INTERNATIONAL STANDARD

**ISO 12167-1**

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# Plain bearings — Hydrostatic plain journal bearings with drainage grooves under steady-state conditions —

## Part 1: Calculation of oil-lubricated plain journal bearings with drainage grooves

*Paliers lisses — Paliers lisses radiaux hydrostatiques avec rainures d'écoulement fonctionnant en régime stationnaire —*

*Partie 1: Calcul pour la lubrification des paliers lisses radiaux avec rainures d'écoulement*

Reference number
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# Foreword

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The committee responsible for this document is ISO/TC 123, *Plain bearings*.

This second edition cancels and replaces the first edition (ISO 12167-1:2001), of which it constitutes a minor revision.

ISO 12167 consists of the following parts, under the general title *Plain bearings — Hydrostatic plain journal bearings with drainage grooves under steady-state conditions*:

— *Part 1: Calculation of oil-lubricated plain journal bearings with drainage grooves*

— *Part 2: Characteristic values for the calculation of oil-lubricated plain journal bearings with drainage grooves*

# Introduction

Hydrostatic bearings use external lubrication to support pressure on the bearings; thus, are less prone to wear and tear, run quietly, and have wide useable speed, as well as high stiffness and damping capacity. These properties also demonstrate the special importance of plain journal bearings in different fields of application such as in machine tools.

Basic calculations described in this part of ISO 12167 may be applied to bearings with different numbers of recesses and different width/diameter ratios for identical recess geometry.

Oil is fed to each bearing recess by means of a common pump with constant pumping pressure (system $p_{en}$ = constant) and through preceding linear restrictors, e.g. capillaries.

The calculation procedures listed in this part of ISO 12167 enable the user to calculate and assess a given bearing design, as well as to design a bearing as a function of some optional parameters. Furthermore, this part of ISO 12167 contains the design of the required lubrication system including the calculation of the restrictor data.

# Plain bearings — Hydrostatic plain journal bearings with drainage grooves under steady-state conditions —

# Part 1: Calculation of oil-lubricated plain journal bearings with drainage grooves

## 1 Scope

This part of ISO 12167 applies to hydrostatic plain journal bearings under steady-state conditions.

In this part of ISO 12167, only bearings with oil drainage grooves between the recesses are taken into account. As compared to bearings without oil drainage grooves, this type needs higher power with the same stiffness behaviour.

## 2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3448, *Industrial liquid lubricants — ISO viscosity classification*

ISO 12167-2:2001, *Plain bearings — Hydrostatic plain journal bearings with drainage grooves under steady-state conditions — Part 2: Characteristic values for the calculation of oil-lubricated plain journal bearings with drainage grooves*

## 3 Bases of calculation and boundary conditions

Calculation in accordance with this part of ISO 12167 is the mathematical determination of the operational parameters of hydrostatic plain journal bearings as a function of operating conditions, bearing geometry and lubrication data. This means the determination of eccentricities, load-carrying capacity, stiffness, required feed pressure, oil flow rate, frictional and pumping power, and temperature rise. Besides the hydrostatic pressure build up, the influence of hydrodynamic effects is also approximated.

Reynolds' differential formula furnishes the theoretical basis for the calculation of hydrostatic bearings. In most practical cases of application, it is, however, possible to arrive at sufficiently exact results by approximation.

The approximation used in this part of ISO 12167 is based on two basic formulae intended to describe the flow through the bearing lands, which can be derived from Reynolds' differential formula when special boundary conditions are observed. The Hagen-Poiseuille law describes the pressure flow in a parallel clearance gap and the Couette formula the drag flow in the bearing clearance gap caused by shaft rotation. A detailed presentation of the theoretical background of the calculation procedure is included in Annex A.

The following important premises are applicable to the calculation procedures described in this part of ISO 12167:

a) all lubricant flows in the lubrication clearance gap are laminar;

b) the lubricant adheres completely to the sliding surfaces;

c) the lubricant is an incompressible Newtonian fluid;

d) in the whole lubrication clearance gap, as well as in the preceding restrictors, the lubricant is partially isoviscous;

e) a lubrication clearance gap completely filled with lubricant is the basis of frictional behaviour;

f) fluctuations of pressure in the lubricant film normal to the sliding surfaces do not take place;

g) bearing and journal have completely rigid surfaces;

h) the radii of curvature of the surfaces in relative motion to each other are large in comparison to the lubricant film thickness;

i) the clearance gap height in the axial direction is constant (axial parallel clearance gap);

j) the pressure over the recess area is constant;

k) there is no motion normal to the sliding surfaces.

The bearing consists of Z cylindrical segments and rectangular recess of the same size and is supplied with oil through restrictors of the same flow characteristics. Each segment consists of a circumferential part between two centre lines of axial drainage grooves. With the aid of the above-mentioned approximation formulae, all parameters required for the design or calculation of bearings can be determined. The application of the similarity principle results in dimensionless similarity values for load-carrying capacity, stiffness, oil flow rate, friction, recess pressures, etc.

The results indicated in this part of ISO 12167 in the form of tables and diagrams are restricted to operating ranges common in practice for hydrostatic bearings. Thus, the range of the bearing eccentricity (displacement under load) is limited to $\varepsilon = 0$ to 0,5.

Limitation to this eccentricity range means a considerable simplification of the calculation procedure as the load-carrying capacity is a nearly linear function of the eccentricity. However, the applicability of this procedure is hardly restricted as in practice eccentricities $\varepsilon > 0,5$ are mostly undesirable for reasons of operational safety. A further assumption for the calculations is the approximated optimum restrictor ratio[1] $\xi = 1$ for the stiffness behaviour.

As for the outside dimensions of the bearing, this part of ISO 12167 is restricted to the range bearing width/bearing diameter $B/D = 0,3$ to 1, which is common in practical cases of application. The recess depth is larger than the clearance gap height by a factor of 10 to 100. When calculating the friction losses, the friction loss over the recess in relation to the friction over the bearing lands can generally be neglected on account of the above premises. However, this does not apply when the bearing shall be optimized with regard to its total power losses.

To take into account the load direction of a bearing, it is necessary to distinguish between the two extreme cases, load in the direction of recess centre and load in the direction of land centre.

Apart from the aforementioned boundary conditions, some other requirements are to be mentioned for the design of hydrostatic bearings in order to ensure their functioning under all operating conditions. In general, a bearing shall be designed in such a manner that a clearance gap height of at least 50 % to 60 % of the initial clearance gap height is ensured when the maximum possible load is applied. With this in mind, particular attention shall be paid to misalignments of the shaft in the bearing due to shaft deflection which may result in contact between shaft and bearing edge and thus in damage of the bearing. In addition, the parallel clearance gap required for the calculation is no longer present in such a case.

In the case where the shaft is in contact with the bearing lands when the hydrostatic pressure is switched off, it might be necessary to check the contact zones with regard to rising surface pressures.

It shall be ensured that the heat originating in the bearing does not lead to a non-permissible rise in the temperature of the oil.

If necessary, a means of cooling the oil shall be provided. Furthermore, the oil shall be filtered in order to avoid choking of the capillaries and damage to the sliding surfaces.

Low pressure in the relieved recess shall also be avoided, as this leads to air being drawn in from the environment and this would lead to a decrease in stiffness (see 5.7).

# 4 Symbols, terms and units

**Table 1 — Symbols, terms and units**

| Symbol | Term | Unit |
|---|---|---|
| $a$ | Inertia factor | 1 |
| $A_{lan}$ | Land area | $m^2$ |
| $A_{lan}^*$ | Relative land area $\left(A_{lan}^* = \frac{A_{lan}}{\pi \times B \times D}\right)$ | 1 |
| $A_p$ | Recess area | $m^2$ |
| $b$ | Width perpendicular to the direction of flow | m |
| $b_{ax}$ | Width of axial outlet $\left[b_{ax} = \frac{\pi \times D}{Z} - \left(l_c + b_G\right)\right]$ | m |
| $b_c$ | Width of circumferential outlet $\left(b_c = B - l_{ax}\right)$ | m |
| $b_G$ | Width of drainage groove | m |
| $B$ | Bearing width | m |
| $c$ | Stiffness coefficient | N/m |
| $c_p$ | Specific heat capacity of the lubricant (p = *constant*) | J/kg·K |
| $C_R$ | Radial clearance $\left[C_R = \left(D_B - D_J\right)/2\right]$ | m |
| $d_{cp}$ | Diameter of capillaries | m |
| $D$ | Bearing diameter ($D_J$: shaft; $D_B$: bearing; $D \approx D_J \approx D_B$) | m |
| $e$ | Eccentricity (shaft displacement) | m |
| $f$ | Relative film thickness [$f = h/C_R$] | 1 |
| $f_{en,i}$ | Relative film thickness at $\varphi = \varphi'_{1,i}$ | 1 |
| $f_{ex,i}$ | Relative film thickness at $\varphi = \varphi'_{2,i}$ | 1 |
| $F$ | Load-carrying capacity (load) | N |
| $F^*$ | Characteristic value of load-carrying capacity [$F^* = F/(B \times D \times p_{en})$] | 1 |
| $F_{eff}^*$ | Characteristic value of effective load-carrying capacity | 1 |
| $F_{eff,0}^*$ | Characteristic value of effective load-carrying capacity for $N = 0$ | 1 |
| $h$ | Local lubricant film thickness (clearance gap height) | m |
| $h_{min}$ | Minimum lubricant film thickness (minimum clearance gap height) | m |
| $h_p$ | Depth of recess | m |
| $K_{rot}$ | Speed-dependent parameter | 1 |
| $l$ | Length in the direction of flow | m |
| $l_{ax}$ | Axial land length | m |

**Table 1** *(continued)*

| Symbol | Term | Unit |
|---|---|---|
| $l_c$ | Circumferential land length | m |
| $l_{cp}$ | Length of capillaries | m |
| $N$ | Rotational frequency (speed) | $s^{-1}$ |
| $p$ | Recess pressure, general | Pa |
| $\bar{p}$ | Specific bearing load [$\bar{p} = F/(B \times D)$] | Pa |
| $p_{en}$ | Feed pressure (pump pressure) | Pa |
| $p_i$ | Pressure in recess *i* | Pa |
| $P_i^*$ | Pressure ratio [$P_i^* = P_i/P_{en}$] | 1 |
| $p_{i,0}$ | Pressure in recess *i*, when $\varepsilon = 0$ | Pa |
| $P^*$ | Power ratio ($P^* = P_f/P_p$) | 1 |
| $P_f$ | Frictional power | W |
| $P_p$ | Pumping power | W |
| $P_{tot}$ | Total power ($P_{tot} = P_p + P_f$) | W |
| $P_{tot}^*$ | Characteristic value of total power | 1 |
| $Q$ | Lubricant flow rate (for complete bearing) | $m^3/s$ |
| $Q^*$ | Lubricant flow rate parameter | 1 |
| $Q_{cp,i}$ | Lubricant flow rate from capillary into recess 1 | $m^3/s$ |
| $R_{cp}$ | Flow resistance of capillaries | $Pa \cdot s/m^3$ |
| $R_{lan,ax}$ | Flow resistance of one axial land $\left( R_{lan,ax} = \dfrac{12 \times \eta \times l_{ax}}{b_{ax} \times C_R^3} \right)$ | $Pa \cdot s/m^3$ |
| $R_{lan,c}$ | Flow resistance of one circumferential land $\left( R_{lan,c} = \dfrac{12 \times \eta \times l_c}{b_c \times C_R^3} \right)$ | $Pa \cdot s/m^3$ |
| $R_{P,0}$ | Flow resistance of one recess, when $\varepsilon = 0$, $\left( R_{p,0} = \dfrac{R_{lan,ax}}{2 \times (1 + \kappa)} \right)$ | $Pa \cdot s/m^3$ |
| $Re$ | Reynolds number | 1 |
| $So$ | Sommerfeld number | 1 |
| $T$ | Temperature | °C |
| $T_B$ | Mean temperature in the bearings; see Formula (15) | °C |
| $\Delta T$ | Temperature difference | °C |
| $u$ | Flow velocity | m/s |
| $U$ | Circumferential speed | m/s |
| $\bar{w}$ | Average velocity in restrictor | m/s |
| $Z$ | Number of recesses | 1 |
| $\alpha$ | Position of first recess related to recess centre measured from load direction; see Figure A.3 | rad |
| $\beta$ | Attitude angle of shaft | ° |
| $g$ | Exponent in viscosity formula | 1 |
| $\varepsilon$ | Relative eccentricity ($\varepsilon = e/C_R$) | 1 |

**Table 1** *(continued)*

| Symbol | Term | Unit |
|---|---|---|
| $\eta$ | Dynamic viscosity | Pa·s |
| $\eta_B$ | Dynamic viscosity for $T = T_B$ | Pa·s |
| $\kappa$ | Resistance ratio $\left(\kappa = \frac{R_{lan,\,ax}}{R_{lan,\,c}} = \frac{l_{ax} \times b_c}{l_c \times b_{ax}}\right)$ | 1 |
| $\xi$ | Restrictor ratio $\left(\xi = \frac{R_{cp}}{R_{P,\,0}}\right)$ | 1 |
| $\pi_f$ | Relative frictional pressure $\left(\pi_f = \frac{\eta_B \times \omega}{P_{en} \times \psi^2}\right)$ | 1 |
| $\rho$ | Density | kg/m$^3$ |
| $\tau$ | Shearing stress | N/m$^2$ |
| $\varphi$ | Angular coordinate measured from radius opposite to eccentricity, e; see Figure A.3 | rad |
| $\psi$ | Relative bearing clearance $\left(\psi = \frac{2 \times C_R}{D}\right)$ | 1 |
| $\omega$ | Angular velocity ($\omega = 2 \times \pi \times N$) | s$^{-1}$ |

# 5 Method of calculation

## 5.1 General

This part of ISO 12167 covers the calculation, as well as the design, of hydrostatic plain journal bearings. In this case, calculation is understood to be the verification of the operational parameters of a hydrostatic bearing with known geometrical and lubrication data. In the case of a design calculation, with the given methods of calculation, it is possible to determine the missing data for the required bearing geometry, the lubrication data and the operational parameters on the basis of a few initial data (e.g. required load-carrying capacity, stiffness, rotational frequency).

In both cases, the calculations are carried out according to an approximation method based on the Hagen-Poiseuille and the Couette formulae, mentioned in Clause 3. The bearing parameters calculated according to this method are given as relative values in the form of tables and diagrams as a function of different parameters. The procedure for the calculation or design of bearings is described in 5.2 to 5.7. This includes the determination of different bearing parameters with the aid of the given calculation formulae or the tables and diagrams. The following calculation items are explained in detail:

a) determination of load-carrying capacity with and without taking into account shaft rotation;

b) calculation of lubricant flow rate and pumping power;

c) determination of frictional power with and without consideration of losses in the bearing recesses;

d) procedure for bearing optimization with regard to minimum total power loss.

For all calculations, it is necessary to check whether the important premise of laminar flow in the bearing clearance gap, in the bearing recess and in the capillary is met. This is checked by determining the Reynolds numbers. Furthermore, the portion of the inertia factor in the pressure differences shall be kept low at the capillary (see A.3.1).

If the boundary conditions defined in Clause 3 are observed, this method of calculation yields results with deviations which can be neglected for the requirements of practice, in comparison with an exact calculation by solving the Reynolds differential formula.

## 5.2 Load-carrying capacity

Unless indicated otherwise, it is assumed in the following that capillaries with a linear characteristic are used as restrictors and that the restrictor ratio is $\xi = 1$. Furthermore, the difference is only made between the two cases, "load in direction of recess centre" and "load in direction of land centre". For this reason, it is no longer mentioned in each individual case that the characteristic values are a function of the three parameters, "restrictor type", "restrictor ratio" and "load direction relative to the bearing".

Even under the abovementioned premises, the characteristic value of load carrying capacity [Formula (1)]

$$F^* = \frac{F}{B \times D \times p_{en}} = \frac{\bar{p}}{p_{en}} \qquad (1)$$

still depends on the following parameters:

— number of recesses, $Z$;

— width/diameter ratio, $B/D$;

— relative axial land width, $l_{ax}/B$;

— relative land width in circumferential direction, $l_c/D$;

— relative groove width, $b_G/D$;

— relative journal eccentricity, $\varepsilon$;

— relative frictional pressure when the difference is only made between the two cases, "load on recess centre" and "load on land centre":

$$\pi_f = \frac{\eta_B \times \omega}{p_{en} \times \psi^2} \qquad (2)$$

NOTE The Sommerfeld number, $So$, common with hydrodynamic plain journal bearings can be set up as follows:

$$So = \frac{\bar{p} \times \psi^2}{\eta_B \times \omega} = \frac{F^*}{\pi_f}$$

In ISO 12167-2:2001, Figures 1 and 2, the functions $F^*(\varepsilon, \pi_f)$ and $\beta(\varepsilon, \pi_f)$ are represented for $Z = 4$, $\xi = 1$, $B/D = 1$, $l_{ax}/B = 0{,}1$, $l_c/D = 0{,}1$, $b_G/D = 0{,}05$, i.e. restriction by means of capillaries, load in direction of centre of bearing recess.

These figures show the influence of rotation on the characteristic value of load-carrying capacity and the attitude angle.

For the calculation of a geometrically similar bearing, it is possible to determine the minimum lubricant film thickness when values are given, e.g. for $F$, $B$, $D$, $p_{en}$, $\omega$, $\psi$, and $\eta_B$ (determination of $\eta_B$ according to 5.6, if applicable).

All parameters are given for the determination of $F^*$ according to Formula (1) and $\pi_f$ according to Formula (2). For this geometry, the relevant values for $\varepsilon$ and $\beta$ can be taken from ISO 12167-2:2001, Figures 1 and 2 and thus, $h_{min} = C_R(1 - \varepsilon)$.

According to the approximation method described in Annex A, it transpires that the characteristic value of effective load-carrying capacity is no longer a function of the ratio $B/D$.

$$F_{\text{eff}}^{*} = \frac{F}{b_{\text{c}} \times Z \times b_{\text{ax}} \times P_{\text{en}}} = \frac{F^{*}}{\dfrac{b_{\text{c}}}{D} \times \dfrac{Z \times b_{\text{ax}}}{\pi \times B}} \tag{3}$$

If the resistance ratio

$$\kappa = \frac{R_{\text{lan, ax}}}{R_{\text{lan, c}}} = \frac{l_{\text{ax}} \times b_{\text{c}}}{l_{\text{c}} \times b_{\text{ax}}} \tag{4}$$

and the speed dependent parameter

$$K_{\text{rot}} = \frac{\xi \times \kappa \times \pi_{\text{f}} \times l_{\text{c}}}{D}$$
$$K_{\text{rot,nom}} = \frac{K_{\text{rot}}}{1+\kappa} \tag{5}$$

is introduced, there remains a dependence on the following parameters:

$$F_{\text{eff}}^{*}\left(Z, \varphi_{\text{G}}, \kappa, K_{\text{rot}}, \varepsilon\right)$$

If, in addition, advantage is taken of the fact that the function $F_{\text{eff}}^{*}(\varepsilon)$ is nearly linear for $\varepsilon \leq 0{,}5$, then it is practically sufficient to know that the function $F_{\text{eff}}^{*}\left(\varepsilon = 0{,}4\right) = f\left(Z, \varphi_{\text{G}}, \kappa, K_{\text{rot}}\right)$ for the calculation of the load carrying capacity.

For $K_{\text{rot}} = 0$, i.e. for the stationary shaft, the characteristic value of effective load-carrying capacity for $\varepsilon = 0{,}4$ only depends on three parameters:

$$F_{\text{eff}}^{*}\left(\varepsilon = 0{,}4\right) = f\left(Z, \varphi_{\text{G}}, \kappa\right)$$

Thus, in ISO 12167-2:2001, Figure 3, $F_{\text{eff,0}}^{*}\left(\varepsilon = 0{,}4\right)$ for $Z = 4$ and 6 can be given via $\kappa$ for different $\varphi_{\text{G}}$ values.

The influence of the rotational movement on the characteristic value of load-carrying capacity is taken into account by the ratio $\dfrac{F_{\text{eff}}^{*}}{F_{\text{eff,0}}^{*}} = f\left(Z, \varphi_{\text{G}}, \kappa, K_{\text{rot}}\right)$.

For $Z = 4$, the ratio $F_{\text{eff}}^{*} / F_{\text{eff,0}}^{*}$ is shown in ISO 12167-2:2001, Figure 4. The hydrodynamically conditioned increase of the load-carrying capacity can be easily recognized when presented in such a manner.

If, e.g. $Z$ and all parameters are given for the determination of $F_{\text{eff}}^{*}$ according to Formula (3), $\kappa$ according to Formula (4) and $K_{\text{rot}}$ according to Formula (5), then the minimum lubricant film thickness developing during operation can be determined.

After having calculated $\varphi_G$, $\kappa$ and $K_{rot, nom}$, the value for $F^*_{eff,0}\left(\varepsilon = 0,4\right)$ is taken from ISO 12167-2:2001, Figure 3 and the value for $F^*_{eff} / F^*_{eff,0}\left(\varepsilon = 0,4\right)$ from ISO 12167-2:2001, Figure 4, $F^*_{eff}$ is calculated according to Formula (3) and then the eccentricity is obtained as follows:

$$\frac{0,4 \times F^*_{eff}}{\frac{F^*_{eff}}{F^*_{eff,\,0}}\left(\varepsilon = 0,4\right) \times F^*_{eff,\,0}\left(\varepsilon = 0,4\right)}$$

and the minimum lubricant film thickness is $h_{min} = C_R \times (1 - \varepsilon)$.

### 5.3 Lubricant flow rate and pumping power

The characteristic value for the lubricant flow rate is given by

$$Q^* = \frac{Q \times \eta_B}{C_R^3 \times p_{en}} \tag{6}$$

It depends only slightly on the relative journal eccentricity $\varepsilon$, the load direction relative to the bearing and the relative frictional pressure $\pi_f$, or the speed dependent parameter $K_{rot}$.

By approximation, the lubricant flow rate can be calculated as follows (see also A.3.4):

$$Q^*\left(\varepsilon \le 0,5\right) \approx Q^*\left(\varepsilon = 0\right) = \frac{Z}{6\left(1+\xi\right)} \times \frac{B}{D} \times \frac{1 - \frac{l_{ax}}{B}}{\frac{l_c}{D}} \times \frac{\kappa + 1}{\kappa} \tag{7}$$

$$\frac{1}{1+\xi} = \frac{P_P}{p_{en}}\left(\varepsilon = 0\right) \text{ with } \xi = \frac{R_{cp}}{R_{P,0}} \text{ and } R_{P,0} = \frac{6 \times \eta_B \times l_{ax}}{b_{ax} \times C_R^3 \left(1+\kappa\right)}$$

The flow resistance of the capillaries according to A.3.2.2 is given by

$$R_{cp} = \frac{128 \times \eta_{cp} \times l_{cp}}{\pi \times d_{cp}^4} \times \left(1 + a\right)$$

with the non-linear portion (inertia factor)

$$a = \frac{1,08}{32} \times \frac{4 \times Q \times \rho}{\eta_{cp} \times l_{cp} \times Z}$$

By converting Formula (6), the lubricant flow rate can be calculated when the parameters $\eta_B$, $C_R$, $p_{en}$, $\xi$, $B/D$, and $l_{ax}/B$ are given.

For optimized bearings, $Q^*$ shall be taken from ISO 12167-2:2001, Table 1. The pumping power, without considering the pump efficiency, is given by

$$P_p = Q \times p_{en} = Q^* \times \frac{p_{en}^2 \times C_R^3}{\eta_B} \tag{8}$$

According to the approximation method, $Q^*$ is again determined according to Formula (7), thus it is the characteristic value of both flow rate and pumping power.

## 5.4 Frictional power

The characteristic value for the frictional power is given by

$$P_{\mathrm{f}}^{*} = \frac{P_{\mathrm{f}} \times C_{\mathrm{R}}}{\eta_{\mathrm{B}} \times U^{2} \times B \times D} \tag{9}$$

Friction generates in the lands as well as in the recess area. The land area related to the total surface of the bearing $\pi \times B \times D$ is given by

$$A_{\mathrm{lan}}^{*} = \frac{2}{\pi} \times \left[ \frac{l_{\mathrm{ax}}}{B} \times \pi + Z \times \frac{l_{\mathrm{c}}}{D} \times \left( 1 - 2 \times \frac{l_{\mathrm{ax}}}{B} \right) - Z \times \frac{l_{\mathrm{ax}}}{B} \times \frac{b_{\mathrm{G}}}{D} \right]$$

According to the approximation method, the characteristic value for the frictional power in the land area is given by

$$P_{\mathrm{f,lan}}^{*} = \frac{\pi}{\sqrt{1 - \varepsilon^{2}}} \times A_{\mathrm{lan}}^{*}$$

and in the recess area

$$P_{\mathrm{f,P}}^{*} = \pi \times 4 \times \frac{C_{\mathrm{R}}}{h_{\mathrm{p}}} \times \left( 1 - A_{\mathrm{lan}}^{*} \right).$$

Thus, the characteristic value for the total amount of friction is given by

$$P_{\mathrm{f}}^{*} = \pi \times A_{\mathrm{lan}}^{*} \times \left[ \frac{1}{\sqrt{1 - \varepsilon^{2}}} + \frac{4 \times C_{\mathrm{R}}}{h_{\mathrm{p}}} \times \left( \frac{1}{A_{\mathrm{lan}}^{*}} - 1 \right) \right] \tag{10}$$

The actual frictional power is obtained by converting Formula (9):

$$P_{\mathrm{f}} = P_{\mathrm{f}}^{*} \times \frac{\eta_{\mathrm{B}} \times U^{2} \times B \times D}{C_{\mathrm{R}}}$$

## 5.5 Optimization

When optimizing according to the power consumption, the total power loss, i.e. the sum of pumping and frictional power, is minimized. According to 5.3 and 5.4, the total power is given by

$$P_{\text{tot}} = P_{\text{p}} + P_{\text{f}} = Q^{*} \times \frac{p_{\text{en}}^{2} \times C_{\text{R}}^{3}}{\eta_{\text{B}}} + P_{\text{f}}^{*} \times \frac{\eta_{\text{B}} \times U^{2} \times B \times D}{C_{\text{R}}}$$

With Formulae (8) and (9), this can be written as follows:

$$P_{\text{tot}} = F \times \omega \times C_{\text{R}} \times \frac{Q^{*}}{4 \times \frac{B}{D} \times F^{*} \times \pi_{\text{f}}} \times \left(1 + \frac{P_{\text{f}}}{P_{\text{p}}}\right) \tag{11}$$

Following a proposal of Vermeulen,[2] the ratio of frictional power to pumping power is introduced as an optional parameter and designated with $P^*$. Thus, the characteristic value for the total power loss is given by

$$P_{\text{tot}}^{*} = \frac{P_{\text{tot}}}{F \times \omega \times C_{\text{R}}} = \frac{Q^{*} \times \left(1 + P^{*}\right)}{4 \times \frac{B}{D} \times F^{*} \times \pi_{\text{f}}} \tag{12}$$

Serial calculations have shown that the power minimum which can be obtained in the relatively wide range, $P^*$ = 1 to 3, depends only slightly on the chosen power ratio, $P^*$. An approximated optimization with the mean value $P^* = 2$ may be carried out.

The relative frictional pressure in Formula (12) cannot be chosen freely, as it is linked to the chosen power ratio, $P^*$:

$$P^{*} = \pi_{\text{f}}^{2} \times 4 \times \frac{B}{D} \times \frac{P_{\text{f}}^{*}}{Q^{*}} \quad \text{or} \quad \pi_{\text{f}} = \frac{1}{2} \times \sqrt{\frac{P^{*} \times Q^{*}}{P_{\text{f}}^{*} \times \frac{B}{D}}} \tag{13}$$

When $P^*$, $B/D$, $\varepsilon$, $\eta_{\text{P}}/C_{\text{R}}$ and $\xi$ are given, the characteristic value of total power according to Formula (12) to be minimized remains only a function of $Z$, $l_{\text{ax}}/B$, $l_{\text{c}}/D$ and $b_{\text{G}}/D$.

In ISO 12167-2:2001, Figures 5 to 12, $P_{\text{tot}}^{*}$ for $P^* = 2$, $b_{\text{G}}/D = 0{,}05$, $\xi = 1$, $\varepsilon = 0{,}4$ is presented for different $B/D$ and $Z$ as a function of $l_{\text{ax}}/B$, $l_{\text{c}}/D$ and $l_{\text{c}}/B$ respectively, taking into account the friction in the recesses. The land widths, $l_{\text{ax}}/B$ and $l_{\text{c}}/B[l_{\text{c}}/D = (l_{\text{c}}/B) \times (B/D)]$, where the total power is reduced to a minimum, result from these figures.

The optimum land widths and the associated values for $B/D$ = 1 to 0,3, as well as the numbers of recesses $Z$ = 4 to 10 obtained by this, are given in ISO 12167-2:2001, Table 1.

With decreasing width, $P_{\text{tot}}^{*}$ and thus, the total need of power increases. For high rotational frequencies and a given diameter, it may, however, be advantageous to use a plain bearing with smaller bearing width.

In the case where the shaft is at a standstill or rotating very slowly, the optimization method with $P^*$ = 1 to 3 can no longer be applied; see Reference [2]. In this case, the pumping power has to be minimized and thus, relatively wide lands are obtained. Therefore, the approximation method also fails and the Reynolds differential formula has to be solved by means of a finite method.

For a bearing with $Z = 4$, $B/D = 1$ the following land widths are recommended as being optimal $l_{\text{ax}}/B = l_{\text{c}}/B = 0{,}25$.

For $\varepsilon = 0{,}4$, the following values can be used for the calculation, $F^* = 0{,}174$ and $Q^* = 1{,}48$.

## 5.6 Temperatures and viscosities

With $\varepsilon = 0$, heating in the capillaries due to dissipation is calculated as follows (heat exchange between lubricant and environment is not considered here):

$$\Delta T_{cp} = \frac{p_{en} - p}{c_p \times \rho} = \frac{p_{en}}{c_p \times \rho} \times \frac{\xi}{1 + \xi}$$

and heating in the bearing, again with $\varepsilon = 0$, as follows:

$$\Delta T_B = \frac{p}{c_p \times \rho} + \frac{P_f}{c_p \times \rho \times Q} = \frac{p_{en}}{c_p \times \rho} \times \left( \frac{1}{1 + \xi} + P^* \right)$$

Thus the mean temperature in the capillaries is given by

$$T_{cp} = T_{en} + \frac{1}{2} \times \Delta T_{cp} \tag{14}$$

and the mean temperature in the bearing

$$T_B = T_{en} + \Delta T_{cp} + \frac{1}{2} \times \Delta T_B \tag{15}$$

It is assumed for the effective viscosities in the capillaries and bearing that $\eta_{cp} = \eta\ (T_{cp})$ and $\eta_B = \eta(T_B)$.

If the dependence of the viscosity on temperature is not completely known, the viscosities, $\eta_{cp}$ and $\eta_B$, can be approximated, following the statement of Reynolds. A precondition is that two viscosities, $\eta_1$ and $\eta_2$, are known at two temperatures, $T_1$ and $T_2$, which should be close to the estimated temperatures, $T_{cp}$ and $T_B$.

$$\eta_{cp} = \eta_1 \times \exp\left[ -\gamma \times \left( T_{cp} - T_1 \right) \right]; \quad \eta_B = \eta_1 \times \exp\left[ -\gamma \times \left( T_B - T_1 \right) \right]$$
$$\text{with } \gamma = \frac{1}{T_2 - T_1} \times \ln \frac{\eta_1}{\eta_2} \tag{16}$$

If only the viscosity class in accordance with ISO 3448 is known, then the course of viscosity for common lubrication oils having a viscosity index of about 100 can be calculated only on the basis of the nominal viscosity, $\eta_{40}$ (dynamic viscosity at 40 °C):

$$\eta(T) = \eta_{40} \times \exp\left[ 160 \times \ln\left( \frac{\eta_{40}}{0{,}18 \times 10^{-3}} \right) \times \left( \frac{1}{T + 95} - \frac{1}{135} \right) \right] \tag{17}$$

Temperature, $T$, shall be taken in degrees Celsius (°C). The dynamic viscosity, $\eta_{40}$, is obtained by multiplying the kinematic viscosity, $\eta_{40}$, based on the viscosity classes, by the density, $\rho$. If this value is not exactly known, it can be calculated by approximation with $\rho = 900$ kg/m$^3$.

Formula (17) is based on the statement of Vogel and empirically determined constants of Cameron and Rost and was transposed by Rodermund[3] to the nominal viscosity at 40 °C.

## 5.7 Minimum pressure in recesses

With high rotational frequencies and high $K_{rot}$ values according to Formula (5), the pressure in the recess, $p_{min}$, on the no-load side of the plain bearing may decrease to zero. Whereas, the recess pressure, $p_{max}$, on the load side may become greater than $p_{en}$. The minimum recess pressure, as well as $F^*$, depends on several variables. For the ratio applies

$$\frac{p_{\text{min}}}{p_{\text{en}}}\left(Z, \varphi_{\text{G}}, \kappa, K_{\text{rot}}\right)$$

In ISO 12167-2:2001, Figure 13, the minimum relative recess pressure over $K_{rot, nom}$ is shown for $Z = 4$, $\varepsilon = 0{,}4$, $\kappa = 1$ to 2 and two $\varphi_G$ values.

# Annex A
(normative)

# Description of the approximation method for the calculation of hydrostatic plain journal bearings

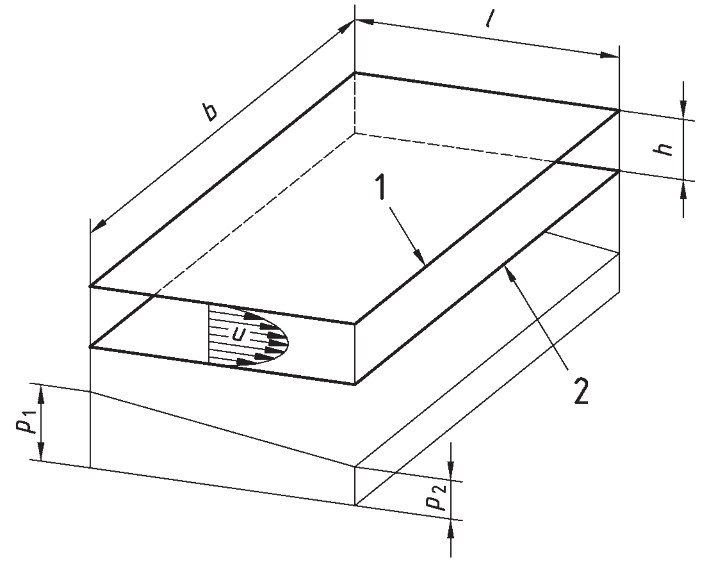
## A.1 General

The calculation is based on an approximation method leading to an almost exact result, especially in cases where small lands are provided (e.g. shaft rotating at high speed). In the case of wider lands, the Reynolds differential formula shall be solved by means of differential formulae.

## A.2 Fundamentals

### A.2.1 General

The approximation method assumes laminar flow, free of inertia, and uses two basic formulae for the flows via the lands (see Figures A.1 and A.2).



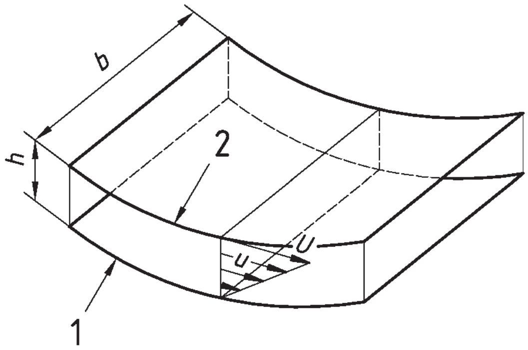

**Key**

1 bearing

2 shaft

**Figure A.1 — Pressure flow between parallel plates**




**Key**

1 bearing

2 shaft

**Figure A.2 — Drag flow due to shaft rotation**

### A.2.2 Hagen-Poiseuille formula

Pressure flow between parallel plates: ($b >> h$)

$$Q = \frac{\left(p_2 - p_1\right) \times b \times h^3}{12 \times \eta \times l}$$

### A.2.3 Couette formula

Drag flow due to shaft rotation:

$$Q = b \times \frac{U \times h}{2}$$

### A.2.4 Further assumptions

a) The pressure is constant over the recess area.

b) The viscosity in the bearing and in the restrictors is constant.

c) Shaft and bearing are rigid, their axes always parallel.

d) For the calculation of the lubricant flow rates, it is assumed that the outlet width extends up to the centre of the adjacent lands and that the pressure drop over the outlet length is linear.

e) For the calculation of the load effects, it is assumed that the pressure in the recesses spreads up to the centre of the adjacent lands.

## A.3 Calculations

### A.3.1 General

At first, the pressures in the recesses are calculated with the aid of the continuity formula for a certain shaft position, defined by

$e$ = eccentricity

$\varepsilon = e/C_R$

$\beta$ = attitude angle

All other parameters are derived from the pressures in the recesses.

The calculation is iterative as the attitude angle $\beta$ is not known in the beginning. This angle is to be varied until the result of the pressures in the recesses and the load has the same direction (see Figure A.3).

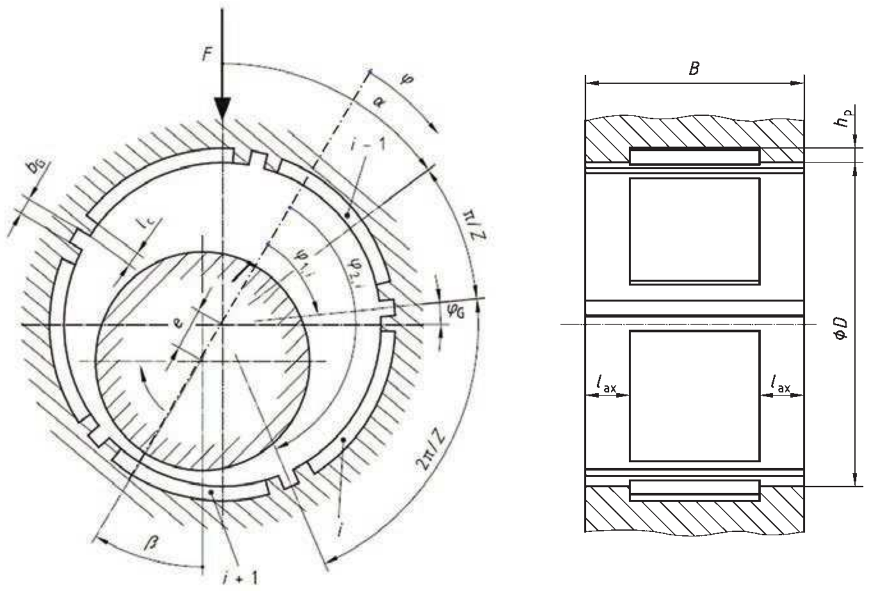



**Figure A.3 — Bearing geometry**

In principle, a vertical load is assumed for the calculation. However, this is no restriction as it can be assumed that the bearing is to be mounted appropriately for other directions of load.

The segment, $i$ ($i$ = 1, 2, ... $Z$), starts at the angle $\varphi_{1,i}$ (centre of drainage groove) and ends at the angle $\varphi_{2,i}$ (centre of next drainage groove), as shown in Figure A.3. Angles $\varphi$, $\varphi_{1,i}$ ,and $\varphi_{2,i}$ are measured from radius opposite to eccentricity, $e$, as shown in Figure A.3.

The centre of the first recess is situated at $a$. The initial angle and the end angle are

$$\varphi_{1,i} = \alpha - \beta + \frac{2\times\pi}{Z}\times\left(i - \frac{3}{2}\right)$$

$$\varphi_{2,i} = \alpha - \beta + \frac{2\times\pi}{Z}\times\left(i - \frac{1}{2}\right)$$

The film thickness, $h$, changes in the land area according to $h = C_R(1 + \varepsilon \times \cos\varphi)$.

### A.3.2 Pressures in the recesses

**A.3.2.1** The continuity principle is used for each recess. The separate recesses are decoupled by axial grooves so that the determination of pressure in the recesses does not depend on the pressure of the adjacent recesses. The grooves themselves are pressureless.

**A.3.2.2** The lubricant flow rate via preresistance ($\varepsilon = 0$) is given by

$$\frac{Q}{Z} = \frac{\left(p_{\text{en}} - p\right)^{k}}{R_{\text{cp}}}$$

where $k = 1$ corresponds to a linear resistance law.

Example of capillary with laminar flow:

$$Re_{\text{cp}} = \frac{\bar{w} \times d_{\text{cp}} \times \rho}{\eta_{\text{cp}}} < 2\,300$$

and with negligible portion of the term of inertia $\frac{\rho}{2} \times \bar{w}^2$.

$k = \frac{1}{2}$ corresponds to a square-law dependency, e.g. of an orifice, the flow coefficient of which can be regarded as independent of the Reynolds number.

When dimensioning a capillary, the portion of the term of inertia shall be kept low and, if applicable, be taken into account. According to the theory of Schiller[4], the pressure drop necessary to generate the velocity, $\bar{w} = \frac{4 \times Q}{Z \times \pi \times d_{\text{cp}}^2}$, at a properly rounded inlet (rounding off radius >0,3 × $d_{\text{cp}}$), is $\Delta p_{\text{en}} = 2{,}16 \times \frac{\rho}{2} \times \bar{w}^2$.

The flow resistance of the capillaries is then

$$R_{\text{cp}} = \frac{p_{\text{en}} - p_i}{\frac{Q}{Z}} = \frac{\Delta p_{\text{lan}}}{\frac{Q}{Z}} + \frac{\Delta p_{\text{en}}}{\frac{Q}{Z}} = \frac{128 \times \eta_{\text{cp}} \times l_{\text{cp}}}{\pi \times d_{\text{cp}}^4} + \frac{2{,}16 \times \frac{\rho}{2} \times \bar{w}^2}{\bar{w} \times \frac{\pi}{4} \times d_{\text{cp}}^2}$$

$$R_{\text{cp}} = \frac{128 \times \eta_{\text{cp}} \times l_{\text{cp}}}{\pi \times d_{\text{cp}}^4} \times \left(1 + a\right)$$

where $Re_{\text{cp}} = \frac{4 \times Q \times \rho}{Z \times \pi \times d_{\text{cp}} \times \eta_{\text{cp}}}$

and $a = \frac{1,08}{32} \times Re_{cp} \times \frac{d_{cp}}{l_{cp}} = \frac{1,08}{32} \times \frac{4 \times Q \times \rho}{Z \times \eta_{cp} \times l_{cp} \times \pi}$

The portion of the non-linear term, $a$ (inertia factor), has the effect that the exponent $k < 1$ in the abovementioned formula for $Q/Z$. Exponent $k$ can be calculated by approximation as follows:

$$k = \frac{1 + a}{1 + 2 \times a}$$

Without greater errors, it is permitted to take $a$ = 0,1 to 0,2 and to calculate with exponent $k$ = 1. With regard to the different lubricant flow rates in the particular recesses ($\varepsilon \neq 0$), a Reynolds number of $Re_{cp}$ = 1 000 to 1 500 shall not be exceeded.

**A.3.2.3** The volume flow from recess $i$ in the axial direction is

$$Q_{ax,i} = 2 \times \int_{\varphi_{1,i} + \varphi_G}^{\varphi_{2,i} - \varphi_G} \frac{h^3}{12 \times \eta_B} \times \frac{p_i}{l_{ax}} \times \frac{D}{2} \times d\varphi$$

where $\varphi_G = \frac{l_c + b_G}{D}$

$h$ is not constant due to the shaft eccentricity.

If, for the initial angle $\varphi'_{1,i} = \varphi_{1,i} + \varphi_G$ or for the end angle $\varphi'_{2,i} = \varphi_{2,i} - \varphi_G$ and

$$a_i = \int_{\varphi'_{1,i}}^{\varphi'_{2,i}} \frac{h^3}{C_R^3} d\varphi = \int_{\varphi'_{1,i}}^{\varphi'_{2,i}} \left(1 + \varepsilon \cos\varphi\right)^3 \times d\varphi$$

$$= \left[ \left(\varphi'_2 - \varphi'_1\right) \times \left(1 + \frac{3}{2} \times \varepsilon^2\right) + \left(\sin\varphi'_2 - \sin\varphi'_1\right) \times \left(3\varepsilon + \varepsilon^3\right) + \frac{3}{4} \times \varepsilon^2 \times \left(\sin 2\varphi'_2 - \sin 2\varphi'_1\right) - \frac{\varepsilon^3}{3} \times \left(\sin^3\varphi'_2 - \sin^3\varphi'_1\right) \right]_i$$

then

$$Q_{ax,i} = \frac{C_R^3 \times D}{12 \times \eta_B \times l_{ax}} \times a_i \times p_i$$

**A.3.2.4** When the volume flow rate in the circumferential direction is calculated, a flow between parallel plates with film thicknesses of $h_{en,i} = h\left(\varphi'_{1,i}\right)$ or $h_{ex,i} = h\left(\varphi'_{2,i}\right)$ is assumed as an approximation.

In the circumferential direction, for the volume flow flowing in:

$$Q_{en,i} = b_c \times \left( \frac{U \times h_{en,i}}{2} - \frac{p_i \times h_{en,i}^3}{12 \times \eta_B \times l_c} \right)$$

and by analogy, the following applies to the volume flow flowing out:

$$Q_{ex,i} = b_c \times \left( \frac{U \times h_{ex,i}}{2} + \frac{p_i \times h_{ex,i}^3}{12 \times \eta_B \times l_c} \right)$$

**A.3.2.5** According to Figure A.4, the continuity equation for recess $i$ results in $Q_{cp,i} = Q_{ax,i} + Q_{ex,i} - Q_{en,i}$.

$Q_{cp,i}$ is the rate of lubricant flow from the capillary restrictor into the recess $i$ and is given by Hagen-Poiseuille law as follows:

$$Q_{cp,\,i} = \left(p_{en} - p_i\right)/R_{cp}$$

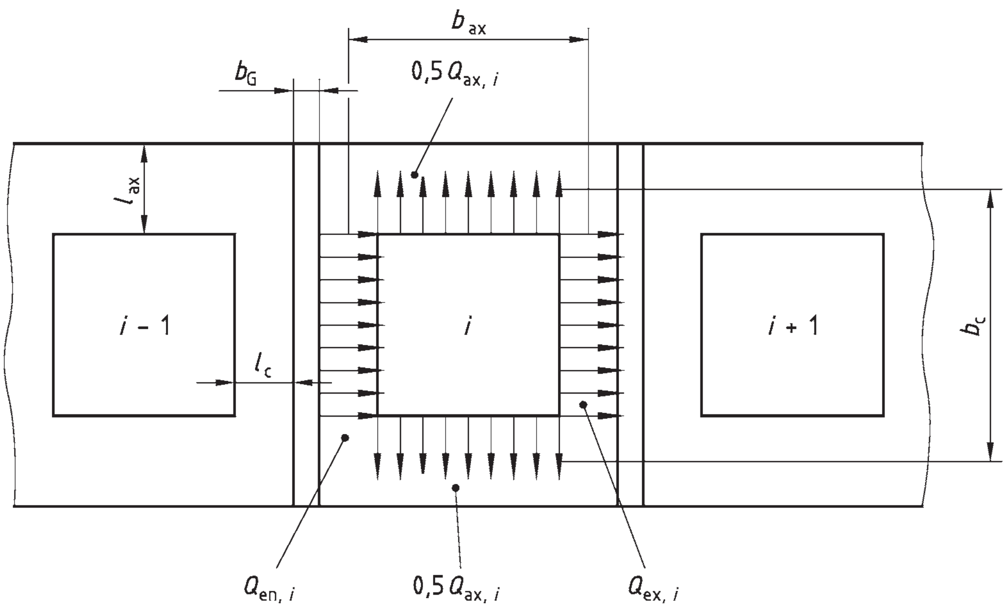



**Figure A.4 — Volume flows for one recess**

If $f_{en,i} = 1 + \varepsilon \times \cos\varphi'_{1,i}$ and $f_{ex,i} = 1 + \varepsilon \times \cos\varphi'_{2,i}$

$$\omega = 2 \times \pi \times N = \frac{2 \times U}{D}$$

$$P_i^* = \frac{p_i}{p_{en}}$$

$$K_{rot} = \frac{\eta_B \times \omega \times \xi \times \kappa \times l_c}{p_{en} \times \psi^2 \times D} = \text{ speed dependent parameter}$$

where

$$\psi = \frac{2 \times C_R}{D} = \text{relative bearing clearance}$$

$$\kappa = \frac{R_{P,ax}}{R_{P,c}} = \frac{l_{ax} \times b_c}{l_c \times b_{ax}} = \text{resistance ratio}$$

$$R_{P,0} = \frac{R_{P,ax}}{2\left(1+\kappa\right)} = \frac{R_{P,c} \times \kappa}{2\left(1+\kappa\right)}$$

$$\xi = \frac{R_{cp}}{R_{P,0}} = \frac{R_{cp} \times b_{ax} \times C_R^3}{6 \times \eta_B \times l_{ax}} \times (1+\kappa) = \text{restrictor ratio}$$

then the pressure $p_i^*$ in the recess $i$ results from

$$p_i^* = \frac{\left[1 - \frac{6K_{\text{rot}}}{1+\kappa}\left(f_{\text{ex},i} - f_{\text{en},i}\right)\right]}{\left\{1 + \frac{a_{i\xi}}{2(1+\kappa)\left(\frac{\pi}{Z} - \varphi_{\text{G}}\right)} + \frac{\kappa\xi\left(f_{\text{en},i}^3 + f_{\text{ex},i}^3\right)}{2(1+\kappa)}\right\}}$$

Thus, the relative pressures in the recesses and all further bearing parameters are determined by:

a) restrictor ratio, $\xi$;

b) bearing geometry:

— number of recesses, $Z$,

— form and position of recesses, ($x$, $\alpha$),

— position of journal, ($\varepsilon$, $\beta$);

c) speed dependent parameter, $K_{\text{rot}}$.

The angle $\beta$ is determined iteratively in the course of the calculation.

### A.3.3 Load *F*, attitude angle *β*, stiffness *c*

The radial load effect on recess $i$ in accordance with Figure A.5 is given by

$$F_i = b_{\text{c}} \times \int_{-\frac{\pi}{Z} - \varphi_G}^{\frac{\pi}{Z} - \varphi_G} p_i \cos\delta \times \frac{D}{2} \times d\delta = b_{\text{c}} \times p_i \times D \times \sin\left(\frac{\pi}{Z} - \varphi_G\right)$$

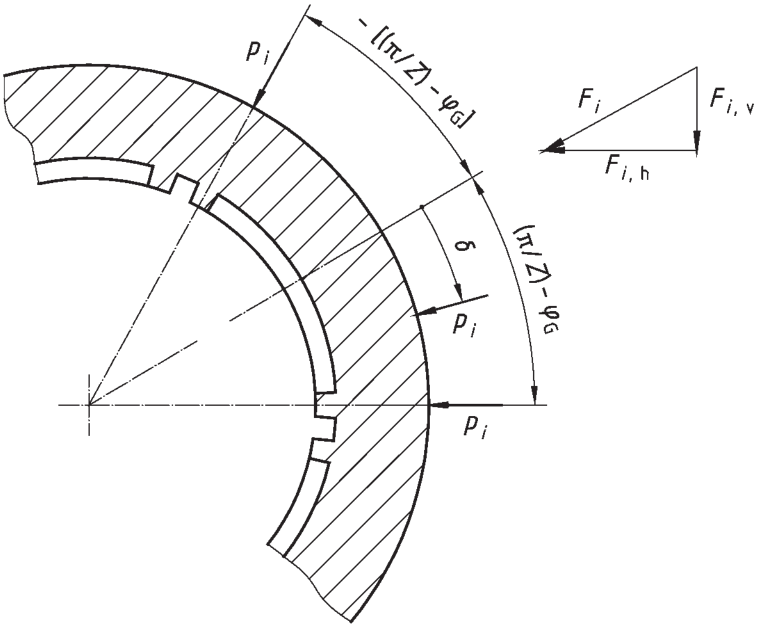



**Figure A.5 — Application of load to one recess**

The direction of $F_i$ is given by

$$\bar{\varphi}_i = \frac{\varphi_{en,i} + \varphi_{ex,\,i}}{2}$$

The horizontal component is the sum of all horizontal projections of $F_i$ is given by

$$F_h = b_c \times D \times \sin\left(\frac{\pi}{Z} - \varphi_G\right) \times \sum_{i=1}^{Z} p_i \times \sin\left(\bar{\varphi}_i + \beta\right)$$

Correspondingly, for the vertical component

$$F_v = b_c \times D \times \sin\left(\frac{\pi}{Z} - \varphi_G\right) \times \sum_{i=1}^{Z} p_i \times \cos\left(\bar{\varphi}_i + \beta\right)$$

where

$$\bar{\varphi}_i + \beta = a + \frac{2\pi}{Z} \times (i-1)$$

The total load is $F = \sqrt{F_h^2 + F_v^2}$

Angle of the resulting force $\varphi_F = \arctan \frac{F_h}{F_v}$

In case of vertical load, the attitude angle for each $\varepsilon$ should be modified in such a way that $\varphi_F = 0$. If the load-carrying capacity $F$ is not applied vertically but at an angle $\varphi_F$ to a perpendicular line, then the results for vertical direction of load can be applied when mounting the plain bearing at the angle $\varphi_F$.

The stiffness $c$ can generally be defined in different ways.

Here, the following definition is used:

$$c = \frac{F}{e} = \frac{F}{\varepsilon \times C_R}$$

### A.3.4 Lubricant flow rate and pumping power

The total lubricant flow rate can be calculated on the basis of the sum of the flow rates through the restrictors, $Q_{cp,\,i}$:

$$Q = \sum_{i=1}^{Z} Q_{cp,i} = \frac{Z \times p_{en} - \sum p_i}{R_{cp}}$$

The lubricant flow rate can also be approximated according to Formula (7).

Pumping power, $P_p = Q \times p_{en}$

### A.3.5 Frictional power

The frictional power is composed of

a) friction in the land area, and

b) friction in the recesses due to secondary flow.

The land area is given by

$$A_{lan} = 2 \times \pi \times l_{ax} \times D + 2 \times Z \times l_c \times (B - 2 \times l_{ax}) - 2 \times Z \times b_G \times l_{ax}$$

$$A_{lan}^* = \frac{A_{lan}}{\pi \times B \times D} = \frac{2}{\pi} \times \left[ \frac{l_{ax}}{B} \times \pi + Z \times \frac{l_c}{D} \times \left( 1 - 2 \times \frac{l_{ax}}{B} \right) - Z \times \frac{b_G}{D} \times \frac{l_{ax}}{B} \right]$$

The shearing stress at the shaft surface is, in general, given by

$$\tau = \eta_B \left( \frac{\partial u}{\partial y} \right)_{y=h} = \frac{1}{2} \times \frac{\partial p}{\partial x} \times h + \frac{U}{h} \times \eta_B$$

As an approximation, the shearing stress $\tau$ is calculated as follows without taking into account the pressure flow rate.

$$\tau = \frac{U}{h} \times \eta_B$$

The result for the land friction is finally given by

$$P_{f,lan} = \int_{A_{lan}} \tau \times U \times dA = \frac{\eta \times U^2}{C_R} \times \int_{A_{lan}} \frac{dA}{1 + \varepsilon \times \cos\varphi}$$

If it is assumed that the lands are uniformly distributed over the periphery, it can be simplified as follows:

$$P_{f,lan} = \frac{\eta \times U^2}{C_R} \times \frac{A_{lan}}{\sqrt{1 - \varepsilon^2}}$$

Although the depth of recess $h_P >> h$ according to Shinkle and Hornung[5], the friction due to the secondary flow in the recesses shall be included in the calculation for shafts running at high speed. This applies especially to wide recesses and small lands.

When the flow in the recesses is still laminar, i.e.

$$Re_p = \frac{U \times h_p \times \rho}{\eta_B} < 1\,000$$

then the friction in the recesses is calculated as follows:

$$P_{f,P} = 4 \times \frac{\eta \times U^2}{h_p} \times A_p$$

where

$$A_p = \pi \times B \times D - A_{lan}$$

When $Re_P > 1\,000$, then the flow is turbulent and the friction increases correspondingly. In that case, the preceding formula for $\tau$ can no longer be used.

### A.3.6 Formulae for dimensioning

The following formulae can be used to determine the dimensions when stiffness $c$ is given:

$$C_\mathrm{R} = \frac{F}{\varepsilon \times c}$$

$$D^2 \times p_\mathrm{en} = \frac{F}{\frac{B}{D} \times F^*}$$

$$\frac{p_\mathrm{en}}{\eta_\mathrm{B}} = \frac{F \times \omega}{C_\mathrm{R}^2} \times \frac{1}{4 \times \frac{B}{D} \times F^* \times \pi_\mathrm{f}}$$

# Annex B
(informative)

# Example of calculation according to the method given in Annex A

## B.1 Example 1 — Calculation of a hydrostatic journal bearing

### B.1.1 General

A bearing with four recesses with given dimensions and operational data are to be examined. The lubricant oil, ISO VG 46, and the temperature in front of the bearing are also stated. The amount of oil, power, stiffness, film thickness, etc. are to be calculated. The following parameters are given.

### B.1.2 Dimensions

— Bearing diameter, $D = 0{,}12$ m

— Bearing width, $B = 0{,}12$ m

— Width of circumferential outlet, $b_c = 0{,}108$ m

— Axial land length, $l_{ax} = 0{,}012$ m

— Circumferential land length, $l_c = 0{,}012$ m

— Width of drainage groove, $b_G = 0{,}006$ m

— Depth of recess, $h_p = 40\ C_R$ m

— Number of recesses, $Z = 4$

— Diameter of capillaries, $d_{cp} = 0{,}003\ 25$ m

— Length of capillaries, $l_{cp} = 1{,}14$ m

— Relative bearing clearance, $\psi = 1{,}5 \times 10^{-3}$

— Radial clearance, $C_R = \psi \times \dfrac{D}{2} = 90 \times 10^{-6}$ m

### B.1.3 Operational data

— Load-carrying capacity (load), $F = 20\ 000$ N

— Rotational frequency (speed), $N = 16{,}66\ s^{-1}$ ($\omega = 104{,}7\ s^{-1}$)

— Inlet temperature, $T_{en} = 45$ °C

— Feed pressure, $p_{en} = 60$ bar $= 6 \times 10^6$ Pa

### B.1.4 Lubricant data

For oil ISO VG 46:

| $T$<br>°C | $h$<br>Pa·s |
|---|---|
| 40 | 0,041 40 |
| 50 | 0,026 58 |
| 60 | 0,018 07 |

— Volume specific heat, $c_p \cdot \rho = 1{,}75 \times 10^6$ J/m³·K

— Density, $\rho = 900$ kg/m³

Exponent, calculated on the basis of the lubricant data, $\gamma = \frac{1}{10} \times \ln \frac{\eta_{40}}{\eta_{50}} = \frac{1}{10} \times \ln \frac{0{,}041\,4}{0{,}026\,58} = 0{,}044\,3$

These data are used to calculate the parameters listed in B.1.5 to B.1.18.

### B.1.5 Temperatures and dynamic viscosities

The frictional power is not yet known from the first calculation. It is therefore approximated as follows with ($\xi = 1$, $P^* = 0$).

$$\Delta T_{cp} = \frac{p_{en}}{c_p \times \rho} \times \left( \frac{\xi}{1+\xi} \right) = \frac{6 \times 10^6}{1{,}75 \times 10^6} \times \left( \frac{1}{1+1} \right) = 1{,}7 \text{ K}$$

$$\Delta T_B = \frac{p_{en}}{c_p \times \rho} \times \left( \frac{1}{1+\xi} + P^* \right) = \frac{6 \times 10^6}{1{,}75 \times 10^6} \times \left( \frac{1}{1+1} + 0 \right) = 1{,}7 \text{ K}$$

$$T_{cp} = T_{en} + \frac{\Delta T_{cp}}{2} = 45 + \frac{1{,}7}{2} = 45{,}85 \text{ °C}$$

$$T_B = T_{en} + \Delta T_{cp} + \frac{\Delta T_B}{2} = 45 + 1{,}7 + \frac{1{,}7}{2} = 47{,}55 \text{ °C}$$

The dynamic viscosities are then given by

$$\eta_{cp} = \eta_{40} \times \exp\left[ -\gamma \times \left( T_{cp} - 40 \right) \right] = 0{,}041\,4 \times \exp\left[ -0{,}044\,3 \times \left( 45{,}85 - 40 \right) \right] = 0{,}031\,9 \text{ Pa} \cdot \text{s}$$

$$\eta_B = \eta_{40} \times \exp\left[ -\gamma \times \left( T_B - 40 \right) \right] = 0{,}041\,4 \times \exp\left[ -0{,}044\,3 \times \left( 47{,}55 - 40 \right) \right] = 0{,}029\,6 \text{ Pa} \cdot \text{s}$$

### B.1.6 Flow resistances

$$R_{cp} = \frac{128 \times \eta_{cp} \times l_{cp}}{\pi \times d_{cp}^4} \times (1+a) = \frac{128 \times 0{,}031\,9 \times 1{,}14}{\pi \times 0{,}003\,25^4} \times (1+0{,}2) = 1{,}594 \times 10^{10}\ \text{Ns/m}^5$$

The inertia factor $a$ cannot yet be calculated in this place, as the oil flow rate is not known. Therefore, it should be started with an estimated value and the exact value of $a$ determined iteratively. Here, the value has been taken from the following calculation.

$$R_{P,0} = \frac{6 \times \eta_B}{C_R^3} \times \frac{l_c/D}{b_c/B} \times \frac{D}{B} \times \frac{\kappa}{1+\kappa} = \frac{6 \times 0{,}029\,6 \times 0{,}1 \times 1 \times 1{,}43}{\left(90 \times 10^{-6}\right)^3 \times 0{,}9 \times 2{,}43} = 1{,}593 \times 10^{10}\ \text{Ns/m}^5$$

$k$ is calculated in B.1.9.

### B.1.7 Restrictor ratio

$$\xi = \frac{R_{cp}}{R_{P,0}} = \frac{1{,}594 \times 10^{10}}{1{,}593 \times 10^{10}} = 1{,}000\,6 \approx 1$$

### B.1.8 Pressure ratio in recesses ($\varepsilon$ = 0)

$$\frac{p_{i,0}}{p_{en}} = \frac{1}{1+\xi} = \frac{1}{1+1} = 0{,}5$$

### B.1.9 Resistance ratio

$$\kappa = \frac{l_{ax} \times b_c}{l_c \times b_{ax}} = \left(\frac{B}{D}\right)^2 \times \frac{\frac{l_{ax}}{B} \times \frac{1-l_{ax}}{B}}{\left(\frac{\pi}{Z} - \frac{l_c + b_G}{D}\right) \times \frac{l_c}{B}} = 1 \times \frac{0{,}1 \times 0{,}9}{0{,}635 \times 0{,}1} = 1{,}416$$

### B.1.10 Relative friction pressure

$$\pi_f = \frac{\eta_B \times \omega}{p_{en} \times \psi^2} = \frac{0{,}029\,6 \times 104{,}7}{6 \times 10^6 \times 1{,}5^2 \times 10^{-6}} = 0{,}229\,6$$

### B.1.11 Speed-dependent parameter

$$K_{rot} = \xi \times \kappa \times \pi_f \times \frac{l_c}{D} = 1 \times 1{,}416 \times 0{,}229\,6 \times \frac{0{,}012}{0{,}12} = 0{,}032\,5$$

According to ISO 12167-2:2001, Figure 4, the speed-dependent parameter in the case of $K_{rot}$ = 0,032 5 and $\varepsilon$ = 0,4 is negligible $\left(F_{eff}^* / F_{eff,0}^* \approx 1\right)$.

### B.1.12 Characteristic values of load carrying capacity and film thicknesses

According to ISO 12167-2:2001, Figure 3 for $\varepsilon = 0{,}4$, $\kappa = 1{,}416$, $\varphi_G = \frac{l_c + b_G}{D} = \frac{0{,}012 + 0{,}006}{0{,}12} = 0{,}15$,

the result is $F_{eff,0}^* = 0{,}357$.

It follows from the load-carrying capacity $F$,

$$F^* = \frac{F}{p_{en} \times B \times D} = \frac{20\ 000}{6 \times 10^6 \times 0{,}12^2} = 0{,}231 \text{ and with}$$

$$b_{ax} = \frac{\pi \times D}{Z} - (l_c + b_G) = \frac{\pi \times 0{,}12}{4} - 0{,}018 = 0{,}076\ 25 \text{ m and}$$

$$F_{eff}^* = \frac{F^* \times \pi \times D}{\left(1 - \frac{l_{ax}}{B}\right) \times b_{ax} \times Z} = \frac{0{,}231 \times \pi \times 0{,}12}{(1 - 0{,}1) \times 0{,}076\ 25 \times 4} = 0{,}317\ 92$$

### B.1.13 Eccentricity and film thickness

$$\varepsilon = \frac{0{,}4 \times F_{eff}^*}{\frac{F_{eff}^*}{F_{eff,0}^*}(\varepsilon = 0{,}4) \times F_{eff,0}^*(\varepsilon = 0{,}4)} = \frac{0{,}4 \times 0{,}317\ 92}{1 \times 0{,}357} = 0{,}356$$

minimum film thickness:

$$h_{min} = (1 - \varepsilon) \times C_R = (1 - 0{,}356) \times 90 = 58\ \mu\text{m}$$

### B.1.14 Frictional power

$$A_{lan}^* = \frac{\pi}{2} \times \left[\frac{l_{ax}}{B} \times \pi + Z \times \frac{l_c}{D} \times \left(1 - 2 \times \frac{l_{ax}}{B}\right) - Z \times \frac{l_{ax}}{B} \times \frac{b_G}{D}\right]$$

$$= \frac{2}{\pi} \times \left[\frac{0{,}012}{0{,}12} \times \pi + 4 \times \frac{0{,}012}{0{,}12} \times \left(1 - 2 \times \frac{0{,}012}{0{,}12}\right) - 4 \times \frac{0{,}012}{0{,}12} \times \frac{0{,}006}{0{,}12}\right] = 0{,}391$$

according to Formula (10):

$$P_f^* = \pi \times A_{lan}^* \times \left[\frac{1}{\sqrt{1 - \varepsilon^2}} + \frac{4 \times C_R}{h_p} \times \left(\frac{1}{A_{lan}^*} - 1\right)\right]$$

$$= \pi \times 0{,}391 \times \left[\frac{1}{\sqrt{1 - 0{,}356^2}} + \frac{4}{40} \times \left(\frac{1}{0{,}391} - 1\right)\right]$$

$$= 1{,}505\ 76$$

with $U = \omega \times \frac{D}{2} = 104{,}7 \times \frac{0{,}12}{2} = 6{,}28 \text{ m/s}$

$$P_f = P_f^* \times \frac{\eta_B \times U^2}{C_R} \times B \times D = 1{,}505\ 76 \times \frac{0{,}029\ 6 \times 6{,}28^2}{90 \times 10^{-6}} \times 0{,}12^2 = 281{,}2 \text{ W}$$

### B.1.15 Pumping power and lubricant flow rate

According to approximate Formula (7):

$$Q^* = \frac{Z}{6 \times (1 + \xi)} \times \frac{B}{D} \times \frac{1 - \frac{l_{ax}}{B}}{\frac{l_c}{D}} \times \frac{\kappa + 1}{\kappa} = \frac{4 \times 0{,}9 \times (1{,}416 + 1)}{6 \times 2 \times 0{,}1 \times 1{,}416} = 5{,}12$$

$$Q = Q^* \times \frac{C_R^3 \times p_{en}}{\eta_B} = 5{,}12 \times \frac{\left(90 \times 10^{-6}\right) \times 6 \times 10^6}{0{,}029\ 6} = 0{,}756 \times 10^{-3}\ m^3/s$$

$$P_p = Q \times p_{en} = 0{,}756 \times 10^{-3} \times 6 \times 10^6 = 4\,540\ W$$

$$R_{cp} = \frac{p_{en} - p_0}{Q/Z} = \frac{3 \times 10^{-6} \times 4}{0{,}756 \times 10^{-3}} = 1{,}587 \times 10^{10}$$

$$P^* = \frac{P_f}{P_p} = \frac{281{,}2}{4\ 540} = 0{,}062$$

$$P_{tot} = P_p + P_f = 4\,540 + 281{,}2 \approx 4\,821\ W$$

### B.1.16 Temperatures and dynamic viscosities

Since the frictional power is smaller compared to the pumping power ($P^*$ = 0,062), the temperatures calculated in B.1.5 are sufficiently exact.

### B.1.17 Reynolds numbers

In the recess:

$$Re_p = \frac{U \times h_p \times \rho}{\eta_B} = \frac{6{,}28 \times 40 \times 90 \times 10^{-6} \times 900}{0{,}029\ 6} = 687$$

$Re_p$ < 1 000 and is thus laminar.

In the capillaries:

$$Re_{cp} = \frac{4 \times Q \times \rho}{Z \times \pi \times d_{cp} \times \eta_{cp}} = \frac{4 \times 0{,}756 \times 10^{-3} \times 900}{4 \times \pi \times 3{,}25 \times 10^{-3} \times 31{,}9 \times 10^{-3}} = 2\ 089$$

$Re_{cp}$ < 2 300 and is thus laminar.

Inertia factor:

$$a = \frac{1{,}08}{32} \times Re_{cp} \times \frac{d_{cp}}{l_{cp}} = \frac{1{,}08}{32} \times 2\,089 \times \frac{0{,}003\,25}{1{,}14} = 0{,}2$$

NOTE The recommendation $Re_{cp}$ < 1 000 to 1 500 given in A.3.2.2 is not observed in this case. Hence, there follows a relatively high non-linear inertia factor, $a$.

### B.1.18 Measures for optimization

As $P_f$ << $P_p$ and thus, $P^*$ << 1 to 3, $\pi_f$ shall be increased, i.e. the clearance shall be decreased or the dynamic viscosity be increased. The optimum friction pressure is calculated and, with a given type of oil, the optimum clearance is calculated on this basis as follows:

Formula (13) with $P^* = 1$ results in

$$\pi_{f,opt} = \frac{1}{2} \times \sqrt{\frac{P^* \times Q^*}{P_f^* \times \frac{B}{D}}} = \frac{1}{2} \times \sqrt{\frac{1 \times 5{,}12}{1{,}505\,76 \times 1}} = 0{,}922$$

with $\pi_f = \frac{\eta_B \times \omega}{p_{en} \times \psi^2}$, one obtains

$$\psi_{opt} = \sqrt{\frac{\eta_B \times \omega}{p_{en} \times \pi_{f,opt}}} = \sqrt{\frac{0{,}029\,6 \times 104{,}7}{6 \times 10^6 \times 0{,}992}} = 0{,}75 \times 10^{-3}$$

$$C_R = \psi_{opt} \times \frac{D}{2} = 0{,}75 \times 10^{-3} \times \frac{0{,}12}{2} = 45\ \mu m$$

By approximation, i.e. without new calculation of $\eta_B$, $\varepsilon$ and $P_f^*$, the following power values are obtained:

$$P_f = P_f^* \times \frac{\eta_B \times U^2 \times B \times D}{C_R} = 1{,}51 \times \frac{0{,}029\,6 \times 6{,}28^2 \times 0{,}12}{45} = 564\ W$$

$$P_p = Q^* \times \frac{C_R^3 \times p_{en}^2}{\eta_B} = 5{,}09 \times \frac{(45 \times 10^{-6})^3 \times (6 \times 10^6)^2}{0{,}029\,6} = 564\ W$$

$$P_{tot} = P_f + P_p = 564 + 564 = 1\,128\ W$$

$$Q = \frac{P_p}{p_{en}} = \frac{564}{6 \times 10^6} = 9{,}4 \times 10^{-5}\ m^3/s = 0{,}094\ l/s$$

## B.2 Example 2 — Design of an optimized hydrostatic journal bearing

### B.2.1 General

An optimized bearing with four recesses, which shall be operated under a load $F$ = 3 000 N and at a rotational frequency (speed) of 50 $s^{-1}$ ($\omega$ = 314 $s^{-1}$) is to be designed. The lubricant given is oil ISO VG 32, the temperature in front of the bearing, $T_{en}$ = 45 °C.

### B.2.2 Given quantities

The following values are taken for the optimized bearing:

$B/D = 1$; $Z = 4$, $l_{ax}/B = 0{,}1$; $l_c/B = 0{,}1$; $b_G/D = 0{,}05$; $\varepsilon = 0{,}4$; $\xi = 1$; $P^* = 2$; $h_p = 40 \cdot C_R$.

The characteristic values indicated in ISO 12167-2:2001, Table 1 are valid for the following data:

$P_{tot}^* = 10{,}349$; $F^* = 0{,}285\,9$; $\pi_f = 1{,}288$; $Q^* = 5{,}08$; $P_f^* = 1{,}531$; $\beta = 23{,}41$

### B.2.3 Feed pressure and dimensions

$$D^2 \times p_{en} = \frac{F}{\frac{B}{D} \times F^*} = \frac{3\,000}{1 \times 0{,}285\,9} = 10\,493\,\text{N}$$

when $D = B = 0{,}06$ m, then

$$p_{en} = \frac{D^2 \times p_{en}}{D^2} = \frac{10\ 493}{0{,}06^2} = 2{,}91 \times 10^6\ Pa \approx 3 \times 10^6\ Pa$$

### B.2.4 Lubricant data

For the oil ISO VG 32:

| $T$<br>°C | $h$<br>Pa·s |
|---|---|
| 40 | 0,028 8 |
| 50 | 0,019 04 |

— Volume specific heat, $c_p \cdot \rho$ = 1,75 × $10^6$ J/$m^3$·K

— Density, $\rho$ = 900 kg/$m^3$

Exponent, calculated from the lubricant data $\gamma = \frac{1}{10} \times \ln \frac{\eta_{40}}{\eta_{50}} = \frac{1}{10} \times \ln \frac{0{,}028\,8}{0{,}019\,04} = 0{,}041\,4$

### B.2.5 Temperatures and dynamic viscosities

$$\Delta T_{cp} = \frac{p_{en}}{c_p \times \rho} \times \frac{\xi}{1 + \xi} = \frac{3 \times 10^6}{1{,}75 \times 10^6} \times \frac{1}{2} = 0{,}86\ K$$

$$\Delta T_B = \frac{p_{en}}{c_p \times \rho} \times \left( \frac{1}{1 + \xi} + P^* \right) = \frac{3 \times 10^6}{1{,}75 \times 10^6} \times \left( \frac{1}{2} + 2 \right) = 4{,}28\ \text{K}$$

$$T_{cp} = T_{en} + \frac{\Delta T_{cp}}{2} = 45 + \frac{0{,}86}{2} = 45{,}43\ °\text{C}$$

$$T_B = T_{en} + \Delta T_{cp} + \frac{\Delta T_B}{2} = 45 + 0{,}86 + \frac{4{,}28}{2} = 48\ °\text{C}$$

The dynamic viscosities are then given by

$$\eta_{cp} = \eta_{40} \times \exp\left[ -\gamma \times \left( T_{cp} - 40 \right) \right] = 0{,}028\,8 \times \exp\left[ -0{,}041\,4 \times 5{,}43 \right] = 0{,}023\ \text{Pa} \cdot \text{s}$$

$$\eta_B = \eta_{40} \times \exp\left[ -\gamma \times \left( T_B - 40 \right) \right] = 0{,}028\,8 \times \exp\left[ -0{,}041\,4 \times 8 \right] = 0{,}020\,68\ \text{Pa} \cdot \text{s}$$

### B.2.6 Bearing clearance

It follows from Formula (2)

$$\psi = \sqrt{\frac{\eta_B \times \omega}{p_{en} \times \pi_f}} = \sqrt{\frac{0{,}020\,68 \times 314}{3 \times 10^6 \times 1{,}288}} = 1{,}3 \times 10^{-3}$$

$$C_R = \psi \times \frac{D}{2} = 1,3 \times 10^{-3} \times \frac{0,06}{2} \times 10^6 = 39\,\mu\text{m}$$

$$h_{min} = C_R \times (1 - \varepsilon) = 39 \times (1 - 0,4) = 23,4\,\mu\text{m}$$

### B.2.7 Power losses and need of lubricant

Total power:

$$P_{tot} = P_{tot}^* \times F \times \omega \times C_R = 10,349 \times 3\,000 \times 314 \times 39 \times 10^{-6} = 380,2\ \text{W}$$

Pumping power:

$$P_p = \frac{P_{tot}}{1 + P^*} = \frac{380,2}{1+2} = 126,7\ \text{W}$$

Frictional power:

$$P_f = P^* \times P_p = 2 \times 126,7 = 253,4\ \text{W}$$

Need of lubricant:

$$Q = \frac{P_p}{P_{en}} = \frac{126,7}{3 \times 10^6} = 4,23 \times 10^{-5}\,\text{m}^3/\text{s} \approx 0,0423\ \text{l/s}$$

### B.2.8 Flow resistance and dimensions of capillaries

Resistance of capillaries:

$$R_{cp} = \frac{P_{en} \times Z}{Q} \times \frac{\xi}{1+\xi} = \frac{3 \times 10^6 \times 4}{4,23 \times 10^{-5}} \times \frac{1}{2} = 1,418 \times 10^{11}\ \text{Pa} \cdot \text{s/m}^3$$

The portion of the inlet pressure drop shall be $a$ = 0,1.

The length of capillaries is then given by

$$l_{cp} = \frac{Q \times 4 \times \rho \times 1,08}{Z \times \pi \times \eta_{cp} \times 32 \times a} = \frac{4,23 \times 10^5 \times 4 \times 900 \times 1,08}{4 \times \pi \times 0,023 \times 32 \times 0,1} = 0,178\ \text{m}$$

and the diameter of the capillaries by

$$d_{cp}^4 = \frac{128 \times \eta_{cp} \times l_{cp}}{R_{cp} \times \pi} \times (1 + a) = \frac{128 \times 0,023 \times 0,178 \times 1,1}{1,418 \times 10^{11} \times \pi} = 1,29 \times 10^{-12}\ \text{m}^4$$

$$d_{cp} = \sqrt[4]{1,29 \times 10^{-12}} = 1,066 \times 10^{-3}\,\text{m} = 1,066\ \text{mm}$$

### B.2.9 Reynolds numbers

In the capillary:

$$Re_{\text{cp}} = \frac{4 \times Q \times \rho}{Z \times \pi \times d_{\text{cp}} \times \eta_{\text{cp}}} = \frac{4,23 \times 10^{-5} \times 900}{\pi \times 1,066 \times 10^{-3} \times 0,023} = 494$$

$Re_{\text{cp}} < 2\,300$ and is thus laminar.

$Re_{\text{cp}} < 1\,000$ to $1\,500$ and thus has a low inertia factor $a$.

In the recess with $h_{\text{p}} = 40 \times C_{\text{R}}$

$$Re_{\text{P}} = \frac{U \times h_{\text{p}} \times \rho}{\eta_{\text{B}}} = \frac{9,42 \times 40 \times 39,3 \times 10^{-6} \times 900}{0,023} = 579$$

$Re_{\text{p}} < 1\,000$ and is thus laminar.

# Bibliography

[1] Opitz H. *Untersuchung der Steifigkeit von Lagern für Hauptspindeln von Werkzeugmaschinen. (A study on the stiffness behaviour of bearings for work spindles of machine tools)*. Westdeutscher Verlag Köln und Opladen, 1967

[2] Vermeulen M. *De invloed van de tweedimensionale stroming op het statisch gedrag van het hydrostatisch radial lager.* Dissertation Rijksuniversiteit, Gent, 1979

[3] Rodermund H. *Berechnung der Temperaturabhängigkeit der Viskosität von Mineralölen aus dem Viskositätsgrad (Calculation of temperature dependence of mineral oils viscosity based on the viscosity grade)*, Schmiertechnik und Tribologie, 25. Jahrgang, 2/1978

[4] Wien-Harms, *Handbuch der Experimentalphysik (Manual of experimental physics)*, Band IV, Teil 4, Leipzig, 1932

[5] Shinkle J.N., & Hornung K.G. *Frictional characteristics of liquid hydrostatic bearings*. Trans. ASME, J. Basic Engng. 1965, H. 2, pp. 163-169

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