INTERNATIONAL STANDARD

ISO 10725

First edition
2000-12-15

Acceptance sampling plans and procedures for the inspection of bulk materials

Plans et procédures d'échantillonnage pour acceptation pour le contrôle de matériaux en vrac

Reference number
ISO 10725:2000(E)

ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.ch
Web www.iso.ch

Printed in Switzerland

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this International Standard may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

International Standard ISO 10725 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 3, *Application of statistical methods in standardization*.

Annexes A and B form a normative part of this International Standard. Annexes C and D are for information only.

Introduction

The application of statistical methods in the field of sampling of bulk materials has been developed since the late 1940s, principally for large quantities of raw materials, such as coals or iron ores, where major interest was to obtain an accurate estimate of the lot mean with reasonable cost, so as to adjust the price and process duly when necessary.

Recently, the need for acceptance sampling of bulk materials has increased especially for industrial products, such as powder chemicals or plastic beads, where the determination of acceptability of a lot is more important than to acquire an accurate estimate of the lot mean. This International Standard has been developed for the former purpose.

The subject of this International Standard is situated on the border line between ISO/TC69/SC 3 dealing with bulk sampling and ISO/TC 69/SC 5 dealing with acceptance sampling, and some SC 5 experts have assisted in the drafting.

Acceptance sampling plans and procedures for the inspection of bulk materials

1 Scope

This International Standard specifies acceptance sampling plans by the determination of variables and use of acceptance inspection procedures for bulk materials. These sampling plans comply with specific operating characteristic curves at reasonable cost.

This International Standard is applicable to the inspection where the lot mean of a single quality characteristic is the principal factor in the determination of lot acceptability, but it also gives special procedures for multiple quality characteristics. This International Standard is applicable to the cases where the values of standard deviations at individual stages of sampling are known or are imprecise.

This International Standard is applicable to various kinds of bulk materials, but is not always applicable to minerals such as iron ores, coals, crude petroleum, etc., where accurate estimation of the lot mean is more important than the determination of lot acceptability.

For special cases when standard procedures are not always adequate and the measurement standard deviation is dominant, this International Standard specifies special acceptance sampling plans and procedures, such as in the case for liquids.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this International Standard. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 2859-1:1999, *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection.*

ISO 3534-1:1993, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.*

ISO 3534-2:1993, *Statistics — Vocabulary and symbols — Part 2: Statistical quality control.*

ISO 5725-1:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 1: General principles and definitions.*

ISO 11648-1:—[1], *Statistical aspects of sampling from bulk materials — Part 1: General principles*.

1) To be published.

3 Terms and definitions

For the purposes of this International Standard, the terms and definitions given in ISO 2859-1, ISO 3534-1, ISO 3534-2, ISO 5725-1 and the following apply.

3.1
acceptance sampling
sampling inspection in which decisions are made to accept or not to accept a lot based on the results of a sample or samples selected from that lot

3.2
acceptance inspection
inspection to determine whether an item or lot delivered or offered for delivery is acceptable

3.3
sampling system
collection of sampling plans, together with criteria by which appropriate sampling plans may be chosen

3.4
sampling plan
combination of sample size and associated acceptability criteria

3.5
sample size
total number of tests or measurements and elements thereof

NOTE 1 In this International Standard, the sample size is, for example, the number of sampling increments in a composite sample, the number of composite samples per lot, the number of test samples prepared from a composite sample, the number of measurements per test sample. The number of measurements is the same as the number of test portions.

NOTE 2 In this International Standard, this term should not be used for sample amount such as the volume or mass of a sampling increment.

3.6
acceptability criteria
criteria or element of the criteria (for instance an acceptance value) for the determination of lot acceptability, i.e. to accept or not to accept a lot

3.7
acceptance quality limit
when a continuing series of lots is considered, a level of the lot mean which for the purposes of sampling inspection is the limit of the satisfactory process average

3.8
non-acceptance quality limit
when a continuing series of lots is considered, a level of the lot mean which for the purposes of sampling inspection is the limit of the unsatisfactory process average

3.9
one-sided specification limit
specification limit of either a lower or an upper limit for the lot mean

3.10
two-sided specification limits
specification limits of both lower and upper limits for the lot mean

3.11
bulk material
amount of material within which component parts are not initially readily distinguishable on the macroscopic level

NOTE This International Standard excludes paper rolls, wire coils, iron scrap or similar materials, because it is difficult to apply the specified sampling procedures.

3.12
sampling increment
amount of bulk material taken in one action by a sampling device

3.13
composite sample
aggregation of two or more sampling increments taken from a lot for inspection of the lot

3.14
test sample
sample, as prepared for testing or analysis, the whole amount or a part of it being used for testing or analysis at one time

3.15
test portion
part of a test sample which is used for testing or for analysis at one time

3.16
acceptance value
limiting value of sample average that permits lot acceptance

3.17
discrimination interval
interval between the acceptance quality limit and the non-acceptance quality limit

3.18
limiting interval
minimum interval between upper and lower acceptance quality limits, when two-sided specification limits are specified

3.19
relative standard deviation
ratio of a standard deviation relative to the discrimination interval

3.20
repeatability
precision under repeatability conditions, i.e. where independent test results are obtained with the same method on identical test items in the same laboratory, by the same operator using the same equipment within short intervals of time

3.21
intermediate precision measurement
precision under intermediate precision conditions, i.e. where test results are obtained with the same method on identical test items in the same laboratory, under some different operating conditions (time, calibration, operator and equipment)

4 Symbols and abbreviated terms

The symbol and the abbreviated terms used in this International Standard are as follows:

C varying cost per lot

C_I sum of costs proportional to total number of sampling increments

C_M sum of costs proportional to total number of measurements

C_T sum of costs proportional to the total number of test samples

c_I cost of drawing a sampling increment

c_M cost of a measurement

c_T cost of preparing a test sample

c_{TM} cost of treating a test sample ($= c_T + n_M c_M$)

D discrimination interval

D_N narrow discrimination interval for multiple characteristics

d_I relative standard deviation between sampling increments ($= \sigma_I / D$)

d_T relative test sample standard deviation ($= \sigma_T / D$)

d_O relative overall standard deviation ($= \sigma_O / D$)

f_D correction factor for multiple characteristics

f_U factor for obtaining upper control limit

G number of lots used for re-estimation of standard deviations

J number of quality characteristics

K_p the upper p-fractile of the standardized normal distribution
(Examples of p are α, β and P_a. For α = 0,05, K_α = 1,644 85. For β = 0,10, K_β,=1,281 55, etc.)

L_{CL} lower control limit

L_{SL} lower specification limit for the lot mean

m lot mean

m_A acceptance quality limit for the lot mean

m_R non-acceptance quality limit for the lot mean

n_I number of sampling increments per composite sample

n_M number of measurements per test sample

n_T number of test samples per composite sample

P_a probability of acceptance

Q_{CR} consumer's risk quality

Q_{PR} producer's risk quality

R_C cost ratio ($= c_{TM} / c_I$)

s_c composite sample standard deviation

s_{cT} combined sample standard deviation

s_{M} measurement standard deviation

s_{T} test sample standard deviation

$t_p(\nu)$ the lower p-fractile of the t-distribution with ν degrees of freedom

U_{SL} upper specification limit for the lot mean

U_{CL} upper control limit

x_{ijk} measured value for the k-th test portion from j-th test sample from the i-th composite sample

$\bar{x}_{...}$ sample grand average

$\bar{x}_L$ lower acceptance value

$\bar{x}_U$ upper acceptance value

α producer's risk

α^* individual producer's risk

β consumer's risk

β^* individual consumer's risk

γ constant for obtaining the acceptance value

Δ interval between the upper and lower acceptance quality limits

δ constant for obtaining the limiting interval

ν degrees of freedom of a standard deviation

ν_{E} degrees of freedom of an estimate standard deviation

σ_{c} composite sample standard deviation

σ_{E} estimate standard deviation for a lot mean

σ_{M} measurement standard deviation

σ_{O} overall standard deviation

σ_{T} test sample standard deviation ($\sigma_{\mathrm{T}}^2 = \sigma_{\mathrm{P}}^2 + \sigma_{\mathrm{M}}^2 / n_{\mathrm{M}}$)

σ_{I}^2 variance component between sampling increments

σ_{M}^2 variance component between measurements

σ_{P}^2 variance component between test samples (variance for test sample preparation).

NOTE 1 The symbols accompanied by a subscript, "L" or "U", denote that they are for the lower or upper specification limit, respectively.

NOTE 2 The symbol σ is used for a population standard deviation, while the symbol s is used for a sample value.

5 Sampling plans

5.1 General

At the beginning of the acceptance sampling, the following items should be established for satisfactory inspection of a lot of bulk material.

5.2 Applicability

5.2.1 Lot mean

This International Standard is applicable when the lot mean of a single quality characteristic is the principal factor in the determination of lot acceptability.

When the material is homogenized through further processing in the consumer's plant, the consumer may be principally interested in the lot mean.

If two or more quality characteristics are specified for a material, then the procedures given in annex A shall be applied. Annex A also provides optional procedures for multiple characteristics to prevent an increase in both the producer's risk and the consumer's risk.

This International Standard is based on the assumption that the lot mean is kept unchanged during acceptance sampling for the lot, or that the expected values of the physical average and the arithmetic mean are equal. Special care is necessary for some unstable characteristics, such as moisture of particulate material. There may be some exceptional cases where this assumption is not true, such as shown in the following example.

EXAMPLE CMC (carboxymethyl cellulose) powder is used as an additive to cement, and in this application one of its most important characteristics is the viscosity of the aqueous solution. If two samples, of equal mass, one having a high value of viscosity and the other a low value, are blended, the viscosity of the blended sample will always be lower than the arithmetic mean of the original two sample values. This International Standard is not applicable to such cases.

5.2.2 Standard deviations

This International Standard is based on the assumption that the values of the individual standard deviation of the specified quality characteristic is known and stable. Guidelines to judge the stability of the individual standard deviation are as follows:

a) in the standard procedure, if both s_{c} and s_{T} control charts have no out-of-control point, and if no other evidence gives doubt about the stability, one can deem that all standard deviations are stable. If σ_{M} is large and unstable, then this fact will probably be detected by the s_{T} control chart. If σ_{M} is sufficiently small, its instability can be neglected, because its precise estimate is unnecessary;

b) in the special procedure in annex B , if the s_{T} control chart has no out-of-control point, and if no other evidence gives doubt about the stability, all standard deviations can be deemed to be stable. In this case, the instability of σ_{I} and σ_{T} can be neglected, because their precise estimates are unnecessary.

However, at the start of acceptance sampling, the precise value and/or the stability of the individual standard deviation may not be sufficiently known. Furthermore, minor and temporary deviation from the stability guidelines given above may occur during application of this acceptance sampling system. In such cases, the procedures for imprecise standard deviations are applicable, where assumed values of standard deviations of the specified quality characteristic are used.

If relevant values of standard deviations are not available at all, this International Standard is not applicable.

5.2.3 Inspection lots

These sampling plans are intended to be used primarily for a continuing series of lots. However, if the requirements for standard deviations are satisfied, these plans may also be used for isolated lots.

5.3 Standardized sampling procedures

5.3.1 General

This International Standard contains the following procedures for inspection of an individual lot:

a) increment sampling;

b) constitution of composite samples;

c) preparation of test samples; and

d) measurements.

Figure 1 illustrates the schematic flow of the above procedures. In order to avoid overcrowding Figure 1, the numbers of unused test samples and test portions drawn are far smaller than the usual values, respectively (see C.2.7).

Representative sampling shall be used throughout the above-mentioned procedures. For example, it is required that individual composite sample can represent the whole lot. In order to obtain reliable results, it is important to specify instructions or standardized procedures. It is recommended that reference be made to ISO 11648-1 beforehand, so that reasonable sampling procedures may be specified.

5.3.2 Increment sampling (see Figure 1)

Take sampling increments of $2n_\mathrm{I}$ from a lot. It is recommended that dynamic sampling be used, where sampling increments are taken from a moving lot. However, the use of static sampling is allowed, where the lot stands still.

It is also recommended that an appropriate sampling device be used. When the material contains coarse lumps, the volume of individual sampling increments should be sufficiently large that representative samples may be obtained.

5.3.3 Constitution of composite samples (see Figure 1)

Pool sampling increments of n_I together and form two composite samples. In this International Standard, two composite samples have been adopted. Each composite sample shall be representive of the whole lot. This requirement may be attained by carrying out systematic duplicate sampling, described as follows:

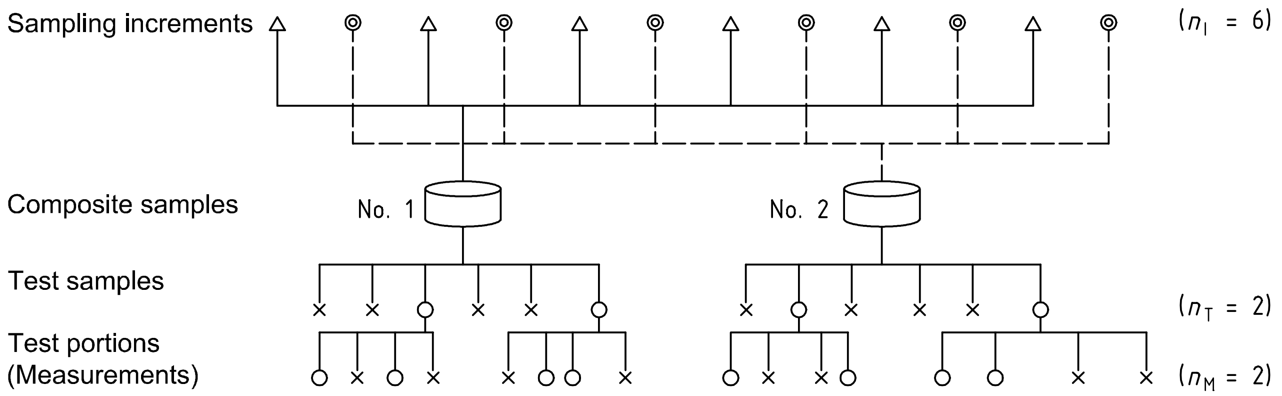
Among $2n_\mathrm{I}$ sampling increments numbered in order, pool those with odd numbers (1, 3, …, $2n_\mathrm{I} - 1$) to form composite sample No. 1, and those with even numbers (2, 4, …., $2n_\mathrm{I}$) to form composite sample No. 2.

5.3.4 Preparation of test samples (see Figure 1)

Prepare n_T test samples from each of the two composite samples. Establish the procedure for test sample preparation beforehand, taking into account the nature of the material to be inspected.

When the material contains coarse lumps, make sure the procedure of the test sample preparation includes one or more stages of particle size reduction (such as crushing and grinding), homogenization (such as mixing) and sample division. The procedure should specify the mass of the test sample and, if necessary, the particle size of the test sample. When the material is liquid, test samples may be taken directly from the composite sample, after sufficient stirring.

NOTE If an adequate procedure for test sample preparation is chosen, then a variance component between test samples, σ_T^2, can be far smaller than the variance component between sampling increments, σ_I^2. On the other hand, economical considerations are also important. For example, grinding of lumps is effective in reducing σ_T^2, but fine grinding of the total amount of composite sample is frequently too expensive.



Key

△ odd numbered sampling increments

◎ even numbered sampling increments

○ used

× not used

Figure 1 — Schematic model of bulk acceptance sampling procedures

5.3.5 **Measurements** (see Figure 1)

Draw test portions of n_M from $2n_T$ test samples respectively, and carry out $2n_Tn_M$ measurements per lot. It is necessary to specify the measurement conditions in detail.

5.4 Standard deviations

5.4.1 General

In the case of imprecise standard deviations, determine the sample standard deviations at various stages in accordance with 6.2.9. In the case of known standard deviations, determine the sample standard deviations at various stages in accordance with 6.2.3. Necessary information common to both cases is given in 5.4.2 to 5.4.4.

If the results of past inspection are available, the estimates of standard deviations can be obtained from them. If knowledge of the standard deviations at their respective stages is insufficient, then it is necessary to obtain relevant values. It is recommended to use one of experimental techniques described in ISO 11648-1 and ISO 11648-2[1]. In many cases a nested experiment is the most suitable.

5.4.2 Standard deviation between sampling increments

The values of the quality characteristics of sampling increments taken from a lot may vary due to differing sources of variation. The standard deviation between sampling increments, σ_I, is the positive square root of the variance component between the sampling increments, σ_I^2.

NOTE If the material is a gas or a low viscosity liquid in one container, the sampling increment standard deviation, σ_I, can be assumed to be zero. However, if the material is a viscous liquid or solid, and if it is in two or more containers, the sampling increment variance, σ_I^2, is usually composed of two components, i.e. the sampling increment variances within the container and between containers.

5.4.3 Standard deviation between test samples

Test samples prepared from a composite sample, in accordance with the specified test sample preparation procedure, may contain variation due to division of the composite sample. The standard deviation between test samples, σ_T, is the positive square root of the variance component between test samples, σ_T^2.

NOTE The standard deviation between test samples, σ_T, composed of solids is not always negligible except when the particle size is sufficiently small. In contrast, in the case of a low viscosity liquid or a gas, σ_T is frequently negligible.

5.4.4 Measurement standard deviation

The results of measurements may vary due to differing sources of variation, including the measurement procedure and the test portion variation within a test sample.

When the total number of measurements is small, the measurement standard deviation usually corresponds to the repeatability. When the total number of measurements is large, it is rather difficult to maintain repeatability conditions, then the appropriate intermediate precision measures shall be used. For more information, see ISO 5725-1, ISO 5725-2[2] and ISO 5725-3[3].

NOTE In this International Standard, the approximate ratio of σ_M/σ_T is more important than σ_M itself. If σ_M/σ_T is sufficiently small (for example, less than 0,2), it is not necessary to know the exact value of σ_M.

5.5 Costs

5.5.1 General

This International Standard uses the following cost values for obtaining the economical sampling plan. When the knowledge of these cost values is insufficient, it is also possible to obtain an applicable sampling plan (see 5.5.6).

5.5.2 Components of costs

The total varying cost per lot, C, consists of the sum of the costs proportional to the total number of sampling increments, to the total number of test samples and to the total number of measurements, as follows:

$$C = C_I + C_T + C_M$$

$$= 2n_I c_I + 2n_T c_T + 2n_T n_M c_M$$

The unit cost values, c_I, c_T and c_M, are used to obtain economical sampling plans.

5.5.3 Cost of taking a sampling increment

The sum of costs proportional to the total number of sampling increments, C_I, contains the following elements:

a) the cost of taking the sampling increments;

b) the cost of pooling to form a composite sample.

The cost of taking one sampling increment, c_I, is given by the following equation:

$$c_I = \frac{C_I}{2n_I}$$

5.5.4 Cost of preparing a test sample

The sum of the cost proportional to the total number of test samples, C_T, contains the following elements:

a) the cost of size reduction and sample division;

b) the cost of preparing test samples.

The cost of preparing a test sample, c_T, is given by the following equation:

$$c_T = \frac{C_T}{2n_T}$$

5.5.5 Cost of a measurement

The cost of a measurement, c_M, is given by the following equation:

$$c_M = \frac{C_M}{2n_T n_M}$$

where the sum of the costs, C_M, is proportional to the total number of measurements.

5.5.6 Procedures for cases when cost values are insufficiently known

At the start of the contract, the knowledge of the above cost values may be insufficient. In such cases, the following procedures should be used in order to obtain an applicable sampling plan.

a) When knowledge of the above cost values is insufficient, assume the approximate ratio, c_I:c_T:c_M, and use each term of the ratio in place of the respective cost value.

   EXAMPLE At the start of the contract, the approximate cost ratio was assumed as follows:

   c_I:c_T:c_M = 3:1:0,5

   Putting c_I = 3, c_T = 1 and c_M = 0,5, the sampling plan was obtained in accordance with the standard procedures.

   After running five lots, the cost ratio was revised as follows:

   c_I:c_T:c_M = 3,5:1:0,4

   Using the new cost values (c_I = 3,5, c_T = 1 and c_M = 0,4), the applicable sampling plan was obtained again, but both the cost ratio level and the sampling plan remained unchanged.

b) If it is difficult to assume the approximate cost ratio, then use the following ratio:

   c_I:c_T:c_M = 1:1:1

5.6 Acceptance quality limit and non-acceptance quality limit

5.6.1 General

The quality measures, the acceptance quality limit, m_A, and the non-acceptance quality limit, m_R, should be specified in accordance with the following procedures.

5.6.2 Interval between m_R and specification limit

It is recommended that the interval between the non-acceptance quality limit and the specification limit ($m_{R,L} - L_{SL}$ or $U_{SL} - m_{R,U}$) be specified taking account of the actual use of an accepted lot. For example, if an accepted lot is divided into sub-lots in actual use, then variation between sub-lots should be considered when determining the interval.

When two-sided specification limits are specified, the two intervals ($m_{R,L} - L_{SL}$ or $U_{SL} - m_{R,U}$) may be different.

This interval can be adjusted to the quality limit of the supplied material. If the quality limit is far from satisfactory, this interval can be increased so that the consumer's risk at the specification limit can be reduced. For this purpose annex D gives useful information. On the contrary, if the quality limit is satisfactory, this interval can be reduced to zero or even can be a negative value.

5.6.3 Discrimination interval

The discrimination interval, D, is the interval between the acceptance quality limit and the non-acceptance quality limit. It is recommended that the value of D be specified, taking account of the values of the standard deviations, σ_I, σ_T and σ_M. If the value of discrimination interval is too small, then this International Standard may not give any applicable sampling plan, and the choice of the acceptance quality limit and/or the non-acceptance quality limit will need to be reconsidered.

When two-sided specification limits, L_{SL} and U_{SL}, are specified, the two discrimination intervals ($m_{A,L} - m_{R,L}$ and $m_{R,U} - m_{A,U}$) shall be equal.

The discrimination interval can be adjusted to the quality limit of the supplied material. If the quality limit is satisfactory, the discrimination interval can be increased to achieve a reduction in costs.

NOTE This interval should be determined mainly from the technical aspects. ISO 10576[4] may give useful information when determining this interval.

5.6.4 Interval between acceptance quality limits

When two-sided specification limits are specified, the interval between the upper and lower acceptance quality limits, Δ, should be equal to or greater than the limiting interval, $\delta \times D$. That is:

$$\Delta = m_{A,U} - m_{A,L} \geqslant \delta \times D$$

For the standard procedures of known standard deviations, $\delta = 0{,}636$, and for the optional procedures, $\delta = 0{,}566$.

For the procedures of imprecise standard deviations, the value of δ can be obtained from Table 1, which is indexed by ν_E. The value of ν_E will be given together with the sample sizes. At the preliminary stage, it is convenient to assume the following interim values:

$$\nu_E = 8 \quad \text{and} \quad \delta = 0{,}566$$

Table 1 — Values of δ for two-sided specification limits (imprecise standard deviations)

ν_E	δ
3,0 to 3,9	0,929
4,0 to 4,9	0,758
5,0 to 5,9	0,670
6,0 to 6,9	0,617
7,0 to 7,9	0,582
$\geqslant$ 8,0	0,566
NOTE The value of δ is used for determining the applicability of the two-sided specification limits.	

5.7 Responsible authority

5.7.1 Functions

The responsible authority has various functions such as:

a) to approve the values of standard deviations;

b) to judge stability of standard deviations;

c) to select between imprecise and known standard deviations;

d) to approve the values of m_A and m_R;

e) to decide whether or not to use optional procedures;

f) other specified or implied functions.

It is desirable that the responsible authority have sufficient knowledge and ability so as to maintain the neutrality of this acceptance sampling system and to smoothly perform the acceptance sampling procedures.

5.7.2 Contractual relationship

The responsible authority may be either of the following:

a) the first party;

b) the second party;

c) the third party; or

d) any of the first, the second or the third party, differing according to function.

The responsible authority should be specified in advance of the acceptance sampling, in the contract or any other relevant document. See also 3.1.12 of ISO 2859-1:1999.

6 Inspection procedures

6.1 General

This clause gives the following procedures:

a) assessment of the standard deviations;

b) determination of sample sizes;

c) selection and preparation of samples;

d) determination of the acceptance value;

e) determination of lot acceptability.

6.2 Assessment of the standard deviations

6.2.1 General

If the results of past inspection are available, the necessary values of standard deviation, σ_I, σ_T and σ_M, can be obtained through the procedures for confirmation and re-estimation given below. If not, see 6.2.9. It is permitted to use the values of standard deviations specified by an agreement between the supplier and the purchaser.

It is especially advantageous to use the values of standard deviations specified by an agreement in the following cases:

a) at the start of the contract, when knowledge of σ values is not always sufficient; or

b) the producer has far more knowledge about the standard deviations than the consumer; or

c) during inspection, when either the s_c or the s_T control chart has given an out-of-control point, but the process has been improved thereafter, and it is believed that the standard deviations have been decreased.

6.2.2 Confirmation procedures

When the procedures are applied to a continuing series of lots, the stability of the standard deviations should be checked lot by lot. For this purpose, it is convenient to use control charts as shown in 6.2.4. If the check results indicate a lack of stability of the standard deviations, then appropriate action should be taken.

On the other hand, values of the population standard deviation should be re-estimated periodically. Unless otherwise specified, the standard deviations should be re-estimated from the results of the immediately preceding 10 lots, and after each five subsequent lots, regardless of lot acceptability. If the standard deviation is re-estimated, then sample sizes, n_M, n_I and n_T, should be re-evaluated. If the re-estimation leads to sample sizes other than the preset ones, then the new sampling plan should be used for the next lot, because the appropriate sample sizes have changed. For additional information, see C.2.3.

6.2.3 Standard deviations

The composite sample standard deviation (sample value, s_c) is obtained from the composite sample averages using the following simpler equation

$$s_c = \sqrt{\frac{(\bar{x}_{1..} - \bar{x}_{2..})^2}{2}}$$

The test sample standard deviation (sample value, s_T) is obtained from the test sample averages using the following equation:

$$s_T = \sqrt{\frac{1}{\nu_T} \sum_{i=1}^{2} \sum_{j=1}^{n_T} (\bar{x}_{ij.} - \bar{x}_{i..})^2}$$

where $\nu_T = 2(n_T - 1)$.

If $n_T = 2$, then the following simpler equation is applicable.

$$s_T = \sqrt{\frac{1}{2} \sum_{i=1}^{2} \frac{(\bar{x}_{i1.} - \bar{x}_{i2.})^2}{2}}$$

If $n_M > 1$, then the measurement standard deviation (sample value, s_M) can be obtained similarly.

$$s_M = \sqrt{\frac{1}{\nu_M} \sum_{i=1}^{2} \sum_{j=1}^{n_T} \sum_{k=1}^{n_M} (\bar{x}_{ijk} - \bar{x}_{ij.})^2}$$

where $\nu_M = 2n_T(n_M - 1)$.

If $n_M = 2$, then the following simpler equation is applicable.

$$s_M = \sqrt{\frac{1}{\nu_M} \sum_{i=1}^{2} \sum_{j=1}^{n_T} \frac{(\bar{x}_{ij1} - \bar{x}_{ij2})^2}{2}}$$

where $\nu_M = 2n_T$.

If the total number of measurements is sufficiently small, then a pocket calculator can be used for the calculation of the above values, otherwise the use of appropriate software is recommended.

6.2.4 Control charts

6.2.4.1 General

Control charts used in this International Standard are a of specific type. They have an upper control limit, U_{CL}, but they do not have a lower control limit, L_{CL} (for more information see C.2.3).

6.2.4.2 s_c control chart

The upper control limit $U_{CL,c}$ of an s_c control chart should be obtained, for each of the 10 lots (or other number of lots used for the check), regardless of lot acceptability, using the following equation:

$$U_{CL,c} = f_U \times \sigma_c$$

where

f_U is a factor given in Table 2, as a function of degrees of freedom ($\nu_c = 1$);

σ_c is a population standard deviation between composite samples.

If none of the standard deviations (sample value, s_c) exceeds the corresponding $U_{CL,c}$, then the s_c control chart may be considered to show a state of control, otherwise it should be considered as showing an out-of-control state.

NOTE 1 Because the number of composite samples is kept to two, the $U_{CL,c}$ is common to all the lots.

NOTE 2 If a sufficient number of preceding inspection results are not available, then σ_c is given by the following equation:

$$\sigma_c = \sqrt{\frac{\sigma_I^2}{n_I} + \frac{\sigma_T^2}{n_T}}$$

6.2.4.3 s_T control chart

The upper control limit, $U_{CL,T}$, of an s_T control chart should be obtained, for each of the 10 lots (or the other number of lots used for the check), regardless of lot acceptability, using the following equation:

$$U_{CL,T} = f_U \times \sigma_T$$

where

f_U is a factor given in Table 2, as a function of degrees of freedom for the test sample, ν_T;

σ_T is a population standard deviation between test samples.

If none of the standard deviations (sample value, s_T) exceeds the corresponding $U_{CL,T}$, then the s_T control chart may be considered to show a state of control, otherwise it should be considered as showing an out-of-control state.

NOTE 1 If n_T is kept constant, then the $U_{CL,T}$ is common to all the lots.

NOTE 2 If a sufficient number of preceding inspection results are not available, then σ_T is given by the following equation:

$$\sigma_T = \sqrt{\sigma_P^2 + \frac{\sigma_M^2}{n_M}}$$

Table 2 — Values of f_U for U_{CL}

ν	f_U	ν	f_U	ν	f_U	ν	f_U
1	2,800	31	1,331	61	1,235	104	1,180
2	2,297	32	1,326	62	1,233	108	1,176
3	2,065	33	1,321	63	1,231	112	1,173
4	1,924	34	1,316	64	1,230	116	1,170
5	1,827	35	1,311	65	1,228	120	1,167
6	1,755	36	1,307	66	1,226	124	1,165
7	1,700	37	1,303	67	1,224	128	1,162
8	1,654	38	1,299	68	1,223	132	1,159
9	1,617	39	1,295	69	1,221	136	1,157
10	1,585	40	1,291	70	1,219	140	1,155
11	1,558	41	1,287	71	1,218	144	1,153
12	1,534	42	1,284	72	1,216	148	1,150
13	1,513	43	1,281	73	1,215	152	1,148
14	1,494	44	1,277	74	1,213	156	1,147
15	1,477	45	1,274	75	1,212	160	1,145
16	1,462	46	1,271	76	1,211	164	1,143
17	1,448	47	1,268	77	1,209	168	1,141
18	1,435	48	1,265	78	1,208	172	1,140
19	1,424	49	1,263	79	1,206	176	1,138
20	1,413	50	1,260	80	1,205	180	1,136
21	1,403	51	1,257	82	1,203	184	1,135
22	1,393	52	1,255	84	1,200	188	1,133
23	1,385	53	1,252	86	1,198	192	1,132
24	1,377	54	1,250	88	1,196	196	1,131
25	1,369	55	1,248	90	1,193	200	1,129
26	1,362	56	1,246	92	1,191	220	1,123
27	1,355	57	1,243	94	1,189	240	1,118
28	1,348	58	1,241	96	1,187	260	1,113
29	1,342	59	1,239	98	1,185	280	1,109
30	1,336	60	1,237	100	1,183	300	1,105

NOTE Where ν is not given, linear interpolation may be used.

6.2.4.4 s_M control chart

If $n_M > 1$, then the upper control limit, $U_{CL,M}$, for an s_M control chart can be calculated similarly.

$$U_{CL,M} = f_U \times \sigma_M$$

where

f_U is a factor given in Table 2, as a function of degrees of freedom for the measurement, ν_M;

σ_M is a population standard deviation of measurement.

If none of the standard deviations (sample values, s_M) exceeds the corresponding $U_{CL,M}$, then the s_M control chart may be considered to show a state of control, otherwise it should be considered as showing an out-of-control state.

NOTE If both of the sample sizes, n_T and n_M, are kept constant, then the $U_{CL,M}$ is common to all the lots.

6.2.5 Re-estimation of population standard deviations

6.2.5.1 General

Re-estimation of population standard deviations shall be made for the immediately preceding G lots. Unless otherwise specified by the responsible authority, G shall be 10. It is recommended that re-estimation be carried out after every five subsequent lots.

6.2.5.2 Constant sample sizes

When sample sizes are constant for i lots, the population standard deviations, σ_c and σ_T, are estimated from the standard deviations (sample values, s_c and s_T) using the following formulae:

$$\sigma_c = \sqrt{\frac{\sum_{i=1}^{G} s_{c,i}^2}{G}}$$

and

$$\sigma_T = \sqrt{\frac{\sum_{i=1}^{G} s_{T,i}^2}{G}}$$

where G is the number of lots used for the re-estimation.

If $n_M > 1$, then the measurement standard deviation, σ_M, can be estimated similarly:

$$\sigma_M = \sqrt{\frac{\sum_{i=1}^{G} s_{M,i}^2}{G}}$$

6.2.5.3 Varying sample sizes

When sample sizes are not constant for G lots, population standard deviations, σ_c and σ_T, are estimated from the standard deviations (sample values, s_c and s_T) using the following equations:

$$\sigma_{\mathrm{c}} = \sqrt{\frac{\sum_{i=1}^{G} \nu_{\mathrm{c},i} \times s_{\mathrm{c},i}^{2}}{\sum_{i=1}^{G} \nu_{\mathrm{c},i}}}$$

and

$$\sigma_{\mathrm{T}} = \sqrt{\frac{\sum_{i=1}^{G} \nu_{\mathrm{T},i} \times s_{\mathrm{T},i}^{2}}{\sum_{i=1}^{G} \nu_{\mathrm{T},i}}}$$

where G is the number of lots used for the re-estimation;

If $n_{\mathrm{M}} > 1$, then the measurement standard deviation, σ_{M}, can be estimated similarly:

$$\sigma_{\mathrm{M}} = \sqrt{\frac{\sum_{i=1}^{G} \nu_{\mathrm{M},i} \times s_{\mathrm{M},i}^{2}}{\sum_{i=1}^{G} \nu_{\mathrm{M},i}}}$$

6.2.6 Re-estimation of variance components

6.2.6.1 General

Population standard deviations, σ_{c} and σ_{T}, vary if the sample sizes, n_{M}, n_{T} or n_{I}, change, whereas σ_{I} and σ_{P} are not affected by the sample size. The variance component between sampling increments, σ_{I}^{2}, and the variance component between test samples, σ_{P}^{2}, are somewhat different from ordinary variances. It is impossible to obtain a variance component directly, but it can be obtained as differences of other variances.

6.2.6.2 Variance component between sampling increments

The variance component between sampling increments, σ_{I}^{2}, is given by the following equation:

$$\sigma_{\mathrm{I}}^{2} = n_{\mathrm{I}}\left(\sigma_{\mathrm{c}}^{2} - \frac{\sigma_{\mathrm{T}}^{2}}{n_{\mathrm{T}}}\right)$$

If $\sigma_{\mathrm{I}}^{2} < 0$, then assume that $\sigma_{\mathrm{I}}^{2} = 0$.

6.2.6.3 Variance component between test samples

If $n_{\mathrm{M}} > 1$, then the variance component between test samples, σ_{P}^{2}, is given by the following equation:

$$\sigma_{\mathrm{P}}^{2} = \sigma_{\mathrm{T}}^{2} - \frac{\sigma_{\mathrm{M}}^{2}}{n_{\mathrm{M}}}$$

If $\sigma_{\mathrm{P}}^{2} < 0$, then assume that $\sigma_{\mathrm{P}}^{2} = 0$;

If $n_{\mathrm{M}} = 1$, then it is not necessary to separate σ_{P}^{2} and σ_{M}^{2}.

6.2.7 Estimate standard deviation

The estimate standard deviation, σ_E, is the standard deviation of the estimate of the lot mean. It is used for obtaining the OC curve. σ_E is given by the following equation:

$$\sigma_E = \sqrt{\frac{\sigma_I^2}{2n_I} + \frac{\sigma_P^2}{2n_T} + \frac{\sigma_M^2}{2n_T n_M}} = \sqrt{\frac{\sigma_I^2}{2n_I} + \frac{\sigma_T^2}{2n_T}}$$

6.2.8 Action for out-of-control

During inspection of a continuing series of lots, if either the s_c or the s_T control chart contains one or more out-of-control points (even after re-estimation of σ and U_{CL}), then some action is necessary. If specific reason(s) have been found and appropriate corrective action is possible, then the corrective action shall be made. Otherwise if the supplier and the purchaser agree, then the following action is applicable, subject to the approval of the responsible authority:

a) to specify adequate value(s) of standard deviation; or

b) to move to optional increased inspection.

If the responsible authority judges that the population standard deviation has become unstable, then the procedures for known standard deviations are no longer applicable.

6.2.9 Imprecise standard deviations

The value of the standard deviations, σ_I, σ_P and σ_M, should be assumed with reference to the most recently available relevant data and the values to be used should be agreed between the supplier and the purchaser.

NOTE If any of the standard deviations appears to be unstable, then a large value should be assumed with reference to the recently available data.

6.3 Determination of sample sizes

6.3.1 Procedures to obtain sample sizes

As it is difficult to obtain all sample sizes at the same time, the following procedures shall be followed:

a) the determination of the economical value of the number of measurements, n_M;

b) the determination of the test sample standard deviation, σ_T;

c) the determination of the relative standard deviations, d_I and d_T;

d) the determination of the cost of treating a test sample, c_T;

e) the determination of the cost ratio, R_C, and the cost ratio level; and

f) the selection of a suitable table and the determination of economical values of n_I and n_T.

6.3.2 Calculation of n_M

6.3.2.1 Use of known standard deviations

For known standard deviations, the first step in determining the economical value for the number of measurements per test sample, n_M, is to calculate an interim value, b, using the following equation:

$$b = \frac{\sigma_M}{\sigma_P}\sqrt{\frac{c_T}{c_M}}$$

Then b should be rounded to an integer, n_M, as follows:

a) If $b < 1{,}5$, then $n_M = 1$;

b) If $1{,}5 \leqslant b < 2{,}5$, then $n_M = 2$;

c) If $b \geqslant 2{,}5$, then $n_M = 3$.

NOTE 1 The above equation gives the most economical value of n_M before rounding, which would minimize c_T.

NOTE 2 The above equation may give a remarkably large value of b. In practice, a large value of n_M is not desirable because generally operators dislike making many measurements on the same test sample, and test results may be less reliable. Therefore, the maximum value of n_M is assumed to be three. If such human factors can be disregarded and a large value of n_M is tolerable, then the value of b calculated using the above equation can be rounded to the nearest integer.

6.3.2.2 Use of imprecise standard deviations

For imprecise standard deviations, the number of measurements per test sample, n_M, should be determined using the following rule:

a) If $\sigma_M/\sigma_T < 0{,}5$, then $n_M = 1$;

b) If $\sigma_M/\sigma_T \geqslant 0{,}5$, then $n_M = 2$.

NOTE In the above rule the effect of cost is negligible.

6.3.3 Test sample standard deviation

The test sample standard deviation, σ_T, should be obtained using the following equation:

$$\sigma_T = \sqrt{\sigma_P^2 + \frac{\sigma_M^2}{n_M}}$$

6.3.4 Relative standard deviations

Standard deviations, σ_I and σ_T, should be converted to the relative standard deviations, d_I and d_T, by dividing by the discrimination interval, D, so that subsequent procedures may be simplified. Thus,

$$d_I = \frac{\sigma_I}{D} \text{ and } d_T = \frac{\sigma_T}{D}$$

6.3.5 Cost of treating a test sample

The cost of treating a test sample, c_{TM}, is given by the following equation:

$$c_{TM} = c_T + n_M c_M$$

6.3.6 Cost ratio and cost ratio level

The cost ratio, R_C, is given by the following equation:

$$R_C = \frac{C_{TM}}{C_I}$$

The cost ratio level should be selected in accordance with the following guideline:

a) level 1: if $R_C \ll 1$ then set R_C = 0,1;

b) level 2: if $R_C < 1$ then set R_C = 0,3;

c) level 3: if $R_C \approx 1$ then set R_C = 1;

d) level 4: if $R_C > 1$ then set R_C = 3;

e) level 5: if $R_C \gg 1$ then set R_C = 10.

6.3.7 Determination of n_I and n_T

6.3.7.1 Structure of tables

Tables 3 to 7 give values for known standard deviations and are provided for determining the number of sampling increments per composite sample, n_I, and the number of test samples per composite sample, n_T. They are indexed by cost ratio level. Each table has two entries, the preferred d_I (relative test sample standard deviation) and the preferred d_T (relative standard deviation between sampling increments). The zone of the actual d_I is shown beside the preferred d_I. The zone of actual d_T is the same as that for the corresponding d_I.

Tables 8 to 12 give values for optional procedures at a risk level of 5 % and have the same structure as those in Tables 3 to 7.

Tables 13 to 22 give values for imprecise standard deviations at a risk level of 5 % separately for $n_M = 1$ and $n_M = 2$. These tables essentially have the same structure as Tables 3 to 7 except that values for ν_E have also been included.

6.3.7.2 Selection of tables

An appropriate table should be selected in accordance with the applicable cost ratio level or combination of n_M and the cost ratio level.

6.3.7.3 Reading tabulated values of n_I and n_T

In the selected table, the values of n_I and n_T should be read as follows:

a) find a line of preferred d_I whose zone (shown in the adjacent column) contains the actual d_I;

b) find a column of preferred d_T whose zone contains the actual d_T. Each zone for the preferred d_T is the same as the preferred d_I, respectively;

c) read n_I and n_T from a cell where the line of preferred d_I crosses the column of preferred d_T.

If any of the following occurs, then move to 6.3.7.4, because either n_I or n_T is too large to be recommended:

— No line of preferred d_I exists.

— No column of preferred d_T exists.

— If "*" is shown in the cell in place of n_I and n_T, then either directly move to 6.3.7.4 or search other sample size tables. If either a subsequent or preceding sample size table for the different cost ratio level gives n_I and n_T for the corresponding cell, these sample sizes are applicable, otherwise move to 6.3.7.4.

6.3.7.4 Reconsideration of the discrimination interval

If it is impossible to obtain n_I and n_T, or if the values obtained for n_I and n_T seem too large, then return to 5.6.3, for reconsideration of the discrimination interval.

6.4 Selection and preparation of samples

6.4.1 General

For inspection of an individual lot, the following procedures should be followed (see Figure 1).

6.4.2 Submission of a lot

As a rule, a production batch of bulk material delivered at one time or offered for delivery shall be submitted as an inspection lot.

6.4.3 Taking of sampling increments

In accordance with the procedure specified in 5.3.2 for taking sampling increments, $2n_I$ increments shall be taken from a lot.

6.4.4 Constitution of composite samples

In accordance with the procedure specified in 5.3.3 for constitution of composite sample, n_I sampling increments shall be pooled together to form a composite sample, and two composite samples shall be formed.

6.4.5 Preparation of test samples

In accordance with the test sample preparation procedure specified in 5.3.4, n_T test samples shall be prepared from each of two composite samples, respectively.

6.4.6 Measurements

In accordance with the measurement procedure specified in 5.3.5, n_M test portions shall be drawn from $2n_T$ test samples, respectively, and $2n_T n_M$ measurements per lot shall be made.

6.4.7 Determination of the lot acceptability

The sample average shall be calculated and the lot acceptability shall be determined in accordance with 6.6.

6.4.8 Disposition of a non-acceptable lot

A non-acceptable lot shall be disposed of in accordance with an agreement made in advance of the acceptance inspection of an individual lot.

Table 3 — Sample sizes ($\alpha \approx$ 5 %, $\beta \approx$ 10 %), cost ratio level 1 for $R_C \approx$ 0,10 (0 to 0,17)

d_I		d_T (preferred value)																											
Preferred value	Zone	0,160		0,200		0,250		0,315		0,400		0,500		0,630		0,800		1,00		1,25		1,60		2,00		2,50		3,15	
		n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T
0,160	0,000 to 0,180	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	4	2	6	2	9	2	14	2	20	2	32	2	48
0,200	0,181 to 0,224	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	4	2	6	2	10	2	15	2	22	2	32	2	50
0,250	0,225 to 0,280	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	7	2	10	2	16	2	22	2	34	2	50
0,315	0,281 to 0,355	2	2	2	2	2	2	2	2	2	2	2	3	2	4	2	6	2	8	2	12	2	17	2	24	2	36	*	
0,400	0,356 to 0,450	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	7	2	9	2	13	2	19	2	26	2	40	*	
0,500	0,451 to 0,560	2	2	2	2	2	2	2	2	2	3	2	4	2	5	2	8	2	11	2	15	2	22	2	32	3	42	*	
0,630	0,561 to 0,710	2	2	2	2	2	2	2	3	2	5	2	6	2	8	2	10	3	12	3	17	3	26	3	36	4	48	*	
0,800	0,711 to 0,900	3	2	3	2	3	3	3	4	3	6	3	7	3	9	4	11	4	15	4	22	4	30	5	38	*		*	
1,00	0,901 to 1,12	5	2	5	3	5	4	5	5	5	6	5	7	5	11	5	14	6	17	6	24	6	34	7	44	*		*	
1,25	1,13 to 1,40	7	3	7	4	7	4	7	6	7	8	8	9	8	12	8	17	8	22	9	26	9	40	10	50	*		*	
1,60	1,41 to 1,80	11	4	11	5	11		12	7	12	9	12	12	12	16	13	20	13	26	14	32	14	46	*		*		*	
2,00	1,81 to 2,24	18	4	18	6	18	7	18	9	18	12	18	15	19	19	19	26	20	30	20	42	22	50	*		*		*	
2,50	2,25 to 2,80	28	6	28	7	28	9	28	11	28	14	28	18	28	24	30	28	30	40	30	50	*		*		*		*	
3,15	2,81 to 3,55	44	7	44	9	44	11	44	14	44	18	44	22	46	28	46	36	46	48	*		*		*		*		*	

NOTE 1 Each zone for the preferred d_T is equal to that of the same preferred d_I, respectively.

NOTE 2 If any of the following occurs, then reconsider the discrimination interval.

a) No line of the preferred d_I exists.

b) No column of the preferred d_T exists.

NOTE 3 If "*" is shown in the cell, then reconsider the discrimination interval, or search other tables, which may give applicable sample sizes.

Table 4 — Sample sizes ($\alpha \approx$ 5 %, $\beta \approx$ 10 %), cost ratio level 2 for $R_C \approx$ 0,32 (0,18 to 0,56)

d_I		d_T (preferred value)																											
Preferred value	Zone	0,160		0,200		0,250		0,315		0,400		0,500		0,630		0,800		1,00		1,25		1,60		2,00		2,50		3,15	
		n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T
0,160	0,000 to 0,180	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	8	2	12	2	19	2	28	2	46
0,200	0,181 to 0,224	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	8	2	13	2	20	2	30	2	46
0,250	0,225 to 0,280	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	4	2	6	2	9	2	14	2	20	2	30	2	50
0,315	0,281 to 0,355	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	4	2	6	2	9	2	14	2	22	2	34	3	50
0,400	0,356 to 0,450	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	7	2	10	2	16	3	22	3	34	4	50
0,500	0,451 to 0,560	2	2	2	2	2	2	2	2	2	2	2	2	2	4	2	6	2	9	3	11	3	17	3	26	4	36	*	
0,630	0,561 to 0,710	2	2	2	2	2	2	2	2	2	3	2	4	3	4	3	6	3	10	4	12	4	19	5	26	5	40	*	
0,800	0,711 to 0,900	3	2	3	2	3	2	4	2	4	3	4	4	4	5	4	8	5	10	5	15	6	20	7	28	8	40	*	
1,00	0,901 to 1,12	5	2	5	2	5	2	5	3	5	4	5	5	6	6	6	9	7	11	7	17	8	24	9	32	10	46	*	
1,25	1,13 to 1,40	7	2	7	2	7	3	8	3	8	4	8	6	9	8	9	11	10	13	10	19	12	26	13	36	14	50	*	
1,60	1,41 to 1,80	12	2	12	3	12	3	12	4	12	6	13	7	14	9	14	12	15	16	16	22	17	30	19	40	*		*	
2,00	1,81 to 2,24	18	3	18	3	18	4	19	5	19	7	19	9	20	11	20	15	22	20	24	24	24	36	26	48	*		*	
2,50	2,25 to 2,80	28	3	28	4	28	5	28	6	30	8	30	11	30	14	32	18	32	24	34	30	36	42	*		*		*	
3,15	2,81 to 3,55	44	4	44	5	44	6	44	8	46	10	46	13	48	17	48	22	50	28	50	36	*		*		*		*	

NOTE See notes of Table 3.

Table 5 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 10$ %), cost ratio level 3 for $R_C \approx 1,0$ (0,57 to 1,7)

d_I		d_T (preferred value)																									
Preferred value	Zone	0,160		0,200		0,250		0,315		0,400		0,500		0,630		0,800		1,00		1,25		1,60		2,00		2,50		3,15	
		n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T
0,160	0,000 to 0,180	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	7	2	12	2	18	2	28	2	44
0,200	0,181 to 0,224	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	7	2	12	2	19	2	30	3	46
0,250	0,225 to 0,280	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	8	2	13	2	20	3	30	4	46
0,315	0,281 to 0,355	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	4	2	5	2	9	3	13	3	20	4	30	5	46
0,400	0,356 to 0,450	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	4	2	6	3	9	3	14	4	20	5	32	6	48
0,500	0,451 to 0,560	2	2	2	2	2	2	2	2	2	2	2	2	2	4	3	4	3	7	4	9	4	15	5	22	6	32	8	50
0,630	0,561 to 0,710	2	2	2	2	2	2	2	2	3	2	4	2	4	4	4	5	4	7	5	10	6	15	7	22	8	34	10	50
0,800	0,711 to 0,900	3	2	3	2	3	2	4	2	4	2	4	3	5	4	6	5	6	8	7	11	8	17	10	24	11	36	*	
1,00	0,901 to 1,12	5	2	5	2	5	2	5	2	6	2	7	3	7	4	8	6	8	9	10	12	11	18	13	26	15	38	*	
1,25	1,13 to 1,40	7	2	7	2	8	2	9	2	9	3	9	4	10	5	11	7	12	10	14	13	15	20	17	28	20	40	*	
1,60	1,41 to 1,80	12	2	12	2	13	2	13	3	14	3	15	4	15	6	17	8	18	11	20	15	22	22	24	32	28	44	*	
2,00	1,81 to 2,24	18	2	19	2	20	2	20	3	20	4	22	5	22	7	24	10	26	13	28	17	32	24	34	34	38	48	*	
2,50	2,25 to 2,80	28	2	30	2	30	3	30	4	32	5	32	6	34	8	36	11	38	15	40	20	44	28	48	38	*		*	
3,15	2,81 to 3,55	44	2	46	3	46	4	46	5	48	6	50	8	50	10	*		*		*		*		*		*		*	
NOTE	See notes of Table 3.																												

Table 6 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 10$ %), cost ratio level 4 for $R_C \approx 3,2$ (1,8 to 5,6)

d_I		d_T (preferred value)																									
Preferred value	Zone	0,160		0,200		0,250		0,315		0,400		0,500		0,630		0,800		1,00		1,25		1,60		2,00		2,50		3,15	
		n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T
0,160	0,000 to 0,180	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	7	2	12	3	18	3	28	4	44
0,200	0,181 to 0,224	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	7	3	12	3	18	4	28	5	44
0,250	0,225 to 0,280	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	3	7	3	12	4	18	5	28	6	44
0,315	0,281 to 0,355	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	4	3	5	3	8	4	12	5	19	6	28	8	44
0,400	0,356 to 0,450	2	2	2		2	2	2	2	2	2	2	2	3	2	3	4	4	5	4	8	6	12	7	19	8	30	10	46
0,500	0,451 to 0,560	2	2	2		2	2	2	2	2	2	2	2	4	2	4	4	5	5	6	8	7	13	9	19	11	30	13	46
0,630	0,561 to 0,710	2	2	2	2	2	2	3	2	3	2	4	2	4	3	5	4	6	6	8	9	9	14	11	20	14	30	17	48
0,800	0,711 to 0,900	3	2	3	2	4	2	4	2	5	2	6	2	6	3	8	4	9	6	11	9	12	14	15	20	18	32	22	48
1,00	0,901 to 1,12	5	2	5	2	6	2	6	2	7	2	9	2	10	3	10	5	11	7	13	10	16	15	20	22	24	32	28	50
1,25	1,13 to 1,40	8	2	8	2	9	2	9	2	10	2	11	3	12	4	15	5	17	7	19	10	22	16	24	24	30	34	36	50
1,60	1,41 to 1,80	12	2	13	2	14	2	14	2	16	2	17	3	19	4	20	6	24	8	26	12	30	17	36	24	42	36	*	
2,00	1,81 to 2,24	19	2	20	2	20	2	22	2	22	3	26	3	26	5	28	7	32	9	36	13	40	19	48	26	*		*	
2,50	2,25 to 2,80	28	2	30	2	30	2	34	2	34	3	36	4	40	5	40	8	46	10	50	14	*		*		*		*	
3,15	2,8 to 3,55	46	2	46	2	50	2	50	3	50	4	*		*		*		*		*		*		*		*		*	
NOTE	See notes of Table 3.																												

Table 7 — Sample sizes ($\alpha \approx$ 5 %, $\beta \approx$ 10 %), cost ratio level 5 for $R_C \approx$ 10 (5,7 or over)

d_I		d_T (preferred value)																									
Preferred value	Zone	0,160		0,200		0,250		0,315		0,400		0,500		0,630		0,800		1,00		1,25		1,60		2,00		2,50		3,15	
		n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T
0,160	0,000 to 0,180	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	3	7	4	11	4	18	6	28	7	44
0,200	0,181 to 0,224	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	5	4	7	5	11	6	18	7	28	9	44
0,250	0,225 to 0,280	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	4	5	4	7	6	11	7	18	9	28	11	44
0,315	0,281 to 0,355	2	2	2	2	2	2	2	2	2	2	2	2	3	2	4	3	5	5	6	7	7	12	9	18	11	28	14	44
0,400	0,356 to 0,450	2	2	2	2	2	2	2	2	2	2	3	2	5	2	6	3	6	5	8	7	9	12	12	18	14	28	18	44
0,500	0,451 to 0,560	2	2	2	2	2	2	3	2	3	2	4	2	6	2	7	3	7	5	9	8	12	12	15	18	18	28	22	44
0,630	0,561 to 0,710	3	2	3	2	3	2	4	2	5	2	5	2	8	2	9	3	11	5	12	8	16	12	19	19	24	28	28	46
0,800	0,711 to 0,900	4	2	4	2	5	2	6	2	7	2	8	2	10	2	11	4	14	5	17	8	20	13	26	19	28	30	36	46
1,00	0,901 to 1,12	6	2	6	2	7	2	8	2	9	2	11	2	12	3	15	4	17	6	22	8	26	13	30	20	40	30	48	46
1,25	1,13 to 1,40	9	2	10	2	10	2	12	2	13	2	15	2	17	3	22	4	24	6	26	9	32	14	42	20	50	30	*	
1,60	1,41 to 1,80	14	2	15	2	16	2	17	2	19	2	22	2	26	3	30	4	34	6	40	9	46	14	50	22	*		*	
2,00	1,81 to 2,24	20	2	22	2	22	2	24	2	26	2	32	2	36	5	38	5	44	7	50	10	*		*		*		*	
2,50	2,25 to 2,80	32	2	32	2	34	2	36	2	40	2	42	3	48	4	*		*		*		*		*		*		*	
3,15	2,81 to 3,55	48	2	50	2	50	2	*		*		*		*		*		*		*		*		*		*		*	

NOTE See notes of Table 3.

Table 8 — Sample sizes ($\alpha \approx$ 5 %, $\beta \approx$ 5 %), cost ratio level 1 for $R_C \approx$ 0,10 (0 to 0,17)

d_I		d_T (preferred value)																								
Preferred value	Zone	0,160		0,200		0,250		0,315		0,400		0,500		0,630		0,800		1,00		1,25		1,60		2,00		2,50	
		n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T
0,160	0,000 to 0,180	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	8	2	11	2	18	2	26	2	40
0,200	0,181 to 0,224	2	2	2	2	2	2	2	2	2	2	2	3	2	4	2	6	2	8	2	12	2	19	2	28	2	42
0,250	0,225 to 0,280	2	2	2	2	2	2	2	2	2	2	2	3	2	4	2	6	2	9	2	13	2	20	2	30	2	44
0,315	0,281 to 0,355	2	2	2	2	2	2	2	2	2	3	2	4	2	5	2	7	2	10	2	15	2	22	2	32	2	46
0,400	0,356 to 0,450	2	2	2	2	2	2	2	2	2	3	2	4	2	6	2	8	2	12	2	17	2	24	2	36	3	50
0,500	0,451 to 0,560	2	2	2	2	2	2	2	3	2	4	2	5	2	7	2	11	2	14	2	20	3	26	3	40	*	
0,630	0,561 to 0,710	2	2	2	3	2	4	2	4	3	5	3	6	3	8	3	12	3	17	3	22	4	30	4	44	*	
0,800	0,711 to 0,900	4	2	4	3	4	4	4	4	4	6	4	9	4	11	5	14	5	19	5	26	6	34	6	50	*	
1,00	0,901 to 1,12	6	3	6	4	6	5	6	6	6	8	6	10	6	13	7	17	7	24	8	28	8	42	*		*	
1,25	1,13 to 1,40	9	4	9	4	9	6	9	7	9	10	10	12	10	15	10	22	11	26	11	36	12	48	*		*	
1,60	1,41 to 1,80	14	5	14	6	15	7	15	9	15	12	15	15	16	19	16	26	17	32	17	44	*		*		*	
2,00	1,81 to 2,24	22	6	22	7	22	9	22	11	24	15	24	18	24	24	24	32	26	38	26	50	*		*		*	
2,50	2,25 to 2,80	3	7	34	9	34	11	34	14	36	18	36	22	36	30	38	36	38	50	*		*		*		*	

NOTE 1 Each zone for the preferred d_T is equal to that of the same preferred d_I, respectively.

NOTE 2 If any of the following occurs, then reconsider the discrimination interval.

a) No line of the preferred d_I exists.

b) No column of the preferred d_T exists.

NOTE 3 If "*" is shown in the cell, then reconsider the discrimination interval, or search other tables, which may give applicable sample sizes.

Table 9 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 5$ %), cost ratio level 2 for $R_C \approx 0,32$ (0,18 to 0,56)

d_I		d_T (preferred value)																									
Preferred value	Zone	0,160		0,200		0,250		0,315		0,400		0,500		0,630		0,800		1,00		1,25		1,60		2,00		2,50	
		n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T
0,160	0,000 to 0,180	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	4	2	6	2	10	2	16	2	24	2	36
0,200	0,181 to 0,224	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	7	2	10	2	16	2	24	2	38
0,250	0,225 to 0,280	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	7	2	11	2	17	2	26	2	40
0,315	0,281 to 0,355	2	2	2	2	2	2	2	2	2	2	2	2	2	4	2	5	2	8	2	12	2	19	2	28	3	42
0,400	0,356 to 0,450	2	2	2	2	2	2	2	2	2	2	2	3	2	4	2	6	2	10	2	14	3	20	3	30	4	44
0,500	0,451 to 0,560	2	2	2	2	2	2	2	2	2	3	2	4	2	6	3	7	3	10	3	15	4	2o	4	32	4	36
0,630	0,561 to 0,710	2	2	2	2	3	2	3	2	3	3	3	5	3	6	4	8	4	12	5	16	5	24	6	34	5	40
0,800	0,711 to 0,900	4	2	4	2	4	2	4	3	4	4	5	5	5	7	5	10	6	13	7	18	7	28	8	38	8	40
1,00	0,901 to 1,12	6	2	6	2	6	3	6	4	7	5	7	6	7	9	8	11	8	16	9	22	10	30	11	42	*	
1,25	1,13 to 1,40	9	2	9	3	10	3	10	4	10	6	10	7	11	10	11	14	12	18	13	24	15	32	16	46	*	
1,60	1,41 to 1,80	15	3	15	3	15	4	16	5	16	7	16	9	17	12	18	16	19	20	20	28	22	38	24	50	*	
2,00	1,81 to 2,24	22	3	22	4	24	5	24	7	24	9	24	11	26	14	26	19	28	24	30	32	32	44	*		*	
2,50	2,25 to 2,80	36	4	36	5	36	6	36	8	38	10	38	13	38	17	40	22	42	30	44	38	46	50	*		*	

NOTE See notes of Table 8.

Table 10 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 5$ %), cost ratio level 3 for $R_C \approx 1,0$ (0,57 to 1,7)

d_I		d_T (preferred value)																									
Preferred value	Zone	0,160		0,200		0,250		0,315		0,400		0,500		0,630		0,800		1,00		1,25		1,60		2,00		2,50	
		n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T
0,160	0,000 to 0,180	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	4	2	6	2	9	2	15	2	24	2	36
0,200	0,181 to 0,224	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	4	2	6	2	10	2	16	2	24	3	36
0,250	0,225 to 0,280	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	4	2	7	2	10	3	16	3	24	4	36
0,315	0,281 to 0,355	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	5	2	7	3	10	3	17	4	26	5	38
0,400	0,356 to 0,450	2	2	2	2	2	2	2	2	2	2	2	2	2	4	3	5	3	8	4	11	4	18	5	26	6	40
0,500	0,451 to 0,560	2	2	2	2	2	2	2	2	2	2	3	3	3	4	4	5	4	8	5	12	6	18	7	26	8	40
0,630	0,561 to 0,710	2	2	2	2	3	2	3	2	4	2	4	3	5	4	5	6	6	8	6	13	8	19	9	28	11	42
0,800	0,711 to 0,900	4	2	4	2	4	2	5	2	5	3	5	4	6	5	7	7	8	10	9	14	10	22	12	30	14	44
1,00	0,901 to 1,12	6	2	6	2	7	2	7	2	8	3	8	4	8	6	10	8	11	11	12	15	14	22	16	32	19	48
1,25	1,13 to 1,40	9	2	10	2	10	2	10	3	11	4	12	5	13	6	14	9	15	12	17	17	19	24	22	36	26	50
1,60	1,41 to 1,80	15	2	16	2	16	3	17	3	18	4	18	6	19	8	22	10	22	14	24	19	28	28	32	38	*	
2,00	1,81 to 2,24	24	2	24	2	24	3	26	4	26	5	26	7	28	9	30	12	32	16	36	22	38	32	42	44	*	
2,50	2,25 to 2,80	36	2	36	3	36	4	38	5	40	6	40	8	42	11	44	14	48	19	50	26	*		*		*	

NOTE See notes of Table 8.

Table 11 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 5$ %), cost ratio level 4 for $R_C \approx 3{,}2$ (1,8 to 5,6)

d_I		d_T (preferred value)																							
Preferred value	**Zone**	0,160		0,200		0,250		0,315		0,400		0,500		0,630		0,800		1,00		1,25		1,60		2,00		2,50	
		n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T
0,160	0,000 to 0,180	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	4	2	6	2	9	3	15	3	22	4	36
0,200	0,181 to 0,224	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	4	2	6	3	9	3	15	4	22	5	36
0,250	0,225 to 0,280	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	4	3	6	3	10	4	15	5	24	6	36
0,315	0,281 to 0,355	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	4	4	6	4	10	5	16	7	24	8	36
0,400	0,356 to 0,450	2	2	2	2	2	2	2	2	2	2	3	2	3	3	4	4	5	7	6	10	7	16	9	24	10	38
0,500	0,451 to 0,560	2	2	2	2	2	2	2	2	3	2	4	2	5	3	5	5	6	7	7	10	9	16	11	24	13	38
0,630	0,561 to 0,710	3	2	3	2	3	2	4	2	4	2	6	2	6	3	7	5	9	7	10	11	12	17	14	26	17	38
0,800	0,711 to 0,900	4	2	5	2	5	2	5	2	6	2	7	3	8	4	10	5	11	8	14	11	16	18	19	26	22	40
1,00	0,901 to 1,12	6	2	7	2	7	2	8	2	10	2	10	3	12	4	13	6	16	8	18	12	20	19	24	28	30	42
1,25	1,13 to 1,40	10	2	10	2	11	2	12	2	14	2	15	3	16	5	18	7	22	9	24	13	28	20	32	30	38	44
1,60	1,41 to 1,80	16	2	16	2	17	2	19	2	20	3	20	4	24	5	28	7	30	10	32	15	38	22	44	32	50	46
2,00	1,81 to 2,24	24	2	24	2	26	2	28	2	30	3	32	4	34	6	38	8	42	11	46	16	50	24	*		*	
2,50	2,25 to 2,80	36	2	38	2	40	2	42	3	44	4	46	5	48	7	50	10	*		*		*		*		*	
NOTE	See notes of Table 8.																								

Table 12 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 5$ %), cost ratio level 5 for $R_C \approx 10$ (5,7 or over)

d_I		d_T (preferred value)																							
Preferred value	**Zone**	0,160		0,200		0,250		0,315		0,400		0,500		0,630		0,800		1,00		1,25		1,60		2,00		2,50	
		n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T	n_I	n_T
0,160	0,000 to 0,180	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	4	3	6	4	9	5	14	6	22	7	34
0,200	0,181 to 0,224	2	2	2	2	2	2	2	2	2	2	2	2	3	2	3	4	4	6	4	9	6	14	7	22	9	34
0,250	0,225 to 0,280	2	2	2	2	2	2	2	2	2	2	2	2	4	2	4	4	5	6	6	9	7	15	9	22	11	34
0,315	0,281 to 0,355	2	2	2	2	2	2	2	2	2	2	3	2	4	2	4	4	6	6	7	9	9	15	11	22	14	36
0,400	0,356 to 0,450	2	2	2	2	2	2	3	2	3	2	4	2	5	3	6	4	8	6	10	9	12	15	15	24	18	36
0,500	0,451 to 0,560	2	2	3	2	3	2	4	2	4	2	5	2	6	3	9	4	10	6	12	10	15	15	18	24	22	36
0,630	0,561 to 0,710	3	2	4	2	4	2	5	2	6	2	7	2	8	3	11	4	13	6	15	10	19	16	24	24	30	36
0,800	0,711 to 0,900	5	2	6	2	6	2	7	2	8	2	11	2	12	3	14	5	17	7	22	10	26	16	32	24	36	38
1,00	0,901 to 1,12	8	2	8	2	9	2	10	2	12	2	14	2	17	3	19	5	24	7	26	11	32	17	38	26	50	38
1,25	1,13 to 1,40	11	2	12	2	13	2	15	2	17	2	20	2	22	3	26	5	28	8	36	11	44	17	50	26	*	
1,60	1,41 to 1,80	18	2	19	2	20	2	22	2	24	2	26	3	30	4	34	6	42	8	48	12	*		*		*	
2,00	1,81 to 2,24	26	2	28	2	30	2	32	2	36	2	40	3	44	4	50	6	*		*		*		*		*	
2,50	2,25 to 2,80	40	2	42	2	44	2	46	2	50	3	*		*		*		*		*		*		*		*	
NOTE	See notes of Table 8.																								

Table 13 — Sample sizes ($\alpha \approx$ 5 %, $\beta \approx$ 5 %) and degrees of freedom for n_M = 1, cost ratio level 1 for $R_C \approx$ 0,10 (0 to 0,17)

d_I		d_T (preferred value)																				
Preferred value	**Zone**	0,160			0,250			0,400			0,630			1,00			1,60			2,50		
		n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E
0,160	0,000 to 0,180	2	2	4,0	2	2	3,4	2	3	5,5	2	4	7,3	2	7	13	2	18	35	2	40	72
0,250	0,181 to 0,280	2	2	3,4	2	2	4,0	2	3	6,0	2	4	8,0	2	8	15	2	20	27	2	44	41
0,400	0,281 to 0,450	3	2	5,5	3	2	6,0	3	3	8,0	3	4	9,9	3	10	18	3	24	26	3	55	33
0,630	0,451 to 0,710	4	2	7,3	4	4	7,8	4	6	9,3	4	9	11	4	17	14	5	32	28	5	65	35
1,00	0,711 to 1,12	7	3	13	7	5	14	7	8	15	8	13	20	8	22	24	9	42	36		*	
1,60	1,13 to 1,80	15	6	29	16	8	32	16	12	34	17	20	40	18	32	49	19	60	60		*	
2,50	1,81 to 2,80	36	7	72	36	12	74	36	19	76	38	30	85	40	48	98		*			*	

NOTE 1 Each zone for the preferred d_T is equal to that of the same preferred d_I, respectively.

NOTE 2 If any of the following occurs, then reconsider the discrimination interval.

a) No line of the preferred d_I exists.

b) No column of the preferred d_T exists.

NOTE 3 If "*" is shown in the cell, then reconsider the discrimination interval, or search other tables, which may give applicable sample sizes.

NOTE 4 The value of v_E is used for two-sided specification limits, multiple quality characteristics or an OC curve.

Table 14 — Sample sizes ($\alpha \approx$ 5 %, $\beta \approx$ 5 %) and degrees of freedom for n_M = 1, cost ratio level 2 for $R_C \approx$ 0,32 (0,18 to 0,56)

d_I		d_T (preferred value)																				
Preferred value	**Zone**	0,160			0,250			0,400			0,630			1,00			1,60			2,50		
		n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E
0,160	0,000 to 0,180	2	2	4,0	2	2	3,4	2	3	5,5	2	4	7,3	2	7	13	2	17	33	2	40	72
0,250	0,181 to 0,280	2	2	3,4	2	2	4,0	2	3	6,0	2	4	8,0	2	8	15	2	18	27	2	44	41
0,400	0,281 to 0,450	3	2	5,5	3	2	6,0	3	3	8,0	3	4	9,9	3	10	18	3	22	27	3	55	33
0,630	0,451 to 0,710	4	2	7,3	4	2	8,0	4	4	10	4	7	12	5	12	23	6	24	43	5	65	35
1,00	0,711 to 1,12	7	2	13	7	4	14	8	5	19	8	9	23	9	16	33	11	30	57	15	55	116
1,60	1,13 to 1,80	16	3	32	16	5	34	17	8	39	18	12	47	20	20	63	22	40	86	26	75	137
2,50	1,81 to 2,80	36	4	74	36	8	75	38	11	85	40	18	97	42	30	114	46	55	147		*	

NOTE See notes of Table 13.

Table 15 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 5$ %) and degrees of freedom for $n_M = 1$, cost ratio level 3 for $R_C \approx 1,0$ (0,57 to 1,7)

d_I		d_T (preferred value)																				
Preferred value	Zone	0,160			0,250			0,400			0,630			1,00			1,60			2,50		
		n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E
0,160	0,000 to 0,180	2	2	4,0	2	2	3,4	2	3	5,5	2	4	7,3	2	7	13	2	16	31	4	36	74
0,250	0,181 to 0,280	2	2	3,4	2	2	4,0	2	3	6,0	2	4	8,0	3	7	14	3	17	35	4	38	79
0,400	0,281 to 0,450	3	2	5,5	3	2	6,0	3	3	8,0	3	4	9,9	4	8	19	5	18	41	7	40	89
0,630	0,451 to 0,710	4	2	7,3	4	2	8,0	4	3	9,9	5	5	16	7	9	27	8	20	51	12	42	103
1,00	0,711 to 1,12	7	2	13	7	3	14	8	4	19	9	7	27	12	11	41	16	22	70	20	48	131
1,60	1,13 to 1,80	16	2	31	17	3	35	18	5	41	20	8	51	22	16	70	28	28	108	38	55	181
2,50	1,81 to 2,80	36	4	74	38	4	79	40	7	89	42	12	103	48	20	131	55	38	181	70	70	276
NOTE See notes of Table 13.																						

Table 16 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 5$ %) and degrees of freedom for $n_M = 1$, cost ratio level 4 for $R_C \approx 3,2$ (1,8 to 5,6)

d_I		d_T (preferred value)																				
Preferred value	Zone	0,160			0,250			0,400			0,630			1,00			1,60			2,50		
		n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E
0,160	0,000 to 0,180	2	2	4,0	2	2	3,4	2	3	5,5	2	4	7,3	2	7	13	3	16	32	4	36	74
0,250	0,181 to 0,280	2	2	3,4	2	2	4,0	2	3	6,0	2	4	8,0	4	7	14	5	16	34	4	36	75
0,400	0,281 to 0,450	3	2	5,5	3	2	6,0	3	3	8,0	4	4	10	5	8	19	8	17	39	11	38	85
0,630	0,451 to 0,710	4	2	7,3	4	2	8,0	4	3	9,9	7	4	12	9	8	23	12	18	47	18	40	97
1,00	0,711 to 1,12	7	2	13	8	2	15	10	3	18	12	5	23	16	9	33	20	20	63	30	42	114
1,60	1,13 to 1,80	17	2	33	18	2	27	22	3	27	24	6	43	30	11	57	40	22	86	55	46	147
2,50	1,81 to 2,80	38	2	70	40	3	74	46	4	65	48	8	90	55	15	116	75	26	137		*	
NOTE See notes of Table 13.																						

Table 17 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 5$ %) and degrees of freedom for $n_M = 1$, cost ratio level 5 for $R_C \approx 10$ (5,7 or over)

d_I		d_T (preferred value)																				
Preferred value	Zone	0,160			0,250			0,400			0,630			1,00			1,60			2,50		
		n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E
0,160	0,000 to 0,180	2	2	4,0	2	2	3,4	2	3	5,5	2	4	7,3	3	7	13	6	15	29	7	36	72
0,250	0,181 to 0,280	2	2	3,4	2	2	4,0	2	3	6,0	4	4	7,8	5	7	14	8	16	32	12	36	74
0,400	0,281 to 0,450	3	2	5,5	3	2	6,0	3	3	8,0	6	4	9,3	8	7	15	12	16	34	19	36	76
0,630	0,451 to 0,710	4	2	7,3	4	2	8,0	5	3	11	9	4	11	13	8	20	20	17	40	30	38	85
1,00	0,711 to 1,12	7	2	13	8	2	15	10	3	18	17	4	14	22	8	24	32	18	49	48	40	98
1,60	1,13 to 1,80	18	2	35	20	2	27	24	3	26	32	5	28	42	9	36	60	19	60		*	
2,50	1,81 to 2,80	40	2	72	44	2	41	55	3	33	65	5	35		*			*			*	
NOTE See notes of Table 13.																						

Table 18 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 5$ %) and degrees of freedom for $n_M = 2$, cost ratio level 1 for $R_C \approx 0,10$ (0 to 0,17)

d_I		d_T (preferred value)																				
Preferred value	**Zone**	0,160			0,250			0,400			0,630			1,00			1,60			2,50		
		n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E
0,160	0,000 to 0,180	2	2	5,3	2	2	6,0	2	2	5,1	2	3	9,0	2	7	24	2	17	56	2	40	112
0,250	0,181 to 0,280	2	2	3,7	2	2	5,3	2	2	5,9	2	4	12	2	8	22	2	20	34	2	44	48
0,400	0,281 to 0,450	3	2	5,8	3	2	7,5	3	3	10	3	5	15	3	10	23	3	24	30	3	50	39
0,630	0,451 to 0,710	4	2	7,5	4	4	7,9	4	6	9,4	4	9	12	4	17	14	5	32	30	5	65	37
1,00	0,711 to 1,12	7	3	13	7	5	14	7	8	15	8	13	21	8	22	25	9	42	37		*	
1,60	1,13 to 1,80	15	6	29	16	8	32	16	12	35	17	20	40	18	32	49	19	60	61		*	
2,50	1,81 to 2,80	36	7	72	36	12	74	36	19	76	38	30	85	40	48	99		*			*	

NOTE See notes of Table 13.

Table 19 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 5$ %) and degrees of freedom for $n_M = 2$, cost ratio level 2 for $R_C \approx 0,32$ (0,18 to 0,56)

d_I		d_T (preferred value)																				
Preferred value	**Zone**	0,160			0,250			0,400			0,630			1,00			1,60			2,50		
		n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E
0,160	0,000 to 0,180	2	2	5,3	2	2	6,0	2	2	5,1	2	3	9,0	2	7	24	2	17	56	2	38	110
0,250	0,181 to 0,280	2	2	3,7	2	2	5,3	2	2	5,9	2	4	12	2	8	22	2	18	36	3	40	111
0,400	0,281 to 0,450	3	2	5,8	3	2	7,5	3	3	10	3	4	14	3	10	23	3	22	32	4	46	81
0,630	0,451 to 0,710	4	2	7,5	4	2	9,0	4	4	10	4	7	13	5	12	27	6	24	52	8	48	117
1,00	0,711 to 1,12	7	2	13	7	3	15	8	5	20	8	9	24	9	16	35	11	30	64	15	55	144
1,60	1,13 to 1,80	16	3	32	16	5	34	17	8	40	18	12	49	20	20	67	22	40	92	26	75	150
2,50	1,81 to 2,80	36	4	74	36	8	76	38	11	86	40	18	99	42	30	118	46	55	153		*	

NOTE See notes of Table 13.

Table 20 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 5$ %) and degrees of freedom for $n_M = 2$, cost ratio level 3 for $R_C \approx 1,0$ (0,57 to 1,7)

d_I		d_T (preferred value)																				
Preferred value	**Zone**	0,160			0,250			0,400			0,630			1,00			1,60			2,50		
		n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E
0,160	0,000 to 0,180	2	2	5,3	2	2	6,0	2	2	5,1	2	3	9,0	2	7	24	2	16	54	3	36	129
0,250	0,181 to 0,280	2	2	3,7	2	2	5,3	2	2	5,9	3	3	9,9	3	7	26	3	17	59	4	38	133
0,400	0,281 to 0,450	3	2	5,8	3	2	7,5	3	2	7,7	3	4	14	4	8	31	5	18	66	7	40	148
0,630	0,451 to 0,710	4	2	7,5	4	2	9,0	4	3	11	5	5	20	6	9	36	8	20	76	12	42	166
1,00	0,711 to 1,12	7	2	13	7	3	15	8	4	21	9	7	30	11	12	51	16	22	103	19	48	189
1,60	1,13 to 1,80	16	2	33	17	3	38	18	5	45	20	8	59	22	16	81	28	28	139	38	55	256
2,50	1,81 to 2,80	36	4	74	38	4	83	40	7	95	42	12	111	48	19	150	55	38	213	70	70	355

NOTE See notes of Table 13.

Table 21 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 5$ %) and degrees of freedom for $n_M = 2$, cost ratio level 4 for $R_C \approx 3,2$ (1,8 to 5,6)

d_I		d_T (preferred value)																				
Preferred value	Zone	0,160			0,250			0,400			0,630			1,00			1,60			2,50		
		n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E
0,160	0,000 to 0,180	2	2	5,3	2	2	6,0	2	2	5,1	2	3	9,0	2	7	24	4	15	54	4	36	132
0,250	0,181 to 0,280	2	2	3,7	2	2	5,3	2	2	5,9	3	3	9,9	3	7	26	5	16	61	8	36	135
0,400	0,281 to 0,450	3	2	5,8	3	2	7,5	3	2	7,7	4	4	17	6	7	29	8	16	65	11	38	150
0,630	0,451 to 0,710	4	2	7,5	4	2	9,0	6	2	10	6	4	20	9	8	39	13	17	76	18	40	169
1,00	0,711 to 1,12	7	2	13	8	2	17	10	3	25	12	5	35	16	9	54	22	19	99	30	42	196
1,60	1,13 to 1,80	17	2	35	18	2	35	22	3	41	24	6	61	30	11	88	40	22	141	55	46	249
2,50	1,81 to 2,80	38	2	77	40	3	84	44	4	88	50	7	108	55	15	157	75	26	214		*	
NOTE See notes of Table 13.																						

Table 22 — Sample sizes ($\alpha \approx 5$ %, $\beta \approx 5$ %) and degrees of freedom for $n_M = 2$, cost ratio level 5 for $R_C \approx 10$ (5,7 or over)

d_I		d_T (preferred value)																				
Preferred value	Zone	0,160			0,250			0,400			0,630			1,00			1,60			2,50		
		n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E	n_I	n_T	v_E
0,160	0,000 to 0,180	2	2	5,3	2	2	6,0	2	2	5,1	2	3	9,0	4	6	19	5	15	53	7	36	131
0,250	0,181 to 0,280	2	2	3,7	2	2	5,3	2	2	5,9	4	3	9,5	5	7	25	8	15	55	12	36	133
0,400	0,281 to 0,450	3	2	5,8	3	2	7,5	4	2	7,7	7	3	10	8	7	28	12	16	63	19	36	138
0,630	0,451 to 0,710	4	2	7,5	4	2	9,0	7	2	10	9	4	20	13	7	31	20	16	67	30	38	154
1,00	0,711 to 1,12	7	2	13	8	2	17	13	2	13	17	4	25	22	8	44	34	17	80	48	40	176
1,60	1,13 to 1,80	17	2	35	20	2	37	24	3	41	34	4	31	42	9	63	55	19	110		*	
2,50	1,81 to 2,80	40	2	80	44	2	62	50	3	60	65	5	62		*			*			*	
NOTE See notes of Table 13.																						

6.5 Determination of the acceptance value

6.5.1 Standard procedures

6.5.1.1 One-sided specification limit

When a lower specification limit, L_{SL}, is specified, the lower acceptance value is given using the following equation:

$$\bar{x}_L = m_A - \gamma \times D = m_A - 0,562 \times D$$

When an upper specification limit, U_{SL}, is specified, the upper acceptance value is given using the following equation:

$$\bar{x}_U = m_A + \gamma \times D = m_A + 0,562 \times D$$

6.5.1.2 Two-sided specification limits

When two-sided specification limits, L_{SL} and U_{SL}, are specified, the lower and the upper acceptance values are given using the following equations:

$$\bar{x}_L = m_{A,L} - \gamma \times D = m_{A,L} - 0{,}562 \times D$$

$$\bar{x}_U = m_{A,U} + \gamma \times D = m_{A,U} + 0{,}562 \times D$$

6.5.2 Optional procedures for 5 % risks and procedures for imprecise standard deviations

a) Set the values of γ and δ as follows:

$$\gamma = 0{,}500$$

$$\delta = 0{,}566$$

b) Set the values of lower and/or upper acceptance values as follows:

$$\bar{x}_L = 0{,}5(m_{A,L} + m_{R,L})$$

$$\bar{x}_U = 0{,}5(m_{A,U} + m_{R,U})$$

6.6 Determination of lot acceptability

6.6.1 Obtaining sample averages

6.6.1.1 Test sample averages

Averages of $2n_T$ test samples, $\bar{x}_{ij.}$, shall be obtained from the n_M measurement results, respectively, using the following equation:

$$\bar{x}_{ij.} = \frac{1}{n_M} \sum_{k=1}^{n_M} x_{ijk}$$

where

x_{ijk} is the result of k-th measurement of j-th test sample of i-th composite sample.

6.6.1.2 Composite sample averages

Two composite sample averages, $\bar{x}_{i..}$, shall be calculated from the n_T test sample averages, respectively, using the following equation:

$$\bar{x}_{i..} = \frac{1}{n_T} \sum_{j=1}^{n_T} \bar{x}_{ij.}$$

6.6.1.3 Sample grand average

The sample grand average, $\bar{x}_{...}$, shall be calculated from two composite sample averages, using the following equation:

$$\bar{x}_{...} = \frac{1}{2} \sum_{i=1}^{2} \bar{x}_{i..}$$

6.6.2 Lot acceptability

Lot acceptability shall be determined in accordance with the following acceptability criteria.

a) When a one-sided lower specification limit, L_{SL}, is specified;

   if $\bar{x}_{...} \geqslant \bar{x}_L$, the lot is acceptable, and

   if $\bar{x}_{...} < \bar{x}_L$, the lot is not acceptable.

b) When a one-sided upper specification limit, U_{SL}, is specified;

   if $\bar{x}_{...} \leqslant \bar{x}_U$, the lot is acceptable, and

   if $\bar{x}_{...} > \bar{x}_U$, the lot is not acceptable.

c) When two-sided specification limits, L_{SL} and U_{SL}, are specified;

   if $\bar{x}_L \leqslant \bar{x}_{...} \leqslant \bar{x}_U$, the lot is acceptable, and

   if either $\bar{x}_{...} < \bar{x}_L$ or $\bar{x}_{...} > \bar{x}_U$, the lot is not acceptable.

7 Examples

7.1 Imprecise standard deviation with one-sided specification limit

An industrial chemical consisting of fine granules will be periodically delivered as either a bulk material or, for each delivery, in a large container. This material will be processed further including homogenization. Therefore, it is desirable to obtain an economical sampling plan, having sufficient assurance for the lot mean.

The characteristic chosen for testing lot acceptability is a physical property, and a lower specification limit, L_{SL} = 90,0 % is specified for the lot mean. Based on the preliminary experiment, standard deviations at their respective stages are assumed as follows:

$$\sigma_I = 4{,}4 ; \quad \sigma_P = 1{,}0 \quad \text{and} \quad \sigma_M = 3{,}0$$

The cost components are as follows: c_I = 25, c_T = 20 and c_M = 60. The quality limits, m_A = 96,0 and m_R = 92,0, are specified. In accordance with the procedures for imprecise standard deviations, the following sampling plan is obtained:

a)	lower specification limit, L_{SL}:	90,0
b)	acceptance quality limit, m_A:	96,0
c)	non-acceptance quality limit, m_R:	92,0
d)	discrimination interval, D:	4,0
e)	lower acceptance value, $\bar{x}_L$: $\bar{x}_L$ = 0,5(m_A + m_R) = 0,5(96,0 + 92,0) =	94,0
f)	standard deviation between sampling increments, σ_I:	4,4
g)	standard deviation between test samples, σ_P:	1,0
h)	measurement standard deviation, σ_M:	3,0
i)	number of composite samples:	2

j) cost of taking an increment, c_I: 25

k) cost of preparing a test sample, c_T: 20

l) cost of a measurement, c_M: 60

m) number of measurements per test sample, n_M: $\sigma_M/\sigma_P = 3{,}0/1{,}0 = 3 \rightarrow n_M =$ 2

n) test sample standard deviation, σ_T: $\sigma_T = \sqrt{\sigma_P^2 + \frac{\sigma_M^2}{n_M}} = \sqrt{1{,}0^2 + \frac{3{,}0^2}{2}} = \sqrt{5{,}50} =$ 2,35

o) cost of treating a test sample, c_{TM}: $c_{TM} = c_T + n_M c_M = 20 + 2 \times 60 =$ 140

p) cost ratio, R_c: $R_c = c_{TM}/c_I = 140/25 =$ 5,60

q) relative standard deviation between sampling increments, d_I: $d_I = \sigma_I/D = 4{,}40/4{,}0 =$ 1,10

r) relative test sample standard deviation, d_T: $d_T = \sigma_T/D = 2{,}35/4{,}0 =$ 0,588

s) table selection: Table 21 ($n_M = 2$, cost ratio level 4)

t) sample sizes, n_I and n_T: $d_I = 1{,}10 \rightarrow 1{,}00$; $d_T = 0{,}588 \rightarrow 0{,}630$: $n_I = 12$, $n_T =$ 5

NOTE 1 The varying cost per lot, C, can be calculated as follows:

$$C = 2(n_I c_I + n_T c_{TM}) = 2(12 \times 25 + 5 \times 140) = 2\,000$$

NOTE 2 The approximate value of the estimate standard deviation, σ_E, can be calculated as follows:

$$\sigma_E = \sqrt{\frac{\sigma_I^2}{2n_I} + \frac{\sigma_T^2}{2n_T}} = \sqrt{\frac{4{,}40^2}{2 \times 12} + \frac{2{,}35^2}{2 \times 5}} = \sqrt{1{,}359} = 1{,}17$$

7.2 Imprecise standard deviation with two-sided specification limits

The conditions are almost the same as those given for 7.1, except that the additional upper specification limit, $U_{SL} = 110{,}0$, is also specified for the lot mean. In this case, the upper specification limit is technically of less importance than the lower specification limit, and hence the quality limits $m_A = 106{,}0$ and $m_R = 110{,}0$ are specified. In accordance with the procedures for imprecise standard deviation, the following sampling plan is obtained.

a) lower specification limit, L_{SL}: 90,0

b) upper specification limit, U_{SL}: 110,0

c) lower acceptance quality limit, $m_{A,L}$: 96,0

d) lower non-acceptance quality limit, $m_{R,L}$: 92,0

e) upper acceptance quality limit, $m_{A,U}$: 106,0

f) upper non-acceptance quality limit, $m_{R,U}$: 110,0

g) discrimination interval, D: 4,0

h) lower acceptance value, $\bar{x}_L$: $\bar{x}_L = 0{,}5(m_{A,L} + m_{R,L}) = 0{,}5(96{,}0 + 92{,}0) =$ 94,0

i) upper acceptance value, $\bar{x}_U$: $\bar{x}_U = 0{,}5(m_{A,U} + m_{R,U}) = 0{,}5(106{,}0 + 110{,}0) =$ 108,0

j) interval between the upper and lower acceptance quality limits, Δ:
$\Delta = m_{A,U} - m_{A,L}$ = 106,0 – 96,0 = 10,0

k) limiting interval, $\delta \times D$ (the interim value): $\delta \times D$ = 0,566 × 4,0 = 2,26 < 10,0

l) standard deviation between sampling increments, σ_I: 4,4

m) standard deviation between test samples, σ_P: 1,0

n) measurement standard deviation, σ_M: 3,0

o) number of composite samples: 2

p) cost of taking a sampling increment, c_I: 25

q) cost of preparing a test sample, c_T: 20

r) cost of a measurement, c_M: 60

s) number of measurements per test sample, n_M: $\sigma_M/\sigma_P = 3{,}0/1{,}0 = 3 \rightarrow n_M =$ 2

t) test sample standard deviation, σ_T: $\sigma_T = \sqrt{\sigma_P^2 + \frac{\sigma_M^2}{n_M}} = \sqrt{1{,}0^2 + \frac{3{,}0^2}{2}} = \sqrt{5{,}50} =$ 2,35

u) cost of treating a test sample, c_{TM}: $c_{TM} = c_T + n_M c_M = 20 + 2 \times 60 =$ 140

v) cost ratio, R_C: $R_C = c_{TM}/c_I = 140/25 =$ 5,60

w) relative standard deviation between sampling increments, d_I: $d_I = \sigma_I/D = 4{,}4/4{,}0 =$ 1,10

x) relative test sample standard deviation, d_T: $d_T = \sigma_T/D = 2{,}35/4{,}0 =$ 0,588

y) table selection: Table 21 (n_M = 2, cost ratio level 4)

z) confirmation of sampling plan:

— sample sizes, n_I and n_T; $d_I = 1{,}10 \rightarrow 1{,}00$; $d_T = 0{,}588 \rightarrow 0{,}630$; $n_I = 12$, $n_T = 5$

— ν_E and δ (Table 1); $\nu_E = 35 \rightarrow \delta = 0{,}566$

— reconfirmation of the limiting interval, $\delta \times D$: $\delta \times D$ = 0,566 × 4,0 = 2,26 < 10,0

NOTE The varying cost and the estimate standard deviation are the same as in 7.1, namely;

— the varying cost, C = 2 000;

— the approximate value of the estimate standard deviation, σ_E = 1,17.

7.3 Optional procedure for known standard deviation with one-sided specification limit

The conditions are almost the same as those given for 7.1, except that standard deviations at their respective stages are known and stable, namely, σ_I = 4,4; σ_P = 1,0 and σ_M = 3,0.

a) lower specification limit, L_{SL}: 90,0

b) acceptance quality limit, m_A: 96,0

c) non-acceptance quality limit, m_R: 92,0

d) discrimination interval, D: 4,0

e) lower acceptance value, $\bar{x}_L$: $\bar{x}_L = 0{,}5(m_{A,L} + m_{R,L}) = 0{,}5(96{,}0 + 92{,}0) =$ 94,0

f) standard deviation between sampling increments, σ_I: 4,4

g) standard deviation between test samples, σ_P: 1,0

h) measurement standard deviation, σ_M 3,0

i) number of composite samples: 2

j) cost of taking a sampling increment, c_I: 25

k) cost of preparing a test sample, c_T: 20

l) cost of a measurement, c_M: 60

m) number of measurements per test sample, n_M: $b = \frac{\sigma_M}{\sigma_P}\sqrt{\frac{c_T}{c_M}} = \frac{3{,}0}{1{,}0}\sqrt{\frac{20}{60}} = 1{,}73 \rightarrow n_M = 2$

n) test sample standard deviation, σ_T: 2,35

o) cost of treating a test sample, c_{TM}: 140

p) cost ratio, R_C: 5,60

q) relative standard deviation between sampling increments, d_I: 1,10

r) relative test sample standard deviation, d_T: 0,588

s) table selection: Table 11 (cost ratio level 4)

t) sample sizes, n_I and n_T: $d_I = 1{,}10 \rightarrow 1{,}00$, $d_T = 0{,}588 \rightarrow 0{,}630$; $n_I = 12$, $n_T = 4$

NOTE 1 The varying cost, C, can be calculated as follows:

$$c = 2(n_I c_I + n_T c_{TM}) = 2(12 \times 25 + 4 \times 140) = 1\ 720$$

NOTE 2 The estimate standard deviation, σ_E, can be calculated using the following equation:

$$\sigma_E = \sqrt{\frac{\sigma_I^2}{2n_I} + \frac{\sigma_T^2}{2n_T}} = \sqrt{\frac{4{,}40^2}{2 \times 12} + \frac{2{,}35^2}{2 \times 4}} = \sqrt{1{,}497} = 1{,}22$$

7.4 Known standard deviation with one-sided specification limit

The conditions are almost the same as those given in 7.3, except that the consumer's risk for m_R is about 10 %.

Standard deviations at their respective stages are known and stable, namely, σ_I = 4,4; σ_P = 1,0 and σ_M = 3,0. The cost components are as follows: c_I = 25, c_T = 20 and c_M = 60. The quality limits, m_A = 96,0 and m_R = 92,0, are specified. In accordance with the procedures for known standard deviation, the following sampling plan is obtained.

a) lower specification limit, L_{SL}: 90,0

b) acceptance quality limit, m_A: 96,0

c) non-acceptance quality limit, m_R: 92,0

d) discrimination interval, D: 4,0

e) lower acceptance value, $\bar{x}_L$: $\bar{x}_L = m_{A,L} - \gamma \cdot D = 96{,}0 - 0{,}562 \times 4{,}0 =$ 93,75

f) standard deviation between sampling increments, σ_I: 4,4

g) standard deviation between test samples, σ_P: 1,0

h) measurement standard deviation, σ_M: 3,0

i) number of composite samples: 2

j) cost of taking a sampling increment, c_I: 25

k) cost of preparing a test sample, c_T: 20

l) cost of a measurement, c_M: 60

m) number of measurements per test sample $b = \frac{\sigma_M}{\sigma_P}\sqrt{\frac{c_T}{c_M}} = \frac{3{,}0}{1{,}0}\sqrt{\frac{20}{60}} = 1{,}73 \rightarrow n_M =$ 2

n) test sample standard deviation, σ_T: $\sigma_T = \sqrt{\sigma_P^2 + \frac{\sigma_M^2}{n_M}} = \sqrt{1{,}0^2 + \frac{3{,}0^2}{2}} = \sqrt{5{,}50} =$ 2,35

o) cost of treating a test sample, c_{TM}: $c_{TM} = c_T + n_M c_M = 20 + 2 \times 60 =$ 140

p) cost ratio, R_C: $R_C = c_{TM}/c_I = 140/25 =$ 5,60

q) relative standard deviation between sampling increments, d_I: $d_I = \sigma_I/D = 4{,}40/4{,}0 =$ 1,10

r) relative test sample standard deviation, d_T: $d_T = \sigma_T/D = 2{,}35/4{,}0 =$ 0,588

s) table selection: Table 6 (cost ratio level 4)

t) sample sizes, n_I and n_T: $d_I = 1{,}10 \rightarrow 1{,}00$; $d_T = 0{,}588 \rightarrow 0{,}630$; $n_I = 10$, $n_T = 3$

NOTE 1 The varying cost, C, can be calculated as follows:

$$c = 2(n_I c_I + n_T c_{TM}) = 2(10 \times 25 + 3 \times 140) = 1\ 340$$

NOTE 2 The estimate standard deviation, σ_E, can be calculated using the following equation:

$$\sigma_E = \sqrt{\frac{\sigma_I^2}{2n_I} + \frac{\sigma_T^2}{2n_T}} = \sqrt{\frac{4{,}40^2}{2 \times 10} + \frac{2{,}35^2}{2 \times 3}} = \sqrt{1{,}888} = 1{,}37$$

7.5 Known standard deviations with two-sided specification limits

The conditions are almost the same as those given for 7.4, except that the additional upper specification limit, $U_{SL} = 110{,}0$, is also specified for the lot mean. In this case, the upper specification limit is technically of less importance than the lower specification limit. Hence the quality limits $m_{A,U}$ = 106,0 and $m_{R,U}$ = 110,0 are specified. In accordance with the procedures for known standard deviations, the following sampling plan is obtained:

a)	lower specification limit, L_{SL}	90,0
b)	upper specification limit, U_{SL}:	110,0
c)	lower acceptance quality limit, $m_{A,L}$:	96,0
d)	lower non-acceptance quality limit, $m_{R,L}$:	92,0
e)	upper acceptance quality limit, $m_{A,U}$:	106,0
f)	upper non-acceptance quality limit, $m_{R,U}$:	110,0
g)	discrimination interval, D:	4,0
h)	lower acceptance value, $\bar{x}_L$: $\bar{x}_L = m_{A,L} - \gamma \cdot D$ = 96,0 − 0,562 × 4,0 =	93,75
i)	upper acceptance value, $\bar{x}_U$: $\bar{x}_U = m_{A,U} + \gamma \cdot D$ = 106,0 + 0,562 × 4,0 =	108,25
j)	interval between the upper and lower acceptance quality limits, Δ: $\Delta = m_{A,U} - m_{A,L}$ =	10,0
k)	limiting interval, $\delta \times D$: $\delta \times D$ = 0,636 × 4,0 =	2,54 < 10,0
l)	standard deviation between sampling increments, σ_I:	4,4
m)	standard deviation between test samples, σ_P:	1,0
n)	measurement standard deviation, σ_M:	3,0
o)	number of composite samples:	2
p)	cost of taking a sampling increment, c_I:	25
q)	cost of preparing a test sample, c_T:	20
r)	cost of a measurement, c_M:	60
s)	number of measurements per test sample (see 7.4), n_M:	2
t)	test sample standard deviation (see 7.2), σ_T:	2,35
u)	cost of treating a test sample, c_{TM}: $c_{TM} = c_T + n_M c_M$ = 20 + 2 × 60 =	140
v)	cost ratio, R_C: $R_C = c_{TM}/c_I$ = 140/25 =	5,60
w)	relative standard deviation between sampling increments, d_I: $d_I = \sigma_I/D$ = 4,40/4,0 =	1,10
x)	relative test sample standard deviation, d_T: $d_T = \sigma_T/D$ = 2,35/4,0 =	0,588
y)	table selection: Table 6 (cost ratio level 4)	
z)	sample sizes, n_I and n_T: d_I = 1,10 → 1,00, d_T = 0,588 → 0,630; n_I = 10, n_T = 3	

NOTE The varying cost and the estimate standard deviation are the same as in 7.4, namely;

a) varying cost, C = 1 340;

b) estimate standard deviation, σ_E = 1,37.

7.6 Revision of discrimination interval

The following application shows how to adjust the discrimination interval for a more economical plan.

In 7.5, five consecutive lots have been acceptable with satisfactory results, and revision of the discrimination interval is investigated so that a more economical sampling plan may be obtained. The procedure is as follows:

a) lower specification limit, L_{SL}: 90,0

b) upper specification limit, U_{SL}: 110,0

c) lower acceptance quality limit, $m_{A,L}$: 97,0

d) lower non-acceptance quality limit, $m_{R,L}$: 91,0

e) upper acceptance quality limit, $m_{A,U}$: 104,0

f) upper non-acceptance quality limit, $m_{R,U}$: 110,0

g) discrimination interval, D: 6,0

h) lower acceptance value, $\bar{x}_L$: $\bar{x}_L = m_{A,L} - \gamma \cdot D$ = 97,0 – 0,562 × 6,0 = 93,63

i) upper acceptance value, $\bar{x}_U$: $\bar{x}_U = m_{A,U} + \gamma \cdot D$ = 104,0 + 0,562 × 6,0 = 107,37

j) interval between the upper and lower acceptance quality limits, Δ: $\Delta = m_{A,U} - m_{A,L}$ = 7,0

k) limiting interval, $\delta \times D$: $\delta \times D$ = 0,636 × 6,0 = 3,82 < 7,0

l) standard deviation between sampling increment, σ_I: 4,4

m) standard deviation between test samples, σ_P: 1,0

n) measurement standard deviation, σ_M: 3,0

o) number of composite samples, 2

p) cost of taking a sampling increment, c_I: 25

q) cost of preparing a test sample, c_T: 20

r) cost of a measurement, c_M: 60

s) number of measurements per test sample (see 7.4), n_M: 2

t) test sample standard deviation (see 7.2), σ_T: 2,35

u) cost of treating a test sample, c_{TM}: $c_{TM} = c_T + n_M c_M$ = 20 + 2 × 60 = 140

v) cost ratio, R_C: $R_C = c_{TM}/c_I$ = 140/25 = 5,60

w) relative standard deviations between sampling increments, d_I:
$d_I = \sigma_I/D$ = 4,40/6,0 = 0,733

x) relative test sample standard deviation, d_T: $d_T = \sigma_T/D$ = 2,35/6,0 = 0,392

y) table selection: Table 6 (cost ratio level 4)

z) sample sizes, n_I and n_T: d_I = 0,733 → 0,800, d_T = 0,392 → 0,400; n_I = 5, n_T = 2

NOTE 1 The varying cost, C, can be calculated as follows:

$$C = 2(n_I c_I + n_T c_{TM}) = 2(5 \times 25 + 2 \times 140) = 810$$

NOTE 2 It is anticipated that the change of discrimination interval from 4,0 to 6,0 (a 1,5 times increase) leads to a cost reduction of $1/1{,}5^2$ (= 0, 444) times. In this case, the actual cost reduction is from 1 340 to 810, or 0,604 times.

NOTE 3 The estimate standard deviation, σ_E, can be calculated using the following equation:

$$\sigma_E = \sqrt{\frac{\sigma_I^2}{2n_I} + \frac{\sigma_T^2}{2n_T}} = \sqrt{\frac{4{,}40^2}{2 \times 5} + \frac{2{,}35^2}{2 \times 2}} = \sqrt{3{,}317} = 1{,}82$$

NOTE 4 It is anticipated that a 1,5 times increase in the discrimination interval leads to a 1,5 times increase in σ_E, also. In this case, the actual change of the estimate standard deviation is from 1,39 to 1,82, or 1,31 times.

7.7 Results from one lot

7.7.1 Determination of acceptability

In accordance with the procedures given in 7.4, the first lot was submitted to acceptance sampling. Results of measurements and sample averages are given in Table 23, n_I = 10, n_T = 3 and n_M = 2. The lot acceptability was determined, and sample standard deviations calculated from the results and compared with the U_{CL} (see Table 23).

Table 23 — Data obtained from one lot

	i =1			i = 2		
	k = 1	k = 2	$\bar{x}_{1j.}$	k = 1	k = 2	$\bar{x}_{2j.}$
j = 1	103,7	106,1	104,90	102,5	99,0	100,75
j = 2	101,9	99,3	100,60	97,3	102,9	100,10
j = 3	97,9	108,7	103,30	101,5	101,5	101,50
$\bar{x}_{i..}$	—	—	102,93	—	—	100,78
$\bar{x}_{...}$	—	—	—	—	—	101,86

7.7.2 Lot acceptability

$$\bar{x}_{...} = 101{,}86 \quad > \quad \bar{x}_L = 93{,}75$$

The lot is acceptable.

7.7.3 Calculation of standard deviations (sample values)

Standard deviations (sample values) were calculated from results of the measurements and sample averages. Degrees of freedom and standard deviations (sample values) were determined as follows:

$$\nu_c = 2-1=1$$
$$\nu_T = 2(n_T - 1) = 2 \times 2 = 4$$
$$\nu_M = 2n_T(n_M - 1) = 2 \times 3 \times 1 = 6$$
$$s_c = \sqrt{\frac{(\bar{x}_{1.} - \bar{x}_{2..})^2}{2}} = \sqrt{\frac{(102,93 - 100,78)^2}{2}} = 1,52$$
$$s_T = \sqrt{\frac{1}{\nu_T}\sum_{i=1}^{2}\sum_{j=1}^{n_T}(\bar{x}_{ij.} - \bar{x}_{i..})^2} = \sqrt{\frac{10,43}{4}} = 1,61$$

where

$$\sum_{i=1}^{2}\sum_{j=1}^{3}(\bar{x}_{ij.} - \bar{x}_{i..})^2 = (104,90 - 102,93)^2 + (100,60 - 102,93)^2 + ... = 10,43$$

and

$$s_M = \sqrt{\frac{1}{\nu_M}\sum_{i=1}^{2}\sum_{j=1}^{n_T}\frac{(\bar{x}_{ij1} - \bar{x}_{ij2})^2}{2}} = \sqrt{\frac{(-2,4)^2 + (2,6)^2 + ... + (-5,6)^2 + (0)^2}{6 \times 2}} = 3,79$$

7.7.4 Comparison with U_{CL}

The following standard deviations were used to obtain U_{CL}:

σ_M = 3,00

σ_T = 2,35 [see 7.4 n)]

and

$$\sigma_c = \sqrt{\frac{\sigma_I^2}{n_I} + \frac{\sigma_T^2}{n_T}} = \sqrt{\frac{4,40^2}{10} + \frac{2,35^2}{3}} = 1,94$$

Values of the U_{CL} and comparison results are as follows:

$U_{CL,c} = f_U \times \sigma_c$ = 2,800 × 1,94 = 5,432 > 1,52

$U_{CL,T} = f_U \times \sigma_T$ = 1,924 × 2,35 = 4,521 > 1,61

and

$U_{CL,M} = f_U \times \sigma_M$ = 1,755 × 3,00 = 5,265 > 3,79

7.8 Results from consecutive lots

7.8.1 Determination of acceptability

After the lot examined in 7.7, nine additional lots were submitted for acceptance sampling. Sample standard deviations for the consecutive 10 lots are given in Table 24.

7.8.2 Comparison with U_{CL}

$U_{CL,c}$ = 5,432 and $U_{CL,T}$ = 4,521 (see 7.7.4). None of the s_c exceeds $U_{CL,c}$ and none of the s_T exceeds $U_{CL,T}$, therefore the number of composite samples, two, will be kept unchanged.

7.8.3 Re-estimation of population standard deviations

σ_C = 1,825, σ_T = 2,229 and σ_M = 2,940.

$$\sigma_I = \sqrt{n_I\left(\sigma_c^2 - \frac{\sigma_T^2}{n_T}\right)} = \sqrt{10\left(1{,}825^2 - \frac{2{,}229^2}{3}\right)} = 4{,}09$$

$$\sigma_P = \sqrt{\sigma_T^2 - \frac{\sigma_M^2}{n_M}} = \sqrt{2{,}229^2 - \frac{2{,}940^2}{2}} = 0{,}804$$

Table 24 — Data of consecutive lots

Lot No.	s_C	s_T	s_M
1	1,52	1,61	3,79
2	2,94	2,36	3,38
3	2,16	3,22	3,02
4	0,521	1,12	1,86
5	1,01	1,52	3,44
6	2,69	2,35	2,45
7	0,843	3,61	2,53
8	1,80	2,16	3,32
9	1,75	1,02	2,85
10	1,46	1,83	2,17
$\sum s^2$	33,29	49,70	86,46
Average of s^2	3,329	4,970	8,646
σ	1,825	2,229	2,940

7.8.4 n_M

$$b = \frac{\sigma_M}{\sigma_P}\sqrt{\frac{c_T}{c_M}} = \frac{2{,}940}{0{,}804}\sqrt{\frac{20}{60}} = 2{,}11 \rightarrow n_M = 2$$

7.8.5 d_I and d_T

$d_I = \sigma_I/D$ = 4,09/4,0 = 1,02

$d_T = \sigma_T/D$ = 2,23/4,0 = 0,558

7.8.6 n_I and n_T

From Table 6 (cost ratio level 4), d_I = 1,02 → 1,00, d_T = 0,558 → 0,500; n_I = 9, n_T = 2

7.8.7 New sampling plan

For the next lot, the new sampling plan (n_I = 9, n_T = 2 and n_M = 2) should be used.

Annex A
(normative)

Special procedures for inspecting multiple characteristics of a material

A.1 General introduction

A material usually has two or more characteristics to be inspected. This annex provides the following procedures for such multiple characteristics:

a) the general procedures; and

b) the optional special procedures to prevent the overall risks.

A.2 General procedures for inspecting multiple characteristics

A.2.1 Composite samples

Unless otherwise specified, use the same composite samples for all the characteristics. If the procedures in this International Standard lead to different numbers of sampling increments per composite sample [n_I, for different characteristics, then the largest one ($n_{\mathrm{I,\,max}}$)] shall be used for the common n_I.

A.2.2 Test samples

In many cases, the same test samples may be used for all the characteristics. If the procedures in this International Standard lead to different numbers of test samples per composite sample, n_T, for different characteristics, the largest number, $n_{\mathrm{T,max}}$, of test samples shall be used for test sample preparation. Then n_T test samples for each characteristic should be selected from $n_{\mathrm{T,max}}$ test samples at random.

Sometimes, different test samples may be required for different characteristics, for example, test samples for particle size are different from those for chemical analysis. In such cases, test samples should be prepared separately. If the test sample preparation procedures are sufficiently simple, test samples may also be prepared separately.

A.2.3 Separate sample preparation

For all samples other than composite samples and test samples, follow the sample preparation procedures for inspecting a single characteristic separately, except when the following optional special procedures apply.

A.3 Overall risks and special procedures

The sampling plans of this International Standard are based on the assumption that a single quality characteristic is to be inspected. If the material has two or more characteristics to be inspected, both the producer's risk and consumer's risk are increased. For two characteristics, both of the overall risks may be doubled. For five characteristics, the overall risks may be about five times higher, and would result in the increased risks not being tolerable.

Special procedures for known standard deviations are given in A.4 and those for imprecise standard deviations are given in A.5. These special procedures provide a handy way to reduce the overall risks to the same levels as for a single characteristic, by means of using a narrow discrimination interval.

It should be noted that sometimes the increased risks may be tolerated, because the same composite samples are used for different characteristics. If different n_I are necessary for inspecting two characteristics, then the larger of the two n_I shall be used for both characteristics, resulting in lower risks for the other characteristic.

A.4 Special procedures for known standard deviations

A.4.1 General procedure for each characteristic

The optional special procedures for each of the characteristics are almost the same as those used when only a single characteristic is inspected. The essential differences are as follows:

a) before determining the sample sizes, find the correction factor, f_D in Table A.1 using the number of characteristics, J. The first row of values in Table A.1 are used for standard procedures ($\alpha \approx 5$ %, $\beta \approx 10$ %) whereas the second row of values are used for optional procedures ($\alpha \approx \beta \approx 5$ %).

Table A.1 — Correction factor, f_D, for J characteristics for known standard deviations

J	2	3	4	5	6	8	10	15	20
$\beta \approx 10$ %	0,816	0,743	0,701	0,672	0,651	0,621	0,600	0,567	0,546
$\beta \approx 5$ %	0,842	0,775	0,736	0,709	0,689	0,661	0,641	0,608	0,588

NOTE 1 First row values are used for standard procedures ($\alpha \approx 5$ %, $\beta \approx 10$ %).

NOTE 2 Second row values are used for optional procedures ($\alpha \approx 5$ %, $\beta \approx 5$ %).

b) convert each discrimination interval, D, to the narrower value, D_N, by multiplying by the correction factor, f_D:

$$D_N = f_D \times D$$

c) obtain the relative standard deviations, d_I and d_T, using D_N for each characteristic;

d) find sample sizes, n_I and n_T, for each characteristic.

Other values, n_M, σ_T, c_T, R_C, acceptance values and the cost ratio level should be kept unchanged. The limiting interval, $\delta \times D$, should also be kept unchanged for simplicity.

The narrower discrimination interval, D_N, leads to the larger sample sizes, n_I and n_T.

The method of converting each discrimination interval to the narrow value is also applicable to the special procedures given in annex B .

A.4.2 Overall adjustment

After obtaining the sampling plan, overall adjustment for each characteristic is necessary.

For composite samples, reconfirm n_I for each characteristic, and use the largest one, $n_{I,max}$, as the common n_I.

As for test samples, if n_I for some characteristic is markedly high when following the procedures described above, then there may be a chance of reducing n_T. The procedure is as follows:

a) move to the next table for the higher cost ratio level;

b) find sample sizes, n_I and n_T, corresponding to d_I and d_T;

c) if $n_I > n_{I,max}$ then this value of n_T cannot be used;

d) if $n_I = n_{I,max}$ then this value of n_T should be used;

e) if $n_I < n_{I,max}$ then this value of n_T can be used, but it may be possible to use a smaller value of n_T. Return to a).

A.4.3 Individual risks and OC curves

The values of the producer's risk and the consumer's risk for each characteristic are given in Table A.2. Information on OC curves for the corrected sampling plans is given in C.7.2.

Table A.2 — Risks at m_A (α*) and at m_R (β*) (for each of J characteristics, in %)

J	**2**	**3**	**4**	**5**	**6**	**8**	**10**	**15**	**20**
$\beta \approx 10$ %	5,13	3,45	2,60	2,09	1,74	1,31	1,05	0,70	0,53
$\beta \approx 5$ %	2,53	1,70	1,27	1,02	0,85	0,64	0,51	0,34	0,26

α* is the individual producer's risk (see C.7)

β* is the individual consumer's risk (see C.7)

NOTE 1 Upper row values are for $\beta \approx 10$ %.

NOTE 2 Lower row values are for $\alpha \approx 5$ %, and are also applicable for $\beta \approx 5$ %.

A.4.4 Example

A.4.4.1 Inspection of three characteristics

The conditions are almost the same as those described in 7.4, except that the material has three characteristics to be inspected. The essential differences are as follows:

a) correction factor, f_D = 0,743 (in Table A.1, J = 3, upper row value);

b) narrow discrimination interval, D_N: $D_N = f_D \times D$ = 0,743 × 4,0 = 2,97;

c) relative standard deviations: $d_I = \sigma_I/D_N$ = 4,40/2,97 = 1,48; $d_T = \sigma_T/D_N$ = 2,35/2,97 = 0,791;

d) sample sizes, n_I and n_T (in Table 6): d_I =1,48 → 1,60, d_T = 0,791 → 0,800; n_I = 20, n_T = 6.

NOTE 1 The common cost: $2n_Ic_I = 2 \times 20 \times 25 = 1\,000$

The proper cost: $2n_Tc_{TM} = 2 \times 6 \times 140 = 1\,680$

NOTE 2 The estimate standard deviation, σ_E, is calculated using the following equation:

$$\sigma_E = \sqrt{\frac{\sigma_I^2}{2n_I} + \frac{\sigma_T^2}{2n_T}} = \sqrt{\frac{4{,}40^2}{2 \times 20} + \frac{2{,}35^2}{2 \times 6}} = \sqrt{0{,}944} = 0{,}972$$

A.4.4.2 Required $n_{I,max}$

Conditions are almost the same as in A.4.4.1, except that another characteristic requires $n_{I,max}$ = 32. The essential differences are as follows:

a) the next table: Table 7;

b) sample sizes (for d_I = 1,60 and d_T = 0,800): n_I = 30, n_T = 4;

c) as $n_{\mathrm{I}} < n_{\mathrm{I,max}}$, $n_{\mathrm{I}} = 32$ and $n_{\mathrm{T}} = 4$ should be used.

NOTE 1 The common cost: $2n_{\mathrm{I}}c_{\mathrm{I}} = 2 \times 32 \times 25 = 1\ 600$

The proper cost: $2n_{\mathrm{T}}c_{\mathrm{TM}} = 2 \times 4 \times 140 = 1\ 120$

NOTE 2 The estimate standard deviation, σ_{E}, is given by the following equation:

$$\sigma_{\mathrm{E}} = \sqrt{\frac{\sigma_{\mathrm{I}}^2}{2n_{\mathrm{I}}} + \frac{\sigma_{\mathrm{T}}^2}{2n_{\mathrm{T}}}} = \sqrt{\frac{4{,}40^2}{2 \times 32} + \frac{2{,}35^2}{2 \times 4}} = \sqrt{0{,}993} = 0{,}996$$

A.5 Special procedures for imprecise standard deviations

A.5.1 For each characteristic

The special procedures for imprecise standard deviations are similar to those given in A.4. The essential differences from A.4 are as follows:

a) obtain ν_{E} for each characteristic;

b) in Table A.3, find the correction factor, f_{D}, for each characteristic, using ν_{E} and the number of characteristics, J;

c) convert each discrimination interval, D, to the narrower value, D_{N}, by multiplying D by the correction factor, f_{D};

d) obtain the relative standard deviations, d_{I} and d_{T}, using D_{N} for each quality characteristic;

e) obtain sample sizes, n_{I} and n_{T}, and ν_{E} for each characteristic;

f) if ν_{E} is too small, return to a), because any change in ν_{E} is not negligible.

A.5.2 Overall adjustment

For overall adjustment, see A.4.2.

A.5.3 Example

The conditions are almost the same as those given in 7.1, except that the material has three characteristics to be inspected. The essential differences are as follows:

a) the initial value of ν_{E}: 35;

b) the correction factor, f_{D}: 0,764 (in Table A.3, $J = 3$, $\nu_{\mathrm{E}} = 30$);

c) the narrower discrimination interval, D_{N}: $D_{\mathrm{N}} = f_{\mathrm{D}} \times D = 0{,}764 \times 4{,}0 = 3{,}06$;

d) relative standard deviations: $d_{\mathrm{I}} = \sigma_{\mathrm{I}}/D_{\mathrm{N}} = 4{,}40/3{,}06 = 1{,}44$; $d_{\mathrm{T}} = \sigma_{\mathrm{T}}/D_{\mathrm{N}} = 2{,}35/3{,}06 = 0{,}768$;

e) sample sizes, n_{I} and n_{T} (in Table 21): $d_{\mathrm{I}} = 1{,}44 \rightarrow 1{,}60$, $d_{\mathrm{T}} = 0{,}768 \rightarrow 1{,}00$; $n_{\mathrm{I}} = 30$, $n_{\mathrm{T}} = 11$.

NOTE 1 The common cost: $2n_{\mathrm{I}}c_{\mathrm{I}} = 2 \times 30 \times 25 = 1\ 500$

The proper cost: $2n_{\mathrm{T}}c_{\mathrm{TM}} = 2 \times 11 \times 140 = 3\ 080$

NOTE 2 The estimate standard deviation, σ_{E}, is given by the following equation:

$$\sigma_{\mathrm{E}} = \sqrt{\frac{\sigma_{\mathrm{I}}^2}{2n_{\mathrm{I}}} + \frac{\sigma_{\mathrm{T}}^2}{2n_{\mathrm{T}}}} = \sqrt{\frac{4{,}40^2}{2 \times 30} + \frac{2{,}35^2}{2 \times 11}} = \sqrt{0{,}574} = 0{,}757$$

Table A.3 — Correction factor, f_D, for J characteristics for imprecise standard deviations

ν_E	J								
	2	**3**	**4**	**5**	**6**	**8**	**10**	**15**	**20**
2	0,683	0,552	0,475	0,424	0,386	0,333	0,298	0,243	0,210
3	0,743	0,633	0,568	0,522	0,488	0,440	0,406	0,352	0,318
4	0,771	0,673	0,614	0,573	0,542	0,497	0,466	0,415	0,383
5	0,787	0,696	0,640	0,602	0,573	0,532	0,503	0,455	0,425
6	0,797	0,710	0,658	0,621	0,594	0,555	0,527	0,481	0,452
7	0,804	0,720	0,670	0,635	0,609	0,571	0,544	0,500	0,472
8	0,809	0,728	0,679	0,645	0,619	0,583	0,557	0,514	0,487
9	0,813	0,733	0,686	0,653	0,628	0,592	0,567	0,525	0,499
10	0,816	0,738	0,691	0,659	0,634	0,599	0,574	0,534	0,508
12	0,821	0,745	0,699	0,668	0,644	0,610	0,586	0,547	0,522
14	0,824	0,749	0,705	0,674	0,651	0,617	0,594	0,556	0,532
16	0,826	0,753	0,709	0,678	0,656	0,623	0,600	0,563	0,539
20	0,829	0,757	0,714	0,685	0,663	0,631	0,608	0,572	0,549
24	0,831	0,760	0,718	0,689	0,667	0,636	0,614	0,578	0,555
30	0,834	0,764	0,722	0,693	0,672	0,641	0,619	0,584	0,562
40	0,836	0,767	0,726	0,697	0,676	0,646	0,625	0,590	0,568
60	0,838	0,770	0,729	0,701	0,681	0,651	0,630	0,596	0,575
120	0,840	0,773	0,733	0,705	0,685	0,656	0,635	0,602	0,581
300	0,841	0,774	0,735	0,708	0,688	0,659	0,638	0,606	0,585

Annex B
(normative)

Acceptance sampling plans and procedures for use where the measurement standard deviation is dominant

B.1 General introduction

In some special cases, where the measurement standard deviation is dominant, standard procedures are not always adequate. This annex provides acceptance sampling plans and procedures for such special cases.

This annex is applicable when both the sampling increment standard deviation, σ_I, and the standard deviation between test samples, σ_P, are far less than the measurement standard deviation, σ_M, which is known and stable.

EXAMPLES

— a low viscosity liquid in a single container or without a container;

— a physical or biological test for which the σ_M is extremely large.

B.2 Standard deviations

B.2.1 Respective standard deviations

This annex is applicable when the value of the measurement standard deviation of a specified quality characteristic, σ_M, is dominant.

It is not always necessary to know the precise values of σ_I and σ_P, but it is sufficient that both be far smaller than σ_M. The guidelines are as follows:

a) $\sigma_I < 0{,}1\sigma_M$ and $d_I < 0{,}1$;

b) $\sigma_P < 0{,}1\sigma_M$.

If either of the above guidelines is not satisfied, then this annex is not applicable and the standard procedures should be used.

B.2.2 Overall standard deviation

After obtaining sample sizes, the overall standard deviation, σ_O, is given by the following equations:

a) when $n_T > 1$,

$$\sigma_O = \sqrt{\frac{n_T n_M}{n_I}\sigma_I^2 + n_M\sigma_P^2 + \sigma_M^2}$$

b) when $n_T = 1$ (and $n_I = 1$),

$$\sigma_O = \sqrt{n_M \sigma_P^2 + \sigma_M^2}$$

At a preliminary investigation, σ_O can be assumed to be $1{,}2\sigma_M$.

B.2.3 Relative overall standard deviation

The overall standard deviation should be converted to a relative one, by dividing by D.

$$d_O = \sigma_O / D$$

B.3 Costs

No knowledge of costs shall be necessary.

B.4 Sample sizes

B.4.1 Number of composite samples

The number of composite samples shall be two.

B.4.2 Determination of n_T and n_M

From Table B.1, the number of test samples per composite sample, n_T, and the number of measurements per test sample, n_M, are obtained from the row corresponding to the calculated value of d_O.

Table B.1 — Sample sizes for special procedures (known standard deviations; $\alpha \approx 5$ %, $\beta \approx 10$ %)

d_O		n_T	n_M	$n_T n_M$
0,683	0,000 to 0,760	1	2	2
0,837	0,761 to 0,901	1	3	3
0,967	0,902 to 1,075	2	2	4
1,184	1,076 to 1,316	2	3	6
1,450	1,317 to 1,561	3	3	9
1,674	1,562 to 1,772	4	3	12
1,872	1,773 to 1,960	5	3	15
2,050	1,961 to 2,132	6	3	18
2,215	2,133 to 2,291	7	3	21
2,367	2,292 to 2,439	8	3	24
2,511	2,440 to 2,579	9	3	27
2,647	2,580 to 2,711	10	3	30

If no line of preferred d_O exists, then return to 5.6.3, because the total number of measurements, $2n_T n_M$ is too large to be practical and reconsideration of the discrimination interval is necessary.

NOTE Table B.1 gives an economical solution to the value of $n_T n_M$. A large value of n_M is not desirable for practical reasons. This annex assumes that the maximum value of n_M is three. However, for a special case where a larger value of n_M is tolerable, another sampling plan having the same value of $n_T n_M$ can be used.

For optional procedures for $\alpha \approx \beta \approx 5$ %, use Table B.2 in place of Table B.1.

In this case, set the values of γ and δ as follows:

$\gamma = 0{,}500$
$\delta = 0{,}566$

In this case, change the values of the lower and/or upper acceptance values as follows:

$\bar{x}_L = 0{,}5(m_{A,L} + m_{R,L})$
$\bar{x}_U = 0{,}5(m_{A,U} + m_{R,U})$

Table B.2 — Sample sizes for special procedures (known standard deviations; $\alpha \approx 5$ %, $\beta \approx 5$ %)

d_O		n_T	n_M	$n_T n_M$
0,608	0,000 to 0,676	1	2	2
0,745	0,677 to 0,802	1	3	3
0,860	0,803 to 0,956	2	2	4
1,053	0,957 to 1,171	2	3	6
1,290	1,172 to 1,389	3	3	9
1,489	1,390 to 1,577	4	3	12
1,665	1,578 to 1,744	5	3	15
1,824	1,745 to 1,896	6	3	18
1,970	1,897 to 2,038	7	3	21
2,106	2,039 to 2,169	8	3	24
2,234	2,170 to 2,294	9	3	27
2,355	2,295 to 2,412	10	3	30

For special procedures for imprecise standard deviations when $\alpha \approx \beta \approx 5$ %, use Table B.3.

NOTE Table B.3 also gives the value of ν_E, which is used when two-sided specification limits are specified.

Table B.3 — Sample sizes for special procedures (imprecise standard deviations; $\alpha \approx 5$ %, $\beta \approx 5$ %)

d_O		n_T	n_M	$n_T n_M$	ν_E
0,425	0,000 to 0,470	1	2	2	3
0,608	0,471 to 0,642	1	3	3	5
0,746	0,643 to 0,800	2	2	4	7
0,964	0,801 to 1,028	2	3	6	11
1,219	1,029 to 1,271	3	3	9	17
1,429	1,272 to 1,474	4	3	12	23
1,612	1,475 to 1,652	5	3	15	29
1,776	1,653 to 1,813	6	3	18	35
1,925	1,814 to 1,960	7	3	21	41
2,065	1,961 to 2,097	8	3	24	47
2,195	2,098 to 2,225	9	3	27	53
2,318	2,226 to 2,346	10	3	30	59
2,434	2,347 to 2,462	11	3	33	65
2,546	2,463 to 2,572	12	3	36	71

NOTE The value of ν_E is used for two-sided specification limits, multiple quality characteristics or an OC curve.

B.4.3 n_I

The number of sampling increments per composite sample, n_I, is usually two. If $n_T = 1$, then $n_I = 1$, also.

B.5 Confirmation of standard deviations

B.5.1 General

If $n_T = 1$ (and $n_I = 1$), then the following special provisions should be applied.

B.5.2 Combined sample standard deviation

If $n_T = 1$, then the sample standard deviations, s_c and s_T, cannot be separated, and a combined sample standard deviation (sample value, s_{cT}) should be obtained from the test sample average given in 6.2.3, by the following simpler equation (for $n_T = 1$, $n_I = 1$ and $\nu_{cT} = 1$):

$$s_{cT} = \sqrt{\frac{\left(\bar{x}_{11.} - \bar{x}_{21.}\right)^2}{2}}$$

B.5.3 Control charts

If $n_T = 1$, then the s_{cT} control chart should be used instead of the s_c and s_T control charts.

B.5.4 Combined variance between test samples

If $n_T = 1$, then the combined variance between test samples, σ_{IP}^2 is given by the following equation:

$$\sigma_{IP}^2 = \sigma_{cT}^2 - \frac{\sigma_M^2}{n_M}$$

B.6 Estimate standard deviation

The estimate standard deviation, σ_E, is given by the following equation:

$$\sigma_E = \frac{\sigma_O}{\sqrt{2 n_T n_M}}$$

B.7 Examples

B.7.1 Known standard deviation

An industrial chemical is produced by a batch process and delivered periodically. It is known that the within-batch variation is negligible, and that the measurement standard deviation, σ_M, is relatively large. Each delivery corresponds to a production batch. The upper specification limit ($U_{SL} = 90{,}0$) is specified for the lot mean. The overall standard deviation, σ_O, is known and stable, and $\sigma_O = 3{,}50$. The quality measures $m_A = 86{,}0$ and $m_R = 90{,}0$ are specified. In accordance with the procedures of this annex, a sampling plan is obtained:

a) upper specification limit, U_{SL}: 90,0

b) acceptance quality limit, m_A: 86,0

c) non-acceptance quality limit, m_R: 90,0

d) discrimination interval, D: 4,0

e) upper acceptance value, $\bar{x}_U$: $\bar{x}_U = m_A + \gamma \cdot D$ = 86,0 + 0,562 × 4,0 = 88,25

f) overall standard deviation, σ_O: 3,50

g) relative overall standard deviation, d_O: $d_O = \sigma_O / D$ = 3,50/4,0 = 0,875

h) number of composite samples: 2

i) table selection: Table B.1

j) sample sizes, n_T and n_M: d_O = 0,875 → 0,837; n_T = 1, n_M = 3

k) number of sampling increments, n_I: 1

NOTE The estimate standard deviation, σ_E, is given by the following equation:

$$\sigma_E = \frac{\sigma_O}{\sqrt{2 n_T n_M}} = \frac{3,50}{\sqrt{2 \times 1 \times 3}} = 1,43$$

B.7.2 Imprecise standard deviation

Conditions are the same as B.7.1 except the measurement standard deviation is imprecise:

a) upper specification limit, U_{SL}: 90,0

b) acceptance quality limit, m_A: 86,0

c) non-acceptance quality limit, m_R: 90,0

d) discrimination interval, D: 4,0

e) upper acceptance value, $\bar{x}_U$: $\bar{x}_U = 0,5(m_A + m_R)$ = 0,5(86,0 + 90) = 88,0

f) overall standard deviation, σ_O: 3,50

g) relative overall standard deviation, d_O: $d_O = \sigma_O / D$ = 3,50/4,0 = 0,875

h) number of composite samples: 2

i) table selection: Table B.3

j) sample sizes, n_T and n_M: d_O = 0,875 → 0,964; n_T = 2, n_M = 3

k) number of sampling increments, n_I: 2

NOTE The approximate value of the estimate standard deviation is calculated by the following equation:

$$\sigma_E = \frac{\sigma_O}{\sqrt{2 n_T n_M}} = \frac{3,50}{\sqrt{2 \times 2 \times 3}} = 1,01$$

Annex C
(informative)

Theoretical background

C.1 General introduction

This annex describes the threoretical background of the procedures for lot inspection using known standard deviations. The procedures are the basis of this International Standard. In C.8, supplementary information of the procedures for inspection for imprecise standard deviations is also given.

C.2 Basic assumptions

C.2.1 General

The procedures for known standard deviations are based on the following assumptions:

a) the specified quality characteristic x is a variable and is measurable on a continuous scale;

b) each standard deviation of x is known and stable;

c) the expected value of the physical average and the arithmetic mean are equal;

d) the averages of x are normally distributed;

e) each composite sample represents the lot;

f) the measurements are carried out in a single laboratory;

g) the population is infinite;

h) the population is simple;

i) a single quality characteristic is considered at one time.

The following clauses give additional information on these assumptions.

C.2.2 "Known" standard deviations

The sampling plans of the standard procedures are based on the assumption of "known" standard deviations. This assumption is hardly satisfied for isolated lots. However, an isolated lot for inspection for the purchaser may be one from a continuous series of production batches of the supplier. In such a case, if the supplier provides sufficient information including control charts to the purchaser, it may be possible to assume "known and stable" standard deviations.

C.2.3 Control charts

Applicability of the procedures for known standard deviations is judged by means of control charts of a specific type. They have an upper control limit, U_{CL}, but they lack a lower control limit, L_{CL}. The reason is as follows.

It is desirable to make efforts to reduce standard deviations, especially when two-sided specification limits are specified. In order to assist these efforts, a version of the one-sided test is applied in this International Standard. The test is an F-test for the following hypotheses:

H_0: σ^2 is stable;

H_1: σ^2 is not stable (the maximum value of the variances is out of the random variation).

Because many users are not familiar with the F-test for variances, it is converted to an equivalent test of control chart type so that standard deviations may be used. The level of risk of 5 % is assumed for each control chart, where the risk is the probability of finding one or more out-of-control points in a series of 10 lots. The factor f_U is given by the following equations:

$$f_U = \sqrt{F_p(\nu, \infty)} \tag{C.1}$$

$$p = \sqrt[10]{0{,}95} = 0{,}994\ 88 \tag{C.2}$$

where F_p (ν_1, ν_2) is the lower p-fractile of the F-distribution with ν_1 and ν_2 degrees of freedom.

C.2.4 Normality

The sampling plans are based on the assumption of normality. However, users usually need not be too concerned about deviation from the normal distribution, because the distribution of the sample grand average is usually very close to a normal distribution, unless sample sizes are very small. This is one of the major differences from other popular sampling plans by variables for percent nonconforming such as ISO 3951[5], where deviation from the normal distribution may bring about an increase or decrease of the producer's risk and/or the consumer's risk, and so the assumption of normality is important in practice.

C.2.5 Representative sampling

The sampling plans are based on the assumption of representative sampling. Though the simplest way of representative sampling is random sampling, systematic duplicate sampling is used in this International Standard.

The systematic sampling shown in the upper half of Figure 1 may give a smaller variance between pooled composite samples than would random sampling. However, if an unbiased estimate of σ_{I}^2 is preferred to an economical sampling plan, random sampling may be used.

Where the number of composite samples is more than three, replicate sampling may be used. However, this International Standard provides sample size tables only for duplicate sampling (two composite samples), for simplicity and economy.

C.2.6 Laboratory

It is assumed that measurements are carried out in a single laboratory, and hence the measurement standard deviation is smaller than the reproducibility. If the results of the purchaser's laboratory and the supplier's laboratory are significantly different, then the difference should be dealt with as a bias in place of a variance (see ISO 11648-1).

C.2.7 Infinite population

The sampling plans are based on the assumption of an infinite population. This assumption is usually satisfied, because;

a) a sampling increment is a very small part of a lot;

b) a test sample is a very small part of a composite sample; and

c) a test portion is a very small part of a test sample.

Even if the assumption of an infinite population were not satisfied, users can generally disregard this because the values of the estimate standard deviation, σ_E, and both risks, α and β, for the finite population would be somewhat smaller than for the infinite population, respectively.

C.2.8 Simple population

The sampling plans are based on the assumption of a simple population. In other words, sampling increments can be directly taken from a lot. This assumption is usually satisfied. However, there may be other cases, for example, the material is in two or more containers. If the variance component between sampling increments, σ_I^2, consists of the variance component between containers, σ_B^2, and the variance component within containers, σ_W^2, and if both are not negligible, this International Standard is not applicable.

C.2.9 Single quality characteristic

Both the producer's risk and the consumer's risk are calculated for a single quality characteristic. For multiple quality characteristics both of the overall risks are increased.

C.3 The simplest model

C.3.1 General

In the standard procedures of this International Standard, economical sampling plans are the objective, but the assumed model is rather complicated. For ease of understanding, the simplest model is based on the following assumptions:

a) n sampling increments are taken from the lot;

b) no composite sample is constituted;

c) one test sample is prepared per sampling increment;

d) one measurement is made per sampling increment;

e) σ_P and σ_M are negligible.

C.3.2 Estimate standard deviation

The estimate standard deviation, σ_E, is the positive square root of the expected variance of the estimate of the lot mean. Under this simplest model, the estimate standard deviation is given by the following equation:

$$\sigma_E = \sqrt{\frac{\sigma_I^2}{n}} \quad \text{(C.3)}$$

where n is both the number of sampling increments and the number of measurements.

C.3.3 Sample sizes

Under this simplest model, the sample size, n, is given by the following equation:

$$n = \left(\frac{(K_\alpha + K_\beta)\sigma_I}{D} \right)^2 \quad \text{(C.4)}$$

The calculated value of n should be rounded to the nearest integer.

C.4 Relationship between m_A, m_R and acceptance value

C.4.1 General

Under the above simplest model, the following relationship between m_A, m_R and acceptance value is given. This relationship also applies to the more complicated models given in C.5 and C.6.

C.4.2 One-sided lower specification limit

When a one-sided lower specification limit is specified, the following equation is obtained:

$$\bar{x}_L = m_A - K_\alpha \sigma_E = m_R + K_\beta \sigma_E \tag{C.5}$$

(see Figure C.1). Furthermore, the following equations are obtained

$$D = m_A - m_R = (K_\alpha + K_\beta)\sigma_E \tag{C.6}$$

$$\bar{x}_L = m_A - \frac{K_\alpha}{K_\alpha + K_\beta} D \tag{C.7}$$

and because α = 0,05 and β = 0,10 are assumed in the standard procedures, the constant γ is given by the following equation:

$$\gamma = \frac{K_\alpha}{K_\alpha + K_\beta} = \frac{1{,}644\ 85}{1{,}644\ 85 + 1{,}281\ 55} = 0{,}562\ 07 \rightarrow 0{,}562 \tag{C.8}$$

For optional procedures where $\alpha = \beta$ is assumed, the constant γ is given by the following equation:

$$\gamma = \frac{K_\alpha}{K_\alpha + K_\beta} = \frac{1}{2} \tag{C.9}$$

C.4.3 One-sided upper specification limit

When a one-sided upper specification limit is specified, the following equation is assumed:

$$\bar{x}_U = m_A + K_\alpha \sigma_E = m_R - K_\beta \sigma_E \tag{C.10}$$

Furthermore, the following equations are obtained.

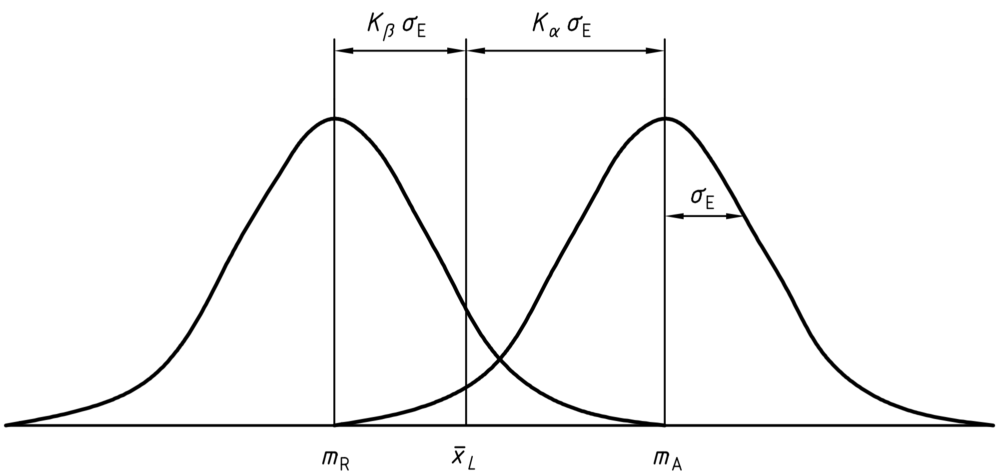


Figure C.1 — Relationship between m_A, m_R and acceptance value (Distribution of $\bar{x}_{...}$; lower specification limit)

$$D = m_R - m_A = (K_\alpha + K_\beta)\sigma_E \quad \text{(C.11)}$$

and

$$\bar{x}_U = m_A + \frac{K_\alpha}{K_\alpha + K_\beta} D \quad \text{(C.12)}$$

(see Figure C.2). The same constant γ is given by equation (C.8).

C.4.4 Two-sided specification limits

When two-sided specification limits are specified, the above equations are applicable to both limits respectively, provided the following constraint is satisfied:

$$\Delta = m_{A,U} - m_{A,L} \geqslant \delta \times D \quad \text{(C.13)}$$

Both Figures C.3 and C.4 show an extreme case of $\Delta = \delta \times D$ (see also Figures C.1 and C.2). In such a case, the maximum probability of acceptance is 0,99 at m = 0,5 ($m_{A,U} + m_{A,L}$), and the following equations are obtained.

$$\delta \times D = m_{A,U} - m_{A,L} = 2(K_{0,005} - K_\alpha)\sigma_E \quad \text{(C.14)}$$

The constant δ is given by the following equation:

$$\delta = \frac{2(K_{0,005} - K_\alpha)}{K_\alpha + K_\beta} = \frac{2(2{,}575\ 83 - 1{,}644\ 85)}{1{,}644\ 85 + 1{,}281\ 55} = 0{,}636 \quad \text{(C.15)}$$

For optional procedures where $\alpha = \beta = 0{,}05$ is assumed, the constant δ is given by the following equation:

$$\delta = \frac{2(K_{0,005} - K_\alpha)}{K_\alpha + K_\beta} = \frac{2(2{,}575\ 83 - 1{,}644\ 85)}{2 \times 1{,}644\ 85} = 0{,}566 \quad \text{(C.16)}$$

C.5 Two-component model

C.5.1 General

In practice, it frequently occurs that σ_M is not negligible. In such cases, the estimate standard deviation, σ_E, consists of two components. Therefore, the following assumptions are to be made for this practical model:

a) n_1 sampling increments are taken from the lot;

b) no composite sample is constituted;

c) one test sample is prepared per sampling increment;

d) n_2 measurements are made per sampling increment;

e) σ_P is negligible.

C.5.2 Estimate standard deviation

Under this model, by rewriting equation (C.6), the estimate standard deviation, σ_E, is given as follows,

$$\sigma_E = \sqrt{\frac{\sigma_I^2}{n_1} + \frac{\sigma_M^2}{n_1 n_2}} = \frac{D}{K_\alpha + K_\beta} \quad \text{(C.17)}$$

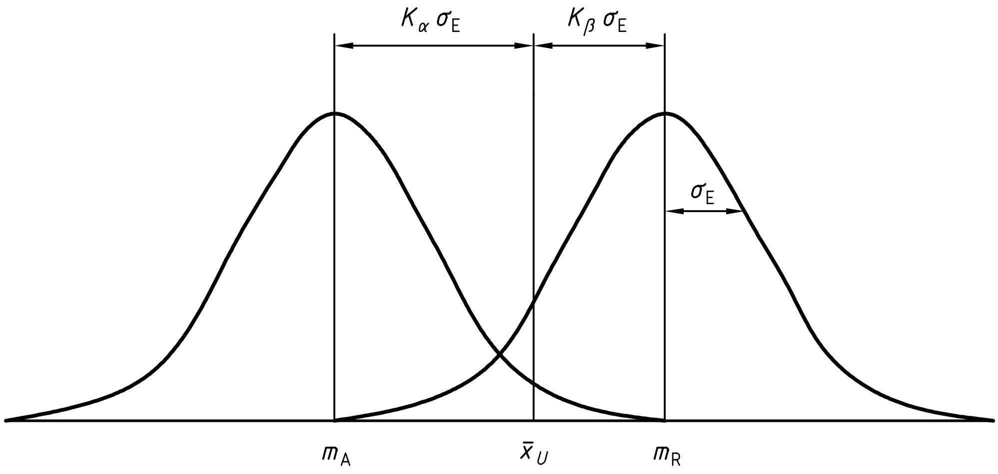


Figure C.2 — Relationship between m_A, m_R and acceptance value (Distribution of $\bar{x}_{...}$; upper specification limit)

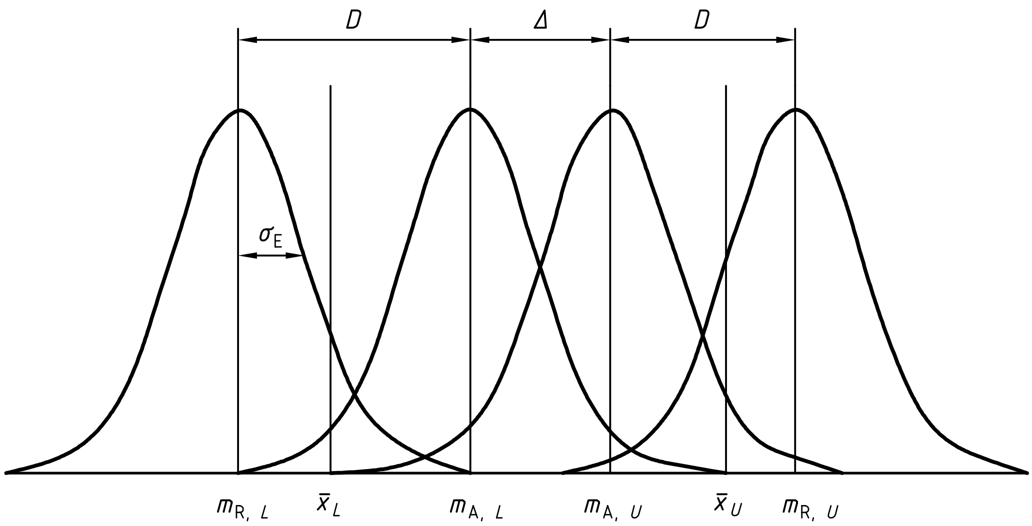


Figure C.3 — Relationship between m_As, m_Rs and acceptance values (Distribution of $\bar{x}_{...}$; two-sided specification limits)

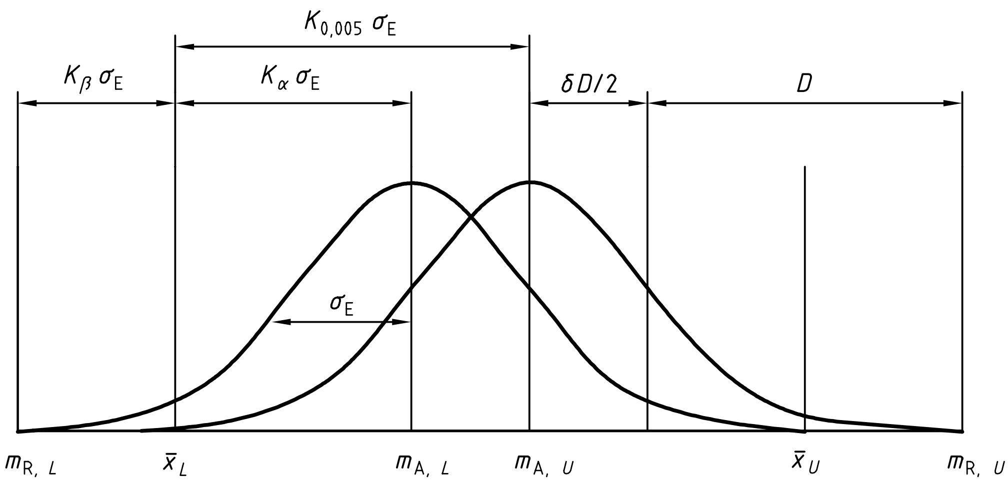


Figure C.4 — Relationship between Δ and D (when $\Delta = \delta \times D$) (Distribution of $\bar{x}_{...}$; two-sided specification limits)

C.5.3 Sample sizes

There may be many combinations of n_1 and n_2, which satisfy equation (C.17), and the most economical value for the number of measurements per sampling increment, n_2, is given by the following equation:

$$n_2 = \frac{\sigma_M}{\sigma_P}\sqrt{\frac{c_1}{c_2}} \qquad \text{(C.18)}$$

where

c_1 is the cost of taking a sampling increment and preparing a test sample;

c_2 is the cost of a measurement (for the lot mean).

Equations (C.17) and (C. 18) give the following solution for n_1:

$$n_1 = \left(\sigma_I^2 + \frac{\sigma_M^2}{n_2}\right)\left(\frac{K_\alpha + K_\beta}{D}\right)^2 \qquad \text{(C.19)}$$

C.6 Three-component model

The model used for the standard procedures of this International Standard is an extension of the above two-component model to the three-component model. Therefore, detailed description is omitted, except for the design procedures.

Simultaneous equations (C.20) and (C.21) provide a solution in n_I and n_T for the standard procedures. The former gives the most economical ratio, and the latter corresponds to the required OC curve.

$$\frac{n_I}{n_T} = \frac{d_I}{d_T}\sqrt{\frac{c_T}{c_I}} \qquad \text{(C.20)}$$

and

$$\sqrt{\frac{d_I^2}{n_I} + \frac{d_T^2}{n_T}} = \frac{1}{K_\alpha + K_\beta} \qquad \text{(C.21)}$$

The mathematical solutions are given by the following equations:

$$n_I = d_I\left(d_I + d_T\sqrt{\frac{c_T}{c_I}}\right)\left(K_\alpha + K_\beta\right)^2 \qquad \text{(C.22)}$$

and

$$n_T = d_T\left(d_T + d_I\sqrt{\frac{c_I}{c_T}}\right)\left(K_\alpha + K_\beta\right)^2 \qquad \text{(C.23)}$$

When n_I or n_T is too small, equations (C.22) and (C.23) do not always give satisfactory results. In this case, the values of n_I and n_T in Tables 3 to 12 can be obtained by the following procedures:

a) interim values of n_I and n_T are obtained;

b) the smaller one is fixed (if $n_I < n_T$ then n_I is fixed by rounding);

c) the other is obtained by trial and error.

C.7 Multiple characteristics

C.7.1 Overall risks

The optional special procedures for multiple characteristics given in A.2 are based on the assumptions that all quality characteristics are independent of each other and have equal importance.

If the number of quality characteristics is J and all of them are independent of each other, the overall producer's risk, α_{O}, is obtained by the following equation:

$$\alpha_{\text{O}} = 1 - \prod_{i=1}^{J} (1 - \alpha_i) \tag{C.24}$$

where

α_i is α for the individual quality characteristic.

If $\alpha_1 = \alpha_2 = \ldots = \alpha_J = \alpha^*$, then

$$\alpha_{\text{O}} = 1 - (1 - \alpha^*)^J \tag{C.25}$$

and

$$\alpha^* = 1 - \sqrt[J]{1 - \alpha_{\text{O}}} \tag{C.26}$$

The similar equation is obtained for the overall consumer's risk, β_{O}.

If $\beta_1 = \beta_2 = \ldots = \beta_J = \beta^*$, then

$$\beta^* = 1 - \sqrt[J]{1 - \beta_{\text{O}}} \tag{C.27}$$

The correction factor, f_{D}, is obtained by the following equation:

$$f_D = \frac{K_{\alpha_{\text{O}}} + K_{\beta_{\text{O}}}}{K_{\alpha^*} + K_{\beta^*}} \tag{C.28}$$

where

α_{O} is the overall producer's risk;

β_{O} is the overall consumer's risk;.

α^* is the individual producer's risk;

β^* is the individual consumer's risk;

The values of correction factor, f_{D}, for known standard deviations are given in Table A.1. The values of the individual risks, α^* and β^*, are given in Table A.2.

C.7.2 OC curve

The procedures for calculating OC curves are given in Annex D and also applicable to corrected sampling plans, but typically they are as follows:

a) if a lot mean is equal to the acceptance value, then P_a = 50 %;

b) if a lot mean is equal to m_A, then $\alpha = 100 \times \alpha^*$, expressed as a percentage;

c) if a lot mean is equal to m_R, then $\beta = 100 \times \beta^*$, expressed as a percentage;

d) if a lot mean is equal to Q_{PR}, then $\alpha = 100 \times \alpha_O$, expressed as a percentage;

e) if a lot mean is equal to Q_{CR}, then $\beta = 100 \times \beta_O$, expressed as a percentage.

When a lower specification limit L_{SL} is specified, the approximate values of Q_{PR} and Q_{CR} are given by the following equations:

$$Q_{PR} = \bar{x}_L + \gamma \times D \quad \text{(C.29)}$$

$$Q_{CR} = Q_{PR} - D \quad \text{(C.30)}$$

When an upper specification limit, U_{SL}, is specified, the approximate values of Q_{PR} and Q_{CR} are given by the following equations:

$$Q_{PR} = \bar{x}_U - \gamma \times D \quad \text{(C.31)}$$

$$Q_{CR} = Q_{PR} + D \quad \text{(C.32)}$$

C.7.3 Independence

The above results are true only when all quality characteristics are independent of each other. If the coefficients of correlation between two or more quality characteristics are significant, then

$$\alpha^* \leqslant \alpha_O \leqslant J\alpha^* \quad \text{(C.33)}$$

and

$$\beta^* \leqslant \beta_O \leqslant J\beta^* \quad \text{(C.34)}$$

It is difficult to provide general sampling plans which can be applied to such cases.

C.8 Supplementary information for imprecise standard deviations

C.8.1 General

This clause provides supplementary information of the procedures for imprecise standard deviations. Sampling plans for them are derived using a model similar to that used for known standard deviations, except that a t-distribution is used in place of a normal distribution. Furthermore, the preferred values of d_I and d_T are chosen from those of Tables 3 to 7 for design calculation of Tables 13 to 22. Satterthwaite's method (see reference [6] in the Bibliography) is used for obtaining approximate values of composed variance and degrees of freedom, using the preferred values of d_I and d_T given above.

C.8.2 Relationship between m_A, m_R and acceptance value

The approximated relationships between m_A, m_R and acceptance value can be developed in the same manner as those described in C.4.

When two-sided specification limits are specified, the approximate equations for the one-sided specification limits are applicable to both limits respectively, provided the following constraint is satisfied:

$$\Delta = m_{A,U} - m_{A,L} \leqslant \delta \times D \tag{C.35}$$

When $\Delta = \delta \times D$ and $\nu_E < 8$, the maximum probability of acceptance is set to be about 0,98 at $m = 0{,}5\ (m_{A,U} + m_{A,L})$, and the approximate value of δ is obtained using the following formula:

$$\delta = \frac{t_{0,99}(\nu_E) - t_{1-\alpha}(\nu_E)}{t_{1-\alpha}(\nu_E)} \tag{C.36}$$

When $\nu_E \geqslant 8$, the above formula gives a smaller value of δ than that for known standard deviations. Therefore, the same value of δ as for known standard deviations is chosen, so that moving from one procedure to the other is facilitated. When $\Delta = \delta \times D$ and ν_E is sufficiently large, the maximum probability of acceptance is close to 0,99.

C.8.3 Multiple characteristics

C.8.3.1 Overall risks

Similar approximate equations to those given in C.7.1 may be obtained by replacing the normal distribution with the t-distribution. The correction factor, f_D, is obtained by the following equation:

$$f_D = \frac{t_{1-\alpha_O}(\nu_E) + t_{1-\beta_O}(\nu_E)}{t_{1-\alpha^*}(\nu_E) + t_{1-\beta^*}(\nu_E)} \tag{C.37}$$

where the symbols are the same as in (C.28).

C.8.3.2 OC curve

The procedures and relationships for the calculation of OC curves can be approximated and developed in a similar manner as those given in C.7.2.

Annex D
(informative)

Operating characteristic curves

D.1 General introduction

This annex describes the procedures for calculating the values of the operating characteristic (OC) curves using the method for known standard deviations in D.2 to D.5. This annex also describes the procedures for calculating the approximate values for OC curves using the method for imprecise standard deviations in D.6.

This annex provides two methods of calculating the values of the operating characteristic (OC) curves for the sampling plan:

a) to convert a lot mean, m, to the probability of acceptance, P_a (see D.3);

b) to obtain a lot mean, m, corresponding to a specified value of probability of acceptance, P_a (see D.4).

Method b) is more convenient than the method a) because there is no need for a normal distribution table or to make interpolations.

D.2 Known standard deviations

D.2.1 Necessity of the estimate standard deviation

Before calculating the values of an OC curve of the procedures for known standard deviations, it is necessary to obtain the estimate standard deviation, σ_E.

D.2.2 Standard procedures

In the case of standard procedures, the estimate standard deviation, σ_E, is given by the following equation:

$$\sigma_E = \sqrt{\frac{\sigma_I^2}{2n_I} + \frac{\sigma_P^2}{2n_T} + \frac{\sigma_M^2}{2n_T n_M}} = \sqrt{\frac{\sigma_I^2}{2n_I} + \frac{\sigma_T^2}{2n_T}} \qquad \text{(D.1)}$$

NOTE An infinite population is assumed in deriving the above equation. If the lot size is not sufficiently large, or if the total volume of the test samples is an important part of the composite sample, then the actual value of σ_E becomes somewhat smaller than the above, and both the producer's risk and the consumer's risk become somewhat smaller than the calculated values.

D.2.3 Special procedures given in annex A

In the case of the special procedures given in annex A, the estimate standard deviation, σ_E, is given by the following equations:

a) when $n_T > 1$, then

$$\sigma_E = \frac{\sigma_O}{\sqrt{2n_T n_M}} = \sqrt{\frac{\sigma_I^2}{2n_I} + \frac{\sigma_P^2}{2n_T} + \frac{\sigma_M^2}{2n_T n_M}} \qquad \text{(D.2)}$$

b) when $n_T = 1$ (and $n_I = 1$), then

$$\sigma_E = \frac{\sigma_O}{\sqrt{2n_M}} = \sqrt{\frac{\sigma_{IP}^2}{2} + \frac{\sigma_M^2}{2n_M}} \tag{D.3}$$

D.3 Converting m to P_a

D.3.1 One-sided specification limit

Values of the upper P_a-fractile of the standardized normal distribution corresponding to arbitrary values of the lot mean, m, are given by the following equations:

a) When the lower specification limit L_{SL} is specified;

$$K_{P_a} = \frac{\bar{x}_L - m}{\sigma_E} \tag{D.4}$$

b) When the upper specification limit, U_{SL}, is specified;

$$K_{P_a} = \frac{m - \bar{x}_U}{\sigma_E} \tag{D.5}$$

The upper P_a-fractile value obtained corresponds to the probability of non-acceptance ($1 - P_a$), and can be readily converted to the probability of acceptance, P_a, using a normal distribution table.

D.3.2 Two-sided specification limits

D.3.2.1 General

In many cases when both the lower and the upper acceptance values are specified, equations (D.4) and (D.5) are applicable separately. If the interval between both values of m_A [Δ (= $m_{A,U} - m_{A,L}$)], is close to the limiting interval, $\delta \times D$, then the following corrections should be considered.

D.3.2.2 Maximum probability of acceptance

At $m = 0{,}5(m_{A,U} + m_{A,L})$, the probability of acceptance is maximum. The value of the ($1 - P_{a,U}$)-fractile of the standardized normal distribution corresponding to the lot mean, m, is given by the following equation:

$$K_{P_{a,U}} = \frac{m - \bar{x}_U}{\sigma_E} \tag{D.6}$$

The ($1 - P_{a,U}$)-fractile obtained can be converted to the probability of non-acceptance ($1 - P_{a,U}$) using a normal distribution table. The value of the probability of acceptance is given by the following equation:

$$P_a = 1 - \left[1 - 2(1 - P_{a,U})\right] \tag{D.7}$$

If the maximum P_a is close to 1,000 (100,0 %) then no more correction is necessary. If $\Delta = \delta \times D$, then the maximum P_a is 0,990 (99,0 %).

D.3.2.3 Probability of acceptance for general values of m

For general values of m, the following method may be used:

a) a few values of m are chosen between $m_{\mathrm{A},L}$ and $m_{\mathrm{A},U}$;

b) the value of the (1 – $P_{\mathrm{a},L}$)-fractile of the standardized normal distribution corresponding to a lot mean, m, is given by the following equation:

$$K_{P_{\mathrm{a}},L} = \frac{\bar{x}_L - m}{\sigma_{\mathrm{E}}} \quad \text{(D.8)}$$

c) the value of the (1 – $P_{\mathrm{a},U}$)-fractile corresponding to m is given by equation (D.6);

d) the (1 – P_{a})-fractiles obtained can be converted to probabilities of non-acceptance, 1 – $P_{\mathrm{a},L}$ and 1 – $P_{\mathrm{a},U}$, respectively, using a normal distribution table. The value of probability of acceptance is given by the following equation:

$$P_{\mathrm{a}} = 1 - (1 - P_{\mathrm{a},L}) - (1 - P_{\mathrm{a},U}) \quad \text{(D.9)}$$

D.4 Converting P_{a} to m

D.4.1 One-sided specification limit

Values of the lot mean, m, corresponding to the specified values of the probability of acceptance are given by the following equations:

a) When the lower specification limit is specified;

$$m = \bar{x}_L - K_{P_{\mathrm{a}}} \times \sigma_{\mathrm{E}} \quad \text{(D.10)}$$

b) When the upper specification limit is specified;

$$m = \bar{x}_U + K_{P_{\mathrm{a}}} \times \sigma_{\mathrm{E}} \quad \text{(D.11)}$$

NOTE In practice, the nine values of P_{a} given in the following applications are sufficient to draw a rough OC curve. These values are also used in OC tables of ISO 2859-1 and ISO 3951[5].

D.4.2 Two-sided specification limits

In many cases when both lower and upper acceptance values are specified, equations (D.10) and (D.11) are applicable separately. If the interval between both values of m_{A} ($m_{\mathrm{A},U} - m_{\mathrm{A},L}$) is close to the limiting interval, $\delta \times D$, the corrections given in D.3.2.3 should be considered.

D.5 Examples for calculating OC curves for known standard deviations

D.5.1 Example 1: Lower specification limit

The OC curve is to be calculated for the example given in 7.4 when a lower specification limit is specified.

The principal parameters are as follows:

a) acceptance quality limit, m_{A}: 96,0

b) non-acceptance quality limit, m_R: 92,0

c) lower acceptance value, $\bar{x}_L$: 93,75

d) estimate standard deviation, σ_E: 1,37

Figure D.1 shows the resulting OC curve and Table D.1 presents the calculation procedures and results. In this example, the producer's risk, α, and the consumer's risk, β, are as follows:

— at $m = m_A = 96,0$, $\alpha = 5,03$ %;

— at $m = m_R = 92,0$, $\beta = 10,1$ %.

Table D.1 — OC values for Example 1

P_a %	K_{Pa}	$K_{Pa} \times \sigma_E$	m
1,0	2,326	3,19	90,56
5,0	1,645	2,25	91,50
10,0	1,282	1,76	91,99
25,0	0,674	0,92	92,83
50,0	0,000	0,00	93,75
75,0	−0,674	−0,92	94,67
90,0	−1,282	−1,76	95,51
95,0	−1,645	−2,25	96,00
99,0	−2,326	−3,19	96,94

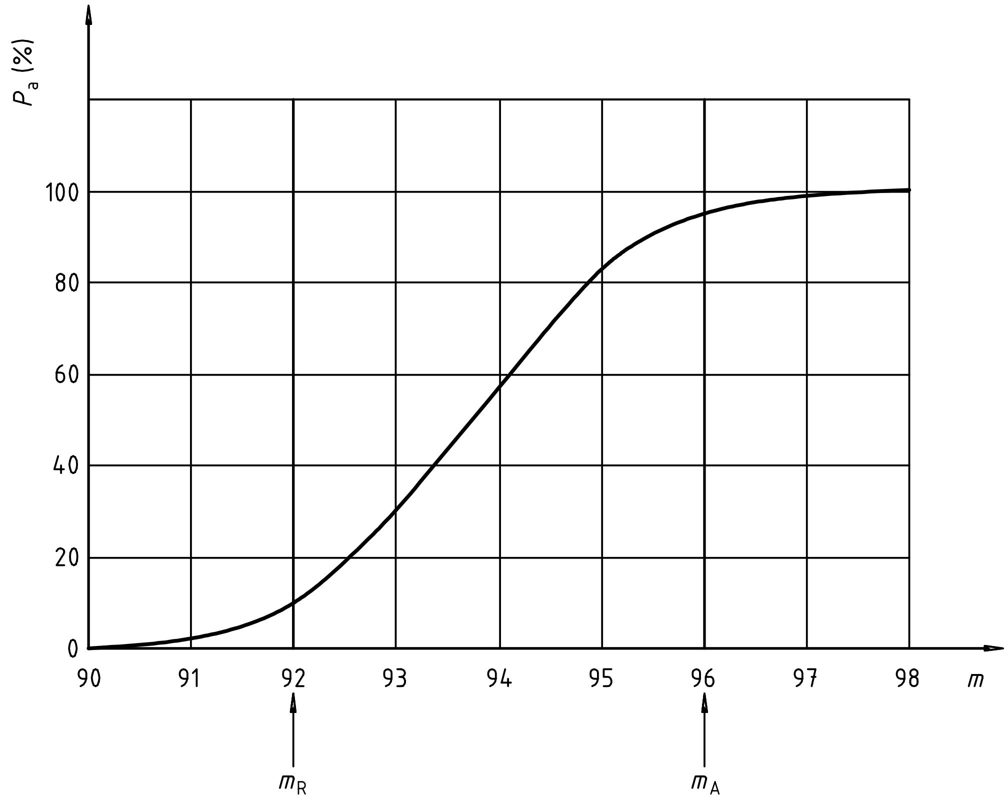


Figure D.1 — OC curve for Example 1

D.5.2 Example 2: Known and dominant standard deviation of measurement

The OC curve is to be calculated for the example given in B.7.1 and for the upper specification limit.

The principal parameters are as follows:

a) acceptance quality limit, m_A: 86,0

b) non-acceptance quality limit, m_R: 90,0

c) upper acceptance value, $\overline{x}_U$: 88,25

d) estimate standard deviation, σ_E: 1,43

Figure D.2 shows the resulting OC curve and Table D.2 presents the calculation procedures and results. In this example, the producer's risk, α, and the consumer's risk, β, are as follows:

— at $m = m_A = 86{,}0$, $\alpha = 5{,}78$ %;

— at $m = m_R = 90{,}0$, $\beta = 11{,}1$ %.

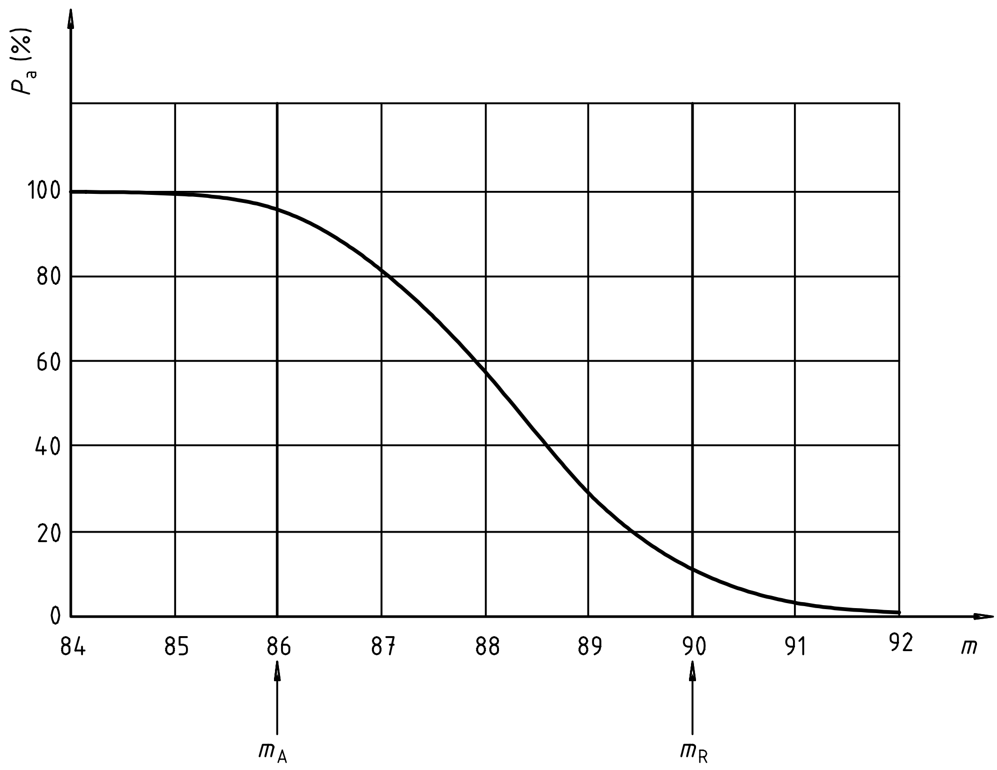


Figure D.2 — OC curve for Example 2

Table D.2 — OC values for Example 2

P_a %	K_{Pa}	$K_{Pa} \times \sigma_E$	m
99,0	−2,326	−3,33	84,92
95,0	−1,645	−2,35	85,90
90,0	−1,282	−1,83	86,42
75,0	−0,674	−0,96	87,29
50,0	0,000	0,00	88,25
25,0	0,674	0,96	89,21
10,0	1,282	1,83	90,08
5,0	1,645	2,35	90,60
1,0	2,326	3,33	91,58

D.5.3 Example 3 — Two-sided specification limits

The OC curve is to be calculated for the example in 7.6 and for two-sided specification limits.

The principal parameters are as follows:

a) lower acceptance quality limit, $m_{A,L}$: 97,0

b) lower non-acceptance quality limit, $m_{R,L}$: 91,0

c) upper acceptance quality limit, $m_{A,U}$: 104,0

d) upper non-acceptance quality limit, $m_{R,U}$: 110,0

e) lower acceptance value, $\bar{x}_L$: 93,63

f) upper acceptance value, $\bar{x}_U$: 107,37

g) estimate standard deviation, σ_E: 1,82

Figure D.3 shows the resulting OC curve for this example. Tables D.3 and D.4 present the calculation procedures and results. In this case, two OC curves for both the lower and upper acceptance values can be obtained separately. In this example, the producer's risk, α, the consumer's risk, β, and the maximum value of probability of acceptance are as follows:

— at $m = m_{A,L}$ = 97,0 or $m = m_{A,U}$ = 104,0, α = 3,20 %;

— at $m = m_{R,L}$ = 91,0 or $m = m_{R,U}$ = 110,0, β = 7,42 %;

— at $m = 0,5(m_{A,L} + m_{A,U})$ = 100,50, the maximum value of P_a, 99,98 %, is reached.

Table D.3 — OC values for Example 3, lower side

P_a %	K_{Pa}	$K_{Pa} \times \sigma_E$	m
1,0	2,326	4,23	89,40
5,0	1,645	2,99	90,64
10,0	1,282	2,33	91,30
25,0	0,674	1,23	92,40
50,0	0,000	0,00	93,63
75,0	–0,674	–1,23	94,86
90,0	–1,282	–2,33	95,96
95,0	–1,645	–2,99	96,62
99,0	–2,326	–4,23	97,86

Table D.4 — OC values for Example 3, upper side

P_a %	K_{Pa}	$K_{Pa} \times \sigma_E$	m
99,0	–2,326	–4,23	103,14
95,0	–1,645	–2,99	104,38
90,0	–1,282	–2,33	105,04
75,0	–0,674	–1,23	106,14
50,0	0,000	0,00	107,37
25,0	0,674	1,23	108,60
10,0	1,282	2,33	109,70
5,0	1,645	2,99	110,36
1,0	2,326	4,23	111,60

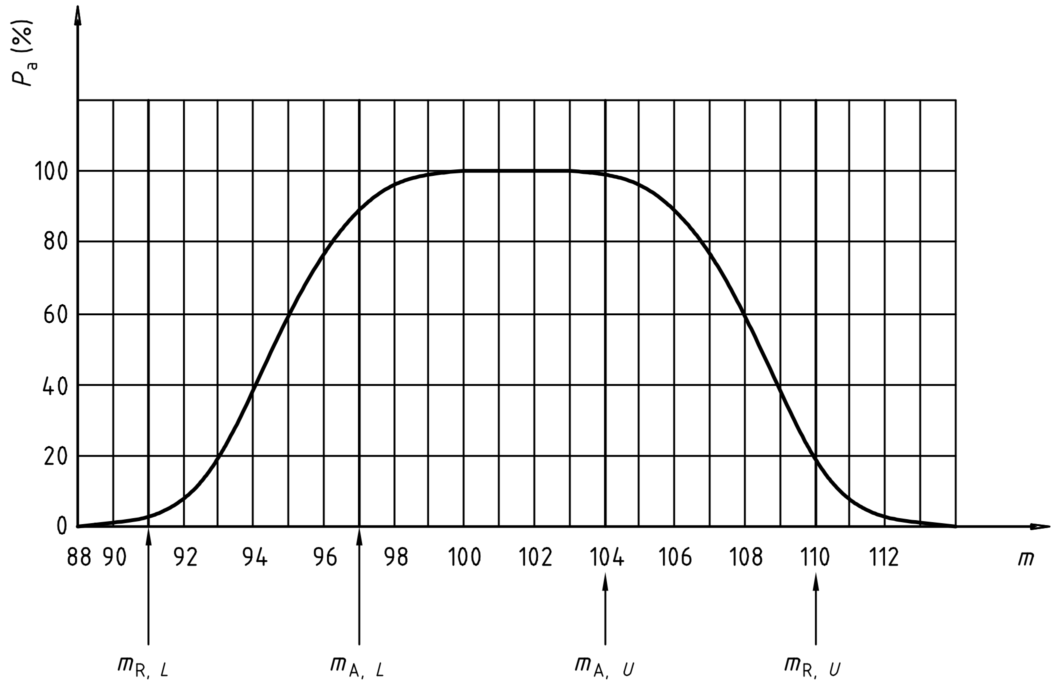


Figure D.3 — OC curve for Example 3

D.6 Imprecise standard deviations

D.6.1 Estimate standard deviation

Before calculating the values of an OC curve using the method for imprecise standard deviations, it is first necessary to obtain the approximate value of the estimate standard deviation, σ_E.

In the case of standard procedures, the approximate value of the estimate standard deviation, σ_E, is given by the following equation:

$$\sigma_E = \sqrt{\frac{\sigma_I^2}{2n_I} + \frac{\sigma_P^2}{2n_T} + \frac{\sigma_M^2}{2n_T n_M}} = \sqrt{\frac{\sigma_I^2}{2n_I} + \frac{\sigma_T^2}{2n_T}} \tag{D.12}$$

Equation (D.12) is used in the procedures for the method for imprecise standard deviations.

In the case of special procedures in annex A for imprecise standard deviations, the approximate value of the estimate standard deviation, σ_E, is also given by the following equation:

$$\sigma_E = \frac{\sigma_O}{\sqrt{2n_T n_M}} \tag{D.13}$$

D.6.2 Converting m to P_a

D.6.2.1 One-sided specification limit

The approximate values of the lower and upper P_a-fractiles of the t-distribution correspond to arbitrary values of the lot mean and are given by the following equations:

a) when the lower specification limit is specified;

$$t_{P_a}(\nu_E) = \frac{m - \bar{x}_L}{\sigma_E} \tag{D.14}$$

b) when the upper specification limit is specified;

$$t_{P_a}(\nu_E) = \frac{\bar{x}_U - m}{\sigma_E} \tag{D.15}$$

NOTE The lower and upper P_a-fractile values of the t-distribution, obtained from equations (D.14) and (D.15), correspond to approximate probabilities of acceptance, P_a, but special software is necessary for the conversion.

D.6.2.2 Two-sided specification limits

In many cases, when both the lower and the upper acceptance values are specified, equations (D.14) and (D.15) are applicable separately. If the interval, Δ, between both values of m_A is close to the limiting interval, $\delta \times D$, some deviations are inevitable. If $\Delta = \delta \times D$, then the maximum P_a is about 0,98 (98 %).

D.6.3 Converting P_a to m

D.6.3.1 One-sided specification limit

The approximate values of the lot mean, m, correspond to specified values of the probability of acceptance, P_a, and are given by the following equations:

a) when the lower specification limit is specified;

$$m = \overline{x}_L + t_{P_a}(\nu_E)\sigma_E \qquad (D.16)$$

b) when the upper specification limit is specified;

$$m = \overline{x}_U - t_{P_a}(\nu_E)\sigma_E \qquad (D.17)$$

NOTE 1 The values of $t_{Pa}(\nu_E)$ are given in a t-distribution table. Some values are given in reference [7] of the Bibliography.

NOTE 2 In practice, the nine values of P_a given in Example 4 are sufficient to draw an approximate OC curve.

D.6.3.2 Two-sided specification limits

In many cases when both lower and upper acceptance values are specified, the above two formulae are applicable separately. If the interval, Δ, between both values of m_A is close to the limiting interval, $\delta \times D$, some deviations are inevitable.

D.6.4 Example 4: Calculation of an OC curve for the method using an estimated standard deviation

The OC curve is to be calculated for the example given in 7.1.

The principal parameters are as follows:

a) acceptance quality limit, m_A: 96,0

b) non-acceptance quality limit, m_R: 92,0

c) lower acceptance value, $\overline{x}_L$: 94,0

d) estimate standard deviation, σ_E: 1,17

e) degrees of freedom of the estimate standard deviation, ν_E: 35

Figure D.4 shows the resulting OC curve for 7.1 and Table D.5 presents the calculation procedures and the results. In this application, the approximate values of the producer's risk, α, and the consumer's risk, β, are as follows;

— at $m = m_A$ = 96,0, α = 4,81 %;

— at $m = m_R$ = 92,0, β = 4,81 %.

Table D.5 — OC values for Example 4

P_a %	K_{Pa}	$K_{Pa} \times \sigma_E$	m
1,0	–2,438	–2,85	91,15
5,0	–1,690	–1,98	92,02
10,0	–1,306	–1,53	92,47
25,0	–0,682	–0,80	93,20
50,0	0,000	0,00	94,00
75,0	0,682	0,80	94,80
90,0	1,306	1,53	95,53
95,0	1,690	1,98	95,98
99,0	2,438	2,85	96,85

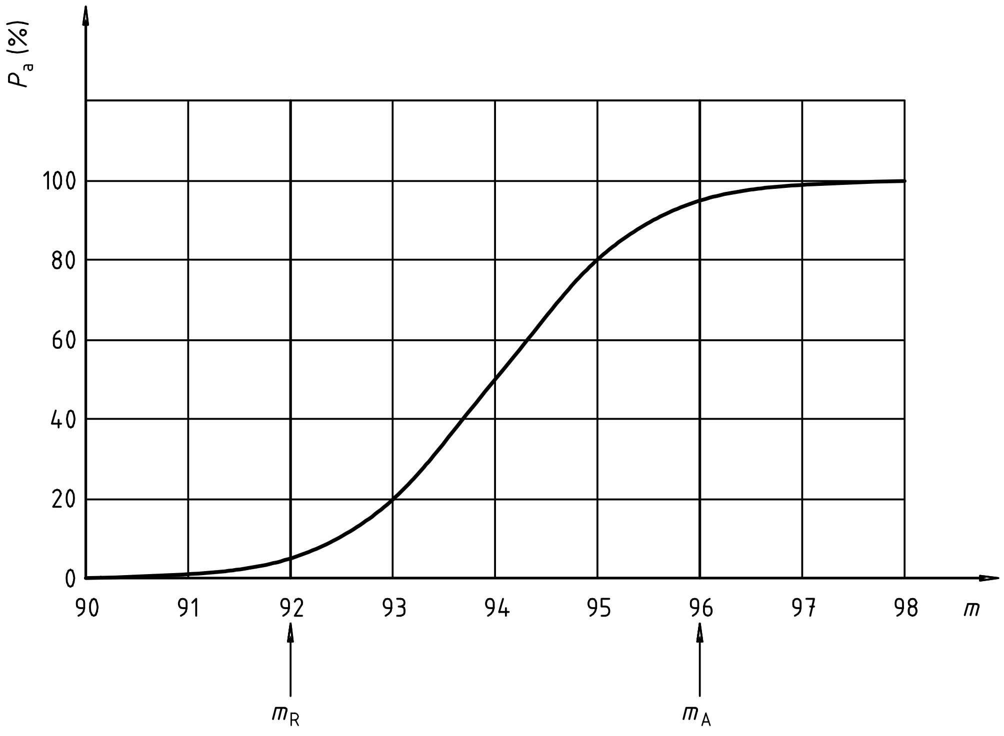


Figure D.4 — OC curve for Example 4

Bibliography

[1] ISO 11648-2:—[1], *Statistical aspects of sampling from bulk materials — Part 2: Sampling of particulate materials.*

[2] ISO 5725-2:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 2: Basic method for the determination of repeatability and reproducibility of a standard measurement method.*

[3] ISO 5725-3:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 3: Intermediate measures of the precision of a standard measurement method.*

[4] ISO 10576 (all parts)[2], *Statistical methods — Guidelines for the evaluation of conformity with specified requirements*.

[5] ISO 3951:1989, *Sampling procedures and charts for inspection by variables for percent nonconforming.*

[6] SNEDECOR G.W. and COCHRAN W.G. *Statistical Methods*., Iowa State University Press, 8th edn., 1989, p. 97.

[7] ISO 2854:1976, *Statistical interpretation of data — Techniques of estimation and tests relating to means and variances.*

[8] ISO 7870:1993, *Control charts — General guide and introduction.*

[9] ISO 9000-2:1997, *Quality management and quality assurance standards — Part 2: Generic guidelines for the application of ISO 9001, ISO 9002 and ISO 9003.*

1) To be published.

2) ISO 10576 may give useful information, but it is still in the course of preparation.

ICS 03.120.30

Price based on 72 pages