
**Photography — Root mean square
granularity of photographic films —
Method of measurement**

*Photographie — Moyenne quadratique granulaire de films
photographiques — Méthode de mesure*



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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 10505 was prepared by Technical Committee ISO/TC 42, *Photography*.

Introduction

This International Standard specifies procedures for measuring and computing the root mean square granularity (rms-granularity) of photographic films. Its purpose is to provide guidance in making accurate measurements, and also to provide an objective basis for comparing films. This International Standard describes a method for making accurate rms-granularity measurements in the presence of instrument noise and minor sample imperfections.

In principle, the measurement of rms-granularity is straightforward, but its determination with accuracy is not a trivial matter. Experience has shown that the preparation of an imperfection-free sample is virtually impossible in usual practice. Therefore, considerable attention has been devoted to the definition of a method of accurately estimating the rms-granularity of a film in the presence of density fluctuations not caused by the intrinsic grain structure of the film.

Research in rms-granularity (see Reference [10]) has pointed out that the inclusion of several “artefact-induced” data values in a set of several thousand “grain-produced” data values may result in large errors in the rms-granularity estimate. Under these circumstances, the traditional method for estimating rms-granularity produces higher rms-granularity estimates than the new method. It can also be shown that this method produces results that are identical to those produced by the traditional method when using artefact-free data.

In either case, it is important to bear in mind that rms-granularity is a statistical estimate which is necessarily reported with its associated confidence intervals. In addition, the measurement process recognizes and accounts for the presence of instrument noise that can affect the accuracy of the rms-granularity estimate.

Photography — Root mean square granularity of photographic films — Method of measurement

1 Scope

This International Standard describes a method for determining the intrinsic root mean square granularity (rms-granularity) of photographic films. Intrinsic rms-granularity refers to those density fluctuations produced solely by the distribution of developed image forming centres in the photographic emulsion.

Continuous-tone monochrome (silver absorbing species) and colour (dye absorbing species) materials coated on a transmitting support can be measured by the procedures described in this International Standard. This International Standard is intended for imaging systems with viewing magnifications between 5× and 12× (see Annex A).

The following kinds of granularity measurements are not covered by this International Standard, even though they are photographically important:

- reflecting materials (photographic papers);
- materials having emulsion coated on both sides of the support (e.g. some X-ray films);
- the estimation of the noise power spectrum (Wiener spectrum).

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 5-2, *Photography — Density measurements — Part 2: Geometric conditions for transmission density*

ISO 5-3, *Photography — Density measurements — Part 3: Spectral conditions*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1

transmittance factor

T

ratio of the measured flux transmitted by a specimen to the measured flux when the specimen is removed from the sampling aperture of the measuring device

NOTE
$$T = \frac{\phi_T}{\phi_J}$$

where

ϕ_T is the transmitted flux;

ϕ_J is the aperture flux.

3.2
transmission density

D_T
logarithm to the base 10 of the reciprocal of the transmittance factor, T

NOTE $D_T = \log_{10} 1/T$.

3.3
microtransmittance factor

transmittance factor of a small area of a film or plate, measured using a suitable instrument such as a microphotometer

NOTE In general, the microtransmittance factor of a uniformly exposed and developed film sample varies from point-to-point on the surface. The measured microtransmittance factor of a given film or plate can depend on the optical geometry of the instrument in which it is measured.

3.4
microdensity

D
transform of the microtransmittance factor in accordance with the usual relation $D = \log_{10} 1/T$

3.5
graininess

sensation produced, in the mind of an observer viewing a photographic image, by random inhomogeneity in what should be structureless areas

NOTE Graininess is a subjective quantity that is necessarily measured by psychophysical methods and, as such, is outside the scope of this International Standard.

3.6
root mean square granularity
rms-granularity

σ_D
objective characterization of the spatial microdensity fluctuation of a uniformly exposed and developed photographic layer, determined in accordance with this International Standard

NOTE 1 See Reference [3].

NOTE 2 In contrast to graininess, rms-granularity is an objective quantity.

NOTE 3 The spatial fluctuation is observed when the microdensity of the layer is measured at various points over the surface and is the result of the random distribution of the absorbent species in the layer. The fluctuation in the microdensity over an area of a specimen is characterized by its standard deviation, σ_D , and is generally a function of the specimen's macrodensity. This quantity is termed the "rms-granularity" and in all conceivable cases of interest, this population parameter cannot be determined exactly because of finite sample size. The method for estimating σ_D is described in Clause 9.

NOTE 4 The relation between graininess and rms-granularity is as follows: rms-granularity is intended to be an objective correlate of graininess. The methods of measuring film rms-granularity as defined in 3.6 have been found to give values that generally correlate with the magnitude of the graininess sensation produced when images produced by the film are viewed under suitable conditions. The just-noticeable differences in graininess, detectable by observers viewing areas having a visual density of about 1,0, correspond to differences in rms-granularity of 6 % for a uniform field, of 16 % for an average scene and of 30 % for a complex or busy scene^[4]. Because rms-granularity does not account for effects encountered in multiple stage imaging processes, methods for evaluating the graininess of final images have been developed^[11]. These methods are particularly useful for comparing the image graininess of different final print formats when produced from different negative formats and film types.

3.7**specimen**

piece of photographic film or photographic plate on which rms-granularity measurements are made

NOTE The specimen can be specific or constitute a sample from a population whose rms-granularity is being determined.

3.8**diffuse conversion factor** g

factor used to convert small transmission (projection) density differences produced by most microdensitometers to small diffuse density differences, as determined in accordance with 6.2

3.9**spatial frequency passband**

part of the spatial frequency spectrum that passes through the measuring system

NOTE The spatial frequency passband is determined by the cut-off frequencies of the system on the low side and the high side of the spatial frequency. The passband characteristics required for the measuring instrument are specified in 5.2.

4 Measurement instrument**4.1 General**

This clause describes the basic elements of an instrument suitable for the measurement of rms-granularity. The generic instrument described in this clause follows the general principles for a linear, incoherent microdensitometer used in an overfilled, image-scanning mode. Microdensitometers of different designs may be employed, provided they are shown to conform to the physical optics principles required for linearity and incoherent illumination, as described in References [6], [7], [8] and [9].

4.2 Microdensitometer**4.2.1 Apparatus**

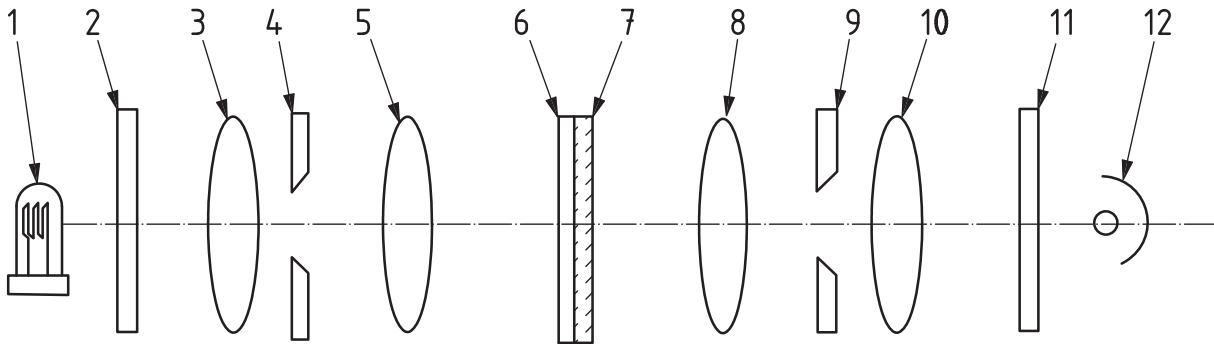
A typical microdensitometer is shown schematically in Figure 1. Its key elements are as follows:

- light source (1): an incoherent source of suitable spectral power distribution;
- illumination filter (2), which produces spectral transmittance necessary to conform to 4.2.2;
- condenser lens (3): optics that fill the influx aperture uniformly with light;
- influx aperture (4): in the optical system shown in Figure 1, this aperture limits the area of the specimen that is illuminated in order to minimize stray light in the optical system;
- influx optics (5), which images the influx aperture (4) on the specimen at point 6;
- specimen (6): emulsion (7) facing the efflux optics;
- efflux optics (8), which collects the light transmitted by the specimen and focuses it upon the efflux aperture of the instrument;
- efflux (measuring) aperture (9): this aperture, projected back onto the specimen, determines the area of the specimen whose density is being measured; its effective size at the specimen is determined by its physical size divided by the optical magnification from the plane at point 6 to the plane at point 9;

- collecting lens (10), which collects the light transmitted by the aperture (9) and relays it to the photodetector;

NOTE The collecting lens is omitted in some designs.

- efflux optical filter (11): spectral transmittance determined by density measurement type;
- photodetector (12), which shall have a spectral responsivity consistent with the spectral products required to produce one or more of the density measurement spectral types defined in ISO 5-3.



Key

- 1 light source
- 2 illumination filter
- 3 condenser lens
- 4 influx aperture
- 5 influx optics
- 6 specimen
- 7 emulsion
- 8 efflux optics
- 9 efflux (measuring) aperture
- 10 collecting lens
- 11 efflux optical filter
- 12 photodetector

Figure 1 — Schematic representation of a typical rms-granularity measuring microdensitometer

4.2.2 Influx spectrum

The influx spectrum for rms-granularity measurements shall be as specified for transmission densitometry in ISO 5-3.

For transmission density measurements, if either the densitometer manufacturer or the user is not sure of the absence of fluorescence in the sample to be measured, the relative spectral power distribution of the incident flux S_H shall be that of the CIE standard illuminant A (ISO 5-3), modified in the infrared region to protect the sample and optical elements from excessive heat, which is typical for most transmission densitometers.

4.2.3 Influx aperture

The influx aperture shall be circular or square in shape and its image shall be concentric with that of the efflux aperture. When both apertures are referred to the plane of the specimen, the linear size of the influx aperture shall not be less than 1,5 times, nor more than 2 times, the linear size of the efflux aperture.

4.2.4 Microdensitometer optics

4.2.4.1 General

The microscope objectives specified in 4.2.4.2 to 4.2.4.4 are the basic objectives for the measurement of rms-granularity and shall be used unless there is a specific reason to employ objectives of different characteristics. The use of alternate objectives is discussed in 4.2.5.3. An analysis of the effects of the microdensitometer optics on the measurement of rms-granularity is presented in Annex B.

4.2.4.2 Influx objective

When films are being measured, the influx objective shall be a high quality microscope objective having a numerical aperture (N_{AI}) of $0,3 \pm 0,1$. It shall not have the same numerical aperture as the efflux objective. If photographic plates are to be measured, the N_{AI} of the influx objective shall be no greater than 0,25, and the optical system shall be carefully focused to accommodate the thickness of the glass support.

4.2.4.3 Efflux objective

The efflux objective of the microdensitometer shall be a high quality microscope objective having a numerical aperture (N_{AE}) equal to $0,25 \pm 0,10$. It shall not have the same numerical aperture as the influx objective (4.2.4.4).

4.2.4.4 Mismatching the numerical apertures of the influx and efflux objectives

In order to maintain incoherence in the optical system over the spatial frequencies of interest, it is necessary to mismatch the numerical aperture of the influx objective, N_{AI} , and the numerical aperture of the efflux objective, N_{AE} . The mismatch criteria are given in References [6], [7], [8] and [9] for the following two cases of interest:

— **Case 1:** overfilled efflux objective, $N_{AI} > N_{AE}$

$$N_{AI} / N_{AE} > 1 + (u_{\max} / u_{\text{co}}) \quad (1)$$

— **Case 2:** underfilled efflux objective, $N_{AE} > N_{AI}$

$$N_{AE} / N_{AI} > 1 + (u_{\max} / u_{\text{co}}) \quad (2)$$

where

u_{\max} is the maximum sample frequency ($u_{\max} = 100 \text{ mm}^{-1}$);

u_{co} is the spatial frequency cut-off of the objective with the smaller numerical aperture, N_A (i.e. $u_{\text{co}} = 2N_A/\lambda$).

EXAMPLE If the efflux objective is underfilled and $N_{AI} = 0,30$, then $N_{AE} > 0,30 (1 + 100/1\ 090) = 0,327$ when λ equals 550 nm.

It can happen then that the mismatch criteria for the numerical apertures of the influx and efflux optics lead to numerical aperture values which are outside the ranges of $0,3 \pm 0,1$ for the influx objective and $0,25 \pm 0,10$ for the efflux objective respectively. Keeping the numerical apertures within their respective ranges would then result in a non-linear behaviour of the microdensitometer leading to a corresponding additional uncertainty.

NOTE The above-mentioned maximum sample frequency, $u_{\max} = 100 \text{ mm}^{-1}$, is true for colour films. However, in the case of black-and-white films, the noise power spectrum can have different contributions well beyond this limit (e.g. microfilm)^[18].

4.2.5 Objectives

4.2.5.1 Objective lens aberrations

The influx and efflux objectives may be achromatic or apochromatic; apochromatic objectives are recommended for use with colour materials. Achromatic objectives can be used either with or without eyepieces, depending on the optical design of the microdensitometer. If achromatic objectives are used to scan colour films, special care shall be taken to optimize the focal settings for each individual colour response.

4.2.5.2 Eyepieces

In some cases, notably that of apochromatic objectives, the optical design is such that the objectives need to be used with certain eyepieces for optimum results. If apochromatic objectives are used in the optical design, they shall be used with eyepieces as directed by the manufacturer. The powers of the eyepieces used may be varied as needed to meet the requirements for the optical magnification. When eyepieces are used, the "tube length" (i.e. the distance between objective and eyepiece) shall be optimized for the objective and eyepiece combination.

4.2.5.3 Use of objectives having other numerical apertures

In general, the purpose of measuring the rms-granularity of the specimen is to predict its behaviour in an imaging application. In some cases, where the film or plate is to be used with an optical system, the numerical apertures of the imaging system will be known and may be significantly different from the values given above. The numerical apertures of a measurement instrument optical system can be changed to simulate a specific imaging application more closely. However, in such cases the simulated application shall be stated and the numerical apertures used shall also be stated.

4.2.6 Efflux aperture

The efflux aperture should be circular in shape. Alternatively, a rectangular or square aperture of similar dimensions (resulting in the same aperture surface) can be used. The effective diameter of the image of the circular efflux aperture at the specimen plane shall be $(48,0 \pm 0,5) \mu\text{m}$ when calculated via the magnification. A $42,5 \mu\text{m}$ square image of the efflux aperture gives comparable results (within 5 %). The physical size of the aperture may be any convenient size.

NOTE In this International Standard, further detailed analyses will all refer to circular apertures.

4.2.7 Photodetector

Any photodetector may be used in the granularity measurement instrument provided that it meets the following requirements:

- a) its spectral responsivity range covers the wavelength range of interest and is continuous over this range,
- b) its frequency response is adequate to evaluate the incoming flux variations,
- c) its sensitivity is sufficient for operation over the required density range (including filters for colour materials), and
- d) the electronic noise it adds to the granularity signal is stable and measurable.

4.3 Spectral products

4.3.1 Specification

The spectral product of the system will depend on the type of measurement desired. ISO 5-3 specifies the spectral products of the types of density referenced below.

4.3.2 Visual density

For films on which the images are truly neutral, the spectral product used is unimportant, because all would give the same result. However, in the interests of standardization and in recognition of the fact that most "neutral" images are not precisely spectrally non-selective, visual density spectral products in accordance with ISO 5-3 shall be used for such films.

Visual density may also be used for films or materials that are viewed directly or by projection (such as colour transparencies).

4.3.3 Status M density

Status M density spectral products in accordance with ISO 5-3 shall be used for films on which spectrally selective images are produced (even when the images are visually neutral), but which are not intended to be viewed directly (such as colour negative films).

4.3.4 Status A density

Status A density spectral products in accordance with ISO 5-3 may be used for films which produce spectrally selective images (even when the images are visually neutral) and which are intended to be viewed directly (such as colour transparencies).

4.4 Spatial frequency response

4.4.1 Spatial frequency response of influx side

The spatial frequency response of the influx side of the microdensitometer is determined by the influx optical system. It shall be of sufficient quality to provide a "hard aperture", as defined in Reference [6].

NOTE A "hard aperture" designates an image that generally resembles the source aperture geometrically.

4.4.2 Spatial frequency response of efflux side

The total spatial frequency response of the measurement instrument efflux side is given by the product of its efflux optical transfer function (OTF) and the transfer function of the circular or square efflux aperture. The transfer function for a circular aperture is a Bessel function of the first kind and of order 1. The first five zero crossings for a 48 μm aperture function occur at frequencies of 25,4 mm^{-1} , 46,5 mm^{-1} , 67,4 mm^{-1} , 88,4 mm^{-1} and 109,2 mm^{-1} , respectively. The OTF includes all optical components except the efflux aperture.

The magnitude of the OTF (which is generally a complex function) is the modulation transfer function (MTF). The minimum on-axis MTF of the efflux optical system of the measuring instrument, as a function of the frequency range, shall be the on-axis diffraction-limited MTF of an objective having a numerical aperture equal to 0,15.

Annex B contains further analysis of the spatial and temporal frequency response characteristics of the measuring instrument.

4.5 Scanning motion

The microdensity of the specimen is measured at various points on the surface, by either translating the specimen with respect to the aperture, or the aperture with respect to the specimen.

5 Instrument electronics

5.1 Conversion to density

The electrical output of the detector system shall be processed to provide a value that is proportional to the microdensity at each sampled point. Signals that are proportional to transmittance can be converted to density-proportional signals by means of a logarithmic amplifier, computation or digital electronics. This conversion shall be done before the rms-granularity is calculated. Note that the electronic bandwidth of logarithmic amplifiers diminishes rapidly with increasing density, so it can be necessary to adjust the scanning velocity to maintain the spatial frequency passband specified in 5.2.

5.2 Temporal frequency response of the instrument

The relationship between the temporal frequency, f , expressed in hertz, and the spatial frequency is as shown in Equation (3):

$$f = u \times v \quad (3)$$

where

u is the spatial frequency, expressed in reciprocal millimetres (mm^{-1});

v is the relative velocity, expressed in millimetres per second (mm/s).

Over 95 % of the image power passed by the (e.g. circular) 48 μm aperture falls within the spatial frequency passband from 0 mm^{-1} to 50 mm^{-1} . It is desirable to restrict the temporal frequency range of the instrument beyond these frequencies. Excess high frequency response adds nothing to the measurement of rms-granularity and can in fact degrade it, by allowing electronic noise to increase the uncertainty of the measurement.

The high frequency response can be controlled through the use of a low-pass filter with a -3 dB point at the temporal frequency corresponding to a spatial frequency of 50 mm^{-1} . If a low-pass filter is used, then it shall have unit response (i.e. 0 dB) at a frequency of 0 Hz, and $(0 \pm 0,5)$ dB within the passband until the -3 dB point is approached and a rolloff of -48 dB per octave thereafter.

5.3 Instrument noise

Electronic noise from sources other than the specimen itself is generated in the detector system and superimposed on the granularity signal. Because this noise can be significant compared to the intrinsic density fluctuations produced by the specimen, it shall be accounted for. The instrument noise shall be less than 1/5 of the granularity signal produced by the specimen.

Clause 9 describes the method for applying a correction for instrument noise to the calculation of the specimen rms-granularity.

6 Diffuse rms-granularity

6.1 Optical geometry

Conventional microdensitometers measure projection densities that are influenced by the geometric configuration of the illumination and imaging optics of the instrument. To correct for effects of the instrument optical geometry, a method of converting projection density to diffuse density shall be applied. The correction process compensates for measurement bias introduced by the optical geometry differences between instruments.

6.2 Diffuse conversion factor g

6.2.1 Purpose

In most microdensitometers, the projection density, D_{proj} , of a photographic film can be greater than the diffuse density, D_{diff} , if flux is scattered out of the collection beam. This is a very important factor for monochrome (i.e. silver) images, but is much less important for most colour (i.e. dye) images that are relatively non-scattering. The value of the diffuse conversion factor, g , is defined as the slope of the curve D_{diff} versus D_{proj} , and is generally a function of a projection density [$g(D_{\text{proj}})$], as shown in Equation (4):

$$g(D_{\text{proj}}) = \frac{\Delta D_{\text{diff}}}{\Delta D_{\text{proj}}} \quad (4)$$

The diffuse conversion factor, g , shall be determined for instrument projection density levels between the minimum and the maximum projection density of interest. A separate diffuse conversion factor shall be determined for each class of black-and-white and colour films under test.

NOTE If a microdensitometer is used that is structured with a diffuse optical system conforming to ISO 5-2, diffuse densities are directly obtained. The diffuse conversion factor, g , is always 1,0 at any density level for these instruments.

6.2.2 Procedure

Various density levels of the film type under test shall be measured using a macro-densitometer that conforms to the specifications for measurement of diffuse transmission density and status density type in accordance with ISO 5. The average diffuse density, \bar{D}_{diff} , is recorded for each of the available density levels. The film sample and the associated densities constitute a "calibration strip" for readout on the microdensity-measuring instrument. The "calibrated strip" shall be placed at the specimen position in the rms-granularity measuring instrument, with the emulsion facing the photodetector; for each density level, the average projection densities, \bar{D}_{proj} , are measured using the appropriate status response.

For each density level, the ratio $\bar{D}_{\text{diff}} / \bar{D}_{\text{proj}}$ is calculated. A plot of this density ratio as a function of the projection density, \bar{D}_{proj} , is helpful in evaluating the nature of the diffuse conversion function, $g(\bar{D}_{\text{proj}})$. Generally speaking, the values of this function are less than 1,0 and approximately constant over the projection density range of interest. If $g(\bar{D}_{\text{proj}})$ differs by 1 % or less over a \bar{D}_{proj} range of interest, then the function $g(\bar{D}_{\text{proj}})$ can be replaced by the constant g , which is computed as the point-to-point average of $g(\bar{D}_{\text{proj}})$ over that range.

For the purpose of determining rms-granularity, it is not necessary to convert the projection densities directly; rather, the conversion is more efficiently performed on the rms-granularity estimate itself. The application of the diffuse conversion factor, g , in the rms-granularity computation process is described in Clause 9.

7 Preparation of specimens

7.1 Sampling and storage

In determining the rms-granularity of a product, it is important that the samples evaluated be representative of those used by the consumer. No fewer than three samples of the product under test shall be obtained from the plant of the manufacturer, or from an accredited distributor if they cannot be obtained directly from the manufacturer. In any case, the samples shall be taken from products stored according to the manufacturer's recommendations and shall be available on the market. Each sample shall represent a different batch of product, and the test specimens shall be prepared from each batch.

7.2 Exposure

An area of the specimen, in accordance with 7.4, shall be exposed to radiant energy of a nature and quality consistent with the intended use of the film material.

7.3 Processing

The chemicals, processing steps, equipment and processing conditions shall be those ordinarily used for the material. Where relevant, the processing may be as the manufacturer recommends for the material being evaluated. Since processing can influence the measured rms-granularity, specifications for the process shall be described when reporting the rms-granularity of a material.

7.4 Specimen uniformity

Utmost care shall be taken to ensure that the areas of the specimen to be measured are both exposed and processed uniformly. Variation in the local mean microdensity level along the scan path and within the spatial frequency passband described in 3.9 inflates the variance of the microdensity values from the value expected when the local mean microdensity level is uniform. An example of the consequent expected effect of non-uniform local mean density level on the granularity estimate is quantified in Annex C. In addition, care shall be taken during handling to keep the processed specimen as free as possible from dirt, pinholes, scratches, developer streaking and anomalous variations in mean density level. The specimen shall be cleaned to eliminate loose surface dirt. In practical operations, most specimens will exhibit some artefacts. Procedures used to reduce the effects of any remaining artefacts or of non-uniform local mean microdensity level on the rms-granularity estimate are discussed in Clause 9.

7.5 Sampled area

The exposed and processed area shall be large enough to provide uniform macrodensity over the region scanned in accordance with 8.2.

8 Operation of the measurement instrument

8.1 Positioning the specimen

The specimen shall be placed in the scanner with the emulsion facing the photodetector. Care shall be taken to ensure that there are no obvious artefacts in the scan path.

8.2 Specimen scanning

The instrument shall be operated in a manner such that it introduces no correlations in the data; this requires that the areas on the specimen, which are sampled by the efflux aperture, be non-overlapping. The total length of the scan, in millimetres, shall not be less than the total number of density observations multiplied by 0,048 mm for a circular aperture, or multiplied by 0,042 5 mm for a square aperture. The path may be longer to any degree consistent with sample uniformity, focus requirements and precision desired. If required by the nature of the specimen or its relative motion, the required scan length may be achieved in non-overlapping segments.

8.3 Control of focus

The optical system of the measuring instrument shall be adjusted such that the influx aperture is sharply focused on the emulsion and the grain pattern is optimally focused on the efflux-measuring aperture at all points in the scan. However, as most specimen imperfections occur at the surface, and in some cases a layer of "matte" (transparent plastic particles) can be coated either on the emulsion or support side of the film, it shall be confirmed through visual means that the final focus is set on the grains themselves and not on another plane.

To ensure optimal focus for each status colour density measured, the focal position of the efflux optics may be varied while replicate rms-granularity measurements of a test specimen are made. The focus position that produces the maximum rms-granularity value should be accepted as the position of optimal focus for the specimen type and status colour density.

8.4 Rate of scan

The rate of scan shall be such that the temporal frequency range of the resulting granularity signal lies within the frequency response range of the electronics used to estimate σ_D . The method of determining the temporal frequency band of the signal is given in 5.2.

8.5 Density mode

All rms-granularity values reported in accordance with this International Standard shall be reported in terms of ISO 5-2 diffuse transmission density. Since the measurement instrument output is generally projection density, the data shall be converted to diffuse macrodensity. Methods of calibration and conversion to diffuse density are given in Clauses 6 and 9.

9 Method of test

9.1 Principle

The rms-granularity of a specimen is calculated as the standard deviation of the specimen's diffuse microdensity fluctuations corrected for instrument noise.

9.2 Statistical background

The usual estimator for the microdensity variance, σ^2 , is given by Equation (5):

$$\sigma^2 = \sum (D_i - \bar{D})^2 / (N - 1) \quad (5)$$

where the estimated mean microdensity, \bar{D} , is given by Equation (6):

$$\bar{D} = \sum D_i / N \quad (6)$$

and where

D_i is the microdensity of the layer as measured through an aperture centred at the point i ;

N is the number of points at which non-overlapping measurements are made.

Both sums in Equations (5) and (6) are performed over N points.

This estimator is not robust against outlying readings caused by dust, pinholes and other artefacts on the specimen. Because such artefacts do not represent intrinsic microdensity fluctuations of the specimen, an alternative estimator, called the median estimator, which is robust against these artefacts, is used (see Reference [10]). This median estimator is also robust against drifts in the mean microdensitometer readings (see Annex C). The median estimator is constructed as described in 9.3 below.

9.3 Construction of the median estimator and the 95 % confidence intervals

9.3.1 The median estimator is constructed by first segmenting the data into M subgroups, each consisting of, for example, $k = 10$ consecutive data values (see Reference [10]).

NOTE If the microdensitometer stage velocity chosen is equal to the product of the power line frequency and the distance traversed in ten consecutive data values, the possibility of power line noise introducing a variation in local mean density is reduced.

9.3.2 For each of the subgroups from $i = 1$ to $i = M$, calculate the usual sample variance, σ_i^2 .

Because the sample areas are non-overlapping, the microdensitometric data may be assumed to be independent and normally distributed.

Hence, for $i = 1, \dots, M$ and where σ_{sp}^2 is the variance of the specimen including the instrument noise, $\frac{9\sigma_i^2}{\sigma_{sp}^2}$ is distributed as a chi-square random variable with nine degrees of freedom.

9.3.3 The computation of the median estimator is carried out as described below.

Let $\sigma_1^2, \sigma_2^2, \dots, \sigma_M^2$, where $\sigma_1^2 < \sigma_2^2 < \dots < \sigma_M^2$, represent the M sample variances ordered from smallest to largest.

The median of the M variances, σ_{med}^2 , is given by Equation (7) if M is odd:

$$\sigma_{med}^2 = \sigma_{(M+1)/2}^2 \tag{7}$$

and by Equation (8) if M is even:

$$\sigma_{med}^2 = \frac{1}{2} \left[\sigma_{M/2}^2 + \sigma_{(M+2)/2}^2 \right] \tag{8}$$

On the basis of these assumptions, Equation (9) is an unbiased estimator for the variance σ_{sp}^2 :

$$\sigma_{sp}^2 = \left[\frac{(k-1)}{c} \right] \sigma_{med}^2 \tag{9}$$

where

- k is the number of consecutive observations within a subgroup;
- c is the critical value of the chi-square distribution for a significance level $\alpha = 0,5$ with $k - 1$ degrees of freedom.

EXAMPLE When $k = 10$, then $\sigma_{sp}^2 = \left(\frac{9}{8,343} \right) \sigma_{med}^2$ (see Annex D).

9.3.4 The computation of the upper and lower 95 % confidence intervals is carried out as described below.

An approximate 95 % confidence interval (see Reference [5]) on σ_{sp}^2 for $k = 10$ is given by Equation (10):

$$\left(\frac{9}{8,343}\right)\sigma_L^2 < \sigma_{\text{sp}}^2 < \left(\frac{9}{8,343}\right)\sigma_U^2 \quad (10)$$

where L and U are integers, such that:

$$L \leq \frac{(M+1) - 1,96M^{1/2}}{2} \quad (11)$$

$$U \geq \frac{(M+1) + 1,96M^{1/2}}{2} \quad (12)$$

9.4 Instrument noise

9.4.1 General

The median estimator method shall be used to determine instrument noise in a manner similar to that described in 9.3. Instrument noise is measured with no specimen present, but with measurement conditions otherwise identical to those when the specimen is present. Since instrument noise is generally a function of detector irradiance levels, it shall be measured over an optical density range equivalent to the projection density range produced by the specimen, and for the appropriate status density type.

A series of spectrally non-selective absorbers positioned in tandem with either the influx or efflux optical filtration may be used to provide the required range of optical densities. Instrument noise values for specific specimen density levels can then be computed, using linear interpolation between these density levels as needed. It is appropriate to calculate the instrument noise using the median estimator because instrument noise values, while not as prone to outlying values as the specimen, can exhibit slight drifts in the mean density.

9.4.2 Estimator for the instrument variance

In the same manner as in 9.3, an unbiased estimator for the instrument variance at optical density level \bar{D} for $k = 10$ is given by Equation (13):

$$\sigma_{\text{instr}}^2(\bar{D}) = \left(\frac{9}{8,343}\right)\sigma_{\text{med}}^2(\bar{D}) \quad (13)$$

An approximate 95 % confidence interval for instrument variance is given by Equation (14)

$$\left(\frac{9}{8,343}\right)\sigma_L^2(\bar{D}) \leq \sigma_{\text{instr}}^2(\bar{D}) \leq \left(\frac{9}{8,343}\right)\sigma_U^2(\bar{D}) \quad (14)$$

Where L and U are computed using Equations (11) and (12) and σ_i^2 , then $i = 1, \dots, M$ represent the M sample variances ordered from smallest to largest.

NOTE The diffuse conversion factor, g , is not required for the instrument noise computation since there is no specimen present to produce optical scattering of the efflux beam. The diffuse conversion factor, g , defined in 3.8 and applied in 9.5, serves only to convert the variance estimates to units of diffuse density.

9.4.3 Measurement capability

It is necessary that the ratio $\sigma_{sp}(\bar{D})/\sigma_{instr}(\bar{D})$ be equal to or greater than 5 for all reported rms-granularity measurements. This condition is necessary to ensure that the expected precision versus sample size estimates given in Annex E are valid for the measurement.

9.4.4 Estimation of instrument noise

To estimate the instrument noise for a mean specific density, \bar{D} , let D_1 and D_2 be those two nearest densities at which instrument noise has been measured satisfying $D_1 \leq \bar{D} \leq D_2$.

Proceed as follows.

Linearly interpolate between the variance estimates $\sigma_{instr}^2(D_1)$ and $\sigma_{instr}^2(D_2)$ to obtain $\sigma_{instr}^2(\bar{D})$.

Similarly, interpolate between the corresponding L and U confidence interval end points to obtain, for $k = 10$, $\left(\frac{9}{8,343}\right)\sigma_L^2(\bar{D})$ and $\left(\frac{9}{8,343}\right)\sigma_U^2(\bar{D})$, respectively.

9.5 Diffuse rms-granularity

The estimated variance of the instrument is subtracted from the estimated variance of the specimen-plus-noise determined in 9.3 to obtain the reported rms-granularity measurement, as shown in Equation (15):

$$\sigma_D = g \left[\sigma_{sp}^2(\bar{D}) - \sigma_{instr}^2(\bar{D}) \right]^{1/2} \quad (15)$$

9.6 Uncertainty of the rms-granularity result

It is recommended that the expanded uncertainty (95 % confidence level) for rms-granularity be less than ± 6 % of the expected value of σ_D . Minimum sample sizes required to obtain this expected uncertainty are found in Annex E.

NOTE This level of confidence will generally be achieved by use of a coverage factor, k , of approximately 2. The association of coverage factor and confidence level is based on assumptions regarding the probability distribution of measurement results. For a more thorough explanation, see Reference [19].

9.7 Reporting results

The rms-granularity measurement reported for a specimen shall include the following minimum information:

- σ_D ,
- the upper and lower 95 % confidence limits,
- \bar{D} ,
- the status density type, and
- the specification of the aperture used (48 μm circle or 42,5 μm square).

The σ_D and confidence limits shall be multiplied by 1 000 and reported to two significant figures; \bar{D} shall be reported to the second significant figure with no scaling factor applied.

These values represent the rms-granularity of the specimen for the specified density level and status density type.

9.8 Summary of rms-granularity characterization parameters

Table 1 provides a summary of rms-granularity characterization parameters.

Table 1 — Summary of rms-granularity characterization parameters

Parameter	Description	Specification
Illumination	Spectral distribution	CIE standard illuminant A
Measuring aperture	Circular	48,0 $\mu\text{m} \pm 0,5 \mu\text{m}$
	Square	42,5 $\mu\text{m} \pm 0,5 \mu\text{m}$
Influx aperture	Circular or square	72 μm to 96 μm
Influx optics	Achromatic/Apochromatic	0,3 \pm 0,1 N_{AI} (film)
	Microscope objective	\leq 0,25 N_{AI} (glass plate)
Efflux optics	Same as above	0,25 \pm 0,1 N_{AE}
Tube length	Standard	160 mm or 210 mm
Efflux magnification	Specimen to detector	measured \pm 5 %
Type of density	Direct viewing colour ^a	Status A Visual
	Colour negative	Status M
	Spectrally non-selective	Visual
k	Number of data points in subgroup	e.g. 10
M	Number of subgroups	Annex E
Sampling interval (Spacing between data points)	Circular	\geq 48 μm
	Square	\geq 42,5 μm
Scan path	Linear, segmented	None
Measurement capability	$\frac{\sigma_{sp}(\bar{D})}{\sigma_{instr}(\bar{D})}$	\geq 5:1
Product sampling	Reporting in accordance with this International Standard	3 batches
$\sigma_D \times 1000$	Reported rms-granularity	2 significant figures
\bar{D}	Reported mean density level	2 significant figures

^a If a single rms-granularity value is desired, a visual filter shall be used for the measurement of colour transparencies (such as colour reversal films).

Annex A (informative)

Typical viewing magnifications for critical naked-eye viewing

Viewing magnification, V_m , is defined as the ratio of the angle subtended at the observer by any image, which is a distance Z from the observer to the angle subtended at the observer, by the image on its original medium when the viewing distance is 355 mm, i.e.

$$V_m = \frac{\theta_2}{\theta_1} \quad (\text{A.1})$$

$$\theta_1 = \arctan\left(\frac{h_1}{710}\right) \quad (\text{A.2})$$

$$\theta_2 = \arctan\left(\frac{h_2}{2Z}\right) \quad (\text{A.3})$$

where

h_1 is the original image height, in millimetres;

h_2 is the image height in millimetres, viewed at a distance Z from the observer.

When both θ_2 and θ_1 are less than 0,1 radians, the small angle approximation yields $V_m \cong \frac{355h_2}{Zh_1}$, with all distances measured in millimetres. Thus, if the viewing distance Z equals 355 mm, then V_m is simply the image magnification.

Some typical imaging systems and viewing magnifications are given below.

EXAMPLE 1 12× enlargements from original media onto a hard copy:

viewing distance $Z = 355$ mm;

$$V_m = 12.$$

EXAMPLE 2 35 mm slide (24 mm × 36 mm) enlarged 28× onto a screen:

viewing distance $Z = 1\,000$ mm;

$$V_m = \frac{28 \times 355}{1\,000} = 9,94.$$

EXAMPLE 3 35 mm negative enlarged on to an 8 × 10 inch print:

viewing distance $Z = 355$ mm;

$$V_m = \frac{355 \times 203,2}{355 \times 24} = 8,47.$$

EXAMPLE 4 35 mm motion picture (aspect ratio 1,66: 1) [2]:

projectable image height $h_1 = 12,62$ mm;

image magnification: M_i ;

viewing distance equal to 3× the image heights: $Z = 3h_2 = 3M_i h_1$.

NOTE Viewing magnification is a constant: $V_m = \left(\frac{M_i h_1}{h_1} \right) \times \frac{355}{h_1} = 9,38$.

EXAMPLE 5 16 mm motion picture [1]:

projectable image height $h_1 = 7,26$ mm;

viewing distance: 3× the image heights;

$V_m = \frac{355}{3h_1} = 16,3$ (same mathematics as for 35 mm motion pictures).

Annex B (informative)

Limiting the temporal frequency response of the measuring instrument

As specified in 5.2, the use of a low-pass filter is required to restrict the temporal frequency response of the instrument to an equivalent spatial frequency response of 50 % at 50 mm⁻¹. The justification for selecting this cut-off characteristic is developed below. It is well known (see Reference [13]) that the density variance of a noise pattern as measured by a circular aperture can be written in terms of the Wiener spectrum, as follows:

$$\sigma_{D_{\text{proj}}}^2 \cong 2\pi \int W(u)A^2(u) u \delta u \quad (\text{B.1})$$

where

$\sigma_{D_{\text{proj}}}^2$ is the variance in projection density for small density fluctuations;

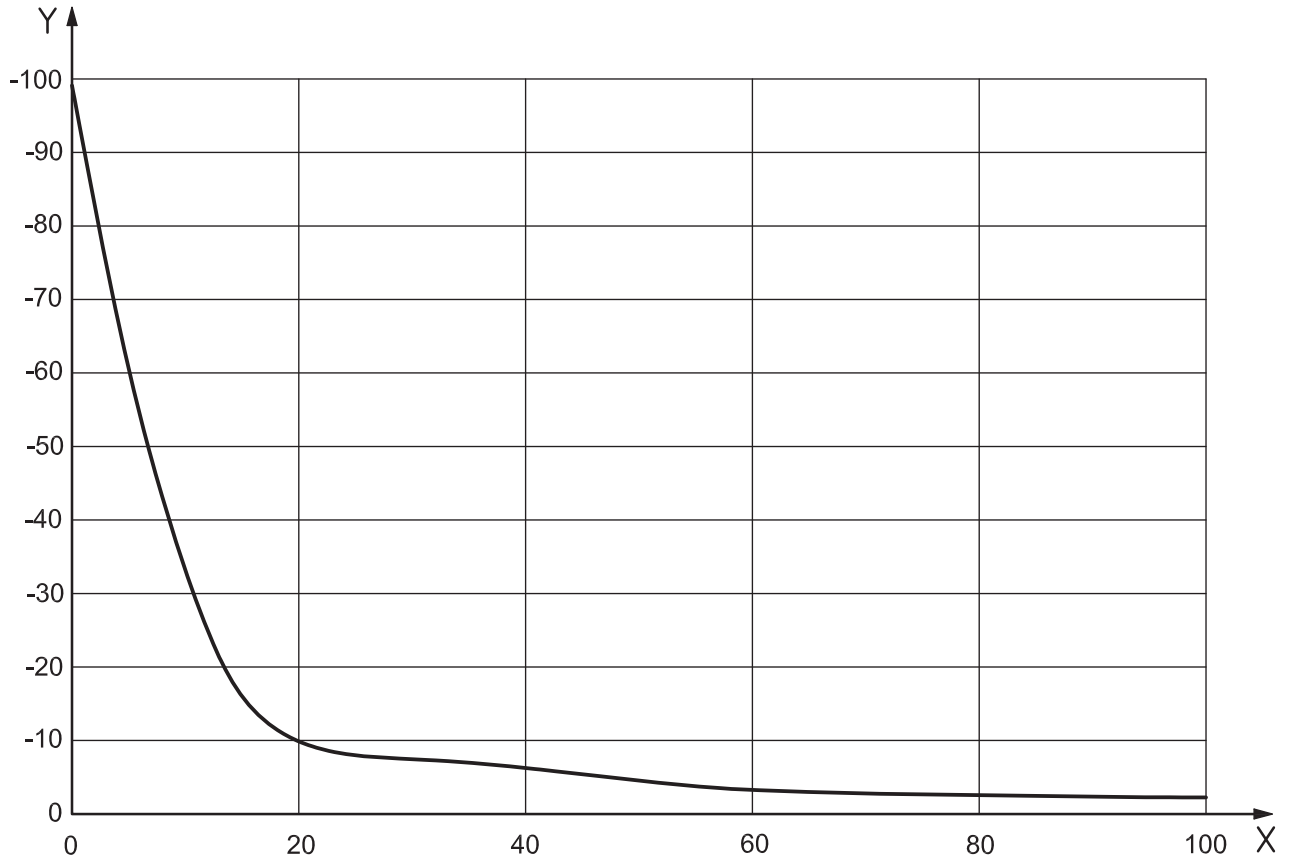
$W(u)$ is the Wiener spectrum;

$A(u)$ is the transfer function of the circular aperture;

u is the spatial frequency.

Here, the integration is carried out over the interval $[0, \infty)$. Consider the effect of truncating this integral to a finite interval $[0, u_{\text{max}}]$. The resulting measured variance σ^2 will now be less than the maximum value of $\sigma_{D_{\text{proj}}}^2$. Figure B.1 shows the percentage error introduced into the calculation of σ as a function of u_{max} for a specimen with a flat Wiener spectrum [$W(u) = \text{constant}$]. Since the function $A(u)$ is sharply peaked in the band of 0 mm⁻¹ to 25 mm⁻¹, it is sometimes assumed that the integration can be truncated to the first zero of $A(u)$, which occurs at 25,4 cycles/mm.

Figure B.1 shows that when the specimen's Wiener spectrum is flat, this introduces a bias of approximately -8 % into the calculation of σ . For $u_{\text{max}} = 50,8 \text{ mm}^{-1}$, which is near the second zero of the aperture function, the bias is approximately -5 %, which is within the statistical precision of the rms-granularity measurement described in this International Standard. As u_{max} approaches 100 cycles/mm, the bias error approaches -2,5 %. Therefore, the instrument temporal frequency cut-off may be limited to an equivalent spatial frequency of not less than 50 cycles/mm to restrict biasing of the rms-granularity.

**Key**X maximum spatial frequency, ν_{max} , in mm^{-1}

Y percentage error

Figure B.1 — Bias in σ for a 48 μm circular aperture as a function of the maximum frequency cut-off

Annex C (informative)

The effects of specimen non-uniformity

Specimen non-uniformity can significantly affect the accuracy in the estimation of rms-granularity, especially if the estimator used is non-robust against non-uniformity and the product is of very fine grain.

Because the effect depends on the magnitude and nature of the non-uniformity, it is impossible to quantify the effect in general. However, the following case is easy to quantify and it demonstrates the significance of specimen non-uniformity.

EXAMPLE 1 A linear change in mean microdensity with distance: suppose the mean microdensity of the specimen changes linearly along the scan path.

Using the usual variance estimator, the variance contribution due to this linear gradient, or “wedge”, equals approximately $(0,29\delta)^2$, where δ is the density difference between the end points of the scan. Hence, for the usual variance estimator, the estimated microdensity variability is given by Equation (C.1):

$$\sigma_{\text{sp,drift}}^2 = \sigma_{\text{sp,nodrift}}^2 + (0,29\delta)^2 \quad (\text{C.1})$$

If $\delta = \frac{\sigma_{\text{sp}}}{2}$, a bias error of 2,1 % is introduced into the estimate of σ_{sp}^2 .

If $\delta = \sigma_{\text{sp}}$, a bias error of 8,4 % is introduced into the estimate of σ_{sp}^2 .

EXAMPLE 2 If, rather than the usual variance estimator, the median estimator with $M = 150$ subgroups of size $k = 10$ is used, then the effect of the linear gradient on the variance estimate is given by Equation (C.2):

$$\sigma_{\text{sp,med,drift}}^2 = \sigma_{\text{sp,med,nodrift}}^2 + \left(\frac{0,29\delta}{150}\right)^2 \quad (\text{C.2})$$

If $\delta = \frac{\sigma_{\text{sp}}}{2}$, an expected bias error of 0,000 09 % is introduced into the estimate of σ_{sp}^2 .

If $\delta = \sigma_{\text{sp}}$, an expected bias error of 0,000 37 % is introduced into the estimate of σ_{sp}^2 .

The advantage of the median estimator over the usual estimator is even greater when there are outliers in the data.

Annex D (informative)

Derived constants c for subgroup sizes 10, 20, ..., 200

The derived constant, c , in Equation (9) (see 9.3.3) is the critical value of the chi-square distribution for a significance level $\alpha = 0,5$ with $k - 1$ degrees of freedom. For convenience, values of the constant c for subgroup sizes k equal to 10, 20, ..., 200 are listed below. For other subgroup sizes, the constant c may be derived as needed. These values were generated using the GAMINV function in SAS, Version 6.

Table D.1 — Derived constants c for subgroup sizes 10, 20, ... , 200

Subgroup size k	Derived constant c
10	8,343 00
20	18,337 65
30	28,336 13
40	38,335 40
50	48,334 97
60	58,334 69
70	68,334 49
80	78,334 34
90	88,334 23
100	98,334 14
110	108,334 10
120	118,334 10
130	128,333 90
140	138,333 90
150	148,333 90
160	158,333 80
170	168,333 80
180	178,333 80
190	188,333 80
200	198,333 70

Annex E (informative)

Determination of sample size for specified precision and subgroup size

The precision of an rms-granularity estimate is determined by the 95 % confidence interval associated with it. The parameters that determine the confidence interval are:

- the size of the subgroups k ,
- the number of subgroups M ,
- the estimated variance itself, and
- the instrument noise variance.

In principle, the confidence interval is limited by the ratio $\frac{\sigma_{sp}}{\sigma_{instr}}$, but for ratios much greater than 1, an arbitrarily short interval may be calculated by selecting M sufficiently large for a fixed value of k . In 9.3, the equations for computing the upper and lower 95 % confidence interval on σ_D are expressed in terms of specific elements of the measured chi-square distributions $\sigma_L^2(\bar{D})$ and $\sigma_U^2(\bar{D})$. Therefore, the 95 % confidence interval for the σ_D estimate $\sigma_{med}(\bar{D})$, can be empirically calculated directly from the distribution of measured subgroup variances. In order to ensure that, on average, these limits will fall within a specified 95 % confidence interval, the total sample size N associated with the specified confidence interval shall be known. The required sample size for a 95 % confidence interval of $2p$ can be expressed as a probability statement on the ratio of $\frac{\sigma_{sp}}{\sigma_{instr}}$, when the subgroup size $k = 10$ and $M_{10} = \frac{N}{10}$, as follows:

$$p = \frac{\sigma_{sp}}{\sigma_{instr}} = \left(1 + \frac{1,219}{M_{10}} \right)^{1/2} \quad (E.1)$$

and

$$N_p = \frac{14,87}{(p^2 - 1)^2} \quad (E.2)$$

Table E.1 gives values of N_p computed from Equation (E.2) for 95 % confidence intervals of $2p$.

Table E.1 — N_p computed from Equation (E.2) for 95 % confidence intervals of $2p$

p	N_p^a	Precision
1,01	36 810	± 1 %
1,02	9 110	—
1,03	4 010	—
1,04	2 240	—
1,05	1 420	—
1,06	980	± 6 %
1,07	710	—
1,08	540	—
1,09	420	—
1,10	340	± 10 %
^a N_p is rounded up in divisions of 10.		

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