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Optics and photonics — Interferometric measurement of optical elements and optical systems —

Part 4:
Interpretation and evaluation of tolerances specified in ISO 10110

Optique et photonique — Mesurage interférométrique de composants et de systèmes optiques —

Partie 4: Directives pour l'évaluation des tolérances spécifiées dans l'ISO 10110

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 14999-4 was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 1, *Fundamental standards*.

ISO 14999 consists of the following parts, under the general title *Optics and photonics — Interferometric measurement of optical elements and optical systems*:

- *Part 1: Terms, definitions and fundamental relationships*
- *Part 2: Measurement and evaluation techniques*
- *Part 3: Calibration and validation of interferometric test equipment and measurements*
- *Part 4: Interpretation and evaluation of tolerances specified in ISO 10110*

Parts 1, 2 and 3 are Technical Reports.

Introduction

This part of ISO 14999 provides a theoretical frame upon which are based indications from ISO 10110-5 and/or ISO 10110-14.

ISO 10110-5 refers to deformations in the form of an optical surface, and provides a means for specifying tolerances for certain types of surface deformations in terms of “fringe spacings”.

ISO 10110-14 refers to deformations of a wavefront transmitted once through an optical system, and provides a means of specifying similar deformation types in terms of optical “wavelengths”.

Because it is common practice to measure the surface form deviation interferometrically as the wavefront deformation caused by a single reflection from the optical surface at normal (90° to surface) incidence, it is possible to describe a single definition of interferometric data reduction that can be used in both cases. One "fringe spacing" (as defined in ISO 10110-5) is equal to a surface deformation that causes a deformation of the reflected wavefront of one wavelength.

Certain scaling factors apply depending on the type of interferometric arrangement – for example, whether the test object is being measured in single pass or double pass.

Because of the potential for confusion and mis-interpretation, units of nanometres rather than units of “fringe spacings” or “wavelengths” should be used for the value of surface form deviation or the value of wavefront deformation, where possible. Where “fringe spacings” or “wavelengths” are used as units, the wavelength should also be specified.

Optics and photonics — Interferometric measurement of optical elements and optical systems —

Part 4:
Interpretation and evaluation of tolerances specified in ISO 10110

1 Scope

This part of ISO 14999 applies to the interpretation of interferometric data relating to the measurement of optical elements.

This part of ISO 14999 gives definitions of the optical functions specified in the preparation of drawings for optical elements and systems, made in accordance with ISO 10110-5 and/or ISO 10110-14 as well as guidance for their interferometric evaluation with visual analysis.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 10110-5:—1), *Optics and photonics — Preparation of drawings for optical elements and systems — Part 5: Surface form tolerances*

ISO 10110-14:—2), *Optics and photonics — Preparation of drawings for optical elements and systems — Part 14: Wavefront deformation tolerance*

3 Terms and definitions

3.1 Mathematical definitions

3.1.1
function
mathematical description of the measured wavefront deformation and its decomposition into components

NOTE The functions used in this part of ISO 14999 are scalar functions.

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1) To be published (Revision of ISO 10110-5:1996 + 10110-5:1996/Cor.1:1996).

2) To be published (Revision of ISO 10110-14:2003).

3.1.2
peak-to-valley value

PV (f)

⟨of a function f⟩ maximum value of the function within the region of interest minus the minimum value of the function within the region of interest

3.1.3
root mean square value

rms (f)

⟨of a function f over a given area A⟩ value given by either of the following integral expressions:

a) Cartesian variables x and y:

$$\text{rms}\,(f) = \left[\frac{\int\limits_{x}\int\limits_{y} \left[f(x,y) \right]^2 \,\mathrm{d}x\,\mathrm{d}y}{\int\limits_{x}\int\limits_{y} \mathrm{d}x\,\mathrm{d}y} \right]^{1/2} \quad \text{where } (x,y) \in A$$

b) Polar variables r and θ:

$$\text{rms}\,(f) = \left[\frac{\int\limits_{\theta}\int\limits_{r} \left[f(r,\theta) \right]^2 r\,\mathrm{d}r\,\mathrm{d}\theta}{\int\limits_{\theta}\int\limits_{r} r\,\mathrm{d}r\,\mathrm{d}\theta} \right]^{1/2} \quad \text{where } (r,\theta) \in A$$

NOTE This integral may be approximated by the standard deviation provided that the measurement resolution is specified and is sufficient.

3.2 Definition of optical functions

NOTE The following optical functions are depicted in Figure 1. For the relationship of interferometric measurements to surface form deviation and transmitted wavefront deformation see Clause 4.

3.2.1
measured wavefront deformation

f_{MWD}

function representing the distances between the measured wavefront and the nominal theoretical wavefront, measured normal to the nominal theoretical wavefront

See Figure 1 a).

3.2.2
tilt

f_{TLT}

plane function representing the best (in the sense of the rms fit) linear approximation to the measured wavefront deformation f_{MWD}

See Figure 1 b).

3.2.3
wavefront deformation

f_{WD}

function resulting after subtraction of the tilt f_{TLT} from the measured wavefront deformation f_{MWD}

$$f_{\mathrm{WD}} = f_{\mathrm{MWD}} - f_{\mathrm{TLT}}$$

See Figure 1 c).

3.2.4
wavefront spherical approximation

f_{WS}

function of spherical form that best (in the sense of the rms fit) approximates the wavefront deformation f_{WD}

See Figure 1 d).

3.2.5
wavefront irregularity

f_{WI}

function resulting after subtraction of the wavefront spherical approximation f_{WS} from the wavefront deformation f_{WD}

$$f_{\mathrm{WI}} = f_{\mathrm{WD}} - f_{\mathrm{WS}}$$

See Figure 1 e).

3.2.6
wavefront aspheric approximation

f_{WRI}

rotationally invariant aspherical function that best (in the sense of the rms fit) approximates the wavefront irregularity, f_{WI}

See Figure 1 f).

3.2.7
rotationally varying wavefront deviation

f_{WRV}

function resulting after subtraction of the wavefront aspheric approximation f_{WRI} from the wavefront irregularity f_{WI}

$$f_{\mathrm{WRV}} = f_{\mathrm{WI}} - f_{\mathrm{WRI}}$$

See Figure 1 g).

3.3 Definition of values related to the optical functions defined in 3.2

3.3.1
sagitta deviation

PV (f_{WS})

peak-to-valley value of the approximating spherical wavefront

NOTE PV (f_{WS}) corresponds to the quantity A in ISO 10110-5:— and ISO 10110-14:—. In the case of ISO 10110-5, if the unit is not fringe spacing, the surface deviation is computed according to the test set-up used.

3.3.2
irregularity
PV (f_{WI})
peak-to-valley value of the wavefront irregularity

NOTE PV (f_{WI}) corresponds to the quantity B in ISO 10110-5:— and ISO 10110-14:—. In the case of ISO 10110-5, if the unit is not fringe spacing, the surface deviation is computed according to the test set-up used.

3.3.3
rotationally invariant irregularity
PV (f_{WRI})
peak-to-valley value of the wavefront aspheric approximation

NOTE PV (f_{WRI}) corresponds to the quantity C in ISO 10110-5:— and ISO 10110-14:—. In the case of ISO 10110-5, if the unit is not fringe spacing, the surface deviation is computed according to the test set-up used.

3.3.4
rotationally varying irregularity
PV (f_{WRV})
peak-to-valley value of the rotationally varying wavefront deviation

3.3.5
rms total
rms (f_{WD})
root-mean-square value of the wavefront deformation

NOTE rms (f_{WD}) corresponds to the quantity RMSt in ISO 10110-5:— and ISO 10110-14:—. In the case of ISO 10110-5, if the unit is not fringe spacing, the surface deviation is computed according to the test set-up used.

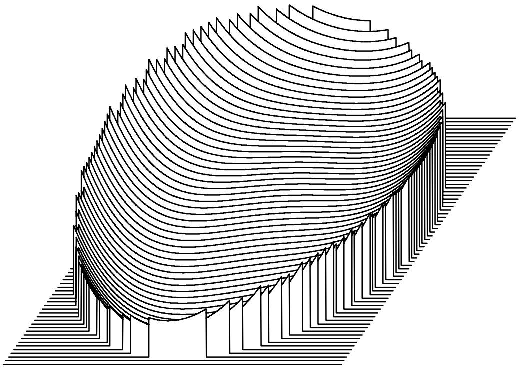
3.3.6
rms irregularity
rms (f_{WI})
root-mean-square value of the wavefront irregularity

NOTE rms (f_{WI}) corresponds to the quantity RMSi in ISO 10110-5:— and ISO 10110-14:—. In the case of ISO 10110-5, if the unit is not fringe spacing, the surface deviation is computed according to the test set-up used.

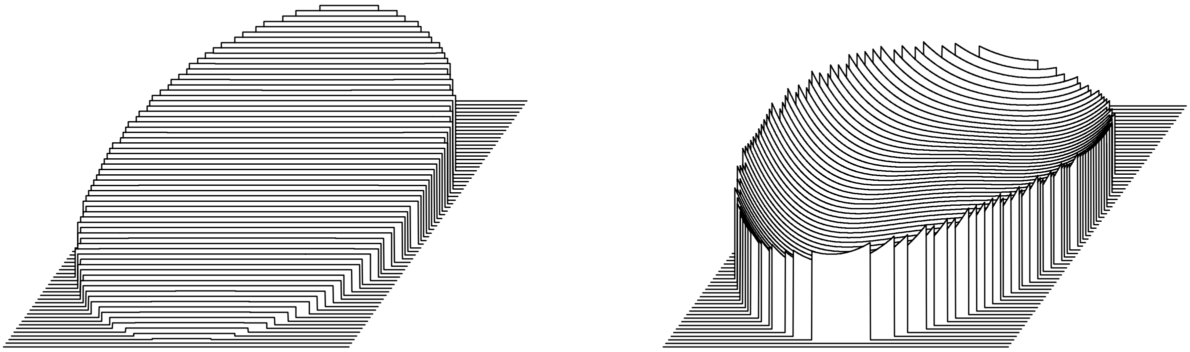
3.3.7
rms rotationally invariant irregularity
rms (f_{WRI})
root-mean-square value of the wavefront aspheric approximation

3.3.8
rms rotationally varying irregularity
rms (f_{WRV})
root-mean-square value of the rotationally varying wavefront deviation

NOTE rms (f_{WRV}) corresponds to the quantity RMSa in ISO 10110-5:— and ISO 10110-14:—. In the case of ISO 10110-5, if the unit is not fringe spacing, the surface deviation is computed according to the test set-up used.

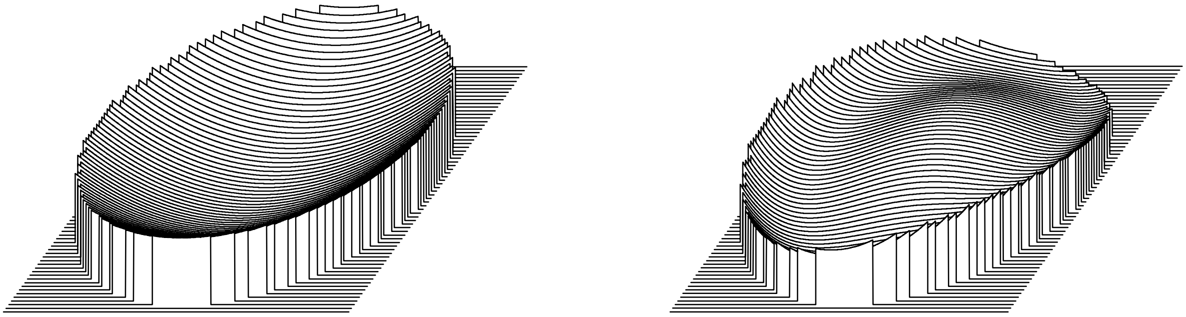


a) Measured wavefront deformation (f_{MWD})



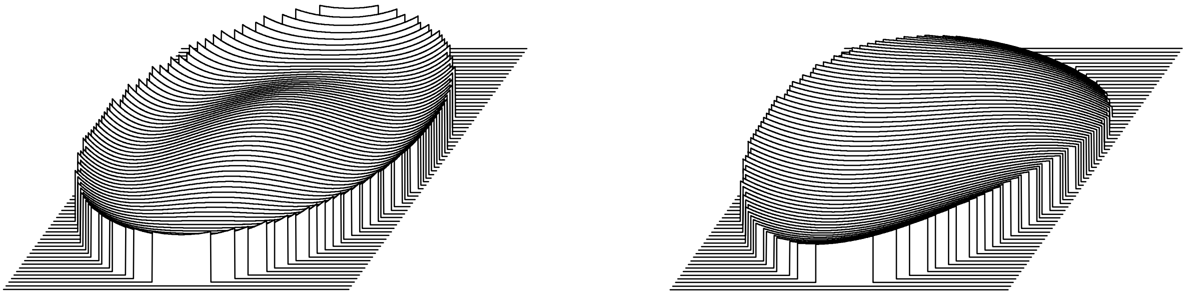
b) Tilt (f_{TLT})

c) Wavefront deformation (f_{WD}) that determines the "RMSt"



d) Wavefront spherical approximation (f_{WS}) that determines the sagitta deviation "A"

e) Wavefront irregularity (f_{WI}) that determines the irregularity "B" **and** "RMSi"



f) (Rotationally invariant) wavefront aspheric approximation (f_{WRI}) that determines the rotationally invariant irregularity "C"

g) Remaining rotationally varying wavefront deviation (f_{WRV}) that determines the "RMSa"

Figure 1 — Measured wavefront deformation and its decomposition into wavefront deformation types

4 Relating interferometric measurements to surface form deviation or transmitted wavefront deformation

4.1 Test areas

The optical functions defined in 3.2 are only defined within the specified test areas.

NOTE If the test area is non-circular, the wavefront deformation decomposition cannot be made by Zernike polynomials.

4.2 Quantities

The quantities defined in 3.3 are used for the indications according to ISO 10110-5 and ISO 10110-14 using the specified fringe spacings or wavelength or nanometre as unit, see ISO 10110-5 or ISO 10110-14, respectively.

An optical path difference of one wavelength in the wavefront (one fringe spacing) corresponds to a surface deviation of half a wavelength when reflected once at normal incidence.

4.3 Single-pass transmitted wavefront

Transmitted wavefront deformation, as defined in ISO 10110-14, is directly measurable using a single-pass arrangement, such as a Mach-Zehnder interferometer, provided that the wavelength of the interferometer is the same as the wavelength of the specification.

4.4 Double-pass transmitted wavefront

Double-pass arrangements are often used to measure the transmitted wavefront deformation of optical elements by common path instruments. In this case, the interferometric measurement is approximately twice as sensitive. The interferometric results shall be divided by two to obtain approximate results for the transmitted wavefront deformation.

NOTE Because diffraction occurs at both passes through the test object, and because the wavefront deformation imparted by the test object on the second pass depends slightly on the wavefront deformation imparted on the first pass, the transmitted wavefront deformation measured in a double-pass arrangement is only approximately half the results reported by the interferometer.

4.5 Surface form deviation

Surface form deviation is commonly measured using an interferometric measurement of a wavefront reflected once from the optical surface under test.

An optical path difference of one wavelength in the wavefront (one fringe spacing) corresponds to a surface deviation of half a wavelength when reflected once at normal incidence.

4.6 Conversion to other wavelengths

If the test wavelength is not equal to the specification wavelength, the results of the interferometric test may be converted using the equation:

$$N_{\lambda 2} = N_{\lambda 1} \times \frac{\lambda_1}{\lambda_2}$$

where $N_{\lambda 1}$ and $N_{\lambda 2}$ are, for example, the numbers of fringe spacings at λ_1 and λ_2.

Annex A
(normative)

Visual interferogram analysis

A.1 General

A.1.1 General remarks

This Annex is intended as an aid to understanding ISO 10110-5 and ISO 10110-14. It is useful for the interpretation of interferograms (including fringe patterns seen when using test glasses). For surface form measurement the form deviation is determined by the resulting wavefront deviation as described in the introduction. The guidelines given for the estimation of the amounts of the various types of wavefront deformation should not be regarded as a definition of those wavefront deformation types.

The purpose of this Annex is to demonstrate the visual appearance of interferograms for the different types of wavefront deformation.

This Annex deals exclusively with the following types of wavefront deformation:

- sagitta deviation;
- irregularity;
- rotationally invariant irregularity.

The rms residual wavefront deformation types (defined in 3.3) cannot be determined accurately by visual inspection.

Clauses A.2 and A.3 describe the analysis of circular test areas. Special consideration for non-circular test areas is given in A.4.

The analysis of interferograms is treated more fully in many textbooks. See, for example, reference [1] in the Bibliography.

A.1.2 Interferometric tilt

Two methods are used for estimating the amounts of sagitta deviation and irregularity, depending on the amount of relative tilt between the test and reference wavefronts. The method without tilt is mainly applied when the wavefront deformation is large. The method employing tilt is generally more accurate.

The relative tilt between the two wavefronts is not a measure of the wavefront deformation.

A.1.3 Determination of the sign of the deformation

In order to determine the sign of the deformation of the wavefront or regions of the wavefront, it is sometimes necessary to shorten slightly or lengthen slightly the test arm of the interferometer, in order to note the behaviour of the interferometric fringes when this is done.

A.2 Estimation of sagitta deviation and irregularity

A.2.1 General

The sagitta deviation can only be determined if the positions of both the object point and the image point are specified. Often, when testing optical elements and systems interferometrically, only one of these two positions is specified, and the sagitta deviation cannot be determined; however, the irregularity can still be determined.

The determination of the sagitta deviation is simplest if the point source of the interferometer is placed at the indicated object point, and the mirror that reflects the beam back toward the interferometer is placed concentric with the indicated image point. In the following, it is assumed that this is the case. If this is not the case, then the distances between the indicated and actual object points and the indicated and actual image points must be taken into account. If dimensional tolerances are associated with the indications of the positions of the object and image points, the source and the reflecting surface may be moved within these tolerances to minimize the sagitta deviation.

Usually, the wavefront deformation is dominated by sagitta deviation and/or by a kind of asymmetry in the sagitta deviation. In the case of asymmetry, cross-sections of the wavefront in different directions show different amounts of sagitta deviation. Other kinds of irregularity are possible; the estimation of their amounts is more difficult. The estimation of the amounts of sagitta deviation and irregularity for the commonly occurring cases is described in A.2.2 and A.2.3, and a more general procedure for unusual types of irregularity is described in A.2.4. The reference given in [1] contains a more thorough discussion of interferogram analysis.

A.2.2 Analysis of interferograms without tilt

In the absence of all other types of wavefront deformation, sagitta deviation causes an interference pattern having concentric, circular fringes. The radii of the fringes increase with the square root of the fringe number, counting from the centre of the interferogram.

If a small amount of asymmetric deformation is present, the circles distort into ellipses, as shown in Figure A.1. If the test wavefront is concave with respect to the reference wavefront, then the fringes will move toward the centre of the fringe pattern when the test arm of the interferometer is shortened. If the reverse is true, then the test wavefront is convex with respect to the reference wavefront.

To estimate the amount of sagitta deviation and irregularity, let m and m' be the numbers of fringe spacings seen in the fringe pattern, counted from the centre to the edge, in the directions that give the largest and smallest numbers of fringes[3]. In the case of elliptical fringes, the sagitta deviation is given by the average of m and m', that is:

$$\text{Sagitta deviation (elliptical fringes)} = \frac{m + m'}{2} \tag{A.1}$$

In the case of elliptical fringes, the wavefront irregularity is equal to the absolute value of the difference of the fringe counts m and m':

$$\text{Irregularity (elliptical fringes)} = \left| m - m' \right| \tag{A.2}$$

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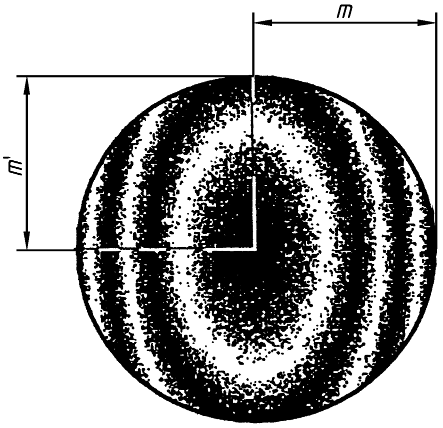
[3] Usually, these two directions are oriented at 90° to one another, but this need not be the case.



Key

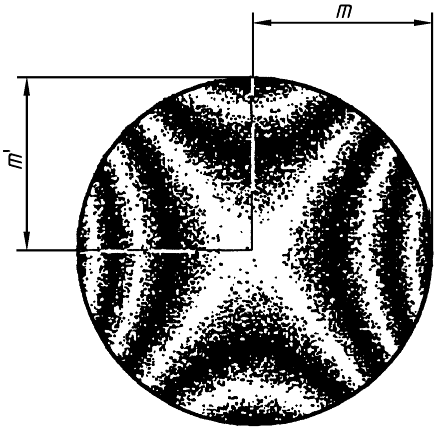
m = 3,0 fringe spacings

m' = 1,0 fringe spacing

Figure A.1 — Example of an interferogram showing 2 wavelengths of sagitta deviation and 2 wavelengths of irregularity (evaluation in Example 1)

EXAMPLE 1 Figure A.1 shows the interferogram of an optical element. The values of m and m' are 3 and 1; therefore the sagitta deviation [Equation (A.1)] is (3 + 1)/2 = 2 wavelengths, the irregularity [Equation (A.2)] is |3 − 1| = 2 wavelengths.

If a large amount of asymmetric deformation is present, the elliptical fringes may be broken into approximately hyperbolic fringes, as shown in Figure A.2. In this case, when the test wavefront is moved slightly toward the interferometric reference wavefront, some of the fringes will move toward the centre of the fringe pattern and some will move away from the centre.



Key

m = 2,5 fringe spacings

m' = 1,5 fringe spacing

Figure A.2 — Example of an interferogram showing 0,5 wavelengths of sagitta deviation and 4 wavelengths of irregularity (evaluation in Example 2)

In the case of hyperbolic fringes, the sagitta deviation is equal to half the difference between the numbers of fringe spacings:

$$\text{Sagitta deviation (hyperbolic fringes)} = \left| \frac{m - m'}{2} \right| \quad \text{(A.3)}$$

The irregularity in the case of hyperbolic fringes is given by the sum of the numbers of the fringe counts:

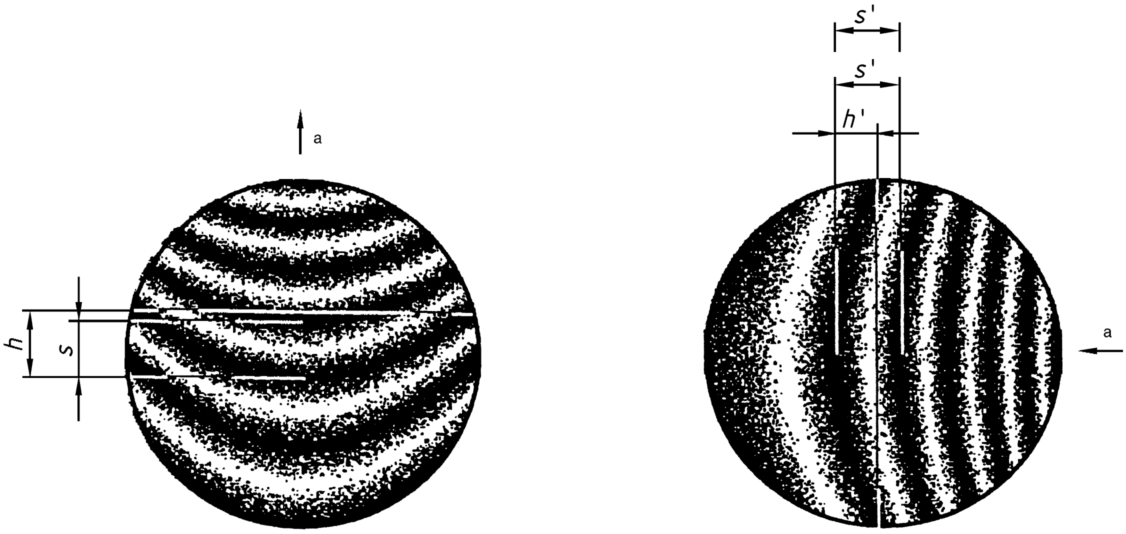
$$\text{Irregularity (hyperbolic fringes)} = m + m' \quad \text{(A.4)}$$

EXAMPLE 2 Figure A.2 shows the interferogram of an optical element. The values of m and m' are 2,5 and 1,5 wavelengths, respectively, so the sagitta deviation [Equation (A.3)] is | 2,5 − 1,5 |/2 = 0,5 wavelengths, and the irregularity [Equation (A.4)] is 2,5 + 1,5 = 4 wavelengths.

A.2.3 Analysis of interferograms with tilt

This method requires the fringes to be observed twice, with the tilt between the test and reference wavefronts adjusted so that the fringes are oriented in two different directions.

When the reference surface is tilted with respect to the test surface, the fringes appear as in Figure A.3. If only sagitta deviation is present, then the fringes appear as parts of concentric circles, with the radii of the fringes increasing with the square root of the fringe number, counting from the apparent centre of the interferogram. If other wavefront deformation types are also present, the fringes are not parts of concentric circles.



a) b)

Key

m = h/s

m' = h'/s'

[a] Motion: the arrows indicate the direction of motion of fringes when the test arm is shortened.

Figure A.3 — Example showing interferograms with 0,3 wavelengths of sagitta deviation and 1,8 wavelengths of irregularity, with the interferometric tilt oriented in two directions
(evaluation in Example 3)

To estimate the sagitta deviation and the irregularity, the curvature of the test wavefront in the cross-section parallel to the fringes is estimated for the two directions of tilt that give the maximum and minimum amounts of curvature. See Figures A.3 a) and A.3 b). In each case, the number of fringe spacings, m, is equal to the curvature, h, of the fringe closest to the centre of the interferogram, divided by the spacing, s, of the fringes, which is also measured as close as possible to the centre of the test area.

In addition, it is necessary to note (for both directions of the tilt) the direction of motion of the fringes when the test arm of the interferometer is shortened (the surface under test is moved slightly towards the reference surface).

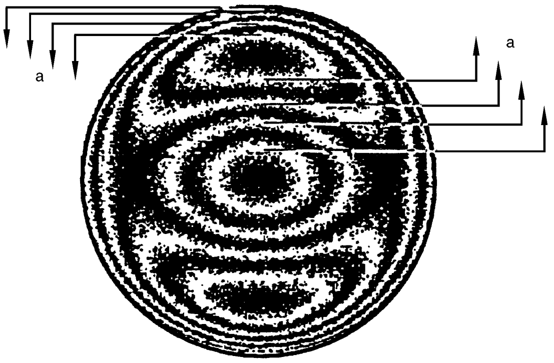
If the fringes for both directions of tilt move toward the apparent centre of curvature of the fringes, or if the fringes for both directions of tilt move away from the apparent centre, then the sagitta deviation exceeds the irregularity, and Equations (A.1) and (A.2) are used to estimate the sagitta deviation and the irregularity, respectively.

If one set of fringes moves towards its apparent centre, and the other fringe pattern moves away from its apparent centre when the test arm is shortened, then the irregularity exceeds the sagitta deviation, and Equations (A.3) and (A.4) are used to estimate the amounts of sagitta deviation and irregularity.

EXAMPLE: Figure A.3 shows interferograms of an optical element tested with tilt. In Figure A.3 a), the fringes move towards the apparent centre when the test arm is shortened, and in Figure A.3 b), the fringes move away from the apparent centre, so Equations (A.3) and (A.4) apply. In Figure A.3 a), the curvature, h, is approximately 1,2 × the fringe spacing, s, so m = 1,2. In Figure A.3 b), the curvature, h', is 60 % the fringe spacing, s', so m' = 0,6. The immediate result of Equation (A.3) is 0,3 wavelengths for the sagitta deviation. Similarly, Equation (A.4) yields a value of 1,8 wavelengths for the irregularity.

A.2.4 Unusual forms of irregularity

It is possible that the wavefront deformation be a maximum at some point inside the test area, rather than at the edge. When testing wavefronts with no tilt between the test and reference wavefronts, this leads to closed fringes which may not be concentric with the centre of the test area, as shown in Figure A.4. In such cases, it is necessary to note which fringes move away from the centre and which toward the centre when the test arm of the interferometer is shortened. Those that move toward the centre may be regarded as "positive", and the others as “negative”.



[a] Motion.

Figure A.4 — Example of an unusual interferogram, showing the direction of motion of the fringes when the test arm of the interferometer is shortened (evaluation in Example 4)

EXAMPLE 4 The sagitta deviation is determined according to Equation (A.1), where m and m' represent the cumulative numbers of fringes measured in two representative directions. In the vertical cross-section of Figure A.4, there are 4 fringe spacings in the negative direction, followed by 4 fringe spacings in the positive direction, giving a value of zero for m. In the horizontal direction, there are 2 negative and 2 positive fringes, again giving m' = 0. According to Equation (A.1), the sagitta deviation is (0 + 0)/2 = 0.

The irregularity is determined by finding the highest and lowest departures from the theoretical expected fringe pattern, which is that the fringes are concentric circles with radii increasing as the square root of the fringe number. The irregularity is the sum of the absolute values of the highest and lowest departures from the pattern. For the pattern of Figure A.4, the sagitta deviation is zero, so the theoretical expected fringe pattern has no fringes. The lowest departure from this is −4 fringe spacings (at the centres of the two outer oval patterns), and the highest departure from this is zero. Therefore, the irregularity is |0| + |−4| = 4 fringes.

A.3 Estimation of rotationally invariant irregularity

The estimation of this deviation by visual methods is difficult if large amounts of other types of wavefront deformation are present. For this reason, digital methods of interferogram analysis are preferred.

If no tilt is present between the test wavefront and the reference wavefront, the fringes appear as concentric circles, but their radii do not increase with the square root of the fringe number, as would be the case with sagitta deviation. Visual observation of this property is difficult and becomes inaccurate for small deviations. Therefore, the assessment of this type of wavefront deformation is practical only in the presence of tilt.

In the presence of tilt, the fringes are W- or M-shaped, depending on the direction of the tilt. In the absence of sagitta deviation, the two ends and the centre of the fringe nearest the centre of the fringe pattern can be joined by a straight line. In this case, the irregularity is represented by the deviation of the fringes from straightness. If sagitta deviation is present in the wavefront, the fringes are curved, as shown in Figure A.5, and the irregularity may be estimated by the deviation of the fringe nearest the centre from a circular arc joining the two ends and the centre of the fringe.

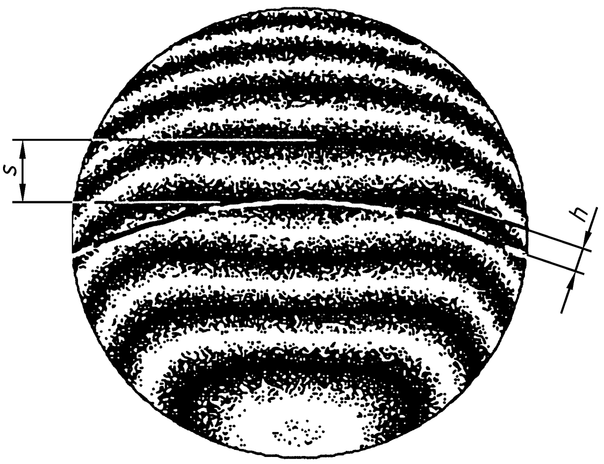


Figure A.5 — Example of an interferogram showing 0,3 wavelengths of rotationally invariant irregularity (evaluation in Example 5)

The irregularity is equal to the maximum deviation, h, of the fringe from a circular arc, divided by the fringe spacing, s. The deviation, h, is measured perpendicular to the circular arc at the point of maximum departure.

The rotationally invariant irregularity is given by:

$$\text{Rotationally invariant irregularity} = \frac{h}{s} \tag{A.5}$$

EXAMPLE 5 In Figure A.5, the deviation, h, of the central fringe from straightness is 30 % of the fringe spacing, s, so the rotationally invariant irregularity is 0,3 fringe spacings [the result of Equation (A.5)].

The degree to which the wavefront deformation is rotationally invariant is observed by repeating the above test with the tilt adjusted so that the fringes are oriented in another direction. The wavefront deformation is rotationally invariant if the appearance of the fringes is the same for all orientations of the fringes. The rotationally invariant irregularity is that part of the deformation which remains the same for all orientations of the fringes.

A.4 Non-circular test areas

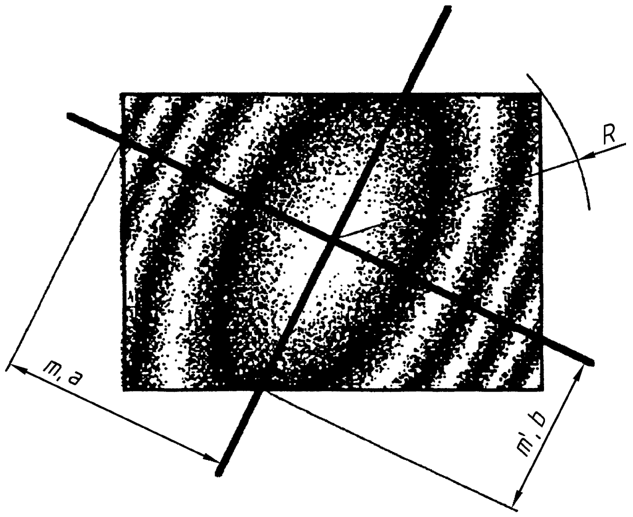
According to the definition, the sagitta deviation (see 3.3.1) is based on the spherical wavefront that best approximates the test wavefront. When using visual analysis methods, the wavefront spherical approximation (see 3.2.4) may be chosen so that the wavefront irregularity (see 3.2.5) is evenly distributed around the boundary of the test area. This requires that sagitta deviation and irregularity be evaluated by a method similar to that described in A.2, except that the calculations shall take into account the dimensions of the test area in the two cross-sections in which m and m' are measured.

This method is reasonably accurate for simple forms of wavefront deformation (i.e., those which are second-order in x and y); for an accurate evaluation of more complex forms, digital methods are necessary.

For non-circular test areas, the "centre" of the test area refers to its centroid ("centre-of-gravity"), and its "radius" is equal to the distance from the centre to the most distant point in the test area.

The cross-sectional curvatures m and m' are determined in the same way as in A.2, using the description of the case with or without tilt, as appropriate. The directions along which m and m' are determined are given by the symmetry of the wavefront deformation; these directions are not necessarily related to the shape of the test area.

Let m and m' be the cross-sectional curvatures in the two directions of symmetry, from the centre to the edge of the test area, as shown in Figure A.6. Let a be the distance from the centre to the edge of the test area in the direction along which the curvature, m, is measured. Similarly, let b be the distance along which the curvature m' is measured. Let R be the radius of the test area, as defined above.



Key

R = 36 mm

m = 3,6 fringe spacings

a = 33 mm

m' = 0,4 fringe spacings

b = 23 mm

Figure A.6 — Example of an interferogram with a non-circular test area, showing 3,2 wavelengths of sagitta deviation and 2,2 wavelengths of irregularity (evaluation in Example 6)

In the case of elliptical fringes and a non-circular test area, the sagitta deviation is determined by:

$$\text{Sagitta deviation (elliptical fringes)} = \frac{R^2\left(m + m'\right)}{a^2 + b^2} \tag{A.6}$$

In the case of elliptical fringes and a non-circular test area, the irregularity is determined by:

$$\text{Irregularity (elliptical fringes)} = \left|\frac{2R^2\left(a^2 m' - b^2 m\right)}{a^2\left(a^2 + b^2\right)}\right| \tag{A.7}$$

EXAMPLE 6 In Figure A.6, the values of m and m' are 3,6 and 0,4 wavelengths, measured over distances of 33 mm and 23 mm respectively. The radius of the test area is 36 mm. The results of Equations (A.6) and (A.7) are 3,2 wavelengths and 2,16 wavelengths, respectively. Thus, the sagitta deviation is 3,2 wavelengths and the irregularity is 2,16 wavelengths.

In the case of hyperbolic fringes, the sagitta deviation is found by the equation:

$$\text{Sagitta deviation (hyperbolic fringes)} = \left|\frac{R^2\left(m - m'\right)}{a^2 + b^2}\right| \tag{A.8}$$

In the case of hyperbolic fringes, the irregularity is found by the equation:

$$\text{Irregularity (hyperbolic fringes)} = \frac{2R^2\left(a^2 m' + b^2 m\right)}{a^2\left(a^2 + b^2\right)} \tag{A.9}$$

If there is tilt between the interferometric reference wavefront and the wavefront under test, then it is necessary to note (for both directions of the tilt) the direction of motion of the fringes when the test arm of the interferometer is shortened.

If the fringes in both cases move towards the apparent centre of the fringe pattern, or if the fringes in both cases move away from the apparent centre of the fringe pattern, then the sagitta deviation exceeds the irregularity, and Equations (A.6) and (A.7) shall be used to estimate the sagitta deviation and the irregularity.

If one set of fringes moves towards its apparent centre, and the other fringe pattern moves away from its apparent centre, then the irregularity exceeds the sagitta deviation, and Equations (A.8) and (A.9) shall be used to estimate the amounts of sagitta deviation and irregularity.

A.5 Target aberrations

The visual analysis of interferograms when target aberrations are specified is difficult and not recommended. In principle, it is possible to draw or otherwise generate the interference pattern corresponding to the target aberrations, and examine the difference between this and the actual interferogram; however, the visual appearance of the actual interferogram depends on the amount and orientation of tilt present. Similarly, the exact choice of the radius of the reference sphere, which often has a generous tolerance, influences the visual appearance of the interference pattern. For these reasons, the precise generation of the theoretical interference pattern with which the actual interferogram should be compared is generally not possible, in which case an accurate evaluation can be obtained only through the use of digital analysis techniques.

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