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Determination of the detection limit and decision threshold for ionizing radiation measurements —

Part 5: Fundamentals and applications to counting measurements on filters during accumulation of radioactive material

Détermination de la limite de détection et du seuil de décision des mesurages de rayonnements ionisants —

Partie 5: Principes fondamentaux et leurs applications aux mesurages par comptage réalisés sur filtres lors d'une accumulation de radioactivité

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

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ISO 11929-5 was prepared by Technical Committee ISO/TC 85, *Nuclear energy*, Subcommittee SC 2, *Radiation protection*.

ISO 11929 consists of the following parts, under the general title *Determination of the detection limit and decision threshold for ionizing radiation measurements*:

— *Part 1: Fundamentals and application to counting measurements without the influence of sample treatment*

— *Part 2: Fundamentals and application to counting measurements with the influence of sample treatment*

— *Part 3: Fundamentals and application to counting measurements with high resolution gamma spectrometry*, *without the influence of sample treatment*

— *Part 4: Fundamentals and applications to measurements by use of linear-scale analogue ratemeters*, *without the influence of sample treatment*

— *Part 5: Fundamentals and applications to counting measurements on filters during accumulation of radioactive material*

— *Part 6: Fundamentals and applications to measurements by use of transient mode*

— *Part 7: Fundamentals and general applications*

— *Part 8: Fundamentals and applications to unfolding of spectrometric measurements without the influence of sample treatment*

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Introduction

This part of ISO 11929 gives basic information on the statistical principles for the determination of the detection limit, of the decision threshold and of the limits of the confidence interval for nuclear radiation measurements when monitoring the concentration of aerosols in exhaust gas, air or waste water.

ISO 11929-1 and ISO 11929-2, respectively, deal with integral counting measurements with or without consideration of the sample treatment. High-resolution spectrometric measurements are covered in ISO 11929-3. ISO 11929-4 deals with measurements using linear-scale analogue ratemeters, ISO 11929-6 with measurements using a transient measuring mode, ISO 11929-7 with general applications and ISO 11929-8 with unfolding of spectrometric measurements.

Whereas the earlier arts 1 to 4 were elaborated for special measuring tasks in nuclear radiation measurements based on the principles defined by Altschuler and Pasternack [1], Nicholson [2], Currie [3], this restriction does not apply to this part, or to parts 6 to 8. The determination of the characteristic limits mentioned above is separated from the evaluation of the measurement. Consequently this part of ISO 11929 is generally applicable and can be applied to any suitable procedure for the evaluation of a measurement. Since the uncertainty of measurement plays a fundamental role in this part of ISO 11929, evaluations of measurements and the determination of the uncertainties of measurement have to be performed according to the Guide for the expression of uncertainty in measurement.

This part, as well as parts 6, 7 and 8, of ISO 11929 is based on methods of Bayesian statistics part of ISO 11929 [4] to [6] in the Bibliography in order to be able to account also for uncertain quantities and influences which do not behave randomly in repeated or counting measurements.

For this purpose, Bayesian statistical methods are used to specify the following statistical values, called “characteristic limit.”

— The *decision threshold*, which allows a decision to be made for a measurement with a given probability of error as to whether the result of the measurement indicates the presence of the physical effect quantified by the measurand.

— The *detection limit*, which specifies the minimum true value of the measurand which can be detected with a given probability of error using the measuring procedure in question. This consequently allows a decision to be made as to whether or not a measuring method checked using this part of ISO 11929 satisfies certain requirements and is consequently suitable for the given purpose of measurement.

— The *limits of the confidence interval*, which define an interval which contains the true value of the measurand with a given probability, in the case that the result of the measurement exceeds the decision threshold.

This part of ISO 11929 concerns the monitoring of the volume concentration and of the increase or decrease of the volume concentration of radioactive particles in exhaust air, gas or (waste) water by multiple counting measurements during accumulation of the particles on a filter. It is assumed that dead-time losses are negligible. Wherever activities, activity concentrations or specific activities are to be determined, it is assumed that the factors for the conversion of pulse rates into activities, activity concentrations or specific activities have to be determined with sufficient accuracy to ignore the influence of their uncertainty in the measurement. The exhaust air, gas or water flow rate and the background are considered to be constant during the measurements.

In counting measurements on filters during accumulation of radioactive material, the measurement in question is evaluated with respect to two measurands:

— the radioactivity per unit volume (activity concentration);

— the variation of the radioactivity per unit volume (variation of activity concentration).

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Determination of the detection limit and decision threshold for ionizing radiation measurements —

Part 5: Fundamentals and applications to counting measurements on filters during accumulation of radioactive material

1 Scope

This part of ISO 11929 specifies suitable statistical values which allow an assessment of the detection capabilities in ionizing radiation measurements and of the physical effect quantified by the measurand. For this purpose, Bayesian statistical methods are used to specify characteristic limits.

This part of ISO 11929 deals with fundamentals and applications to counting measurements on filters during accumulation of radioactive material.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 11929-7:2005, *Determination of the detection limit and decision threshold for ionizing radiation measurements — Part 7: Fundamentals and general applications*

BIPM/IEC/IFCC/ISO/IUPAC/OIML, *Guide to the Expression of Uncertainty in Measurement*, *Geneva, 1993*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1
measuring method
any logical sequence of operations, described generically, used in the performance of measurements

NOTE 1 Adapted from the International Vocabulary of Basic and General Terms in Metrology:1993.

NOTE 2 A method as defined for the purpose of this part of ISO 11929 shall be an integral, a dual channel or a spectrometric counting measurement under specified conditions during accumulation of particles on a filter.

3.2
measurand
particular quantity subject to measurement

[International Vocabulary of Basic and General Terms in Metrology:1993]

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NOTE 1 Measurand in this part of ISO 11929 is non-negative and quantifies a nuclear radiation effect. The effect is not present if the value of the measurand is zero.

NOTE 2 In this standard, two measurands are distinguished, namely the radioactivity per unit volume (activity concentration) and the variation of the radioactivity per unit volume (variation of the activity concentration). For both measurands the characteristic limits are specified.

3.3
uncertainty (of measurement)
parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand

[Guide for the Expression of Uncertainty in Measurement:1993]

NOTE The uncertainty of a measurement derived according to the Guide for the Expression of Uncertainty in Measurement comprises, in general, many components. Some of these components may be evaluated from the statistical distribution of the results of series of measurements and can be characterized by experimental standard deviations. The other components, which also can be characterized by standard deviations, are evaluated from assumed or known probability distributions based on experience and other information.

3.4
mathematical model of the evaluation
a set of mathematical relationships between all measured and other quantities involved in the evaluation of the measurement

3.5
decision quantity
random variable for the decision whether or not the physical effect to be measured is present

3.6
series of measurements
number of independent measurement cycles following one another giving information about the volume concentration of radioactivity and about the trend of the results versus time.

NOTE Each new result will be checked to decide whether it is in the trend or it is significantly different from the expected value.

3.7
decision threshold
fixed value of the decision quantity by which, when exceeded by the result of an actual measurement of a measurand quantifying a physical effect, one decides that the physical effect is present

NOTE The decision threshold is the critical value of a statistical test to decide between the hypothesis that the physical effect is not present and the alternative hypothesis that it is present. When the critical value is exceeded by the result of an actual measurement this is taken to indicate that the hypothesis should be rejected. The statistical test will be designed such that the probability of wrongly rejecting the hypothesis (error of the first kind) is equal at most to a given value α.

3.8
detection limit
smallest true value of the measurand which is detectable by the measuring method

NOTE 1 The detection limit is the smallest true value of the measurand which is associated with the statistical test and hypotheses according to 3.7 by the following characteristics: if in reality the true value is equal to or exceeds the detection limit, the probability of wrongly not rejecting the hypothesis (error of the second kind) will be at most equal to a given value β.

NOTE 2 The difference between using the decision threshold and using the detection limit is that measured values are to be compared with the decision threshold, whereas the detection limit is to be compared with the guideline value.

3.9
confidence limits
values which define confidence intervals to be specified for the measurand in question which, if the result exceeds the decision threshold, includes the true value of the measurand with the given probability $(1-\gamma)$

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3.10
sample
radioactive material accumulated on a filter absorbed from the whole amount of exhausted gas, air or water or an aliquot of it in a bypass and measured in a particular cycle

3.11
background counting rate
measured counting rate without radioactivity of interest

NOTE This is the counting rate caused by external sources, and radioactivity in detector and shielding and detector noise.

3.12
gross counting rate
measured counting rate due to both the radioactivity on the filter (sample contribution) and the background counting rate

3.13
net counting rate(1)
difference between the gross counting rate of the first measuring cycle and the background counting rate

3.14
net counting rate(2)
difference between the gross counting rates of two consecutive measurements

3.15
guideline value
value which corresponds to scientific, legal or other requirements for which the measuring procedure is intended to assess

EXAMPLE Activity, specific activity or activity concentration, surface activity, or dose rate

4 Quantities and symbols

$\hat{\xi}$ Random variable as an estimator for a non-negative measurand quantifying a physical effect

ξ Value of the estimator $\hat{\xi}$; true value of the measurand

X Random variable as decision quantity; estimator of the measurand

x Result of a measurement of the decision quantity X

$u(x)$ Standard uncertainty of the measurand associated with the measured result x of a measurement

$\tilde{u}(\xi)$ Standard uncertainty of the decision quantity X as a function of the true value ξ of the measurand

z Best estimate of the measurand

$u(z)$ Standard uncertainty of the measurand associated with the best estimate z

x^* Decision threshold for the measurand

ξ^* Detection limit for the measurand

ξ_l, ξ_u Respectively the lower and upper limit of the confidence interval for the measurand

α Probability of the error of the first kind; the probability of rejecting the hypothesis if it is true

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β	Probability of the error of the second kind; the probability of accepting the hypothesis if it is false
$1 - \gamma$	Probability attributed to the confidence interval of the measurand; probability that the true value of the measurand is included by the confidence interval
k_p	Quantiles of the standardized normal distribution for the probability p (see Table 1); $p = 1 - \alpha$, $1 - \beta$, $1 - \gamma/2$
Y_k	Output quantity Y_k derived from the measured results; ($k = 1, ..., n$)
y_k	Estimate of an output quantity Y_k; ($k = 1, ..., n$)
$u(y_k)$	Standard uncertainty associated with y_k
G_k	Function of the input quantities X_i; ($i = 1, ..., m$); model of the evaluation
X_i	Input quantities; ($i = 1, \ldots, m$)
x_i	Estimate of an input quantity; ($i = 1, \ldots, m$)
$u(x_i, x_j)$	Covariance associated with x_i and x_j; ($i, j = 1, \ldots, m$)
E	Operator for the formation of the expectation of a random variable
Var	Operator for the formation of the variance of a random variable
κ	Parameter
A_V	Activity concentration
α_V	True value of the activity concentration
A^*_V	Decision threshold of the activity concentration
α^*_V	Detection limit of the activity concentration
ΔA_V	Variation of the activity concentration
$\Delta\alpha_\mathrm{V}$	True value of the variation of the activity concentration
ΔA^*_V	Decision threshold of the variation of the activity concentration
$\Delta\alpha^*_\mathrm{V}$	Detection limit of the variation of the activity concentration
$N_{\mathrm{g},i}$	Gross number of events counted in the i-th measuring cycle of duration t
t	Duration of each measurement cycle
$R_{\mathrm{g},i}$	Gross counting rate during i-th measuring cycle
ε	Detection efficiency
V	Volume of gas or water passing the filter during a measuring cycle
R_0	Background count rate during the first measuring cycle
$R_{\mathrm{n},i}$	Net counting rate, difference of gross counting rate of measuring cycles i and i–1; $R_{\mathrm{n},i} = R_{\mathrm{g},i} - R_{\mathrm{g},i-1}$
$\overline{R_{\mathrm{n},i}}$	Mean net counting rate averaged over k measuring cycles preceding the i-th measuring cycle

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A_i Activity collected on the filter during the i-th measuring cycle

$\overline{A}$ Average activity collected during k measuring cycles

$\Phi(t)$ Distribution function of the standardized normal distribution

$\varphi(z)$ Standardized normal distribution

5 Statistical values and confidence interval

5.1 Principles

5.1.1 General aspects

For a particular task involving nuclear radiation measurements, first the particular physical effect which is the objective of the measurement has to be described. Then a non-negative measurand has to be defined which quantifies the physical effect and which assumes the value zero if the effect is not present in an actual case.

A random variable, called a decision quantity X, has to be attributed to the measurand. It is also an estimator of the measurand. It is required that the expectation value EX of the decision quantity X equals the true value ξ of the measurand. A value x of the estimator X derived from measurements is a primary estimate of the measurand. The primary estimate x of the measurand, and its associated standard uncertainty $u(x)$, have to be calculated as the primary complete result of the measurement, according to the Guide for the Expression of Uncertainty in Measurement, by evaluation of measured quantities and of other information using a mathematical model of the evaluation which takes into account all relevant quantities. Generally, the fact that the measurand is non-negative will not be explicitly made use of. Therefore, x may become negative, in particular, if the true value of the measurand is close to zero.

NOTE The model of the evaluation of the measurement need not necessarily be given in the form of explicit mathematical formulas. It can also be represented by an algorithm or a computer code [see Equation (2)].

For the determination of the decision threshold and the detection limit, the standard uncertainty of the decision quantity has to be calculated, if possible, as a function $\tilde{u}(\xi)$ of the true value ξ of the measurand. In the case that this is not possible, approximate solutions are described below.

ξ is the value of another, non-negative estimator $\hat{\xi}$ of the measurand. The estimator $\hat{\xi}$, in contrast to X, makes use of the knowledge that the measurand is non-negative. The limits of the confidence interval to be determined refer to this estimator $\hat{\xi}$ (compare 5.4). Besides the limits of the confidence interval, the expectation value E$\hat{\xi}$ of this estimator as best estimate z of the measurand, and the standard deviation [Var $(\hat{\xi})]^{1/2}$ as a the standard uncertainty $u(z)$ associated with the best estimate z of the measurand, have to be calculated (see 6.3).

For the numerical calculation of the decision threshold and the detection limit, the function $\tilde{u}(\xi)$ is needed, which is the standard uncertainty of the decision quantity X as a function of the true value ξ of the measurand. The function $\tilde{u}(\xi)$ generally has to be determined by the user of this part of ISO 11929, in the course of the evaluation of the measurement according to the Guide for the Expression of Uncertainty in Measurement. For examples see Annex A. This function is often only slowly increasing. Therefore, it is justified in many cases to use the approximation $\tilde{u}(\xi) = u(x)$.This applies, in particular, if the primary estimate x of the measurand is not much larger than its standard uncertainty $u(x)$ associated with x. If the value x is calculated as the difference (net effect) of two approximately equal values y_1 and y_0 obtained from independent measurements, that is $x = y_1 - y_0$, one gets $u^2(\xi) = u^2(y_1) + u^2(y_0)$ with the standard uncertainties $u(y_1)$ and $u(y_0)$ associated with y_1 and y_0, respectively.

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If only $\tilde{u}(0)$ and $u(x)$ are known, an approximation by linear interpolation is often sufficient for $x > 0$ according to:

$$\tilde{u}^2(\xi) = \tilde{u}^2(0) \cdot (1 - \xi / x) + u^2(x) \cdot \xi / x \tag{1}$$

NOTE In many practical cases, $\tilde{u}^2(\xi)$ is a slowly increasing linear function of ξ. This justifies the approximations above, in particular, the linear interpolation of $\tilde{u}^2(\xi)$ instead of $\tilde{u}(\xi)$ itself.

For setting up the mathematical model of the evaluation of the measurement, one has to distinguish two types of physical quantities, input and output quantities. The output quantities Y_k ($k = 1, ..., n$) are viewed as measurands (for example, the parameters of an unfolding or fitting procedure) which have to be determined by the evaluation of a measurement. The decision quantity X is one of them. They depend on the input quantities x_i ($i = 1, ..., m$) which are the quantities obtained by repeated measurements, influence quantities and results of previous measurements and evaluations. (Compare chapter 4.1.2 of the ISO Guide for the Expression of Uncertainty in Measurement:1993.) One has to calculate the estimates y_k of the output quantities (measurands) as the results of the measurement and the standard uncertainties $u(y_k)$ associated with y_k.

The model of the evaluation is given by a set of functional relationships:

$$Y_k = G_k(x_1, ..., X_m); (x = 1, ..., n) \tag{2}$$

Estimates of the measurands Y_k, denoted y_k, are obtained from Equation (2) using input estimates $x_1, ..., x_m$ for the values of the m quantities $X_1, ..., X_m$. Thus, the output estimates y_k and the standard uncertainties $u(y_k)$ associated with y_k are given by

$$y_k = G_k(x_1, ..., x_m); (k = 1, ..., n) \tag{3}$$

$$u(y_k, y_l) = \sum_{i, j = 1}^{m} \frac{\partial G_k}{\partial X_i} \cdot \frac{\partial G_l}{\partial X_j} \cdot u(x_i, x_j); (k, l = 1, ..., n) \tag{4}$$

where x_i and x_j are the estimates of X_i and X_j and $u(x_i, x_j) = u(x_j, x_i)$ are the estimated covariances associated with x_i and x_j. The standard uncertainty $u(y_k)$ is given by:

$$u^2(y_k) = u(y_k, y_k) \tag{5}$$

In cases when the partial derivatives are not explicitly available, they can be numerically approximated in a sufficiently exact way using the standard uncertainty $u(x_k)$ as an increment of x_k by

$$\frac{\partial G_k}{\partial X_i} \approx \frac{1}{u(x_i)} \left\{ G_k \left[x_1, ..., x_i + u(x_i) / 2, ..., x_m \right] - G_k \left[x_1, ..., x_i - u(x_i) / 2, ..., x_m \right] \right\} \tag{6}$$

5.1.2 Model

When evaluating any measurements of filters during accumulation of radioactive material, an output quantity Y will be calculated from input data X_i using the general model

$$Y = G(X_1, ..., X_n) \tag{7}$$

The general model of Equation (7) will be specified in this part of ISO 11929. Determine the values of two measurands:

a) the concentration of radioactivity per unit volume of gas or liquid A_V; and

b) the variation of the concentration of radioactivity per unit volume of gas or liquid ΔA_V.

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The measurements are performed as follows. A volume V of a gas or liquid is flowing through a filter during a time t. During the accumulation of radioactive materials on the filter, cyclic counting measurements are performed. Each measuring cycle i has a duration t during which a gross number of events $N_{g,i}$ is counted. The values of both measurands A_V and ΔA_V are to be determined from these measurements taking into account a calibration factor ε. For the sake of simplicity it is assumed that the calibration factor ε and the volume V can be determined with negligible uncertainties.

For the evaluation of the measurements with respect to the two measurands, one needs two different models of evaluation which are described in 5.1.2.1 and 5.1.2.2.

5.1.2.1 Model for the measurement of concentration of radioactivity

The measurand in this counting measurement is the activity concentration A_V calculated from the difference between the gross counting rates of two consecutive measurements:

$$A_V = \frac{R_{g,i} - R_{g,i-1}}{\varepsilon \cdot V} = \frac{N_{g,i} - N_{g,i-1}}{\varepsilon \cdot V \cdot t} \tag{8}$$

where

$R_{g,i}$ is the i-th gross counting rate;

$R_{g,i-1}$ is the (i–1)-th gross counting rate;

$N_{g,i}$ are the gross pulses counted in the i-th measuring cycle of duration t;

$N_{g,i-1}$ are the gross pulses counted in the (i–1)-th measuring cycle of duration t;

t is the duration of each measurement;

ε is the detection efficiency;

V is the volume of gas or water passing the filter while the measurement is running.

Ignoring the uncertainty of t, and for simplicity, those of ε and V, the standard uncertainty $u(A_V)$ associated with A_V is given by:

$$\tilde{u}^2(A_V) = \frac{N_{g,i} + N_{g,i-1}}{(\varepsilon \cdot V \cdot t)^2} \tag{9}$$

For the calculation of the decision threshold A^*_V, one needs the standard uncertainty $\tilde{u}(0)$ of A_V for a true value $\alpha_V = 0$. For $\alpha_V = 0$, one expects $N_{g,i} = N_{g,i-1}$. This yields:

$$\tilde{u}^2(0) = \frac{2 \cdot N_{g,i-1}}{(\varepsilon \cdot V \cdot t)^2} \tag{10}$$

$\tilde{u}^2(0)$ depends on $N_{g,i-1}$ which increases with increasing radioactivity on the filter. The smallest value of $\tilde{u}^2(0)$ is:

$$\tilde{u}^2(0) = \frac{2 \cdot N_0}{(\varepsilon \cdot V \cdot t)^2} = \frac{2 \cdot R_0}{(\varepsilon \cdot V)^2 \cdot t} \tag{11}$$

with N_0 being the measured counts using a fresh filter.

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For the calculation of the detection limit, the standard uncertainty $\tilde{u}(\alpha_V)$ as a function of the true value α_V is needed. For a true value α_V of the measurand, one expects:

$$N_{g,i} = N_{g,i-1} + \alpha_V \cdot \varepsilon \cdot V \cdot t \tag{12}$$

and with Equation (9) one gets

$$\tilde{u}^2(\alpha_V) = \frac{2 \cdot N_{g,i-1}}{(\varepsilon \cdot V \cdot t)^2} + \frac{\alpha_V \cdot \varepsilon \cdot V \cdot t}{(\varepsilon \cdot V \cdot t)^2} = \frac{2 \cdot N_{g,i-1}}{(\varepsilon \cdot V \cdot t)^2} + \frac{\alpha_V}{\varepsilon \cdot V \cdot t} \tag{13}$$

5.1.2.2 Model for the evaluation of the measurement of the variation of concentration of radioactivity ΔA_V

The variation (increase or decrease) of the volume concentration of radioactivity ΔA_V in an exhaust air, gas or water stream will be determined from the variation of the amount of radioactivity A_i collected on the filter during a measuring cycle in comparison to the mean amount of radioactivity $\overline{A}$ collected on the filter during preceding measuring cycles of the same duration, divided by the volume V of air, gas or water passing the filter during a measuring cycle.

$$\Delta A_V = (A_i - \overline{A})/V \tag{14}$$

$(A_i - \overline{A})$ will be determined by comparing the variation (increase or decrease) of the net counting rate $R_{n,i} = R_{g,i} - R_{g,i-1}$ of the last measuring cycle with the mean of the net counting rates $\overline{R_{n,i}}$ measured in a chosen number $k > 1$ of preceding measuring cycles.

$$\Delta A_V = \frac{R_{n,i} - \overline{R_{n,i}}}{\varepsilon \cdot V} \tag{15}$$

where

ε is the detection efficiency;

V is the volume of gas or water passing the filter while the measurement is running.

and

$$\overline{R_{n,i}} = \frac{1}{k} \cdot \sum_{j=i-k}^{i-1} R_{n,j} = \frac{1}{k} \cdot \sum_{j=i-k}^{i-1} \left(R_{g,j} - R_{g,j-1}\right) = \frac{1}{k} \cdot (R_{g,i-1} - R_{g,i-k-1}) \tag{16}$$

Ignoring the uncertainties of t, ε and V, one obtains with a model according to Equation (15)

$$\Delta A_V = \frac{1}{\varepsilon \cdot V} \cdot \left[R_{g,i} - R_{g,i-1} - \frac{1}{k}(R_{g,i-1} - R_{g,i-k-1})\right] = \frac{1}{\varepsilon \cdot V} \cdot \left[R_{g,i} - (1 + \frac{1}{k}) \cdot R_{g,i-1} + \frac{1}{k} \cdot R_{g,i-k-1}\right] \tag{17}$$

$$u^2(\Delta A_V) = \frac{1}{(\varepsilon \cdot V)^2 \cdot t} \cdot \left[R_{g,i} + (1 + \frac{1}{k})^2 \cdot R_{g,i-1} + \frac{1}{k^2} \cdot R_{g,i-k-1}\right] \tag{18}$$

For the calculation of the decision threshold ΔA^*_V, one needs the standard uncertainty $\tilde{u}(0)$ for a true value $\Delta\alpha_V = 0$. For $\Delta\alpha_V = 0$ one expects:

$$R_{g,i} = R_{g,i-1} + \frac{1}{k} \cdot (R_{g,i-1} - R_{g,i-k-1}) \text{ with } u^2(R_{g,i}) = (1 + \frac{1}{k})^2 \cdot \frac{R_{g,i-1}}{t} + \frac{1}{k^2} \cdot \frac{R_{g,i-k-1}}{t} \tag{19}$$

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This yields

$$\tilde{u}^2(0) = \frac{2}{(\varepsilon \cdot V)^2 \cdot t} \cdot \left[(1+\frac{1}{k})^2 \cdot R_{g,i-1} + \frac{1}{k^2} \cdot R_{g,i-k-1}) \right] \qquad (20)$$

For the calculation of the detection limit $\Delta\alpha^*_V$, one needs the standard uncertainty $\tilde{u}(\Delta\alpha_V)$ as a function of the true value $\Delta\alpha_V$. For $\Delta\alpha_V$ one expects

$$R_{g,i} = \Delta\alpha_V \cdot \varepsilon \cdot V + R_{g,i-1} + \frac{1}{k}(R_{g,i-1} - R_{g,i-k-1}) \qquad (21)$$

With Equation (18), one obtains from Equation (21)

$$\tilde{u}^2(\Delta\alpha_V) = \frac{\Delta\alpha_V}{\varepsilon \cdot V \cdot t} + \frac{2}{(\varepsilon \cdot V)^2 \cdot t} \left[(1+\frac{1}{k})^2 \cdot R_{g,i-1} + \frac{1}{k^2} \cdot R_{g,i-k-1} \right] \qquad (22)$$

5.2 Decision threshold

The decision threshold x^* of a non-negative measurand quantifying the physical effect, according to 5.1 is a value of the decision quantity X which, when it is exceeded by a result x of a measurement, indicates that the physical effect is present. If $x \leqslant x^*$ one decides that the physical effect is not present. If this decision rule is observed, a wrong decision in favour of the presence of the physical effect occurs with the probability α (error of the first kind). The primary result of the measurement x must be greater than the decision threshold.

The decision threshold is given by:

$$x^* = k_{1-\alpha} \cdot \tilde{u}(0) \qquad (23)$$

Values of the quantiles $k_{1-\alpha}$ of the standardized normal distribution are given in Table 1. It is $\Phi(k_{1-\alpha}) = 1-\alpha$.

If the approximation $\tilde{u}(\xi) = u(x)$ is sufficient, one gets: $x^* = k_{1-\alpha} \cdot u(x)$.

5.2.1 Decision threshold for the measurement of concentration of radioactivity

This decision threshold A^*_V is calculated using Equations (10) and (23):

$$A^*_V = \frac{k_{1-\alpha}}{\varepsilon \cdot V \cdot t} \cdot \sqrt{2 \cdot N_{g,i-1}} \qquad (24)$$

With the background measured on a fresh filter, one gets:

$$A^*_V = \frac{k_{1-\alpha}}{\varepsilon \cdot V \cdot t} \sqrt{2 \cdot N_0} \qquad (25)$$

This gives the best decision threshold. The decision threshold increases with increasing activity on the filter and therefore the measurement becomes less and less sensitive.

5.2.2 Decision threshold for the measurement of the variation of the concentration of radioactivity

This decision threshold ΔA^*_V is calculated using Equations (20) and (23):

$$\Delta A^*_V = \frac{2 \cdot k_{1-\alpha}}{(\varepsilon \cdot V)^2 \cdot t} \cdot \left[(1+\frac{1}{k})^2 \cdot R_{g,i-1} + \frac{1}{k^2} \cdot R_{g,i-k-1} \right] \qquad (26)$$

Here the increase of the decision threshold with increasing activity of the filter is even more profound.

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5.3 Detection limit

5.3.1 General

The detection limit ξ^*, which is the smallest true value of the measurand detectable with the measuring method, is so much larger than the decision threshold that the probability of an error of the second kind is not greater than β. The detection limit is given by:

$$\xi^* = x^* + k_{1-\beta} \cdot \tilde{u}(\xi^*) \tag{27}$$

Equation (27) is an implicit one. The detection limit can be calculated from it by iteration using, for example, the starting approximation $\xi^* = 2x^*$. The iteration converges in most cases.

5.3.2 Detection limit for the measurement of the concentration of radioactivity

This detection limit α^*_V is calculated using Equations (27) and (13) with $\alpha_V = \alpha^*_V$:

$$\alpha_V^* = A_V^* + k_{1-\beta} \cdot \sqrt{\frac{\alpha_V^*}{\varepsilon \cdot V \cdot t} + \frac{2 \cdot N_{g,i-1}}{(\varepsilon \cdot V \cdot t)^2}} \tag{28}$$

This detection limit can be calculated from Equation (28) by iteration using the starting approximation $\alpha^*_V = 2A^*_V$.

5.3.3 Detection limit for the measurement of the variation of the concentration of radioactivity

This detection limit $\Delta\alpha^*_V$ is calculated using Equations (27) and (22) with $\Delta\alpha_V = \Delta\alpha^*_V$:

$$\Delta\alpha_V^* = \Delta A_V^* + k_{1-\beta} \cdot \sqrt{\frac{\Delta\alpha_V^*}{\varepsilon \cdot V \cdot t} + \frac{2}{(\varepsilon \cdot V)^2 \cdot t} \cdot \left[(1+\frac{1}{k})^2 \cdot R_{g,i-1} + \frac{1}{k^2} \cdot R_{g,i-k-1}\right]} \tag{29}$$

The detection limit can be calculated from Equation (29) by iteration using the starting approximation $\Delta\alpha^*_V = 2\Delta A^*_V$.

5.4 Confidence limits

For a result x of a measurement which exceeds the decision threshold x^*, the confidence interval includes the true value of the measurand with the given probability $1 - \gamma$. It is enclosed by the confidence limits ξ_l and ξ_u according to:

$$\xi_l = x - k_p \cdot u(x) \quad \text{with} \quad p = \kappa \cdot (1 - \gamma/2) \tag{30}$$

$$\xi_u = x + k_q \cdot u(x) \quad \text{with} \quad q = 1 - (\kappa \cdot \gamma/2) \tag{31}$$

κ is given by:

$$\kappa = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x/u(x)} \exp(-z^2/2)\,\mathrm{d}z = \Phi\left[x/u(x)\right] \tag{32}$$

Values of the function $\Phi(t)$ are tabulated in Table 1. It is $\Phi(k_p) = p$ and $\Phi(k_q) = q$.

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The confidence limits are not symmetrical around the expectation $\mathrm{E}\hat{\xi}$. The probabilities of $\hat{\xi} < \xi_l$ and $\hat{\xi} > \xi_u$, however, are both equal to $\gamma/2$ and the relationship $0 < \xi_l < \xi_u$ is valid. For $x \gg u(x)$ the approximation

$$\xi_{l,u} = x \pm k_{1-\gamma/2} \cdot u(x) \tag{33}$$

is applicable if $x > 2k_{1-\gamma/2} \cdot u(x)$.

ξ_l, ξ_u can be calculated by Equations (30) to (32) or (33) using $u(A_V)$ from Equation (9) for the confidence limits of measured concentration of activity and using $u(\Delta A_V)$ from Equation (18) for the confidence limits of measured variations of activity concentration.

6 Application of this part of ISO 11929

6.1 Specific values

The probabilities α, β and $(1 - \gamma)$ shall be specified in advance by the user of this part of ISO 11929. Commonly used values are $\alpha = \beta = 0{,}05$ and $\gamma = 0{,}05$.

6.2 Assessment of a measuring method

To check whether a measuring method (see 3.1) is suitable for the measurement of a physical effect, the detection limit shall be compared with a specified guideline value (e.g. specified requirements on the sensitivity of the measuring procedure for scientific, legal or other reasons; see 3.15).

The detection limit shall be calculated by means of Equation (27). If the detection limit thus determined is greater than the guideline value, the measuring procedure is not suitable for the measurement.

6.3 Assessment of measured results

A measured result has to be compared with the decision threshold calculated by means of Equation (23). If the result of the measurement x is larger than the decision threshold x^*, it is decided that the physical effect quantified by the measurand is present.

If this is the case, the best estimate z of the measurand is calculated using κ from Equation (32) by:

$$z = \mathrm{E}\hat{\xi} = x + \frac{u(x) \cdot \exp\left\{-x^2 / \left[2 \cdot u^2(x)\right]\right\}}{\kappa \cdot \sqrt{2\pi}} \tag{34}$$

with the standard uncertainty $u(z)$ associated with z:

$$u(z) = \sqrt{\mathrm{Var}(\hat{\xi})} = \sqrt{u^2(x) - (z - x) \cdot z} \tag{35}$$

The following relationships: $z \geqslant x$ and $z \geqslant 0$, as well as $u(z) \leqslant u(x)$, are valid and for $x \gg u(x)$, i.e. $x > 4 \cdot u(x)$, the approximations $z = x$ and $u(z) = u(x)$ hold true.

6.4 Documentation

The documentation of measurements in accordance with this part of ISO 11929 shall contain details of the probabilities α, β and $(1 - \gamma)$, the decision threshold x^*, the detection limit ξ^*, and the guideline value.

For a result x of the measurement exceeding the decision threshold x^*, the standard uncertainty $u(x)$ associated with x and the limits of the confidence interval $\xi_{l,u}$ have to be given. If the result x of the measurement is below the decision threshold x^*, it shall be documented as “below the decision threshold”.

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If the detection limit exceeds the guideline value, it shall be documented that the method is not suitable for the measurement purpose.

In addition, the best estimate z of the measurand and the standard uncertainty $u(z)$ associated with z may be specified if $x/u(x) < 4$.

7 Values of the distribution function of the standardized normal distribution

Values $\Phi(t) = \int_{-\infty}^{t} \varphi(z)\,\mathrm{d}z$ with $\varphi(z) = (1/\sqrt{2\pi}) \cdot \exp(-z^2/2)$ are given in Table 1. For the distribution function of the standardized normal distribution, $\Phi(-t) = 1 - \Phi(t)$ is valid. Quantiles of the standardized normal distribution can also be obtained from this Table 1, since $t = k_p$ for $p = \Phi(t)$, i.e. $\Phi(k_p) = k_p$.

For $t \geqslant 0$, the approximation (see [8] in the Bibliography):

$$\Phi(t) = 1 - \frac{\exp(-t^2/2)}{\sqrt{2\pi}} (\alpha_1\zeta + \alpha_2\zeta^2 + \alpha_3\zeta^3) + \varepsilon; \zeta = \frac{1}{1 + \alpha_0 t}$$

is valid with $|\varepsilon| < 10^{-5}$ and

$\alpha_0 = 0{,}332\,67$; $\alpha_1 = 0{,}436\,183\,6$; $\alpha_2 = -0{,}120\,167\,6$; $\alpha_3 = 0{,}937\,298\,0$

For $t < 0$, one obtains $\Phi(t)$ from the relationship $\Phi(t) = 1 - \Phi(-t)$.

For $0{,}5 \leqslant p < 1$ the approximation (see [8] in the Bibliography):

$$k_p \approx t - \frac{b_0 + b_1 t + b_2 t^2}{1 + c_1 t + c_2 t^2 + c_3 t^3} + \varepsilon ; t = \sqrt{-2\ln(1-p)}$$

is valid with $|\varepsilon| < 4{,}5 \times 10^{-4}$ and

$b_0 = 2{,}515\,517$; $b_1 = 0{,}802\,853$; $b_2 = 0{,}010\,328$;

$c_1 = 1{,}432\,788$; $c_2 = 0{,}189\,269$; $c_3 = 0{,}001\,308$.

For $0 < p < 0{,}5$, one obtains k_p from the relationship $k_p = -k_{1-p}$.

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Table 1 — Values of the distribution function of the standardized normal distribution $\Phi(t)$
(see [7] in the Bibliography)

t	$\Phi(t)$	t	$\Phi(t)$	t	$\Phi(t)$	t	$\Phi(t)$	t	$\Phi(t)$
0,00	0,500 0	0,70	0,758 0	1,40	0,919 2	2,10	0,982 1	2,80	0,997 4
0,02	0,508 0	0,72	0,764 2	1,42	0,922 2	2,12	0,983 0	2,90	0,998 1
0,04	0,516 0	0,74	0,770 4	1,44	0,925 1	2,14	0,983 8	3,00	0,998 6
0,06	0,523 9	0,76	0,776 4	1,46	0,927 8	2,16	0,984 6	3,10	0,999 0
0,08	0,531 9	0,78	0,782 3	1,48	0,930 6	2,18	0,985 4	3,20	0,999 3
0,10	0,539 8	0,80	0,788 1	1,50	0,933 2	2,20	0,986 1	3,30	0,999 5
0,12	0,547 8	0,82	0,793 9	1,52	0,935 7	2,22	0,986 8	3,40	0,999 7
0,14	0,555 7	0,84	0,799 6	1,54	0,938 2	2,24	0,987 4	3,50	0,999 8
0,16	0,563 6	0,86	0,805 1	1,56	0,940 6	2,26	0,988 1	3,60	0,999 8
0,18	0,571 4	0,88	0,810 6	1,58	0,943 0	2,28	0,988 7	3,80	0,999 9
0,20	0,579 3	0,90	0,815 9	1,60	0,945 2	2,30	0,989 3	4,00	1,000 0
0,22	0,587 1	0,92	0,821 2	1,62	0,947 4	2,32	0,989 8		
0,24	0,594 8	0,94	0,826 4	1,64	0,949 5	2,34	0,990 4		
0,26	0,602 6	0,96	0,831 5	1,66	0,951 5	2,36	0,990 9		
0,28	0,610 3	0,98	0,836 5	1,68	0,953 5	2,38	0,991 3		
0,30	0,617 9	1,00	0,841 3	1,70	0,955 4	2,40	0,991 8		
0,32	0,625 5	1,02	0,846 1	1,72	0,957 3	2,42	0,992 2		
0,34	0,633 1	1,04	0,850 8	1,74	0,959 1	2,44	0,992 7		
0,36	0,640 6	1,06	0,855 4	1,76	0,961 0	2,46	0,993 0		
0,38	0,648 0	1,08	0,859 9	1,78	0,962 5	2,48	0,993 4		
0,40	0,655 4	1,10	0,864 3	1,80	0,964 1	2,50	0,993 8		
0,42	0,662 8	1,12	0,868 6	1,82	0,965 6	2,52	0,994 1		
0,44	0,670 0	1,14	0,872 9	1,84	0,967 1	2,54	0,994 5		
0,46	0,677 2	1,16	0,877 0	1,86	0,968 6	2,56	0,994 8		
0,48	0,684 4	1,18	0,881 0	1,88	0,970 0	2,58	0,995 1		
0,50	0,691 5	1,20	0,884 9	1,90	0,971 3	2,60	0,995 3		
0,52	0,698 5	1,22	0,888 8	1,92	0,972 6	2,62	0,995 6		
0,54	0,705 4	1,24	0,892 5	1,94	0,973 8	2,64	0,995 9		
0,56	0,712 3	1,26	0,896 1	1,96	0,975 0	2,66	0,996 1		
0,58	0,719 0	1,28	0,899 7	1,98	0,976 2	2,68	0,996 3		
0,60	0,725 8	1,30	0,903 2	2,00	0,977 2	2,70	0,996 5		
0,62	0,732 4	1,32	0,906 6	2,02	0,978 3	2,72	0,996 7		
0,64	0,738 9	1,34	0,909 9	2,04	0,979 3	2,74	0,996 9		
0,66	0,745 4	1,36	0,913 1	2,06	0,980 3	2,76	0,997 1		
0,68	0,751 8	1,38	0,916 2	2,08	0,981 2	2,78	0,997 3		

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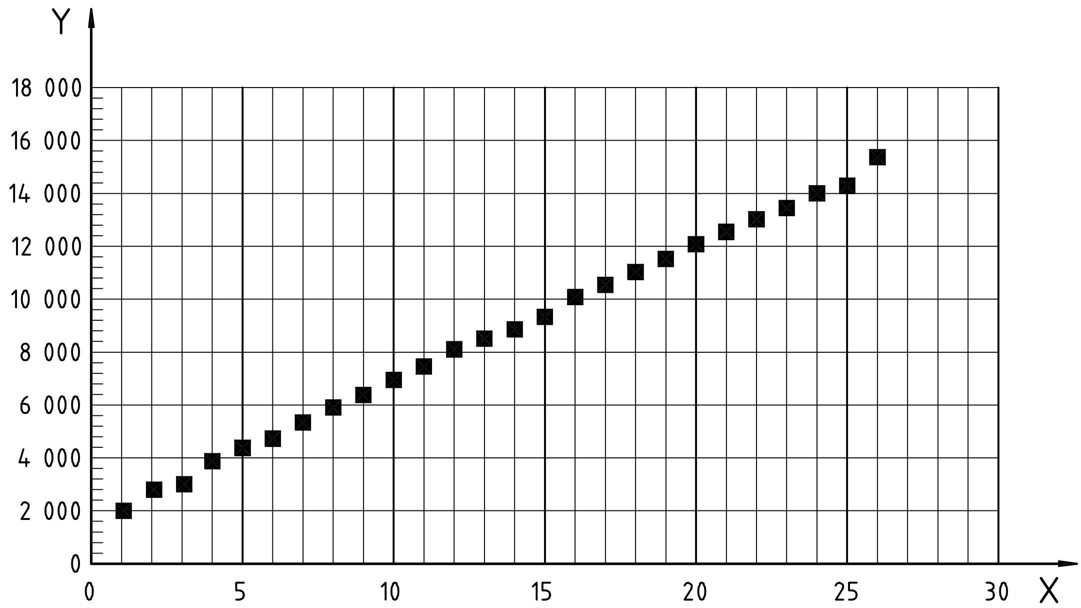
Annex A
(informative)

Example of application of this part of ISO 11929

A.1 Examples of the application of this part of ISO 11929

To exemplify the application of this part of ISO 11929, it is assumed that a radiochemical laboratory is working exclusively with ^{131}I. In order to comply with legal regulations a limit of 20 Bq·m^{-3} must not be exceeded by the activity concentration of the exhaust air. To ensure this, part of the exhaust air is send via a bypass over a filter which is continuously measured by an integral-counting method. The calibration factor was independently determined to be $\varepsilon = 0{,}37$ s^{-1}/Bq. The counting intervals are chosen to be $t = 1$ h, during which time an air volume of $V = 3$ m^3 passes the filter. For simplicity, a 100 % effectiveness of the filter is assumed.

Starting with a new filter at zero time the following counts $N_{g,i}$ ($i = 0, ..., 25$) were measured during 26 measuring cycles: 2 124, 2 691, 3 037, 3 895, 4 475, 4 835, 5 338, 5 987, 6 453, 6 912, 7 577, 8 145, 8 589, 8 998, 9 450, 10 104, 10 537, 11 023, 11 601, 12 035, 12 459, 12 998, 13 456, 14 001, 14 356, 15 438. The counted events are displayed in Figure A.1.



Key

Y Counts

X Time, h

Figure A.1 — Number of counts recorded during 26 measuring cycles during accumulation of ^{131}I on a filter

These measurements shall be evaluated in accordance with this International Standard to ISO 11929-5 with respect to two measurands:

— the activity concentration A_V in the exhaust air, as a function of time for the entire measurement; and

— the change of the activity concentration ΔA_V for the last measurement (cycle $i = 25$).

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For the calculation of characteristic limits, the parameters $\alpha = \beta = 0{,}05$ and $1 - \gamma = 0{,}95$ were chosen yielding $k_{1-\alpha} = k_{1-\beta} = 1{,}645$ and $k_{1-\gamma/2} = 1{,}960$.

A guideline value for the activity concentration of 2 Bq·m^{-3} is chosen in order to be able to detect at least activity concentrations equal to 10 % of the activity concentration limit. For the variation of the activity concentration, a guideline value of 0,2 Bq·m^{-3} is chosen in order to allow for technical measures to reduce the ^{131}I emissions to values well below 10 % of the activity concentration limit.

A.2 Measurement of the activity concentration

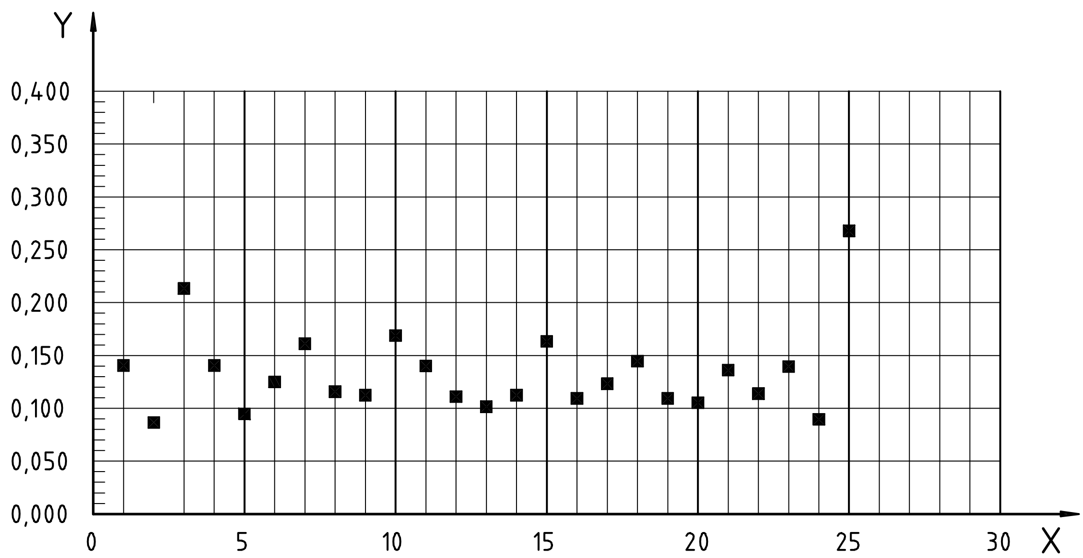
The calculations of the results of the measurement of the activity concentration and of its characteristic limits will be explicitly performed only for the last measurement cycle $i = 25$. With the input data mentioned in A.1 (see also Table A.2), the activity concentration for measuring cycle $i = 25$ is calculated according to Equation (8):

$$A_\mathrm{V} = \frac{N_{\mathrm{g},i} - N_{\mathrm{g},i-1}}{\varepsilon \cdot V \cdot t} = \frac{15\,438 - 14\,356}{0{,}37 \times 3 \times 3\,600} = 0{,}271\ \mathrm{Bq{\cdot}m^{-3}} \quad \text{(A.1)}$$

and its standard uncertainty according to Equation (9):

$$u(A_\mathrm{V}) = \sqrt{\frac{N_{\mathrm{g},i} + N_{\mathrm{g},i-1}}{(\varepsilon \cdot V \cdot t)^2}} = \sqrt{\frac{15\,438 + 14\,356}{(0{,}37 \times 3 \times 3\,600)^2}} = 0{,}043\ \mathrm{Bq{\cdot}m^{-3}} \quad \text{(A.2)}$$

The results for the activity concentrations for all measuring cycles are given in Table A.2 and displayed in Figure A.2.



Key

Y Activity, Bq·m^{-3}

X Time, *h*

Figure A.2 — Activity concentrations measured during 26 measuring cycles during accumulation of ^{131}I on a filter

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A.2.1 Calculation of the decision threshold

For the calculation of the decision threshold A^*_V one needs the standard uncertainty $\tilde{u}(0)$ of A_V for a true value $\alpha_V = 0$.

According to Equation (10) one calculates:

$$\tilde{u}^2(0) = \frac{2 \cdot N_{g,i-1}}{(\varepsilon \cdot V \cdot t)^2} = \frac{2 \cdot 14\,356}{(0{,}37 \times 3 \times 3\,600)^2} = 0{,}001\,798\,1 \text{ Bq}^2 \cdot \text{m}^{-6} \tag{A.3}$$

and consequently with $k_{1-\alpha} = 1{,}645$ and Equation (24), the decision threshold:

$$A^*_V = k_{1-\alpha} \cdot \tilde{u}(0) = 1{,}645 \times 0{,}042\,403\,9 = 0{,}069\,76 \text{ Bq m}^{-3} \tag{A.4}$$

NOTE In this example, numbers are given with too high a precision in order to facilitate recalculation.

Since the result of measurement $A_V = 0{,}271$ Bq m^{-3} exceeds the decision threshold of $A^*_V = 0{,}070$ Bq m^{-3}, an activity concentration of ^{131}I has been observed in the exhaust air in the $i = 25$ measuring cycle.

A.2.2 Calculation of the detection limit

According to Equation (28), one obtains an implicit equation for the detection limit of the activity concentration:

$$\begin{aligned} \alpha^*_V &= A^*_V + k_{1-\beta} \cdot \sqrt{\frac{\alpha^*_V}{\varepsilon \cdot V \cdot t} + \frac{2 \cdot N_{g,i-1}}{(\varepsilon \cdot V)^2 \cdot t}} \\ &= 0{,}069\,76 + 1{,}645 \times \left[\frac{\alpha^*_V}{(0{,}37 \times 3 \times 3\,600)} + \frac{2 \times 14\,356}{(0{,}37 \times 3)^2 \times 3\,600} \right]^{1/2} \end{aligned} \tag{A.5}$$

As an alternative to an iterative solution of this implicit equation, this example gives an explicit solution of this quadratic equation using Equations (1) and (4) of ISO 11929-7:2005.

Since $\tilde{u}(0)$ and $u(A_V)$ are known, one can use the interpolation formula of Equation (1) of ISO 11929-7 to calculate $\tilde{u}(\alpha_V)$.

$$\begin{aligned} \tilde{u}^2(\alpha_V) &= \tilde{u}^2(0) \cdot (1 - \alpha_V / A_V) + u^2(A_V) \cdot \alpha_V / A_V \\ &= \tilde{u}^2(0) \times (1 - \alpha_V / 0{,}271) + 0{,}001\,866 \times \alpha_V / 0{,}271 \end{aligned} \tag{A.6}$$

Then the detection limit can be explicitly calculated using Equation (4) of ISO 11929-7.

$$\alpha^*_V = a + \sqrt{a^2 + (k^2_{1-\beta} - k^2_{1-\alpha}) \cdot \tilde{u}^2(0)} \tag{A.7}$$

with

$$\begin{aligned} a &= k_{1-\alpha} \cdot \tilde{u}(0) + \frac{1}{2}(k^2_{1-\beta} / A_V) \cdot \left[u^2(A_V) - \tilde{u}^2(0) \right] \\ &= 1{,}645 \times 0{,}042\,403\,9 + \frac{1}{2}(1{,}645^2 / 0{,}271) \times (0{,}001\,866 - 0{,}001\,798\,1) \\ &= 0{,}070\,10 \text{ Bq} \cdot \text{m}^{-3} \end{aligned} \tag{A.8}$$

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In this example $\alpha = \beta$, one obtains:

$$\alpha_V^* = 2a = 2 \times 0{,}070\,10 = 0{,}140\,20\ \text{Bq} \cdot \text{m}^{-3} \tag{A.9}$$

Since the detection limit of the activity concentration $\alpha^*_V = 0{,}14\ \text{Bq} \cdot \text{m}^{-3}$ is smaller than the guideline value of $2\ \text{Bq} \cdot \text{m}^{-3}$, the measuring method is suitable for the measurement of the activity concentration.

A.2.3 Calculation of the confidence limits

In order to calculate the confidence limits, the parameter κ has to be calculated:

$$\begin{aligned}\kappa &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{A_V/u(A_V)} \exp(-z^2/2)\,\mathrm{d}z = \Phi[A_V/u(A_V)] \\ &= \Phi(0{,}271/0{,}043) = \Phi(6{,}268) = 1{,}0\end{aligned} \tag{A.10}$$

Then one calculates with Equations (30) and (31):

$$p = \kappa \cdot (1 - \gamma/2) = 1{,}0 \cdot (1 - 0{,}025) = 0{,}975 \tag{A.11}$$

$$q = 1 - (\kappa \cdot \gamma/2) = 1 - (1{,}0 \times 0{,}025) = 0{,}975 \tag{A.12}$$

From Table A.1, one obtains the quantiles for the probabilities p and q of the standardized normal distribution $k_p = 1{,}96$ and $k_q = 1{,}96$ and calculates the confidence limits with Equations (30) and (31):

$$\alpha_{V,l} = A_V - k_p \cdot u(A_V) = 0{,}270\,77 - 1{,}96 \times 0{,}043\,20 = 0{,}186\,10\ \text{Bq} \cdot \text{m}^{-3} \tag{A.13}$$

$$\alpha_{V,u} = A_V - k_q \cdot u(A_V) = 0{,}270\,77 + 1{,}96 \times 0{,}043\,20 = 0{,}355\,44\ \text{Bq} \cdot \text{m}^{-3} \tag{A.14}$$

Since the uncertainty of measurement result is small compared to the measurement result, the calculation of the best estimate z and its standard uncertainty $u(z)$ is not needed.

A.2.4 Documentation

Table A.1 — Complete documentation of the measurement of the activity concentration during accumulation of ^{131}I on a filter for measuring cycle 25

1	2	3	4
Quantity	**Symbol**	**Value**	**Unit**
activity concentration (cycle 25)	A_V	0,271	$\text{Bq} \cdot \text{m}^{-3}$
uncertainty of the activity concentration	$u(A_V)$	0,043	$\text{Bq} \cdot \text{m}^{-3}$
probability of the error of 1st kind	α	0,050	1
probability of the error of 2nd kind	β	0,050	1
confidence level	$1 - \gamma$	0,950	1
guideline value	—	2,000	$\text{Bq} \cdot \text{m}^{-3}$
decision threshold	A_V^*	0,070	$\text{Bq} \cdot \text{m}^{-3}$
detection limit	α_V^*	0,140	$\text{Bq} \cdot \text{m}^{-3}$
lower confidence limit	$\alpha_{V,l}$	0,186	$\text{Bq} \cdot \text{m}^{-3}$
upper confidence limit	$\alpha_{V,u}$	0,355	$\text{Bq} \cdot \text{m}^{-3}$

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Table A.2 summarizes the experimental data and all the results of the activity concentration measurements of all the measuring cycles, as well as their characteristic limits.

One clearly observes the increase of the decision thresholds and of the detection limits with increasing time and filter load. In all the measurements, the measured activity concentrations are above the decision thresholds. Therefore, one concludes that ^{131}I has been measured in all the measurement cycles.

In spite of their increase with time, the detection limits remain below the guideline value of 2 Bq·m^{-3}. Thus, the measurement procedure is suitable for its purpose.

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Table A.2 — Complete documentation of 26 measuring cycles of a measurement of the activity concentration during accumulation of ^{131}I on a filter, of its evaluation and of the calculated characteristic limits

1	2	3	4	5	6	7	8	9	10	11
Measuring cycle	$N_{g,i}$	A_V Bq·m^{-3}	$u(A_V)$ Bq·m^{-3}	A_V^* Bq·m^{-3}	α_V^* Bq·m^{-3}	$A_V/u(A_V)$	k_p	$\alpha_{V,l}$ Bq·m^{-3}	k_q	$\alpha_{V,u}$ Bq·m^{-3}
0	2 124	—	—	—	—	—	—	—	—	—
1	2 691	0,142	0,017	0,027	0,054	8,171	1,960	0,108	1,960	0,176
2	3 037	0,087	0,019	0,030	0,061	4,572	1,960	0,049	1,960	0,124
3	3 895	0,215	0,021	0,032	0,065	1,305	1,960	0,174	1,960	0,256
4	4 457	0,141	0,023	0,036	0,073	6,150	1,960	0,096	1,960	0,185
5	4 835	0,095	0,024	0,039	0,078	3,921	1,959	0,047	1,960	0,142
6	5 338	0,126	0,025	0,040	0,082	4,987	1,960	0,076	1,960	0,175
7	5 987	0,162	0,027	0,043	0,086	6,099	1,960	0,110	1,960	0,215
8	6 453	0,117	0,028	0,045	0,091	4,178	1,960	0,062	1,960	0,171
9	6 912	0,115	0,029	0,047	0,094	3,970	1,959	0,058	1,960	0,172
10	7 577	0,166	0,030	0,048	0,097	5,525	1,960	0,107	1,960	0,225
11	8 145	0,142	0,031	0,051	0,102	4,530	1,960	0,081	1,960	0,204
12	8 589	0,111	0,032	0,053	0,106	3,432	1,955	0,048	1,960	0,175
13	8 998	0,102	0,033	0,054	0,109	3,084	1,944	0,038	1,960	0,167
14	9 450	0,113	0,034	0,055	0,111	3,328	1,953	0,047	1,960	0,180
15	10 104	0,164	0,035	0,057	0,114	4,677	1,960	0,095	1,960	0,232
16	10 537	0,108	0,036	0,059	0,118	3,014	1,939	0,039	1,961	0,179
17	11 023	0,122	0,037	0,060	0,120	3,310	1,952	0,050	1,960	0,194
18	11 601	0,145	0,038	0,061	0,123	3,843	1,959	0,071	1,960	0,218
19	12 035	0,109	0,038	0,063	0,126	2,823	1,923	0,035	1,961	0,184
20	12 459	0,106	0,039	0,064	0,128	2,709	1,908	0,031	1,961	0,183
21	12 998	0,135	0,040	0,065	0,131	3,378	1,954	0,057	1,960	0,213
22	13 456	0,115	0,041	0,066	0,131	2,816	1,922	0,036	1,961	0,194
23	14 001	0,136	0,041	0,068	0,136	3,289	1,952	0,055	1,960	0,218
24	14 356	0,089	0,042	0,069	0,138	2,108	1,732	0,016	1,968	0,172
25	15 438	0,271	0,043	0,070	0,140	6,268	1,960	0,186	1,960	0,355

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A.3 Measurement of the variation of the activity concentration

Based on the measured counts given in A.1 and Table A.2, the measurement of cycle $i = 25$ shall be evaluated with respect to the variation of activity concentration ΔA_V. Equation (17) yields, with a number of preceding measurements $k = 24$, the result for the variation of the activity concentration:

$$\begin{aligned}\Delta A_\mathrm{V} &= \frac{1}{\varepsilon \cdot V} \cdot \left[R_{\mathrm{g},i} - R_{\mathrm{g},i-1} - \frac{1}{k} \cdot \left(R_{\mathrm{g},i-1} - R_{\mathrm{g},i-k-1} \right) \right] \\ &= \frac{1}{0{,}37 \times 3} \times \left[\frac{15\,438}{3\,600} - \frac{14\,356}{3\,600} - \frac{1}{24} \times \left(\frac{14\,356}{3\,600} - \frac{2\,124}{3\,600} \right) \right] \\ &= 0{,}143\,23\ \mathrm{Bq \cdot m^{-3}}\end{aligned} \qquad \text{(A.15)}$$

Equation (18) gives its standard uncertainty

$$\begin{aligned}u(\Delta A_\mathrm{V}) &= \sqrt{\frac{1}{(\varepsilon \cdot V)^2 \cdot t} \cdot \left[R_{\mathrm{g},i} + (1 + \frac{1}{k})^2 \cdot R_{\mathrm{g},i-1} + \frac{1}{k^2} \cdot R_{\mathrm{g},i-k-1} \right]} \\ &= \sqrt{\frac{1}{(0{,}37 \times 3)^2 \times 3\,600} \times \left[\frac{15\,438}{3\,600} + \left(1 + \frac{1}{24} \right)^2 \frac{14\,356}{3\,600} + \frac{1}{24^2} \times \frac{2\,124}{3\,600} \right]} \\ &= 0{,}044\,07\ \mathrm{Bq \cdot m^{-3}}\end{aligned} \qquad \text{(A.16)}$$

A.3.1 Calculation of the decision threshold

For the calculation of the decision threshold ΔA^*_V, one needs the standard uncertainty $\tilde{u}(0)$ of ΔA_V for a true value $\Delta \alpha_\mathrm{V} = 0$. With Equation (20) one calculates:

$$\begin{aligned}\tilde{u}(0) &= \sqrt{\frac{2}{(\varepsilon \cdot V)^2 \cdot t} \cdot \left[(1 + \frac{1}{k})^2 \cdot R_{\mathrm{g},i-1} + \frac{1}{k^2} \cdot R_{\mathrm{g},i-k-1}) \right]} \\ &= \sqrt{\frac{2}{(0{,}37 \times 3)^2 \times 3\,600} \times \left[\left(1 + \frac{1}{24} \right)^2 \times \frac{14\,356}{3\,600} + \frac{1}{24^2} \times \frac{2\,124}{3\,600} \right]} \\ &= 0{,}044\,176\ \mathrm{Bq \cdot m^{-3}}\end{aligned} \qquad \text{(A.17)}$$

and consequently with $k_{1-\alpha} = 1{,}645$ and Equation (28), the decision threshold:

$$\Delta A^*_\mathrm{V} = k_{1-\alpha} \cdot \tilde{u}(0) = 1{,}645 \times 0{,}044\,176 = 0{,}072\,670\ \mathrm{Bq \cdot m^{-3}} \qquad \text{(A.18)}$$

Since the result of the measurement of the variation of the activity concentration $\Delta A_\mathrm{V} = 0{,}143\ \mathrm{Bq \cdot m^{-3}}$ exceeds the decision threshold for this measurand $\Delta A^*_\mathrm{V} = 0{,}073\ \mathrm{Bq \cdot m^{-3}}$, a variation of the activity concentration of $^{131}\mathrm{I}$ in the exhaust air has been observed in the $i = 25$ measuring cycle compared to the preceding 24 cycles.

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A.3.2 Calculation of the detection limit

According to Equation (29), one obtains an implicit equation for the detection limit of variation of the activity concentration:

$$\Delta\alpha_V^* = \Delta A_V^* + k_{1-\beta} \cdot \sqrt{\frac{\Delta\alpha_V^*}{\varepsilon \cdot V \cdot t} + \frac{2}{(\varepsilon \cdot V)^2 \cdot t} \cdot \left[(1+\frac{1}{k})^2 \cdot R_{g,i-1} + \frac{1}{k^2} \cdot R_{g,i-k-1} \right]}$$

$$= 0{,}072\,670 + 1{,}645 \times \left\{ \frac{\Delta\alpha_V^*}{0{,}37 \times 3 \times 3\,600} + \frac{2}{(0{,}37 \times 3)^2 \times 3\,600} \times \left[\left(1+\frac{1}{24}\right)^2 \times \frac{14\,356}{3\,600} + \frac{1}{24^2} \times \frac{2\,124}{3\,600} \right] \right\}^{1/2}$$

$$= 0{,}14\,827 \text{ Bq·m}^{-3} \quad \text{(A.19)}$$

As an alternative to an iterative solution of this implicit equation, this example also gives an explicit solution of this quadratic equation using Equations (1) and (4) of ISO 11929-7:2005.

Since $\tilde{u}(0)$ and $u(\Delta A_V)$ are known, one can use the interpolation formula of Equation (1) of ISO 11929-7 to calculate $\tilde{u}(\Delta\alpha_V)$.

$$\tilde{u}^2(\Delta\alpha_V) = \tilde{u}^2(0) \cdot (1 - \Delta\alpha_V / \Delta A_V) + u^2(\Delta A_V) \cdot \Delta\alpha_V / \Delta A_V$$
$$= 0{,}001\,951\,52 \times (1 - \Delta\alpha_V / 0{,}143\,227) + 0{,}001\,942\,57 \times \Delta\alpha_V / 0{,}143\,227 \quad \text{(A.20)}$$

Then the detection limit can be explicitly calculated using Equation (4) of ISO 11929-7:2005.

$$\Delta\alpha_V^* = a + \sqrt{a^2 + (k_{1-\beta}^2 - k_{1-\alpha}^2) \cdot \tilde{u}^2(0)} \quad \text{(A.21)}$$

with

$$a = k_{1-\alpha} \cdot \tilde{u}(0) + \frac{1}{2}(k_{1-\beta}^2 / \Delta A_V) \cdot \left[u^2(\Delta A_V) - \tilde{u}^2(0) \right]$$
$$= 1{,}645 \times 0{,}044\,176 + \frac{1}{2}(1{,}645^2 / 0{,}143\,23) \times (0{,}001\,942\,57 - 0{,}001\,951\,52) \quad \text{(A.22)}$$
$$= 0{,}072\,585\,0 \text{ Bq·m}^{-3}$$

Since in this example $\alpha = \beta$, one obtains with Equation (56) the following:

$$\Delta\alpha_V^* = 2a = 2 \times 0{,}072\,585\,0 = 0{,}145\,170 \text{ Bq·m}^{-3} \quad \text{(A.23)}$$

Since the detection limit of the variation of the activity concentration $\Delta\alpha_V^* = 0{,}15$ Bq·m^{-3} is smaller than the guideline value of 0,2 Bq·m^{-3}, the measuring method is suitable for the purpose of the measurement.

Note that the decision threshold and the detection limit for the change of the activity concentration are each higher than those for the activity concentration itself. This is due to changes with time of ^{131}I concentration in the exhaust air which is not due to the Poisson process of the radioactive decay and which enlarge the standard uncertainty of the measurand "variation of the activity concentration".

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A.3.3 Calculation of the confidence limits

In order to calculate the confidence limits, the parameter κ has to be calculated according to Equation (32) using Table A.1.

$$\kappa = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\Delta A_V/u(\Delta A_V)} \exp(-z^2/2)\,dz = \Phi\left[\Delta A_V/u(\Delta A_V)\right] \tag{A.24}$$
$$= \Phi(0{,}143\ 23/0{,}044\ 07) = \Phi(3{,}250) = 0{,}999\ 4$$

Then, one calculates with Equations (30) and (31):

$$p = \kappa \cdot (1-\gamma/2) = 0{,}999\ 4 \cdot (1-0{,}025) = 0{,}975 \tag{A.25}$$

$$q = 1-(\kappa \cdot \gamma/2) = 1-(0{,}999\ 4 \times 0{,}025) = 0{,}975 \tag{A.26}$$

From Table A.1, one obtains the quantiles for the probabilities p and q of the standardized normal distribution k_p = 1,96 and k_q = 1,96 and calculates the confidence limits with Equations (30) and (31):

$$\alpha_{V,l} = A_V - k_p \cdot u(A_V)$$
$$= 0{,}143\ 23 - 1{,}96 \times 0{,}044\ 07 = 0{,}056\ 84\ \text{Bq·m}^{-3} \tag{A.27}$$

$$\alpha_{V,u} = A_V + k_q \cdot u(A_V)$$
$$= 0{,}143\ 23 + 1{,}96 \times 0{,}044\ 07 = 0{,}229\ 6\ \text{Bq·m}^{-3} \tag{A.28}$$

Since the uncertainty of measurement result is small compared to the measurement result, the calculation of the best estimate z and its standard uncertainty $u(z)$ is not needed.

A.3.4 Documentation

Table A.3 — Complete documentation of the measurement of the variation of the activity concentration for measuring cycle i = 25

1	2	3	4
Quantity	**Symbol**	**Value**	**Unit**
change of activity concentration (cycle 25)	ΔA_V	0,143	Bq·m^{-3}
uncertainty of the change of activity concentration	$u(\Delta A_V)$	0,045	Bq·m^{-3}
probability of the error of first kind	α	0,050	1
probability of the error of second kind	β	0,050	1
confidence level	$1-\gamma$	0,950	1
guideline value	—	0,20	Bq·m^{-3}
decision threshold	ΔA_V^*	0,073	Bq·m^{-3}
detection limit	$\Delta \alpha_V^*$	0,148	Bq·m^{-3}
lower confidence limit	$\Delta \alpha_{V,l}$	0,057	Bq·m^{-3}
upper confidence limit	$\Delta \alpha_{V,u}$	0,230	Bq·m^{-3}

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A.4 Measurements using the alpha-beta-pseudo-coincidence method

A.4.1 General aspects

In addition to the examples in A.2 and A.3 which make use of the relatively simple models of evaluation on the basis of Equation 8 and 14, respectively, this clause gives two examples with more complicated models for which the characteristic limits can be calculated on the basis of Equation 7, using the methodology laid down in ISO 11929-7. These examples describe measurements of artificial alpha- and beta-radioactivity concentrations in air, discriminating against natural alpha- and beta-radioactivity by the so-called alpha-beta-pseudo-coincidence method, its principles being first described in reference [9] in the Bibliography.

In these measurements, aerosols are collected on a filter during a well-defined sampling time. Then, the filter is moved by a step-motor in another position to a detector system consisting of three detectors and measured for a well-defined counting time. A scheme of the measurement system is given in Figure A.3.

The three detectors are as follows:

a) detector 1 which is sensitive to alpha-particles only, measuring the alpha-radiation emerging from the filter;

b) detector 2 which is sensitive to beta- and gamma-radiation only, measuring the beta- and gamma-radiation emerging from the filter and events due to cosmic radiation;

c) detector 3 serving as a guard detector and measuring events due to cosmic radiation.

The system makes use of five counters under the following conditions

— Counter 1 records information given by detector 1.

— Counter 2 records the beta- and gamma-radiation emerging from the filter including events caused by interactions of the cosmic radiation informations given by detector 2.

— Counter 3 records the coincidence events from the so-called alpha-beta-pseudo-coincidence between detector 1 and 2 of which is a measure of the natural alpha- respectively beta-radiation on the filter. In addition it counts random coincidences between detector 1 and 2.

— Counter 4 records the random coincidences between detector 1 and 2 by a time-delayed coincidence between these detectors.

— Counter 5 records the events given by detector 3.

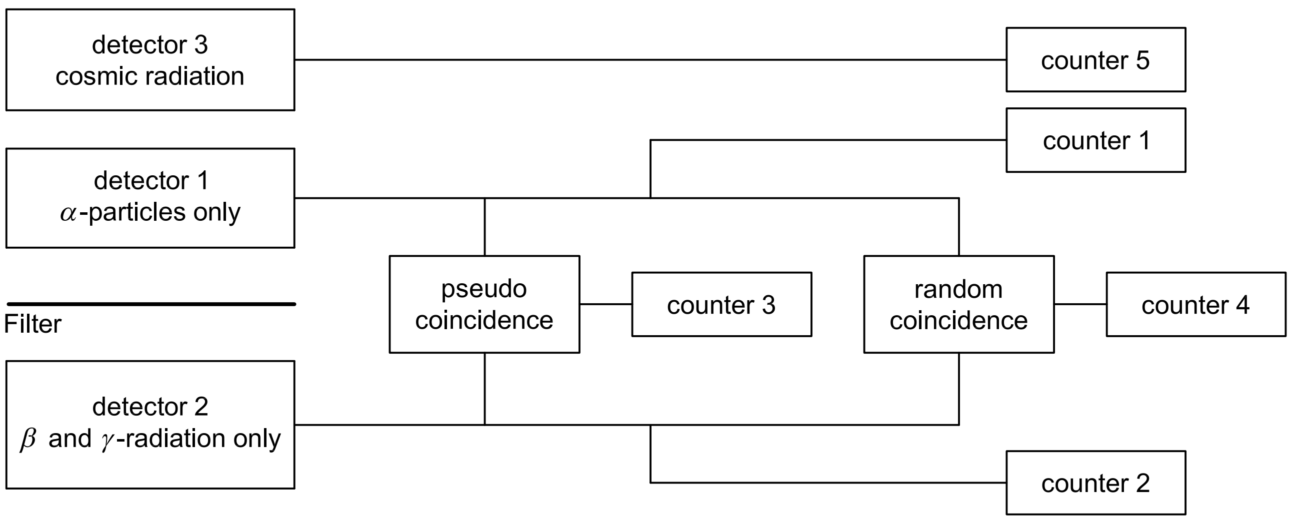

Figure A.3 — Scheme of the measurement of artificial alpha- and beta-radioactivity by means of the alpha-beta-pseudo-coincidence method

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For these measurements, the following quantities and symbols are used:

c_α activity concentration of the artificial alpha-radioactivity in air;

c_β activity concentration of the artificial beta-radioactivity in air;

R_1 count rate of counter 1; alpha-radiation from the filter;

R_2 count rate of counter 2; beta- and gamma-radiation of the filter including cosmic ray events;

R_3 count rate of counter 3; total pseudo-coincidence count rate of detectors 1 and 2;

R_4 count rate of counter 4; random coincidence rate of counters 1 and 2 (measured by time-delayed coincidence between counters 1 and 2);

R_5 count rate of counter 5; beta-, gamma-count rate of the cosmic-ray guard detector 3;

t_m counting time; its uncertainty is neglected;

t_s sampling time; its uncertainty is neglected;

F_α compensation factor of the alpha-count rate;

$u(F_\alpha)$ standard uncertainty associated with the compensation factor of the alpha-count rate;

F_β compensation factor of the beta-count rate;

$u(F_\beta)$ standard uncertainty associated with the compensation factor of the beta-count rate;

ε_α alpha-detection efficiency;

$u(\varepsilon_\alpha)$ standard uncertainty associated with the alpha-detection efficiency;

ε_β beta-detection efficiency;

$u(\varepsilon_\beta)$ standard uncertainty associated with the beta-detection efficiency;

$\dot{V}$ rate of air volume sampled during sampling time.

Using this method, the activity concentration of artificial alpha-radioactivity in the air sampled is determined by the model of evaluation according to:

$$c_\alpha = \frac{R_1 - F_\alpha \cdot (R_3 - R_4)}{\varepsilon_\alpha \cdot \dot{V} \cdot t_s} \tag{A.29}$$

The standard uncertainty associated with c_α is calculated by:

$$u^2(c_\alpha) = \frac{\frac{u^2(\varepsilon_\alpha)}{\varepsilon_\alpha^2} \cdot \left[R_1 - F_\alpha \cdot (R_3 - R_4)\right]^2 + u^2(F_\alpha) \cdot (R_3 - R_4)^2 + u^2(R_1) + \left[u^2(R_3) + u^2(R_4)\right] \cdot F_\alpha^2}{(\varepsilon_\alpha \cdot \dot{V} \cdot t_s)^2}$$

(A.30)

$$= \frac{\frac{u^2(\varepsilon_\alpha)}{\varepsilon_\alpha^2} \cdot \left[R_1 - F_\alpha \cdot (R_3 - R_4)\right]^2 + u^2\left[(F_\alpha) \cdot (R_3 - R_4)^2 + \frac{R_1}{t_m} + \left(\frac{R_3}{t_m} + \frac{R_4}{t_m}\right) \cdot F_\alpha^2\right]}{(\varepsilon_\alpha \cdot \dot{V} \cdot t_s)^2}$$

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$$u(c_\alpha) = \frac{\sqrt{\frac{u^2(\varepsilon_\alpha)}{\varepsilon_\alpha^2} \cdot \left[R_1 - F_\alpha \cdot (R_3 - R_4)\right]^2 + u^2\left[(F_\alpha)\cdot(R_3 - R_4)^2 + \frac{R_1 + F_\alpha^2 \cdot (R_3 + R_4)}{t_\text{m}}\right]}}{\varepsilon_\alpha \cdot \dot{V} \cdot t_\text{s}} \tag{A.31}$$

The compensation factor F_α is not constant for different measurements. It is set, however, to a nominal value, the uncertainty of which has to be determined by separate experiments.

Using the alpha-beta-pseudo-coincidence method, the activity concentration of artificial beta-radioactivity in the air sampled is determined by the model of evaluation according to:

$$c_\beta = \frac{(R_2 - R_5) - F_\beta \cdot (R_3 - R_4)}{\varepsilon_\beta \cdot \dot{V} \cdot t_\text{s}} \tag{A.32}$$

The standard uncertainty associated with c_β is calculated by:

$$u^2(c_\beta) = \frac{\frac{u^2(\varepsilon_\beta)}{\varepsilon_\beta^2} \cdot \left[(R_2 - R_5) - F_\beta \cdot (R_3 - R_4)\right]^2 + u^2(F_\beta)\cdot(R_3 - R_4)^2 + u^2(R_2) + u^2(R_5) + F_\beta^2 \cdot \left[u^2(R_3) + u^2(R_4)\right]}{(\varepsilon_\beta \cdot \dot{V} \cdot t_\text{s})^2}$$

$$= \frac{\frac{u^2(\varepsilon_\beta)}{\varepsilon_\beta^2} \cdot \left[(R_2 - R_5) - F_\beta \cdot (R_3 - R_4)\right]^2 + u^2\left[(F_\beta)\cdot(R_3 - R_4)^2 + \frac{R_2}{t_\text{m}} + \frac{R_5}{t_\text{m}} + F_\beta^2 \cdot \left(\frac{R_3}{t_\text{m}} + \frac{R_4}{t_\text{m}}\right)\right]}{(\varepsilon_\beta \cdot \dot{V} \cdot t_\text{s})^2} \tag{A.33}$$

$$u(c_\beta) = \frac{\sqrt{\frac{u^2(\varepsilon_\beta)}{\varepsilon_\beta^2} \cdot \left[(R_2 - R_5) - F_\beta \cdot (R_3 - R_4)\right]^2 + u^2\left[(F_\beta)\cdot(R_3 - R_4)^2 + \frac{R_2 + R_5 + F_\beta^2 \cdot (R_3 + R_4)}{t_\text{m}}\right]}}{\varepsilon_\beta \cdot \dot{V} \cdot t_\text{s}} \tag{A.34}$$

The compensation factor F_β is not constant for different measurements. It is set, however, to a nominal value, the uncertainty of which has to be determined by separate experiments.

For these examples, the data in Table A.4 apply.

A.4.2 Measurement of the activity concentration of the artificial alpha-radioactivity in Air, c_α

With the data of Table A.4, one calculates the activity concentration of artificial alpha-radioactivity.

$$c_\alpha = \frac{R_1 - F_\alpha \cdot (R_3 - R_4)}{\varepsilon_\alpha \cdot \dot{V} \cdot t_\text{s}} \tag{A.35}$$

$$c_\alpha = \frac{30{,}0 - 4{,}0 \times (3{,}55 - 0{,}12)}{0{,}28 \times 52{,}71 \times 24} = 0{,}045\ 96\ \text{Bq·m}^{-3} \tag{A.36}$$

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The standard uncertainty associated with this result, is calculated according to the Guide for the Expression of Uncertainty in Measurement. One obtains:

$$u^2(c_\alpha) = \frac{\frac{u^2(\varepsilon_\alpha)}{\varepsilon_\alpha^2} \cdot \left[R_1 - F_\alpha \cdot (R_3 - R_4)\right]^2 + u^2(F_\alpha) \cdot (R_3 - R_4)^2 + u^2(R_1) + \left[u^2(R_3) + u^2(R_4)\right] \cdot F_\alpha^2}{(\varepsilon_\alpha \cdot \dot{V} \cdot t_s)^2}$$

$$= \frac{\frac{u^2(\varepsilon_\alpha)}{\varepsilon_\alpha^2} \cdot \left[R_1 - F_\alpha \cdot (R_3 - R_4)\right]^2 + u^2\left[(F_\alpha) \cdot (R_3 - R_4)^2 + R_1 / t_m + (R_3 / t_m + R_4 / t_m) \cdot F_\alpha^2\right]}{(\varepsilon_\alpha \cdot \dot{V} \cdot t_s)^2} \quad \text{(A.37)}$$

Table A.4 — Data used in this example of an alpha-beta-pseudo-coincidence measurement

Quantity X_i	**Description**	**Experimental results** x_i **in this example**	**Source of uncertainty**	**Type**	**Uncertainties** $u(x_i)$	**Unit**
R_1	count rate of counter 1; alpha-count rate of the filter	30,0	Poisson	A	$\sqrt{R_1/t_m}$	s^{-1}
R_2	count rate of counter 2; beta- and gamma-radiation of the filter including cosmic ray events	70,17	Poisson	A	$\sqrt{R_2/t_m}$	s^{-1}
R_3	count rate of counter 3; total pseudo-coincidence rate (coincidence of detectors 1 and 2)	3,55	Poisson	A	$\sqrt{R_3/t_m}$	s^{-1}
R_4	count rate of counter 4; random coincidence rate (time-delayed coincidence of detectors 1 and 2)	0,12	Poisson	A	$\sqrt{R_4/t_m}$	s^{-1}
R_5	count rate of counter 5; beta-, gamma-count rate of the cosmic-ray guard detector 3	9,49	Poisson	A	$\sqrt{R_5/t_m}$	s^{-1}
t_m	counting time	600	uncertainty neglected	—	—	s
t_s	sampling time	24	uncertainty neglected	—	—	h
F_α	compensation factor of the alpha-count rate	4,0	rectangular distribution $2a = 2 \times 0{,}2F_\alpha = 1{,}6$	B	0,461 6 $= 0{,}577a$ [a]	1
F_β	compensation factor of the beta-count rate	8,5	rectangular distribution $2a = 2 \times 0{,}1F_\beta = 1{,}7$	B	0,490 5 $= 0{,}577a$ [a]	1
ε_α	alpha-detection efficiency	0,28	from certificate of α-calibration source	B	3 %	$Bq^{-1} \cdot s^{-1}$
ε_β	beta-detection efficiency	0,3	from certificate of β-calibration source	B	3 %	$Bq^{-1} \cdot s^{-1}$
φ_α	α-calibration factor $(\varepsilon_\alpha \cdot \dot{V} \cdot t_s)^{-1}$	0,002 8	not needed in this example	—	—	$Bq^{-1} \cdot s^{-1} \cdot m^{-3}$
φ_β	β-calibration factor $(\varepsilon_\beta \cdot \dot{V} \cdot t_s)^{-1}$	0,002 6	not needed in this example	—	—	$Bq^{-1} \cdot s^{-1} \cdot m^{-3}$
$\dot{V}$	rate of air volume during sampling	52,71	uncertainty neglected	—	—	$m^3 \cdot h^{-1}$

[a] a is the half-width of a rectangular distribution [(upper limit – lower limit)/2].

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$$u(c_\alpha) = \frac{1}{\varepsilon_\alpha \cdot \dot{V} \cdot t_s} \cdot \sqrt{\frac{u^2(\varepsilon_\alpha)}{\varepsilon_\alpha^2} \cdot \left[R_1 - F_\alpha \cdot (R_3 - R_4)\right]^2 + u^2(F_\alpha) \cdot (R_3 - R_4)^2 + \frac{R_1}{t_m} + \frac{F_\alpha^2 \cdot R_3}{t_m} + \frac{F_\alpha^2 \cdot R_4}{t_m}} \quad \text{(A.38)}$$

$u(c_\alpha) = 0{,}002\,8 \times$

$$\sqrt{0{,}000\,9 \times \left[30{,}0 - 4{,}0 \times (3{,}55 - 0{,}12)\right]^2 + 0{,}213\,1 \times (3{,}55 - 0{,}12)^2 + \frac{30{,}0}{600} + \frac{16 \times 3{,}55}{600} + \frac{16 \times 0{,}12}{600}} \quad \text{(A.39)}$$

$$= 0{,}002\,8 \times \sqrt{0{,}238\,5 + 2{,}507 + 0{,}05 + 0{,}094\,67 + 0{,}003\,2} = 0{,}004\,763\ \text{Bq·m}^{-3}$$

A.4.2.1 Calculation of the decision threshold

For the calculation of the decision threshold, $\tilde{u}(\xi_\alpha = 0)$ has to be determined. For $\xi_\alpha = 0$ one expects:

$R_1 - F_\alpha \cdot (R_3 - R_4) = 0$ and one obtains

$$\tilde{u}(0) = \frac{1}{\varepsilon_\alpha \cdot \dot{V} \cdot t_s} \cdot \sqrt{u^2(F_\alpha) \cdot (R_3 - R_4)^2 + \frac{R_1}{t_m} + \frac{F_\alpha^2 \cdot R_3}{t_m} + \frac{F_\alpha^2 \cdot R_4}{t_m}} \quad \text{(A.40)}$$

$$\tilde{u}(0) = 0{,}0028 \times \sqrt{0{,}2131 \times (3{,}55 - 0{,}12)^2 + \frac{30}{600} + \frac{16 \times 3{,}55^2}{600} + \frac{16 \times 0{,}12^2}{600}}$$
$$= 0{,}002\,8 \times \sqrt{2{,}507 + 0{,}022\,87 + 0{,}336\,1 + 0{,}000\,384} = 0{,}004\,562\ \text{Bq·m}^{-3} \quad \text{(A.41)}$$

and with this the decision threshold

$$c_\alpha^* = k_{1-\alpha} \cdot \tilde{u}(0) = k_{1-\alpha} \cdot \frac{1}{\varepsilon_\alpha \cdot \dot{V} \cdot t_s} \cdot \sqrt{u^2(F_\alpha) \cdot (R_3 - R_4)^2 + \frac{R_1}{t_m} + \frac{F_\alpha^2 \cdot R_3}{t_m} + \frac{F_\alpha^2 \cdot R_4}{t_m}} \quad \text{(A.42)}$$

With $\alpha = \beta = 0{,}001$ and hence $k_{1-\alpha} = k_{1-\beta} = 3{,}09$, one obtains

$$c_\alpha^* = 3{,}09 \times 0{,}004\,562 = 0{,}014\,097\ \text{Bq·m}^{-3} \quad \text{(A.43)}$$

Since the result of the measurement $c_\alpha = 0{,}046\,0\ \text{Bq·m}^{-3}$ exceeds the decision threshold $c_\alpha^* = 0{,}014\,1\ \text{Bq·m}^{-3}$, a non-zero activity concentration in air of the artificial alpha-radioactivity was observed.

A.4.2.2 Calculation of the detection limit

For the calculation of the detection limit, one needs $\tilde{u}(\xi_\alpha)$. The calculation makes use of the interpolation formula of Equation (1):

$$\tilde{u}^2(\xi_\alpha) = \tilde{u}^2(0) \cdot (1 - \xi_\alpha / c_\alpha) + u^2(c_\alpha) \cdot \xi_\alpha / c_\alpha \quad \text{(A.44)}$$

which in this example yields

$$\tilde{u}^2(\xi_\alpha) = 2{,}081 \times 10^{-5} \times (1 - \xi_\alpha / 0{,}045\,96) + 2{,}269 \times 10^{-5} \times \xi_\alpha / 0{,}045\,96 \quad \text{(A.45)}$$

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With this, one obtains the detection limit

$$\xi_\alpha^* = c_\alpha^* + k_{1-\beta} \cdot \tilde{u}(\xi_\alpha^*) = c_\alpha^* + k_{1-\beta} \cdot \sqrt{\tilde{u}^2(0) \cdot (1 - \xi_\alpha^* / c_\alpha) + u^2(c_\alpha) \cdot \xi_\alpha^* / c_\alpha} \quad \text{(A.46)}$$

$$\xi_\alpha^* = 0{,}014\ 65 + 3{,}09 \times \sqrt{2{,}081 \times 10^{-5} \times (1 - \xi_\alpha^* / 0{,}045\ 96) + 2{,}269 \times 10^{-5} \times \xi_\alpha^* / 0{,}045\ 96} \quad \text{(A.47)}$$

This implicit equation is solved by iteration. With the starting value $\xi_\alpha^* = 2c_\alpha^* = 2 \times 0{,}0146\,5\ \text{Bq·m}^{-3}$, one obtains after three iterative steps, the detection limit.

$$\xi_\alpha^* = 0{,}029\,1\ \text{Bq·m}^{-3} \quad \text{(A.48)}$$

The detection limit qualifies that this method is suitable for the measurement purpose of measuring the activity concentration in air of the artificial alpha-radioactivity down to a concentration of $0{,}029\,1\ \text{Bq·m}^{-3}$.

A.4.2.3 Calculation of the confidence limits

Since the measured value of the radioactivity concentration of the artificial alpha-radioactivity exceeds the decision threshold, the lower and upper confidence limits ξ_l and ξ_u, respectively, for a confidence level $1 - \gamma = 0{,}95$, are calculated according to Equations (30) to (31). One obtains with Equation (32):

$$\kappa = \Phi\left[x / u(x)\right] = \Phi(0{,}045\ 96/0{,}004\ 763) = \Phi(9{,}649) = 1{,}0 \quad \text{(A.49)}$$

Then, Equations (30) and (31) yield $p = q = (1 - \gamma / 2)$ with $k_{1-\gamma/2} = 1{,}96$, and hence one obtains the confidence limits:

$$\xi_\text{l} = c_\alpha - 1{,}96 \cdot u(c_\alpha) = (0{,}045\ 96 - 1{,}96 \times 0{,}004\ 763) = 0{,}036\ 62\ \text{Bq·m}^{-3} \quad \text{(A.50)}$$

$$\xi_\text{u} = c_\alpha + 1{,}96 \cdot u(c_\alpha) = (0{,}045\ 96 + 1{,}96 \cdot 0{,}004\ 763) = 0{,}055\ 30\ \text{Bq·m}^{-3} \quad \text{(A.51)}$$

Since in this case $c_\alpha \gg u(c_\alpha)$ holds, the confidence interval is symmetric around the measured value and a calculation of the best estimate is not necessary.

A.4.2.4 Documentation

Table A.5 — Complete documentation of the measurement of the concentration in air of the artificial alpha-radioactivity

1	2	3	4
Quantity	**Symbol**	**Value**	**Unit**
concentration of the artificial alpha-radioactivity	c_α	0,046	Bq·m^{-3}
uncertainty of the concentration of the artificial alpha-radioactivity	$u(c_\alpha)$	0,005	Bq·m^{-3}
probability of the error of first kind	α	0,001	1
probability of the error of second kind	β	0,001	1
confidence level	$1 - \gamma$	0,95	1
guideline value	—	—	Bq·m^{-3}
decision threshold	c_α^*	0,014	Bq·m^{-3}
detection limit	ξ_α^*	0,029	Bq·m^{-3}
lower confidence limit	ξ_l	0,037	Bq·m^{-3}
upper confidence limit	ξ_u	0,055	Bq·m^{-3}

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A.4.3 Measurement of the activity concentration of the artificial beta-radioactivity in air c_β

With the data of Table A.5, one calculates the activity concentration of artificial beta-radioactivity.

$$c_\beta = \frac{(R_2 - R_5) - F_\beta \cdot (R_3 - R_4)}{\varepsilon_\beta \cdot \dot{V} \cdot t_s} \tag{A.52}$$

$$c_\beta = \frac{(70{,}17 - 9{,}49) - 8{,}5 \times (3{,}55 - 0{,}12)}{0{,}3 \times 52{,}71 \times 24} = 0{,}083\ 07\ \text{Bq·m}^{-3} \tag{A.53}$$

The standard uncertainty associated with this result is calculated according to the Guide for the Expression of Uncertainty in Measurement. One obtains:

$$u^2(c_\beta) = \frac{1}{(\varepsilon_\beta \cdot \dot{V} \cdot t_s)^2} \cdot \left\{ \begin{array}{l} \dfrac{u^2(\varepsilon_\beta)}{\varepsilon_\beta^2} \cdot \left[(R_2 - R_5) - F_\beta \cdot (R_3 - R_4) \right]^2 + u^2(F_\beta) \cdot (R_3 - R_4)^2 \\ + u^2(R_2) + u^2(R_5) + F_\beta^2 \cdot \left[u^2(R_3) + u^2(R_4) \right] \end{array} \right\}$$

$$= \frac{\dfrac{u^2(\varepsilon_\beta)}{\varepsilon_\beta^2} \cdot \left[(R_2 - R_5) - F_\beta \cdot (R_3 - R_4) \right]^2 + u^2(F_\beta) \cdot (R_3 - R_4)^2 + \dfrac{R_2}{t_m} + \dfrac{R_5}{t_m} + F_\beta^2 \cdot \left(\dfrac{R_3}{t_m} + \dfrac{R_4}{t_m} \right)}{(\varepsilon_\beta \cdot \dot{V} \cdot t_s)^2} \tag{A.54}$$

$$u(c_\beta) = \frac{1}{\varepsilon_\beta \cdot \dot{V} \cdot t_s} \cdot \sqrt{\frac{u^2(\varepsilon_\beta)}{\varepsilon_\beta^2} \cdot \left[(R_2 - R_5) - F_\beta \cdot (R_3 - R_4) \right]^2 + u^2(F_\beta) \cdot (R_3 - R_4)^2 + \frac{R_2}{t_m} + \frac{R_5}{t_m} + \frac{F_\beta^2 \cdot R_3}{t_m} + \frac{F_\beta^2 \cdot R_4}{t_m}} \tag{A.55}$$

$$u(c_\beta) = 0{,}002\ 6 \times \sqrt{0{,}000\ 9 \times \left[(70{,}17 - 9{,}49) - 8{,}5 \times (3{,}55 - 0{,}12) \right]^2 + 0{,}240\ 6 \times (3{,}55 - 0{,}12)^2 + \frac{70{,}17 + 9{,}49 + 72{,}25 \cdot 3{,}55 + 72{,}25 \times 0{,}12}{600}} \tag{A.56}$$

$$= 0{,}002\ 6 \times \sqrt{0{,}894\ 4 + 2{,}831 + 0{,}117\ 0 + 0{,}015\ 82 + 0{,}4275 + 0{,}014\ 45}$$

$$= 0{,}005\ 392\ \text{Bq·m}^{-3}$$

A.4.3.1 Calculation of the decision threshold

For the calculation of the decision threshold, one needs $\tilde{u}(\xi_\beta = 0)$. For $\xi_\beta = 0$, one expects $(R_2 - R_5) - F_\beta \cdot (R_3 - R_4) = 0$ and one obtains:

$$\tilde{u}(0) = \frac{1}{\varepsilon_\beta \cdot \dot{V} \cdot t_s} \cdot \sqrt{u^2(F_\beta) \cdot (R_3 - R_4)^2 + \frac{R_2}{t_m} + \frac{R_5}{t_m} + \frac{F_\beta^2 \cdot R_3}{t_m} + \frac{F_\beta^2 \cdot R_4}{t_m}} \tag{A.57}$$

$$\tilde{u}(0) = 0{,}002\ 6 \times \sqrt{0{,}240\ 6 \times (3{,}55 - 0{,}12)^2 + \frac{70{,}17}{600} + \frac{9{,}49}{600} + \frac{72{,}25 \times 3{,}55}{600} + \frac{72{,}25 \times 0{,}12}{600}}$$

$$= 0{,}002\ 6 \times \sqrt{2{,}831 + 0{,}117\ 0 + 0{,}015\ 82 + 0{,}427\ 5 + 0{,}014\ 45} = 0{,}004\ 798\ \text{Bq·m}^{-3} \tag{A.58}$$

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and with this the decision threshold:

$$c_\beta^* = k_{1-\alpha} \cdot \tilde{u}(0) = k_{1-\alpha} \cdot \frac{1}{\varepsilon_\beta \cdot \dot{V} \cdot t_s} \cdot \sqrt{u^2(F_\beta) \cdot (R_3 - R_4)^2 + \frac{R_2}{t_m} + \frac{R_5}{t_m} + \frac{F_\beta^2 \cdot R_3}{t_m} + \frac{F_\beta^2 \cdot R_4}{t_m}} \qquad (A.59)$$

and with $\alpha = \beta = 0{,}001$ and $k_{1-\alpha} = k_{1-\beta} = 3{,}09$

$$c_\beta^* = 3{,}09 \times 0{,}004\,798 = 0{,}014\,83\ \text{Bq·m}^{-3} \qquad (A.60)$$

Since the result of the measurement $c_\beta = 0{,}083\,1\ \text{Bq·m}^{-3}$ exceeds the decision threshold $c_\beta^* = 0{,}014\,8\ \text{Bq·m}^{-3}$, a non-zero activity concentration in air of the artificial beta-radioactivity was observed.

A.4.3.2 Calculation of the detection limit

For the calculation of the detection limit, one needs $\tilde{u}(\xi_\beta)$. In this example, one obtains with the interpolation formula of Equation (1):

$$\tilde{u}^2(\xi_\beta) = 2{,}302 \times 10^{-5} \times (1 - \xi_\beta / 0{,}083\,07) + 2{,}907 \times 10^{-5} \times \xi_\beta / 0{,}083\,07 \qquad (A.61)$$

With this, one obtains the detection limit with Equation (27):

$$\begin{aligned} \xi_\beta^* &= c_\beta^* + k_{1-\beta} \cdot \tilde{u}(\xi_\beta^*) = c_\beta^* + k_{1-\beta} \cdot \sqrt{\tilde{u}^2(0) \cdot (1 - \xi_\beta^* / c_\beta) + u^2(c_\beta) \cdot \xi_\beta^* / c_\beta} \\ &= 0{,}014\,83 + 3{,}09 \times \sqrt{2{,}302 \times 10^{-5} \times (1 - \xi_\beta / 0{,}083\,07) + 2{,}907 \times 10^{-5} \times \xi_\beta / 0{,}083\,07} \end{aligned} \qquad (A.62)$$

This implicit equation is solved by iteration with the starting value $\xi_\beta^* = 2c_\beta^* = 2 \times 0{,}014\,83\ \text{Bq·m}^{-3}$.

After four iteration steps, one obtains the solution:

$$\xi_\beta^* = 0{,}03\ \text{Bq·m}^{-3} \qquad (A.63)$$

The detection limit qualifies that this method is suitable for the measurement purpose of measuring the activity concentration in air of the artificial beta-radioactivity down to a concentration of $0{,}03\ \text{Bq·m}^{-3}$.

A.4.3.3 Calculation of the confidence limits

Since the measured value of the radioactivity concentration of the artificial beta-radioactivity exceeds the decision threshold, the lower and upper confidence limits ξ_l and ξ_u, respectively, for the confidence level $1 - \gamma = 0{,}95$, are calculated according to Equations (30) to (31). One obtains with Equation (32):

$$\kappa = \Phi\left[x / u(x)\right] = \Phi(0{,}083\,07 / 0{,}005\,392) = \Phi(15{,}406) = 1{,}0 \qquad (A.64)$$

Then, Equations (30) and (31) yield $p = q = (1 - \gamma / 2)$ with $k_{1-\gamma/2} = 1{,}96$, and hence one obtains the confidence limits:

$$\xi_l = c_\beta - 1{,}96 \cdot u(c_\beta) = (0{,}083\,07 - 1{,}96 \times 0{,}005\,392) = 0{,}072\,50\ \text{Bq·m}^{-3} \qquad (A.65)$$

$$\xi_u = c_\beta + 1{,}96 \cdot u(c_\beta) = (0{,}083\,07 + 1{,}96 \times 0{,}005\,392) = 0{,}093\,64\ \text{Bq·m}^{-3} \qquad (A.66)$$

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Since in this case $c_\beta \gg u(c_\beta)$ holds, the confidence interval is symmetric around the measured value and a calculation of the best estimate is not necessary.

A.4.3.4 Documentation

Table A.6 — Complete documentation of the measurement of the concentration in air of the artificial beta-radioactivity

1	2	3	4
Quantity	**Symbol**	**Value**	**Unit**
concentration of the artificial beta-radioactivity	c_β	0,083	$Bq \cdot m^{-3}$
uncertainty of the concentration of the artificial beta-radioactivity	$u(c_\beta)$	0,005	$Bq \cdot m^{-3}$
probability of the error of first kind	α	0,001	1
probability of the error of second kind	β	0,001	1
confidence level	$1 - \gamma$	0,95	1
guideline value	—	—	$Bq \cdot m^{-3}$
decision threshold	c_β^*	0,015	$Bq \cdot m^{-3}$
detection limit	ξ_β^*	0,030	$Bq \cdot m^{-3}$
lower confidence limit	ξ_l	0,073	$Bq \cdot m^{-3}$
upper confidence limit	ξ_u	0,094	$Bq \cdot m^{-3}$

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Bibliography

[1] ALTSCHULER, B. AND PASTERNACK, B. *Health Physics 9*, (1963), pp. 293-298

[2] NICHOLSON, W. L. *Fixed time estimation of counting rates with background corrections.* Hanford Laboratories, Richland (1963)

[3] CURRIE, L. A. Limits for Qualitative Detection and Quantitative Determination. Anal. Chem. 40, (1968), pp. 586-593

[4] LEE, P.M. *Bayesian Statistics An Introduction.* Oxford University Press, New York (1989)

[5] WEISE, K., WÖGER, W. A Bayesian theory of measurement uncertainty, *Meas. Sci. Technol. 4*, (1993), pp. 1-11

[6] WEISE, K. Bayesian-statistical decision threshold, detection limit and confidence interval in nuclear radiation measurement, *Kerntechnik 63* (1998), pp. 214-224

[7] KOHLRAUSCH, F. *Praktische Physik*, Band 3, 24th edition, p. 613, Teubner, Stuttgart (1996)

[8] ABRAMOWITZ, M., STEGUN, I. *Handbook of Mathematical Function*, 5th ed., Chapter 26, Dover Publication, New York (1968)

[9] SPAA, J.H. *A continuously operated instrument for the stepwise measurement of the radioactivity of aerosols with a special background compensation*, Proc. 2nd Conf. Peaceful Use of Atomic Energy, (1958), pp. 169-173

[10] *International Vocabulary of Basic and General Terms in Metrology.* 2nd edition, Geneva (1993)

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