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Plain bearings — Hydrostatic plain journal bearings without drainage grooves under steady-state conditions —

Part 1:
Calculation of oil-lubricated plain journal bearings without drainage grooves

Paliers lisses — Paliers lisses radiaux hydrostatiques sans rainure d'écoulement fonctionnant en régime stationnaire —

Partie 1: Calcul pour la lubrification des paliers lisses radiaux sans rainure d'écoulement

Reference number
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Foreword

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this part of ISO 12168 may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 12168-1 was prepared by Technical Committee ISO/TC 123, *Plain bearings*, Subcommittee SC 4, *Methods of calculation of plain bearings*.

ISO 12168 consists of the following parts, under the general title *Plain bearings — Hydrostatic plain journal bearings without drainage grooves under steady-state conditions*:

— *Part 1: Calculation of oil-lubricated plain journal bearings without drainage grooves*

— *Part 2: Characteristic values for the calculation of oil-lubricated plain journal bearings without drainage grooves*

Annexes A and B form a normative part of this part of ISO 12168.

Introduction

The functioning of hydrostatic bearings is characterized by the fact that the supporting pressure of the bearing is generated by external lubrication. The special advantages of hydrostatic bearings are lack of wear, quiet running, wide useable speed range as well as high stiffness and damping capacity. These properties are also the reason for the special importance of hydrostatic bearing units in different fields of application such as e.g. machine tools.

The bases of calculation described in this part of ISO 12168 apply to bearings with different numbers of recesses and different width/diameter ratios for identical recess geometry. In this part of ISO 12168 only bearings without oil drainage grooves between the recesses are taken into account. As compared to bearings with oil drainage grooves, this type needs less power with the same stiffness behaviour.

The oil is fed to each bearing recess by means of a common pump with constant pump pressure (system p_{en} = constant) and via preceding linear restrictors (e.g. in the form of capillaries).

The calculation procedures listed in this part of ISO 12168 enable the user to calculate and assess a given bearing design as well as to design a bearing as a function of some optional parameters. Furthermore, this part of ISO 12168 contains the design of the required lubrication system including the calculation of the restrictor data.

Plain bearings — Hydrostatic plain journal bearings without drainage grooves under steady-state conditions —

Part 1:
Calculation of oil-lubricated plain journal bearings without drainage grooves

1 Scope

This part of ISO 12168 applies to hydrostatic plain journal bearings under steady-state conditions.

In this part of ISO 12168 only bearings without oil drainage grooves between the recesses are taken into account.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this part of ISO 12168. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this part of ISO 12168 are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 3448:1992, *Industrial liquid lubricants — ISO viscosity classification*

ISO 12168-2:2001, *Plain bearings — Hydrostatic plain journal bearings without drainage grooves under steady-state conditions — Part 2: Characteristic values for the calculation of oil-lubricated plain journal bearings without drainage grooves*

3 Bases of calculation and boundary conditions

Calculation within the meaning of this part of ISO 12168 is the mathematical determination of the operational parameters of hydrostatic plain journal bearings as a function of operating conditions, bearing geometry and lubrication data. This means the determination of eccentricities, load-carrying capacity, stiffness, required feed pressure, oil flow rate, frictional and pumping power, and temperature rise. Besides the hydrostatic pressure build-up, the influence of hydrodynamic effects is also approximated.

Reynolds' differential equation furnishes the theoretical bases for the calculation of hydrostatic bearings. In most practical cases of application it is, however, possible to arrive at sufficiently exact results by approximation.

The approximation used in this part of ISO 12168 is based on two basic equations for describing the flow via the bearing lands, which can be derived from Reynolds' differential equation when special boundary conditions are observed. The Hagen-Poiseuille law describes the pressure flow in a parallel clearance gap and the Couette equation the drag flow in the bearing clearance gap caused by shaft rotation. A detailed presentation of the theoretical background of the calculation procedure is included in annex A.

The following important premises apply to the calculation procedures described in this part of ISO 12168:

a) all lubricant flows in the lubrication clearance gap are laminar;

b) the lubricant adheres completely to the sliding surfaces;

c) the lubricant is an incompressible Newtonian fluid;

d) in the whole lubrication clearance gap, as well as in the preceding restrictors, the lubricant is partially isoviscous;

e) a lubrication clearance gap completely filled with lubricant is the basis for the frictional behaviour;

f) fluctuations of pressure in the lubricant film normal to the sliding surfaces do not take place;

g) half bearing and journal have completely rigid surfaces;

h) the radii of curvature of the surfaces in relative motion to each other are large in comparison to the lubricant film thickness;

i) the clearance gap height in the axial direction is constant (axial parallel clearance gap);

j) the pressure over the recess area is constant;

k) there is no motion normal to the sliding surfaces.

With the aid of the above-mentioned approximation equations, all parameters required for the design or calculation of bearings can be determined. The application of the similarity principle results in dimensionless similarity values for load-carrying capacity, stiffness, oil flow rate, friction, recess pressures, etc.

The results indicated in this part of ISO 12168 in the form of tables and diagrams are restricted to operating ranges common in practice for hydrostatic bearings. Thus the range of the bearing eccentricity (displacement under load) is limited to $\varepsilon = 0$ to 0,5.

Limitation to this eccentricity range means a considerable simplification of the calculation procedure as the load-carrying capacity is a nearly linear function of the eccentricity. However, the applicability of this procedure is hardly restricted as in practice eccentricities $\varepsilon > 0,5$ are mostly undesirable for reasons of operational safety. A further assumption for the calculations is the approximated optimum restrictor ratio [1] $\xi = 1$ for the stiffness behaviour.

As for the outside dimensions of the bearing, this part of ISO 12168 is restricted to the range bearing width/bearing diameter $B/D = 0,3$ to 1 which is common in practical cases of application. The recess depth is larger than the clearance gap height by the factor 10 to 100. When calculating the friction losses, the friction loss over the recess in relation to the friction over the bearing lands can generally be neglected on account of the above premises. However, this does not apply when the bearing shall be optimized with regard to its total power losses.

To take into account the load direction of a bearing, difference is made between the two extreme cases, load in the direction of recess centre and load in the direction of land centre.

Apart from the afore-mentioned boundary conditions, some other requirements are to be mentioned for the design of hydrostatic bearings in order to ensure their functioning under all operating conditions. In general, a bearing shall be designed in such a manner that a clearance gap height of at least 50 % to 60 % of the initial clearance gap height is assured when the maximum possible load is applied. With this in mind, particular attention shall be paid to misalignments of the shaft in the bearing due to shaft deflection which may result in contact between shaft and bearing edge and thus in damage of the bearing. In addition, the parallel clearance gap required for the calculation is no longer present in such a case.

As the shaft is in contact with the bearing lands when the hydrostatic pressure is switched off, it might be necessary to check the contact zones with regard to rising surface pressures.

It shall be assured that the heat originating in the bearing does not lead to a non-permissible rise in the temperature of the oil.

If necessary, a means of cooling the oil shall be provided. Furthermore, the oil shall be filtered in order to avoid choking of the capillaries and damage to the sliding surfaces.

Low pressure in the relieved recess shall also be avoided, as this leads to air being drawn in from the environment and this would lead to a decrease in stiffness (see 5.7).

4 Symbols, terms and units

See Table 1.

Table 1 — Symbols, terms and units

Symbol	Term	Unit
a	Inertia factor	1
A_{lan}	Land area	m^2
A^*_{lan}	Relative land area $\left(A^*_{\mathrm{lan}} = \frac{A_{\mathrm{lan}}}{\pi \times B \times D}\right)$	1
A_{p}	Recess area	m^2
b	Width perpendicular to the direction of flow	m
b_{ax}	Width of axial outlet $\left[b_{\mathrm{ax}} = \frac{\pi \times D}{Z}\right]$	m
b_{c}	Width of circumferential outlet $(b_{\mathrm{c}} = B - l_{\mathrm{ax}})$	m
B	Bearing width	m
c	Stiffness coefficient	N/m
c_{p}	Specific heat capacity of the lubricant (p = *constant*)	J/kg·K
C_{R}	Radial clearance $\left[C_{\mathrm{R}} = (D_{\mathrm{B}} - D_{\mathrm{J}})/2\right]$	m
d_{cp}	Diameter of capillaries	m
D	Bearing diameter (D_{J}: shaft; D_{B}: bearing; $D \approx D_{\mathrm{J}} \approx D_{\mathrm{B}}$)	m
e	Eccentricity (shaft displacement)	m
F	Load-carrying capacity (load)	N
F^*	Characteristic value of load-carrying capacity [$F^* = F/(B \times D \times p_{\mathrm{en}})$]	1
F^*_{eff}	Characteristic value of effective load-carrying capacity	1
$F^*_{\mathrm{eff,0}}$	Characteristic value of effective load-carrying capacity for $N = 0$	1
h	Local lubricant film thickness (clearance gap height)	m
h_{min}	Minimum lubricant film thickness (minimum clearance gap height)	m
h_{p}	Depth of recess	m
K_{rot}	Speed-dependent parameter	1
l	Length in the direction of flow	m

Table 1 — (*continued*)

Symbol	Term	Unit
l_{ax}	Axial land length	m
l_c	Circumferential land length	m
l_{cp}	Length of capillaries	m
N	Rotational frequency (speed)	s^{-1}
p	Recess pressure, general	Pa
$\overline{p}$	Specific bearing load $\left[\overline{p} = F/(B \times D)\right]$	Pa
p_{en}	Feed pressure (pump pressure)	Pa
p_i	Pressure in recess i	Pa
$p_{i,0}$	Pressure in recess i, when $\varepsilon = 0$	Pa
P^*	Power ratio ($P^* = P_f/P_p$)	1
P_f	Frictional power	W
P_p	Pumping power	W
P_{tot}	Total power ($P_{tot} = P_p + P_f$)	W
P_{tot}^*	Characteristic value of total power	1
Q	Lubricant flow rate (for complete bearing)	m^3/s
Q^*	Lubricant flow rate parameter	1
R_{cp}	Flow resistance of capillaries	$Pa \cdot s/m^3$
$R_{lan,ax}$	Flow resistance of one axial land $\left(R_{lan,ax} = \dfrac{12 \times \eta \times l_{ax}}{b_{ax} \times C_R^3}\right)$	$Pa \cdot s/m^3$
$R_{lan,c}$	Flow resistance of one circumferential land $\left(R_{lan,c} = \dfrac{12 \times \eta \times l_c}{b_c \times C_R^3}\right)$	$Pa \cdot s/m^3$
$R_{P,0}$	Flow resistance of one recess, when $\varepsilon = 0$, $\left(R_{P,0} = 0{,}5 R_{lan,ax}\right)$	$Pa \cdot s/m^3$
Re	Reynolds number	1
So	Sommerfeld number	1
T	Temperature	°C
ΔT	Temperature difference	K
u	Flow velocity	m/s
U	Circumferential speed	m/s
$\overline{w}$	Average velocity in restrictor	m/s
Z	Number of recesses	1
α	Position of 1st recess related to recess centre	rad
β	Attitude angle of shaft	°
γ	Exponent in viscosity formula	1
ε	Relative eccentricity ($\varepsilon = e/C_R$)	1

Table 1 — (*continued*)

Symbol	Term	Unit
η	Dynamic viscosity	Pa·s
κ	Resistance ratio $\left(\kappa = \frac{R_{\text{lan,ax}}}{R_{\text{lan,c}}} = \frac{l_{\text{ax}} \times b_{\text{c}}}{l_{\text{c}} \times b_{\text{ax}}}\right)$	1
ξ	Restrictor ratio $\left(\xi = \frac{R_{\text{cp}}}{R_{\text{P,0}}}\right)$	1
π_{f}	Relative frictional pressure $\left(\pi_{\text{f}} = \frac{\eta_{\text{B}} \times \omega}{p_{\text{en}} \times \psi^2}\right)$	1
ρ	Density	kg/m^3
τ	Shearing stress	N/m^2
φ	Angular coordinate	rad
ψ	Relative bearing clearence $\left(\psi = \frac{2 \times C_{\text{R}}}{D}\right)$	1
ω	Angular velocity $(\omega = 2 \times \pi \times N)$	s^{-1}

5 Method of calculation

5.1 General

This part of ISO 12168 covers the calculation as well as the design of hydrostatic plain journal bearings. In this case, calculation is understood to be the verification of the operational parameters of a hydrostatic bearing with known geometrical and lubrication data. In the case of a design calculation, with the given methods of calculation it is possible to determine the missing data for the required bearing geometry, the lubrication data and the operational parameters on the basis of a few initial data (e.g. required load-carrying capacity, stiffness, rotational frequency).

In both cases, the calculations are carried out according to an approximation method based on the Hagen-Poiseuille and the Couette equations, mentioned in clause 3. The bearing parameters calculated according to this method are given as relative values in the form of tables and diagrams as a function of different parameters. The procedure for the calculation or design of bearings is described in 5.2 to 5.7. This includes the determination of different bearing parameters with the aid of the given calculation formulae or the tables and diagrams. The following calculation items are explained in detail:

a) determination of load-carrying capacity with and without consideration of shaft rotation;

b) calculation of lubricant flow rate and pumping power;

c) determination of frictional power with and without consideration of losses in the bearing recesses;

d) procedure for bearing optimization with regard to minimum total power loss.

For all calculations, it shall be checked in addition whether the important premise of laminar flow in the bearing clearance gap, in the bearing recess and in the capillary is met. This is checked by determining the Reynolds numbers. Furthermore, the portion of the inertia factor in the pressure differences shall be kept low at the capillary (see A.3.2.2).

If the boundary conditions defined in clause 3 are observed, this method of calculation yields results with deviations which can be neglected for the requirements of practice, in comparison with an exact calculation by solving the Reynolds differential equation.

5.2 Load-carrying capacity

Unless indicated otherwise, it is assumed in the following that capillaries with a linear characteristic are used as restrictors and that the restrictor ratio is $\xi = 1$. Furthermore, difference is only made between the two cases “load in direction of recess centre” and “load in direction of land centre”. For this reason, it is no longer mentioned in each individual case that the characteristic values are a function of the three parameters “restrictor type”, “restrictor ratio” and “load direction relative to the bearing”.

Even under the above mentioned premises, the characteristic value of load carrying capacity

$$F^* = \frac{F}{B \times D \times p_{en}} = \frac{\overline{p}}{p_{en}} \tag{1}$$

still depends on the following parameters:

— the number of recesses Z;

— the width/diameter ratio B/D;

— the relative axial land width l_{ax}/B;

— the relative land width in circumferential direction l_c/B;

— the relative journal eccentricity ε;

the relative frictional pressure $\pi_f = \dfrac{\eta_B \times \omega}{p_{en} \times \psi^2}$ (2)

NOTE The Sommerfeld number, So, common with hydrodynamic plain journal bearings can be set up as follows:

$$So = \frac{\overline{p} \times \psi^2}{\eta_B \times \omega} = \frac{F^*}{\pi_f}$$

In Figures 1 and 2 of ISO 12168-2:2001, the functions $F^*(\varepsilon, \pi_f)$ and $\beta(\varepsilon, \pi_f)$ are represented for $Z = 4$, $\xi = 1$, $B/D = 1$, $l_{ax}/B = 0,16$, $l_c/B = 0,26$, i.e. restriction by means of capillaries, load in direction of centre of bearing recess.

These figures represent a comparison between the approximation and the more precise solution by means of Reynolds equation. Further, the influence of rotation on the characteristic value of the load-carrying capacity and on the attitude angle can be realized.

For the calculation of a geometrically similar bearing, it is possible to determine the minimum lubricant film thickness when values are given e.g. for F, B, D, p_{en}, ω, ψ and η_B (determination of η_B according to 5.6, if applicable):

All parameters are given for the determination of F^* according to equation (1) and π_f according to equation (2). For this geometry, the relevant values for ε and β can be taken from Figures 1 and 2 in ISO 12168-2:2001 and thus $h_{min} = C_R(1 - \varepsilon)$.

According to the approximation method described in annex A, this results in a dependence of the characteristic value of effective load-carrying capacity formed with the so-called “effective bearing width” $B - l_{ax}$

$$F^*_{\text{eff}} = \frac{F}{(B - l_{\text{ax}}) \times D \times p_{\text{en}}} \tag{3}$$

on lesser parameters. In the case of this definition, espacially the width/diameter ratio B/D can be dropped as parameter. Maintained are the number of recesses Z, the resistance ratio:

$$\kappa = \frac{R_{\text{lan,ax}}}{R_{\text{lan,c}}} = \frac{l_{\text{ax}} \times b_{\text{c}}}{l_{\text{c}} \times b_{\text{ax}}} = \left(\frac{B}{D}\right)^2 \times \frac{Z}{\pi} \times \frac{\frac{l_{\text{ax}}}{B} \times \left(1 - \frac{l_{\text{ax}}}{B}\right)}{l_{\text{c}}/D} \tag{4}$$

the relative journal eccentricity ε, and the speed dependent parameter determining the ratio of hydrodynamic to hydrostatic pressure build-up:

$$K_{\text{rot}} = \pi_{\text{f}} \times \kappa \times \xi \frac{l_{\text{c}}}{D} = \frac{\eta_{\text{B}} \times \omega}{p_{\text{en}} \psi^2} \times \kappa \times \xi \frac{l_{\text{c}}}{D} \tag{5}$$

If, in addition, advantage is taken of the fact that the function $F^*_{\text{eff}}(\varepsilon)$ is nearly linear for $\varepsilon \leqslant 0{,}5$, then it is practically sufficent to know the function $F^*_{\text{eff}}(\varepsilon = 0{,}4) = f(Z, \kappa, K_{\text{rot}})$ for the calculation of the load carrying capacity.

In Figure 3 of ISO 12168-2:2001, the function $F^*_{\text{eff,0}}(\varepsilon = 0{,}4) = F^*_{\text{eff}}(\varepsilon = 0{,}4); (K_{\text{rot}} = 0) = f(Z, \kappa)$ and in Figure 4 the function $\frac{F^*_{\text{eff}}}{F^*_{\text{eff,0}}} = f(Z = 4, \kappa, K_{\text{rot}})$ are presented for the case “load in direction of recess centre”. The hydrodynamically conditioned increase of the load carrying capacity can be recognized well when presented in such manner.

If, e.g, Z and all parameters are given for the determination of F^*_{eff} according to equation (3), κ according to equation (4) and K_{rot} according to equation (5), then the minimum lubricant film thickness developing during operation can be determined.

After having calculated κ and K_{rot}, $F^*_{\text{eff,0}}(\varepsilon = 0{,}4)$ is taken from Figure 3 of ISO 12168-2:2001 and $F^*_{\text{eff}} / F^*_{\text{eff,0}}(\varepsilon = 0{,}4)$ from Figure 4 of ISO 12168-2, F^*_{eff} is calculated according to equation 3 and with

$$\varepsilon = \frac{0{,}4 \times F^*_{\text{eff}}}{\left(F^*_{\text{eff}} / F^*_{\text{eff,0}}\right) \times (\varepsilon = 0{,}4) \times F^*_{\text{eff,0}}(\varepsilon = 0{,}4)}$$

the minimum lubricant film thickness $h_{\text{min}} = C_{\text{R}}(1 - \varepsilon)$ is obtained.

5.3 Lubricant flow rate and pumping power

The characteristic value for the lubricant flow rate is given by

$$Q^* = \frac{Q \times \eta_{\text{b}}}{C_{\text{R}}^3 \times p_{\text{en}}} \tag{6}$$

It depends only slightly on the relative journal eccentricity ε, the load direction relative to the bearing and the relative frictional pressure π_{f} or the speed dependent parameter K_{rot}.

By approximation, the lubricant flow rate can be calculated as follows (see also A.3.3):

$$Q^*(\varepsilon \leqslant 0{,}5) \approx Q^*(\varepsilon = 0) = \frac{1}{1+\xi} \times \frac{\pi}{6(B/D)} \times \frac{1}{l_{ax}/B} \tag{7}$$

where $\xi = \frac{R_{cp}}{R_{P,0}}$ and $R_{P,0} = \frac{6 \times \eta_B \times l_{ax}}{b_{ax} \times C_R^3}$.

The flow resistance of the capillaries according to A.3.2.2 is given by

$$R_{cp} = \frac{128 \times \eta_{cp} \times l_{cp}}{\pi \times d^4_{cp}} \times (1 + a)$$

with the non-linear portion (inertia factor):

$$a = \frac{1{,}08}{32} \times \frac{4 \times Q \times \rho}{\eta_{cp} \times l_{cp} \times Z}$$

By converting equation (6), the lubricant flow rate can be calculated when the parameters η_B, C_R, p_{en}, ξ, B/D, and l_{ax}/B are given.

For optimized bearings, Q^* shall be taken from Table 1 of ISO 12168-2:2001. The pumping power, without considering the pump efficiency, is given by

$$P_p = Q \times p_{en} = Q^* \times \frac{p_{en}^2 \times C_R^3}{\eta_B} \tag{8}$$

According to the approximation method, Q^* is again determined according to equation (7), thus it is the characteristic value of both flow rate and pumping power.

5.4 Frictional power

The characteristic value for the frictional power is given by

$$P_f^* = \frac{P_f \times C_R}{\eta_B \times U^2 \times B \times D} \tag{9}$$

Friction is generated in the lands as well as in the recess area. The land area related to the total surface of the bearing $\pi \times B \times D$ is given by

$$A_{lan}^* = 2 \times \frac{l_{ax}}{B} + \frac{Z}{\pi} \times \frac{l_c}{D} \times \left(1 - 2 \times \frac{l_{ax}}{B}\right)$$

According to the approximation method, the characteristic value for the frictional power in the land area is given by

$$P_{f,lan}^* = \frac{\pi}{\sqrt{1 - \varepsilon^2}} \times A_{lan}^*,$$

and in the recess area by

$$P^*_{f,P} = \pi \times 4 \times \frac{C_R}{h_p} \times (1 - A^*_{lan}).$$

Thus the characteristic value for the total amount of friction is given by

$$P^*_f = \pi \times A^*_{lan} \times \left[\frac{1}{\sqrt{1 - \varepsilon^2}} + \frac{4 \times C_R}{h_p} \times \left(\frac{1}{A^*_{lan}} - 1 \right) \right] \tag{10}$$

The actual frictional power is obtained by converting equation (9) as follows

$$P_f = P_f{}^* \times \frac{\eta_B \times U^2 \times B \times D}{C_R}$$

5.5 Optimization

When optimizing according to the power consumption, the total power loss, i.e. the sum of pumping and frictional power, is minimized. According to 5.3 and 5.4, the total power is given by

$$P_{tot} = P_p + P_f = Q^* \times \frac{p_{en}^2 \times C_R^3}{\eta_B} + P_f^* \times \frac{\eta_B \times U^2 \times B \times D}{C_R}$$

With equations (1) and (2) this can be written as follows

$$P_{tot} = F \times \omega \times C_R \times \frac{Q^*}{4 \times (B/D) \times F^* \times \pi_f} \times \left(1 + \frac{P_f}{P_p} \right). \tag{11}$$

Following a proposal of Vermeulen [2], the ratio of frictional to pumping power is introduced as an optional parameter P^* and designated with ($P^* = P_f/P_p$). Thus the characteristic value for the total power loss is given by

$$P^*_{tot} = \frac{P_{tot}}{F \times \omega \times C_R} = \frac{Q^* \times (1 + P^*)}{4 \times (B/D) \times F^* \times \pi_f} \tag{12}$$

Serial calculations have shown that the power minimum which can be obtained in the relatively wide range $P^* = 1$ to 3 depends only slightly on the chosen power ratio P^*. It is proposed to carry out an approximated optimization with the mean value $P^* = 2$.

The relative frictional pressure in equation (12) cannot be chosen freely as it is linked to the chosen power ratio P^*:

$$P^* = \pi_f^2 \times 4 \times \frac{B}{D} \times \frac{P_f^*}{Q^*} \quad \text{or} \quad \pi_f = \frac{1}{2} \sqrt{\frac{P^* \times Q^*}{P_f^* \times \frac{B}{D}}} \tag{13}$$

When P^*, B/D, ε, h_p/C_R and ξ are given, the characteristic value of total power according to equation (12) becomes a function of Z, l_{ax}/B, and l_c/B.

In Figures 5 and 6 of ISO 12168-2:2001, P^*_{tot} for $P^* = 2$, $Z = 4$, $\xi = 1$, $B/D = 1$, $\varepsilon = 0{,}4$ with or without friction in the recess ($h_p/C_R = 40$) is presented as a function of the geometrical parameters l_{ax}/B and l_c/B.

In Figures 7 to 12 of ISO 12168-2:2001, P^*_{tot} for P^* = 2, ξ = 1, ε = 0,4, h_p = 40 C_R, is presented for different B/D and Z as a function of l_{ax}/B and l_c/B, taking into account friction in the recesses. The land widths l_{ax}/B and l_c/B, where the total power is reduced to a minimum, result from these figures.

The optimum land widths and the associated values for B/D = 1 to 0,3 as well as the numbers of recesses from Z = 4 up to 10 obtained by this are given in Table 1 of ISO 12168-2:2001.

With decreasing width, P^*_{tot}, and thus the total need of power, increases. For high rotational frequencies and a given wide diameter it may, however, be advantageous to use a plain bearing with smaller bearing width.

In the case where the shaft is at a standstill or rotating very slowly, the optimization method with P^* = 1 to 3 can no longer be applied, see [2]. In this case, the pumping power has to be minimized and thus relatively wide lands are obtained. Therefore, the approximation method also fails and the Reynolds differential equation is to be solved by means of a finite method.

For a bearing with Z = 4, B/D = 1 and ε = 0,4 the following values are obtained under optimum conditions according to [6]:

l_{ax}/B = 0,25; l_c/B = 0,4; F^* = 0,202; Q^* = 1,003.

In Figures 13 to 18 of ISO 12168-2:2001 the characteristic value of effective load carrying capacity $F^*_{eff,0}$ is given according to the results of [1] for various numbers of recesses as a function of ε with κ as the parameter for load on centre of recess and centre of land.

5.6 Temperatures and viscosities

When ε = 0, the heating in the capillaries due to dissipation (heat exchange between lubricant and environment is not considered here) is given by:

$$\Delta T_{cp} = \frac{p_{en} - p}{c_p \times \rho} = \frac{p_{en}}{c_p \times \rho} \times \frac{\xi}{1+\xi}$$

that in the bearing, again with ε = 0, as follows:

$$\Delta T_B = \frac{p}{c_p \times \rho} + \frac{P_f}{c_p \times \rho \times Q} = \frac{p_{en}}{c_p \times \rho} \times \left(\frac{1}{1+\xi} + P^* \right).$$

Thus the mean temperature in the capillaries is given by

$$T_{cp} = T_{en} + \frac{1}{2} \times \Delta T_{cp} \qquad (14)$$

and that in the bearing by

$$T_B = T_{en} + \Delta T_{cp} + \frac{1}{2} \times \Delta T_B. \qquad (15)$$

It is assumed for the effective viscosities in the capillaries and bearing respectively:

$\eta_{cp} = \eta(T_{cp})$; $\eta_B = \eta(T_B)$.

If the dependence of the viscosity on temperature is not completely known, the viscosities η_{cp} and η_B can be approximated following the statement of Reynolds'. A precondition is that two viscosities η_1 and η_2 are known at two temperatures T_1 and T_2, which should be close to the temperatures T_{cp} and T_B to be expected.

$$\eta_{cp} = \eta_1 \times \exp[-\gamma \times (T_{cp} - T_1)] \ ; \ \eta_B = \eta_1 \times \exp[-\gamma \times (T_B - T_1)]$$

$$\text{where } \gamma = \frac{1}{T_2 - T_1} \times \ln \frac{\eta_1}{\eta_2} \tag{16}$$

If only the viscosity class according to ISO 3448 is known, then the course of viscosity for common lubrication oils having a viscosity index of about 100 can be calculated only on the basis of the nominal viscosity η_{40} (dynamic viscosity at 40 °C):

$$\eta(T) = \eta_{40} \times \exp\left[160 \times \ln\left(\frac{\eta_{40}}{0{,}18 \times 10^{-3}}\right) \times \left(\frac{1}{T + 95} - \frac{1}{135}\right)\right] \tag{17}$$

Temperature T is to be taken in °C. The dynamic viscosity η_{40} is obtained by multiplying the kinematic viscosity ν_{40}, based on the viscosity classes, by the density ρ. If this value is not exactly known, it can be calculated by approximation with ρ = 900 kg/m^3.

Equation (17) is based on the statement of Vogel and empirically determined constants of Cameron and Rost and was transposed by Rodermund [3] to the nominal viscosity at 40 °C.

5.7 Minimum pressure in recesses

With high rotational frequencies and high K_{rot} values according to equation (5) the pressure in the recess p_{min} on the no-load side of the plain bearing may decrease to zero, whereas the pressure in the recess p_{max} on the load side may become greater than p_{en}. The minimum recess pressure as well as F^* depends on several variables. For the ratio the following applies

$$\frac{p_{min}}{p_{en}} \ (Z, \kappa, K_{rot})$$

In Figure 19 of ISO 12168-2:2001 the minimum relative recess pressure over K_{rot} is shown for Z = 4, ε = 0,4 and 3 κ-values.

Annex A
(normative)

Description of the approximation method for the calculation of hydrostatic plain journal bearings

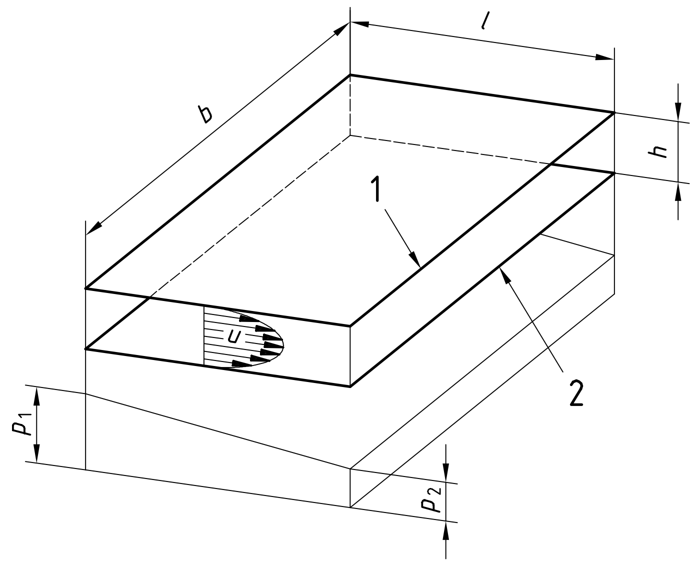
A.1 Introduction

The calculation is based on an approximation method leading to rather exact results especially in such cases where small lands are provided (e.g. shaft rotating at high speed). In case of wider lands the Reynolds differential equation shall be solved, e.g. by means of numerical difference equations.

A.2 Fundamentals

A.2.1 General

The approximation method assumes laminar flow, free of inertia, and uses two basic equations for the flows via the lands (see Figures A.1 and A.2).

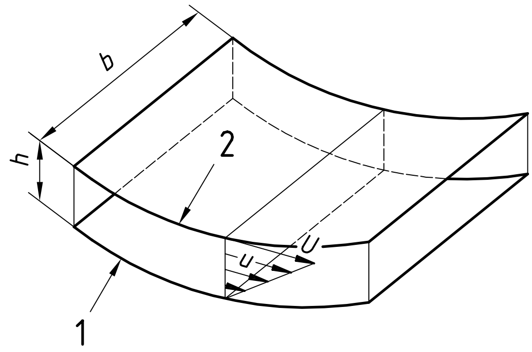


Key

1 Bearing

2 Shaft

Figure A.1 — Pressure flow between parallel plates



Key

1 Bearing

2 Shaft

Figure A.2 — Drag flow due to shaft rotation

A.2.2 Hagen-Poiseuille equation

Pressure flow between parallel plates: ($b >> h$)

$$Q = \frac{(p_2 - p_1) \times b \times h^3}{12 \times \eta \times l}$$

A.2.3 Couette equation

Drag flow due to shaft rotation:

$$Q = b \times \frac{U \times h}{2}$$

A.2.4 Further assumptions

a) The pressure is constant over the recess area.

b) The viscosity in the bearing and in the restrictors is constant.

c) Shaft and bearing are rigid, their axes always parallel.

d) That for the calculation of the lubricant flow rates, the outlet width extends up to the centre of the adjacent lands and the pressure drop over the outlet length is linear.

e) That for the calculation of the load effects, the pressure in the recesses spreads up to the centre of the adjacent lands.

A.3 Calculation

A.3.1 General

At first, the pressures in the recesses are calculated with the aid of the continuity equation for a certain shaft position, defined by

e = eccentricity

ε = e/C_R

β = attitude angle

All other parameters are derived from the pressures in the recesses.

The calculation is iterative as the attitude angle β is not known in the beginning. This angle is to be varied until the result of the pressures in the recesses and the load have the same direction (see Figure A.3).

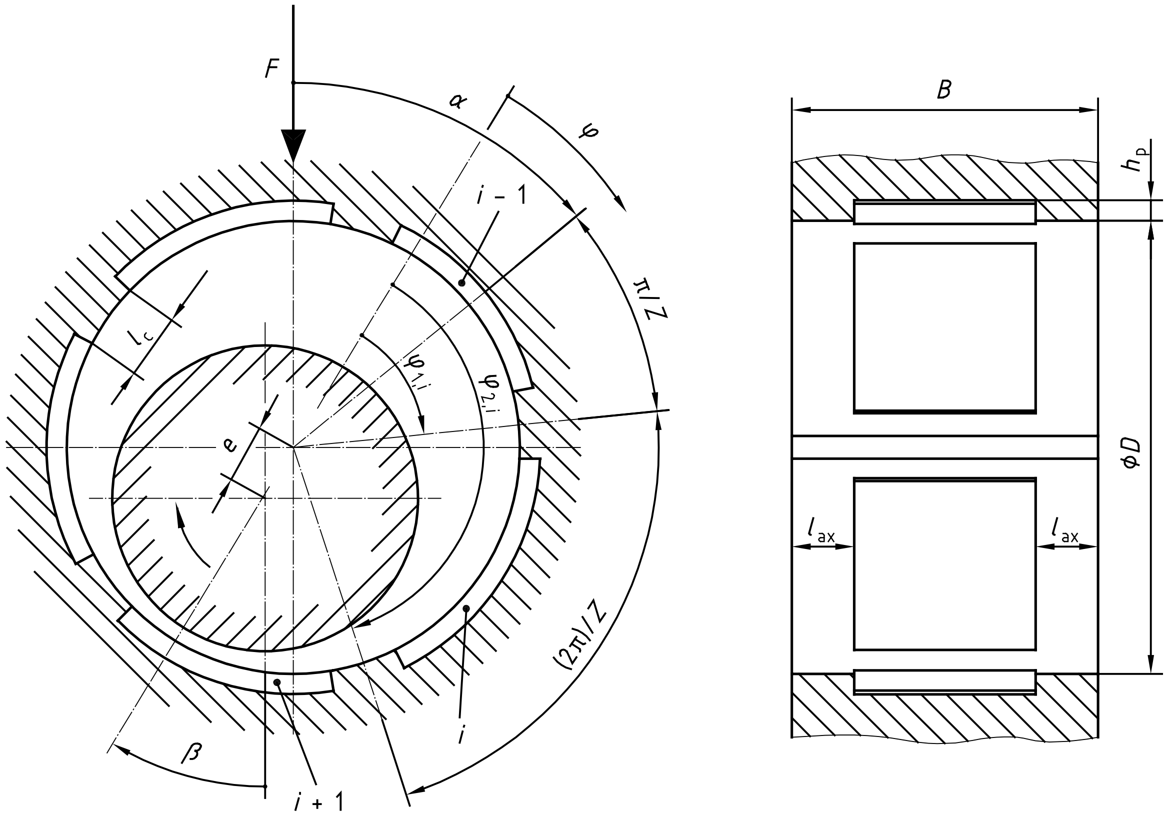


Figure A.3 — Bearing geometry

In principle, a vertical load is assumed for the calculation. However, this is no restriction as we can assume the bearing is to be mounted appropriately for other directions of load.

The recess i (i = 1, 2, … Z) starts at the angle $\varphi_{1,i}$ and ends at the angle $\varphi_{2,i}$.

The centre of the first recess is situated at α. The initial angle and the end angle are

$$\varphi_{1,i} = \alpha - \beta + \frac{2 \times \pi}{Z} \times \left(i - \frac{3}{2} \right)$$

$$\varphi_{2,i} = \alpha - \beta + \frac{2 \times \pi}{Z} \times \left(i - \frac{1}{2} \right)$$

The film thickness h changes in the land area according to $h = C_R(1 + \varepsilon \times \cos \varphi)$

A.3.2 Pressures in the recesses

A.3.2.1 The continuity principle is used for each recess. An equation covering the three pressures p_{i-1}, p_i and p_{i+1} applies to each recess i. This results in a system of equations furnishing all pressures (see Figure A.4).

A.3.2.2 The lubricant flow rate via preresistance ($\varepsilon = 0$) is given by

$$\frac{Q}{Z} = \frac{\left(p_{\text{en}} - p\right)^{k}}{R_{\text{cp}}}$$

$k = 1$ corresponds to a linear resistance law.

E.g. of a capillary with laminar flow:

$$Re_{\text{cp}} = \frac{\overline{w} \times d_{\text{cp}} \times \rho}{\eta_{\text{cp}}} < 2\,300$$

and with negligible portion of the term of inertia $\frac{\rho}{2} \times \overline{w}^2$.

$k = 1/2$ corresponds to a square-law dependency, e.g. of an orifice, the flow coefficient of which can be regarded as independent of the Reynolds number.

When dimensioning a capillary, the portion of the term of inertia shall be kept low and, if applicable, be taken into account. According to the theory of Schiller [4] the pressure drop necessary to generate the velocity $\overline{w} = \frac{4 \times Q}{Z \times \pi \times d_{\text{cp}}^2}$ at a properly rounded inlet (rounding off radius > 0,3 × d_{cp}) is $\Delta p_{\text{en}} = 2{,}16 \times \frac{\rho}{2} \times \overline{w}^2$

The flow resistance of the capillaries is then

$$R_{\text{cp}} = \frac{p_{\text{en}} - p_{\text{i}}}{\frac{Q}{Z}} = \frac{\Delta p_{\text{lam}}}{\frac{Q}{Z}} + \frac{\Delta p_{\text{en}}}{\frac{Q}{Z}} = \frac{128 \times \eta_{\text{cp}} \times l_{\text{cp}}}{\pi \times d_{\text{cp}}^4} + \frac{2{,}16 \times \frac{\rho}{2} \times \overline{w}^2}{\overline{w} \times \frac{\pi}{4} \times d_{\text{cp}}^2}$$

$$R_{\text{cp}} = \frac{128 \times \eta_{\text{cp}} \times l_{\text{cp}}}{\pi \times d_{\text{cp}}^4} \times (1 + a) \text{ where } Re_{\text{cp}} = \frac{4 \times Q \times \rho}{Z \times \pi \times d \times \eta_{\text{cp}}} \text{ and}$$

$$a = \frac{1{,}08}{32} \times Re_{\text{cp}} \times \frac{d_{\text{cp}}}{l_{\text{cp}}} = \frac{1{,}08}{32} \times \frac{4 \times Q \times \rho}{Z \times \eta_{\text{cp}} \times l_{\text{cp}} \times \pi}$$

The portion of the non-linear term a (inertia factor) has the effect that the exponent $k < 1$ in the above-mentioned equation for Q/Z. Exponent k can be calculated by approximation as follows:

$$k = \frac{1 + a}{1 + 2 \times a}$$

Without greater errors it is permitted to take a = 0,1 to 0,2 and to calculate with exponent $k = 1$. With regard to the different lubricant flow rates in the particular recesses ($\varepsilon \neq 0$), a Reynolds number of Re_{cp} = 1 000 to 1 500 shall not be exceeded.

A.3.2.3 The volume flow from recess i in the axial direction is

$$Q_{ax,i} = 2 \times \int_{\varphi'_{1,i}}^{\varphi'_{2,i}} \frac{h^3}{12 \times \eta_B} \times \frac{P_i}{l_{ax}} \times \frac{D}{2} \times d\varphi$$

h is not constant due to the shaft eccentricity.

If

$$a_i = \int_{\varphi'_{1,i}}^{\varphi'_{2,i}} \frac{h^3}{C_R^3} d\varphi = \int_{\varphi'_{1,i}}^{\varphi'_{2,i}} (1 + \varepsilon \cos \varphi)^3 \times d\varphi$$

$$= \left[(\varphi'_2 - \varphi'_1) \times \left(1 + \frac{3}{2} \times \varepsilon^2\right) + (\sin\varphi'_2 - \sin\varphi'_1) \times (3\varepsilon + \varepsilon^3) + \frac{3}{4} \times \varepsilon^2 \times (\sin 2\varphi'_2 - \sin 2\varphi'_1) - \frac{\varepsilon^3}{3} \times (\sin^3\varphi'_2 - \sin^3\varphi'_1) \right]_i$$

then

$$Q_{ax,i} = \frac{C_R^3 \times D}{12 \times \eta_B \times l_{ax}} \times a_i \times p_i$$

A.3.2.4 When the volume flow rate in the circumferential direction is calculated, then a flow between parallel plates with a film thickness of $\bar{h}_i = h\,(\varphi_{2,i})$ is assumed as an approximation.

For the volume flow from recess i to recess i +1, one obtains the following results:

$$Q_{c,i+1} = \frac{\bar{h}^3 \times b_c}{12 \times \eta_B \times l_c} \times (p_i - p_{i+1}) + \frac{U \times \bar{h}_i \times b_c}{2}$$

where $\bar{h}_i = C_R \times (1 + \varepsilon \times \cos \varphi_{2,i})$ and $U = \pi \times D \times N$

By analogy, the following applies to the flow rate from recess $i - 1$ to recess i:

$$Q_{c,i-1} = \frac{\bar{h}_{i-1}^3 \times b_c}{12 \times \eta_B \times l_c} \times (p_{i-1} - p_1) + \frac{U \times \bar{h}_{i-1} \times b_c}{2}$$

A.3.2.5 According to Figure A.4 the continuity equation for recess i results in

$$Q_{R,i} = Q_{ax,i} + Q_{c,i+1} - Q_{c,i-1}$$

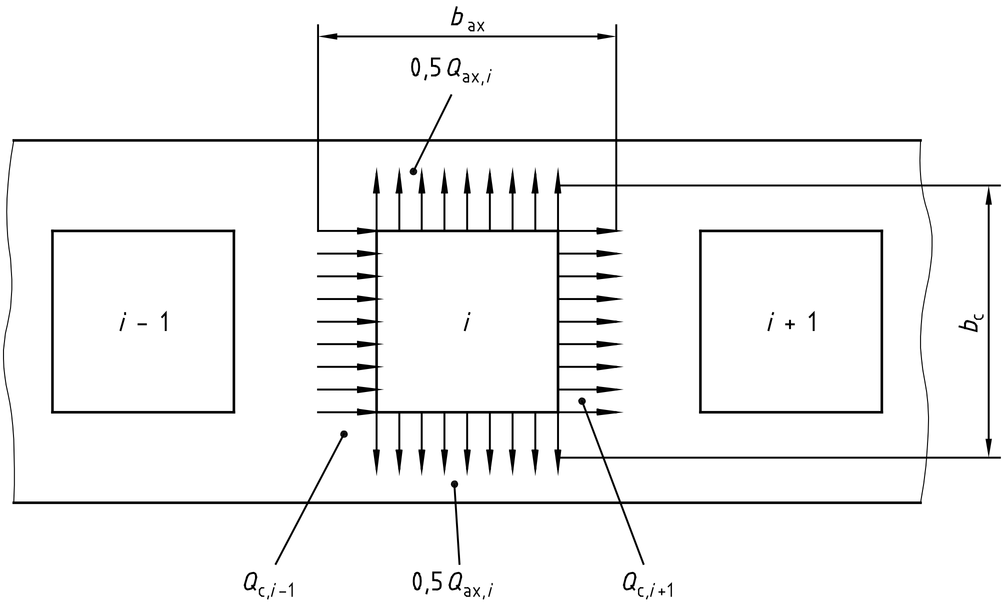


Figure A.4 — Volume flows for one recess

If $f_i = 1 + \varepsilon \times \cos \varphi_{2,i}$

$$\omega = 2 \times \pi \times N = \frac{2 \times U}{D}$$

$$p_i^* = \frac{p_i}{p_{en}}:$$

$$K_{rot} = \frac{\eta_B \times \omega \times \xi \times \kappa \times l_c}{p_{en} \times \psi^2 \times D} \text{ = speed dependent parameter}$$

where

$$\psi = \frac{2 \times C_R}{D} \text{ = relative bearing clearance}$$

$$\kappa = \frac{R_{P,ax}}{R_{P,c}} = \frac{l_{ax} \times b_c}{l_c \times b_{ax}} \text{ = resistance ratio}$$

$$R_{P,0} = \frac{R_{P,ax}}{2}$$

$$\xi = \frac{R_{cp}}{R_{P,0}} = \frac{R_{cp} \times b_{ax} \times C_R^3}{6 \times \eta_B \times l_{ax}} \text{ = restrictor ratio}$$

then the equation system is

$$-p_{i-1}^* \times \frac{\kappa \times \xi}{2} f_{i-1}^3 + p_i^* \times \left[1 + \frac{a_i}{2 \times \pi} Z \times \xi + \frac{\kappa \times \xi}{2} \times (f_i^3 + f_{i-1}^3) \right] - p_{i+1}^* \times \frac{\kappa \times \xi}{2} \times f_i^3 = 1 - 6 \times K_{rot} \times (f_i - f_{i-1})$$

Thus the relative pressures in the recesses and all further bearing parameters are determined by:

a) restrictor ratio ξ;

b) bearing geometry:

— number of recesses Z

— form and position of recesses (κ, α)

— position of journal (ε, β);

c) speed dependent parameter K_{rot}.

The angle β is determined iteratively in the course of the calculation.

A.3.3 Load F, attitude angle β, stiffness c

The radial load effect on recess i in accordance with Figure A.5 is given by

$$F_{\text{i}} = \text{b}_{\text{c}} \times \int_{-\frac{\pi}{Z}}^{\frac{\pi}{Z}} p_{\text{i}} \cos \delta \times \frac{D}{2} \times \text{d}\delta = b_{\text{c}} \times p_{\text{i}} \times D \times \sin\left(\frac{\pi}{Z}\right)$$

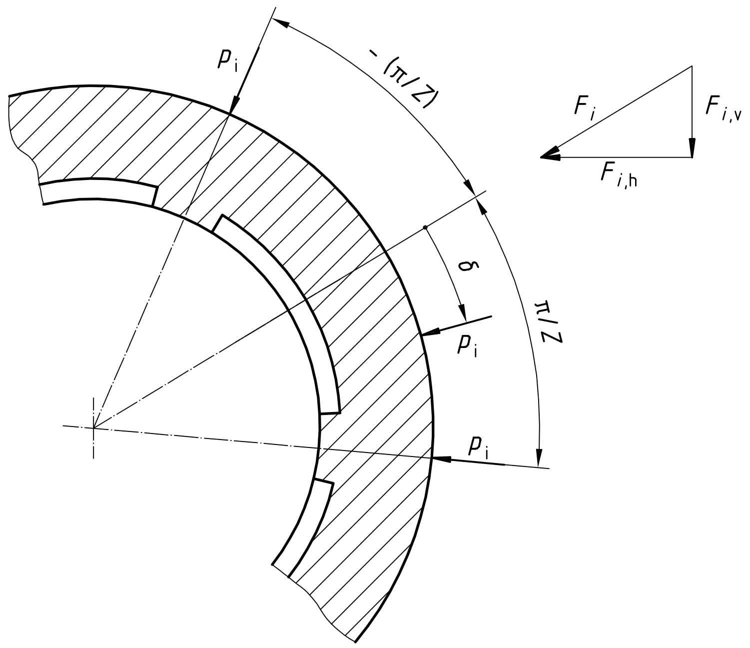


Figure A.5 — Application of load to one recess

The direction of F_i is given by

$$\bar{\varphi}_i = \frac{\varphi_{1,i} + \varphi_{2,i}}{2}$$

The horizontal component is the sum of all horizontal projections of $F_{i:}$ is given by

$$F_\mathrm{h} = b_\mathrm{c} \times D \times \sin\left(\frac{\pi}{Z}\right) \times \sum_{i=1}^{Z} p_i \times \sin(\bar{\varphi}_i + \beta)$$

Correspondingly for the vertical component

$$F_\mathrm{v} = b_\mathrm{c} \times D \times \sin\left(\frac{\pi}{Z}\right) \times \sum_{i=1}^{Z} p_i \times \cos(\bar{\varphi}_i + \beta)$$

where

$$\bar{\varphi}_i + \beta = a + \frac{2\pi}{Z} \times (i-1)$$

The total load is $F = \sqrt{F_\mathrm{h}^2 + F_\mathrm{v}^2}$

Angle of the resulting force $\varphi_\mathrm{F} = \arctan \frac{F_\mathrm{h}}{F_\mathrm{v}}$

NOTE In case of vertical load, the attitude angle for each ε should be modified in such a way that $\varphi_F = 0$. If the load-carrying capacity F is not applied vertically but at an angle φ_F to a perpendicular line, then the results for vertical direction of load can be applied when mounting the plain bearing at the angle φ_F.

The stiffness c can generally be defined in different ways:

Here the following definition is used:

$$c = \frac{F}{e} = \frac{F}{\varepsilon \times C_\mathrm{R}}$$

A.3.4 Lubricant flow rate and pumping power

The total lubricant flow rate can be calculated on the basis of the sum of the flow rates through the restrictors $Q_{\mathrm{cp},i}$:

$$Q = \sum_{i=1}^{Z} Q_{\mathrm{cp},i} = \frac{Z \times p_\mathrm{en} - \sum p_i}{R_\mathrm{cp}}$$

The lubricant flow rate can also be approximated according to equation (7).

Pumping power, $P_\mathrm{p} = Q \times p_\mathrm{en}$

A.3.5 Frictional power

The frictional power is composed of

a) friction in the land area;

b) friction in the recesses due to secondary flow.

The land area is given by

$$A_{lan} = 2 \times \pi \times l_{ax} \times D + Z \times l_c \times (B - 2 \times l_{ax})$$

$$A_{lan}^* = \frac{A_{lan}}{\pi \times B \times D}$$

The shearing stress at the shaft surface is in general given by

$$\tau = \eta_B \left(\frac{\partial u}{\partial y} \right)_{y=h} = \frac{1}{2} \times \frac{\partial p}{\partial x} \times h + \frac{U}{h} \times \eta_B$$

As an approximation, the shearing stress τ is calculated as follows without taking into account the pressure flow rate.

$$\tau = \frac{U}{h} \times \eta_B$$

The result for the land friction finally is given by

$$P_{f,lan} = \int_{A_{lan}} \tau \times U \times dA = \frac{\eta \times U^2}{C_R} \times \int_{A_{lan}} \frac{dA}{1 + \varepsilon \times \cos \varphi}$$

If it is assumed that the lands are uniformly distributed over the periphery, it can be simplified as follows

$$P_{f,lan} = \frac{\eta \times U^2}{C_R} \times \frac{A_{lan}}{\sqrt{1 - \varepsilon^2}}$$

Although the depth of recess $h_p >> h$, according to Shinkle and Hornung [5] the friction due to the secondary flow in the recesses shall be included in the calculation for shafts running at high speed. This applies especially to wide recesses and small lands.

When the flow in the recesses is still laminar, i.e.

$$Re_p = \frac{U \times h_p \times \rho}{\eta_B} < 1\,000$$

then the friction in the recesses is calculated as follows:

$$P_{f,P} = 4 \times \frac{\eta \times U^2}{h_p} \times A_p$$

where

$$A_p = \pi \times B \times D - A_{lan}$$

When Re_P > 1 000, then the flow is turbulent and the friction increases correspondingly. In that case, the preceding equation for τ can no longer be used.

A.3.6 Equations for dimensioning

The following equations can be used to determine the dimensions when stiffness c is given:

$$C_\mathrm{R} = \frac{F}{\varepsilon \times c}$$

$$D^2 \times p_\mathrm{en} = \frac{F}{\frac{B}{D} \times F^*}$$

$$\frac{p_\mathrm{en}}{\eta_\mathrm{B}} = \frac{F \times \omega}{C_\mathrm{R}^2} \times \frac{1}{4 \times \frac{B}{D} \times F^* \times \pi_\mathrm{f}}$$

Annex B
(normative)

Examples of calculation

B.1 Example 1 — Calculation of a hydrostatic journal bearing

B.1.1 General

A bearing with four recesses with given dimensions and operational data is to be examined. The lubricant oil ISO VG 46 and the temperature in front of the bearing are also stated. The amount of oil, power, stiffness, film thickness etc are to be calculated. The following parameters are given:

B.1.2 Dimensions

- Bearing diameter, D = 0,12 m
- Bearing width, B = 0,12 m
- Width of circumferential outlet, b_c = 0,018 m
- Axial land length, l_{ax} = 0,018 m
- Circumferential land length, l_c = 0,012 m
- Depth of recess, h_p = 40 C_Rm
- Number of recesses, Z = 4
- Diameter of capillaries, d_{cp} = 0,002 38 m
- Length of capillaries, l_{cp} = 0,74 m
- Relative bearing clearance, ψ = $1,6 \times 10^{-3}$
- Radial clearance, C_R = $\psi \times \frac{D}{2} = 96 \times 10^{-6}$ m

B.1.3 Operational data

- Load-carrying capacity (load), F = 40 000 N
- Rotational frequency (speed), N = 16,66 s^{-1} (ω = 104,7 s^{-1})
- Inlet temperature, T_{en} = 45 °C
- Feed pressure, p_{en} = 116 bar = $11,6 \times 10^6$ Pa

B.1.4 Lubricant data

For oil ISO VG 46:

T °C	η Pa·s
40	0,041 40
50	0,026 58
60	0,018 07

— Volume specific heat, $c_p \cdot \rho$ = 1,75 × 10^6 W·s/m^3·K

— Density, ρ = 900 kg/m^3

Exponent, calculated on the basis of the lubricant data, $\gamma = \frac{1}{10} \times \ln \frac{\eta_{40}}{\eta_{50}} = \frac{1}{10} \times \ln \frac{0{,}041\,4}{0{,}026\,58} = 0{,}044\,3$

These data are used to calculate the parameters listed in B.1.5 to B.1.18.

B.1.5 Temperatures and dynamic viscosities

The first calculation is carried out with the following approximate temperatures and dynamic viscosities (without frictional powers P_f and with ξ = 1).

$$\Delta T_{cp} = \frac{p_{en}}{c_p \times \rho} \times \frac{\xi}{1+\xi} = \frac{11{,}6 \times 10^6}{1{,}75 \times 10^6} \times \frac{1}{1+1} = 3{,}3\,\mathrm{K}$$

$$\Delta T_B = \frac{p_{en}}{c_p \times \rho} \times \left(\frac{1}{1+\xi} + P^* \right) = \frac{11{,}6 \times 10^6}{1{,}75 \times 10^6} \times \left(\frac{1}{1+1} + 0 \right) = 3{,}3\,\mathrm{K}$$

$$T_{cp} = T_{en} + \frac{\Delta T_{cp}}{2} = 41 + \frac{3{,}3}{2} = 42{,}65\,°\mathrm{C}$$

$$T_B = T_{en} + \Delta T_{cp} + \frac{\Delta T_B}{2} = 41 + 3{,}3 + \frac{3{,}3}{2} = 46\,°\mathrm{C}$$

The dynamic viscosities are then given by

$$\eta_{cp} = \eta_{40} \times \exp\left[-\gamma (T_{cp} - 40)\right] = 0{,}041\,4 \times \exp\left[-0{,}044\,3 \times (42{,}65 - 40)\right] = 0{,}036\,8\,\mathrm{Pa \cdot s}$$

$$\eta_B = \eta_{40} \times \exp\left[-\gamma (T_B - 40)\right] = 0{,}041\,4 \times \exp\left[-0{,}044\,3 \times (46 - 40)\right] = 0{,}031\,8\,\mathrm{Pa \cdot s}$$

B.1.6 Flow resistances

$$R_{cp} = \frac{128 \times \eta_{cp} \times l_{cp}}{\pi \times d_{cp}^4} \times (1 + a) = \frac{128 \times 0{,}036\,8 \times 0{,}74}{\pi \times 32{,}1 \times 10^{-12}} \times (1 + 0{,}2) = 4{,}15 \times 10^{10}\,\mathrm{N\,s/m^5}$$

NOTE The inertia factor a cannot yet be calculated in this place, as the oil flow rate is not known. Therefore, it should be started with an estimated value and the exact value of a determined iteratively. Here, the value has been taken from the following calculation.

$$R_{P,0} = \frac{R_{ax,0}}{2} = \frac{6 \times \eta_B \times l_{ax}}{C_R^3 \times D \times (\pi/Z)} = \frac{6 \times 0{,}031\,8 \times 0{,}018}{\left(96 \times 10^{-6}\right)^3 \times 0{,}12 \times (\pi/4)} = 4{,}12 \times 10^{10}\ \mathrm{N\,s/m^5}$$

B.1.7 Restrictor ratio

$$\xi = \frac{R_{cp}}{R_{P,0}} = \frac{4{,}15 \times 10^{10}}{4{,}12 \times 10^{10}} = 1{,}007 \approx 1$$

B.1.8 Pressure ratio in recesses (ε = 0)

$$\frac{p_{i,0}}{p_{en}} = \frac{1}{1 + \xi} = \frac{1}{1 + 1} = 0{,}5$$

B.1.9 Resistance ratio

$$\kappa = \frac{l_{ax} \times b_c}{l_c \times b_{ax}} = \frac{0{,}018 \times 0{,}102}{0{,}018 \times 0{,}094\,2} = 1{,}082$$ where $b_{ax} = \pi \times D/Z$ and $b_c = D - l_{ax}$

B.1.10 Relative friction pressure

$$\pi_f = \frac{\eta_B \times \omega}{p_{en} \times \psi^2} = \frac{0{,}031\,8 \times 104{,}7}{11{,}6 \times 10^6 \times 1{,}6^2 \times 10^{-6}} = 0{,}112$$

B.1.11 Speed dependent parameter

$$K_{rot} = \xi \times \kappa \times \frac{l_c}{D} \times \pi_f = 1 \times 1{,}082 \times \frac{0{,}018}{0{,}12} \times 0{,}112 = 0{,}018\,3;$$

Figure 4 in ISO 12168-2:2001 shows that in the case where K_{rot} = 0,018 3 and ε = 0,4, the influence of the speed is so small that $F^*_{eff} / F^*_{eff,0} \approx 1$.

B.1.12 Characteristic values of load carrying capacity and film thicknesses

$$F^*_{eff,0} = \frac{F}{p_{en} \times (B - b_{ax}) \times D} = \frac{40\,000}{11{,}6 \times 10^6 \times (0{,}12 - 0{,}018) \times 0{,}12} = 0{,}281\,7$$

According to Figure 3 in ISO 12168-2:2001, $F^*_{eff,0}$ over κ for κ = 1,082 and ε = 0,4 gives $F^*_{eff,0} = 0{,}253$.

B.1.13 Eccentricity and film thickness

$$\varepsilon = 0{,}4 \times \frac{F^*_{eff,0}}{F^*_{eff,0}\ (\varepsilon = 0{,}4)} = 0{,}4 \times \frac{0{,}281\,7}{0{,}253} = 0{,}445$$

Minimum film thickness $h_{min} = (1 - \varepsilon) \times C_R = (1 - 0{,}445) \times 96 = 53{,}25\ \mu m$

B.1.14 Frictional power

$$A_{\text{lan}}^{*} = 2 \times \frac{l_{\text{ax}}}{B} + \frac{Z}{\pi} \times \frac{l_{\text{c}}}{D} \times \left(1 - 2 \times \frac{l_{\text{ax}}}{B}\right) = 2 \times \frac{0{,}018}{0{,}12} + \frac{4}{\pi} \times \frac{0{,}018}{0{,}12} \times \left(1 - 2 \times \frac{0{,}018}{0{,}12}\right) = 0{,}433$$

according to equation (10):

$$P_{\text{f}}^{*} = \pi \times A_{\text{lan}}^{*} \left[\frac{1}{\sqrt{1-\varepsilon^2}} + \frac{4 \times C_R}{h_{\text{p}}} \times \left(\frac{1}{A_{\text{lan}}^{*}} - 1 \right) \right] = \pi \times 0{,}433 \left[\frac{1}{\sqrt{1-0{,}445^2}} + \frac{4}{40} \times \left(\frac{1}{0{,}433} - 1 \right) \right] = 1{,}697$$

where $U = \omega \times \frac{D}{2} = 104{,}7 \times \frac{0{,}12}{2} = 6{,}28$ m/s

$$P_{\text{f}} = P_{\text{f}}^{*} \times \frac{\eta_{\text{B}} \times U^2}{C_{\text{R}}} \times B \times D = 1{,}697 \times \frac{0{,}031\,8 \times 6{,}28^2}{96 \times 10^{-6}} \times 0{,}12 \times 0{,}12 = 319{,}6 \approx 320 \text{ W}$$

B.1.15 Pumping power and lubricant flow rate

According to equation (7):

$$Q^{*} = \frac{1}{1+\xi} \times \frac{\pi}{6 \times \frac{B}{D}} \times \frac{1}{\frac{l_{\text{ax}}}{B}} = \frac{1}{1+1} \times \frac{\pi}{6 \times (0{,}12/0{,}12)} \times \frac{1}{(0{,}018/0{,}12)} = 1{,}745$$

$$Q = Q^{*} \times \frac{C_{\text{R}}^3 \times p_{\text{en}}}{\eta_{\text{B}}} = 1{,}745 \times \frac{(96 \times 10^{-6})^3 \times 11{,}6 \times 10^6}{0{,}031\,8} = 5{,}63 \times 10^{-4} \text{ m}^3\text{/s} = 0{,}563 \text{ l/s}$$

$$P_{\text{p}} = Q \times p_{\text{en}} = 5{,}63 \times 10^{-4} \times 11{,}6 \times 10^6 = 6\,533 \text{ W}$$

$$P^{*} = \frac{P_{\text{f}}}{P_{\text{p}}} = \frac{320}{6\,533} = 0{,}049$$

$$P_{\text{tot}} = P_{\text{f}} + P_{\text{p}} = 320 + 6\,533 = 6\,853 \text{ W}$$

B.1.16 Temperatures and dynamic viscosities

$$\Delta T_{\text{cp}} = 3{,}3 \text{ K}$$

$$\Delta T_{\text{B}} = 3{,}3 + \frac{P_{\text{f}}}{c_{\text{p}} \times \rho \times Q} = 3{,}3 + \frac{320}{1{,}75 \times 10^6 \times 5{,}63 \times 10^{-4}} = 3{,}3 + 0{,}32 = 3{,}62 \text{ K}$$

A further iteration of the temperatures and dynamic viscosities is not necessary in this case.

B.1.17 Reynolds numbers

In the recess

$$Re_{\text{p}} = \frac{U \times h_{\text{p}} \times \rho}{\eta_{\text{B}}} = \frac{6{,}28 \times 40 \times 96 \times 10^{-6} \times 900}{0{,}031\,8} = 682$$

Re_p < 1000 and is thus laminar.

In the capillaries:

$$Re_{cp} = \frac{4 \times Q \times \rho}{\eta_{cp} \times \pi \times d_{cp} \times Z} = \frac{4 \times 5{,}63 \times 10^{-4} \times 900}{0{,}036\,8 \times \pi \times 2{,}38 \times 10^{-3} \times 4} = 1\,842$$

Re_{cp} < 2 300 and is thus laminar.

Inertia factor:

$$a = \frac{1{,}08}{32} \times Re_{cp} \times \frac{d_{cp}}{l_{cp}} = \frac{1{,}08}{32} \times 1\,842 \times \frac{2{,}38 \times 10^{-3}}{0{,}74} = 0{,}2$$

NOTE The recommendation Re_{cp} < 1 000 to 1 500 given in A.3.2.2 is not observed in this case. There hence follows a relatively high non-linear inertia factor a.

B.1.18 Measures for optimization

As P_f << P_p and thus P^* << 1 to 3, π_f is to be increased, i.e. the clearance shall be decreased or the dynamic viscosity increased. The optimum relative friction pressure is calculated with an assumed value of P^* = 1 as follows:

$$\pi_{f,opt} = \frac{1}{2}\sqrt{\frac{P^* \times Q^*}{P_f^* \times \frac{B}{D}}} = \frac{1}{2}\sqrt{\frac{1 \times 1{,}745}{1{,}699 \times 1}} = 0{,}506\,7$$

With this value of relative friction pressure and a dynamic viscosity ε_B = 0,045 5 Pa·s which, for the sake of simplicity, is assumed to be greater it follows that:

$$\psi_{opt} = \sqrt{\frac{\eta_B \times \omega}{p_{en} \times \pi_{f,opt}}} = \sqrt{\frac{0{,}045\,5 \times 104{,}7}{11{,}6 \times 10^6 \times 0{,}506\,7}} = 0{,}9 \times 10^{-3}$$

$$C_R = \psi_{opt} \times \frac{D}{2} = 0{,}9 \times 10^{-3} \times \frac{0{,}12}{2} = 54\ \mu m$$

$$P_f = P_f^* \times \frac{\eta_B \times U^2 \times B \times D}{C_R} = 1{,}697 \times \frac{0{,}045\,5 \times 6{,}28^2 \times 0{,}12 \times 0{,}12}{54 \times 10^{-6}} = 812\ W$$

$$P_p = Q^* \times \frac{C_R^3 \times p_{en}^2}{\eta_B} = 1{,}745 \times \frac{\left(54 \times 10^{-6}\right)^3 \times \left(11{,}6 \times 10^6\right)^2}{0{,}045\,5} = 812{,}6\ W$$

$P_{tot} = P_f + P_p$ = 812 + 812,6 = 1 624,6 W

$$Q = \frac{P_p}{p_{en}} = \frac{812{,}6}{11{,}6 \times 10^6} = 7 \times 10^{-5}\ m^3/s = 0{,}07\ l/s$$

B.2 Example 2 — Design of an optimized hydrostatic journal bearing

B.2.1 General

An optimized bearing with four recesses, which shall be operated under a load F = 15 000 N at a rotational frequency (speed) of 31,88 s^{-1} (ω = 200 s^{-1}), is to be designed. A stiffness of c = 500 N/µm is required.

B.2.2 Given quantities

The following values are taken for the optimized bearing:

B/D = 1; Z = 4; l_{ax}/B = 0,15; l_c/B = 0,15; ε = 0,4; ξ = 1; P^* = 2; α = 0° (load directed to recess)

The following characteristic values have been calculated for these data:

P^*_{tot} = 6,907; F^* = 0,264 8; π_f = 0,732 7; Q^* = 1,787

NOTE It is also possible in this case to take the data given in Table 1 in ISO 12168-2:2001.

B.2.3 Lubricant data

An oil with η_{40} = 0,061 2 Pa·s and an inlet temperature of T_{en} = 44 °C is chosen. The values given in the following table shall apply to this oil.

T °C	η Pa·s
40	0,061 20
50	0,038 06

Volume specific heat, $c_p \cdot \rho$ = 1,75 × 10^6 W/m^3 K

Density, ρ = 900 kg/m^3

Exponent, calculated from the lubricant data $\gamma = \frac{1}{10} \times \ln \frac{\eta_{40}}{\eta_{50}} = \frac{1}{10} \times \ln \frac{0{,}061\,2}{0{,}038\,06} = 0{,}047\,5$

B.2.4 Temperatures and dynamic viscosities

$$\Delta T_{cp} = \frac{p_{en}}{c_p \times \rho} \times \frac{\xi}{1 + \xi} = \frac{5{,}24 \times 10^6}{1{,}75 \times 10^6} \times \frac{1}{2} = 1{,}5\ \text{K}$$

$$\Delta T_B = \frac{p_{en}}{c_p \times \rho} \times \left(\frac{1}{1 + \xi} + P^* \right) = \frac{5{,}24 \times 10^6}{1{,}75 \times 10^6} \times \left(\frac{1}{2} + 2 \right) = 7{,}5\ \text{K}$$

$$T_{cp} = T_{en} + \frac{\Delta T_{cp}}{2} = 44 + 0{,}75 = 44{,}75\ °\text{C}$$

$$T_B = T_{en} + \Delta T_{cp} + \frac{\Delta T_B}{2} = 44 + 1{,}5 + 3{,}75 = 49{,}25\ °\text{C}$$

The dynamic viscosities are then given by

$$\eta_{cp} = \eta_{40} \times \exp[-\gamma \times (T_{cp} - 40)] = 0{,}061\,2 \times \exp[-0{,}047\,5 \times 4{,}75] = 0{,}048\,8 \approx 0{,}05 \text{ Pa·s}$$

$$\eta_{B} = \eta_{40} \times \exp[-\gamma \times (T_{B} - 40)] = 0{,}061\,2 \times \exp[-0{,}047\,5 \times 9{,}25] = 0{,}039\,4 \approx 0{,}04 \text{ Pa·s}$$

B.2.5 Feed pressure and dimensions

$$C_R = \frac{F}{\varepsilon \times c} = \frac{15\ 000}{0{,}4 \times 500} = 75 \text{ µm}$$

$$D^2 \times p_{en} = \frac{F}{\frac{B}{D} \times F^*} = \frac{15\ 000}{1 \times 0{,}264\ 8} = 56\ 646 \text{ N}$$

$$\frac{p_{en}^2}{\eta_B} = \frac{F \times \omega}{C_R^2} \times \frac{1}{4 \times \frac{B}{D} \times F^* \times \pi_f} = \frac{15\ 000 \times 200}{(75 \times 10^{-6})^2 \times 4 \times 1 \times 0{,}264\ 8 \times 0{,}732\ 7} = 6{,}873 \times 10^{14}$$

With the dynamic viscosity calculated before, one obtains:

$$p_{en} = \sqrt{\frac{p_{en}^2}{\eta_B} \times \eta_B} = \sqrt{6{,}873 \times 10^{14} \times 0{,}04} = 5{,}24 \times 10^6 \text{ Pa}$$

$$D = \sqrt{\frac{D^2 \times p_{en}}{p_{en}}} = \sqrt{\frac{56\ 646}{5{,}24 \times 10^6}} = 0{,}104 \text{ m}$$

B.2.6 Power losses and need of lubricant

Total power:

$$P_{tot} = P_{tot}^* \times F \times \omega \times C_R = 6{,}907 \times 15\ 000 \times 200 \times 75 \times 10^{-6} = 1\,554 \text{ W}$$

Pumping power:

$$P_p = \frac{P_{tot}}{1 + P^*} = \frac{1\,554}{1 + 2} = 518 \text{ W}$$

Frictional power:

$$P_f = P^* \times P_p = 2 \times 518 = 1\,036 \text{ W}$$

Need of lubricant:

$$Q = \frac{P_p}{p_{en}} = \frac{518}{5{,}24 \times 10^6} = 9{,}83 \times 10^{-5} \text{ m}^3/\text{s} = 0{,}098\ 3 \text{ l/s}$$

As a check, Q is calculated from Q^*

$$Q = Q^* \times \frac{C_R^3 \times p_{en}}{\eta_B} = 1{,}787 \times \frac{(75 \times 10^{-6})^3 \times 5{,}24 \times 10^6}{0{,}04} = 9{,}88 \times 10^{-5} \text{ m}^3/\text{s} = 0{,}098\ 8$$

B.2.7 Flow resistance and dimensions of capillaries

The flow resistance of the capillaries is determined from

$$R_{cp} = \frac{p_{en} \times Z}{Q} \times \frac{\xi}{1+\xi} = \frac{5,24 \times 10^{6} \times 4}{9,83 \times 10^{-5}} \times \frac{1}{2} = 1,066 \times 10^{11}$$

For a capillary length of l_{cp} = 0,45 m the portion of the inlet pressure drop results in:

$$a = \frac{1,08 \times 4 \times Q \times \rho}{32 \times Z \times \eta_{cp} \times l_{cp} \times \pi} = \frac{1,08 \times 4 \times 9,83 \times 10^{-5} \times 900}{32 \times 4 \times 0,05 \times 0,45 \times \pi} = 0,042\ 3$$

The diameter of capillaries is determined from

$$d_{cp}^{4} = \frac{128 \times \eta_{cp} \times l_{cp}}{R_{cp} \times \pi} \times (1+a) = \frac{128 \times 0,05 \times 0,45 \times 1,042\ 3}{1,066 \times 10^{11} \times \pi} = 8,968 \times 10^{-12}$$

$$d_{cp} = \sqrt[4]{8,968 \times 10^{-12}} = 1,73 \times 10^{-3}\ \text{m} = 1,73\ \text{mm}$$

B.2.8 Reynolds numbers

In the capillary

$$Re_{cp} = \frac{4 \times Q \times \rho}{Z \times \eta_{cp} \times \pi \times d_{cp}} = \frac{4 \times 9,83 \times 10^{-5} \times 900}{4 \times 0,05 \times \pi \times 1,73 \times 10^{-3}} = 326$$

$Re_{cp} < 2\ 300$ and is thus laminar

$Re_{cp} < 1\ 000$ to 1 500, and thus has a low inertia factor a

In the recess with $h_p = 40 \times C_R$

$$Re_p = \frac{U \times h_p \times \rho}{\eta_B} = \frac{10,4 \times 40 \times 75 \times 10^{-6} \times 900}{0,04} = 702$$

$Re_p < 1\ 000$ and is thus laminar

where

$$U = \omega \times \frac{D}{2} = 200 \times \frac{0,104}{2} = 10,4\ \text{m/s}$$

and

$h_p = 40 \times C_R$

Table B.1 shows various combinations of C_R, η_B, p_{en}, D and ψ. The third column contains the calculated data. The values in the fourth column are based on a higher viscosity. In the fifth column, the dynamic viscosity is given instead of stiffness. In the sixth column, the relative bearing clearance is chosen.

Table B.1 — Table of calculation results

Dimension	Unit	Calculated data	Data based on a higher η	η given instead of c	ψ
F	N	15 000	15 000	15 000	15 000
ω	s^{-1}	200	200	200	200
c	N/µm	500	500	642	961
C_R	µm	75	75	58,4	39,0
P^*_{tot}	1	6,907	6,907	6,907	6,907
F^*	1	0,264 8	0,264 8	0,264 8	0,264 8
Q^*	1	1,787	1,787	1,787	1,787
π_f	1	0,732 7	0,732 7	0,732 7	0,732 7
$D^2 \cdot p_{en}$	N	56 645	56 645	56 645	56 645
P^2_{en}/η_B	Pa/s	$6,873 \times 10^{14}$	$6,873 \times 10^{14}$	$11,33 \times 10^{14}$	$25,42 \times 10^{14}$
η_B	Pa·s	0,04	0,063	0,031 5	0,05
p_{en}	Pa	$52,4 \times 10^5$	$64,2 \times 10^5$	$59,7 \times 10^5$	$112,8 \times 10^5$
D	m	0,104	0,094	0,097 4	0,070 8
ψ	1	$1,44 \times 10^{-3}$	$1,6 \times 10^{-3}$	$1,2 \times 10^{-3}$	$1,1 \times 10^{-3}$
P_{tot}	W	1 554	1 554	1 211	807
Q	l/s	0,098 75	0,078 7	0,067 5	0,023 9
$p_{en} \cdot \psi^2$	Pa	10,86	16,43	8,598	13,65
p_{en}/η_B	s^{-1}	131×10^6	$101,9 \times 10^6$	$187,3 \times 10^6$	$225,6 \times 10^6$

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