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Determination of the detection limit and decision threshold for ionizing radiation measurements —

Part 6: Fundamentals and applications to measurements by use of transient mode

Détermination de la limite de détection et du seuil de décision des mesurages de rayonnements ionisants —

Partie 6: Principes fondamentaux et leurs applications aux mesurages réalisés en mode transitoire

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

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ISO 11929-6 was prepared by Technical Committee ISO/TC 85, *Nuclear energy*, Subcommittee SC 2, *Radiation protection*.

ISO 11929 consists of the following parts, under the general title *Determination of the detection limit and decision threshold for ionizing radiation measurements*:

- *Part 1: Fundamentals and application to counting measurements without the influence of sample treatment*
- *Part 2: Fundamentals and application to counting measurements with the influence of sample treatment*
- *Part 3: Fundamentals and application to counting measurements with high resolution gamma spectrometry, without the influence of sample treatment*
- *Part 4: Fundamentals and applications to measurements by use of linear-scale analogue ratemeters, without the influence of sample treatment*
- *Part 5: Fundamentals and applications to counting measurements on filters during accumulation of radioactive material*
- *Part 6: Fundamentals and applications to measurements by use of transient mode*
- *Part 7: Fundamentals and general applications*
- *Part 8: Fundamentals and applications to unfolding of spectrometric measurements without the influence of sample treatment*

Introduction

This part of ISO 11929 gives basic information on the statistical principles for the determination of the detection limit and decision threshold (and directives for specification of the confidence limits) for nuclear radiation measurements.

This part of ISO 11929 applies to monitoring systems for checking materials moved on vehicles, lorries, ships, in containers, on moving belts, etc. for hidden radioactivity (contamination, activation products, radioactive sources), while passing gates, borders or other check points. The purpose of the measurement is to detect suspicious goods or vehicles and to stop them for a more detailed inspection.

Whereas the earlier parts 1 to 4 were elaborated for special measuring tasks in nuclear radiation measurements based on the principles defined by Altschuler and Pasternack [1], Nicholson [2], Currie [3], this restriction does not apply to this part, or to parts 5, 7 and 8. The determination of the characteristic limits mentioned above is separated from the evaluation of the measurement. Consequently, this part of ISO 11929 is generally applicable and can be applied to any suitable procedure for the evaluation of a measurement. Since the uncertainty of measurement plays a fundamental role in this part of ISO 11929, evaluations of measurements and the determination of the uncertainties of measurement have to be performed according to the Guide for the Expression of Uncertainty in Measurement.

This part, as well as parts 5, 7 and 8, of ISO 11929 is based on methods of Bayesian statistics (see [4] to [6]) in the Bibliography in order to be able to account also for such uncertain quantities and influences which do not behave randomly in repeated or counting measurements.

For this purpose, Bayesian statistical methods are used to specify the following statistical values, called “characteristic limits”.

— The *decision threshold*, which allows a decision to be made for a measurement with a given probability of error as to whether the result of the measurement indicates the presence of the physical effect quantified by the measurand.

— The *detection limit*, which specifies the minimum true value of the measurand which can be detected with a given probability of error using the measuring procedure in question. This consequently allows a decision to be made as to whether or not a measuring method checked using this part of ISO 11929 satisfies certain requirements and is consequently suitable for the given purpose of measurement.

— The *limits of the confidence interval*, which define an interval which contains the true value of the measurand with a given probability, in the case that the result of the measurement exceeds the decision threshold.

Determination of the detection limit and decision threshold for ionizing radiation measurements —

Part 6: Fundamentals and applications to measurements by use of transient mode

1 Scope

This part of ISO 11929 specifies suitable statistical values which allow an assessment of the detection capabilities in ionizing radiation measurements by use of a transient mode. For this purpose, statistical methods are used to specify two statistical values characterizing given probabilities of error.

This part of ISO 11929 deals with fundamentals and applications to measurements by use of transient mode.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML, *Guide to the Expression of Uncertainty in Measurement*, Geneva, 1993

ISO 11929-7:2005, *Determination of the detection limit and decision threshold for ionizing radiation measurements — Part 7: Fundamentals and general applications*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1
measuring method
any logical sequence of operations, described generically, used in the performance of measurements

NOTE 1 Adapted from the International Vocabulary of Basic and General Terms in Metrology:1993.

NOTE 2 In this part of ISO 11929, the measuring method is the application of any radiation detection systems suitable for measuring the radiation emitted from materials while transported on vehicles, lorries, ships, moving belts or in containers, and its evaluation.

3.2
measurand
particular quantity subject to measurement [International Vocabulary of Basic and General Terms in Metrology:1993]

NOTE In this part of ISO 11929, a measurand is non-negative and quantifies a nuclear radiation effect. The effect is not present if the value of the measurand is zero. It is a characteristic of this part of ISO 11929 that it can be applied to any measurand suitable to indicate radioactivity of the materials investigated.

3.3
uncertainty (**of measurement**)
parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand [Guide for the Expression of Uncertainty in Measurement:1993]

NOTE The uncertainty of a measurement derived according to the ISO Guide for the Expression of Uncertainty in Measurement comprises, in general, many components. Some of these components may be evaluated from the statistical distribution of the results of series of measurements and can be characterized by experimental standard deviations. The other components, which also can be characterized by standard deviations, are evaluated from assumed or known probability distributions based on experience and other information.

3.4
mathematical model of the evaluation
a set of mathematical relationships between all measured and other quantities involved in the evaluation of measurements

3.5
decision quantity
random variable for the decision whether the physical effect to be measured is present or not

3.6
decision threshold
fixed value of the decision quantity by which, when exceeded by the result of an actual measurement of a measurand quantifying a physical effect, one decides that the physical effect is present

NOTE The decision threshold is the critical value of a statistical test to decide between the hypothesis that the physical effect is not present and the alternative hypothesis that it is present. When the critical value is exceeded by the result of an actual measurement, this is taken to indicate that the hypothesis should be rejected. The statistical test will be designed such that the probability of wrongly rejecting the hypothesis (error of the first kind) is at most equal to a given value α.

3.7
detection limit
smallest true value of the measurand which is detectable by the measuring method

NOTE 1 The detection limit is the smallest true value of the measurand which is associated with the statistical test and hypotheses according to 3.6 by the following characteristics: if in reality the true value is equal to or exceeds the detection limit, the probability of wrongly not rejecting the hypothesis (error of the second kind) will be at most equal to a given value β.

NOTE 2 The difference between using the decision threshold and using the detection limit is that measured values are to be compared with the decision threshold, whereas the detection limit is to be compared with the guideline value.

3.8
confidence limits
values which define a confidence interval to be specified for the measurand in question which, if the result exceeds the decision threshold, includes the true value of the measurand with the given probability $1 - \gamma$

3.9
background counting rate
measured counting rate without radioactivity of interest

NOTE 1 This is the counting rate caused by external sources, and radioactivity in detector and shielding and detector noise.

NOTE 2 The shielding effect by the object to be measured can reduce the background counting rate by a factor f.

3.10
gross counting rate
measured counting rate due to both the object to be measured and the background counting rate

3.11
net counting rate
⟨for transient measurements⟩ gross counting rate minus the background counting rate, taking into account shielding of the background counting rate by the object

3.12
measuring time
⟨for transient measurements⟩ time the object of measurement needs to pass the detector area

NOTE 1 The time starts when it interrupts the entrance light beam or the device receives a “go” signal and stops, when it leaves the exit light beam or the device receives a “stop” signal.

NOTE 2 The entrance angle of the detector can be limited by a collimator.

3.13
guideline value
value which corresponds to scientific, legal or other requirements for which the measuring procedure is intended to assess

EXAMPLE Activity, specific activity or activity concentration, surface activity, or dose rate.

4 Quantities and symbols

$\hat{\xi}$	Random variable as an estimator for a non-negative measurand quantifying a physical effect
ξ	True value of the estimator $\hat{\xi}$ of the non-negative measurand quantifying a physical effect; true value of the measurand
X	Random variable as decision quantity; estimator of the measurand
x	Result of a measurement of the decision quantity X
$u(x)$	Standard uncertainty of the measurand associated with the measured result x of a measurement
$\tilde{u}(\xi)$	Standard uncertainty of the decision quantity X as a function of the true value ξ of the measurand
z	Best estimate of the measurand
$u(z)$	Standard uncertainty of the measurand associated with the best estimate z
x^*	Decision threshold for the measurand
ξ^*	Detection limit for the measurand
ξ_l, ξ_u	Respectively, the lower and upper limit of the confidence interval for the measurand
α	Probability of the error of the first kind; the probability of rejecting the hypothesis if it is true
β	Probability of the error of the second kind; the probability of accepting the hypothesis if it is false
$1 - \gamma$	Probability attributed to the confidence interval of the measurand; probability that the true value of the measurand is included by the confidence interval
k_p	Quantiles of the standardized normal distribution for the probability p (see Table 1); ($p = 1 - \alpha$), ($1 - \beta$), ($1 - \gamma/2$)

Y_k Output quantity derived from the measured results; ($k = 1, ..., n$)

y_k Estimate of an output quantity Y_k; ($k = 1, ..., n$)

$u(y_k)$ Standard uncertainty associated with y_k

G_k Function of the input quantities X_i; ($i = 1, ..., m$); model of the evaluation; ($k = 1, ..., n$)

X_i Input quantities; ($i = 1, ..., m$)

x_i Estimate of an input quantity; ($i = 1, ..., m$)

$u(x_i, x_j)$ Covariance associated with x_i and x_j

N_0 Background counts

N_g Gross counts

t_0 Measuring time for background effect measurement

t_g Gross measuring time $t_g = l/v$

l Length of measuring path

v Moving velocity

R_0 Background count rate $R_0 = N_0/t_0$

R_g Gross count rate $R_g = N_g/t_g$

f Factor specifying the reduction of the background counting rate due to the shielding by the object of measurement

R_n Net count rate (difference between gross and background count rate taking into account a shielding factor f for the reduction of the background count rate due to the shielding by the object), $R_n = R_g - f \cdot R_0$

R_n^* Decision threshold for the net count rate R_n

ρ_n^* Detection limit for the expectation value of the net count rate R_n

$\rho_{n,l}$, $\rho_{n,u}$ Lower, respectively upper, confidence limit of the net count rate

$\Phi(t)$ Distribution function of the standardized normal distribution

$\phi(z)$ Standardized normal distribution

κ Parameter

E Operator for the formation of the expectation of a random variable

Var Operator for the formation of the variance of a random variable

5 Statistical values and confidence interval

5.1 Principles

5.1.1 General aspects

For a particular task involving nuclear radiation measurements, first the particular physical effect which is the objective of the measurement has to be described. Then a non-negative measurand has to be defined which quantifies the physical effect and which assumes the value zero if the effect is not present in an actual case.

A random variable, called a decision quantity X, has to be attributed to the measurand. It is also an estimator of the measurand. It is required that the expectation value EX of the decision quantity X equals the true value ξ of the measurand. A value x of the estimator X derived from measurements is a primary estimate of the measurand. The primary estimate x of the measurand, and its associated standard uncertainty $u(x)$, have to be calculated as the primary complete result of the measurement according to the Guide for the Expression of Uncertainty in Measurement, by evaluation of measured quantities and of other information using a mathematical model of the evaluation which takes into account all relevant quantities. Generally, the fact that the measurand is non-negative will not be explicitly made use of. Therefore, x may become negative, in particular, if the true value of the measurand is close to zero.

NOTE The model of the evaluation of the measurement need not necessarily be given in the form of explicit mathematical formulas. It can also be represented by an algorithm or a computer code [see Equation (2)].

For the determination of the decision threshold and the detection limit, the standard uncertainty of the decision quantity has to be calculated, if possible, as a function $\tilde{u}(\xi)$ of the true value ξ of the measurand. In the case that this is not possible, approximate solutions are described below.

ξ is the value of another, non-negative estimator $\hat{\xi}$ of the measurand. The estimator $\hat{\xi}$, in contrast to X, makes use of the knowledge that the measurand is non-negative. The limits of the confidence interval to be determined refer to this estimator $\hat{\xi}$ (compare 5.4). Besides the limits of the confidence interval, the expectation value E$\hat{\xi}$ of this estimator as a best estimate z of the measurand, and the standard deviation $[\mathrm{Var}(\hat{\xi})]^{1/2}$ as the standard uncertainty $u(z)$ associated with the best estimate z of the measurand, have to be calculated (see 6.3).

For the numerical calculation of the decision threshold and the detection limit, the function $\tilde{u}(\xi)$ is needed, which is the standard uncertainty of the decision quantity X as a function of the true value ξ of the measurand. The function $\tilde{u}(\xi)$ generally has to be determined by the user of this part of ISO 11929, in the course of the evaluation of the measurement according to the Guide for the Expression of Uncertainty in Measurement. For examples see Annex A. This function is often only slowly increasing. Therefore, it is justified in many cases to use the approximation $\tilde{u}(\xi) = u(x)$.This applies, in particular, if the primary estimate x of the measurand is not much larger than its standard uncertainty $u(x)$ associated with x. If the value x is calculated as the difference (net effect) of two approximately equal values y_1 and y_0 obtained from independent measurements, that is $x = y_1 - y_0$, one gets $\tilde{u}^2(0) = u^2(y_1) + u^2(y_0)$ with the standard uncertainties $u(y_1)$ and $u(y_0)$ associated with y_1 and y_0, respectively.

If only $\tilde{u}(0)$ and $u(x)$ are known, an approximation by linear interpolation is often sufficient for $x > 0$ according to:

$$\tilde{u}^2(\xi) = \tilde{u}^2(0) \cdot (1 - \xi / x) + u^2(x) \cdot \xi / x \tag{1}$$

NOTE In many practical cases, $\tilde{u}^2(\xi)$ is a slowly increasing linear function of ξ. This justifies the approximations above, in particular, the linear interpolation of $\tilde{u}^2(\xi)$ instead of $\tilde{u}(\xi)$ itself.

For setting up the mathematical model of the evaluation of the measurement, one has to distinguish two types of physical quantities, input and output quantities. The output quantities Y_k ($k = 1, ..., n$) are viewed as measurands (for example, the parameters of an unfolding or fitting procedure) which have to be determined by the evaluation of a measurement. The decision quantity X is one of them. They depend on the input quantities x_i ($i = 1, ..., m$) which are the quantities obtained by repeated measurements, influence quantities and results of previous measurements and evaluations. (Compare chapter 4.1.2 of the Guide for the Expression of Uncertainty in Measurement:1993.) One has to calculate the estimates y_k of the output quantities (measurands) as the results of the measurement and the standard uncertainties $u(y_k)$ associated with y_k.

The model of the evaluation is given by a set of functional relationships:

$$Y_k = G_k(X_1, ..., X_m); (k = 1, ..., n) \tag{2}$$

Estimates of the measurands Y_k, denoted y_k, are obtained from Equation (2) using input estimates $x_1, ..., x_m$ for the values of the m quantities $X_1, ..., X_m$. Thus, the output estimates y_k and the standard uncertainties $u(y_k)$ associated with y_k are given by:

$$y_k = G_k(x_1, ..., x_m); (k = 1, ..., n) \tag{3}$$

$$u(y_k, y_l) = \sum_{i,j=1}^{m} \frac{\partial G_k}{\partial X_i} \cdot \frac{\partial G_l}{\partial X_j} \cdot u(x_i, x_j); (k, l = 1, ..., n) \tag{4}$$

where x_i and x_j are the estimates of X_i and X_j and $u(x_i, x_j) = u(x_j, x_i)$ are the estimated covariances associated with x_i and x_j. The standard uncertainty $u(y_k)$ is given by:

$$u^2(y_k) = u(y_k, y_k) \tag{5}$$

In cases when the partial derivatives are not explicitly available, they can be numerically approximated in a sufficiently exact way using the standard uncertainty $u(x_k)$ as an increment of x_k by

$$\frac{\partial G_k}{\partial X_i} \approx \frac{1}{u(x_i)} \left\{ G_k\left[x_1, ..., x_i + u(x_i)/2, ..., x_m\right] - G_k\left[x_1, ..., x_i - u(x_i)/2, ..., x_m\right] \right\} \tag{6}$$

5.1.2 Model

When evaluating transient measurements of radioactivity, an output quantity Y is calculated using the following model:

$$Y = G\left[(R_{g,i}; i = 1, ..., n) - (R_{0,j}; j = 1, ..., m); (t_k; k = 1, ..., r)\right] \tag{7}$$

with

$R_{g,i}$ is the gross counting rate with index i;

$R_{0,j}$ is the background counting rate with index j;

t are the other input quantities.

In this model, the $R_{g,i}$ are input quantities derived from gross measurements of the object under investigation. The $R_{0,j}$ are input quantities derived from measurements of the background of the measuring equipment. The t_k are other input quantities which may not be directly connected to one of those measurements and which may or may not have uncertainties. Summarizing, the $R_{g,i}$, $R_{0,i}$ and t_k, as input quantities X_i in Equation (2), give the general model for transient measurements.

For this model, the characteristic limits are described in 5.2 to 5.4. Consequently, this part of ISO 11929 is applicable to each system for which the evaluation can be formally described by Equation (2).

In addition, a simple version of this model is used to exemplify the application of this part of ISO 11929 and to give explicit formulas for the characteristic limits in 5.2, 5.3 and 5.4.

This simple model is that of a system by which one gross and one background measurement are performed at different times.

The gross measurement is assumed to be performed on a vehicle which is moving with constant velocity v, while it passes the detection area, of length l, with a constant efficiency.

It is assumed that start and stop of the gross counting rate measurement is externally triggered with negligible uncertainties of start and stop times (i.e. of measuring duration). The measuring time will be $t_g = l/v$.

If the object to be measured is very big and heavy, it can shield the background radiation and therefore reduce the background counting rate by a factor f.

During the gross measurement, N_g counts are registered. The measurements are evaluated by a model according to Equation (8)

$$R_n = \frac{N_g}{t_g} - f \cdot \frac{N_0}{t_0} = R_g - f \cdot R_0 \tag{8}$$

The net count rate R_n has a variance of

$$u^2(R_n) = \frac{R_g}{t_g} + f^2 \cdot \frac{R_0}{t_0} + u^2(f) \cdot R_0^2 = \frac{N_g}{t_g^2} + f^2 \cdot \frac{N_0}{t_0^2} + u^2(f) \cdot \frac{N_0^2}{t_0^2} \tag{9}$$

For the calculation of the decision threshold, one needs the standard uncertainty of the measurand for a true value $\rho_n = 0$. For $\rho_n = 0$, one expects $R_g = f \cdot R_0$ and $N_g = f \cdot N_0 \cdot t_g / t_0$. This yields

$$\tilde{u}^2(0) = \frac{f \cdot R_0}{t_g} + f^2 \frac{R_0}{t_0} + u^2(f) \cdot R_0^2 \tag{10}$$

For the calculation of the detection limit, one needs the standard uncertainty $\tilde{u}(\rho_n)$ of the measurand as a function of its true value ρ_n. For a true value ρ_n, one expects

$$R_g = \rho_n + f \cdot R_0 \text{ and } N_g = \rho_n \cdot t_g + f \cdot R_0 \cdot t_g = \rho_n \cdot t_g + f \cdot N_0 \cdot \frac{t_g}{t_0} \tag{11}$$

Hence one gets from Equation (9)

$$\tilde{u}^2(\rho_n) = \frac{\rho_n + f \cdot R_0}{t_g} + f^2 \cdot \frac{R_0}{t_0} + u^2(f) \cdot R_0^2 \tag{12}$$

5.2 Decision threshold

The decision threshold x^* of a non-negative measurand quantifying the physical effect, according to 5.1, is a value of the decision quantity X which, when it is exceeded by a result x of a measurement, indicates that the physical effect is present. If $x \leqslant x^*$, one decides that the physical effect is not present. If this decision rule is observed, a wrong decision in favour of the presence of the physical effect occurs with a probability not greater than α (error of the first kind).

The decision threshold is given by

$$x^* = k_{1-\alpha} \cdot \tilde{u}(0) \qquad (13)$$

Values of the quantiles $k_{1-\alpha}$ of the standardized normal distribution are given in Table 1. It is $\Phi(k_{1-\alpha}) = 1 - \alpha$.

If the approximation $\tilde{u}(\xi) = u(x)$ is sufficient, the decision threshold is given by

$$x^* = k_{1-\alpha} \cdot u(x) \qquad (14)$$

For a model according to Equation (8), one expects with Equation (11) for $\rho_n = 0$, a number of gross counts

$$N_g = f \cdot N_0 \cdot t_g / t_0 = f \cdot R_0 \cdot t_g$$

From this, one obtains with Equation (9)

$$\tilde{u}^2(0) = \frac{f \cdot R_0}{t_g} + f^2 \cdot \frac{R_0}{t_0} + u^2(f) \cdot R_0^2$$

and hence the decision threshold can be calculated using Equation (15):

$$R_n^* = k_{1-\alpha} \cdot \sqrt{\frac{f \cdot R_0}{t_g} + f^2 \cdot \frac{R_0}{t_0} + u^2(f) \cdot R_0^2} \qquad (15)$$

If $R_0 \cdot t_g$ and $R_0 \cdot t_0$ are sufficiently large for the first two terms under the square root of Equation (15) to be neglected, the following approximation holds:

$$R_n^* = k_{1-\alpha} \cdot R_0 \cdot u(f) \qquad (16)$$

5.3 Detection limit

The detection limit ξ^*, which is the smallest true value of the measurand detectable with the measuring method, is so much larger than the decision threshold that the probability of an error of the second kind is not greater than β. The detection limit is given by

$$\xi^* = x^* + k_{1-\beta} \cdot \tilde{u}(\xi^*) \qquad (17)$$

Equation (17) is an implicit one. The detection limit can be calculated from it by iteration using, for example, the starting approximation $\xi^* = 2x^*$. The iteration converges in most cases.

With R_n^* and $\tilde{u}(\rho_n)$ according to Equation (15) and (12), respectively, the detection limit for a model according to Equation (8) can be calculated using Equation (18).

$$\rho_n^* = R_n^* + k_{1-\beta} \cdot \sqrt{\frac{\rho_n^* + f \cdot R_0}{t_g} + f^2 \cdot \frac{R_0}{t_0} + u^2(f) \cdot R_0^2} \qquad (18)$$

Using this implicit equation, ρ_n^* can be calculated by iteration using the starting approximation $\rho_n^* = 2 \cdot R_n^*$.

If $R_0 \cdot t_g$ and $R_0 \cdot t_0$ are sufficiently large for the first two terms under the square root of Equation (18) to be neglected, the following approximation holds:

$$\rho_n^* = (k_{1-\alpha} + k_{1-\beta}) \cdot R_0 \cdot u(f) \qquad (19)$$

5.4 Confidence limits

For a result x of a measurement which exceeds the decision threshold x^*, the confidence interval includes the true value of the measurand with the given probability $1 - \gamma$. It is enclosed by the confidence limits ξ_l and ξ_u according to

$$\xi_\mathrm{l} = x - k_p \cdot u(x) \text{ with } p = \kappa \cdot (1 - \gamma / 2) \tag{20}$$

$$\xi_\mathrm{u} = x + k_p \cdot u(x) \text{ with } q = 1 - (\kappa \cdot \gamma / 2) \tag{21}$$

κ is given by

$$\kappa = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x/u(x)} \exp(-z^2/2)\,\mathrm{d}z = \Phi[x/u(x)] \tag{22}$$

Values of the function $\Phi(t)$ are tabulated [7] and given in Table 1. It is $\Phi(k_p) = p$ and $\Phi(k_q) = q$.

The confidence limits are not symmetrical around the expectation $\mathrm{E}\hat{\xi}$. The probabilities of $\hat{\xi} < \xi_\mathrm{l}$ and $\hat{\xi} > \xi_\mathrm{u}$, however, both are equal to $\gamma/2$ and the relationship $0 < \xi_\mathrm{l} < \xi_\mathrm{u}$ is valid. For $x \gg u(x)$ the approximation

$$\xi_\mathrm{l,u} = x \pm k_{1-\gamma/2} \cdot u(x) \tag{23}$$

is applicable if $x > \approx 2k_{1-\gamma/2}\, u(x)$.

6 Application of this part of ISO 11929

6.1 Specific values

The probabilities α, β and $(1 - \gamma)$ shall be specified in advance by the user of this part of ISO 11929. Commonly used values are $\alpha = \beta = 0{,}05$ and $\gamma = 0{,}05$.

6.2 Assessment of a measuring method

To check whether a measuring method (see 3.1) is suitable for the measurement of a physical effect, the detection limit shall be compared with a specified guideline value (e.g. specified requirements on the sensitivity of the measuring procedure for scientific, legal or other reasons; see 3.13).

The detection limit shall be calculated by means of Equation (18). If the detection limit thus determined is greater than the guideline value, the measuring procedure is not suitable for the measurement.

6.3 Assessment of measured results

A measured result has to be compared with the decision threshold calculated by means of Equation (15) or (16). If a result of the measurement x is larger than the decision threshold x^*, it is decided that the physical effect quantified by the measurand is present.

If this is the case, the best estimate z of the measurand is calculated using κ from Equation (22) by

$$z = \mathrm{E}\,\hat{\xi} = x + \frac{u(x) \cdot \exp\left\{-x^2 / [2u^2(x)]\right\}}{\kappa \cdot \sqrt{2\pi}} \tag{24}$$

with the standard uncertainty $u(z)$ associated with z:

$$u(z) = \sqrt{\mathrm{Var}(\hat{\xi})} = \sqrt{u^2(x) - (z - x) \cdot z} \tag{25}$$

The following relationships: $z \geqslant x$ and $z \geqslant 0$, as well as $u(z) \leqslant u(x)$, are valid and for $x \gg u(x)$, i.e. $x > 4\,u(x)$, the approximations $z = x$ and $u(z) = u(x)$ hold true.

6.4 Documentation

The documentation of measurements in accordance with this part of ISO 11929 shall contain details of the probabilities α, β and $(1 - \gamma)$, the decision threshold x^*, the detection limit ξ^*, and the guideline value.

For a result x of the measurement exceeding the decision threshold x^*, the standard uncertainty $u(x)$ associated with x and the limits of the confidence interval $\xi_{\mathrm{l,u}}$ have to be given. If the result x of the measurement is below the decision threshold ξ^*, it shall be documented as “below the decision threshold”.

If the detection limit exceeds the guideline value, it shall be documented that the method is not suitable for the measurement purpose.

In addition, the best estimate z of the measurand and the standard uncertainty $u(z)$ associated with z may be specified if $x/u(x) < 4$.

7 Values of the distribution function of the standardized normal distribution

Values $\Phi(t) = \int_{-\infty}^{t} \varphi(z)\,\mathrm{d}z$ with $\varphi(z) = (1/\sqrt{2\pi}) \cdot \exp(-z^2/2)$ are given in Table 1. For the distribution function of the standardized normal distribution, $\Phi(-t) = 1 - \Phi(t)$ is valid. Quantiles of the standardized normal distribution can also be obtained from Table 1 since $t = k_p$ for $p = \Phi(t)$, i.e. $\Phi(k_p) = p$.

For $t \geqslant 0$, the approximation (see [8] in the Bibliography):

$$\Phi(t) = 1 - \frac{\exp(-t^2/2)}{\sqrt{2\pi}}(\alpha_1\zeta + \alpha_2\zeta^2 + \alpha_3\zeta^3) + \varepsilon; \zeta = \frac{1}{1 + \alpha_0 t}$$

is valid with $|\varepsilon| < 10^{-5}$ and

$\alpha_0 = 0{,}332\ 67$; $\alpha_1 = 0{,}436\ 183\ 6$; $\alpha_2 = -0{,}120\ 167\ 6$; $\alpha_3 = 0{,}937\ 298\ 0$

For $t < 0$, one obtains $\Phi(t)$ from the relationship $\Phi(t) = 1 - \Phi(-t)$.

For $0{,}5 \leqslant p < 1$, the approximation (see [8] in the Bibliography):

$$k_p \approx t - \frac{b_0 + b_1 t + b_2 t^2}{1 + c_1 t + c_2 t^2 + c_3 t^3} + \varepsilon; t = \sqrt{-2\ln(1 - p)}$$

is valid with $|\varepsilon| < 4{,}5 \times 10^{-4}$ and

$b_0 = 2{,}515\ 517$; $b_1 = 0{,}802\ 853$; $b_2 = 0{,}010\ 328$;

$c_1 = 1{,}432\ 788$; $c_2 = 0{,}189\ 269$; $c_3 = 0{,}001\ 308$.

For $0 < p < 0{,}5$, one obtains k_p from the relationship $k_p = -k_{1-p}$.

Table 1 — Values of the distribution function of the standardized normal distribution $\Phi(t)$
(see [7] in the Bibliography)

t	$\Phi(t)$	t	$\Phi(t)$	t	$\Phi(t)$	t	$\Phi(t)$	t	$\Phi(t)$
0,00	0,500 0	0,70	0,758 0	1,40	0,919 2	2,10	0,982 1	2,80	0,997 4
0,02	0,508 0	0,72	0,764 2	1,42	0,922 2	2,12	0,983 0	2,90	0,998 1
0,04	0,516 0	0,74	0,770 4	1,44	0,925 1	2,14	0,983 8	3,00	0,998 6
0,06	0,523 9	0,76	0,776 4	1,46	0,927 8	2,16	0,984 6	3,10	0,999 0
0,08	0,531 9	0,78	0,782 3	1,48	0,930 6	2,18	0,985 4	3,20	0,999 3
0,10	0,539 8	0,80	0,788 1	1,50	0,933 2	2,20	0,986 1	3,30	0,999 5
0,12	0,547 8	0,82	0,793 9	1,52	0,935 7	2,22	0,986 8	3,40	0,999 7
0,14	0,555 7	0,84	0,799 6	1,54	0,938 2	2,24	0,987 4	3,50	0,999 8
0,16	0,563 6	0,86	0,805 1	1,56	0,940 6	2,26	0,988 1	3,60	0,999 8
0,18	0,571 4	0,88	0,810 6	1,58	0,943 0	2,28	0,988 7	3,80	0,999 9
0,20	0,579 3	0,90	0,815 9	1,60	0,945 2	2,30	0,989 3	4,00	1,000 0
0,22	0,587 1	0,92	0,821 2	1,62	0,947 4	2,32	0,989 8		
0,24	0,594 8	0,94	0,826 4	1,64	0,949 5	2,34	0,990 4		
0,26	0,602 6	0,96	0,831 5	1,66	0,951 5	2,36	0,990 9		
0,28	0,610 3	0,98	0,836 5	1,68	0,953 5	2,38	0,991 3		
0,30	0,617 9	1,00	0,841 3	1,70	0,955 4	2,40	0,991 8		
0,32	0,625 5	1,02	0,846 1	1,72	0,957 3	2,42	0,992 2		
0,34	0,633 1	1,04	0,850 8	1,74	0,959 1	2,44	0,992 7		
0,36	0,640 6	1,06	0,855 4	1,76	0,961 0	2,46	0,993 0		
0,38	0,648 0	1,08	0,859 9	1,78	0,962 5	2,48	0,993 4		
0,40	0,655 4	1,10	0,864 3	1,80	0,964 1	2,50	0,993 8		
0,42	0,662 8	1,12	0,868 6	1,82	0,965 6	2,52	0,994 1		
0,44	0,670 0	1,14	0,872 9	1,84	0,967 1	2,54	0,994 5		
0,46	0,677 2	1,16	0,877 0	1,86	0,968 6	2,56	0,994 8		
0,48	0,684 4	1,18	0,881 0	1,88	0,970 0	2,58	0,995 1		
0,50	0,691 5	1,20	0,884 9	1,90	0,971 3	2,60	0,995 3		
0,52	0,698 5	1,22	0,888 8	1,92	0,972 6	2,62	0,995 6		
0,54	0,705 4	1,24	0,892 5	1,94	0,973 8	2,64	0,995 9		
0,56	0,712 3	1,26	0,896 1	1,96	0,975 0	2,66	0,996 1		
0,58	0,719 0	1,28	0,899 7	1,98	0,976 2	2,68	0,996 3		
0,60	0,725 8	1,30	0,903 2	2,00	0,977 2	2,70	0,996 5		
0,62	0,732 4	1,32	0,906 6	2,02	0,978 3	2,72	0,996 7		
0,64	0,738 9	1,34	0,909 9	2,04	0,979 3	2,74	0,996 9		
0,66	0,745 4	1,36	0,913 1	2,06	0,980 3	2,76	0,997 1		
0,68	0,751 8	1,38	0,916 2	2,08	0,981 2	2,78	0,997 3		

Annex A
(informative)

Example of application of this part of ISO 11929

A.1 Example of application of this part of ISO 11929

This example describes a measurement in which a truck is monitored for radioactivity in its load by an integral counting measurement. The truck moves with a velocity $v = 2$ m/s along the detector. Due to appropriate shielding of the detector, the length of the measurement path is $l = 6$ m. Thus, the gross measurement time is $t_g = l/v = 3$ s. During this time, a number $N_g = 366$ of gross events was measured. With this, one calculates a gross counting rate of $R_g = N_g/t_g = 122\ s^{-1}$ and a standard uncertainty $u(R_g) = \sqrt{N_g/t_g^2} = 6{,}377\ s^{-1}$.

NOTE In this example, numbers are given with too high a precision in order to facilitate recalculation.

The background count rate was measured independently. During a measurement time $t_0 = 1\,000$ s, a number $N_0 = 132\,267$ of background events was measured resulting in a background count rate $R_0 = N_0/t_0 = 132{,}267\ s^{-1}$ with a standard uncertainty $u(R_0) = \sqrt{N_0/t_0^2} = 0{,}364\ s^{-1}$.

In independent test measurements with trucks carrying non-radioactive loads, a range of shielding factors from $f = 0{,}7$ to $f = 0{,}9$ was observed. Lacking further information on the actual shielding factor, a rectangular distribution is assumed to estimate the mean shielding factor and its uncertainty. According to the Guide for the Expression of Uncertainty in Measurement, the standard uncertainty of the value of a quantity, with a rectangular probability distribution with a range of values of $2a$ is given as $u(f) = 0{,}577a$. Consequently, a mean shielding factor $f = 0{,}8$ with its standard uncertainty $u(f) = 0{,}577 \times 0{,}1 = 0{,}057\,7$ is used in this example.

The measurand is the net counting rate R_n, using a model according to Equation (8). This yields the result of the measurement:

$$R_n = \frac{N_g}{t_g} - f \cdot \frac{N_0}{t_0} = \frac{366}{3\ s} - 0{,}8 \times \frac{132\,267}{1\,000\ s} = 16{,}186\ s^{-1} \quad (A.1)$$

and with Equation (9), its standard uncertainty:

$$u(R_n) = \sqrt{\frac{N_g}{t_g^2} + f^2 \cdot \frac{N_0}{t_0^2} + u^2(f) \cdot \frac{N_0^2}{t_0^2}} = \sqrt{\frac{366}{3^2} + 0{,}8^2 \times \frac{132\,267}{1\,000^2} + 0{,}057\,7^2 \times \frac{132\,267^2}{1\,000^2}} = 9{,}950\ s^{-1} \quad (A.2)$$

For the calculation of the characteristic limits, the parameters $\alpha = \beta = 0{,}05$ and $1 - \gamma = 0{,}95$ were chosen, yielding $k_{1-\alpha} = k_{1-\beta} = 1{,}645$ and $k_{1-\gamma/2} = 1{,}960$. A guideline value of $35\ s^{-1}$ is arbitrarily assumed for this example.

A.2 Calculation of the decision threshold

For a true value $\rho_n = 0$ of the measurand, one expects with Equation (11) a number of gross counts $N_g = f \cdot N_0 \cdot t_g/t_0 = f \cdot R_0 \cdot t_g$ and one calculates $\tilde{u}(0)$ with Equation (9) as

$$\tilde{u}^2(0) = \frac{f \cdot R_0}{t_g} + f^2 \cdot \frac{R_0}{t_0} + u^2(f) \cdot R_0^2 = \frac{0{,}8 \times 132{,}267}{3} + 0{,}8^2 \times \frac{132{,}267}{1\,000} + 0{,}057\,7^2 \times 132{,}267^2 = 93{,}600 \text{ s}^{-2} \quad \text{(A.3)}$$

and hence the decision threshold R_n^*, according to Equation (14)

$$R_n^* = k_{1-\alpha} \cdot \tilde{u}(0) = 1{,}645 \times 9{,}675 = 15{,}917 \text{ s}^{-1} \quad \text{(A.4)}$$

Since the result of the measurement $R_n = 16{,}186 \text{ s}^{-1}$ exceeds the decision threshold of $R_n^* = 15{,}917 \text{ s}^{-1}$, one decides that a contribution to the counted events originating from activity in the load of the truck has been observed.

A.3 Calculation of the detection limit

According to Equation (18), one obtains an implicit equation for the detection limit

$$\rho_n^* = R_n^* + k_{1-\beta} \cdot \sqrt{\frac{\rho_n^* + f \cdot R_0}{t_g} + f^2 \cdot \frac{R_0}{t_0} + u^2(f) \cdot R_0^2} = 15{,}917 + 1{,}645 \times \sqrt{\frac{\rho_n^* + 0{,}8 \times 132{,}267}{3} + 0{,}8^2 \times \frac{132{,}267}{1000} + 0{,}057\,7^2 \times 132{,}267^2} \quad \text{(A.5)}$$

As an alternative to an iterative solution of this implicit equation, this example gives an explicit solution using Equation (1).

Since $\tilde{u}(0)$ and $u(R_n)$ are known, one can use Equation (1) to calculate $\tilde{u}(\rho_n)$.

$$\tilde{u}^2(\rho_n^*) = \tilde{u}^2(0) \cdot (1 - \rho_n^* / R_n) + u^2(R_n) \cdot \rho_n^* / R_n = 93{,}600 \times (1 - \rho_n^* / 16{,}186) + 14{,}299^2 \times \rho_n^* / 16{,}186 \quad \text{(A.6)}$$

Then the detection limit can be explicitly calculated using Equation (17).

$$\rho_n^* = a + \sqrt{a^2 + (k_{1-\beta}^2 - k_{1-\alpha}^2) \cdot \tilde{u}^2(0)} \quad \text{(A.7)}$$

with

$$a = k_{1-\alpha} \cdot \tilde{u}(0) + \frac{1}{2}(k_{1-\beta}^2 / R_n) \cdot [u^2(R_n) - \tilde{u}^2(0)] \quad \text{(A.8)}$$

Since in this example $\alpha = \beta = 0{,}5$, one obtains

$$\rho_n^* = 2a = 2 \cdot \left\{ k_{1-\alpha} \cdot \tilde{u}(0) + \frac{1}{2}(k_{1-\beta}^2 / R_n) \cdot [u^2(R_n) - \tilde{u}^2(0)] \right\} = 2 \times \left[1{,}645 \times 9{,}675 + 0{,}5 \times (1{,}645^2 / 16{,}186) \times (9{,}950^2 - 9{,}675^2) \right] = 32{,}282 \text{ s}^{-1} \quad \text{(A.9)}$$

Since the detection limit $\rho_n^* = 32{,}282 \text{ s}^{-1}$ is smaller than the guideline value of 35 s^{-1}, the measuring method is suitable for the purpose of the measurement.

A.4 Calculation of the confidence limits

For the calculation of the confidence limits $\rho_{n,l}$ and $\rho_{n,u}$, first the parameter κ has to be calculated according to Equation (22):

$$\kappa = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{R_n/u(R_n)} \exp(-z^2/2)\ \mathrm{d}z = \Phi[R_n / u(R_n)] \quad (A.10)$$

With the actual results, one obtains from Table A.1:

$$\kappa = \Phi(16{,}186\ \mathrm{s}^{-1}/9{,}950\ \mathrm{s}^{-1}) = \Phi(1{,}627) = 0{,}948$$

In addition, calculations with Equations (20) and (21) give

$$p = \kappa \cdot (1 - \gamma/2) = 0{,}948 \times (1 - 0{,}025) = 0{,}924\ 3 \quad (A.11)$$

$$q = 1 - (\kappa \cdot \gamma/2) = 1 - 0{,}948 \times 0{,}025 = 0{,}976\ 3 \quad (A.12)$$

From Table A.1 one obtains the quantiles for the probabilities p and q of the standardized normal distribution k_p = 1,444 3 and k_q = 1,983 1 and calculates the confidence limits with equations (20) and (21)

$$\rho_{n,l} = R_n - k_p \cdot u(R_n) = 16{,}186 - 1{,}444\ 3 \times 9{,}950 = 1{,}815\ \mathrm{s}^{-1} \quad (A.13)$$

$$\rho_{n,u} = R_n + k_q \cdot u(R_n) = 16{,}186 + 1{,}983\ 1 \times 9{,}950 = 35{,}918\ \mathrm{s}^{-1} \quad (A.14)$$

A.5 Calculation of the best estimate of the measurand

The best estimate of the measurand and its associated standard uncertainty are calculated according to Equations (24) and (25).

$$\begin{aligned} z &= R_n + \frac{u(R_n) \cdot \exp\{-R_n^2/[2 \cdot u^2(R_n)]\}}{\kappa \cdot \sqrt{2\pi}} \\ &= 16{,}186 + \frac{9{,}950 \times \exp(-16{,}186^2/2 \times 9{,}950^2)}{0{,}948\ 11 \times \sqrt{2\pi}} = 17{,}301\ \mathrm{s}^{-1} \end{aligned} \quad (A.15)$$

$$\begin{aligned} u(z) &= \sqrt{u^2(R_n) - (z - R_n) \cdot z} \\ &= \sqrt{9{,}950^2 - (17{,}301 - 16{,}186) \times 17{,}301} = 8{,}928\ \mathrm{s}^{-1} \end{aligned} \quad (A.16)$$

Table A.1 — Complete documentation of the measurement, of its evaluation and of the calculated characteristic limits

1	2	3	4
Quantity	**Symbol**	**Value**	**Unit**
net count rate (measurand)	R_n	16,2	s^{-1}
uncertainty of the net count rate	$u(R_n)$	9,9	s^{-1}
probability of the error of first kind	α	0,05	1
probability of the error of second kind	β	0,05	1
confidence level	$1 - \gamma$	0,95	1
guideline value	–	35	s^{-1}
decision threshold	R_n^*	15,9	s^{-1}
detection limit	ρ_n^*	32,3	s^{-1}
lower confidence limit	$\rho_{n,l}$	1,8	s^{-1}
upper confidence limit	$\rho_{n,u}$	35,9	s^{-1}
best estimate of the measurand	z	17,3	s^{-1}
uncertainty of the best estimate	$u(z)$	8,9	s^{-1}

Bibliography

[1] ALTSCHULER, B. and PASTERNACK, B. *Health Physics 9*, (1963), pp. 293-298

[2] NICHOLSON, W. L. *Fixed time estimation of counting rates with background corrections*. Hanford Laboratories, Richland, (1963)

[3] CURRIE, L. A. Limits for Qualitative Detection and Quantitative Determination. Anal. Chem. 40, (1968), pp. 586-593

[4] LEE, P.M. *Bayesian Statistics: An Introduction*. Oxford University Press, New York (1989)

[5] WEISE, K., WÖGER, W. Eine Bayessche Theorie der Meßunsicherheit. PTB-Bericht N-11, Physikalisch Technische Bundesanstalt, Braunschweig, (1992); A Bayesian theory of measurement uncertainty, *Meas. Sci. Technol. 4*, (1993), pp. 1-11

[6] WEISE, K. Bayesian-statistical decision threshold, detection limit and confidence interval in nuclear radiation measurement, *Kerntechnik 63*, (1998), pp. 214-224

[7] KOHLRAUSCH, F. *Praktische Physik*, Band 3, 24th edition, p. 613, Teubner, Stuttgart (1996)

[8] ABRAMOWITZ, M. *Handbook of Mathematical Function*, 5th Edition, Chapter 26, Dove Publication, New York (1968)

[9] DIN 1319-3, *Grundlagen der Messtechnik — Teil 3: Auswertung von Messungen einer einzelnen Messgröße, Messunsicherheit (in German)*, Beuth, Berlin

[10] DIN 1319-4, *Grundlagen der Messtechnik — Teil 4: Auswertung von Messungen, Messunsicherheit (in German)*, Beuth, Berlin

[11] DIN 25482-8, *Detection limit and decision threshold for nuclear radiation measurements — Part 8: Counting measurements at moving objects*, in preparation

[12] DIN 25482-10:2000-05, *Detection limit and decision threshold for nuclear radiation measurements — Part 10: General applications*

[13] BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML, *International Vocabulary of Basic and General Terms in Metrology*; 2nd edition, Geneva, 1993

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