.
NPSHA	=	Net positive suction head in m (ft) available to the pump
NPSHR	=	Net positive suction head in m (ft) required by the pump based on the standard 3 % head drop criterion
$\text{NPSHR}_{\text{BEP-W}}$	=	Net positive suction head in m (ft) required for water at the maximum efficiency rate of flow, based on the standard 3 % head drop criterion
$\text{NPSHR}_{\text{vis}}$	=	Viscous net positive suction head in m (ft) required in a viscous liquid
NPSHR_W	=	Net positive suction head in m (ft) required on water, based on the standard 3 % head drop criterion

P_{vis}	=	Viscous power in kW (hp); the shaft input power required by the pump for the viscous conditions
Q_{BEP-W}	=	Water rate of flow in m ³ /h (gpm) at which maximum pump efficiency is obtained
Q_{vis}	=	Viscous rate of flow in m ³ /h (gpm); the rate of flow when pumping a viscous liquid
Q_W	=	Water rate of flow in m ³ /h (gpm); the rate of flow when pumping water
s	=	Specific gravity of pumped liquid in relation to water at 20 °C (68 °F)
V_{vis}	=	Kinematic viscosity in centistokes (cSt) of the pumped liquid
V_W	=	Kinematic viscosity in centistokes (cSt) of water reference test liquid
η_{BEP-W}	=	Water best efficiency
η_{vis}	=	Viscous efficiency; the efficiency when pumping a viscous liquid
η_W	=	Water pump efficiency; the pump efficiency when pumping water

NOTE 1 Other technical expressions are defined in HI standards.

NOTE 2 Equations for converting kinematic viscosity from SSU to cSt units and vice versa are shown in Annex A.

NOTE 3 Pump viscosity corrections are determined by the procedures outlined in the following 6.4, 6.5, and 6.6.

6.4 Overview of procedure to estimate effects of viscosity on pump performance

The procedure is in three parts: first, to establish whether or not the document is applicable; second, to calculate the pump performance on a viscous liquid when performance on water is known; and third, to select a pump for given head, rate of flow and viscous conditions.

The procedure for the first part is illustrated in Figure 2.

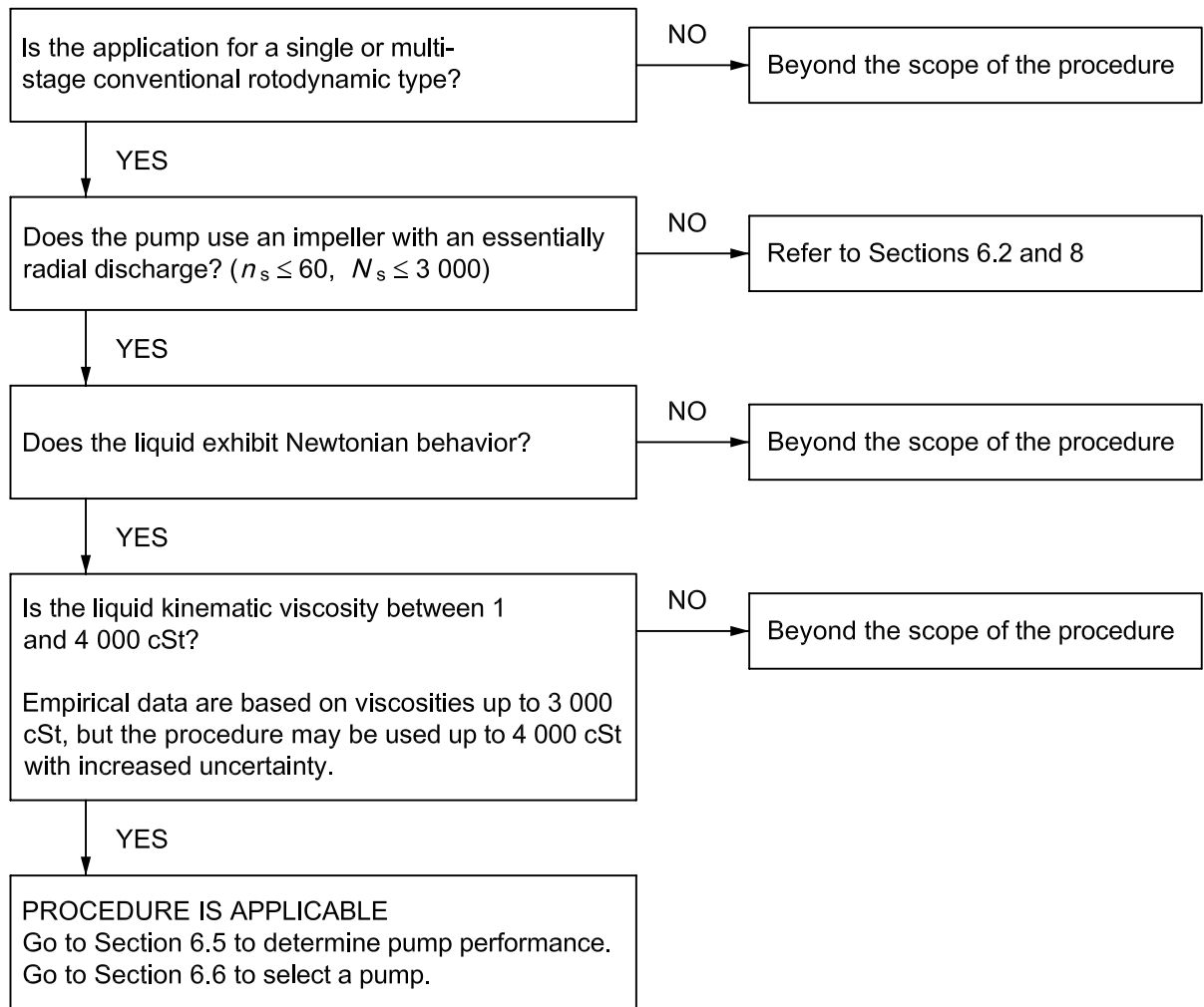


Figure 2 — Flowchart to establish if the procedure is applicable

The procedure for the second part is defined in 6.5 and summarized in Figure 3.

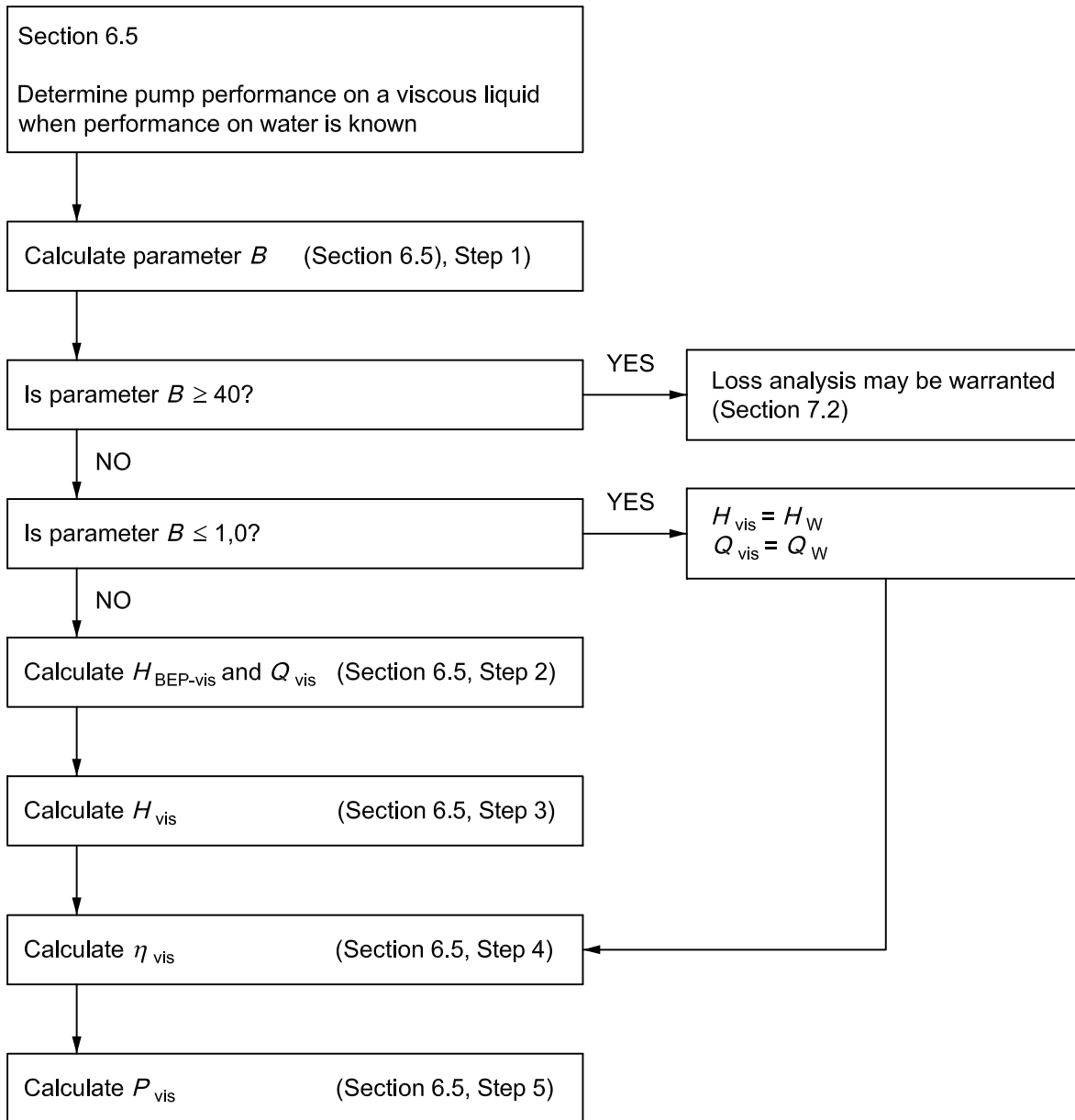


Figure 3 — Flowchart to determine pump performance on a viscous liquid when performance on water is known

The procedure for the third part is defined in 6.6 and summarized in Figure 4.

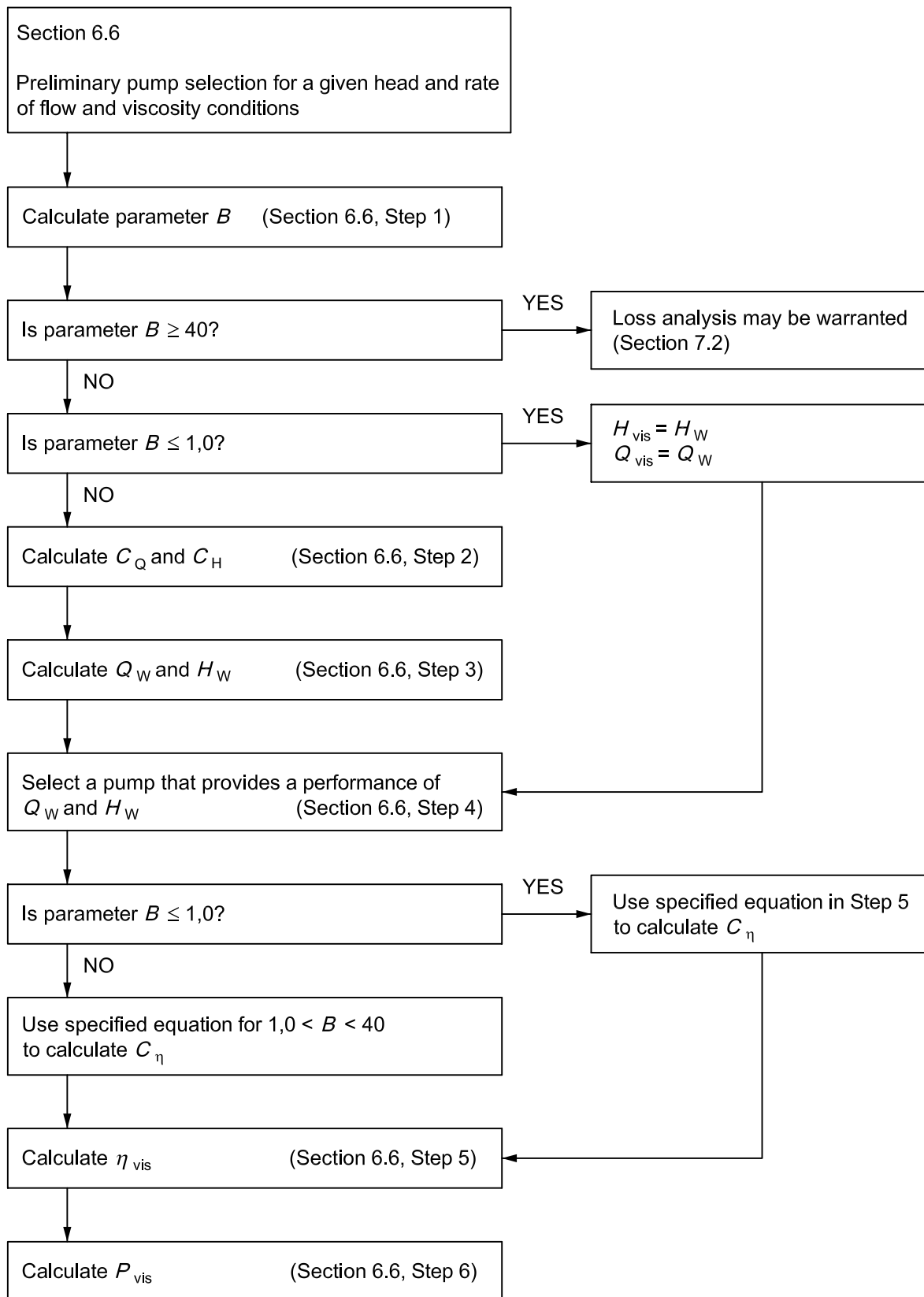


Figure 4 — Flowchart to select a pump for given head, rate of flow, and viscous conditions

6.5 Instructions for determining pump performance on a viscous liquid when performance on water is known

The following equations are used for developing the correction factors to adjust pump water performance characteristics of rate of flow, total head, efficiency, and input power to the corresponding viscous liquid performance.

Step 1

Calculate parameter B based on the water performance best efficiency flow ($Q_{\text{BEP-W}}$)

Given metric units of $Q_{\text{BEP-W}}$ in m^3/h , $H_{\text{BEP-W}}$ in m, N in rpm, and V_{vis} in cSt, use Equation (2):

$$B = 16,5 \times \frac{(V_{\text{vis}})^{0,50} \times (H_{\text{BEP-W}})^{0,0625}}{(Q_{\text{BEP-W}})^{0,375} \times N^{0,25}} \quad (2)$$

Given USCS units of $Q_{\text{BEP-W}}$ in gpm, $H_{\text{BEP-W}}$ in ft, N in rpm, and V_{vis} in cSt, use Equation (3):

$$B = 26,6 \times \frac{(V_{\text{vis}})^{0,50} \times (H_{\text{BEP-W}})^{0,0625}}{(Q_{\text{BEP-W}})^{0,375} \times N^{0,25}} \quad (3)$$

If $1,0 < B < 40$, go to Step 2.

If $B \geq 40$, the correction factors derived using the equations in 6.5 and 6.6 are highly uncertain and should be avoided. Instead a detailed loss analysis method may be warranted. See 7.2.

If $B \leq 1,0$, set $C_H = 1,0$ and $C_Q = 1,0$, and then skip to Step 4.

Step 2

Calculate correction factor for flow (C_Q) (which is also equal to the correction factor for head at BEP [$C_{\text{BEP-H}}$]) corresponding to the water performance best efficiency flow ($Q_{\text{BEP-W}}$) using Equation (4). Correct the other water performance flows (Q_W) to viscous flows (Q_{vis}). These two equations are valid for all rates of flow (Q_W).

$$C_Q = (2,71)^{-0,165 \times (\log B)^{3,15}} \quad (4)$$

$$Q_{\text{vis}} = C_Q \times Q_W$$

Correct the water performance total head ($H_{\text{BEP-W}}$) that corresponds to the water performance best efficiency flow ($Q_{\text{BEP-W}}$) using Equation (5).

$$C_{\text{BEP-H}} = C_Q \quad (5)$$

$$H_{\text{BEP-vis}} = C_{\text{BEP-H}} \times H_{\text{BEP-W}}$$

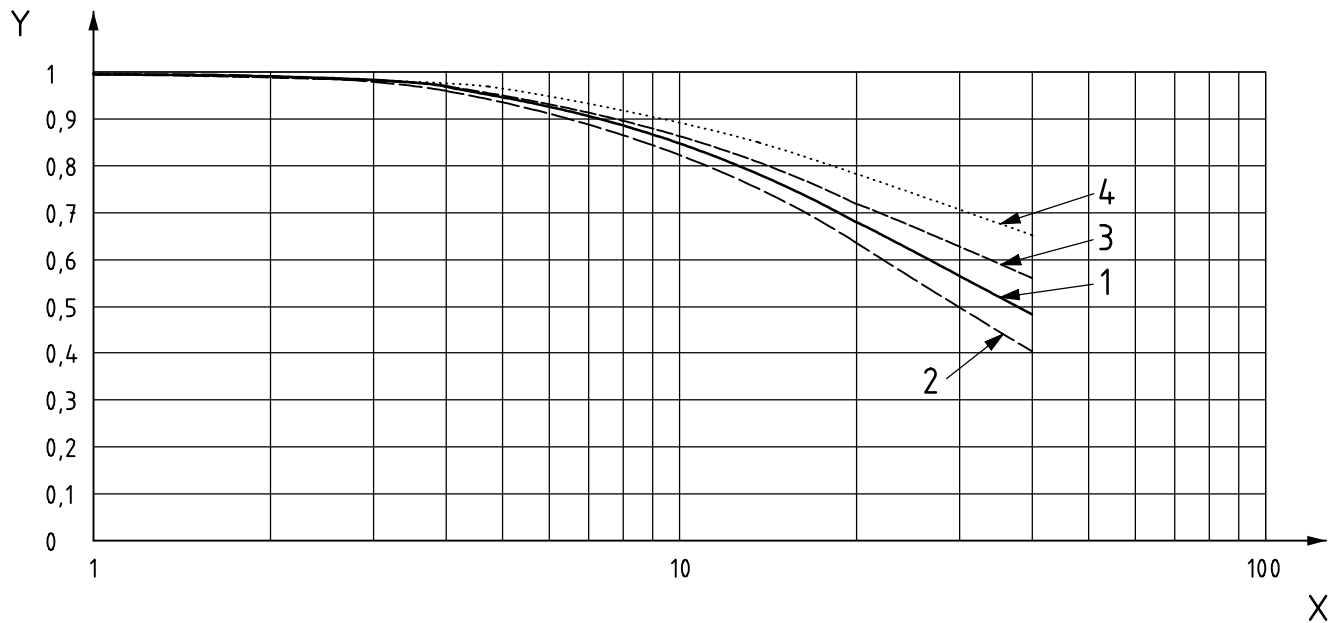
Step 3

Calculate head correction factors (C_H) using Equation (6), and then corresponding values of viscous head (H_{vis}) for flows (Q_W) greater than or less than the water best efficiency flow ($Q_{\text{BEP-W}}$).

$$C_H = 1 - \left[(1 - C_{\text{BEP-H}}) \times \left(\frac{Q_W}{Q_{\text{BEP-W}}} \right)^{0,75} \right] \quad (6)$$

$$H_{\text{vis}} = C_H \times H_W$$

NOTE An optional means of determining the values for C_Q , $C_{\text{BEP-H}}$, and C_H is to read them from the chart in Figure 5.

**Key**

- X parameter B
 Y correction factors C_H and C_Q
 1 C_H and C_Q vs. B at Q_{BEP-W}
 2 C_H vs. B at $1,2 \times Q_{BEP-W}$
 3 C_H vs. B at $0,8 \times Q_{BEP-W}$
 4 C_H vs. B at $0,6 \times Q_{BEP-W}$

Figure 5 — Chart of correction factors C_Q and C_H vs. parameter B

Step 4

Calculate the correction factor for efficiency (C_η) using Equation (7) or Equation (8) and the corresponding values of viscous pump efficiency (η_{vis}). The following equations are valid for flows (Q_W) greater than, less than, and equal to the water best efficiency flow Q_{BEP-W} :

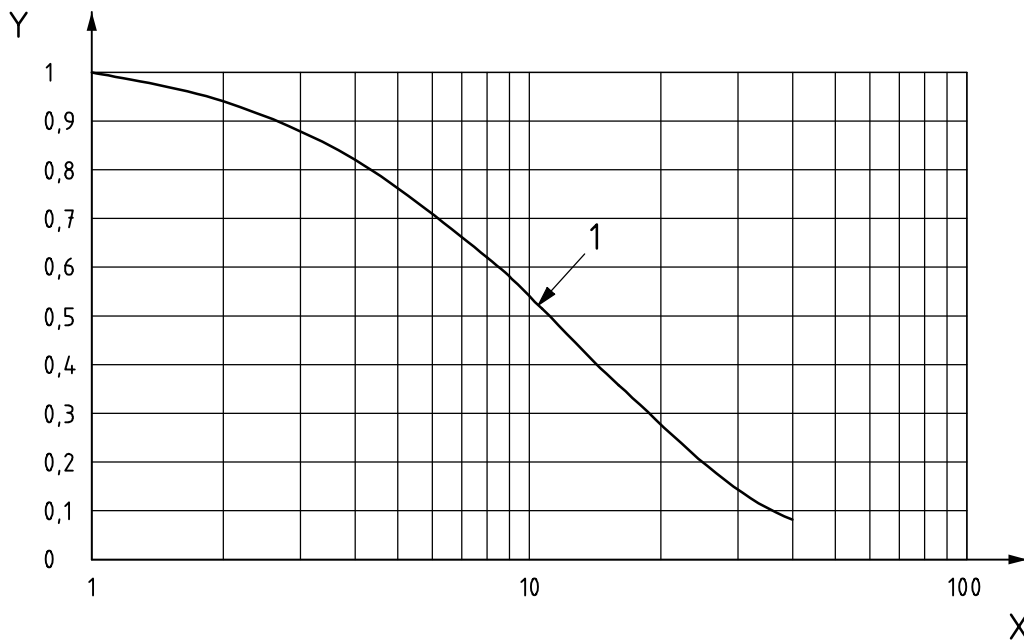
$$\text{For } 1,0 < B < 40, C_\eta = B^{-(0,0547 \times B^{0,69})} \quad (7)$$

NOTE An optional means of determining the value for C_η is to read it from the chart in Figure 6.

For $B \leq 1,0$, estimate the efficiency correction (C_η) from the following Equation (8):

$$C_\eta = \frac{1 - \left[(1 - \eta_{BEP-W}) \times \left(\frac{V_{vis}}{V_W} \right)^{0,07} \right]}{\eta_{BEP-W}} \quad (8)$$

$\eta_{vis} = C_\eta \times \eta_w$ where η_w is the water pump efficiency at the given rate of flow.



X parameter *B*
 Y correction factors C_{η}

Figure 6 — Chart of correction factors C_{η} vs. parameter *B*

Step 5

Calculate the values for viscous pump shaft input power (P_{vis}). The following equations are valid for all rates of flow.

For flow in m³/h, head in m, shaft power in kW, and efficiency in percent, use Equation (9):

$$P_{vis} = \frac{Q_{vis} \times H_{vis-tot} \times s}{367 \times \eta_{vis}} \tag{9}$$

For flow in gpm, head in ft, and shaft power in hp, use Equation (10):

$$P_{vis} = \frac{Q_{vis} \times H_{vis-tot} \times s}{3\,960 \times \eta_{vis}} \tag{10}$$

EXAMPLE (Metric units): Refer to Figure 7 and Table 1. The given single-stage pump has a water performance best efficiency flow of 110 m³/h at 77 m total head at 2 950 rpm and has a pump efficiency of 0,680. The procedure below illustrates how to correct the pump performance characteristics for a viscous liquid of 120 centistokes and specific gravity of 0,90.

Step 1

Calculate parameter *B* based on the water performance best efficiency flow conditions using Equation (2). If the pump is a multi-stage configuration, calculate parameter *B* using the head per stage.

Given units of Q_{BEP-W} in m³/h, H_{BEP-W} in m, *N* in rpm, and V_{vis} in cSt:

$$B = 16,5 \times \frac{(120)^{0,50} \times (77)^{0,0625}}{(110)^{0,375} \times (2\,950)^{0,25}} = 5,52$$

Step 2

Calculate correction factor for flow (C_Q) using Equation (4) and correct the flows corresponding to ratios of water best efficiency flow (Q_W/Q_{BEP-W}).

$$C_Q = (2,71)^{-0,165 \times (\log 5,52)^{3,15}} = 0,938$$

$$\text{At: } \frac{Q_W}{Q_{BEP-W}} = 1,00$$

$$Q_{vis} = 0,938 \times 110,0 \times 1,00 = 103,2 \text{ m}^3/\text{h}$$

$$\text{At: } \frac{Q_W}{Q_{BEP-W}} = 0,60$$

$$Q_{vis} = 0,938 \times 110,0 \times 0,60 = 61,9 \text{ m}^3/\text{h}$$

According to Equation (5), the correction factor for head (C_{BEP-H}) is equal to (C_Q) at Q_{BEP-W} .

$$C_{BEP-H} = C_Q = 0,938$$

At Q_{BEP-W} , the corresponding viscous head ($H_{BEP-vis}$) is:

$$H_{BEP-vis} = 0,938 \times 77,0 = 72,2 \text{ m}$$

Step 3

Calculate head correction factors (C_H) and corresponding values of viscous head (H_{vis}) for flows (Q_W) greater than or less than the water best efficiency flow (Q_{BEP-W}).

At 60 % of Q_{BEP-W} , the corresponding head correction factor (C_H) and viscous head (H_{vis}) are calculated using Equation (6):

$$C_H = 1 - (1 - 0,938) \times (0,60)^{0,75} = 0,958$$

$$H_{vis} = 0,958 \times 87,3 = 83,6 \text{ m}$$

Step 4

Calculate the correction factor for efficiency (C_η) and the corresponding values of viscous pump efficiency (η_{vis}) for flows (Q_W) greater than, less than, and equal to the water best efficiency flow (Q_{BEP-W}). Equation (7) is used to calculate C_η as the value of parameter $B = 5,52$ calculated in Step 1 falls within the range of 1 to 40:

$$C_\eta = (5,52)^{-[0,0547 \times (5,52)^{0,69}]} = 0,738$$

$$\text{At: } \frac{Q_W}{Q_{BEP-W}} = 1,00$$

$$\text{Where: } \eta_W = 0,680$$

$$\eta_{vis} = 0,738 \times 0,680 = 0,502$$

$$\text{At: } \frac{Q_W}{Q_{BEP-W}} = 0,60$$

$$\text{Where: } \eta_W = 0,602$$

$$\eta_{vis} = 0,738 \times 0,602 = 0,444$$

Step 5

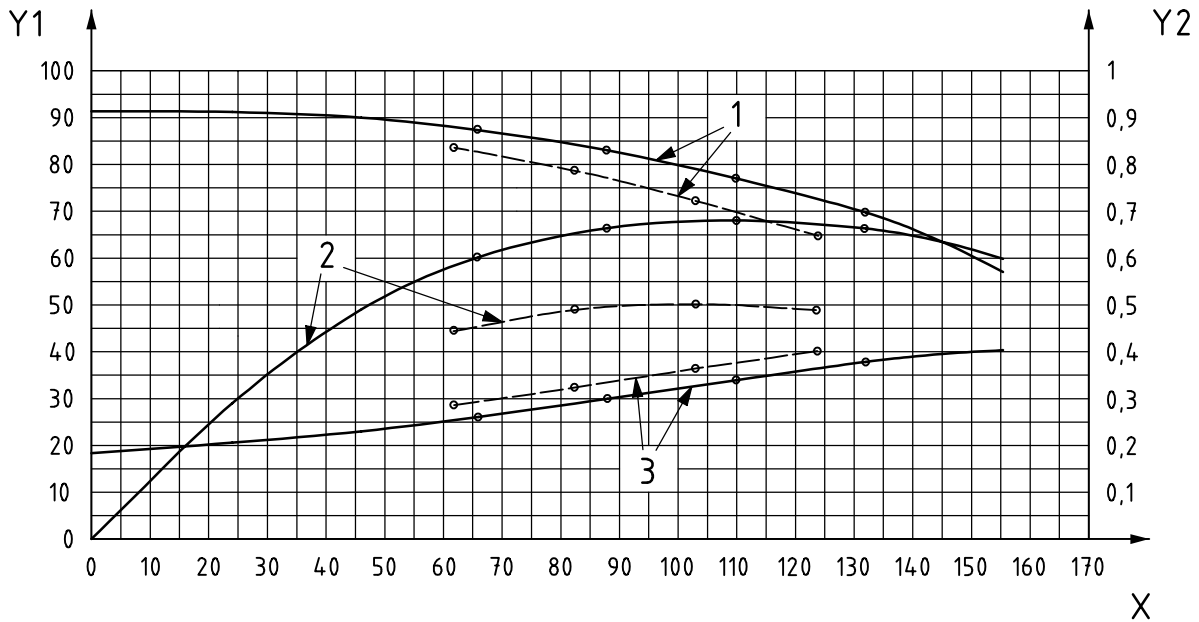
Calculate the values for viscous pump shaft input power (P_{vis}) for flows (Q_W) greater than, less than, or equal to the water best efficiency flow (Q_{BEP-W}) using Equation (9).

$$\text{At: } \frac{Q_W}{Q_{BEP-W}} = 1,00$$

$$P_{vis} = \frac{103,2 \times 72,2 \times 0,90}{367 \times 0,502} = 36,4 \text{ kW}$$

$$\text{At: } \frac{Q_W}{Q_{BEP-W}} = 0,60$$

$$P_{vis} = \frac{61,9 \times 83,6 \times 0,90}{367 \times 0,444} = 28,6 \text{ kW}$$



Key

- X rate of flow, cubic meters per hour, at 2 950 r/min
- Y1 total head in meters or power in kilowatts
- Y2 efficiency
- water
- viscous liquid, with $V_{vis} = 120 \text{ cSt}$ and $s = 0,90$
- 1 head vs. flow
- 2 efficiency vs. flow
- 3 shaft input power vs. flow

Figure 7 — Example performance chart of a single-stage pump, expressed in metric units

Table 1 — Example calculations (metric units)

Viscosity of liquid to be pumped (V_{vis}) — cSt	120			
Specific gravity of viscous liquid (s)	0,90			
Pump shaft speed (N) — rpm	2 950			
Ratio of water best efficiency flow Q_W/Q_{BEP-W}	0,60	0,80	1,00	1,20
Water rate of flow (Q_W or Q_{BEP-W}) — m ³ /h	66,0	88,0	110,0	132,0
Water head per stage (H_W or H_{BEP-W}) — m	87,3	83,0	77,0	69,7
Water pump efficiency (η_W)	0,60	0,66	0,68	0,66
Parameter B	5,52			
Correction factor for flow (C_Q)	0,938			
Correction factors for head (C_H or C_{BEP-H})	0,958	0,947	0,938	0,929
Correction factor for efficiency (C_η)	0,738			
Corrected flow (Q_{vis}) — m ³ /h	61,9	82,5	103,2	123,8
Corrected head per stage (H_{vis} or $H_{BEP-vis}$) — m	83,6	78,6	72,2	64,8
Corrected efficiency (η_{vis})	0,44	0,49	0,50	0,48
Viscous shaft input power (P_{vis}) — kW	28,6	32,5	36,4	40,2

EXAMPLE (USCS units): Refer to Figure 8 and Table 2. The given single-stage pump has a water performance best efficiency flow of 440 gpm at 300 ft total head at 3 550 rpm and has a pump efficiency of 0,68. The procedure below illustrates how to correct the pump performance characteristics for a viscous liquid of 120 centistokes and specific gravity of 0,90.

Step 1

Calculate parameter B based on the water performance best efficiency flow conditions using Equation (3). If the pump is a multi-stage configuration, calculate parameter B using the head per stage.

Given units of Q_{BEP-W} in gpm, H_W in ft, N in rpm, and V_{vis} in cSt:

$$B = 26,6 \times \frac{(120)^{0,50} \times (300)^{0,0625}}{(440)^{0,375} \times (3\ 550)^{0,25}} = 5,50$$

Step 2

Calculate correction factor for flow (C_Q) using Equation (4) and correct the flows corresponding to ratios of water best efficiency flow (Q_W/Q_{BEP-W}).

$$C_Q = (2,71)^{-0,165 \times (\log 5,50)^{3,15}} = 0,938$$

$$\text{At: } \frac{Q_W}{Q_{BEP-W}} = 1,00$$

$$Q_{vis} = 0,938 \times 440 \times 1,00 = 413 \text{ gpm}$$

$$\text{At: } \frac{Q_W}{Q_{BEP-W}} = 0,60$$

$$Q_{vis} = 0,938 \times 440 \times 0,60 = 248 \text{ gpm}$$

From Equation (5), the correction factor for head (C_{BEP-H}) is equal to (C_Q) at Q_{BEP-W} .

$$C_{BEP-H} = C_Q = 0,938$$

At Q_{BEP-W} , the corresponding viscous head ($H_{BEP-vis}$) is

$$H_{BEP-vis} = 0,938 \times 300 = 281 \text{ ft}$$

Step 3

Calculate head correction factors (C_H) and corresponding values of viscous head (H_{vis}) for flows (Q_W) greater than or less than the water best efficiency flow (Q_{BEP-W}).

At 60 % of Q_{BEP-W} , the corresponding head correction factor (C_H) and viscous head (H_{vis}) are calculated using Equation (6):

$$C_H = 1 - (1 - 0,938) \times (0,60)^{0,75} = 0,958$$

$$H_{vis} = 0,958 \times 340 = 326 \text{ ft}$$

Step 4

Calculate the correction factor for efficiency (C_η) and the corresponding values of viscous pump efficiency (η_{vis}) for flows (Q_W) greater than, less than, and equal to the water best efficiency flow Q_{BEP-W} . Equation (7) is used to calculate C_η as the value of parameter $B = 5,50$ calculated in Step 1 falls within the range of 1 to 40:

$$C_\eta = (5,50)^{- (0,0547 \times (5,50)^{0,69})} = 0,738$$

$$\text{At: } \frac{Q_W}{Q_{BEP-W}} = 1,00$$

$$\text{Where: } \eta_W = 0,680$$

$$\eta_{vis} = 0,738 \times 0,680 = 0,502$$

$$\text{At: } \frac{Q_W}{Q_{BEP-W}} = 0,60$$

$$\text{Where: } \eta_W = 0,602$$

$$\eta_{vis} = 0,738 \times 0,602 = 0,444$$

Step 5

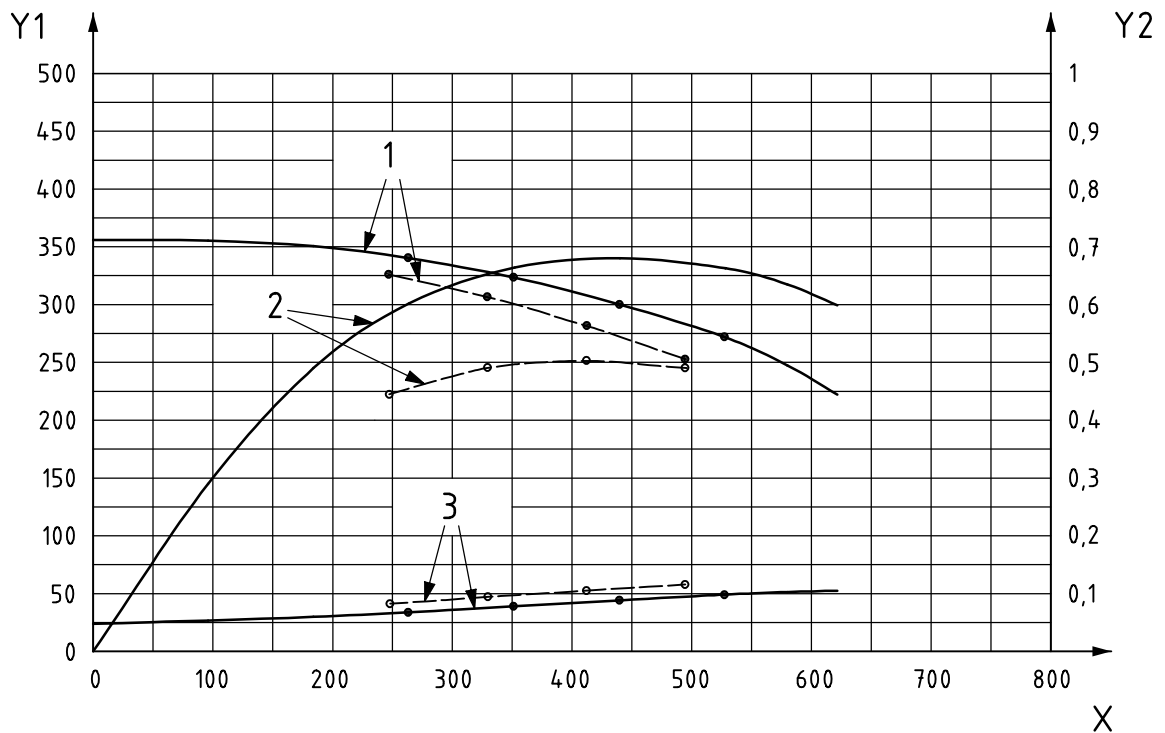
Calculate the values for viscous pump shaft input power (P_{vis}) for flows (Q_W) greater than, less than, or equal to the water best efficiency flow Q_{BEP-W} using Equation (10):

$$\text{At: } \frac{Q_W}{Q_{BEP-W}} = 1,00$$

$$P_{vis} = \frac{413 \times 281 \times 0,90}{3\,960 \times 0,502} = 52,5 \text{ hp}$$

$$\text{At: } \frac{Q_W}{Q_{BEP-W}} = 0,60$$

$$P_{vis} = \frac{248 \times 326 \times 0,90}{3\,960 \times 0,444} = 41,1 \text{ hp}$$

**Key**

- X rate of flow, gallons per minute at 3 550 r/min
- Y1 total head in feet or power in horsepower
- Y2 efficiency
- water
- viscous liquid, with $V_{vis} = 120$ cSt and $s = 0,90$
- 1 head vs. flow
- 2 efficiency vs. flow
- 3 shaft input power vs. flow

Figure 8 — Example performance chart of a single-stage pump, expressed in USCS units

Table 2 — Example calculations (USCS units)

Viscosity of liquid to be pumped (V_{vis}) — cSt	120			
Specific gravity of viscous liquid (s)	0,90			
Pump shaft speed (N) — rpm	3 550			
Ratio of water best efficiency flow Q_W/Q_{BEP-W}	0,60	0,80	1,00	1,20
Water rate of flow (Q_W or Q_{BEP-W}) — gpm	264	352	440	528
Water head per stage (H_W or H_{BEP-W}) — ft	340	323	300	272
Water pump efficiency (η_W)	0,602	0,66	0,680	0,66
Parameter B	5,50			
Correction factor for flow (C_Q)	0,938			
Correction factors for head (C_H or C_{BEP-H})	0,958	0,948	0,938	0,929
Correction factor for efficiency (C_η)	0,739			
Corrected flow (Q_{vis}) — gpm	248	330	413	495
Corrected head per stage (H_{vis} or $H_{BEP-vis}$) — ft	326	306	281	252
Corrected efficiency (η_{vis})	0,44	0,49	0,50	0,49
Viscous shaft input power (P_{vis}) — hp	41,4	46,7	52,5	57,9

6.6 Instructions for preliminary selection of a pump for given head, rate of flow, and viscosity conditions

Given the desired rate of flow and head of the viscous liquid to be pumped, and the viscosity and specific gravity at the pumping temperature, the following equations are used for finding the approximate equivalent water performance and estimating the viscous pump input power. Note that starting with the viscous conditions to determine the required water performance is less accurate than starting with a known water performance, unless iterations are done.

Step 1

Calculate parameter B given metric units of Q_{vis} in m^3/h , H_{vis} in m , and V_{vis} in cSt using Equation (11):

$$B = 2,80 \times \frac{(V_{vis})^{0,50}}{(Q_{vis})^{0,25} \times (H_{vis})^{0,125}} \tag{11}$$

or, given USCS units of Q_{vis} in gpm , H_{vis} in ft , and V_{vis} in cSt using Equation (12):

$$B = 4,70 \times \frac{(V_{vis})^{0,50}}{(Q_{vis})^{0,25} \times (H_{vis})^{0,125}} \tag{12}$$

If $1,0 < B < 40$, go to Step 2.

If $B \geq 40$, the correction factors derived using the equations in 6.5 and 6.6 are highly uncertain and should be avoided. Instead a detailed loss analysis method may be warranted.

If $B \leq 1,0$, set $C_H = 1,0$ and $C_Q = 1,0$, and then skip to Step 4.

NOTE The numerical constants in 6.6 for calculating parameter B are different than those in 6.5 due to the omission of the pump speed (N) variable from the equations.

Step 2

Calculate correction factors for flow (C_Q) and head (C_H). These two correction factors are approximately equal at a given rate of flow when they are derived from the water performance at the best efficiency flow $Q_{\text{BEP-W}}$.

Reference Equation (4).

$$C_Q \approx C_H \approx (2,71)^{-0,165 \times (\log B)^{3,15}}$$

NOTE An optional means of determining the values for C_Q and C_H is to read them from the chart in Figure 5.

Step 3

Calculate the approximate water performance rate of flow and total head.

$$Q_W = \frac{Q_{\text{vis}}}{C_Q}$$

$$H_W = \frac{H_{\text{vis}}}{C_H}$$

Step 4

Select a pump that provides a water performance of Q_W and H_W .

Step 5

Calculate the correction factor for efficiency (C_η) and the corresponding value of viscous pump efficiency (η_{vis}).

Reference Equation (7):

$$\text{For } 1,0 < B < 40 : C_\eta = B^{-(0,0547 \times B^{0,69})}$$

NOTE An optional means of determining the value for C_η is to read it from the chart in Figure 6.

If $B \leq 1,0$, estimate the efficiency correction (C_η) from the following equation.

Reference Equation (8):

$$C_\eta = \frac{1 - \left[(1 - \eta_{\text{BEP-W}}) \times \left(\frac{V_{\text{vis}}}{V_W} \right)^{0,07} \right]}{\eta_{\text{BEP-W}}}$$

$$\eta_{\text{vis}} = C_\eta \times \eta_W$$

Step 6

Calculate the approximate viscous pump-shaft input power.

For rate of flow in m^3/h , total head in m, and shaft input power in kW use the following equation.

Reference Equation (9):

$$P_{\text{vis}} = \frac{Q_{\text{vis}} \times H_{\text{vis-tot}} \times s}{367 \times \eta_{\text{vis}}}$$

For rate of flow in gpm, total head in ft, and shaft input power in hp use the following equation.

Reference Equation (10):

$$P_{\text{vis}} = \frac{Q_{\text{vis}} \times H_{\text{vis-tot}} \times s}{3\,960 \times \eta_{\text{vis}}}$$

EXAMPLE (Metric units): Select a pump to deliver 100 m³/h rate of flow at 70 m total head of a liquid having a kinematic viscosity of 120 cSt and a specific gravity of 0,90 at the pumping temperature.

Step 1

Calculate parameter *B* given units of *Q*_{vis} in m³/h, *H*_{vis} in m, and *V*_{vis} in cSt using Equation (11):

$$B = 2,80 \times \frac{(120)^{0,50}}{(100)^{0,25} \times (70)^{0,125}} = 5,70$$

Step 2

Calculate correction factors for rate of flow (*C*_Q) and total head (*C*_H). These two correction factors are approximately equal at a given rate of flow when they are derived from the water performance at the best efficiency rate of flow (*Q*_{BEP-W}).

Reference Equation (4):

$$C_Q \approx C_H \approx (2,71)^{-0,165 \times (\log 5,70)^{3,15}} = 0,934$$

Step 3

Calculate the approximate water performance rate of flow and total head.

$$Q_W = \frac{100}{0,934} = 107,1 \text{ m}^3/\text{h}$$

$$H_W = \frac{70}{0,934} = 74,9 \text{ m}$$

Step 4

Select a pump that provides a water performance of 107,1 m³/h rate of flow and 74,9 m total head. The selection should preferably be at or close to the maximum efficiency point for water performance. Assume the selected pump has an efficiency (*η*_{BEP-W}) of 0,680.

Step 5

Calculate the correction factor for efficiency using Equation (7) and the approximate viscous pump efficiency, or refer to Figure 6.

$$C_{\eta} = (5,70)^{-[0,0547 \times (5,70)^{0,69}]} = 0,729$$

$$\eta_{\text{vis}} = 0,729 \times 0,680 = 0,496$$

Step 6

Calculate the approximate pump-shaft input power for the viscous liquid. For rate of flow in m³/h, total head in m, and shaft input power in kW, use Equation (9):

$$P_{\text{vis}} = \frac{100 \times 70 \times 0,90}{367 \times 0,496} = 34,6 \text{ kW}$$

EXAMPLE (USCS units): Select a pump to deliver 440 gpm rate of flow at 230 ft total head of a liquid having a kinematic viscosity of 120 cSt and a specific gravity of 0,90 at the pumping temperature.

Step 1

Calculate parameter *B* given units of *Q*_{vis} in gpm, *H*_{vis} in ft, and *V*_{vis} in cSt using Equation (12):

$$B = 4,70 \times \frac{(120)^{0,50}}{(440)^{0,25} \times (230)^{0,125}} = 5,70$$

Step 2

Calculate correction factors for rate of flow (C_Q) and total head (C_H). These two correction factors are approximately equal at a given rate of flow when they are derived from the water performance at the best efficiency rate of flow (Q_{BEP-W})

Reference Equation (4):

$$C_Q \approx C_H \approx (2,71)^{-0,165 \times (\log 5,70)^{3,15}} = 0,934$$

Step 3

Calculate the approximate water performance rate of flow and total head.

$$Q_W = \frac{440}{0,934} = 471 \text{ gpm}$$

$$H_W = \frac{230}{0,934} = 246 \text{ ft}$$

Step 4

Select a pump that provides a water performance of 471 gpm rate of flow and 246 ft total head. The selection should be at or close to the maximum efficiency point for water performance. Assume the selected pump has an efficiency (η_{BEP-W}) of 0,680.

Step 5

Calculate the correction factor for efficiency using Equation (7) and the approximate viscous pump efficiency, or refer to Figure 6.

$$C_\eta = (5,70)^{-[0,0547 \times (5,70)^{0,69}]} = 0,729$$

$$\eta_{vis} = 0,729 \times 0,680 = 0,496$$

Step 6

Calculate the approximate pump-shaft input power for the viscous liquid. For rate of flow in gpm, total head in ft, and shaft input power in hp, use Equation (10):

$$P_{vis} = \frac{440 \times 230 \times 0,90}{3\,960 \times 0,496} = 46,4 \text{ hp}$$

The preceding procedure has sufficient accuracy for typical pump selection purposes. When working with a given pump's water performance curves, the procedure per 6.5 above can be used to obtain an improved estimate of the viscous performance corrections at all rates of flow.

7 Further theoretical explanations**7.1 Scope**

In this section the theoretical basis of loss analysis methods is explained. An analytical method of predicting NPSHR when pumping viscous liquids is also developed. This method is not supported by any known test data.

7.2 Power balance and losses

The power balance of a pump operating without recirculation is shown in Equation (13), which applies when pumping water as well as viscous liquids:

$$P = f \left(\frac{\rho g H Q}{\eta_{\text{vol}} \eta_{\text{h}}} \right) + P_{\text{RR}} + P_{\text{m}} \quad (13)$$

In this equation (P) is the power input at the coupling of the pump; (η_{vol}) is the volumetric efficiency; (η_{h}) is the hydraulic efficiency; (P_{RR}) is the sum of all disc friction losses on the impeller side shrouds and axial thrust balancing drum or disc, if any; and (P_{m}) is the sum of all mechanical losses from radial and axial bearings as well as from shaft seals.

When the viscosity of the liquid pumped increases, the Reynolds number decreases, which causes the friction factors in the hydraulic passages of the pump to increase just as would be the case with flow through a pipe. The increase in viscosity affects pump losses in the following ways

Mechanical losses, P_{m} , are essentially independent of the viscosity of the liquid being pumped.

Hydraulic losses similar to pipe friction losses occur at the inlet, in the impeller, in the volute or diffuser, and in the discharge of a pump. In basic rotodynamic pump theory, the useful head (H) is the difference of the impeller theoretical head (H_{th}) minus the hydraulic losses (H_{L}). In accordance with References [9], [10] and [18] of the bibliography, the flow deflection or slip factor of the impeller is not generally influenced by the viscosity and therefore the theoretical head (H_{th}) is not affected. Thus head reduction due to viscous flow is primarily a function of the hydraulic viscous flow losses.

These hydraulic losses consist of friction losses, which are a function of the Reynolds number (pump size, rotor speed, and viscosity effects), surface roughness of the hydraulic passageways, and mixing losses caused by the exchange of flow momentum due to nonuniform velocity distributions. Such nonuniformities or mixing losses are caused by the action of work transfer from the blades, decelerations of the liquid, angle of incidence between liquid flow and blades, and even local flow separations.

Volumetric losses are caused by leakage flows through the tight running clearances between pump rotor and stator parts. Such leakages decrease with increasing viscosity because the friction factors in the clearances increase with decreasing Reynolds number. The rate of flow through the pump is thus increased, resulting in a higher head. This shift of the H - Q curve caused by reduced leakage compensates to some extent the hydraulic losses mentioned above. The effect may be appreciable for low-specific-speed small pumps with relatively large clearances when operating with viscosities below about 100 cSt. This may be the reason why a moderate increase in viscosity does not have much effect on the head. In fact a slight increase in head has been observed occasionally with increased viscosity. See Reference [23] in the Bibliography, for example.

The information contained in Reference [25] has been used successfully to calculate the leakage flows across axial wear rings.

Disc friction losses are another type of friction loss occurring on all wetted surfaces rotating in the pump. The associated power losses (P_{RR}) strongly influence pump efficiency with viscous liquids. Disc friction losses are generated mainly on the side shrouds of a closed impeller, and in devices for balancing axial thrust. Such losses also increase with decreasing Reynolds number or increasing viscosity; they can be calculated from standard textbooks. State of the art data are given in Reference [8] in the Bibliography.

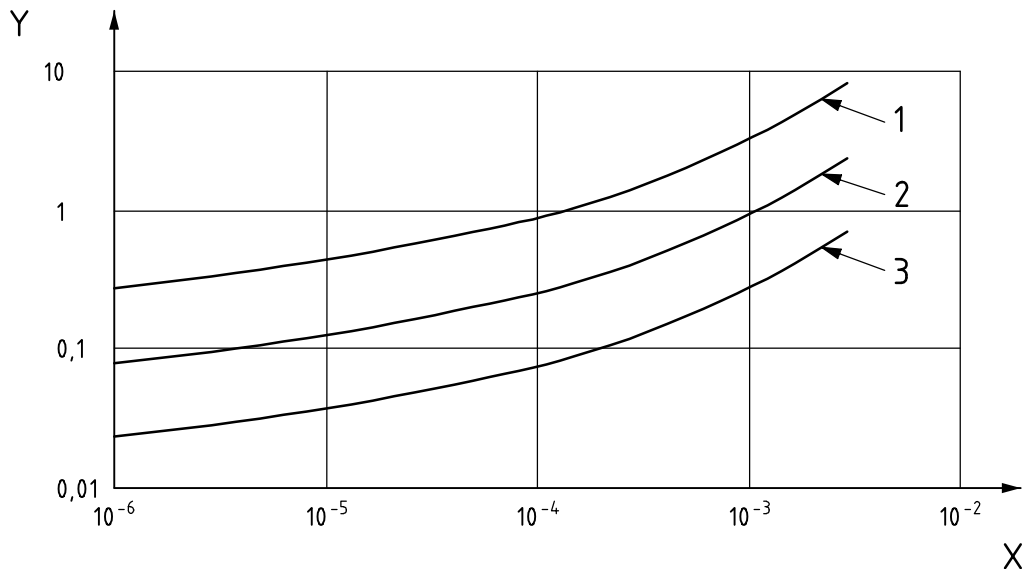
Useful information on the calculation of disc friction and drum friction, which have given good correlation with experimental results, can also be found in References [25], [26] and [27], respectively.

Boundary layers leaving impeller side shrouds also add some useful energy to the liquid being pumped. This effect compensates for some of the hydraulic losses discussed above and may also explain part of the head increase occasionally observed at moderate viscosities.

Disc friction losses have a strong impact on power absorbed by the pump in viscous service. The influences of impeller diameter (d_2), rotational speed (N), specific speed (n_{S}), and head coefficient (ψ) are shown in Equation (14):

$$P_{\text{RR}} = f \left(\frac{d_2^5 N^3}{n_{\text{S}}^2 \psi^{2.5}} \right) \quad (14)$$

The influence of viscosity on efficiency is demonstrated in Figure 9 where the ratio of the disc friction losses (P_{RR}) to the useful power, P_u , is plotted against the viscosity, with the specific speed n_s also as a parameter. In this particular case, the disc friction losses increase by a factor of about 30 when the viscosity rises from 10^{-6} to $3 \times 10^{-3} \text{ m}^2/\text{s}$ (1 to 3 000 cSt). With a viscosity of 3 000 cSt the disc friction power is nearly 10 times larger than the useful power for a specific speed of $n_s = 10$ ($N_s = 500$) and accounts for 50 % of P_u for $n_s = 45$ ($N_s = 2\,300$).



Key

X	kinematic viscosity [m^2/s]
Y	P_{RR}/P_u
1	$n_s = 10$ ($N_s = 500$)
2	$n_s = 20$ ($N_s = 1\,000$)
3	$n_s = 45$ ($N_s = 2\,300$)

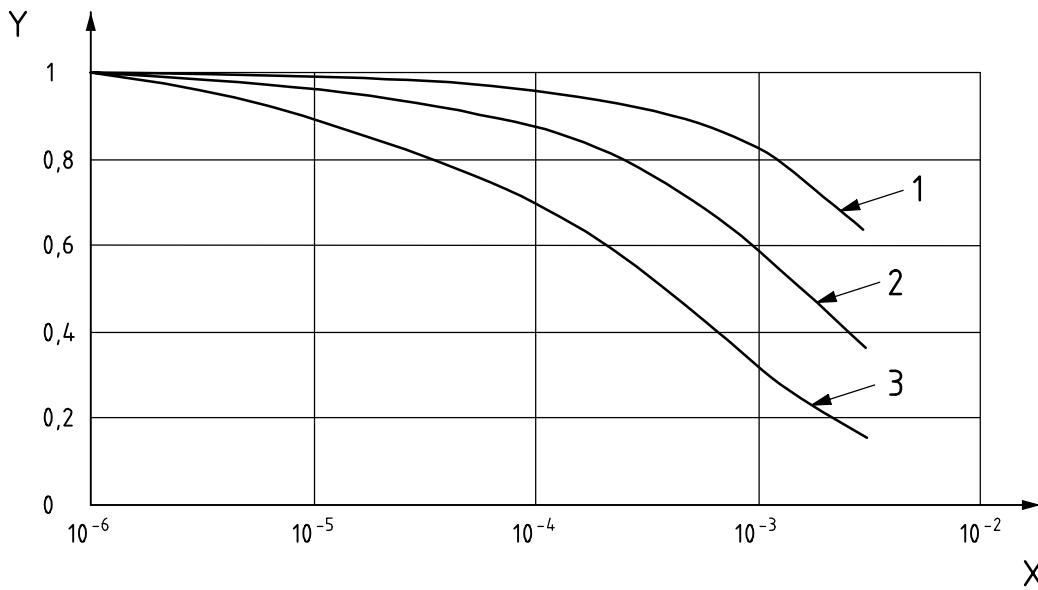
Figure 9 — Ratio of disc friction losses to useful power (References [7] and [8] in Bibliography)

Considering only the effect of the disc friction losses on the efficiency, a multiplier $C_{\eta_{1-RR}}$ can be derived, which is plotted in Figure 10. This demonstrates that efficiency when pumping viscous liquids depends strongly on specific speed, due solely to the effects of disc friction. Absorbed power is likewise affected.

Thermal effects: All power losses, with the exception of external mechanical losses, are dissipated as heat added to the liquid. This increases the local temperature of the liquid and lowers the viscosity compared with the bulk viscosity at pump suction temperature. Local heating of the liquid by high shear stresses mainly affects disc friction losses and volumetric efficiency. At viscosities above about 1 000 cSt, local heating of the liquid may be expected to be appreciable, but the effects cannot be easily quantified.

Power curves $P = f(Q)$: Because theoretical head and mechanical losses are essentially not affected by viscosity, increase in absorbed power when pumping viscous liquids is predominantly caused by disc friction losses. The power for viscous liquids, $P_{vis} = f(Q)$, is therefore shifted relative to the power for water, $P_w = f(Q)$, by an essentially constant amount equivalent to the increase in disc friction losses, except at low flow conditions; Figure 1.

Net positive suction head required (NPSHR) is influenced by the pressure distribution near the leading edge of impeller blades. The pressure distribution depends on both the Reynolds number and hydraulic losses between the pump suction flange and impeller inlet. These losses increase with viscosity and affect NPSHR. Other factors that influence NPSHR are liquid thermodynamic properties and the presence of entrained or dissolved gas. The interaction of these factors is discussed in 7.3. A method of estimating the NPSHR on viscous liquids based on analytical considerations is also outlined in 7.3.



Key

X	kinematic viscosity [m ² /s]
Y	$C_{\eta-RR}$
1	$n_s = 45 (N_s = 2\,300)$
2	$n_s = 20 (N_s = 1\,000)$
3	$n_s = 10 (N_s = 500)$

Figure 10 — Influence of disc friction losses on viscosity correction factor for efficiency (References [7] and [8] in Bibliography)

The effects of viscosity on the pressure drop in the suction piping, hence on NPSHA, need also to be considered.

7.3 Method for estimating net positive suction head required (NPSHR)

NPSHR as a characteristic of rotodynamic pump suction performance represents the total absolute suction head, minus the head corresponding to the vapor pressure at the pump intake, required to prevent more than 3 % loss in total head caused by blockage from cavitation vapor. It depends on the pump operating conditions, the geometry of both pump and intake, as well as the physical properties of the pumped liquid.

There is a dual influence of the pumped liquid viscosity on NPSHR. With increased viscosity the friction goes up, which results in an increase of NPSHR. At the same time, higher viscosity results in a decrease of air and vapor particle diffusion in the liquid. This slows down the speed of bubble growth and there is also a thermodynamic effect, which leads to some decrease of NPSHR.

The effect of viscosity on NPSHR is substantially a function of the Reynolds number. However, this effect cannot be expressed by a single relationship for all of the different pumps designs and types. As a general rule, larger size pumps and pumps with smooth and sweeping impeller inlets are less susceptible to changes in the pumped liquid viscosity.

Gas dissolved in the liquid and gas entrained by the pumped liquid in the form of finely dispersed bubbles influence NPSHR differently than large bubbles of gas. If the flow velocity at the pump inlet is high enough, a small amount of entrained gas does not separate and essentially has no or very little influence on the NPSHR. The presence of larger gas accumulations greatly affects the pump suction performance. It causes the total head – NPSHR characteristic curves to change shape from exhibiting a well-defined “knee” to having a gradual sloping decay in head. This increases the point of 3 % head loss or, in other words, moves the NPSHR to a higher value.

When handling viscous liquids at lower shaft rotational speeds, the NPSHR has been observed to be higher than would be predicted by the affinity rules.

Overall the development of vaporization and gas release depends to a great extent on the time of exposure to lower pressure. In general, a cavitation test at constant rate of flow and speed with variable suction conditions cannot be applied to viscous liquids if variation in suction pressure is obtained by lowering the pressure in the whole test loop. This is because, unlike water, the liquid in the tank will not be rapidly deaerated. Rather, air will gradually diffuse out of the liquid in the suction line and will cause blockage at the impeller inlet.

The following generalized method is provided for approximation purposes but the user is cautioned that it is based on an analytical approach and is not based on actual NPSHR test data. When pumping highly viscous liquids, ample margins of NPSHA over the NPSHR are required and the advice of the pump manufacturer should be sought.

This generalized method should not be applied to hydrocarbons without consideration of thermal effects on the liquid properties. See ANSI/HI 1.3.4.1.16.3 [24].

The following equations are used for developing the correction factor to adjust the pump water performance NPSHR, based on the standard 3% head drop criteria, to the corresponding viscous liquid NPSHR_{vis} performance.

Given units of $Q_{\text{BEP-W}}$ in m³/h, NPSHR_{BEP-W} in m, and N in rpm, use Equation (15):

$$C_{\text{NPSH}} = 1 + \left\{ A \times \left(\frac{1}{C_H} - 1 \right) \times 274\,000 \times \left[\frac{\text{NPSHR}_{\text{BEP-W}}}{(Q_{\text{BEP-W}})^{0,667} \times N^{1,33}} \right] \right\} \quad (15)$$

Given units of $Q_{\text{BEP-W}}$ in gpm, NPSHR_{BEP-W} in ft, and N in rpm, use Equation (16):

$$C_{\text{NPSH}} = 1 + \left\{ A \times \left(\frac{1}{C_H} - 1 \right) \times 225\,000 \times \left[\frac{\text{NPSHR}_{\text{BEP-W}}}{(Q_{\text{BEP-W}})^{0,667} \times N^{1,33}} \right] \right\} \quad (16)$$

The value of the suction inlet geometry variable (A) is selected as follows.

For end suction pumps: $A = 0,1$

For side inlet pumps (flow passageway bends approximately 90 degrees from suction nozzle into the impeller): $A = 0,5$

Values of NPSHR are adjusted by the NPSHR correction factor, C_{NPSH} .

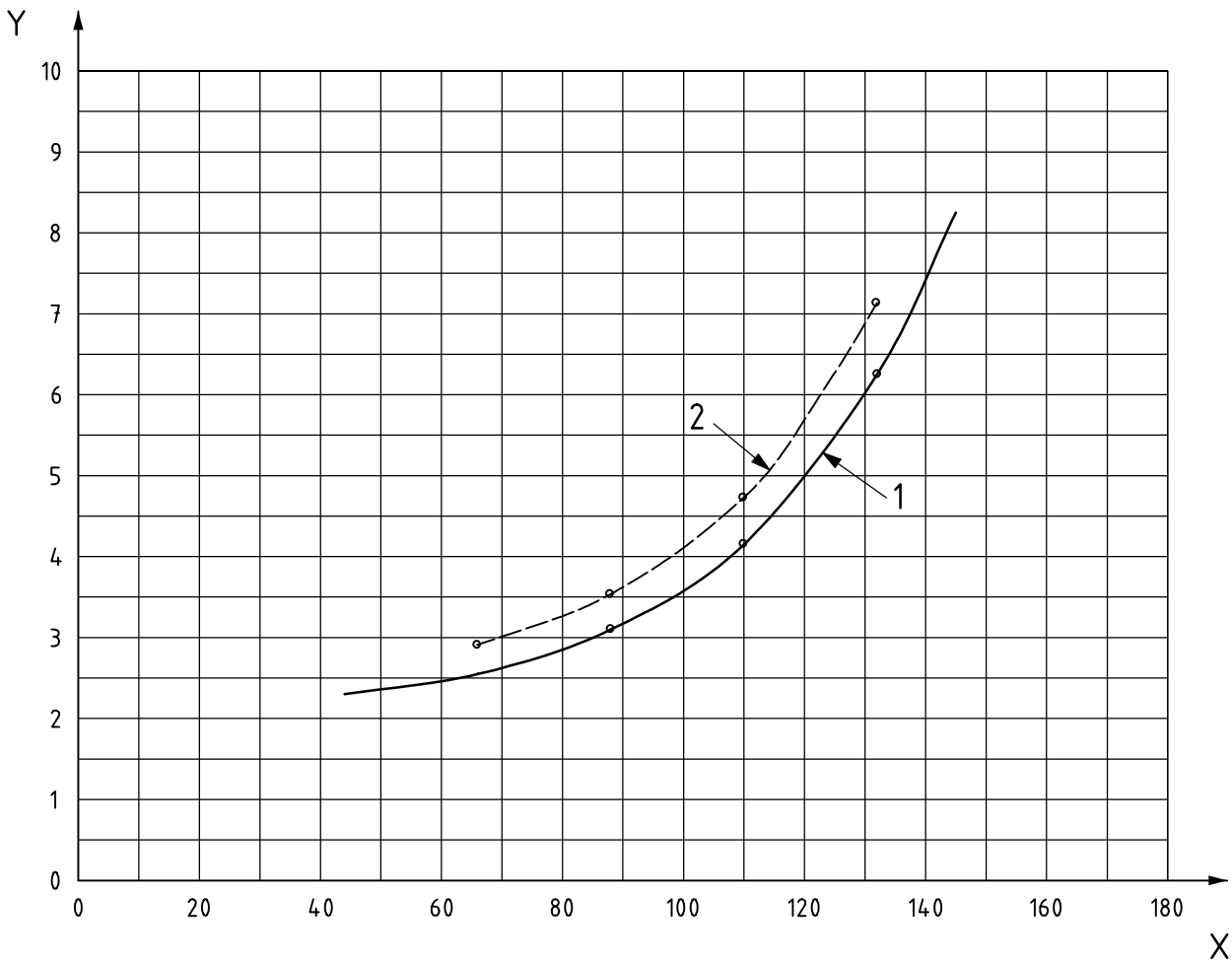
$$\text{NPSHR}_{\text{vis}} = C_{\text{NPSH}} \times \text{NPSHR}$$

Rate of flow is not corrected in this NPSHR correction method. For rate of flow corresponding to corrected values of NPSHR_{vis}, use uncorrected values of Q_W .

An example of this NPSHR correction method is illustrated in Figures 11 and 12.

EXAMPLE (Metric units): Refer to Figure 11 and Table 3. Assume that the example pump has a radial suction inlet configuration with $A = 0,5$. Assume the $Q_{\text{BEP-W}}$ rate of flow is 110 m³/h, the NPSHR_{BEP-W} is 4,15 m, the speed N is 2 950 rpm, and the B factor is 12,0 yielding a head correction factor C_H of 0,81. Calculate the NPSHR correction factor using Equation (15):

$$C_{\text{NPSH}} = 1 + 0,5 \times \left(\frac{1}{0,81} - 1 \right) \times 274\,000 \times \left(\frac{4,15}{110^{0,667} \times 2\,950^{1,33}} \right) = 1,14$$



- Key**
- X rate of flow, expressed in cubic meters per hour at 2 950 rev/min
 - Y NPSH - meters
 - 1 water
 - 2 viscous liquid, with $s = 0,90$ and $B = 12,0$

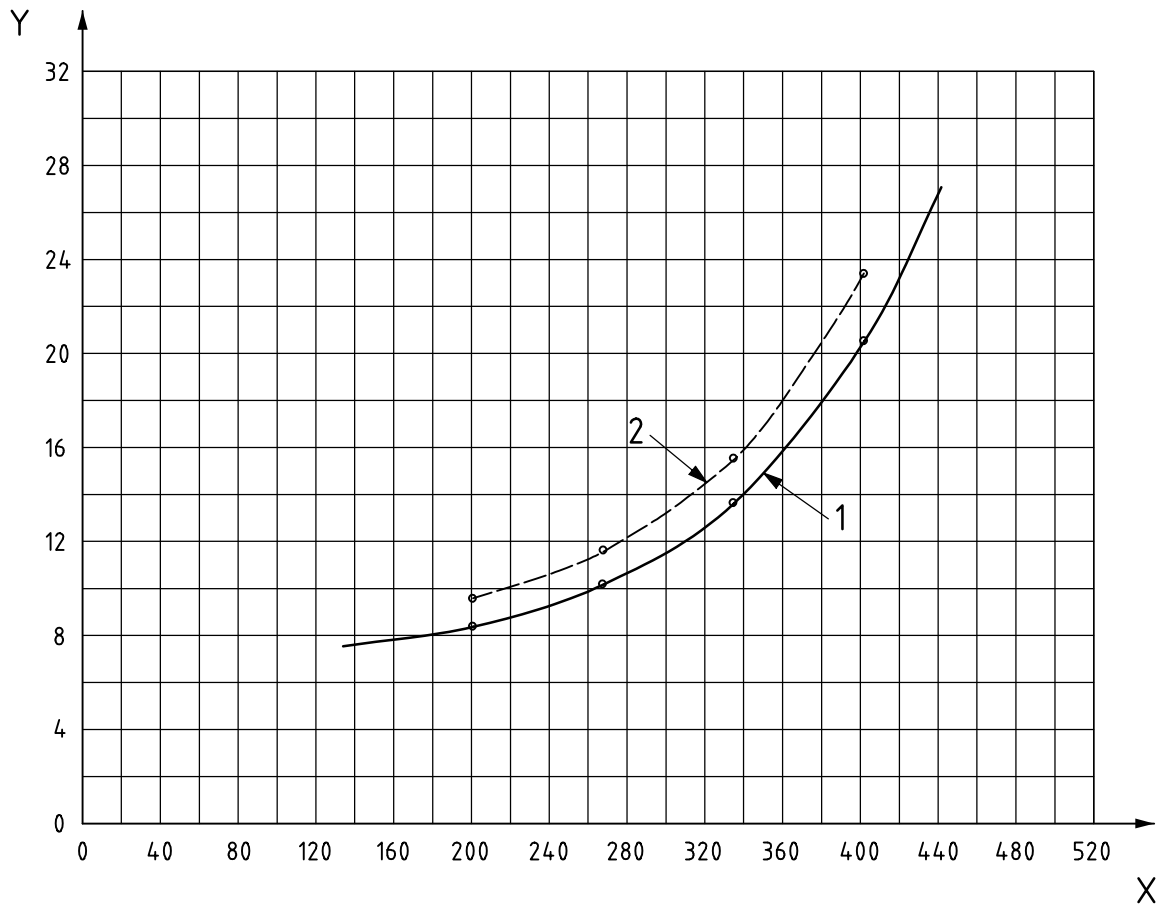
Figure 11 — Example NPSHR vs. rate-of-flow chart, expressed in metric units

Table 3 — Example calculations (metric units)

<i>B</i> factor	12,0			
Specific gravity of viscous liquid (<i>s</i>)	0,90			
Pump shaft speed (<i>N</i>) — rpm	2 950			
Ratio of water best efficiency flow Q_W/Q_{BEP-W}	0,60	0,80	1,00	1,20
Water rate of flow (Q_W) — m ³ /h	66	88	110	132
Water net positive suction head required (NPSHR _W) — m	2,55	3,10	4,15	6,25
Correction factor for head at best efficiency flow (C_H)	0,81			
Correction factor for NPSHR (C_{NPSH})	1,14			
Corrected net positive suction head required (NPSHR _{vis}) — m	2,91	3,53	4,73	7,13

EXAMPLE (USCS units): Refer to Figure 12 and Table 4. Assume that the example pump has a radial suction inlet configuration with $A = 0,5$. Assume the $Q_{\text{BEP-W}}$ rate of flow is 335 gpm, the $\text{NPSHR}_{\text{BEP-W}}$ is 13,6 ft, the speed N is 3 550 rpm, and the B factor is 12,0 yielding a head correction factor C_H of 0,81. Calculate the NPSHR correction factor using Equation (16):

$$C_{\text{NPSH}} = 1 + 0,5 \times \left(\frac{1}{0,81} - 1 \right) \times 225\,000 \times \left(\frac{13,6}{335^{0,667} \times 3\,550^{1,33}} \right) = 1,14$$



Key

- X rate of Flow, expressed in gallons per minute at 3 550 rev/min
- Y NPSH - feet
- 1 water
- 2 viscous liquid, with $s = 0,90$ and $B = 12,0$

Figure 12 — Example NPSHR vs. rate-of-flow chart, expressed in USCS units

Table 4 — Example calculations (USCS units)

B factor	12,0			
Specific gravity of viscous liquid (s)	0,90			
Pump shaft speed (N) — rpm	3 550			
Ratio of water best efficiency flow Q_W/Q_{BEP-W}	0,60	0,80	1,00	1,20
Water rate of flow (Q_W) — gpm	201	268	335	402
Water net positive suction head (NPSHR _W) — ft	8,37	10,2	13,6	20,5
Correction factor for head at best efficiency flow (C_H)	0,81			
Correction factor for NPSHR (C_{NPSH})	1,14			
Corrected net positive suction head required (NPSHR _{vis}) — ft	9,54	11,6	15,5	23,4

8 Additional considerations

8.1 General

This section explains some limitations of the correction method, particular pump design effects, some mechanical considerations, and sealing issues when pumping viscous liquids. Information is in general qualitative due to the lack of quantitative facts.

8.2 Limitations

The correction formulas in Clause 6 are based on test data with parameter B values up to approximately $B = 35$. Extrapolation with B values higher than 40 is not advisable as the calculated pump-shaft input power may be excessively high. In such cases, the loss analysis method may be necessary to more accurately predict the viscous hydraulic performance and power requirements.

Due to limited available test data above $n_s = 40$ ($N_s = 2\,000$), the performance predictions using the generalized method for pumps with specific speeds above this value may involve greater uncertainties.

Performance guarantees are normally based on water performance. All methods for viscous corrections are subject to uncertainty and adequate margins need to be considered, especially with respect to the pump driver rating.

The prediction procedures discussed are based on tests with Newtonian liquids. Non-Newtonian liquids may behave quite differently.

A few studies indicate that pump head slightly increases over that of water when operating with viscosities up to 180 cSt. There is substantial data scatter in viscous flow investigations, and this phenomenon is observed only occasionally. It might be explained by the factors that tend to increase head with increasing viscosity, such as disc pumping and reduced leakage losses, which overcome, up to a certain point, the bulk viscosity effect tending to reduce head.

8.3 Pump design effects

Pumps in the range of $20 \leq n_s \leq 40$ ($1\,000 \leq N_s \leq 2\,000$) can be expected, based on available data, to give the highest efficiencies when viscous liquids are being pumped.

This publication provides viscosity performance corrections only for the pumping element. Pumps that incorporate external piping, a suction barrel for vertical can type pumps, a discharge column, or other appurtenances for liquid conveyance to or from the pumping element, require additional consideration for viscous losses. Traditional piping liquid flow viscosity calculations could be adapted for this purpose.

Impellers with auxiliary pump-out vanes are likely to require additional power in viscous pumping applications. Thermal effects, however, may tend to limit the added power by reducing disc friction

High head coefficient impeller designs (with higher vane numbers and steeper vane discharge angles) tend to have higher efficiencies but also tend to exhibit flat or drooping $H-Q$ curves towards shut off in water tests. The $H-Q$ curve becomes steeper when high viscosity liquids are pumped. High head coefficient designs may therefore be acceptable if the head curve with viscous liquids rises to shut off.

The axial clearances between the impeller shrouds and the pump casing have a strong impact on disc friction losses and efficiency in laminar flow (viscous pumping) but are insignificant in turbulent flow. Two otherwise identical pumps with different axial clearances may have the same efficiency with water, but different efficiencies with viscous liquids if operation should extend into the laminar flow regime.

While the surface roughness (casting quality) has a significant influence on the efficiency when pumping water, its impact is diminished in viscous applications and is theoretically zero in laminar flow.

8.4 Mechanical considerations

Mechanical design of pumps, drivers and couplings should consider the increased viscosity and resulting torque that will occur if pumps start with liquid temperatures below the normal operating temperature.

Internal pump components, such as the pump shaft and associated drive mechanisms, should be checked to ensure they are adequate for the additional torque that the pump will experience.

Externally, proper sizing of the pump driver needs to be considered as increased starting and operating torque will be required. It is recommended that a speed–torque curve specific to the application be supplied by the pump vendor if there is concern regarding the driver size and design.

The coupling between the pump and driver needs to be sized for the higher torque and starting cycles demanded by the service.

8.5 Sealing issues

Sealing issues related to viscous liquids are complex. Seal manufacturers should be consulted for detailed information.

Mechanical seals or sealing devices must be capable of sealing the pump for the range of anticipated viscous conditions, including transient or upset conditions. Mechanical seal components may not perform as anticipated and may experience higher loads than with water.

Associated with the mechanical seal(s) are the seal flushing arrangement and associated piping. In many cases auxiliary systems include secondary components, such as orifices and filters, that may plug or cease to function correctly when handling viscous liquids. The piping is normally external to the pump case and may require heat tracing or other consideration to ensure proper seal flushing.

8.6 Sealless pumps

The use of sealless pumps requires additional consideration. There are two basic kinds of sealless pumps: canned motor pumps and magnetic drive pumps. In canned motor pumps, the motor rotor and sleeve bearings are immersed in the pumped liquid. In magnetic drive pumps, the shaft magnetic coupling and bearings are immersed in the pumped liquid. The additional viscous drag due to the immersion of these components will lead to higher losses, resulting in increased power consumption and increased starting torque requirements. Heating of the viscous fluid in the rotor chamber may be a mitigating factor in sealless pump losses. Furthermore, cooling flow to the motor or magnetic coupling and bearings will be decreased. The temperature rise caused by the increased losses and decreased cooling flow must also be considered. In addition, the ability of the liquid to lubricate the sleeve bearings must be evaluated.

Annex A (informative)

Conversion of kinematic viscosity units

Definitions

ν_{cSt} = Kinematic viscosity in centistokes (cSt) of the pumped liquid

ν_{SSU} = Kinematic viscosity in Seconds Saybolt Universal (SSU)

For convenience, the following Equation (A.1) is provided for converting kinematic viscosity in Seconds Saybolt Universal (SSU; also known as Saybolt Universal Seconds, SUS) to centistokes (cSt). This SSU to cSt conversion equation has been derived from a set of values produced by Equation (A.2) below.

Equation A.1

For $32 \text{ SSU} \leq \nu_{\text{SSU}} \leq 2316 \text{ SSU}$

$$\nu_{\text{cSt}} = 0,2159\nu_{\text{SSU}} - \left[\frac{10\,000 \times (\nu_{\text{SSU}} + 17,06)}{(0,9341\nu_{\text{SSU}}^3 + 9,01\nu_{\text{SSU}}^2 - 83,62\nu_{\text{SSU}} + 53\,340)} \right] \quad (\text{A.1})$$

cSt to SSU

The following equation, as given in ASTM Designation D 2161 - 93 (Reapproved 1999)^{e2[28]}, based on the 38 °C (100 °F) data, can be used to convert kinematic viscosity in cSt to SSU.

Equation A.2

For $1,81 \text{ cSt} \leq \nu_{\text{cSt}} \leq 500 \text{ cSt}$

$$\nu_{\text{SSU}} = 4,6324\nu_{\text{cSt}} + \left[\frac{1,0 + 0,03264\nu_{\text{cSt}}}{(3930,2 + 262,7\nu_{\text{cSt}} + 23,97\nu_{\text{cSt}}^2 + 1,646\nu_{\text{cSt}}^3) \times 10^{-5}} \right] \quad (\text{A.2})$$

Conversion of dynamic (absolute) viscosity to kinematic viscosity

If viscosity of pumped liquid is given in terms of dynamic, or absolute, viscosity, it should be converted to kinematic viscosity to use the pump performance correction method. Numerical values of dynamic viscosity are usually expressed in centipoise (cP) or Pascal-seconds (Pa-s). Kinematic viscosity is obtained by dividing the dynamic (absolute) viscosity by the mass density.

$$\nu = \frac{\mu}{\rho}$$

To convert dynamic viscosity in centipoise (cP), divide by the mass density in grams per cubic centimeter (g/cm³) to obtain the kinematic viscosity in centistokes (cSt).

To convert dynamic viscosity in Pascal-seconds (Pa-s), divide by the mass density in kilograms per cubic meter (kg/m³) to obtain kinematic viscosity in square meters per second (m²/s).

Conversion from CGS units to SI units

Quantity	CGS units	Conversion ratio to SI units	SI units
Viscosity (μ)	Poise (P) [g/(cm-s)]	10^{-1}	Pa-s
	Centipoise (cP)	10^{-3}	Pa-s
Kinematic Viscosity (ν)	Stokes (St) (cm ² /s)	10^{-4}	m ² /s
	Centistokes (cSt)	10^{-6}	m ² /s

Conversion from SI units to CGS units

Quantity	SI units	Conversion ratio to CGS units	CGS units
Viscosity (μ)	Pa-s	10^1	Poise (P) [g/(cm-s)]
	Pa-s	10^3	Centipoise (cP)
Kinematic Viscosity (ν)	m ² /s	10^4	Stokes (St) (cm ² /s)
	m ² /s	10^6	Centistokes (cSt)

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