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Plastics piping systems — Glass-reinforced thermosetting plastics (GRP) pipes and fittings — Methods for regression analysis and their use

Systèmes de canalisation en matières plastiques — Tubes et raccords plastiques thermodurcissables renforcés de verre (PRV) — Méthodes pour une analyse de régression et leurs utilisations

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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The committee responsible for this document is ISO/TC 138, *Plastics pipes, fittings and valves for the transport of fluids*, Subcommittee SC 6, *Reinforced plastics pipes and fittings for all applications*.

This third edition cancels and replaces the second edition (ISO 10928:2009), which has been technically revised and includes the following changes. It also incorporates the Amendment ISO 10928:2009/Amd 1:2013:

— Annex A (GRP pressure pipe design procedure) has been removed from the document;

— several bibliographical references have been removed.

Introduction

This document describes the procedures intended for analysing the regression of test data, usually with respect to time and the use of the results in design and assessment of conformity with performance requirements. Its applicability is limited to use with data obtained from tests carried out on samples. The referring standards require estimates to be made of the long-term properties of the pipe for such parameters as circumferential tensile strength, long-term ring deflection, strain corrosion and creep or relaxation stiffness.

A range of statistical techniques that could be used to analyse the test data produced by destructive tests was investigated. Many of these simple techniques require the logarithms of the data to

a) be normally distributed,

b) produce a regression line having a negative slope, and

c) have a sufficiently high regression correlation (see Table 1).

While the last two conditions can be satisfied, analysis shows that there is a skew to the distribution and hence this primary condition is not satisfied. Further investigation into techniques that can handle skewed distributions resulted in the adoption of the covariance method of analysis of such data for this document.

However, the results from non-destructive tests, such as long-term creep or relaxation stiffness, often satisfy all three conditions and hence a simpler procedure, using time as the independent variable, can also be used in accordance with this document.

These data analysis procedures are limited to analysis methods specified in ISO product standards or test methods. However, other analysis procedures can be useful for the extrapolation and prediction of long-term behaviour of some properties of glass-reinforced thermosetting plastics (GRP) piping products. For example, a second-order polynomial analysis is sometimes useful in the extrapolation of creep and relaxation data. This is particularly the case for analysing shorter term data, where the shape of the creep or relaxation curve can deviate considerably from linear. A second-order polynomial analysis is included in Annex A. In Annex B, there is an alternative non-linear analysis method. These non-linear methods are provided only for information and the possible use in investigating the behaviour of a particular piping product or material therefore they might not be generally applicable to other piping products.

Plastics piping systems — Glass-reinforced thermosetting plastics (GRP) pipes and fittings — Methods for regression analysis and their use

1 Scope

This document specifies procedures suitable for the analysis of data which, when converted into logarithms of the values, have either a normal or a skewed distribution. It is intended for use with the test methods and referring standards for glass-reinforced thermosetting plastics (GRP) pipes or fittings for the analysis of properties as a function of time. However, it can be used for the analysis of other data.

Depending upon the nature of the data, two methods are specified. The extrapolation using these techniques typically extends the trend from data gathered over a period of approximately 10 000 h to a prediction of the property at 50 years, which is the typical maximum extrapolation time.

This document only addresses the analysis of data. The test procedures to collect the data, the number of samples required and the time period over which data are collected are covered by the referring standards and/or test methods. Clause 6 discusses how the data analysis methods are applied to product testing and design.

2 Normative references

There are no normative references in this document.

3 Terms and definitions

No terms and definitions are listed in this document.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— IEC Electropedia: available at http://www.electropedia.org/

— ISO Online browsing platform: available at http://www.iso.org/obp

4 Principle

Data are analysed for regression using methods based on least squares analysis which can accommodate the incidence of a skew and/or a normal distribution. The two methods of analysis used are the following:

— method A: covariance using a first-order relationship;

— method B: least squares, with time as the independent variable using a first-order relationship.

The methods include statistical tests for the correlation of the data and the suitability for extrapolation.

5 Procedures for determining the linear relationships — Methods A and B

5.1 Procedures common to methods A and B

Use method A (see 5.2) or method B (see 5.3) to fit a straight line of the form given in Formula (1):

$$y = a + b \times x \quad (1)$$

where

- y is the logarithm, lg, of the property being investigated;
- a is the intercept on the y-axis;
- b is the slope;
- x is the logarithm, lg, of the time, in hours.

5.2 Method A — Covariance method

5.2.1 General

For method A, calculate the following variables in accordance with 5.2.2 to 5.2.5, using Formulae (2), (3) and (4):

$$Q_{\mathrm{y}} = \frac{\sum \left(y_i - Y \right)^2}{n} \quad (2)$$

$$Q_{\mathrm{x}} = \frac{\sum \left(x_i - X \right)^2}{n} \quad (3)$$

$$Q_{\mathrm{xy}} = \frac{\sum \left[\left(x_i - X \right) \times \left(y_i - Y \right) \right]}{n} \quad (4)$$

where

- Q_{y} is the sum of the squared residuals parallel to the y-axis, divided by n;
- Q_{x} is the sum of the squared residuals parallel to the x-axis, divided by n;
- Q_{xy} is the sum of the squared residuals perpendicular to the line, divided by n;
- Y is the arithmetic mean of the y data, i.e. given as Formula (5):

$$Y = \frac{\sum y_i}{n} \quad (5)$$

- X is the arithmetic mean of the x data, i.e. given as Formula (6):

$$X = \frac{\sum x_i}{n} \quad (6)$$

x_i, y_i are individual values;

n is the total number of results (pairs of readings for x_i, y_i).

NOTE If the value of Q_{xy} is greater than zero, the slope of the line is positive and if the value of Q_{xy} is less than zero, then the slope is negative.

5.2.2 Suitability of data

Calculate the linear coefficient of correlation, r, using Formulae (7) and (8):

$$r^2 = \frac{Q_{xy}^2}{Q_x \times Q_y} \quad (7)$$

$$r = \left| \left(r^2 \right)^{0,5} \right| \quad (8)$$

If the value of r is less than $\frac{\text{Student's } t(f)}{\sqrt{n-2+\left[\text{Student's } t(f)\right]^2}}$, then the data are unsuitable for analysis.

Table 1 gives the minimum acceptable values of the correlation coefficient, r, as a function of the number of variables, n. The Student's t value is based on a two-sided 0,01 level of significance.

Table 1 — Minimum values of the correlation coefficient, *r*, for acceptable data from *n* pairs of data

Number of variables n	Degrees of freedom $n - 2$	Student's $t(0,01)$	Minimum r
13	11	3,106	0,683 5
14	12	3,055	0,661 4
15	13	3,012	0,641 1
16	14	2,977	0,622 6
17	15	2,947	0,605 5
18	16	2,921	0,589 7
19	17	2,898	0,575 1
20	18	2,878	0,561 4
21	19	2,861	0,548 7
22	20	2,845	0,536 8
23	21	2,831	0,525 6
24	22	2,819	0,515 1
25	23	2,807	0,505 2
26	24	2,797	0,495 8
27	25	2,787	0,486 9
32	30	2,750	0,448 7
37	35	2,724	0,418 2
42	40	2,704	0,393 2
47	45	2,690	0,372 1
52	50	2,678	0,354 2
62	60	2,660	0,324 8
72	70	2,648	0,301 7
82	80	2,639	0,283 0
92	90	2,632	0,267 3
102	100	2,626	0,254 0

5.2.3 Functional relationships

Find a and b for the functional relationship line using Formula (1).

First, set Γ as given in Formula (9):

$$\Gamma = \frac{Q_y}{Q_x} \tag{9}$$

then calculate a and b using Formulae (10) and (11):

$$b = -\left(\Gamma\right)^{0,5} \tag{10}$$

$$a = Y - b \times X \tag{11}$$

5.2.4 Calculation of variances

If t_u is the applicable time to failure, then set x_u as given in Formula (12):

$$x_u = \lg t_u \tag{12}$$

Using Formulae (13), (14) and (15) respectively, calculate for i = 1 to n, the following sequence of statistics:

— the best fit x_i' for true x_i;

— the best fit y_i' for true y_i;

— the error variance, σ_δ^2 for x.

$$x_i' = \frac{\Gamma \times x_i + b \times \left(y_i - a\right)}{2 \times \Gamma} \tag{13}$$

$$y_i' = a + b \times x_i' \tag{14}$$

$$\sigma_\delta^2 = \frac{\left[\sum\left(y_i - y_i'\right)^2 + \Gamma \times \sum\left(x_i - x_i'\right)^2\right]}{\left(n-2\right) \times \Gamma} \tag{15}$$

Calculate quantities E and D using Formulae (16) and (17):

$$E = \frac{b \times \sigma_\delta^2}{2 \times Q_{xy}} \tag{16}$$

$$D = \frac{2 \times \Gamma \times b \times \sigma_\delta^2}{n \times Q_{xy}} \tag{17}$$

Calculate the variance, C, of the slope b, using Formula (18):

$$C = D \times \left(1 + E\right) \tag{18}$$

5.2.5 Check for the suitability of data for extrapolation

If it is intended to extrapolate the line, calculate T using Formula (19):

$$T = \frac{b}{\left(\text{var } b\right)^{0,5}} = \frac{b}{C^{0,5}} \tag{19}$$

If the absolute value, $|T|$ (i.e. ignoring signs), of T is equal to or greater than the applicable value for Student's t, t_v, shown in Table 2 for $(n - 2)$ degrees of freedom, then consider the data suitable for extrapolation.

Calculation of confidence limits is not required by the test methods or referring standards; however, the calculation of lower confidence limit, LCL, and lower prediction limit, LPL, is given in Annex C.

Table 2 — Percentage points of Student's t distribution (upper 2,5 % points; two-sided 5 % level of confidence; t_v for 97,5 %)

Degree of freedom $(n - 2)$	Student's t value t_v	Degree of freedom $(n - 2)$	Student's t value t_v	Degree of freedom $(n - 2)$	Student's t value t_v
1	12,706 2	36	2,028 1	71	1,993 9
2	4,302 7	37	2,026 2	72	1,993 5
3	3,182 4	38	2,024 4	73	1,993 0
4	2,776 4	39	2,022 7	74	1,992 5
5	2,570 6	40	2,021 1	75	1,992 1
6	2,446 9	41	2,019 5	76	1,991 7
7	2,364 6	42	2,018 1	77	1,991 3
8	2,306 0	43	2,016 7	78	1,990 8
9	2,262 2	44	2,015 4	79	1,990 5
10	2,228 1	45	2,014 1	80	1,990 1
11	2,201 0	46	2,012 9	81	1,989 7
12	2,178 8	47	2,011 2	82	1,989 3
13	2,160 4	48	2,010 6	83	1,989 0
14	2,144 8	49	2,009 6	84	1,988 6
15	2,131 5	50	2,008 6	85	1,988 3
16	2,119 9	51	2,007 6	86	1,987 9
17	2,109 8	52	2,006 6	87	1,987 6
18	2,100 9	53	2,005 7	88	1,987 3
19	2,093 0	54	2,004 9	89	1,987 0
20	2,086 0	55	2,004 0	90	1,986 7

Table 2 *(continued)*

Degree of freedom $(n-2)$	Student's t value t_v	Degree of freedom $(n-2)$	Student's t value t_v	Degree of freedom $(n-2)$	Student's t value t_v
21	2,079 6	56	2,003 2	91	1,986 4
22	2,073 9	57	2,002 5	92	1,986 1
23	2,068 7	58	2,001 7	93	1,985 8
24	2,063 9	59	2,001 0	94	1,985 5
25	2,059 5	60	2,000 3	95	1,985 3
26	2,055 5	61	1,999 6	96	1,985 0
27	2,051 8	62	1,999 0	97	1,984 7
28	2,048 4	63	1,998 3	98	1,984 5
29	2,045 2	64	1,997 7	99	1,984 2
30	2,042 3	65	1,997 1	100	1,984 0
31	2,039 5	66	1,996 6		
32	2,036 9	67	1,996 0		
33	2,034 5	68	1,995 5		
34	2,032 2	69	1,994 9		
35	2,030 1	70	1,994 4		

5.2.6 Validation of statistical procedures by an example calculation

The data given in Table 3 are used in the following example to aid in verifying that statistical procedures, as well as computer programs and spreadsheets adopted by users, will produce results similar to those obtained from the formulae given in this document. For the purposes of the example, the property in question is represented by V, the values for which are of a typical magnitude and in no particular units. Because of rounding errors, it is unlikely that the results will agree exactly, so for a calculation procedure to be acceptable, the results obtained for r, r^2, b, a, and the mean value of V, and V_m, shall agree to within ±0,1 % of the values given in this example. The values of other statistics are provided to assist the checking of the procedure.

Sums of squares:

$Q_x = 0{,}798\ 12$;

$Q_y = 0{,}000\ 88$;

$Q_{xy} = -0{,}024\ 84$.

Coefficient of correlation:

$r^2 = 0{,}879\ 99$;

$r = 0{,}938\ 08$.

Functional relationships:

$\Gamma = 0{,}001\ 10$;

$b = -0{,}033\ 17$;

$a = 1{,}627\ 31$.

Table 3 — Basic data for example calculation and statistical analysis validation

n	V	Y lg V	**Time** h	X lg h
1	30,8	1,488 6	5 184	3,714 7
2	30,8	1,488 6	2 230	3,348 3
3	31,5	1,498 3	2 220	3,346 4
4	31,5	1,498 3	12 340	4,091 3
5	31,5	1,498 3	10 900	4,037 4
6	31,5	1,498 3	12 340	4,091 3
7	31,5	1,498 3	10 920	4,038 2
8	32,2	1,507 9	8 900	3,949 4
9	32,2	1,507 9	4 173	3,620 4
10	32,2	1,507 9	8 900	3,949 4
11	32,2	1,507 9	878	2,943 5
12	32,9	1,517 2	4 110	3,613 8
13	32,9	1,517 2	1 301	3,114 3
14	32,9	1,517 2	3 816	3,581 6
15	32,9	1,517 2	669	2,825 4
16	33,6	1,526 3	1 430	3,155 3
17	33,6	1,526 3	2 103	3,322 8
18	33,6	1,526 3	589	2,770 1
19	33,6	1,526 3	1 710	3,233 0
20	33,6	1,526 3	1 299	3,113 6
21	35,0	1,544 1	272	2,434 6
22	35,0	1,544 1	446	2,649 3
23	35,0	1,544 1	466	2,668 4
24	35,0	1,544 1	684	2,835 1
25	36,4	1,561 1	104	2,017 0
26	36,4	1,561 1	142	2,152 3
27	36,4	1,561 1	204	2,309 6
28	36,4	1,561 1	209	2,320 1
29	38,5	1,585 5	9	0,954 2
30	38,5	1,585 5	13	1,113 9
31	38,5	1,585 5	17	1,230 4
32	38,5	1,585 5	17	1,230 4
Means:		Y = 1,530 1		X = 2, 930 5

Calculated variances (see 5.2.4):

$E = 3{,}520\ 2 \times 10^{-2}$;

$D = 4{,}842\ 2 \times 10^{-6}$;

$C = 5{,}012\ 7 \times 10^{-6}$ (the variance of b);

$\sigma_\delta{}^2 = 5{,}271\ 1 \times 10^{-2}$ (the error variance of x).

Check for the suitability for extrapolation (see 5.2.5):

n = 32;

t_v = 2,042 3;

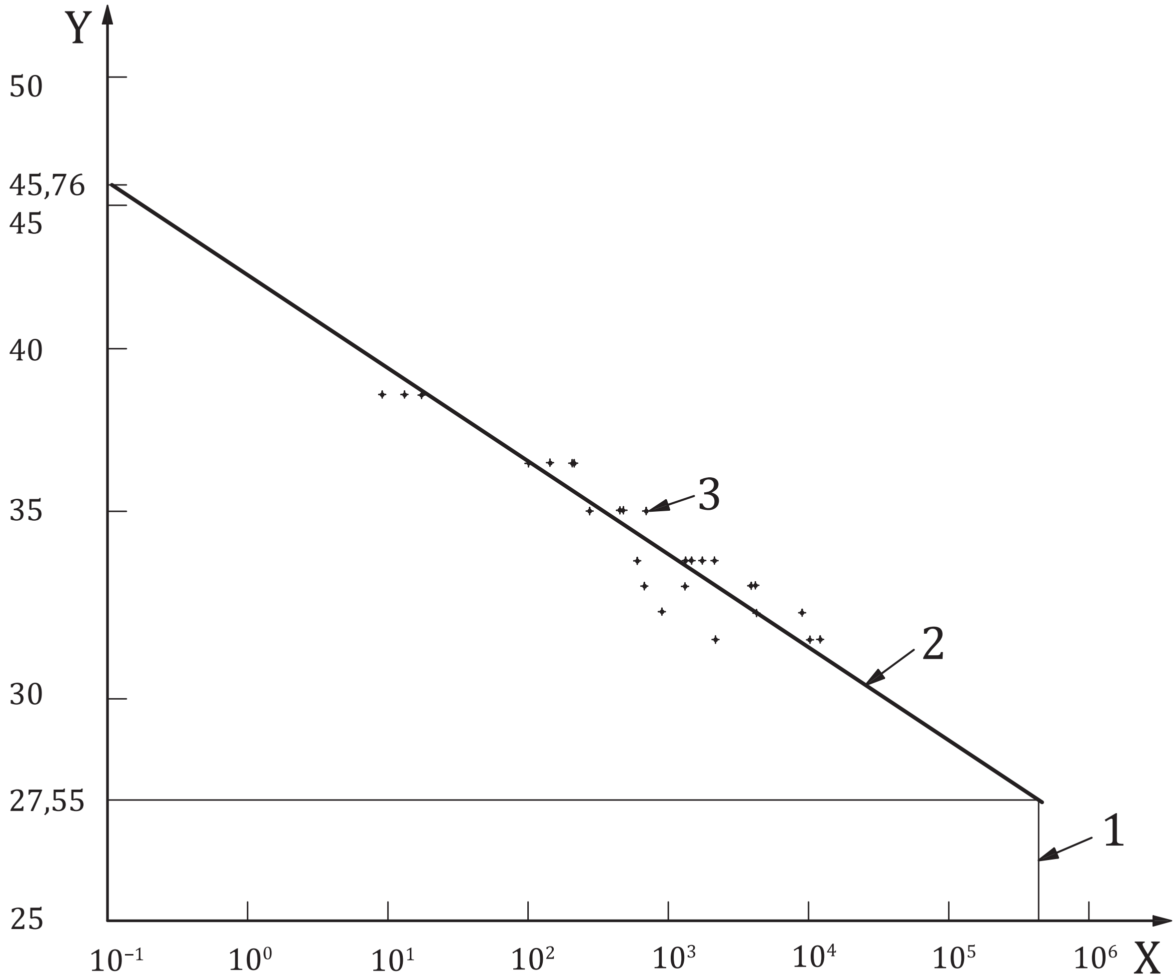
$T = -0{,}033\ 17 / (5{,}012\ 7 \times 10^{-6})^{0{,}5} = -14{,}816\ 7$;

$|T| = 14{,}816\ 7 > 2{,}042\ 3$.

The estimated mean values for V at various times are given in Table 4 and shown in Figure 1.

Table 4 — Estimated mean values, V_m, for V

Time h	V_m
0,1	45,76
1	42,39
10	39,28
100	36,39
1 000	33,71
10 000	31,23
100 000	28,94
438 000	27,55



Key

1	438 000 h (50 years)
2	regression line from Table 4
3	data point
X-axis	lg scale of time, in hours
Y-axis	lg scale of property

Figure 1 — Regression line from the results in Table 4

5.3 Method B — Regression with time as the independent variable

5.3.1 General

For method B, calculate the sum of the squared residuals parallel to the Y-axis, S_y, using Formula (20):

$$S_y = \sum \left(y_i - Y\right)^2 \tag{20}$$

Calculate the sum of the squared residuals parallel to the X-axis, S_x, using Formula (21):

$$S_x = \sum \left(x_i - X\right)^2 \tag{21}$$

Calculate the sum of the squared residuals perpendicular to the line, S_{xy}, using Formula (22):

$$S_{xy} = \sum \left[\left(x_i - X\right) \times \left(y_i - Y\right)\right] \tag{22}$$

where

Y is the arithmetic mean of the y data, i.e.

$$Y = \frac{\sum y_i}{n};$$

X is the arithmetic mean of the x data, i.e.

$$X = \frac{\sum x_i}{n};$$

x_i, y_i are individual values;

n is the total number of results (pairs of readings for x_i, y_i).

NOTE If the value of S_{xy} is greater than zero, the slope of the line is positive and if the value of S_{xy} is less than zero, then the slope is negative.

5.3.2 Suitability of data

Calculate the squared, r^2, and the linear coefficient of correlation, r, using Formulae (23) and (24):

$$r^2 = \frac{S_{xy}^2}{S_x \times S_y} \tag{23}$$

$$r = \left|\left(r^2\right)^{0,5}\right| \tag{24}$$

If the value of r^2 or r is less than the applicable minimum value given in Table 1 as a function of n, consider the data unsuitable for analysis.

5.3.3 Functional relationships

Calculate a and b for the functional relationship line [see Formula (1)], using Formulae (25) and (26):

$$b = \frac{S_{xy}}{S_x} \tag{25}$$

$$a = Y - b \times X \tag{26}$$

5.3.4 Check for the suitability of data for extrapolation

If it is intended to extrapolate the line, calculate M using Formula (27):

$$M = \frac{S_x^2}{S_{xy}^2} - \frac{t_v^2 \times \left(S_x \times S_y - S_{xy}^2\right)}{\left(n-2\right) \times S_y^2} \tag{27}$$

where

t_v is the applicable value for Student's t determined from Table 2.

If M is equal to or less than zero, consider the data unsuitable for extrapolation.

5.3.5 Validation of statistical procedures by an example calculation

The data given in Table 5 are used in the following example to aid in verifying that statistical procedures, as well as computer programs and spreadsheets adopted by users, will produce results similar to those obtained from the formulae given in this document. Use the data given in Table 5 for the calculation procedures described in 5.3.2 to 5.3.4 to ensure that the statistical procedures to be used in conjunction with this method will give results for r, r^2, a, b and V_m to within ±0,1 % of the values given in this example.

Table 5 — Basic data for example calculation and statistical validation

n	**Time** T in h	*X* lg T	*V*	*Y* lg *V*
1	0,10	−1,000 0	7 114	3,852 1
2	0,27	−0,568 6	6 935	3,841 0
3	0,50	−0,301 0	6 824	3,834 1
4	1,00	0	6 698	3,825 9
5	3,28	0,515 9	6 533	3,815 1
6	7,28	0,862 1	6 453	3,809 8
7	20,0	1,301 0	6 307	3,799 9
8	45,9	1,661 8	6 199	3,792 3
9	72,0	1,857 3	6 133	3,787 7
10	166	2,220 1	5 692	3,755 2
11	219	2,340 4	5 508	3,741 0
12	384	2,584 3	5 393	3,731 8
13	504	2,702 4	5 364	3,729 5
14	3 000	3,477 1	5 200	3,716 0
15	10 520	4,022 0	4 975	3,696 8
Means:		X = 1,445 0		Y = 3,781 9

Sums of squares:

S_x = 31,681 1;

S_y = 0,034 7;

S_{xy} = −1,024 2.

Coefficient of correlation:

r^2 = 0,955 6;

r = 0,977 5.

Functional relationships (see 5.3.3):

a = 3,828 6;

b = −0,032 3.

Check for the suitability for extrapolation (see 5.3.4):

t_v = 2,160 4;

M = 942,21.

The estimated mean values, V_m, for V at various times are given in Table 6.

Table 6 — Estimated mean values, V_m, for V

Time h	V_m
0,1	7 259
1	6 739
10	6 256
100	5 808
1 000	5 391
10 000	5 005
100 000	4 646
438 000	4 428

6 Application of methods to product design and testing

6.1 General

The referring standards specify limiting requirements for the long-term properties and performance of a product. Some of these are based on destructive tests, for example, hoop tensile strength, while others are based on actual or derived physical properties, such as creep stiffness.

These properties require an extrapolated long-term (e.g. 50 years) value for the establishment of a product design or comparison with the requirement. This extrapolated value is determined by inserting, as necessary, the values for a and b, determined in accordance with 5.2 and 5.3 as appropriate, into Formula (28).

$$\lg y = a + b \times t_L \qquad (28)$$

where

t_L is the logarithm, lg, of the long-term period, in hours [for 50 years (438 000 h), t_L = 5,641 47].

Solving Formula (28) for y gives the extrapolated value.

The use of the data and the specification of requirements in the product standards are in three distinct categories.

6.2 Product design

In the first category, the data are used for design or calculation of a product line. This is the case for long-term circumferential strength testing (ISO 7509)[1].The long-term destructive test data are analysed using method A.

6.3 Comparison to a specified value

The second category is where the long-term extrapolated value is compared to a minimum requirement given in the product standard. This is the case for long-term ring bending (ISO 10471)[3] and strain corrosion (ISO 10952)[4]. The long-term destructive test data are analysed using method A and can be used to establish a value to compare to the product standard requirement. As the analysis of the data does provide a long-term stress or strain value, this value may also be utilized in analysis of product suitability for a range of installation and application conditions.

6.4 Declaration of a long-term value

The third category is when the long-term extrapolated value is used to calculate a long-term property and this value is then declared by the manufacturer. This is the case for long-term creep (ISO 10468)[2] stiffness. The long-term non-destructive test data are analysed using method B.

Annex A
(informative)

Second-order polynomial relationships

A.1 General

This method fits a curved line of the form given in Formula (A.1):

$$y = c + d \times x + e \times x^2 \quad (A.1)$$

where

y is the logarithm, lg, of the property being investigated;

c is the intercept on the y-axis;

d, e are the coefficients to the two orders of x;

x is the logarithm, lg, of the time, in hours.

A.2 Variables

Calculate the following variables:

- x_i , the sum of all individual x data;
- x_i^2 , the sum of all squared x data;
- x_i^3 , the sum of all x data to the third power;
- x_i^4 , the sum of all x data to the fourth power;
- y_i , the sum of all individual y data;

$\left(\sum y_i\right)^2$, the squared sum of all individual y data;

- y_i^2 , the sum of all squared y data;

$\sum(x_i \times y_i)$, the sum of all products x_i,y_i;

$\sum\left(x_i^2 \times y_i\right)$, the sum of all products x_i2y_i;

$S_x = \sum(x_i - X)^2$, the sum of the squared residuals parallel to the X-axis for the linear part;

$S_{xx} = \sum \left(x_i^2 - X^2 \right)^2$, the sum of the squared residuals parallel to the X-axis for the quadratic part;

$S_y = \sum \left(y_i - Y \right)^2$, the sum of the squared residuals parallel to the Y-axis;

$S_{xy} = \sum \left[\left(x_i - X \right) \times \left(y_i - Y \right) \right]$, the sum of the squared residuals perpendicular to the line for the linear part;

$S_{xxy} = \sum \left[\left(x_i^2 - X^2 \right) \times \left(y_i - Y \right) \right]$, the sum of the squared residuals perpendicular to the line for the quadratic part

where

Y is the arithmetic mean of the y data, i.e. $Y = \frac{\sum y_i}{n}$;

X is the arithmetic mean of the x data, i.e. $X = \frac{\sum x_i}{n}$.

A.3 Solution system

Determine c, d and e using the following matrix:

$$\sum y_i = c \times n + d \times \sum x_i + e \times \sum x_i^2 \quad (A.2)$$

$$\sum \left(x_i \times y_i \right) = c \times \sum x_i + d \times \sum x_i^2 + e \times \sum x_i^3 \quad (A.3)$$

$$\sum \left(x_i^2 \times y_i \right) = c \times \sum x_i^2 + d \times \sum x_i^3 + e \times \sum x_i^4 \quad (A.4)$$

A.4 Suitability of data

Calculate the squared, r^2, and the linear coefficient of correlation, r, using Formulae (A.5) and (A.6):

$$r^2 = \frac{c \times \sum y_i + d \times \sum \left(x_i \times y_i \right) + e \times \sum \left(x_i^2 \times y_i \right) - \left[\left(\sum y_i \right)^2 / n \right]}{\sum y_i^2 - \left[\left(\sum y_i \right)^2 / n \right]} \quad (A.5)$$

$$r = \left| \left(r^2 \right)^{0,5} \right| \quad (A.6)$$

If the value of r^2 or r is less than the applicable minimum value given in Table 1 as a function of n, consider the data unsuitable for analysis.

A.5 Check for the suitability of data for extrapolation

If it is intended to extrapolate the line, calculate M using Formula (A.7):

$$M=\frac{S_x^{2}}{S_{xy}^{2}}+\frac{S_{xx}^{2}}{S_{xxy}^{2}}-\frac{t_v^{2}\times\left(S_x\times S_y-S_{xy}^{2}+S_{xx}\times S_y-S_{xxy}^{2}\right)}{(n-2)\times S_y^{2}} \tag{A.7}$$

If M is equal to or less than zero, consider the data unsuitable for extrapolation.

A.6 Validation of statistical procedures by an example calculation

Use the data given in Table 5 for the calculation procedures described in A.1 to A.5 to ensure that the statistical procedures to be used in conjunction with this method will give results for r, r^2, a, b and V_m to within ±0,1 % of the values given in this example (n = 15).

- x_i = 21,671;
- x_i^2 = 62,989;
- x_i^3 = 180,623;
- x_i^4 = 584,233;
- y_i = 56,728;

$\left(\sum y_i\right)^2$ = 3 218,09;

- y_i^2 = 214,574;

$\sum(x_i\times y_i)$ = 80,932;

$\sum\left(x_i^2\times y_i\right)$ = 235,175;

$S_x=\sum(x_i-X)^2$ = 31,681;

$S_{xx}=\sum\left(x_i^2-X^2\right)^2$ = 386,638;

$S_y=\sum(y_i-Y)^2$ = 0,034 7;

$S_{xy}=\sum\left[(x_i-X)\times(y_i-Y)\right]$ = −1,024 2;

$S_{xxy}=\sum\left[\left(x_i^2-X^2\right)\times(y_i-Y)\right]$ = −3,041 8.

Solution system:

c = 3,828 8;

d = −0,026 2;

e = −0,002 2.

Coefficient of correlation:

r^2 = 0,964 7;

r = 0,982 2.

Check for the suitability for extrapolation:

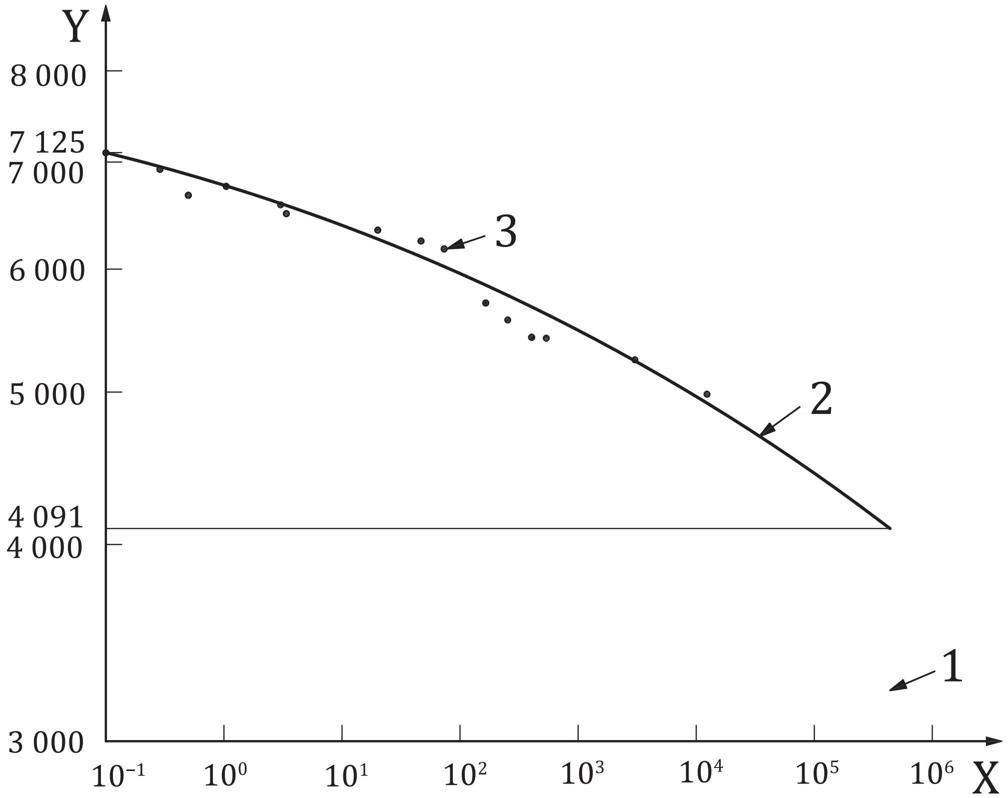
t_v = 2,160 4;

M = 15 859,6.

The estimated mean values, V_m, for V at various times are given in Table A.1 and shown in Figure A.1.

Table A.1 — Estimated mean values, V_m, for V

Time h	V_m
0,1	7 125
1	6 742
10	6 315
100	5 856
1 000	5 375
10 000	4 884
100 000	4 393
438 000	4 091



Key

1 438 000 h (50 years)

2 regression line from Table A.1

3 data point

X-axis lg scale of time, in hours

Y-axis lg scale of property

Figure A.1 — Regression line from the results in Table A.1

Annex B
(informative)

Non-linear relationships

B.1 General

Given a model for a set of data obtained from a long-term stiffness test on GRP test pieces, the objective of this annex is to produce explicit formulae for

a) estimating all four parameters in the model, i.e. a, b, c and d, and

b) calculating confidence and prediction intervals about the curve.

These formulae are presented in this annex, along with associated graphical displays for a typical data set.

NOTE While the data and procedures refer to a long-term stiffness test, the method can also be applied to data which fit the mathematical model and require extrapolation to 50 years.

B.2 Model

As described in B.4, the model shown can be re-expressed as two linked straight-line regression models called Line 1 and Line 2. Answers obtained from the procedures described for Line 1 are then applied in the Line 2 procedures to obtain the four parameters in the model, which is then used to obtain the long-term value for the property under investigation.

B.2.1 Procedure for Line 1

The formulae given in the following subclauses are used to develop the models for Lines 1 and 2.

NOTE Many of the formulae make reference to a subscript i, which is the count value shown in the tables. For the data used in the tables, i runs from 0 to 16, where data indexed 1 to 15 are the experimental measured values, and those indexed 0 or 16 are the calculated values. When using the procedures described in this annex, the amount of data that can be analysed is not limited to 15 data sets but can be any number.

B.2.1.1 Determination of derived values for Y_i, x_i and y_i

Calculate Y_i, x_i, y_i, $\bar{x}$ and $\bar{y}$ using Formulae (B.1), (B.2), (B.3), (B.4) and (B.5):

$$Y_i = \lg_{10}\left(S_i\right) \tag{B.1}$$

$$x_i = \lg_{10}(\text{minutes} + 1) \tag{B.2}$$

$$y_i = \ln\left[\left(a + b - Y_i\right) / \left(Y_i - a\right)\right] \tag{B.3}$$

$$\bar{x} = \left(\sum x_i\right) / n \tag{B.4}$$

$$\bar{y} = \left(\sum y_i\right) / n \tag{B.5}$$

Formulae (B.1), (B.2), (B.3), (B.4) and (B.5) relate to values obtained for the property, S_i, after various periods of time under test, x_i.

B.2.1.2 Determination of parameters a and b

Calculate the following using Formulae (B.6), (B.7) and (B.8):

$$a_0 = 0{,}995\ \text{min.}\left(Y_i\right) \tag{B.6}$$

$$a_0 + b_0 = 1{,}005\ \text{max.}\left(Y_i\right) \tag{B.7}$$

$$b_0 = \left(a_0 + b_0\right) - a_0 \tag{B.8}$$

Formulae (B.6), (B.7) and (B.8) produce the initial estimates for two of the parameters in the model, a_0 and b_0.

B.2.1.3 Determination of the least squares estimates for A and B and unbiased estimate for $\tilde{\sigma}_1^2$

Calculate the following using Formulae (B.9), (B.10), (B.11) and (B.12):

$$\hat{A} = \overline{y} - \hat{B}\overline{x} \tag{B.9}$$

$$\hat{B} = \sum\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right) / \sum\left(x_i - \overline{x}\right)^2 \tag{B.10}$$

$$\tilde{\sigma}_1^2 = \sum\left(y_i - \hat{y}_i\right)^2 / \left(n-2\right) = \text{RSS} / \left(n-2\right) \tag{B.11}$$

where

$$\text{RSS} = \sum y_i^{\,2} - \hat{A} \times \sum y_i - \hat{B} \times \sum x_i \times y_i \tag{B.12}$$

NOTE RSS is the residual sum of squares.

B.2.1.4 Determination of estimates for parameters c and d

Calculate the following using Formulae (B.13) and (B.14):

$$\hat{c} = \left(\hat{A}\hat{B}^{-1} + \lg_{10} 60\right) \tag{B.13}$$

$$\hat{d} = -\hat{B}^{-1} \tag{B.14}$$

B.2.2 Procedure for Line 2

B.2.2.1 Determination of X_i, Y_i, $\overline{X}$ and $\overline{Y}$

Calculate X_i using Formula (B.15):

$$X_i = 1 / \left\{1 + \exp\left[-\lg_{10}\left(\text{Time}\right) - \hat{c}\right] / \hat{d}\right\} \tag{B.15}$$

NOTE Values for c and d are given as Formulae (B.13) and (B.14).

$$Y_i = \lg_{10}\left(\text{stiffness}\right) \tag{B.16}$$

$$\bar{X} = \left(\sum X_i\right) / n \qquad \text{(B.17)}$$

$$\bar{Y} = \left(\sum Y_i\right) / n \qquad \text{(B.18)}$$

B.2.2.2 Determination of the least squares estimates for $\hat{a}$ and $\hat{b}$ and unbiased estimate for $\tilde{\sigma}_1^2$

The estimates are derived using Formulae (B.19), (B.20), (B.21) and (B.22):

$$\hat{b} = \sum\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right) / \sum\left(X_i - \bar{X}\right)^2 =$$

$$\left(n \times \sum XY - \sum X \times \sum Y\right) / \left(n \times \sum X^2 - \sum X \times \sum X\right) \qquad \text{(B.19)}$$

$$\hat{a} = \bar{Y} - \hat{b} \times \bar{X} \qquad \text{(B.20)}$$

$$\tilde{\sigma}_1^2 = \sum\left(Y_i - \hat{Y}_i\right)^2 / \left(n-2\right) = \text{RSS} / \left(n-2\right) \qquad \text{(B.21)}$$

where

$$\text{RSS} = \sum Y_i^2 - \hat{a} \times \sum Y_i - \hat{b} \times \sum X_i \times Y_i \qquad \text{(B.22)}$$

Check that the following constraints are met: $\hat{a} + \hat{b} > Y_i > \hat{a}$.

B.2.2.3 Determination of confidence and prediction intervals

Calculate the variances of $\hat{a}$ and $\hat{b}$ using Formulae (B.23) and (B.24):

$$\text{var}\left(\hat{a}\right) = \left(\tilde{\sigma}_2^2 \sum X_i^2\right) \Big/ \left[n \sum X_i^2 - \left(\sum X_i\right)^2\right] = \left(\tilde{\sigma}_2{}^2 \sum X_i{}^2\right) \Big/ \left[n \sum \left(X_i - \bar{X}\right)^2\right] \qquad \text{(B.23)}$$

$$\text{var}\left(\hat{b}\right) = \left(n\tilde{\sigma}_2^2\right) \Big/ \left[n \sum X_i^2 - \left(\sum X_i\right)^2\right] = \left(n\tilde{\sigma}_2{}^2\right) \Big/ \left[n \sum \left(X_i - \bar{X}\right)^2\right] \qquad \text{(B.24)}$$

Calculate the estimated standard error, ε, of $\hat{a}$ and $\hat{b}$ given as Formulae (B.25) and (B.26):

$$\varepsilon\left(\hat{a}\right) = \sqrt{\text{var}\left(\hat{a}\right)} \qquad \text{(B.25)}$$

$$\varepsilon\left(\hat{b}\right) = \sqrt{\text{var}\left(\hat{b}\right)} \qquad \text{(B.26)}$$

Formulae for 100 % confidence and prediction intervals about the fitted Line 2 as functions of X are given as Formulae (B.27) and (B.28), respectively:

$$\text{Confidence interval } \mu_X = \hat{\mu}_X \pm t_P \tilde{\sigma}_2 \sqrt{\left[\frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{\sum\left(X_i - \bar{X}\right)^2}\right]} \qquad \text{(B.27)}$$

Prediction interval $Y_X = \hat{\mu}_X \pm t_P \tilde{\sigma}_2 \sqrt{\left[1 + \frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{\sum\left(X_i - \bar{X}\right)^2}\right]}$ (B.28)

where

$$\hat{Y}_X = \hat{\mu}_X = \hat{a} + \hat{b}X \quad \text{(B.29)}$$

B.2.2.4 Student's *t* test for *a* and *b*

To test whether $\hat{a}$ or $\hat{b}$ is equal to 0, perform the following calculations:

$$\Pr\left(\left|t\right| < t_P\right) = P;$$

t has Student's *t*-distribution on $(n - 2)$ degrees of freedom.

From Statistical tables *t* boundary for P = 90 % = 1,771.

From Statistical tables *t* boundary for P = 95 % = 2,160.

If Formulae (B.30) and (B.31) give *t* values greater than the applicable one of the above bounds, it is certain that $\hat{a}$ and $\hat{b}$ are not equal to 0.

$$t \text{ for } \hat{a} = \hat{a} / \varepsilon(\hat{a}) \quad \text{(B.30)}$$

$$t \text{ for } \hat{b} = \hat{b} / \varepsilon(\hat{b}) \quad \text{(B.31)}$$

B.2.2.5 Calculation of long-term (50-year) stiffness

All the formulae in B.2.1 and B.2.2 are standard formulae for straight-line regressions. From the values given in Tables B.4 and B.5, the estimated long-term stiffness and its confidence and prediction limits can be determined using Formulae (B.32), (B.33), (B.34), (B.35) and (B.36):

Using Formula (B.29), the extrapolated long-term stiffness is given as Formula (B.32):

$$\hat{Y}_X = \hat{\mu}_X = \hat{a} + \hat{b}X \quad \text{(B.32)}$$

Using Formula (B.27), the confidence limits for $\mu_{50 \text{ years}}$ are given as Formula (B.33):

$$\mu_{50 \text{ years}} = \hat{\mu}_X \pm \mu_{\text{bounds}} \quad \text{(B.33)}$$

Using Formula (B.28), the prediction limits are given as Formula (B.34):

$$\hat{Y}_{50 \text{ years}} = Y_X \pm Y_{Y \text{ bounds}} \quad \text{(B.34)}$$

Transforming these values back to stiffness gives:

Extrapolated long-term stiffness = $10^{\hat{Y}_{50 \text{ years}}}$

90 % confidence limits for extrapolated long-term stiffness are given as Formula (B.35):

$$\mu\left(S\right)_{50\ \text{years}} = 10^{\hat{\mu}_X \pm \mu_{\text{bounds}}} \tag{B.35}$$

90 % prediction limits for extrapolated long-term stiffness are given as Formula (B.36):

$$S_{50\ \text{years}} = 10^{Y_X \pm Y_{Y\ \text{bounds}}} \tag{B.36}$$

B.3 Validation of statistical procedures by an example calculation

Use the data given in Table B.1 for the calculation procedures described in B.2.1 to B.2.2.5 to ensure that the statistical procedures to be used in conjunction with this method will give results to within ±0,1 % of the values given in this example ($n = 15$).

B.3.1 Procedure for Line 1

B.3.1.1 Determination of derived values for Y_i, x_i and y_i

$$Y_i = \lg_{10}\left(S_i\right) \tag{B.37}$$

NOTE 1 See derived values in Table B.1.

$$x_i = \lg_{10}(\text{minutes} + 1) \tag{B.38}$$

NOTE 2 See derived values in Table B.1.

$$y_i = \ln\left[\left(a + b - Y_i\right) / \left(Y_i - a\right)\right] \tag{B.39}$$

NOTE 3 See derived values in Table B.1.

$$\bar{x} = \left(\sum x_i\right) / n = 48{,}465\ 540/15 = 3{,}231\ 036 \tag{B.40}$$

$$\bar{y} = \left(\sum y_i\right) / n = -2{,}512\ 828/15 = -0{,}167\ 522 \tag{B.41}$$

B.3.1.2 Determination of parameters a and b

For values of Y_i, see Table B.1.

$$a_0 = 0{,}995\ \text{min.}\left(Y_i\right) = 0{,}995 \times 3{,}696\ 793 = 3{,}678\ 309 \tag{B.42}$$

$$a_0 + b_0 = 1{,}005\ \text{max.}\left(Y_i\right) = 1{,}005 \times 3{,}852\ 114 = 3{,}871\ 375 \tag{B.43}$$

$$b_0 = \left(a_0 + b_0\right) - a_0 = 3{,}871\ 375 - 3{,}678\ 309 = 0{,}193\ 066 \tag{B.44}$$

B.3.1.3 Determination of the least squares estimates for A and B and unbiased estimate for $\tilde{\sigma}_1^2$

Calculate the following:

$$\hat{A} = \overline{y} - \hat{B}\overline{x} = -0{,}167\,5 - 0{,}831\,9 \times 3{,}231\,036 = -2{,}855\,5 \quad \text{(B.45)}$$

$$\hat{B} = \sum\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right) / \sum\left(x_i - \overline{x}\right)^2 = \left(n \times \sum xy - \sum x \times \sum y\right) \div \left[n \times \sum x^2 - \left(\sum x\right)^2\right] \quad \text{(B.46)}$$

Using values from Table B.2,

$$\hat{B} = \left(15 \times 17{,}8 - 48{,}47 \times 2{,}51\right) / \left[15 \times 187{,}75 - \left(2\,349{,}34\right)\right] = 0{,}831\,9$$

Using values for A and B in this subclause and from Table B.2,

$$\tilde{\sigma}_1^2 = \sum\left(y_i - \hat{y}_i\right)^2 / \left(n-2\right) = \text{RSS} / \left(n-2\right) = 0{,}665\,3 / 13 = 0{,}051\,2 \quad \text{(B.47)}$$

where

$$\text{RSS} = \sum {y_i}^2 - \hat{A} \times \sum y_i - \hat{B} \times \sum x_i \times y_i \quad \text{(B.48)}$$

B.3.1.4 Determination of estimates for parameters c and d

Using the values for $\hat{A}$ and $\hat{B}$ calculated above, the estimated values for $\hat{c}$ and $\hat{d}$ are:

$$\hat{c} = \left(\hat{A}\hat{B}^{-1} + \lg_{10} 60\right) = 1{,}653\,53 \quad \text{(B.49)}$$

$$\hat{d} = -\hat{B}^{-1} = -\,1{,}202 \quad \text{(B.50)}$$

B.3.2 Procedure for Line 2

B.3.2.1 Determination of X_i, Y_i, $\overline{X}$ and $\overline{Y}$

Calculate the following:

$$X_i = 1 / \left\{1 + \exp\left[-\lg_{10}\left(\text{Time}\right) - \hat{c}\right] / \hat{d}\right\} \quad \text{(B.51)}$$

NOTE 1 See derived values in Table B.3.

NOTE 2 Values for c and d are given in Formulae (B.49) and (B.50).

$$Y_i = \lg_{10}\left(\text{stiffness}\right) \quad \text{(B.52)}$$

NOTE 3 See derived values in Table B.3.

$$\overline{X} = \left(\sum X_i\right) / n = 7{,}966\,259 / 15 = 0{,}531\,084 \quad \text{(B.53)}$$

$$\overline{Y} = \left(\sum Y_i\right) / n = 56{,}728\,211 / 15 = 3{,}781\,881 \quad \text{(B.54)}$$

B.3.2.2 Determination of the least squares estimates for $\hat{a}$ and $\hat{b}$ and unbiased estimate for $\tilde{\sigma}_1^2$

The estimates are derived using Formulae (B.55), (B.56), (B.57) and (B.58):

$$\hat{b} = \sum\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right) / \sum\left(X_i - \bar{X}\right)^2 =$$

$$\left(n \times \sum XY - \sum X \times \sum Y\right) / \left(n \times \sum X^2 - \sum X \times \sum X\right) \tag{B.55}$$

$$\hat{b} = \left(15 \times 30{,}302\ 557 - 7{,}966\ 259 \times 56{,}728\ 211\right) \Big/ \left(15 \times 5{,}146\ 065 - 7{,}966\ 259 \times 7{,}966\ 259\right)$$
$$= 2{,}626\ 734 \Big/ 13{,}729\ 693 = 0{,}191\ 318$$

$$\hat{a} = \bar{Y} - \hat{b} \times \bar{X} \tag{B.56}$$

$$\hat{a} = 3{,}781\ 881 - 0{,}191\ 318 \times 0{,}531\ 083\ 4 = 3{,}680\ 275$$

$$\tilde{\sigma}_1^2 = \sum\left(Y_i - \hat{Y}_i\right)^2 / \left(n - 2\right) = \text{RSS} / \left(n - 2\right) \tag{B.57}$$

$$\text{RSS} = \sum Y_i^2 - \hat{a} \times \sum Y_i - \hat{b} \times \sum X_i \times Y_i =$$

$$\sum Y^2 - \hat{a} \times \sum Y - \hat{b} \times \sum XY = 214{,}573\ 977 - 3{,}680\ 275 \times 56{,}728\ 211 - 0{,}191\ 318 \times 30{,}302\ 557 =$$
$$= 0{,}001\ 136$$

$$\tilde{\sigma}_2^2 = 0{,}001\ 136 / 13 = 0{,}000\ 087 \tag{B.58}$$

Check that the following constraints are met: $\hat{a} + \hat{b} > Y_i > \hat{a}$

max. $Y_i = 3{,}852\ 114$, min. $Y_i = 3{,}696\ 793$, $\hat{a} + \hat{b} = 3{,}680\ 275 + 0{,}191\ 318 = 3{,}871\ 593$ and $\hat{a} = 3{,}680\ 275$

From inspection of these values, the constraints are satisfied.

B.3.2.3 Determination of confidence and prediction intervals

Using Formulae (B.59) and (B.60), determine the variances of $\hat{a}$ and $\hat{b}$:

$$\text{var}\left(\hat{a}\right) = \left(\tilde{\sigma}_2^2 \sum X_i^2\right) \Big/ \left[n \sum X_i^2 - \left(\sum X_i\right)^2\right] = \left(\tilde{\sigma}_2{}^2 \sum X_i{}^2\right) \Big/ \left[n \sum \left(X_i - \bar{X}\right)^2\right] \tag{B.59}$$

$$\text{var}\left(\hat{a}\right) = 0{,}000\ 033$$

$$\text{var}\left(\hat{b}\right) = \left(n\tilde{\sigma}_2^2\right) \Big/ \left[n \sum X_i^2 - \left(\sum X_i\right)^2\right] = \left(n\tilde{\sigma}_2{}^2\right) \Big/ \left[n \sum \left(X_i - \bar{X}\right)^2\right] \tag{B.60}$$

$$\text{var}\left(\hat{b}\right) = 0{,}000\ 097$$

Using Formulae (B.61) and (B.62), compute the estimated standard errors, ε, of $\hat{a}$ and $\hat{b}$:

$$\varepsilon\left(\hat{a}\right) = \sqrt{\mathrm{var}\left(\hat{a}\right)} \tag{B.61}$$

$$\varepsilon\left(\hat{a}\right) = \sqrt{0{,}000\ 033} = 0{,}005\ 756$$

$$\varepsilon\left(\hat{b}\right) = \sqrt{\mathrm{var}\left(\hat{b}\right)} \tag{B.62}$$

$$\varepsilon\left(\hat{b}\right) = \sqrt{0{,}000\ 097} = 0{,}009\ 828$$

Formulae for 100 % confidence and prediction intervals about the fitted Line 2 as functions of X are given as Formulae (B.63) and (B.64), respectively:

$$\text{Confidence interval } \mu_X = \hat{\mu}_X \pm t_P \tilde{\sigma}_2 \sqrt{\left[\frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{\sum\left(X_i - \bar{X}\right)^2}\right]} \tag{B.63}$$

$$\text{Prediction interval } Y_X = \hat{\mu}_X \pm t_P \tilde{\sigma}_2 \sqrt{\left[1 + \frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{\sum\left(X_i - \bar{X}\right)^2}\right]} \tag{B.64}$$

where

$$\text{extrapolated long-term stiffness is } \hat{\mu}_X = \hat{a} + \hat{b}X \tag{B.65}$$

$t_P = 1{,}771$ for $P = 90$ %

In Table B.4, the confidence intervals are shown as μ_L and μ_U and the prediction intervals as Y_L and Y_U.

In Table B.5, the confidence intervals are shown as $S\mu_L$ and $S\mu_U$ and the prediction intervals as S_L and S_U.

B.3.2.4 Student's *t* test for *a* and *b*

To test whether $\hat{a}$ or $\hat{b}$ is equal to 0, the following calculations are performed:

$$\Pr\left(\left|t\right| < t_P\right) = P;$$

t has Student's t-distribution on $(n - 2)$ degrees of freedom.

From Statistical tables t boundary for $P = 90$ % = 1,771.

From Statistical tables t boundary for $P = 95$ % = 2,160.

If Formulae (B.66) and (B.67) give t values greater than the applicable one of the above bounds, it is certain that $\hat{a}$ and $\hat{b}$ are not equal to 0.

$$t \text{ for } \hat{a} = \hat{a} / \varepsilon(\hat{a}) \tag{B.66}$$

$$t \text{ for } \hat{a} = 3{,}680\ 3 / 0{,}005\ 756 = 639{,}337\ 53$$

$$t \text{ for } \hat{b} = \hat{b} / \varepsilon(\hat{b}) \quad \text{(B.67)}$$

$$t \text{ for } \hat{b} = 0{,}191\ 3 / 0{,}009\ 828 = 19{,}466\ 632\ 3$$

By inspection, $\hat{a}$ and $\hat{b}$ are not equal to 0.

B.3.2.5 Calculation of long-term (50-year) stiffness

All of the formulae in B.3 are standard formulae for straight-line regressions. From the values given in Tables B.4 and B.5, the estimated long-term stiffness and its confidence and prediction limits are:

Using Formula (B.65), the extrapolated long-term stiffness is given as Formula (B.68):

$$\hat{Y}_X = \hat{\mu}_X = \hat{a} + \hat{b}X \quad \text{(B.68)}$$

$$\hat{Y}_X = \hat{\mu}_X = 3{,}680\ 3 + 0{,}191\ 3 \times 0{,}35 = 3{,}686\ 968$$

Using Formula (B.63), the confidence limits for $\mu_{50\ \text{years}}$ are given as Formula (B.69):

$$\mu_{50\text{years}} = \hat{\mu}_X \pm \mu_{\text{bounds}} \quad \text{(B.69)}$$

$$\mu_{50\text{years}} = 3{,}686\ 968 \pm 0{,}009\ 6 = \left(3\ 677\ 322 \quad 3\ 696\ 614\right)$$

Using Formula (B.64), the prediction limits for $\hat{Y}_{50\text{years}}$ are given as Formula (B.70):

$$\hat{Y}_{50\text{years}} = Y_X \pm Y_{Y\ \text{bounds}} \quad \text{(B.70)}$$

$$\hat{Y}_{50\text{years}} = 3{,}686\ 968 \pm 0{,}019\ 2 = \left(3\ 667\ 725 \quad 3\ 706\ 21\right)$$

Transforming these values back to stiffness gives:

Extrapolated long-term stiffness is equal to:

$$S_{50\ \text{years}} = 10^{\hat{Y}_{50\ \text{years}}} \quad \text{(B.71)}$$

$$S_{50\,\text{years}} = 10^{3{,}686\,968} = 486\;4\;\text{N/m}^2$$

90 % confidence limits for extrapolated long-term stiffness, $\mu(S)_{50\text{ years}}$, are given as Formula (B.72):

$$\mu\left(S\right)_{50\text{ years}} = 10^{\hat{\mu}_X \pm \mu_{\text{bounds}}} \qquad \text{(B.72)}$$

90 % confidence limits for extrapolated long-term stiffness, $\mu(S)_{50\text{ years}}$, are given as Formula (B.73):

$$\mu\left(S\right)_{50\text{ years}} = 10^{3{,}686\,968 \pm 0{,}009\,6} = \left(4\;757{,}497\;3\right) \qquad \text{(B.73)}$$

90 % prediction limits for extrapolated long-term stiffness, $Y(S)_{50\text{ years}}$, are given as Formula (B.74):

$$Y(S)_{50\text{ years}} = 10^{Y_X \pm Y_{Y\text{ bounds}}} \qquad \text{(B.74)}$$

90 % prediction limits for extrapolated long-term stiffness, $Y(S)_{50\text{ years}}$, are given as Formula (B.75):

$$Y(S)_{50\text{ years}} = 10^{3{,}686\,968 \pm 0{,}019\,2} = \left(4\;653 \quad 508\;4\right) \qquad \text{(B.75)}$$

B.4 Description and comments on data and model

A sequential linearized procedure is utilized in this annex so as to meet the requirement for easily accessible explicit formulae. This procedure is almost certainly only marginally sub-optimal for the particular purposes of this document. In particular, it can be observed that for predicting the value of S at $S_{50\text{ years}}$, which is the extrapolated long-term stiffness normally at 50 years, only the parameter a plus the associated estimates of measurement error for S and a are important. The four-parameter model for this procedure is shown as Formula (B.76):

$$Y_i = \lg_{10}\left(S_i\right) = a + b\Big/\left\langle 1 + \exp\left\{-\left[\lg_{10}\left(T_i\right) - c\right]\big/d\right\}\right\rangle \quad i = 1,\ldots,n \qquad \text{(B.76)}$$

where

S is the stiffness, expressed in newtons per square metre (N/m^2);

T is time, expressed in hours (h);

i is the index for observations.

This model is linear in the parameters a and b but non-linear in the parameters c and d. This means that a fully developed statistical analysis designed to produce all the required estimates and intervals would require much algebraic development coupled with a “black-box” use of professional statistical packages. However, an alternative sequential linearized procedure is utilized in this annex to meet the requirement for easily accessible explicit formulae.

B.4.1 Line 1

Line 1 is a rewrite of the model given as Formula (B.76) in order to display time as a function of stiffness, followed by the addition of a simple random error term so as to complete the full specification for a standard straight-line regression model. It is obtained by transforming the Y-axis using preliminary estimates for a and b [see Formula (B.77)].

$$\text{Line 1} \quad y_i = \lg_e\left[\left(a + b - Y_i\right)/\left(Y_i - a\right)\right] = A + Bx_i + e_{1,i} \qquad i = 1, n \tag{B.77}$$

where

$e_{1,i} \approx N\left(0, \sigma_1^2\right)$ is the added random error term.

NOTE 1 This represents normally distributed measurement and sample piece variability under nominally constant experimental conditions.

$$x_i = \lg_{10}\left(60T_i + 1\right) \approx \lg_{10} 60 + \lg_{10} T$$

NOTE 2 The additional “1 min” is a variant on the standard adaptation to ensure that zeroes on the Time and lg (Time) axes roughly correspond.

So that

$$A = \left(c + \lg_{10} 60\right)/d;$$

$$B = -1/d.$$

And hence

$$c = -\left(AB^{-1} + \lg_{10} 60\right);$$

$$d = -B^{-1}.$$

Given initial estimates for a and b, which are easily obtained via the observed maximum and minimum values for stiffness, this straight-line regression model can then be used to estimate c and d.

However, as it stands, this model for Line 1 requires $a + b > Y_i > a$ and this might not hold for the initial estimates. But assuming the model fits the data well, in which case measurement errors are small, it seems reasonable to replace y_i with Formula (B.78):

$$y_i = \lg_e\left\{\text{abs}\left[\left(a + b - Y_i\right)/\left(Y_i - a\right)\right]\right\} \tag{B.78}$$

Suitable initial estimates for a and b can be obtained from the data by setting:

$$a_0 + b_0 = 1{,}005 \max.\left(Y_i\right);$$

$$a_0 = 0{,}995 \min.\left(Y_i\right);$$

$$\text{So } b_0 = \left(a_0 + b_0\right) - a_0.$$

Alternatively, values for a, b, c and d can be obtained using suitable statistical software which can compute a set of best-fit values for these variables from the data by iteration on an appropriately specified least-squares criterion. Such software may not, however, supply standard errors or confidence intervals for these best-fit values.

B.4.2 Line 2

Line 2 is obtained by rewriting the basic model [Formula (B.76)] as a simple straight-line dependence of stiffness on a transformed Time axis using the Line 1 estimates for c and d [see Formula (B.79)]:

$$Y_i = a + bX_i + e_{2i} \tag{B.79}$$

where

$X_i = \left\langle 1 + \exp\left\{-\left[\lg_{10}\left(T_i\right) - c\right] / d\right\}\right\rangle^{-1}$;

$e_{2,i} \approx N\left(0, \sigma_2^2\right)$ represents random measurement and sample pierce variability error.

NOTE This represents normally distributed measurement and sample piece variability under nominally constant experimental conditions.

Given the estimates for c and d obtained using either the Line 1 procedure described above or a suitable statistics package, this straight-line regression model can then be used to re-estimate a and b. Confidence and prediction intervals about this line can then be constructed using standard statistical techniques for linear models and hence, by back transformation, about the curve for S as a function of X or Time.

B.4.3 Analysis and formulae

Using the data set given in Table B.1, the sequential procedure is demonstrated and fully described in this subclause.

WARNING — The calculations described in this annex should only be performed using suitable statistical software packages or appropriate spreadsheet software. This is to avoid errors which are known to occur when attempts are made to perform them using pocket calculators.

B.4.3.1 Line 1

The least-squares estimates for A and B and the unbiased estimate for $\tilde{\sigma}_1^2$ are given as Formulae (B.80) and (B.81).

B.4.3.1.1 Computation of $\hat{B}$

$\hat{B}$ is computed using Formula (B.80):

$$\begin{aligned}\hat{B} &= \sum(x_i - \bar{x})(y_i - \bar{y}) \Big/ \sum(x_i - \bar{x})^2 = \left(n \times \sum xy - \sum x \times \sum y\right) \Big/ \left[n \times \sum x^2 - \left(\sum x\right)^2\right] \\ &= \left(15 \times 17{,}80 - 48{,}47 \times 2{,}51\right) \Big/ \left[15 \times 187{,}75 - \left(2\,349{,}34\right)\right] = 0{,}831\,9\end{aligned} \tag{B.80}$$

B.4.3.1.2 Computation of $\hat{A}$

$\hat{A}$ is computed using Formula (B.81):

$$\hat{A} = \bar{y} - \hat{B}\bar{x} \tag{B.81}$$

where

$$\bar{y} = \sum y/n = -2{,}51/15 = -0{,}167\ 5;$$

$$\bar{x} = \sum x/n = 48{,}47/15 = 3{,}231\ 036;$$

$$\hat{B} = 0{,}831\ 9.$$

NOTE See Formula (B.80).

$$\hat{A} = 0{,}167\ 5 - 0{,}831\ 9 \times 3{,}231\ 036 = -2{,}855\ 5 \quad \text{(B.82)}$$

B.4.3.1.3 Computation of $\tilde{\sigma}_1^2$

$\tilde{\sigma}_1{}^2$ is given as Formula (B.83):

$$\tilde{\sigma}_1^2 = \sum (y_i - \hat{y}_i)^2 \Big/ (n-2) \quad \text{(B.83)}$$

where

$$\sum (y_i - \hat{y}_i)^2 = \sum y^2 - A \times \sum y - B \times \sum xy \Big/ n - 2 = 22{,}65 - -2{,}855\ 5 \times -2{,}51 - 0{,}831\ 9 \times 17{,}8 = 0{,}665\ 3$$

$$\tilde{\sigma}_1^2 = 0{,}665\ 3/13 = 0{,}051\ 2 \quad \text{(B.84)}$$

Table B.1 — Raw data and derived values

Count i	Raw data		Derived values				
	Time h	Stiffness S	$\lg_{10}(S)$ Y_i	$\lg_{10}$(mins+1) x_i	Linearized Y y_i	Linearized time X_i	$\lg_{10}$(Time) T_i
0	0	*	*	0,000 000	*	*	*
1	0,10	7 114	3,852 114	0,845 098	−2,199 873	0,900 981 089	−1,000 000
2	0,27	6 935	3,841 046	1,235 528	−1,680 067	0,864 045 469	−0,568 636
3	0,50	6 824	3,834 039	1,491 362	−1,428 181	0,835 713 323	−0,301 030
4	1,00	6 698	3,825 945	1,785 330	−1,178 593	0,798 385 481	0,000 000
5	3,28	6 533	3,815 113	2,296 226	−0,888 531	0,720 523 173	0,515 874
6	7,28	6 453	3,809 762	2,641 276	−0,757 777	0,659 034 121	0,862 131
7	20,0	6 307	3,799 823	3,079 543	−0,529 608	0,572 939 842	1,301 030
8	45,9	6 199	3,792 322	3,440 122	−0,366 192	0,498 426 628	1,661 813
9	72,0	6 133	3,787 673	3,635 584	−0,267 424	0,457 861 654	1,857 332
10	166	5 692	3,755 265	3,998 303	0,411 303	0,384 436 030	2,220 108
11	219	5 508	3,740 994	4,118 628	0,732 338	0,361 035 235	2,340 444
12	384	5 393	3,731 830	4,362 501	0,958 300	0,315 663 484	2,584 331

NOTE 1 Linearized $Y = y_i = \ln\left\langle ABS\left\{\left[a_0 + b_0 - \lg_{10}(S)\right] / \left[\lg_{10}(S) - a_0\right]\right\}\right\rangle$.

NOTE 2 Linearized time $X_i = 1 / \left\langle 1 + \exp\left\{-\left[\lg_{10}(\text{Time})\right] - c / d\right\}\right\rangle$.

NOTE 3 The raw data in this table are the same as those in Table 5.

NOTE 4 The values given in rows 0 and 16 are calculated values, while the values in rows 1 to 15 inclusive are measured values or derived from measured values.

Table B.1 *(continued)*

Count i	Raw data		Derived values				
	Time h	Stiffness S	$\lg_{10}(S)$ Y_i	$\lg_{10}$(mins+1) x_i	Linearized Y y_i	Linearized time X_i	$\lg_{10}$(Time) T_i
13	504	5 364	3,729 489	4,480 596	1,019 680	0,294 833 214	2,702 431
14	3 000	5 200	3,716 003	5,255 275	1,416 309	0,179 974 497	3,477 121
15	10 520	4 975	3,696 793	5,800 168	2,245 487	0,122 406 128	4,022 016
16	438 300	*	*	7,419 923	*	0,034 979 805	5,641 771

NOTE 1 Linearized $Y = y_i = \ln\left\langle ABS\left\{\left[a_0 + b_0 - \lg_{10}\left(S\right)\right] / \left[\lg_{10}\left(S\right) - a_0\right]\right\}\right\rangle$.

NOTE 2 Linearized time $X_i = 1 / \left\langle 1 + \exp\left\{-\left[\lg_{10}\left(\text{Time}\right)\right] - c / d\right\}\right\rangle$.

NOTE 3 The raw data in this table are the same as those in Table 5.

NOTE 4 The values given in rows 0 and 16 are calculated values, while the values in rows 1 to 15 inclusive are measured values or derived from measured values.

B.4.3.1.4 Computation of *a*, *b*, *c* and *d*

The values for *a*, *b*, *c* and *d* can be found by either using the procedure described in B.4 or by the use of suitable statistical software.

B.4.3.1.4.1 Using estimation procedure

Assuming measurements of stiffness for up to 10 000 h are available, suitable initial estimates for *a* and *b* are obtained from the data by using Formulae (B.85), (B.86), and (B.87) (see also B.2.1):

$$a + b = 1{,}005 \times \max.\left(Y_i\right) = 1{,}005 \times 3{,}852\ 114 = 3{,}871\ 375 \tag{B.85}$$

$$a = 0{,}995 \min.\left(Y_i\right) = 0{,}995 \times 3{,}696\ 793 = 3{,}678\ 309 \tag{B.86}$$

$$b = 3{,}871\ 375 - 3{,}678\ 309 = 0{,}193\ 066 \tag{B.87}$$

For these data:

a is $\lg_{10}$ [stiffness at 50 years (S_{50})], therefore S_{50} = 10^3,678 309 = 4 768 N/m²;

b is change in $\lg_{10}$(stiffness) between initial and 50 years so, $b = \lg_{10}$ [of the ratio (S_0/S_{50})];

but 10^*b* = 10^0,193 066 = 1,559 79

therefore, initial stiffness, S_0 = 1,559 79 × S_{50} = 1,559 79 × 4 768 = 7 437 N/m².

Alternatively, $a + b$ is $\lg_{10}(S_0)$ so S_0 = 10^3,871 375 = S_0 = 7 437 N/m².

Hence, the implied estimates for *c* and *d* are:

$$\hat{c} = -\left[A / B - \lg_{10}\left(60\right)\right] = 3{,}432\ 504 - 1{,}778\ 151\ 3 = 1{,}653\ 53 \tag{B.88}$$

$$\hat{d} = -B^{-1} = -0{,}831\ 9^{-1} = -1{,}202\ 011 \tag{B.89}$$

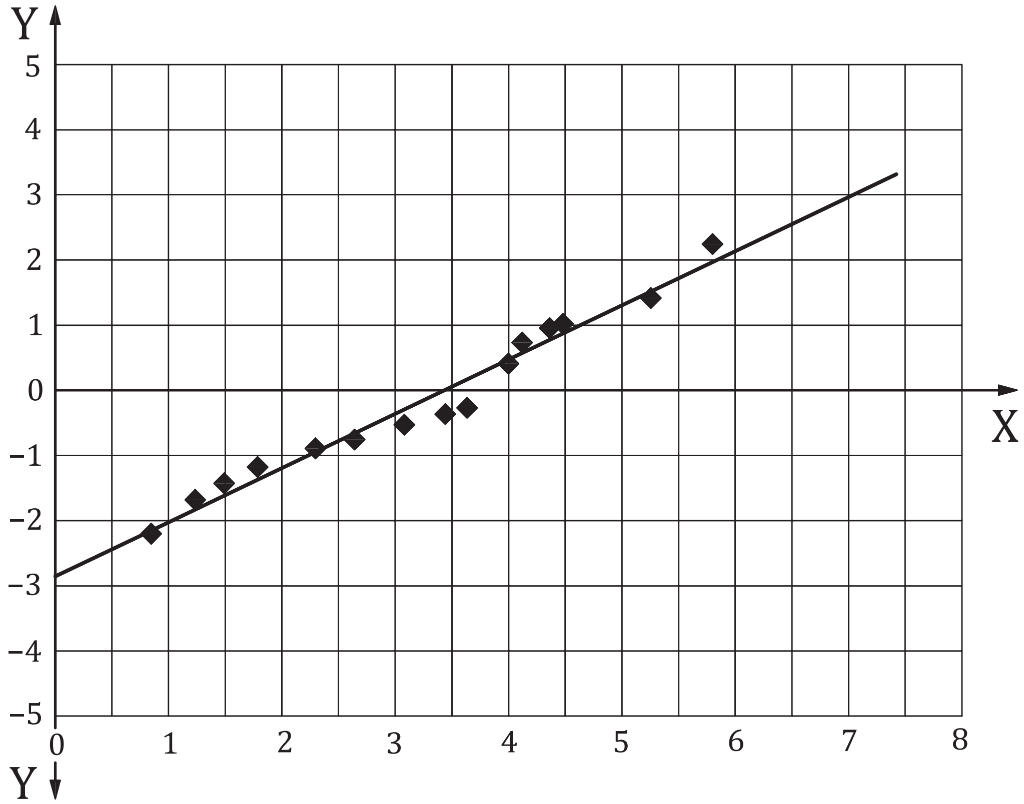
where

$\hat{c} = \lg_{10}\left(\text{half-period}\right)$;

NOTE The half-period is the time at which $\lg_{10}$(stiffness) achieves 50 % of its overall loss.

$\hat{d}$ is the negative inverse of the slope of the line in Figure B.1.

The fitted parameters are statistically significant and the fitted line is shown in Figure B.1.



Key

X-axis $\lg_{10}$(minutes + 1)

Y-axis transformed stiffness

trend-line $(\hat{y} = A + Bx)$

■ data points

NOTE See Table B.2 for details of the data used to produce this graph.

Figure B.1 — Line 1 — Straight-line fit

B.4.3.1.4.2 Using statistical software

By using initial estimates for a, b and c of 1 and −1 for d and the data in Table B.1, the software was run for up to 1 000 iterations and the following values were produced:

a = 3,675

b = 0,193

$a + b$ = 3,871

c = 1,654

d = −1,202

These values can be seen to be very similar to the estimated values obtained in B.3.1.4 and B.3.1.3.

Table B.2 — Line 1 data results for Figure B.1

Count i	x_i	y_i	x^2	y^2	xy		y_{hat} $A + Bx$	**Residuals** $y - y_{\text{hat}}$
0	0,000 000	−2,855 548	0,000 000	8,154 155	0,000 000		−2,855 548	0,000 000
1	0,845 098	−2,199 873	0,714 191	4,839 439	−1,859 108		−2,152 478	−0,047 395
2	1,235 528	−1,680 067	1,526 531	2,822 626	−2,075 771		−1,827 663	0,147 596
3	1,491 362	−1,428 181	2,224 160	2,039 700	−2,129 934		−1,614 826	0,186 645
4	1,785 330	−1,178 593	3,187 403	1,389 081	−2,104 177		−1,370 262	0,191 669
5	2,296 226	−0,888 531	5,272 655	0,789 487	−2,040 267		−0,945 227	0,056 696
6	2,641 276	−0,757 777	6,976 338	0,574 226	−2,001 498		−0,658 167	−0,099 610
7	3,079 543	−0,529 608	9,483 585	0,280 485	−1,630 951		−0,293 555	−0,236 053
8	3,440 122	−0,366 192	11,834 437	0,134 097	−1,259 746		0,006 425	−0,372 617
9	3,635 584	−0,267 424	13,217 473	0,071 516	−0,972 242		0,169 038	−0,436 462
10	3,998 303	0,411 303	15,986 426	0,169 170	1,644 515		0,470 798	−0,059 495
11	4,118 628	0,732 338	16,963 100	0,536 319	3,016 228		0,570 901	0,161 437
12	4,362 501	0,958 300	19,031 418	0,918 339	4,180 586		0,773 789	0,184 512
13	4,480 596	1,019 680	20,075 742	1,039 746	4,568 772		0,872 036	0,147 643
14	5,255 275	1,416 309	27,617 914	2,005 933	7,443 096		1,516 522	−0,100 213
15	5,800 168	2,245 487	33,641 945	5,042 213	13,024 202		1,969 840	0,275 647
16	7,419 923	3,317 378	55,055 253	11,004 998	24,614 690		3,317 378	0,000 000
Totals	48,465 540	−2,512 828	187,753 317	22,652 377	17,803 704		37,464 846	0,000 000

NOTE 1 $x_i = \lg_{10}(\text{minutes} + 1)$.

NOTE 2 y_i = linearized Y.

NOTE 3 The values given in rows 0 and 16 are calculated values, while the values in rows 1 to 15 inclusive are measured values or derived from measured values. The values in rows 0 and 16 are not included in the totals below row 16.

B.4.3.1.5 Other derived variables associated with Line 1

$$S_x = n \times \sum x_i^2 - \left(\sum x_i\right)^2 = 467,39 \quad \text{(B.90)}$$

$$S_y = n \times \sum y_i^2 - \left(\sum y_i\right)^2 = 333,47 \quad \text{(B.91)}$$

$$S_{xy} = n \times \sum x_i y_i - \left(\sum x_i \times \sum y_i\right) = 388,84 \quad \text{(B.92)}$$

$$\bar{x} = \sum x_i / n = 3,231\ 036 \quad \text{(B.93)}$$

$$\bar{y} = \sum y_i / n = -0,167\ 522 \quad \text{(B.94)}$$

B.4.3.2 Line 2

B.4.3.2.1 Computation of a, b, and $\hat{\sigma}_2^{\,2}$

NOTE 1 The data used in these calculations are shown in Table B.3.

The least-squares estimate for a and b and the unbiased estimate for $\tilde{\sigma}_2^2$ are given as Formula (B.95):

$$\hat{b} = \sum(X_i - \bar{X})(Y_i - \bar{Y}) \Big/ \sum(X_i - \bar{X})^2 = \left(n \times \sum XY - \sum X \times \sum Y\right) \Big/ \left(n \times \sum X^2 - \sum X \times \sum X\right);$$

$$\hat{b} = \left(15 \times 30{,}302\ 557 - 7{,}966\ 259 \times 56{,}728\ 211\right) \Big/ \left(15 \times 5{,}146\ 065 - 7{,}966\ 259 \times 7{,}966\ 259\right) =$$

$$= 2{,}626\ 734 \Big/ 13{,}729\ 693 = 0{,}191\ 318 \tag{B.95}$$

Using the estimates for c and d, Formulae (B.96), (B.97), (B.98), (B.99), (B.100) and (B.101) are derived using Line 1:

$$\bar{Y} = \sum Y / n = 56{,}728\ 211 \Big/ 15 = 3{,}781\ 881 \tag{B.96}$$

$$\bar{X} = \sum X / n = 7{,}966\ 259 \Big/ 15 = 0{,}531\ 084 \tag{B.97}$$

$$\hat{a} = \bar{Y} - \hat{b} \times \bar{X} = 3{,}781\ 881 - 0{,}191\ 318 \times 0{,}531\ 083\ 4 = 3{,}680\ 275 \tag{B.98}$$

$$\tilde{\sigma}_2^2 = \sum(Y_i - \hat{Y}_i)^2 \Big/ \left(n - 2\right) = \mathrm{RSS} \Big/ n - 2 \tag{B.99}$$

$$\begin{aligned} \mathrm{RSS} &= \sum Y^2 - \hat{a} \times \sum Y - \hat{b} \times \sum XY = \\ &= 214{,}573\ 977 - 3{,}680\ 275 \times 56{,}728\ 211 - 0{,}191\ 318 \times 30{,}302\ 557 = 0{,}001\ 136 \end{aligned} \tag{B.100}$$

$$\hat{\sigma}_2{}^2 = 0{,}001\ 136/13 = 0{,}000\ 087 \tag{B.101}$$

NOTE 2 For the data observed, these estimates satisfy the model constraint $a + b > Y_i > a$.

Table B.3 — Line 2 data results

Count i	X_i	Y_i	X^2	Y^2	XY		Y_{hat} $(A + BXi)$	**Residu-als** $(Y - Y_{hat})$	Y_{bounds}	Y_L $(Y_{hat} - Y_{bounds})$	Y_U $(Y_{hat} + Y_{bounds})$	μ_{bounds}	μ_L $Y_{hat} - \mu_{bounds}$	μ_U $Y_{hat} + \mu_{bounds}$	Error bound	Y_{hat} − error	Y_{hat} + error
0	1,000 000	3,871 592	1,000 000	14,989 224	3,871 592		3,871 592	0,000 000	0,019 036	3,852 556	3,890 628	0,009 224	3,862 367	3,880 816	0,016 651	3,854 941	3,888 243
1	0,900 981	3,852 114	0,811 767	14,838 781	3,470 682		3,852 648	0,000 534	0,018 363	3,834 285	3,871 011	0,007 742	3,844 906	3,860 389	0,016 651	3,835 997	3,869 299
2	0,864 045	3,841 046	0,746 575	14,753 638	3,318 839		3,845 582	0,004 535	0,018 148	3,827 434	3,863 729	0,007 216	3,838 366	3,852 797	0,016 651	3,828 930	3,862 233
3	0,835 713	3,834 039	0,698 417	14,699 855	3,204 157		3,840 161	0,006 122	0,017 996	3,822 165	3,858 157	0,006 826	3,833 335	3,846 987	0,016 651	3,823 510	3,856 813
4	0,798 385	3,825 945	0,637 419	14,637 856	3,054 579		3,833 020	0,007 075	0,017 816	3,815 204	3,850 835	0,006 335	3,826 685	3,839 354	0,016 651	3,816 369	3,849 671
5	0,720 523	3,815 113	0,519 154	14,555 085	2,748 877		3,818 123	0,003 011	0,017 511	3,800 613	3,835 634	0,005 418	3,812 705	3,823 542	0,016 651	3,801 472	3,834 775
6	0,659 034	3,809 762	0,434 326	14,514 284	2,510 763		3,806 360	0,003 402	0,017 341	3,789 019	3,823 701	0,004 842	3,801 518	3,811 201	0,016 651	3,789 708	3,823 011
7	0,572 940	3,799 823	0,328 260	14,438 654	2,177 070		3,789 888	0,009 934	0,017 213	3,772 676	3,807 101	0,004 361	3,785 528	3,794 249	0,016 651	3,773 237	3,806 540
8	0,498 427	3,792 322	0,248 429	14,381 703	1,890 194		3,775 633	0,016 689	0,017 207	3,758 426	3,792 840	0,004 337	3,771 296	3,779 970	0,016 651	3,758 982	3,792 284
9	0,457 862	3,787 673	0,209 637	14,346 466	1,734 230		3,767 872	0,019 801	0,017 245	3,750 628	3,785 117	0,004 484	3,763 388	3,772 356	0,016 651	3,751 221	3,784 523
10	0,384 436	3,755 265	0,147 791	14,102 014	1,443 659		3,753 825	0,001 440	0,017 386	3,736 439	3,771 210	0,005 000	3,748 825	3,758 824	0,016 651	3,737 173	3,770 476
11	0,361 035	3,740 994	0,130 346	13,995 036	1,350 631		3,749 348	0,008 354	0,017 450	3,731 897	3,766 798	0,005 220	3,744 128	3,754 567	0,016 651	3,732 696	3,765 999
12	0,315 663	3,731 830	0,099 643	13,926 558	1,178 003		3,740 667	0,008 837	0,017 601	3,723 066	3,758 269	0,005 705	3,734 963	3,746 372	0,016 651	3,724 016	3,757 319
13	0,294 833	3,729 489	0,086 927	13,909 086	1,099 577		3,736 682	0,007 193	0,017 682	3,719 000	3,754 364	0,005 949	3,730 733	3,742 631	0,016 651	3,720 031	3,753 333
14	0,179 974	3,716 003	0,032 391	13,808 681	0,668 786		3,714 708	0,001 296	0,018 251	3,696 457	3,732 959	0,007 472	3,707 236	3,722 180	0,016 651	3,698 057	3,731 359

NOTE 1 Linearized time, $X_i = 1 \Big/ \left\{ 1 + \exp\left[-\lg_{10}\left(\text{Time}\right) - C \right] / d \right\}$.

NOTE 2 $Y_i = \lg_{10}\left(\text{Stiffness}\right)$.

NOTE 3 $Y_{\text{bounds}} = t - dn_{\text{bound}} \times \sqrt{\left\{ \sigma^2 \left[1 + \left(1/n \right) + \left(X_i - \bar{X} \right) \times \left(X_i - \bar{X} \right) / \sum X_i^2 - \sum X_i \times \sum X_i \; / \; n \right] \right\}}$.

NOTE 4 $\mu_{\text{bounds}} = t - dn_{\text{bound}} \times \sqrt{\left\langle \sigma^2 \left\{ \left[\left(1/n \right) + \left(x_i - \bar{x} \right) \right] / \left[\sum x_i^2 - \sum x_i \times \sum x_i \; / \; n \right] \right\} \right\rangle}$.

NOTE 5 Error bounds $= t - dn_{\text{bound}} \times \sqrt{\left(\sigma^2 \right)}$.

NOTE 6 The values given in rows 0 and 16 are calculated values, while the values in rows 1 to 15 inclusive are measured values or derived from measured values. The values in rows 0 and 16 are not included in the totals below row 16.

Table B.3 *(continued)*

Count i	X_i	Y_i	X^2	Y^2	XY		Y_{hat} $(A + BXi)$	**Residu-als** $(Y - Y_{hat})$	Y_{bounds}	Y_L $(Y_{hat} - Y_{bounds})$	Y_U $(Y_{hat} + Y_{bounds})$	μ_{bounds}	μ_L $Y_{hat} - \mu_{bounds}$	μ_U $Y_{hat} + \mu_{bounds}$	Error bound	Y_{hat} − error	Y_{hat} + error
15	0,122 406	3,696 793	0,014 983	13,666 279	0,452 510		3,703 694	0,006 901	0,018 610	3,685 084	3,722 304	0,008 311	3,695 383	3,712 005	0,016 651	3,687 043	3,720 345
16	0,034 980	3,686 968	0,001 224	13,593 733	0,128 969		3,686 968	0,000 000	0,019 243	3,667 725	3,706 211	0,009 646	3,677 322	3,696 614	0,016 651	3,670 317	3,703 619
Totals	7,966 259	56,728 211	5,146 065	214,573 977	30,302 557		5,204 350	0,000 000									

NOTE 1 Linearized time, $X_i = 1\Big/\Big\{1 + \exp\Big[-\lg_{10}\Big(\text{Time}\Big) - C\Big] / d\Big\}$.

NOTE 2 $Y_i = \lg_{10}\Big(\text{Stiffness}\Big)$.

NOTE 3 $Y_{\text{bounds}} = t - dn_{\text{bound}} \times \sqrt{\Big\{\sigma^2\Big[1 + \big(1/n\big) + \big(X_i - \overline{X}\big) \times \big(X_i - \overline{X}\big) / \sum X_i^{\,2} - \sum X_i \times \sum X_i \,/\, n\Big]\Big\}}$.

NOTE 4 $\mu_{\text{bounds}} = t - dn_{\text{bound}} \times \sqrt{\Big\langle\sigma^2\Big\{\Big[\big(1/n\big) + \big(x_i - \overline{x}\big)\Big] / \Big[\sum x_i^{\,2} - \sum x_i \times \sum x_i \,/\, n\Big]\Big\}\Big\rangle}$.

NOTE 5 Error bounds $= t - dn_{\text{bound}} \times \sqrt{\big(\sigma^2\big)}$.

NOTE 6 The values given in rows 0 and 16 are calculated values, while the values in rows 1 to 15 inclusive are measured values or derived from measured values. The values in rows 0 and 16 are not included in the totals below row 16.

B.4.3.2.2 Prediction and confidence intervals

Estimates for the variances of $\hat{a}$ and $\hat{b}$ are given as Formulae (B.102) and (B.103):

$$\operatorname{var}\left(\hat{a}\right) = \left(\tilde{\sigma}_2^2 \sum X_i^2\right) \Big/ \left[n \sum X_i^2 - \left(\sum X_i\right)^2 \right] = \left(\tilde{\sigma}_2{}^2 \sum X_i{}^2\right) \Big/ \left[n \sum \left(X_i - \bar{X}\right)^2 \right] = 0{,}000\ 033 \quad \text{(B.102)}$$

$$\operatorname{var}\left(\hat{b}\right) = \left(n\tilde{\sigma}_2^2\right) \Big/ \left[n \sum X_i^2 - \left(\sum X_i\right)^2 \right] = \left(n\tilde{\sigma}_2{}^2\right) \Big/ \left[n \sum \left(X_i - \bar{X}\right)^2 \right] = 0{,}000\ 097 \quad \text{(B.103)}$$

The square root of these variances is called the estimated standard error, ε, and for Line 2, the data set gives Formulae (B.104) and (B.105):

$$\varepsilon\left(\hat{a}\right) = \sqrt{0{,}000\ 033} = 0{,}005\ 756 \quad \text{(B.104)}$$

$$\varepsilon\left(\hat{b}\right) = \sqrt{0{,}000\ 097} = 0{,}009\ 828 \quad \text{(B.105)}$$

The formulae for 100 % confidence and prediction intervals about the fitted Line 2 as functions of X are given as Formulae (B.106) and (B.107), respectively:

Confidence interval:

$$\mu_X = \hat{\mu}_X \pm t_P \tilde{\sigma}_2 \sqrt{\left[\frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{\sum \left(X_i - \bar{X}\right)^2} \right]} \quad \text{(B.106)}$$

Prediction interval:

$$Y_X = \hat{\mu}_X \pm t_P \tilde{\sigma}_2 \sqrt{\left[1 + \frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{\sum \left(X_i - \bar{X}\right)^2} \right]} \quad \text{(B.107)}$$

where

$$\hat{\mu}_X = \hat{a} + \hat{b}X \quad \text{(B.108)}$$

In Table B.4, the confidence intervals are shown as μ_L and μ_U and the prediction intervals are Y_L and Y_U.

In Table B.5, the confidence intervals are shown as $S\mu_L$ and $S\mu_U$ and the prediction intervals are S_L and S_U.

B.4.3.2.3 Student’s *t* test for *a* and *b*

To test whether a or b is equal to 0, perform the following calculations:

$$\Pr\left(\left|t\right| < t_P\right) = P$$

t has Student’s t-distribution on $(n - 2)$ degrees of freedom.

t boundary for $P = 90\ \% = 1{,}771$

t for $\hat{a} = \hat{a} \big/ \varepsilon(\hat{a}) = 3{,}680\ 3 \big/ 0{,}005\ 756 = 639{,}337\ 53$

t for $\hat{b} = \hat{b} / \varepsilon(\hat{b}) = 0{,}191\ 3 / 0{,}009\ 828 = 19{,}466\ 632\ 3$

As both of these values are greater than 1,771, it is certain that $\hat{a}$ and $\hat{b}$ are not equal to 0.

B.4.3.2.4 Calculation of long-term (50-year) stiffness

All the formulae in this subclause are standard formulae for straight-line regressions. The fitted parameters are statistically significant and graphs of the fitted line and implied curves, with these intervals superimposed, can be seen in Figures B.2, B.3 and B.4. In particular, at the specified 50-year point (which has X = 0,035) and for P = 90 %, these intervals become Formulae (B.109) and (B.110), respectively:

$$\mu_{50\text{years}} = \hat{a} + \hat{b}X = 3{,}680\ 3 + 0{,}191\ 3 \times 0{,}035 = 3{,}686\ 996 \pm 0{,}009\ 6 \quad \text{(B.109)}$$

$$Y_{50\text{years}} = 3{,}686\ 996 \pm 0{,}019\ 2 \quad \text{(B.110)}$$

Transforming back to stiffness gives Formulae (B.111) and (B.112):

$$\mu(S)_{50\text{years}} = 10^{3{,}687 \pm 0{,}009\ 6} = (4\ 757 \quad 497\ 3) \quad \text{(B.111)}$$

$$S_{50\text{years}} = 10^{3{,}687 \pm 0{,}019\ 2} = (4\ 653 \quad 508\ 4) \quad \text{(B.112)}$$

Table B.4 — Data for Figure B.2

Count i	T	X_i	Y_i	Y_{hat} ($A + BXi$)	**Residuals** ($Y - Y_{hat}$)	Y_{bounds}	Y_L ($Y_{hat} -$ Y_{bounds})	Y_U ($Y_{hat} +$ Y_{bounds})	μ_{bounds}	μ_L $Y_{hat} -$ μ_{bounds}	μ_U $Y_{hat} +$ μ_{bounds}	Error bound	Y_{hat} − error	Y_{hat} + error
0	0,0	1,000 000	3,871 592	3,871 592	0,000 000	0,019 036	3,852 556	3,890 628	0,009 224	3,862 367	3,880 816	0,016 651	3,854 941	3,888 243
1	0,1	0,900 981	3,852 114	3,852 648	−0,000 534	0,018 363	3,834 285	3,871 011	0,007 742	3,844 906	3,860 389	0,016 651	3,835 997	3,869 299
2	0,3	0,864 045	3,841 046	3,845 582	−0,004 535	0,018 148	3,827 434	3,863 729	0,007 216	3,838 366	3,852 797	0,016 651	3,828 930	3,862 233
3	0,5	0,835 713	3,834 039	3,840 161	−0,006 122	0,017 996	3,822 165	3,858 157	0,006 826	3,833 335	3,846 987	0,016 651	3,823 510	3,856 813
4	1,0	0,798 385	3,825 945	3,833 020	−0,007 075	0,017 816	3,815 204	3,850 835	0,006 335	3,826 685	3,839 354	0,016 651	3,816 369	3,849 671
5	3,3	0,720 523	3,815 113	3,818 123	−0,003 011	0,017 511	3,800 613	3,835 634	0,005 418	3,812 705	3,823 542	0,016 651	3,801 472	3,834 775
6	7,3	0,659 034	3,809 762	3,806 360	0,003 402	0,017 341	3,789 019	3,823 701	0,004 842	3,801 518	3,811 201	0,016 651	3,789 708	3,823 011
7	20,0	0,572 940	3,799 823	3,789 888	0,009 934	0,017 213	3,772 676	3,807 101	0,004 361	3,785 528	3,794 249	0,016 651	3,773 237	3,806 540
8	45,9	0,498 427	3,792 322	3,775 633	0,016 689	0,017 207	3,758 426	3,792 840	0,004 337	3,771 296	3,779 970	0,016 651	3,758 982	3,792 284
9	72,0	0,457 862	3,787 673	3,767 872	0,019 801	0,017 245	3,750 628	3,785 117	0,004 484	3,763 388	3,772 356	0,016 651	3,751 221	3,784 523
10	166,0	0,384 436	3,755 265	3,753 825	0,001 440	0,017 386	3,736 439	3,771 210	0,005 000	3,748 825	3,758 824	0,016 651	3,737 173	3,770 476
11	219,0	0,361 035	3,740 994	3,749 348	−0,008 354	0,017 450	3,731 897	3,766 798	0,005 220	3,744 128	3,754 567	0,016 651	3,732 696	3,765 999
12	384,0	0,315 663	3,731 830	3,740 667	−0,008 837	0,017 601	3,723 066	3,758 269	0,005 705	3,734 963	3,746 372	0,016 651	3,724 016	3,757 319
13	504,0	0,294 833	3,729 489	3,736 682	−0,007 193	0,017 682	3,719 000	3,754 364	0,005 949	3,730 733	3,742 631	0,016 651	3,720 031	3,753 333
14	3 000,0	0,179 974	3,716 003	3,714 708	0,001 296	0,018 251	3,696 457	3,732 959	0,007 472	3,707 236	3,722 180	0,016 651	3,698 057	3,731 359
15	10 520,0	0,122 406	3,696 793	3,703 694	−0,006 901	0,018 610	3,685 084	3,722 304	0,008 311	3,695 383	3,712 005	0,016 651	3,687 043	3,720 345
16	438 300,0	0,034 980	3,686 968	3,686 968	0,000 000	0,019 243	3,667 725	3,706 211	0,009 646	3,677 322	3,696 614	0,016 651	3,670 317	3,703 619

NOTE 1 Linearized time, $X_i = 1/\left\{1 + \exp\left[-\lg_{10}\left(\text{Time}\right) - C\right] / d\right\}$.

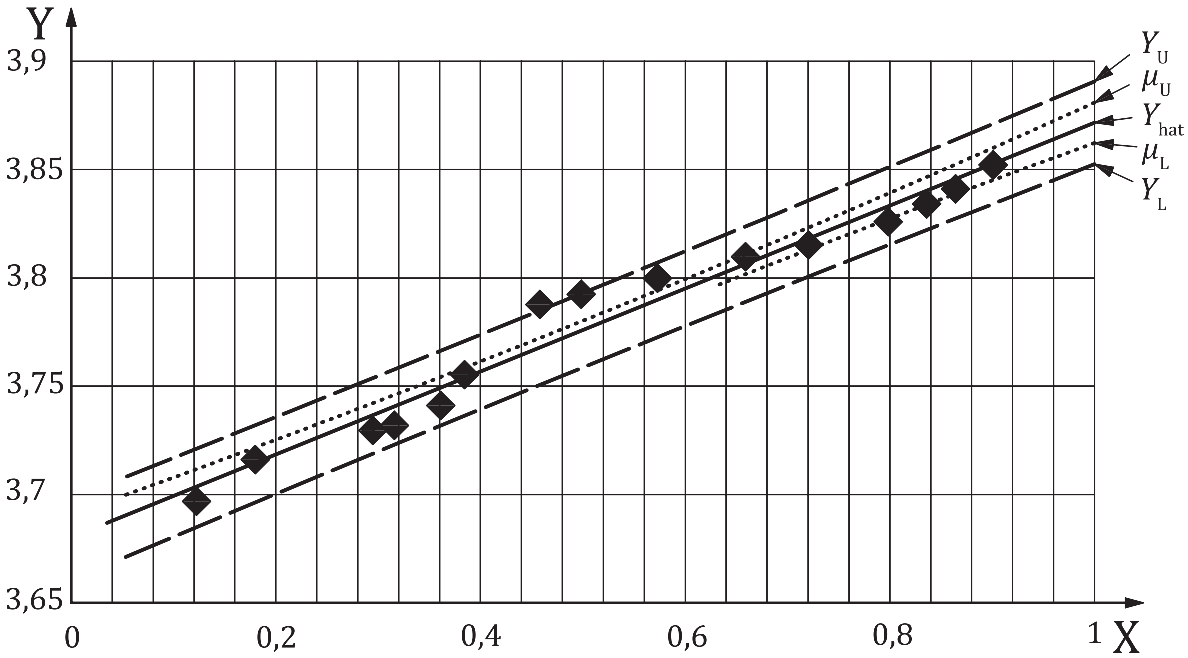
NOTE 2 $\lg_{10}\left(\text{Stiffness}\right), Y_i$.

NOTE 3 $Y_{\text{bounds}} = t - dn_{\text{bound}} \times \sqrt{\left\{\sigma^2\left[1 + \left(1/n\right) + \left(X_i - \bar{X}\right) \times \left(X_i - \bar{X}\right) / \left(\sum X_i^2 - \sum X_i \times \sum X_i / n\right)\right]\right\}}$.

NOTE 4 $\mu_{\text{bounds}} = t - dn_{\text{bound}} \times \sqrt{\left\{\sigma^2\left[\left(1/n\right) + \left(x_i - \bar{x}\right) / \left(\sum x_i^2 - \sum x_i \times \sum x_i / n\right)\right]\right\}}$.

NOTE 5 Error bounds $= t - dn_{\text{bound}} \times \sqrt{\left(\sigma^2\right)}$.

NOTE 6 The values given in rows 0 and 16 are calculated values, while the values in rows 1 to 15 inclusive are measured values or derived from measured values.



Key

X transformed time, *X*

Y $Y_i = \lg_{10}$(stiffness)

μ_L per cent lower confidence interval for line

μ_U per cent upper confidence interval for line

Y_{hat} Line 2 trend-line, X

Y_L per cent lower prediction interval for future *Y* values

Y_U per cent upper prediction interval for future *Y* values

■ data points, X,Y

NOTE See Table B.4 for details of the data used to produce this graph.

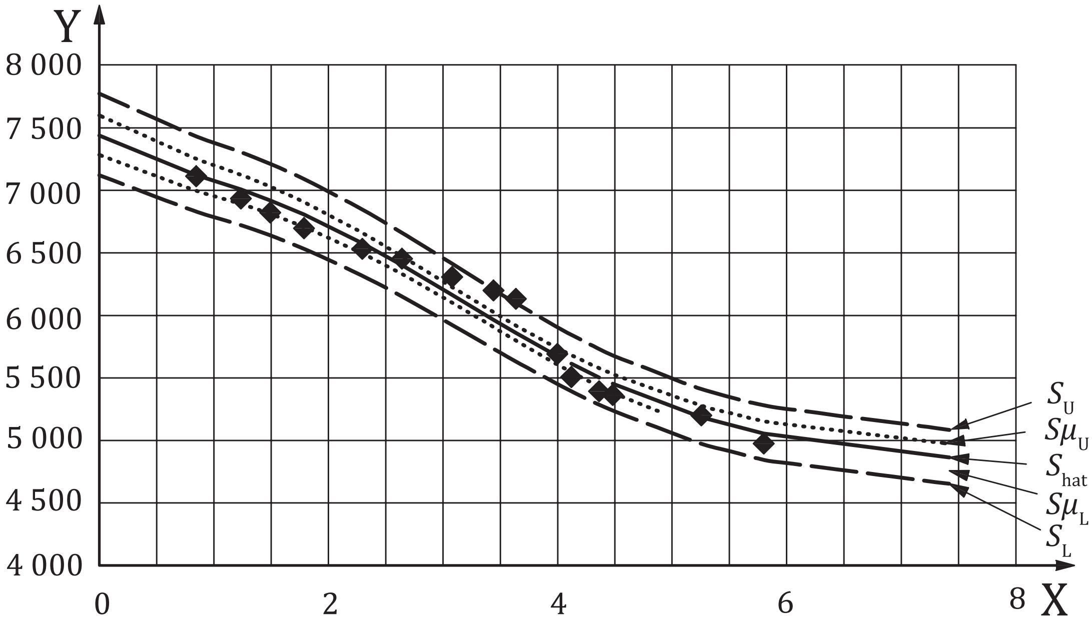
Figure B.2 — Plot of $\lg_{10}$(stiffness) **vs. transformed time for Line 2, including confidence and prediction intervals**

Table B.5 — Data for Figures B.3 and B.4

Count i	T	x_i	S_i $(10\text{^}Y_i)$	S_{hat} $(10\text{^}Y_{hat})$	**Residuals** $(S - S_{hat})$	S_{bounds} $(10\text{^}Y_{bounds})$	S_L $(10\text{^}Y_L)$	S_U $(10\text{^}Y_U)$	$S\mu_{bounds}$ $\left(10^{\hat{\mu}_{bounds}}\right)$	$S\mu_L$ $\left(10^{\hat{\mu}_L}\right)$	$S\mu_U$ $\left(10^{\hat{\mu}_U}\right)$	S error bound (10^ error bound)	S − error [10^(Y_{hat} − error)]	S + error [10^(Y_{hat} + error)]
0	0,0	0,000 000	7 440	7 440	0	1,044 806	7 121	7 774	1,021 467	7 284	7 600	1,039 085	7 160	7 731
1	0,1	0,845 098	7 114	7 123	−9	1,043 189	6 828	7 430	1,017 985	6 997	7 251	1,039 085	6 855	7 401
2	0,3	1,235 528	6 935	7 008	−73	1,042 672	6 721	7 307	1,016 754	6 892	7 125	1,039 085	6 744	7 282
3	0,5	1,491 362	6 824	6 921	−97	1,042 308	6 640	7 214	1,015 842	6 813	7 031	1,039 085	6 661	7 191
4	1,0	1,785 330	6 698	6 808	−110	1,041 875	6 534	7 093	1,014 693	6 709	6 908	1,039 085	6 552	7 074
5	3,3	2,296 226	6 533	6 578	−45	1,041 143	6 318	6 849	1,012 554	6 497	6 661	1,039 085	6 331	6 836
6	7,3	2,641 276	6 453	6 403	50	1,040 737	6 152	6 663	1,011 211	6 332	6 474	1,039 085	6 162	6 653
7	20,0	3,079 543	6 307	6 164	143	1,040 430	5 925	6 414	1,010 091	6 103	6 227	1,039 085	5 932	6 405
8	45,9	3,440 122	6 199	5 965	234	1,040 415	5 734	6 206	1,010 036	5 906	6 025	1,039 085	5 741	6 198
9	72,0	3,635 584	6 133	5 860	273	1,040 506	5 632	6 097	1,010 379	5 799	5 920	1,039 085	5 639	6 089
10	166,0	3,998 303	5 692	5 673	19	1,040 844	5 451	5 905	1,011 579	5 608	5 739	1,039 085	5 460	5 895
11	219,0	4,118 628	5 508	5 615	−107	1,040 999	5 394	5 845	1,012 091	5 548	5 683	1,039 085	5 404	5 834
12	384,0	4,362 501	5 393	5 504	−111	1,041 361	5 285	5 732	1,013 222	5 432	5 577	1,039 085	5 297	5 719
13	504,0	4,480 596	5 364	5 454	−90	1,041 555	5 236	5 680	1,013 793	5 379	5 529	1,039 085	5 248	5 667
14	3 000,0	5,255 275	5 200	5 185	15	1,042 920	4 971	5 407	1,017 353	5 096	5 274	1,039 085	4 989	5 387
15	10 520,0	5,800 168	4 975	5 055	−80	1,043 783	4 843	5 276	1,019 322	4 959	5 152	1,039 085	4 865	5 252
16	438 300,0	7,419 923	4 864	4 864	0	1,045 306	4 653	5 084	1,022 458	4 757	4 973	1,039 085	4 681	5 054

NOTE 1 $\lg_{10}$(minutes + 1), x_i.

NOTE 2 The values given in rows 0 and 16 are calculated values, while the values in rows 1 to 15 inclusive are measured values or derived from measured values.



Key

X x = $\lg_{10}$(minutes + 1)

Y stiffness, S

$S\mu_L$ per cent lower confidence interval for line

$S\mu_U$ per cent upper confidence interval for line

S_{hat} Line 2 trend-line, X

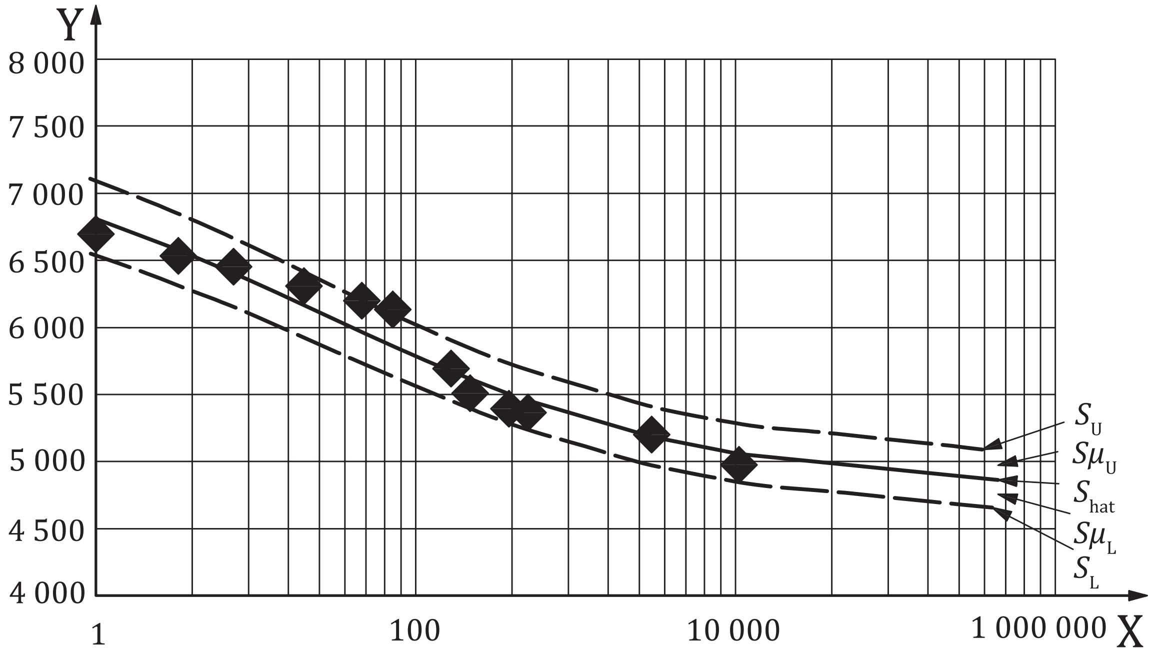
S_L per cent lower prediction interval for future S values

S_U per cent upper prediction interval for future S values

■ data points, X,Y

NOTE See Table B.5 for details of the data used to produce this graph.

Figure B.3 — Plot of stiffness vs. $\lg_{10}$(Time) **for fitted model with confidence and prediction intervals**



Key

X minutes + 1

Y stiffness, *S*

$S\mu_L$ per cent lower confidence interval for line

$S\mu_U$ per cent upper confidence interval for line

S_{hat} Line 2 trend-line, X

S_L per cent lower prediction interval for future *S* values

S_U per cent upper prediction interval for future *S* values

■ data points, X,Y

NOTE See Table B.5 for details of the data used to produce this graph.

Figure B.4 — Plot of stiffness vs. lg of Time in minutes for fitted model with confidence and prediction intervals

Annex C
(normative)

Calculation of lower confidence and prediction limits for method A

C.1 General

The calculation of confidence limits is not required by any of the ISO or CEN test methods or referring standards. However, the calculation of lower confidence limit (LCL) and lower prediction limit (LPL) is required by other standards (ASTM, for example) using the same basic covariant analysis procedures of the test data collected by similar test methods.

C.2 Calculation of quantities and variances

Calculate the quantity B using Formula (C.1):

$$B = -D \times X(1 + E) \tag{C.1}$$

Calculate the variance A of α using Formula (C.2):

$$A = D\left[X^2(1 + E) + Q_{xy} / b\right] \tag{C.2}$$

Calculate the variance, σ_n^2, of the fitted line at x_L using Formula (C.3):

$$\sigma_n^2 = A + Bx_L + Cx_L^2 \tag{C.3}$$

Calculate the error variance, σ_ε^2, using Formula (C.4):

$$\sigma_\varepsilon^2 = 2\Gamma\sigma_\delta^2 \tag{C.4}$$

Calculate the total variance, σ_y^2, or future values of y_L at x_L using Formula (C.5):

$$\sigma_y^2 = \sigma_n^2 + \sigma_\varepsilon^2 \tag{C.5}$$

Calculate the estimated standard deviation, σ_y, for y_L using Formula (C.6):

$$\sigma_y = (\sigma_n^2 + \sigma_\varepsilon^2)^{0,5} \tag{C.6}$$

C.3 Calculation of confidence intervals

Calculate the predicted value y_L for y at x_L using Formula (C.7):

$$y_L = a + bx_L \tag{C.7}$$

where

a and b are as calculated by Formulae (10) and (11) (see 5.2.3).

Calculate the lower 95 % prediction interval $y_{L0,95}$ predicted for y_L using Formula (C.8):

$$y_{L0,95} = y_L - t_v \sigma_y \qquad (C.8)$$

where

t_v is the value from Table 2.

Calculate the corresponding lower 95 % prediction limit for x_L using Formula (C.9):

$$x_{L0,95} = 10^{y_{L0,95}} \qquad (C.9)$$

Setting ${\sigma_y}^2 = {\sigma_n}^2$ in Formula (C.5) will calculate a confidence interval for the regression line, rather than a prediction interval for a future observation.

C.4 Validation of procedures by a sample calculation

The data given in Table 3, analysed in 5.2.6 and summarized in Table 4, are extended for the sample calculation of confidence intervals.

Quantities and variances:

$B = -1{,}469 \times 10^{-5}$

$A = 4{,}667\ 3 \times 10^{-5}$

at 50 years

$\sigma_n{}^2 = 4{,}046\ 6 \times 10^{-5}$

$\sigma_\varepsilon{}^2 = 1{,}160\ 1 \times 10^{-4}$

The estimated values for LCL and LPL are given in Table C.1 (see Table 4).

Table C.1 — Estimated values, V_m, LCL and LPL for V

Time h	V_m	LCL	LPL
0,1	45,76	43,86	42,83
1	42,39	41,05	39,93
10	39,28	38,41	37,16
100	36,39	35,91	34,53
1 000	33,71	33,41	32,03
10 000	31,23	30,79	29,63
100 000	28,94	28,26	27,36
438 000	27,55	26,74	25,98

Bibliography

[1] ISO 7509, *Plastics piping systems — Glass-reinforced thermosetting plastics (GRP) pipes — Determination of time to failure under sustained internal pressure*

[2] ISO 10468, *Glass-reinforced thermosetting plastics (GRP) pipes — Determination of the long-term specific ring creep stiffness under wet conditions and calculation of the wet creep factor*

[3] ISO 10471, *Glass-reinforced thermosetting plastics (GRP) pipes — Determination of the long-term ultimate bending strain and the long-term ultimate relative ring deflection under wet conditions*

[4] ISO 10952, *Plastics piping systems — Glass-reinforced thermosetting plastics (GRP) pipes and fittings — Determination of the resistance to chemical attack for the inside of a section in a deflected condition*

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