TECHNICAL REPORT

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Geometrical product specifications (GPS) — Systematic errors and contributions to measurement uncertainty of length measurement due to thermal influences

Spécifications géométriques des produits (GPS) — Erreurs systématiques et contributions à l'incertitude de mesure de la longueur, dues aux influences thermiques

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 16015 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

Introduction

This Technical Report is a geometrical product specification (GPS) document and is to be regarded as a global GPS document (see ISO/TR 14638). It influences chain links 4, 5 and 6 of the chains of standards.

For more detailed information on the relationship of this Technical Report to other standards and to the GPS matrix model, see Annex C.

Uncertainty in temperature measurement and measurement at other than the standard reference temperature lead to uncertainty in the length measurement results. In addition, measurements at other than the standard reference temperature leads to systematic error in the measurement result.

The principle addressed by this Technical Report is that most materials expand or contract when their temperatures are changed. If the temperature at which the measurement is made is the standard reference temperature, the nominal thermal expansion is zero but uncertainty in the measurement of temperature leads to uncertainty in the measurement result. If length measurements are made at temperatures other than the standard reference temperature, there will be a resulting differential thermal expansion. This can arise both when the measuring instrument is adjusted, as by comparison with a working standard, and when it is used to measure the workpiece.

If the temperatures and the response to thermal changes of the workpiece, the working standard, and the measuring instrument are known, a correction can be made for differential thermal expansion. It is impossible to know exactly either the temperatures or the response; thus, there will be an uncertainty in the correction and in the measurement result. This Technical Report shows how to calculate the relevant systematic error and evaluate the thermal contribution to the measurement uncertainty.

The resulting standard uncertainty component due to thermal effects shall be combined in the usual manner (see GUM) in order to evaluate the combined standard uncertainty for a measurement.

When necessary, an appropriate decision rule (e.g. an acceptable fraction of workpiece tolerance or that embodied in ISO 14253-1) can be invoked so that the consequence of the thermally-induced dimensional uncertainty on workpiece conformance decisions can be determined.

ISO/TR 16015 is developed in support of ISO 1.

It is recognized that this Technical Report, developed in support of ISO 1, will have to be brought in line with ISO/TS 17450-2, but at the time of publication this presentation was the only practical one possible.

Geometrical product specifications (GPS) — Systematic errors and contributions to measurement uncertainty of length measurement due to thermal influences

1 Scope

This Technical Report defines procedures for the determination of errors, appropriate corrections and the evaluation of measurement uncertainty contributors due to thermal influences in the performance of length measurements for geometrical product specification (GPS), when

— the mean temperature is the standard reference temperature,

— the mean temperature is not the standard reference temperature and where the mean is taken over time and space,

— temperature varies with time.

Three cases are considered:

a) the mean temperature is the standard reference temperature;

b) the mean temperature is not the standard reference temperature and the user makes corrections;

c) the mean temperature is not the standard reference temperature and the user makes no corrections.

This Technical Report provides the specialized details for the analysis of thermal influence when broader analyses, such as those in accordance with ISO/TS 14253-2, show that thermal effects are significant contributors to the measurement uncertainty, systematic error or both. While temperature gradients are not covered in detail, some information is given (see Annex A).

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 1, *Geometrical Product Specifications (GPS) — Standard reference temperature for geometrical product specification and verification*

Guide to the Expression of Uncertainty in Measurement (*GUM*). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 1st edition, 1993, corrected and reprinted in 1995

International Vocabulary of Basic and General Terms in Metrology (VIM). BIPM, IFCC, IEC, ISO, IUPAC, IUPAP, OIML, 2nd edition, 1993

3 Terms and definitions

For the purposes of this document, the terms and definitions given in GUM and VIM and the following apply.

3.1 Terms concerning coefficient of thermal expansion

3.1.1
coefficient of thermal expansion

α

ratio of the fractional change of length to the change in temperature

NOTE 1 In general, this coefficient is a function of temperature. For the purposes of this Technical Report, however, a temperature range averaged value of the coefficient is used in accordance with

$$\alpha(20,T) = \frac{L_T - L_{20}}{L_{20}(T-20)} \tag{1}$$

where the temperature range is relative to 20°C in accordance with ISO 1.

NOTE 2 To the symbol α are added subscripts to denote the coefficient of thermal expansion for the workpiece (i.e. α_w) and the working standard (i.e. α_s).

NOTE 3 *Coefficient of thermal expansion* can describe the behaviour of a non-homogeneous body such as a composite working standard or a workpiece that is an assembly of different materials. No distinction is made in this document between coefficients of expansion for homogeneous and composite working standards or workpieces.

3.1.2
measured coefficient of thermal expansion

α_m

experimentally-determined coefficient of thermal expansion of a specific individual object

NOTE 1 This coefficient can be obtained from the calibration service of an accredited measurement laboratory or from properly performed experiments.

NOTE 2 To the symbol α_m are added subscripts to denote the measured coefficient of thermal expansion for the workpiece (i.e. α_{mw}) and the working standard (i.e. α_{ms}).

3.1.3
nominal coefficient of thermal expansion

α_n

approximate value for the coefficient of thermal expansion over a range of temperature from 20 °C to T

NOTE To the symbol α_n are added subscripts to denote the nominal coefficient of thermal expansion for the workpiece (i.e. α_{nw}) and the working standard (i.e. α_{ns}). Estimated values for α_{nw} and α_{ns} can be obtained from experiments on like objects or from published data.

3.1.4
uncertainty of coefficient of thermal expansion

$u(\alpha)$

parameter that characterizes the dispersion of the values that could reasonably be attributed to the coefficient of thermal expansion

NOTE 1 To the symbol $u(\alpha)$ are added subscripts to denote the uncertainty of coefficient of thermal expansion for the workpiece [i.e. $u(\alpha_w)$] and the working standard [i.e. $u(\alpha_s)$]. Such values, like that of α itself, shall either be estimates, based on an assumed probability distribution, or the results of experiments performed to measure α.

NOTE 2 The uncertainty for α_n is usually much larger than the uncertainty for α_m.

3.2 Terms concerning thermal expansion

3.2.1
thermal expansion

Δ_{E}

change in length of a body, such as a workpiece or working standard, in response to a temperature change

3.2.2
thermal expansion based on nominal or measured coefficients of thermal expansion

Δ_{nE} or Δ_{mE}

estimate of the thermal expansion of an object from 20 °C to its average temperature at the time of measurement

NOTE 1 The estimate is based on either nominal or measured coefficients of thermal expansion:

$$\Delta_{nE} = \alpha_{n} L(T - 20\ °C) = \alpha_{n} L\theta$$

$$\Delta_{mE} = \alpha_{m} L(T - 20\ °C) = \alpha_{m} L\theta$$

$$\Delta_{E} = \Delta_{nE} \text{ or } \Delta_{mE} \qquad (2)$$

NOTE 2 To the symbol Δ_{nE} or Δ_{mE} are added subscripts to denote the thermal expansion based on nominal/measured coefficients of thermal expansion for the workpiece (i.e. Δ_{nEw} or Δ_{mEw}) and the working standard (i.e. Δ_{nEs} or Δ_{mEs}).

3.2.3
corrected length

L_{c}

measured length adjusted for the computed thermal expansion based on either the measured or nominal thermal expansion

NOTE The corrected length can be calculated to

$$L_{c} = L_{m} - \Delta_{mE}$$

or

$$L_{c} = L_{m} - \Delta_{nE} \qquad (3)$$

3.2.4
differential thermal expansion

difference between the changes in the lengths of the workpiece and the working standard in response to temperature changes from 20 °C to their temperatures at the times of the measurements

NOTE If not corrected, differential thermal expansion can cause a systematic error in a measurement result.

3.2.5
differential thermal expansion based on nominal or measured coefficients of thermal expansion

Δ_{nDE} or Δ_{mDE}

difference between the thermal expansion of the workpiece and the working standard(s)

NOTE The differential thermal expansion based on nominal or measured coefficients of thermal expansion can be calculated by

$$\Delta_{nDE} = \Delta_{nEw} - \Delta_{nEs}$$

or

$$\Delta_{mDE} = \Delta_{mEw} - \Delta_{mEs} \qquad (4)$$

3.2.6
uncertainty of thermal expansion due to uncertainty of α
$u_{\mathrm{E}}(L)$
uncertainty in the thermal expansion arising from uncertainty in the coefficient of thermal expansion

NOTE 1 The uncertainty of thermal expansion due to uncertainty of α can be calculated by

$$u_{\mathrm{E}}(L) = L\theta\, u(\alpha) \tag{5}$$

NOTE 2 An additional subscript is added to each symbol to specify values for a workpiece ("… $_{\mathrm{w}}$") or working standard ("… $_{\mathrm{s}}$").

3.2.7
uncertainty of differential thermal expansion due to uncertainties of α_{w} and α_{s}
$u_{\mathrm{DE}}(L)$
combined standard uncertainty of the uncertainties of thermal expansion of the workpiece and working standard due to uncertainties in their coefficients of expansion

NOTE 1 Assuming that the coefficients of thermal expansion of the workpiece and the working standard are uncorrelated, the value of $u_{\mathrm{DE}}(L)$ is given by

$$\begin{aligned} u_{\mathrm{DE}}(L) &= \sqrt{u_{\mathrm{E}}^2(L_{\mathrm{w}}) + u_{\mathrm{E}}^2(L_{\mathrm{s}})} \\ &= \sqrt{\theta_{\mathrm{w}}^2 L_{\mathrm{w}}^2 u^2(\alpha_{\mathrm{w}}) + \theta_{\mathrm{s}}^2 L_{\mathrm{s}}^2 u^2(\alpha_{\mathrm{s}})} \end{aligned} \tag{6}$$

NOTE 2 The estimates of the coefficients of thermal expansion of the workpiece and working standard can be assumed to be uncorrelated when they are obtained from different sources.

NOTE 3 When nominal values are used to estimate the coefficients of thermal expansion, this quantity has historically been called the *uncertainty of nominal differential expansion (UNDE)*.

3.3 Dimensional consequences of temperature variation

3.3.1
dimensional thermal response
amplitude of length variation of an object as a thermal response to the magnitude and time-dependency of a temperature fluctuation

3.3.2
thermal response time
soak-out time
time interval between the instant when the environmental temperature is subjected to a specified abrupt change and the instant when the temperature of an object reaches and remains within specified limits around its final steady value

NOTE When a change in environmental temperature is experienced, such as occurs when an object is transported from one room to another, there will be some period of time during which the object equilibrates, within specified limits, to its new environmental temperature.

3.3.3
differential thermal response
length difference between any two objects measured simultaneously in an environment with varying temperature and caused solely by the variation in temperature of the objects with time

3.3.4
dimensional variation due to environmental temperature variation
E_{ETV}
estimate of the possible length measurement variation induced solely by deviation of the environment from average conditions over a time interval equivalent to the adjustment cycle time

NOTE The dimensional variation due to environmental temperature variation, E_{ETV}, is usually determined from the results of two **drift test(s)** (3.4.5), one of the working standard and comparator and the other of the workpiece and comparator. However, drift tests performed at a single position cannot reveal all thermally-induced errors, particularly where spatial temperature gradients exist. It is the responsibility of the instrument user to ensure that such effects are detected and the appropriate uncertainties added.

3.3.5
uncertainties of workpiece and working standard temperatures
$u(\theta_w)$ and $u(\theta_s)$
uncertainty of time-averaged temperature of the workpiece and working standard

NOTE 1 These uncertainties arise from the calibration of the thermometer, thermometer mounting procedures, and instrumental variations.

NOTE 2 The time over which the temperature is averaged is usually the period of a measurement cycle.

3.3.6
uncertainty of length due to temperature measurement
$u_{TM}(L)$
uncertainty in a measured length due to uncertainty in the measurement of the temperature at which the length measurement was made

NOTE 1 In the case where there is a workpiece and a working standard, it is given by

$$u_{TM}(L) = \sqrt{\alpha_w^2 L_w^2 u^2(\theta_w) + \alpha_s^2 L_s^2 u^2(\theta_s)} \tag{7}$$

where $u(\theta_w)$ and $u(\theta_s)$ are the **uncertainties of the workpiece and the working standard temperatures** (3.3.5) in measurement. The temperature measurements of the workpiece and the working standard are assumed to be uncorrelated, an assumption which is valid if the two temperatures are measured using two different thermometers, each traceable to the international temperature scale (ITS-90) through different calibration processes.

NOTE 2 In cases where the standard is a laser interferometer, the coefficient relating wavelength to the temperature of the propagation medium is used for the coefficient of expansion of the working standard. In the case where the medium is "standard" air, the value for α_{ns} is 0,93 × 10^{-6}/°C.

3.3.7
uncertainty of length due to environmental temperature variation
$u_{ETV}(L)$
uncertainty in the estimate of the possible length measurement variation induced solely by deviation of the environmental temperature from average conditions over a time interval equivalent to the adjustment cycle time

NOTE 1 The dimensional uncertainty of length due to environmental temperature variation, $u_{ETV}(L)$, arises from the range of the drift of the instrument/working standard/workpiece system over times equivalent to the adjustment cycle time. It is established from measurements or estimates of dimensional variation due to environmental temperature variation, E_{ETV}.

NOTE 2 In computing $u_{ETV}(L)$, note that the maximum error is the range observed in the drift test, depending on whether the setting was done at the lowest temperature and the measuring at the highest temperature, or vice versa. The probability density function may be "U-shaped" as, for example, for a sinusoidally varying temperature. However, the probability that the setting and measuring are exactly displaced in time by the period of the sinusoidally varying temperature is low. The setting–measuring process involves taking pairs of points from the U-shaped probability distribution function and then differencing them. The consequence of this procedure is that the measurement process has a different probability distribution function from “U-shaped”. The default is to use a uniform distribution [see 5.4 and Equation (13)].

3.4 Measuring instruments, measuring procedures and metrology

3.4.1
comparator
measuring device used to perform a comparison of a workpiece and a working standard

NOTE A comparator can be a simple short-range indicating device such as a gauge block comparator or a complex comparator such as a coordinate measuring machine.

3.4.2
working standard
standard that is used routinely to calibrate or check material measures, measuring instruments or reference materials

[VIM:1993, definition 6.7]

NOTE 1 A working standard is usually calibrated against a *reference standard* (see VIM:1993, definition 6.6).

NOTE 2 A working standard used routinely to ensure that measurements are being carried out correctly is called a check standard. While it is recognized that the use of *working standard* versus *check standard* has not yet been perfectly resolved, at the time of publication this terminology remained the only practical solution possible. The subject is to be re-examined at the next revision of this Technical Report.

NOTE 3 For procedures in accordance with this Technical Report, such working standards are dimensional and can be in the form of the wavelength of light, a gauge block, a line standard, a lead screw, etc.

3.4.3
adjustment
operation of bringing a measuring instrument into a state of performance suitable for its use

[VIM:1993, definition 4.30]

EXAMPLE The action of nulling or setting a comparator with a working standard.

NOTE Adjustment may be automatic, semi-automatic or manual.

3.4.4
adjustment cycle time
period between successive adjustments of a comparator

3.4.5
drift test
experiment conducted to determine the slow change in the metrological characteristic of a measuring instrument

3.4.6
tolerance
TOL
difference between the upper and lower tolerance limits

[ISO 3534-2:1993, definition 1.4.4]

NOTE For the purpose of this Technical Report, the abbreviation TOL is used instead of the standard “T” (for target value), in order to avoid confusion with other symbols or abbreviations using the same letter.

3.4.7
target uncertainty
U_T
uncertainty determined as the optimum for the measuring task

[ISO/TS 14253-2:1999, definition 3.10]

3.4.8
systematic error
mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions minus a true value of the measurand

NOTE 1 Systematic error is equal to error minus random error.

NOTE 2 Like true value, systematic error and its causes cannot be completely known.

NOTE 3 For a measuring instrument, see *bias* (VIM:1993, definition 5.25).

[VIM:1993, definition 3.14]

3.5 Dimensional quantities related to thermal effects

3.5.1
standard uncertainty component due to thermal effects
$u_{cT}(L)$
standard uncertainty component for a length measurement made at a temperature other than 20 °C

NOTE The standard uncertainty component due to thermal effects can be calculated by

$$u_{cT}(L) = \sqrt{u_{ETV}^2(L) + u_{DE}^2(L) + u_{TM}^2(L)} \tag{8}$$

A simplified form arises in the case where: $\alpha_s = \alpha_{ns} = \alpha_w = \alpha$ with a standard uncertainty $u(\alpha)$; $\theta_w = \theta_s = \theta$ with a standard uncertainty $u(\theta)$; and where the nominal lengths $L_w = L_s = L$. In this case, $\Delta_{nDE} = 0$ and

$$u_{cT}(L) = \sqrt{u_{ETV}^2(L) + 2\theta^2 L^2 u^2(\alpha) + 2\alpha^2 L^2 u^2(\theta)}$$

3.5.2
thermal error
TE
estimate of the maximum error that might reasonably be expected to occur if a length measurement is not corrected for a nominal differential expansion

NOTE 1 The thermal error can be calculated by

$$\mathrm{TE} = \left[\left|\Delta_{nDE}\right| + 2u_{cT}(L)\right] \tag{9}$$

NOTE 2 If length measurements are made at other than 20 °C, and in the special case where corrections are not made for the nominal differential thermal expansion between working standard and workpiece, then, in order to be in accordance with this Technical Report, the thermal error has to be evaluated and reported.

3.5.3
thermal error index
TEI
fraction of the tolerance that is consumed by the thermal error

NOTE 1 The thermal error index can be calculated by

$$\mathrm{TEI} = 2 \times \frac{\mathrm{TE}}{\mathrm{TOL}} \times 100\ \% \tag{10}$$

NOTE 2 In the case where a target uncertainty, U_{T}, exists, the thermal error index is computed using Equation (10), with TOL replaced by 2 U_{T}.

NOTE 3 The TEI is a component of uncertainty management; large values of the TEI may require that a measurement result has to be corrected for the differential thermal expansion in order to prove workpiece conformance.

4 Symbols and abbreviated terms

The standard reference temperature is 20 °C (see ISO 1). All temperatures are assumed to be in degrees Celsius. The majority of the symbols used in this Technical Report are based on the conventions given in Table 1, while Table 2 provides an overview of the symbols used.

Table 1 — Symbol conventions

Symbol	Explanation
α_{ij}	Coefficient of thermal expansion, where: i is the subscript m or n, where m is measured and n is nominal, and j = w or s, where w represents the workpiece and s the working standard
$\Delta_{(\ldots)\mathrm{DE}}$	Difference between thermally-induced expansions of the workpiece and working standard, where (…) may contain the subscript m or n, where m is for measured and n for nominal
$\Delta_{(\ldots)\mathrm{E}}$	Thermally-induced expansions, where (…) may contain the subscript m or n, where m is for measured and n for nominal, and the subscript w or s, where w represents the workpiece and s the working standard
L_j	Length where j is the subscript: m for measured, uncorrected for temperature; n for nominal; c for corrected for nominal expansion; T is at temperature T; or w or s, where w represents the workpiece and s the working standard
T_j	Temperature where j is the subscript w or s, where w represents the workpiece and s the working standard
θ_j	Temperature difference (T_j – 20 °C) where j is the subscript w or s, where w represents the workpiece and s the working standard
u (symbol)	Uncertainty of the quantity associated with the symbol given in parentheses

Table 2 — List of symbols and abbreviations

Symbol	Explanation
d	Reading of instrument
E_{ETV}	Dimensional variation due to environmental temperature variation
k	Coverage factor
L	Length
L_{c}	Corrected length
L_{i}	Length of a measuring instrument or comparator measured at temperature, T, and corrected for nominal thermal expansion
L_{m}	Length of an object measured at temperature, T, and not corrected for nominal thermal expansion
L_{n}	Nominal length of a workpiece
L_{w}	Length of a workpiece at 20 °C
L_{s}	Length of a working standard at 20 °C

Table 2 — ***(continued)***

Symbol	Explanation
L_T	Length of an object at temperature, T
t	Time
T_w	Temperature of a workpiece
T_s	Temperature of a working standard
TE	Thermal error
TEI	Thermal error index
TOL	Tolerance
$U(\alpha)$	Uncertainty of the coefficient of thermal expansion
$U(\alpha_i)$	Uncertainty of the coefficient of thermal expansion of the measuring instrument
$U(\alpha_w)$	Uncertainty of the coefficient of thermal expansion of the workpiece
$U(\alpha_s)$	Uncertainty of the coefficient of thermal expansion of the working standard
$U(\theta_I)$	Uncertainty of the temperature of the measuring instrument
$U(\theta_w)$	Uncertainty of the temperature of the workpiece
$U(\theta_s)$	Uncertainty of the temperature of the working standard
$u_{cT}(L)$	Standard uncertainty component due to thermal effects
$u_{DE}(L)$	Uncertainty of differential thermal expansion due to uncertainties in α_s and α_w
$u_E(L)$	Uncertainty of thermal expansion due to uncertainty in α
$u_{ETV}(L)$	Uncertainty of length due to the environmental temperature variation
$u_{TM}(L)$	Uncertainty of length due to temperature measurement
U_T	Target uncertainty
α	Coefficient of thermal expansion
α_m	Measured coefficient of thermal expansion
α_{mw}	Measured coefficient of thermal expansion of the workpiece
α_{ms}	Measured coefficient of thermal expansion of the working standard
α_n	Nominal coefficient of thermal expansion
α_{nw}	Nominal coefficient of thermal expansion of the workpiece
α_{ns}	Nominal coefficient of thermal expansion of the working standard
α_w	Average coefficient of thermal expansion of the workpiece
α_s	Average coefficient of thermal expansion of the working standard
Δ_{DE}	Differential expansion between a workpiece and the working standard
Δ_E	Thermal expansion
Δ_{mDE} Δ_{nDE}	Differential thermal expansion based on nominal or measured coefficients of thermal expansion
Δ_{mE} Δ_{nE}	Thermal expansion based on nominal or measured coefficients of thermal expansion
τ	Time constant of a physical quantity
θ_w	Temperature difference (T_W – 20 °C) of the workpiece from 20 °C
θ_s	Temperature difference (T_W – 20 °C) of the working standard from 20 °C

5 Procedure

5.1 General

If a dimensional measurement is performed at a temperature other than the standard reference temperature of 20 °C, there will be a systematic effect due to differential thermal expansion between the workpiece and the working standard. This effect will cause a systematic error if the measurement result is not corrected. The decision whether or not to correct a measurement result for the effect of differential thermal expansion is a management decision based on costs and risks. In any case, it is necessary to evaluate the thermally-related components of the measurement uncertainty budget. A general procedure is as follows:

a) Measure the length of the workpiece, yielding an uncorrected result L_{m}.

b) Evaluate the relevant temperatures and their uncertainties (see 5.2).

c) Evaluate the thermal expansion coefficients and their uncertainties (see 5.3).

d) Evaluate the length uncertainty due to environmental temperature variation, $u_{\mathrm{ETV}}(L)$ (see 5.4).

e) Calculate the standard uncertainty component due to thermal effects, $u_{\mathrm{cT}}(L)$ (see 5.5).

f) Calculate the differential thermal expansion, Δ_{DE}.

g) If the measured length is to be corrected, perform the correction.

h) Report the corrected length, L_{c}, and the standard uncertainty component due to thermal effects, $u_{\mathrm{cT}}(L)$.

i) If no correction is to be performed, report the uncorrected result, L_{m}, the differential thermal expansion (systematic error), Δ_{DE}, and the standard uncertainty component due to thermal effects, $u_{\mathrm{cT}}(L)$. If there is a tolerance or a target uncertainty, also compute and report the thermal error index (TEI).

This procedure is illustrated in the flow chart of Figure 1. It should be recognized that thermal influences are only some of the effects that must be taken into account in the evaluation of the combined standard uncertainty of a dimensional measurement. Once the standard uncertainty component due to thermal effects has been computed, it may be

a) combined with the other standard uncertainties, following the recommendations of GUM, in order to evaluate the combined standard uncertainty of the measurement, and, if this is the case,

b) combined with other standard uncertainties, following the recommendations of GUM, in order to evaluate an estimated uncertainty that can be compared with a target uncertainty in support of a business decision.

5.2 Evaluate uncertainties of workpiece and working standard temperatures

These uncertainties may be evaluated by various methods:

a) the temperature of the body may be measured and the uncertainty computed from this measurement in accordance with GUM;

b) the evaluation may be based on the distribution found among results of measurements conducted on a number of like objects, using the same thermometers and the same procedures;

c) the evaluation may be based on the distribution found in published data regarding the use of such thermometers and specific procedures;

d) if the temperature cycles between two limits, the u-distribution shall be used;

e) the evaluation may be based upon judgment regarding the range of possible errors, where for the purposes of this Technical Report and in the absence of other information, it is recommended that temperature measurement uncertainty be evaluated using rectangular distributions for the relevant temperatures (see Annex B).

For Method e), where the knowledge of the temperatures is represented by a rectangular (uniform) distribution, the standard uncertainty in temperature measurement is given by

$$u(\theta) = \frac{a^{+} - a^{-}}{2\sqrt{3}} \tag{11}$$

where a^{+} and a^{-} are the upper and lower limits of the rectangular distribution, respectively.

5.3 Evaluate uncertainties of coefficients of thermal expansion

These uncertainties may be evaluated by various methods:

a) the thermal expansion of the objects may be measured and the uncertainties associated with these measurements adopted;

b) the evaluation may be based on the distribution found among results of actual experiments conducted on a number of like objects;

c) the evaluation may be based on the distribution found among published data;

d) the evaluation may be based upon judgment regarding the range of possible errors, where, for the purposes of this Technical Report and in the absence of other information, it is recommended that the uncertainty in the coefficient of thermal expansion be estimated using a rectangular distribution to represent knowledge of the quantity (see Annex B).

For Method d), the standard uncertainty in the coefficient of thermal expansion is given by

$$u(\alpha) = \frac{a^{+} - a^{-}}{2\sqrt{3}} \tag{12}$$

where a^{+} and a^{-} are the upper and lower limits of the rectangular distribution, respectively.

5.4 Evaluate length uncertainty due to environmental temperature variation

The dimensional uncertainty due to environmental temperature variation, u_{ETV}, is obtained from E_{ETV}, i.e. from a drift test using one of the following procedures.

a) Measure the E_{ETV} for a given measurement system to obtain a distribution of E_{ETV} over the required measurement time. This distribution could then be analysed in accordance with its actual form and a standard uncertainty computed following accepted statistical procedures.

b) Assume that the possible values for the environmental error are uniformly distributed within the range of the E_{ETV} obtained from a single test for a given measurement. The resulting standard uncertainty, $u_{\mathrm{ETV}}(L)$, is then given by

$$u_{\mathrm{ETV}}(L) = \frac{E_{\mathrm{ETV}}}{2\sqrt{3}} \tag{13}$$

5.5 Evaluate standard uncertainty component due to other thermal effects

The standard uncertainty component $u_{\mathrm{cT}}(L)$ due to thermal effects is evaluated using the following procedure.

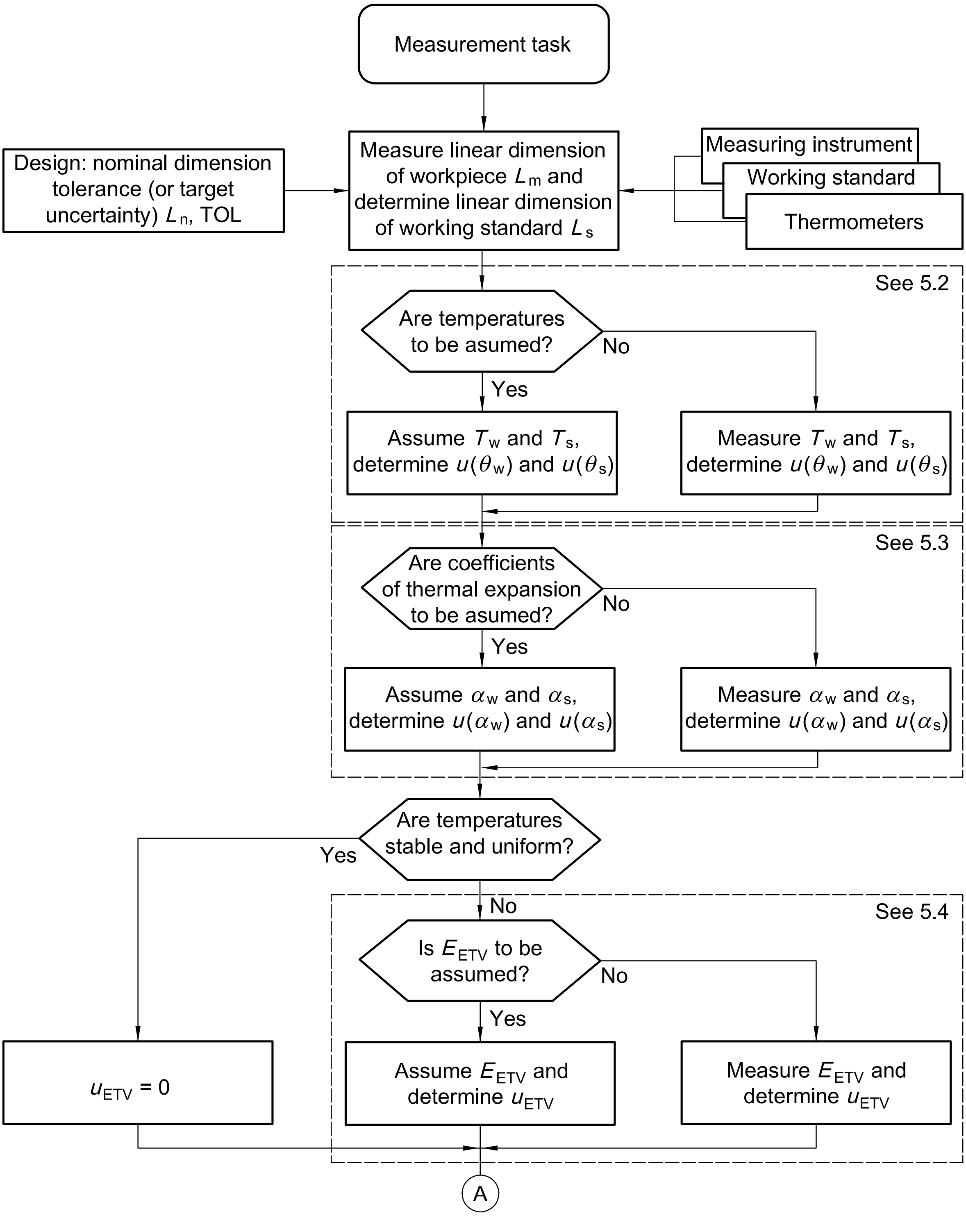
a) Evaluate the dimensional uncertainty due to environmental temperature variation, $u_{\mathrm{ETV}}(L)$ (see 5.4).

b) Evaluate the uncertainty in the differential thermal expansion due to uncertainties in the coefficients of thermal expansion, $u_{\mathrm{DE}}(L)$.

c) Evaluate the uncertainty in the differential thermal expansion due to uncertainties in the measurement of temperatures, $u_{TM}(L)$.

d) Evaluate the standard uncertainty component due to thermal effects for a length measurement made at a temperature other than 20 °C in a changing environment, $u_{cT}(L)$.

The uncertainties obtained shall be combined in accordance with the rules given in GUM.



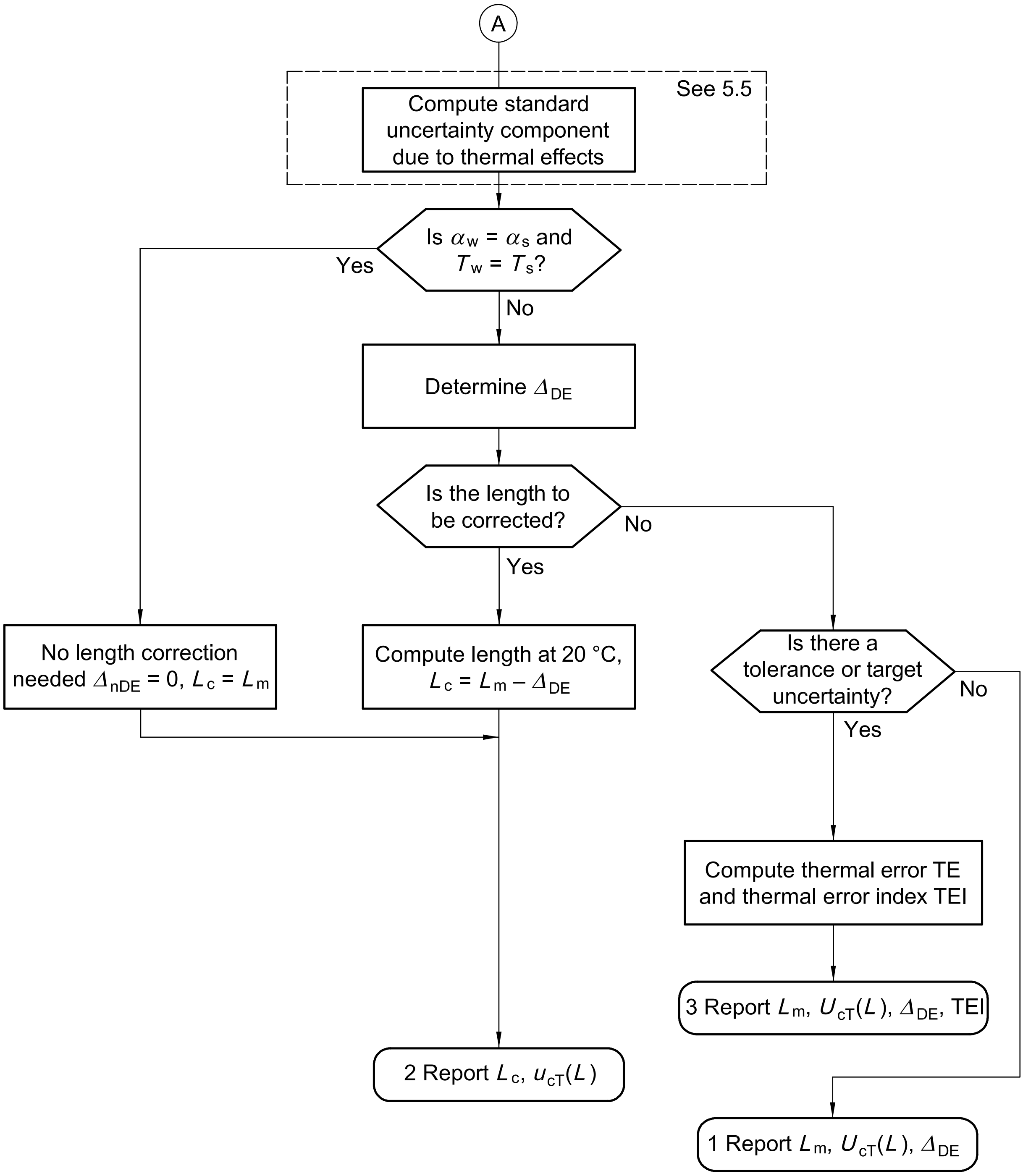


Figure 1 — Flow chart for dealing with systematic errors and uncertainty of dimensional measurement due to thermal influences

Annex A
(informative)

Advisory information on temperature environment for length measurement

A.1 General

In this annex, it is assumed that the measuring equipment and the thermal environment exist, and that normal or expected operating conditions are in force. The object of the discussion is to describe the manner in which the extent of systematic error and uncertainty resulting from non-ideal temperature conditions is determined.

The ideas and methods described are those found in fairly common usage by metrologists everywhere. Some of the concepts presented could at first appear strange and unrelated to previous experience. The three-element system concept, for example, will probably fall in this category. However, the concept will be seen to correspond to common thoughts, and its utility in a disciplined investigation will become clear.

The other concepts are those of the uncertainty of the coefficient of thermal expansion and the uncertainty of the temperature measurement. These concepts are examined and reduced to a practical procedure (see A.3 to A.4).

A.5 is devoted to explaining the thermal error index and its use.

A.2 Estimating consequences of mean environmental temperatures other than 20 °C

A.2.1 Length measurements

The consequences of temperature variation are easily obtained by means of equations that give the nominal differential thermal expansion in terms of the nominal thermal expansions of the workpiece and the working standard [see Equations (2) and (4)]

$$\Delta_{\mathrm{nDE}} = \Delta_{\mathrm{n}E\mathrm{w}} - \Delta_{\mathrm{n}E\mathrm{s}} \tag{A.1}$$

and

$$\Delta_{\mathrm{nE}} = \alpha L \left(T - 20\ ^\circ\mathrm{C} \right) \tag{A.2}$$

Combining these equations, assuming that $L \approx L_{\mathrm{s}} \approx L_{\mathrm{w}}$, yields

$$\begin{aligned} \Delta_{\mathrm{nDE}} &= \alpha_{\mathrm{w}} L_{\mathrm{w}} \left(T_{\mathrm{w}} - 20\ ^\circ\mathrm{C} \right) - \alpha_{\mathrm{s}} L_{\mathrm{s}} \left(T_{\mathrm{s}} - 20\ ^\circ\mathrm{C} \right) \\ &= L \left[\alpha_{\mathrm{w}} \left(T_{\mathrm{w}} - 20\ ^\circ\mathrm{C} \right) - \alpha_{\mathrm{s}} \left(T_{\mathrm{s}} - 20\ ^\circ\mathrm{C} \right) \right] \end{aligned} \tag{A.3}$$

Assuming that both the workpiece and working standard are at the mean temperature, $T_{\mathrm{w}} = T_{\mathrm{s}} = T_{\mathrm{mean}}$ (the only reasonable assumption unless thermometers are attached to both the workpiece and working standard), we see that the systematic error is reduced to insignificance if the coefficients of thermal expansion approach equality. This is true even with a large deviation of the mean temperature from 20 °C.

Because the great majority of manufactured workpieces and measuring equipment are of ferrous materials having similar coefficients of thermal expansion, many companies, particularly those dealing with tolerances are in tenths of millimetres or where the workpieces are small, have successfully functioned without concern over the effect of mean temperature on manufacturing accuracy. In many such situations, an arbitrary insistence on 20 °C temperature control leads to an unjustified increase in the cost of manufacture.

As tolerances become tighter, as the workpieces become bigger, and as the materials of workpieces and working standards become more dissimilar, the consequences of mean temperatures other than 20 °C become correspondingly greater. In recognition of the possible consequences of mean temperatures other than 20 °C, it is not uncommon to find the following procedures in use:

a) special measuring equipment or working standards made of nominally the same material as the workpieces;

b) computation of corrections which are applied to the indicated values of linear dimensions.

As the working tolerance decreases, both of these procedures fail to be satisfactory because of the magnitude of the uncertainty of nominal differential thermal expansion (see A.2) and the uncertainty of length due to temperature measurement.

A.2.2 Measurements other than length

Procedures and formulae for the assessment of the effects of mean environmental temperatures other than 20 °C as simple and straightforward as those presented in this Technical Report are not usually possible in cases other than linear dimension measurements.

For example, consider the case of an iron bedway casting of a machine. Because the casting can have both thick- and thin-walled sections, the physical composition of the material may not be homogeneous, resulting in a non-uniform coefficient of thermal expansion. The magnitude of such a variation in thermal expansion coefficient may be as much as 5 %. If the non-uniformity is distributed as a vertical gradient, raising or lowering the mean temperature will result in a bending such as that produced by a vertical temperature gradient. This effect is the same as that observed in the well-known bi-metal strip and can be called a *bi-metal effect*.

This effect in structures of nominally one material is relatively small compared with the effect of temperature gradients. For example, a base casting such as that mentioned above would have to be subjected to a temperature offset of 10 °C before the bending approached that induced by an upper and lower surface temperature difference of only approximately 0,5 °C. However, in structures composed of two or more greatly dissimilar materials that are assembled at 20 °C, the bi-metal effect can be quite significant. In such cases, the effect of mean temperature other than 20 °C can be properly estimated only by taking into account the existing thermal stresses.

The use of machines with granite tables and large reference flats that are subjected to vertical temperature gradients — as might be created by the temperature variations of the laboratory floor or the radiation from overhead lights etc. — can result in significant variations in the flatness of the reference surfaces.

Evaluation of the effects of mean temperatures other than 20 °C requires that the net effect of the distortions of both working standard and workpiece be determined.

A.3 Consequences of uncertainties of coefficients of thermal expansion and temperature

There are two kinds of uncertainties that arise when the effects of mean temperatures other than 20 °C are computed: uncertainties in the values of the temperatures and those in the coefficients of thermal expansion used in the computations.

Values of temperatures used in computations can be in error because of errors in measurements, defects in the instruments used in making the measurements or because of the location at which the measurement is made. For example, the thermometer used may be inaccurately calibrated or have a built-in source of error such as the self-heating effect found in resistance thermometers. Because of the self-heating effect, resistance thermometers can be very precisely calibrated in liquid baths and give erroneous readings on metal surfaces or in air because of the quite different heat transfer processes involved.

Location of the temperature-measuring sensor is important because of possible gradients. Use of environmental temperature values may introduce errors of 1 °C or more. Readings of direct-contact probes are more reliable but are still subject to error because of gradients within the object whose temperature is being measured or improper mounting of thermometers to the object. An effective means of assessing the validity of a given location is to compare temperatures at several locations.

The standard procedure for estimating the effects of uncertainty of nominal differential thermal expansion is to require that workpiece and working standard temperatures be measured to determine worst-case deviations from 20 °C. This procedure takes into account the effects of gradients in the apparatus, as well as in the environment in which it is located.

If workpiece and working standard temperatures are not measured, the computations shall include consideration of the larger uncertainties in the temperatures used in computing the estimation of the effects of temperatures other than 20 °C.

With proper attention to the simple, well-established rules of precision thermometry, uncertainties due to temperature measurement can be reduced. There are, however, many questions regarding the proper mounting of thermometers, and in actual measurement situations in the factory, temperature measurement errors can be large. The effects of uncertainties of coefficient of thermal expansion values are also difficult to overcome.

Coefficient of thermal expansion data are published in tables in many handbooks and other sources. These values cannot be used without consideration of uncertainties which arise because of the following.

a) The material of the elements of the measuring equipment — workpiece or working standard or both — differ from the material for which the data are given. The differences may be in chemical composition, physical conditions or in both.

b) The published values are usually the result of the averaging of data from several experiments and from several experimenters. Consequently, the data reflect the effect of experimental systematic errors.

c) The published values are valid only for temperatures other than 20 °C or for a range of temperature other than that of the computation.

National laboratories when calibrating steel gauge blocks often assume an uncertainty of the coefficients of thermal expansion of ± 5% when the heat and mechanical treatment of the steel is known. The variation of the coefficient is about

— ± 3 % among many heat treatments of steel of nominally the same chemical composition,

— ± 10 % among several heat treatments of the same steel, and

— ± 2 % among samples cut from different locations in a large workpiece of steel that has been fully annealed.

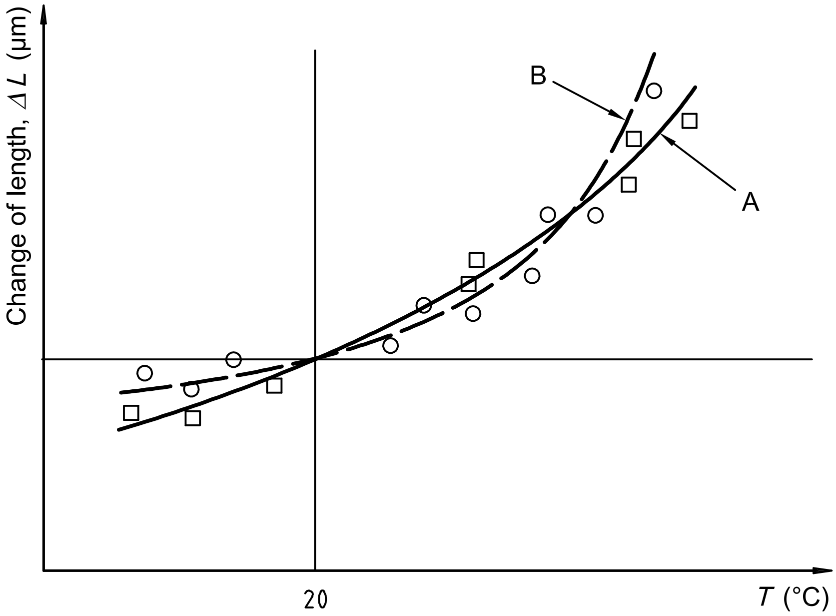
Hot or cold rolling will cause a difference of about ± 5 %.

Other materials have uncertainties due to the effects of chemical composition or physical conditions. Some materials have grain structure effects which cause thermal expansion coefficients to vary with direction. Moreover, large variations in thermal expansion coefficients have to be expected, for example, polymers, ceramics or composite structures.

The typical thermal expansion measurement is conducted with an apparatus called a dilatometer in which a specimen, usually rod-shaped, is heated and its change of length measured. Another form of dilatometer measures change of volume by liquid displacement, resulting in a coefficient of volumetric thermal expansion. For isotropic materials, the coefficient of volumetric thermal expansion has a value approximately three times that of the coefficient of thermal expansion.

The fact that the typical test specimen bears little resemblance to real workpieces, with consequent uncertainties in composition and treatment not reflected in experimental data scatter, suggests that uncertainties can be reduced by direct measurement of each specific object, or full-scale dilatometry.

Figures A.1 and A.2 represent two common ways in which thermal expansion data are presented in the literature. Figure A.1 is a synthetic case deliberately oversimplified for the purposes of this discussion. Figure A.2 is an actual case[10]. Note that Figure A.1 is a plot of change of length, ΔL, as a function of temperature, where ΔL is defined as zero when the temperature is 20 °C. This is the usual form of raw data from dilatomer experiments.



A is "Experimenter A": average line, data points denoted by rectangles.

B is "Experimenter B": average line, data points denoted by circles.

Figure A.1 — Synthetic experimental results of thermal expansion measurements

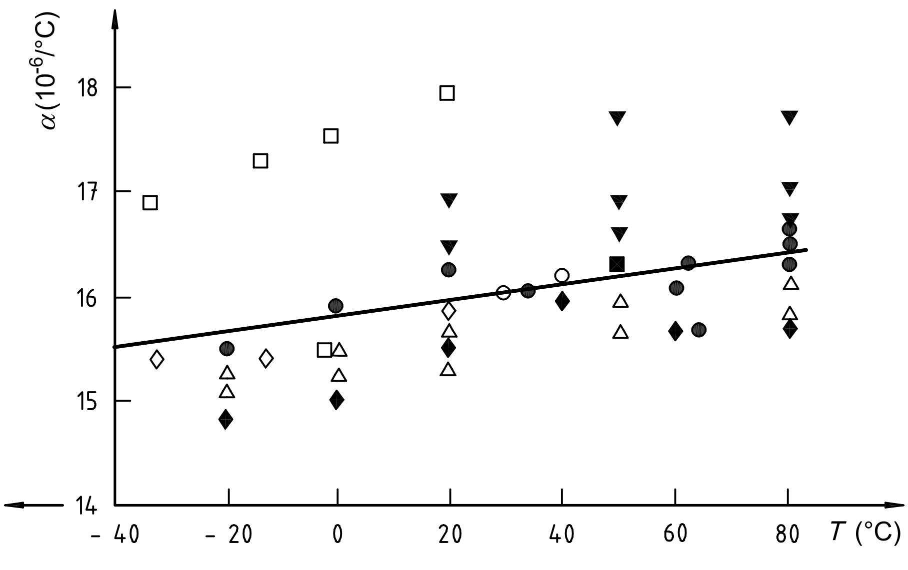


Figure A.2 — Experimental data and average line on the thermal expansion coefficient

Figure A.2, on the other hand, is a plot of the mean (or average) coefficient of thermal expansion from 20 °C [see Equation (1)],

$$\alpha(20,T) = \frac{L_T - L_{20}}{L_{20}(T-20)} \tag{A.4}$$

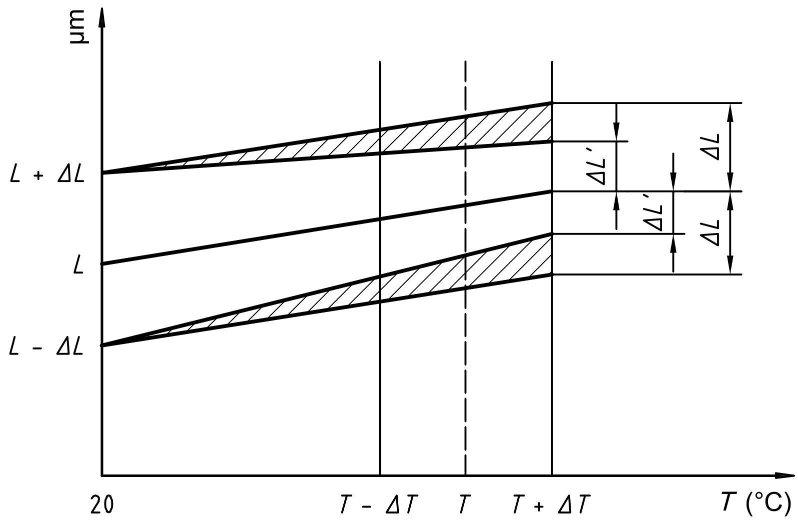
plotted at T. The data for $T = 20$ °C are derived from the slope of the thermal expansion, $d\,\Delta L/dT$, at that special temperature.

Figure A.2 gives results from several investigators, while Figure A.1 shows how two investigators may obtain differing results that are reflected in Figure A.2. Both figures show

a) the scatter of experimental data, and

b) the non-linear nature of thermal expansion relative to temperature.

Data of this type are the source of all tabulated coefficient of thermal expansion data. The published value, however, varies in accordance with how the experimental data are interpreted. For a single investigation, the value depends on how the trend is interpreted (i.e. how the average curve is fitted). For multiple investigations, the value depends on how the data are averaged. In any case, these variations lead to increased uncertainties due to thermal effects (see Figure A.3).



L is the length.

ΔL is the change of length at 20 °C.

ΔT is the change of temperature.

$\Delta L'$ is the change of length at temperature.

Figure A.3 — Effects of uncertainties of coefficients of expansion on length measurement uncertainties

A.4 Estimation of consequences of temperature variation

A.4.1 General

A good estimation of the consequences of temperature variation can very seldom be obtained by direct calculation. Therefore, the procedures described in this clause are based on an experimental approach to the estimation.

The basic experimental procedure used to measure the effect of temperature variation is the drift test (see A.4.2). Drift test results can be interpreted in a variety of ways to obtain the error of dimensional variation due to environmental temperature variation, E_{ETV}. One method is described in more detailed in A.4.3, along with other methods of interpreting drift test results which could be useful because less conservative and which can provide guidelines for negotiating the acceptability of thermal effects, systematic error, and uncertainty in special cases.

The rationale for both the drift test and the E_{ETV} estimate is given in A.4.4 as an explanation of the concept of the three-element system.

A.4.2 Drift test procedure

A.4.2.1 Equipment

The object of a drift test is to record relative displacement in a two-element system (see A.4.3). The most direct method utilizes electronic indicators whose output is recorded by a computer. Some measurement processes, such as the measurement of flatness with an optical flat and monochromatic light or an indicating micrometer, do not lend themselves to the use of automatic recording. Therefore, in some cases it will be necessary for a human operator to observe the drift and record numerical values and corresponding times. These data can be subsequently hand-plotted.

It is strongly recommended, however, that, sensitive electronic indicators and automatic data-acquisition equipment be used wherever possible.

Though a drift test can be performed without any necessity for knowledge of temperature variation, it is often advisable to record one or more temperatures either for use in later correlation of two drift tests or for reference if temperature variation is to be later accepted as a method of monitoring the process for validation of the temperature variation error estimate.

Just as in the case of displacement measurements, it is strongly recommended that all temperatures be automatically recorded. For this purpose, recording resistance element thermometers, especially those with thermistor sensors, may be used.

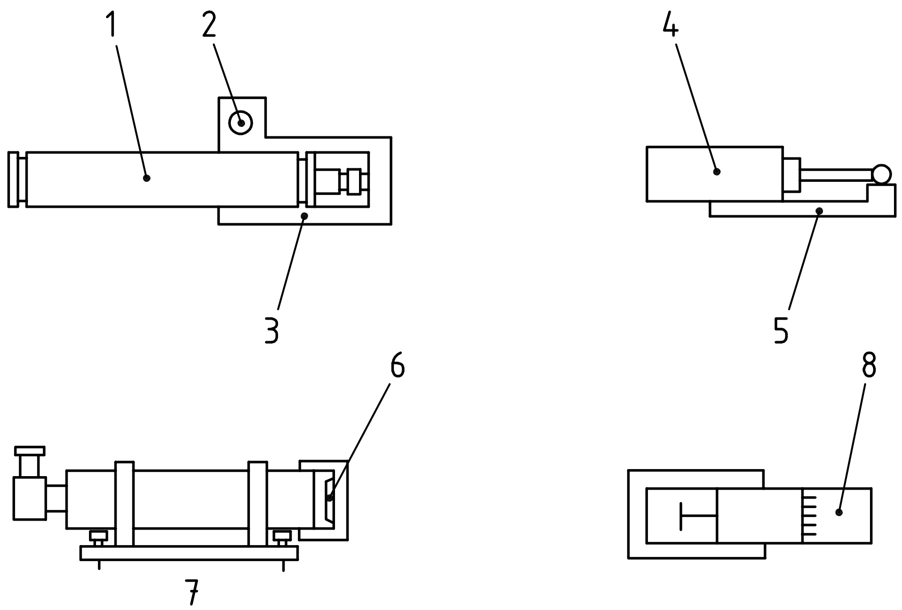
A.4.2.2 Equipment testing

A.4.2.2.1 Displacement transducers

Aside from the usual calibration, electronic indicators should be checked for possible sensitivity to the thermal environment in which the drift test is to be performed. An “electronics drift check” should be conducted by blocking the transducer and recording the output for at least the same period of time as that of the drift test to be performed. “Blocking” a transducer is to make it effectively indicate on its own frame, base or cartridge. Figure A.4 shows, among others, a cartridge-type linear variable differential transformer blocked by means of a cap or capture device which holds the indicator armature in a fixed position relative to the cartridge.

During the electronics drift test, the entire displacement recording system should be located as nearly as possible to where it will be during the drift test.

Electronic drift tests have been useful in proving in many cases where electronic indicators were the suspected source of drift that they were in fact innocent and the real cause was thermal drift. The commercially available cartridge-type LVDT gauge heads have been proven many times to be especially free from drift.



Key

1 cartridge type (LVDT) electronic transducer
2 clamping screw
3 capture device
4 finger type electronic transducer
5 clamp
6 mirror
7 autocollimator
8 capacitance transducer

Figure A.4 — Schematics for conducting "blocking" tests on transducers

A.4.2.2.2 Temperature recording systems

The temperature-measuring and recording apparatus should be calibrated and characterized for response and drift.

For many practical situations, a resolution of 0,1 °C is adequate. This resolution and the time constants of sensing elements for air temperature sensors should be chosen to suit the measurement task and environmental conditions. Air probes shall be shielded from possible radiation effects.

A.4.2.2.3 Preparation of system for test

An essential feature of the drift test is that conditions during the test should approximate the normal conditions for the process. Therefore, before the test is started, normal conditions shall be determined. The step-by-step procedure followed in the subject process shall be followed in the same sequence and with the same timing as in the drift test. This is especially important in terms of the actions of human operators in mastering and all preliminary set-up steps.

With as little deviation from normal procedure as possible, the displacement transducers should be introduced between the workpiece (or working standard, depending on the type of drift test) and the remains of the measurement loop such that it measures relative displacement along the line of action of the subject measurement process.

The temperature sensor should be placed so as to measure a temperature able to be correlated with the drift. Some trial and error could be necessary. In the extreme case, temperature sensors could have to be placed to measure the temperatures of all of the active elements of the measurement loop.

A.4.2.2.4 Representative time period for a drift test

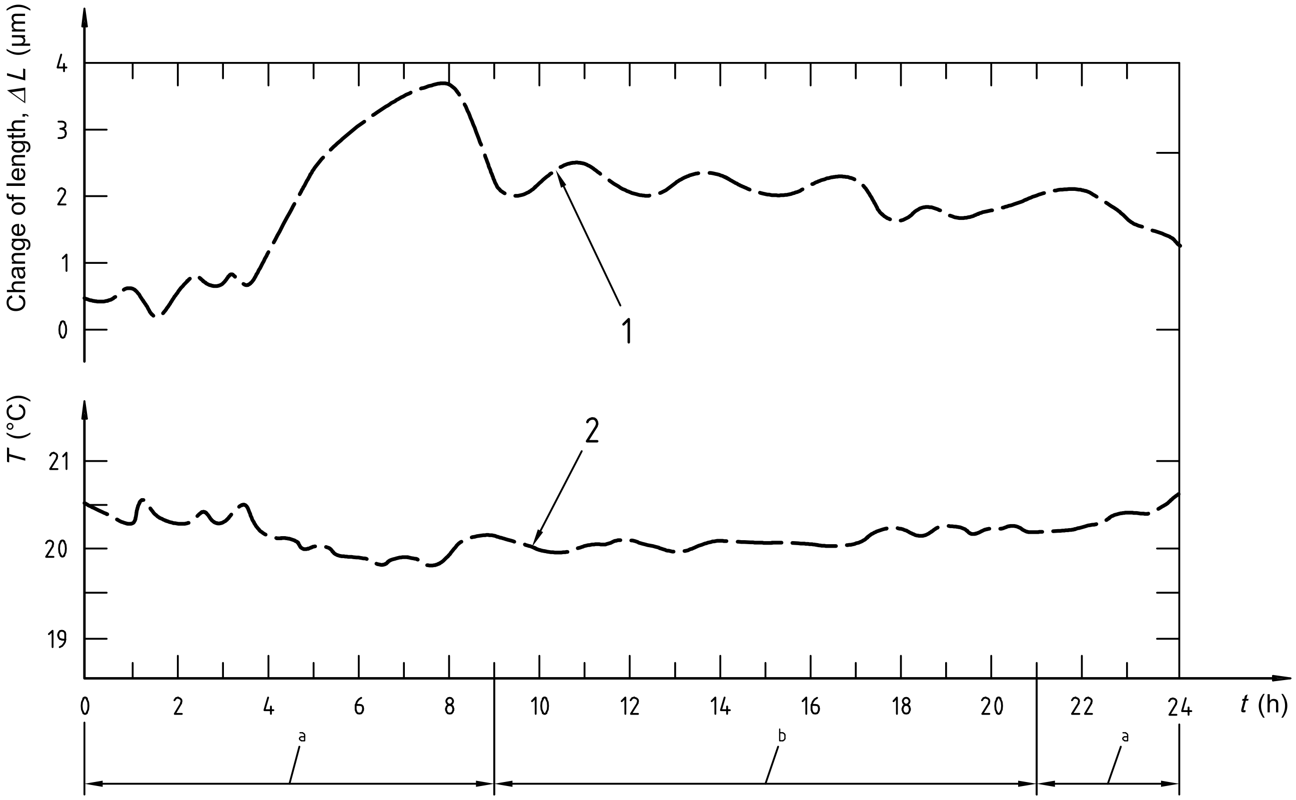
Once set up, the drift test should be allowed to continue as long as possible, using normal operating conditions. In situations where a set pattern of activity is observed, its duration should be over some period of time during which most events are repeated. For example, if the acceptance tests on a measuring instrument are to take a day, the drift test should be run for a day.

A.4.2.2.5 Procedure

E_{ETV} is the range of the dimensional variation due to environmental temperature variation. After the drift test, the displacement transducers and the temperature recording apparatus should be readjusted.

A.4.2.2.6 Example drift test results

Figures A.5 and A.6 are results from drift tests conducted on a measuring instrument. Figure A.5 is the drift recorded over a 24-hour period for a system consisting of the working standard and comparator. Figure A.6 is the drift recorded over the succeeding 24-hour period for a system consisting of the workpiece to be measured and the comparator. In both cases, ambient temperature at a point near the gauge was recorded and is plotted in the corresponding figures.



Key

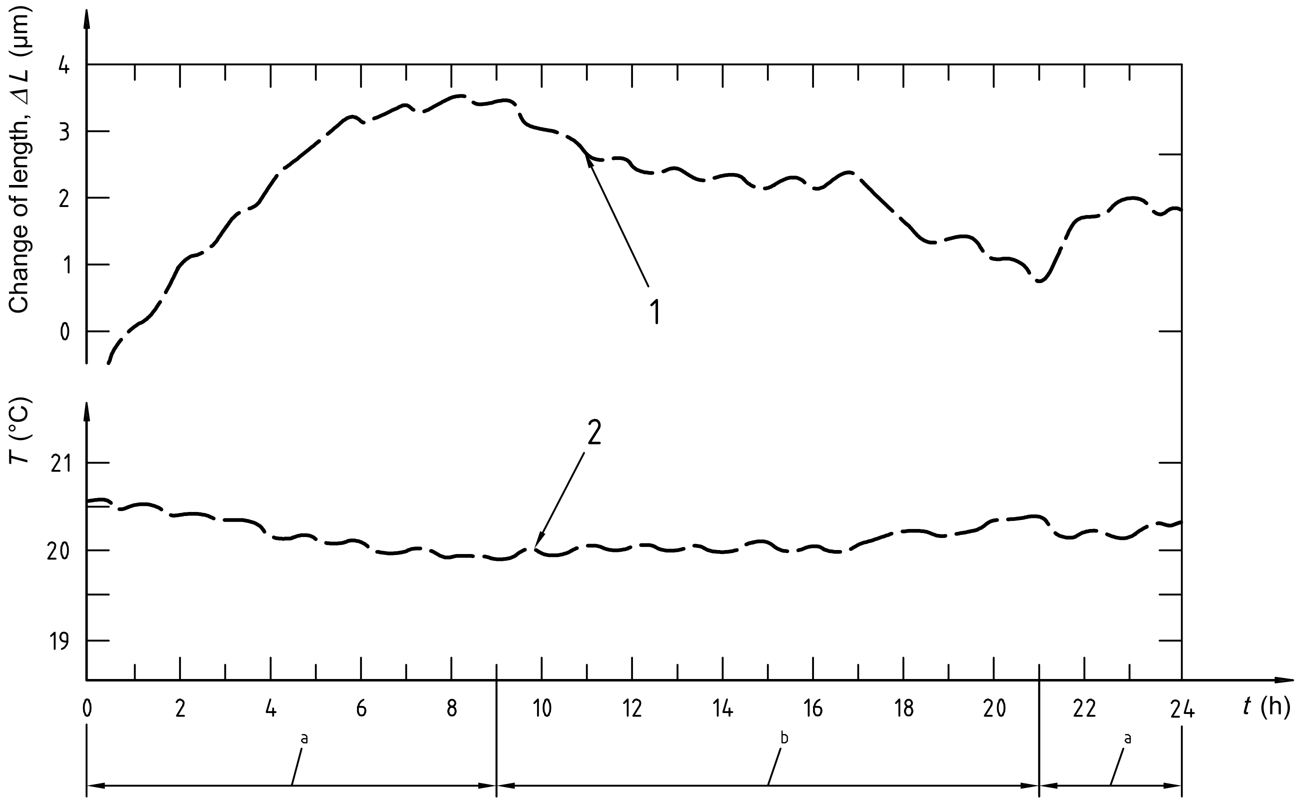
1 displacement

2 ambient temperature

[a] Before midnight.

[b] After midnight.

Figure A.5 — Drift of a working standard and comparator system



Key

1 displacement

2 ambient temperature

[a] Before midnight.

[b] After midnight.

Figure A.6 — Drift of a workpiece and comparator system

A.4.2.2.7 Other drift tests

For specific instruments or machines, other standards [1] recommend different types of drift tests.

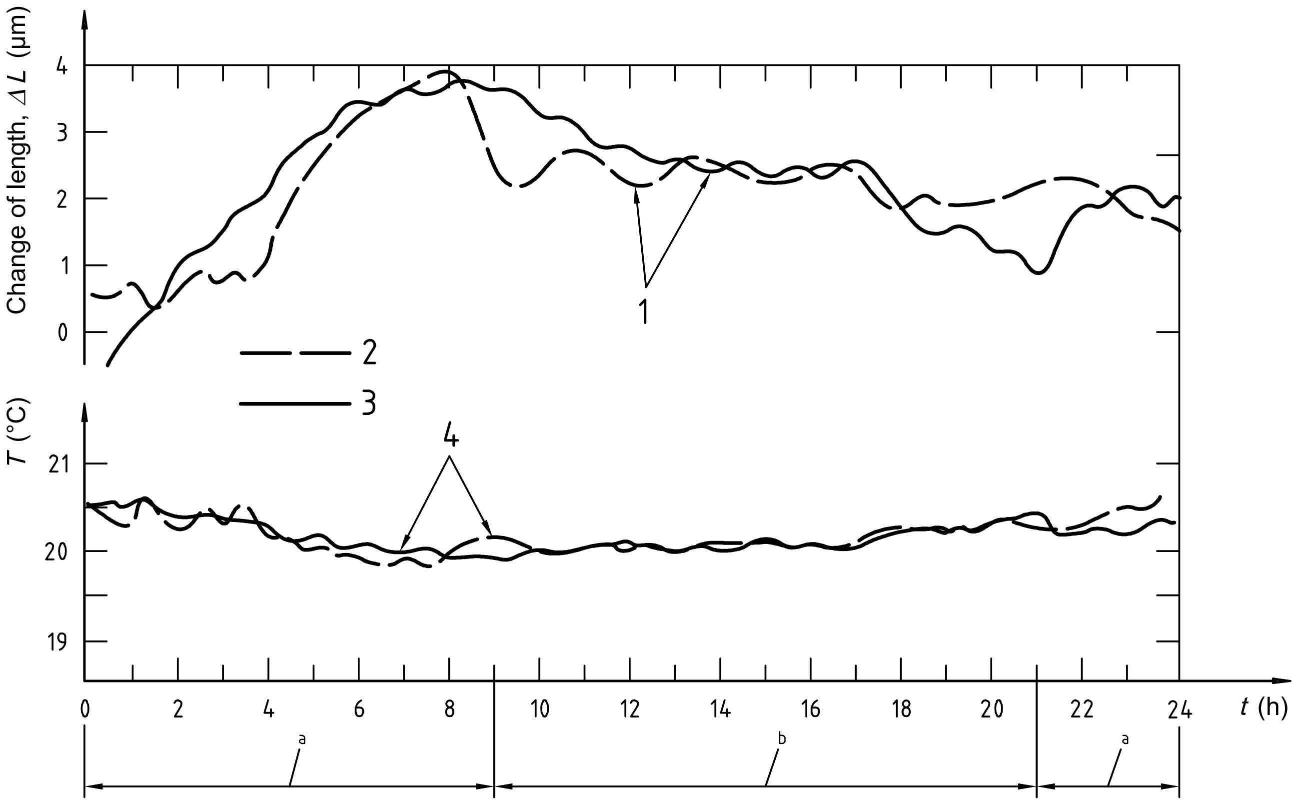
A.4.3 Ambient temperature variation

Figure A.7 shows the results of both workpiece/comparator and working standard/comparator drift tests from Figures A.5 and A.6 superimposed on each other. In this case, ambient temperature readings were obtained simultaneously with each drift test for the purpose of approximating the proper phase relationship. The two sets of data were superimposed in accordance with the time of day, which appears to give a good overall agreement in ambient temperature variation.

The ambient temperature variation on the two successive days has a well-defined twenty-four-hour component with an amplitude of about 0,8 °C. Superimposed on this are higher frequency components with periods of from 30 min to 1,5 h. From these data, it is possible to compare the twenty-four-hour cycle characteristics because of the repeatability of the environment at this frequency; but phase relationships at the higher frequencies are not easily discernible.

At the twenty-four-hour frequency, the working standard/comparator and workpiece/comparator drift curves are in phase and have very nearly the same amplitude. This is a classic example that shows the importance of measuring cycle time owing to the larger amplitudes of drift being associated with the low frequency, whereas the smaller amplitudes of drift are associated with the higher frequencies.

For short measurement cycle times, say 1 h, the procedure for evaluating environmental temperature variation given in 5.4 results in an E_{ETV} equal to 1,5 µm; for measurement cycle times of 12 h or more, an E_{ETV} equal to 3 µm results.



Key

1 displacement

2 working standard/comparator drift

3 workpiece/comparator drift

4 ambient temperature

[a] Before midnight.

[b] After midnight.

Figure A.7 — Results of workpiece/comparator and working standard/comparator drift tests using inspection shop gauge

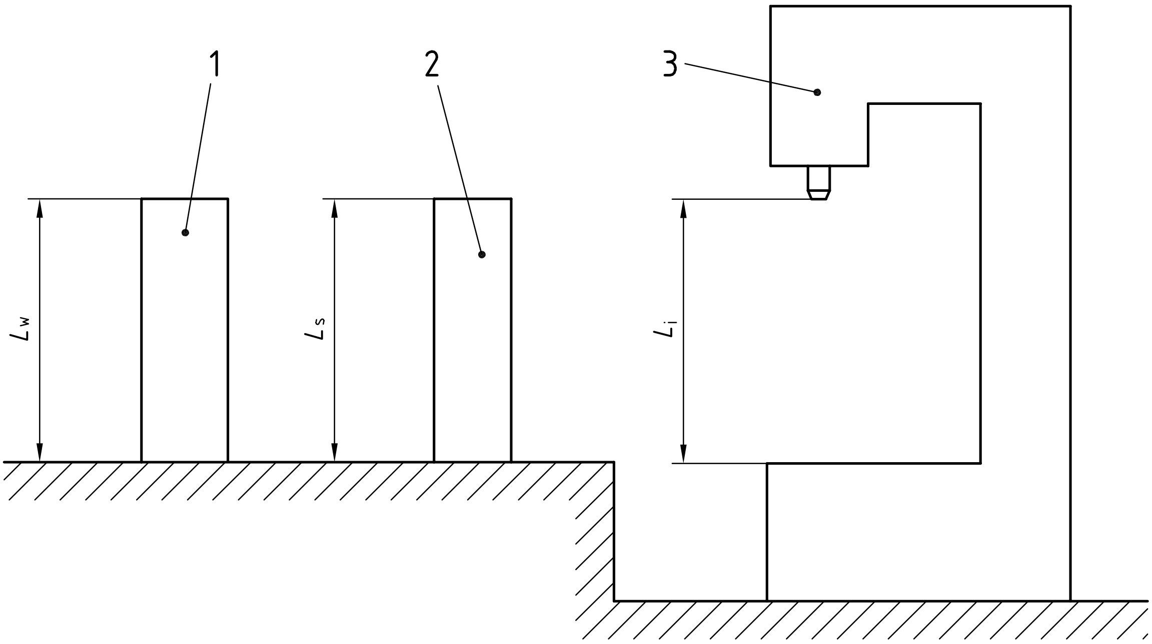
When the quality of the drift data permits, it is sometimes possible to apply the more precise evaluation methods discussed in this subclause which are less conservative. In the example of Figure A.7, little is gained by this procedure because the maximum difference between the two drift curves, which corresponds to the possible error for short measurement cycle times, is still about 1,5 µm. This is probably because of non-repeatable components of temperature variation in the two days' testing. The day on which the working standard/comparator drift test was performed appears to have had more severe high-frequency temperature components. This discrepancy appears to exaggerate the true workpiece/working standard relative drift. Further drift tests to obtain results for more consistent temperature variations would be advisable in this case were environmental temperature variation the major thermal effect in this measurement process.

A.4.4 Three-element system concept

The magnitudes of the effects of temperature variation are dependent on the structure of the measurement instrument, and not only on the size and composition of the workpiece and working standard as was true in the previous subclauses. Also, unlike the other components of thermal error, temperature variation can be influenced as well by the work procedures of the person making the measurements.

One of the simplest structures is that encountered in the measurement of the length of an object with a gauge block and a column comparator. Figure A.8 shows a schematic representation of such a system. As can be seen, it consists of a workpiece, a working standard (the gauge block) and a comparator. Thus the system consists of three elements. This is called the comparator method; it may also be referred to as the substitution method.

In Figure A.8, each element is shown to have a characteristic length. In the measurement process, the characteristic length of the comparator L_i is first set equal to the length of the working standard L_s, then the length of the workpiece L_w is checked to see that $L_w = L_i$.



Key

1 workpiece

2 working standard

3 comparator

L_w workpiece length

L_s working standard length

L_i characteristic length of the comparator

Figure A.8 — The three elements of a length-measuring system

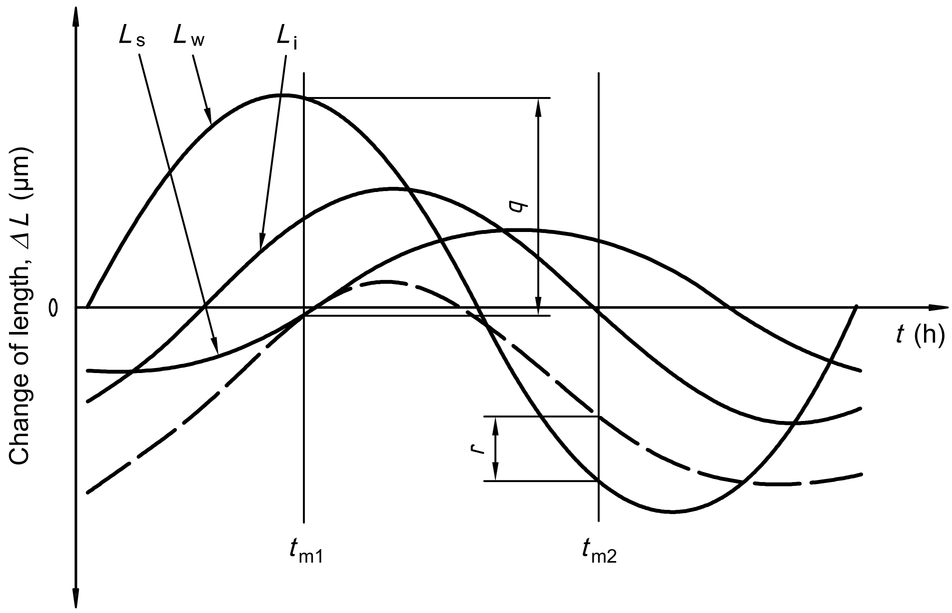
If there were no temperature variations, the measurement process would be straightforward. However, because of temperature variations, heat is constantly being exchanged between the three elements and the changing environment.

If the time constants of all three elements are not the same, they may respond to temperature variations in such a way that all three elements will never simultaneously have the same temperature. Even if the time constants are all the same and their temperatures always equal, they may not have the same length, except when all are set at 20 °C, because of different coefficients of thermal expansion.

For each element, the time constant, length or characteristic length, respectively, and coefficient of thermal expansion determine its dimensional response to temperature variation.

Figure A.9 shows the dimensional response of the three elements of Figure A.8 for an assumed sinusoidal ambient temperature variation. For simplicity, the hypothetical system consists of three elements of the same material but different time constants, the largest being that of the working standard, the smallest that of the workpiece, with the time constant of the comparator between those of the other elements.

As can be seen, the thermal response of the three elements differ in amplitude and phase. It should be noted that dimensional response data in this form are rarely obtainable because they require the use of an independent apparatus that must itself be unaffected by temperature variation.



Key

L_w workpiece length

L_s working standard length

L_i characteristic length of the comparator

q deviation of workpiece length at time t_{m1}

r deviation of workpiece length at time t_{m2}

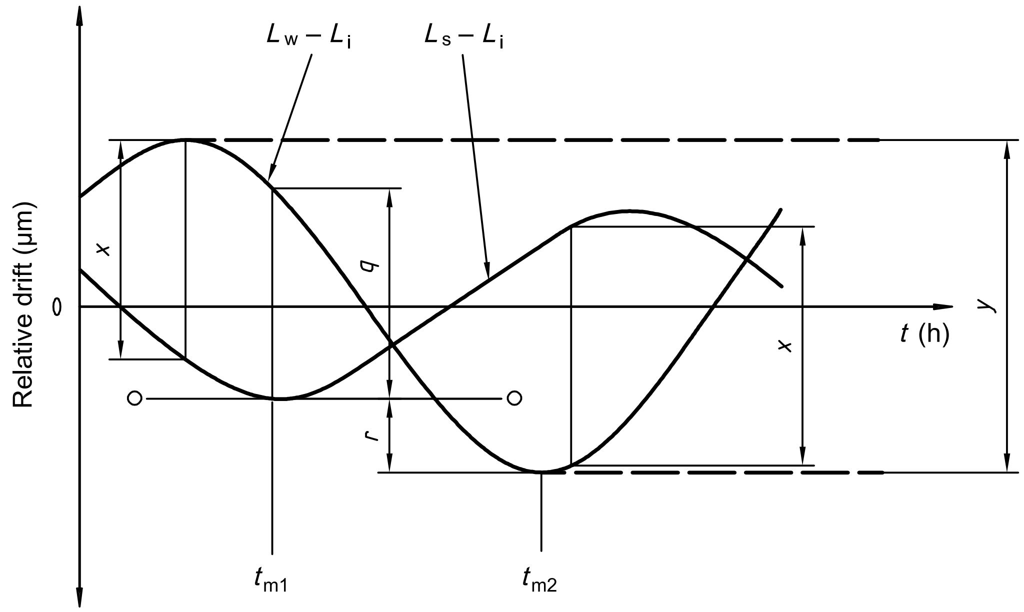
Figure A.9 — Sample steady-state dimensional response of three-element system shown in Figure A.8 to sinusoidal ambient temperature variation

The data of Figure A.9, if they were obtainable, could easily be interpreted for an estimate of the temperature variation. It would only be necessary to consider the effect of the measurement cycle as follows.

Suppose that at time t_{m1} the comparator is set by the working standard. The act of making L_i and L_s equal causes the dimensional response curve of the comparator to be shifted parallel to itself (the comparator is "zero shifted") as shown by the dashed curve in Figure A.9. If the workpiece is checked without delay after setting of the comparator, it will be found to be too large by the amount q. If, instead, the workpiece is checked much later, say at time t_{m2}, the workpiece will be found to be too small by the amount r. If the comparator were to be reset at time t_{m2}, the comparator curve would be shifted again, resulting in new magnitudes of possible deviation (i.e. new values of q and r).

Because temperature variation causes a variation in the differences of the length or characteristic lengths, respectively, it is possible to separate the three-element system into two, two-element subsystems. For example, Figure A.10 shows the two curves that result when the characteristic length of the comparator L_i is subtracted from the workpiece length L_w and working standard length L_s, resulting in the relative drift $L_w - L_i$

and $L_s - L_i$. These data might have been obtained by recording the output of an electronic indicator, such as is used on modern column comparators, when the workpiece and working standard are, successively, in the comparator with the indicator contacting the workpiece and working standard, respectively. Data such as these are obtained using the drift tests described in A.4.2.



Key

q difference of relative drifts at time t_{m1}

r difference of relative drifts at time t_{m2}

x difference of relative drifts at any time

Figure A.10 — Relative drift components for a three-element system

The main problem in interpreting such data results from the fact that with a set-up in accordance with Figure A.8 it is not possible to conduct simultaneous workpiece/comparator and working standard/comparator drift tests. Consequently, additional data are required to determine the proper phase relationship between the two recorded drift curves or the possible consequences of unknown phase relationship must be considered in the estimate of temperature variation.

Because the data of Figure A.10 have been constructed from the hypothetical data of Figure A.9, no phase uncertainty exists and the temperature variation error can be extracted easily. For example, for a setting cycle occurring between times t_{m1} and t_{m2} and a measurement of the workpiece without delay, the possible error, q, is simply the signed difference between the two curves.

The effect of setting is to establish a new baseline for the workpiece/comparator drift curve ($L_w - L_i$). This new baseline for setting the comparator at t_{m1} is shown in Figure A.10 as the line (0–0). If the measurement of the workpiece takes place at time t_{m2}, the resultant deviation is r, as previously shown.

If a workpiece is inspected between times t_{m1} and t_{m2}, the deviations due to temperature variation range from $+q$ to $-r$.

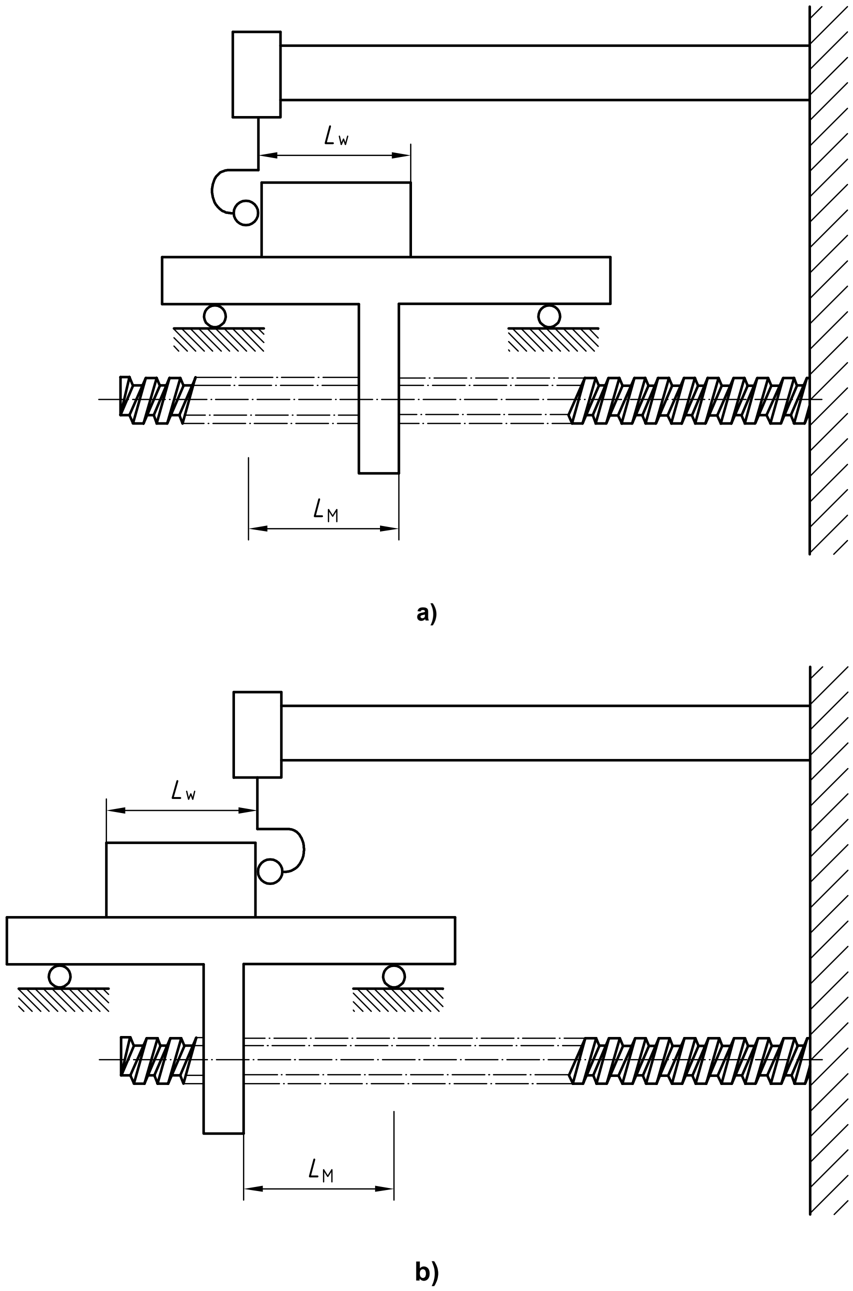
If the times at which setting occurs are unknown and unpredictable and the measurement cycle time is very short (setting with each measurement and negligible delay before the workpiece is inspected), the possible deviation is $\pm x$, or the maximum difference between the two drift curves at any given time. Because of the short measuring cycle time, the comparator is slaved to the working standard so that the comparator, in principle, contributes nothing to the deviation. The deviation, therefore, is $(L_w - L_i) - (L_s - L_i) = L_w - L_s$.

This deviation is dependent only on the difference between the working standard/comparator drift and the workpiece/comparator drift and the time at which the measurement is made. If the measurement cycle time is longer than the period of the temperature oscillation, the maximum possible deviation is $\pm E_{\mathrm{ETV}}$, or the maximum difference between the two drift curves regardless of time. Note that E_{ETV} is generally larger than x.

In length-measuring processes, however, a three-element system is always found. For example, consider the case shown in Figure A.11 of a measuring machine or machine tool with a leadscrew (or a linescale) serving as virtual working standard having a characteristic length, $L_{\mathrm{M.}}$ In Figure A.11, the measurement process is shown to consist of changing from position a) to position b). The analogy between this case and the simple three-element system of Figure A.8 is seen if it is realised that in the two configurations the comparator is composed of a portion of the lead-screw (or a linescale), the nut and the table support for the workpiece. These elements, though appearing to change, remain in a structural loop, while the workpiece and working standard exchange places as members of the loop. Note that since the working standard and the workpiece are simultaneously in the comparator, only one drift check is needed.

The case shown in Figure A.12 is that of a 25 mm indicating micrometer used as a comparator. The working standard is a gauge block.

In Figure A.13, the same micrometer is brought to its null position and a zero correction is made before the workpiece is measured. In this case, the working standard is that portion of the screw that is withdrawn to make room for the workpiece. The remainder of the micrometer forms the comparator.



Key

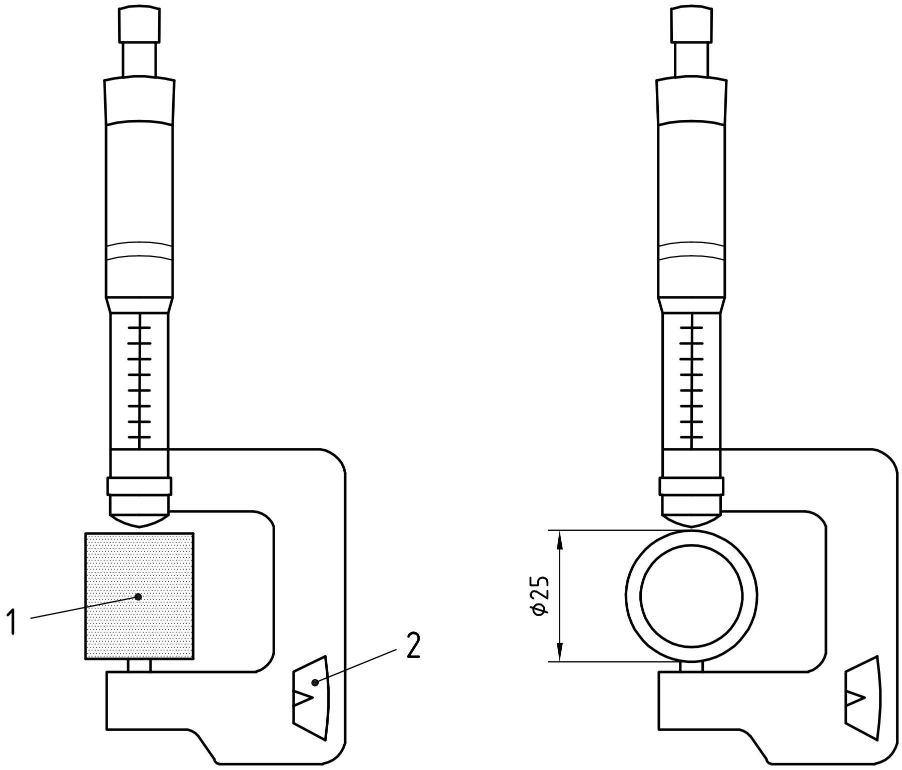
L_W workpiece length

L_M virtual working standard length

NOTE The measuring sequence is a change from position a) to position b). The indicator is highly idealized.

Figure A.11 — Schematic of set-up used to measure workpiece on gauge with lead-screw working standard

Dimensions in millimetres



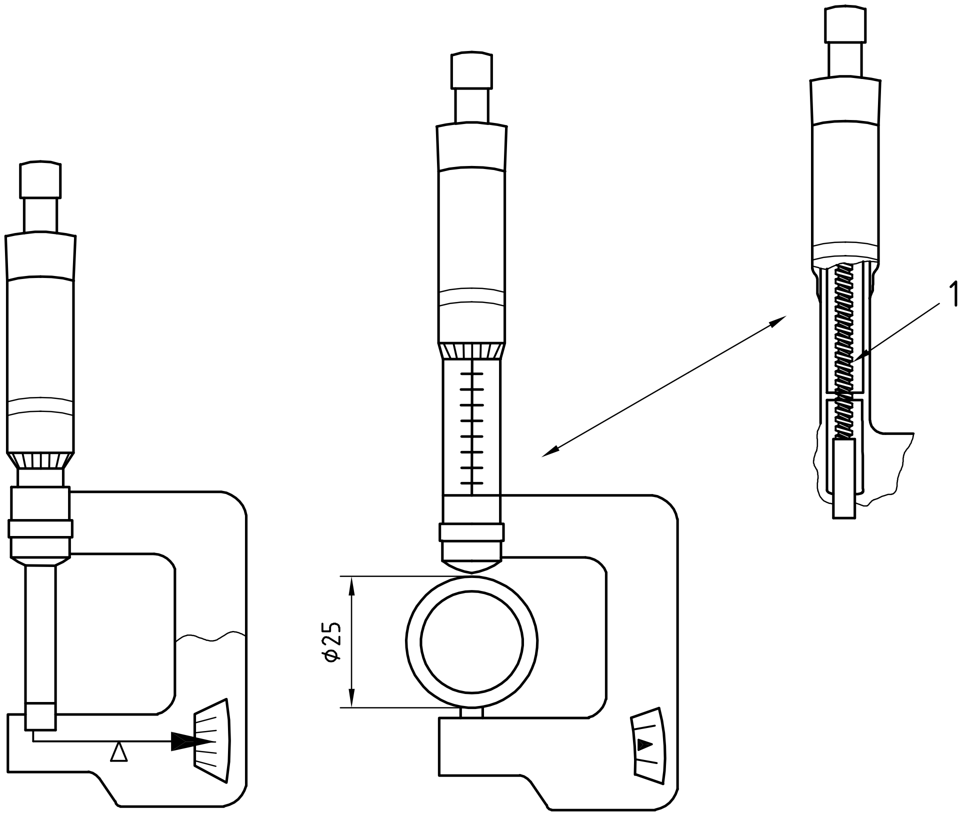
Key

1 gauge block

2 indicator

Figure A.12 — Use of micrometer as comparator

Dimensions in millimetres



Key

1 portion of screw

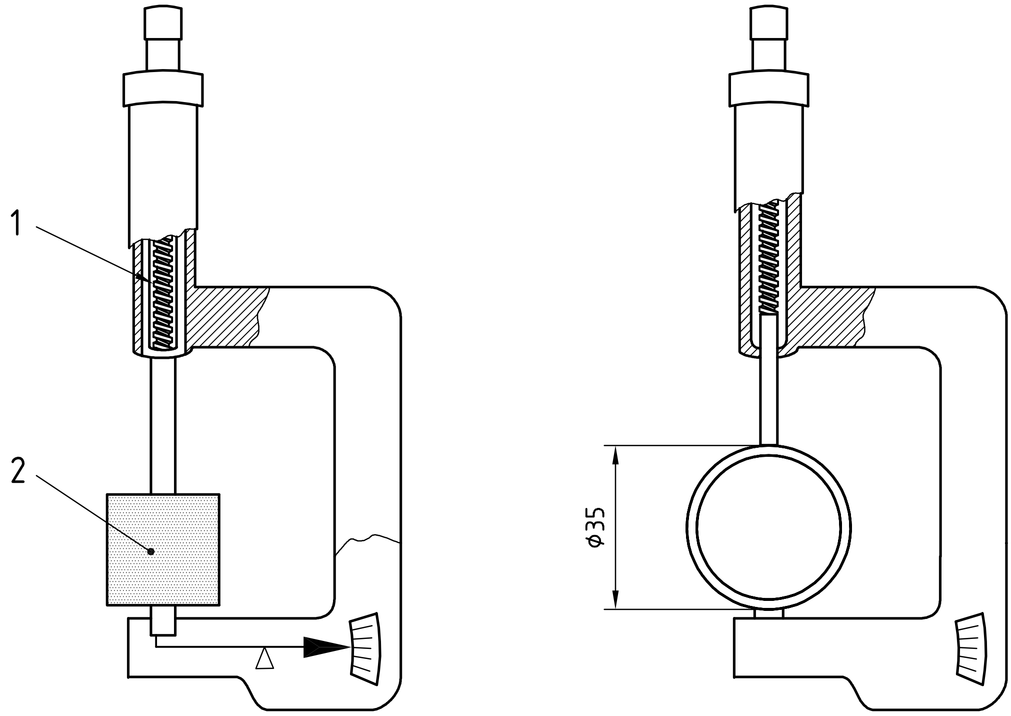
NOTE The working standard is now a portion of the screw.

Figure A.13 — Measurement made with zeroed micrometer

Consider now a 50 mm indicating micrometer and the following case. The workpiece is 35 mm in diameter. A 25 mm gauge block is used to master the micrometer. The working standard in this case is the gauge block plus that portion of the screw, approximately 10 mm long, withdrawn to make room for the workpiece (see Figure A.14).

These cases show how the working standard and comparator can be changed by changes in the operating procedure.

Dimensions in millimetres



Key

1 portion of screw (10 mm)

2 gauge block

Figure A.14 — Combination case where a portion of the screw is used

A.5 Thermal error index (TEI)

This Technical Report does not recommend values for the thermal error index (TEI). Such values cannot be stated without regard to other sources of error and uncertainty in the measurement process. For example, a TEI of 10 % assigns to thermal effects that fraction of the tolerance that is usually considered to include the composite effects of all error sources. In any given case, the permissible values depend on the degree of control maintained over all aspects of the measurement process, including the skill level of personnel.

One way to reduce a TEI is to increase the working tolerance. Consequently, it serves as a feedback device to inform management and designers of the degree of difficulty posed by a specified tolerance. The TEI does nothing more than estimate the maximum possible uncertainty due to the thermal influences affecting a particular measurement process. It does not establish the true magnitude of error in any measurement. It serves to remove doubt about the existence of errors and to establish a system of rewards and penalties to processes that are combinations of techniques and conditions — some good, some bad.

A TEI evaluation penalizes a measurement process on four counts:

a) existence of temperatures other than 20 °C;

b) existence of temperature variations;

c) existence of temperature gradients;

d) not applying the differential thermal expansion correction.

The same evaluation rewards good practice by decreasing the TEI for

- attempting a correction for temperatures other than 20 °C,
- keeping temperature variations to a minimum,
- maintaining acceptable small temperature gradients, and
- applying the differential thermal expansion correction.

The act of performing the evaluation results in the knowledge of what techniques or conditions can be changed to achieve the greatest improvement with the least effort. For example, if temperature variation is found to be the greatest source of error, the measurement cycle time may be reduced such that the TEI is reduced to an acceptable small value. Thus, by more frequent setting, at some nominal increase in operating expense, possible misapplication of capital to improve temperature control is avoided.

Annex B
(informative)

Uncertainty in length measurement due to thermal effects — Example

B.1 Example problem — Measurement of workpiece using comparator

A workpiece of nominal length L_W is to be measured by comparing it with a known working standard of length L_S. The comparison is performed by a comparator, using a short-range displacement indicator whose change in output between setting and measurement is d. In general, the working standard, the comparator and the workpiece are not at 20 °C, and their temperatures are not the same and are changing with time. The workpiece and the working standard also have different coefficients of thermal expansion. It is desired to evaluate the standard uncertainty component in this measurement resulting from thermal errors alone.

The comparator measures the difference in lengths:

$$d = L_W(1+\alpha_W\theta_W) - L_S(1+\alpha_S\theta_S) \tag{B.1}$$

where

L_W is the measurand, that is, the length at 20 °C of the workpiece being measured;

L_S is the length of the working standard at 20 °C as given on its calibration certificate;

α_W and α_S are the coefficients of thermal expansion of the workpiece and the working standard;

θ_W and θ_S are the deviations in temperature from 20 °C of the workpiece and the working standard.

B.2 The mathematical model

From Equation (B.1), the measurand is given by

$$L_W = [L_S(1+\alpha_S\theta_S)+d] / (1+\alpha_W\theta_W)$$

or, assuming the product $\alpha_\omega\theta_\omega$ is small:

$$L_W \approx L_S + d + L_S(\alpha_S\theta_S - \alpha_W\theta_W) \tag{B.2}$$

The third term on the right-hand side of the last expression is the differential thermal expansion between the working standard and the workpiece. In the treatment that follows, it is assumed that this differential expansion has been accounted for in the calculation of the length of the workpiece.

B.3 Uncertainty evaluation

B.3.1 General

Assuming that the quantities α_W, α_S, θ_W and θ_S are uncorrelated, applying the law of propagation of uncertainty (see GUM) to Equation (B.2) yields (let $L_W = L$):

$$u_c^2(L) = [1+(\alpha_S\theta_S - \alpha_W\theta_W)]u^2(L_S) + u^2(d) + u_{DE}^2(L) + u_{TM}^2(L) \tag{B.3}$$

where

$$u_{\mathrm{DE}}^2(L) = L_{\mathrm{s}}^2 \theta_{\mathrm{s}}^2 u^2(\alpha_{\mathrm{s}}) + L_{\mathrm{w}}^2 \theta_{\mathrm{w}}^2 u^2(\alpha_{\mathrm{w}}) \tag{B.4}$$

is the component due to uncertainties in the coefficients of thermal expansion (see 3.2.5), and

$$u_{\mathrm{TM}}^2(L) = L_{\mathrm{s}}^2 \alpha_{\mathrm{s}}^2 u^2(\theta_{\mathrm{s}}) + L_{\mathrm{w}}^2 \alpha_{\mathrm{w}}^2 u^2(\theta_{\mathrm{w}}) \tag{B.5}$$

is the component due to uncertainties in the temperatures (uncertainty of length due to temperature measurement).

As a numerical example, consider a steel workpiece ($\alpha_{\mathrm{W}} = 12 \times 10^{-6}$/°C) of nominal length $L_{\mathrm{W}} = 500$ mm measured by comparison with a working standard ($\alpha_{\mathrm{S}} = 8 \times 10^{-6}$/ °C). The environment is assumed to be varying about a mean temperature of 25 °C. A one-hour drift test has been conducted, revealing a peak-to-valley range of $E_{\mathrm{ETV}} = 12$ µm. The temperatures of the workpiece and the working standard are measured using calibrated thermistors; the mean temperature of the workpiece is 26 °C ($\theta_{\mathrm{W}} = 6$ °C) and the mean temperature of the working standard is 24 °C ($\theta_{\mathrm{S}} = 4$ °C). The workpiece tolerance is TOL = 50 µm (a typical production tolerance), and the adjustment cycle is assumed to be 1 h.

B.3.2 Uncertainty of calibration of length of working standard, $u(L_{\mathrm{s}})$

If a working standard in the classical sense were used for comparison purposes, this uncertainty would be taken from the calibration certificate. In the following it is assumed that $u(L_{\mathrm{s}}) = 0$.

B.3.3 Uncertainty of the measured difference in lengths, $u(d)$

In the ideal situation, multiple measurements would be made on the workpiece and spanning a period of time. The distribution of these measurements would be examined and, depending upon the distribution, the standard uncertainty could be calculated in accordance with the guidelines. In the present situation where the workpiece, the working standard and the comparator are at different temperatures, with different time constants and where the temperature is changing, the values obtained for d would have a wide range. The complexity of this situation is discussed in Annex A.

In the absence of such a definitive series of measurements, this uncertainty, which will have several components, can only be estimated. One of these components, $u_{\mathrm{ETV}}(L)$, would be attributed to the changing temperatures of workpiece, working standard and comparator, with their different time constants and environmental couplings.

As a general procedure, this component should be assessed by performing drift tests over a period of time comparable to the duration of the measurement. These drift tests, one for the working standard and one for the workpiece, will result in a range of values for d which is identical with the quantity E_{ETV} described in Annex A. Assuming that a uniform (rectangular) distribution (see GUM) of E_{ETV} characterizes the knowledge of the effects of temperature variation, then [cf. Equation (13)]:

$$u(d) = u_{\mathrm{ETV}}(L) = \frac{E_{\mathrm{ETV}}}{2\sqrt{3}} \tag{B.6}$$

In this example, then

$$u_{\mathrm{ETV}}(L) \approx 3{,}5 \text{ µm}.$$

NOTE This assumes that the measurement process takes one hour: the same duration as that of the drift test.

B.3.4 Uncertainties of thermal expansion coefficients $u(\alpha_w)$ and $u(\alpha_s)$

The coefficients of thermal expansion have considerable uncertainty. The values of α_w and α_s are assumed to be uniformly distributed with a range of $\pm 2 \times 10^{-6}$ / °C (i.e. $a^+ - a^- = 4 \times 10^{-6}$ / °C. The contribution to the measurement uncertainty from this source follows from [see Equation (6)]

$$u_{DE}(L) = \sqrt{L_s^2 \theta_s^2 u^2(\alpha_s) + L_w^2 \theta_w^2 u^2(\alpha_w)} \tag{B.7}$$

where [in accordance with Equation (12)]

$$u^2(\alpha_s) = u^2(\alpha_w) = [(4 \times 10^{-6})^2/12] \text{ °C}^{-2}$$

For this example, taking $L_s \approx L_w = L = 500$ mm, it follows from Equation (B.7) that

$$u_{DE}(L) = Lu(\alpha)\sqrt{\theta_s^2 + \theta_w^2}$$

so that

$$u_{DE}(L) \approx 4{,}2 \ \mu\text{m}.$$

B.3.5 Uncertainties in temperatures $u(\theta_w)$ and $u(\theta_s)$

The measurements of the temperatures of the workpiece and the working standard also have uncertainties arising from thermometer calibration, sensor mounting and electronic noise. Since the thermal expansion coefficients of workpiece and working standard are different, the uncertainty component from this source is [see Equation (7)]

$$u_{TM}(L) = \sqrt{L^2 \alpha_s^2 u^2(\theta_s) + L^2 \alpha_w^2 u^2(\theta_w)} \tag{B.8}$$

thus, with $L_s \approx L_w = L$ and the assumption that $u^2(\theta_s) = u^2(\theta_w) = u^2(\theta)$:

$$u_{TM}(L) = Lu(\theta)\sqrt{\alpha_s^2 + \alpha_w^2} \tag{B.9}$$

The terms $u^2(\theta_w)$ and $u^2(\theta_s)$ shall be evaluated. In absence of other guidelines, it is suggested that one should assume that these errors are uniformly distributed with a range of ± 1 °C for thermocouples and ± 0,5 °C for properly calibrated thermistors[1]. For this example, this uncertainty becomes:

$$u_{TM}^2(L) = (0{,}5\text{ m})^2\,(0{,}5\,\text{°C})^2\,\frac{\left[\left(8\times10^{-6}\,\text{°C}^{-1}\right)^2 + \left(12\times10^{-6}\,\text{°C}^{-1}\right)^2\right]}{3}$$

or

$$u_{TM}(L) \approx 2{,}1\,\mu\text{m}.$$

B.3.6 Uncertainty in length measurement due to thermal effects

Combining the terms from the previous expressions [Equations (B.6), (B.7) and (B.9)] yields a standard uncertainty component for the measurement of length, $u_{cT}(L)$, of

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1) Standards laboratories can, of course, measure temperatures much more accurately than this, but this is a reasonable number for industrial measurements.

$$u_{cT}(L) = \sqrt{u_{ETV}^2(L) + u_{DE}^2(L) + u_{TM}^2(L)} \quad \text{(B.10)}$$

The first term in the square root is the contribution due to environmental temperature variation, the second term is the contribution due to uncertainty in the thermal expansion coefficients, and the third term is the contribution due to uncertainty in the temperature measurement. For this example, the combination yields

$$u_{cT}(L) \approx \sqrt{(12 + 17{,}3 + 4{,}3)\ \mu\text{m}^2}$$

or

$$u_{cT}(L) \approx 5{,}8\ \mu\text{m}.$$

None of the uncertainties is dominant.

B.3.7 Thermal error index (TEI)

If the user chooses not to perform the correction for nominal differential thermal expansion, then, if there is a tolerance on the workpiece dimension, a TEI calculation shall be performed. In this example, a steel workpiece ($\alpha_W = 12 \times 10^{-6}/\ ^\circ\text{C}$) with a length of $L_W = 500$ mm is compared with a working standard ($\alpha_S = 8 \times 10^{-6}/\ ^\circ\text{C}$). The temperature is at 25 °C and changing, the working standard at T_S = 24 °C, and the workpiece at $T_W = 26$ °C. In accordance with Equation (A.3), nominal differential thermal expansion $\Delta_{nDE} = 20$ µm will be obtained.

Combining Equations (9) and (10), the thermal error index is given by

$$\text{TEI} = \frac{2\left|\Delta_{nDE}\right| + 4u_{cT}(L)}{\text{TOL}} \times 100\ \% \quad \text{(B.11)}$$

Thus, for this example, the TEI is 126 %. In such a case, it would be impossible to prove that the workpiece was acceptable, since the range of reasonably probable errors exceeds the width of the tolerance zone.

Annex C
(informative)

Relationship to the GPS matrix model

For full details about the GPS matrix model, see ISO/TR 14638.

C.1 Information about this Technical Report and its use

This Technical Report deals with systematic errors, corrections and uncertainty contributions due to thermal influences on length measurements. It provides the specialized details for analysing thermal influence when broader analyses, such as those in accordance with the methods of ISO/TS 14253-2, show that thermal effects are significant contributors to the measurement uncertainty, systematic error or both.

C.2 Position in the GPS matrix model

This Technical Report is a global GPS technical report that influences chain links 4, 5 and 6 in all chains of standards in the GPS matrix structure, as graphically illustrated in Figure C.1.

Global GPS standards						

	General GPS standards						
	Chain link number	1	2	3	4	5	6
Fundamental GPS standards	Size						
	Distance						
	Radius						
	Angle						
	Form of line independent of datum						
	Form of line dependent of datum						
	Form of surface independent of datum						
	Form of surface dependent of datum						
	Orientation						
	Location						
	Circular run-out						
	Total run-out						
	Datums						
	Roughness profile						
	Waviness profile						
	Primary profile						
	Surface imperfections						
	Edges						

Figure C.1

C.3 Related International Standards

The related International Standards are those of the chains of standards indicated in Figure C.1.

Bibliography

[1] ISO 230-3, *Test code for machine tools — Part 3: Determination of thermal effects*

[2] ISO 3534-1:1993, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*

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[4] ISO 14253-1, *Geometrical Product Specifications (GPS) — Inspection by measurement of workpieces and measuring equipment — Part 1: Decision rules for proving conformance or non-conformance with specifications*

[5] ISO/TS 14253-2, *Geometrical Product Specifications (GPS) — Inspection by measurement of workpieces and measuring equipment — Part 2: Guide to the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification*

[6] ISO/TR 14638, *Geometrical product specifications (GPS) — Masterplan*

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