INTERNATIONAL STANDARD

ISO 10360-6

First edition
2001-12-15

Geometrical Product Specifications (GPS) — Acceptance and reverification tests for coordinate measuring machines (CMM) —

Part 6: Estimation of errors in computing Gaussian associated features

Spécification géométrique des produits (GPS) — Essai de réception et de vérification périodique des machines à mesurer tridimensionnelles (MMT)

Partie 6: Estimation des erreurs dans le calcul des éléments associés gaussiens

Reference number
ISO 10360-6:2001(E)

PDF disclaimer

This PDF file may contain embedded typefaces. In accordance with Adobe's licensing policy, this file may be printed or viewed but shall not be edited unless the typefaces which are embedded are licensed to and installed on the computer performing the editing. In downloading this file, parties accept therein the responsibility of not infringing Adobe's licensing policy. The ISO Central Secretariat accepts no liability in this area.

Adobe is a trademark of Adobe Systems Incorporated.

Details of the software products used to create this PDF file can be found in the General Info relative to the file; the PDF-creation parameters were optimized for printing. Every care has been taken to ensure that the file is suitable for use by ISO member bodies. In the unlikely event that a problem relating to it is found, please inform the Central Secretariat at the address given below.

ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.ch
Web www.iso.ch

Printed in Switzerland

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this part of ISO 10360 may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

International Standard ISO 10360-6 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO 10360 consists of the following parts, under the general title *Geometrical Product Specifications (GPS) — Acceptance and reverification tests for coordinate measuring machines (CMM)*:

— *Part 1: Vocabulary*

— *Part 2: CMMs used for measuring size*

— *Part 3: CMMs with the axis of a rotary table as the fourth axis*

— *Part 4: CMMs used in scanning measuring mode*

— *Part 5: CMMs using multiple-stylus probing systems*

— *Part 6: Estimation of errors in computing Gaussian associated features*

Annex A forms a normative part of this part of ISO 10360. Annex B is for information only.

Introduction

This part of ISO 10360 is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO/TR 14638). It influences link 5 of the chains of standards on size, distance, radius, angle, form, orientation, location, run-out and datums.

For more detailed information of the relation of this part of ISO 10360 to other standards and the GPS matrix model see annex B.

Coordinate measurement technology is widely used in industrial metrology to assess features of a workpiece. A common requirement is to fit an associated feature to a data set consisting of coordinate measurements of a real feature. This fitting is carried out by software.

Software for calculating an associated feature provides values of parameters of the associated feature that are descriptive of the size, shape, location and orientation of the feature. These parameters are useful

— for the purpose of carrying out calculations involving the feature, often in conjunction with other associated features and other information, and

— in determining the extent to which a workpiece satisfies dimensional and positional specifications.

The reliability of information about features that is determined from associated features is influenced by the quality of the software for computing these features.

The tests defined in this part of ISO 10360 are concerned with assessing the correctness of the parameters of computed associated features as measured by a coordinate measuring machine (CMM) or other coordinate measuring system. Although different criteria may be used to compute associated features, for example, by minimizing the Euclidean or Chebyshev norm of residuals, this test is applicable for software designed for unconstrained Gaussian (least-squares) features.

In the case of reverification tests of CMMs, the software test of this part of ISO 10360 usually does not provide new or different information in comparison with that obtained by an acceptance test, since software is supposed to be stable over time. However, a reverification test of the software may be useful following possible corruption or alteration of the software under test.

For software already in existence, the evaluation of the performance may not be obtained only by fulfilling the requirements of this part of ISO 10360. However, such cases do not necessarily exclude the ability of the software to perform correct computation of measurements.

This part of ISO 10360 is applicable to software submitted for test in respect of the values it provides for the parameters of an associated feature. The test procedure is based on applying the software under test to reference data sets, and comparing the results obtained with reference results.

Geometrical Product Specifications (GPS) — Acceptance and reverification tests for coordinate measuring machines (CMM) —

Part 6: Estimation of errors in computing Gaussian associated features

1 Scope

This part of ISO 10360 specifies a method for testing software used for computing associated features from coordinate measurements. The features of concern are the line (in two and three dimensions), the plane, the circle (in two and three dimensions), the sphere, the cylinder, the cone and the torus.

One or more separate tests are required for each feature claimed to be covered by the software.

The test is of the software alone and therefore independent of the coordinate measuring system.

NOTE 1 If the result of the test indicates that the performance values for linear size parameters of the associated feature are significant compared with the error of indication of a CMM for size measurement (see ISO 10360-2), as provided by the CMM manufacturer, the software is inadequate for application on that measuring system. However, small performance values, obtained as a result of this test, do not provide complete assurance that the software is totally suitable for computing associated features.

This part of ISO 10360 is concerned with complete features and non-extremely partial features; however, the test for complete features and that for partial features are separate, and software may be submitted for either or both tests.

Cones with very large apex angles are not covered by the test.

NOTE 2 Associated cones with very large angles are unusual in practice and the software for their stable computation is not widely available.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this part of ISO 10360. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this part of ISO 10360 are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 10360-1:2000, *Geometrical product specifications (GPS) — Acceptance and reverification tests for coordinate measuring machines (CMM) — Part 1: Vocabulary*

ISO 10360-2:2001, *Geometrical Product Specifications (GPS) — Acceptance and reverification tests for coordinate measuring machines (CMM) — Part 2: CMMs used for measuring size*

ISO 14253-1:1998, *Geometrical product specifications (GPS) — Inspection by measurement of workpieces and measuring equipment — Part 1: Decision rules for proving conformance or non-conformance with specifications*

ISO 14660-1:1999, *Geometrical product specifications (GPS) — Geometrical features — Part 1: General terms and definitions*

ISO 14660-2:1999, *Geometrical product specifications (GPS) — Geometrical features — Part 2: Extracted median line of a cylinder and a cone, extracted median surface, local size of an extracted feature*

International Vocabulary of Basic and General Terms in Metrology (VIM). BIPM, IFCC, IEC, ISO, IUPAC, IUPAP, OIML, 2nd edition, 1993

3 Terms and definitions

For the purposes of this part of ISO 10360, the terms and definitions given in ISO 10360-1, ISO 14660-1, ISO 14660-2 and VIM apply.

4 Basic requirements

The following basic requirements shall be met by the software supplier.

a) The software under test shall have an unambiguous and unique identification (e.g. a release number).

Improper applications of the test result to other versions of the software under test are forbidden. The testing body is allowed to satisfy the request by an owner of (a license of) the software under test and its test certificate to re-run the test based on the reference data sets identified by the release number reported in the test certificate.

b) The software under test shall provide a means of

1) direct input of a reference data set and output of test parameter values to adequate numerical precision (see clause 8), bypassing the measurement and software correction parts of the system, and

2) inputting 2D coordinates to the software under test for computing 2D associated features (line and circle in two dimensions); if this is not available, it is tolerated to add a dummy null z coordinate to each point in the reference data sets, thus projecting the feature onto the xy coordinate plane.

NOTE 1 The input and output procedures associated with some measuring systems may be limited in terms of the numerical precision of the values transmitted. This limitation may disadvantage the software under test in terms of the test results obtained.

c) The method of input to, and output from, the processor is to be agreed with the testing body.

NOTE 2 It may be convenient to use a standard computer-readable medium in a standard format (e.g. ASCII on a 3,5" disk).

d) Corresponding to each feature for which the software under test is to be tested, a statement of the parametrization of the feature used by the software under test shall be provided.

NOTE 3 Reference parametrizations are given in Table 3.

e) Corresponding to each feature for which the software under test is to be tested and to the test type (see Table 2), a statement of the maximum permissible errors, MPEq, of the relevant parameter classes (see 9.3) shall be provided.

5 Reference data sets and reference parameter values

5.1 General

The reference data sets and the corresponding reference parameter values used for the purposes of testing the software under test shall be generated according to the procedure specified in annex A. The reference data sets are designed to simulate a range of sizes, shapes, locations, orientations and sampling of features. They are also designed to simulate typical CMM errors of measurement, including probing errors, and feature form deviations.

The reference data sets and the reference parameter values generated according to annex A shall be used only once for verification of any software under test (see A.1).

5.2 Initial estimates of parameter values

Software under test may require that a subset of the points input to the software, usually the first ones in the set, has a predefined sampling pattern. This subset is used to determine the initial estimates of the parameter values. When this requirement is written in the operating instructions of the software under test, and upon request of the software supplier, the testing body shall generate additional points consistent with the predefined sampling pattern. These additional points form the subset added to the data as generated according to annex A to form a reference data set. These circumstances shall be noted on the test certificate [see clause 11, e)].

NOTE 1 The software under test typically employs iterative methods of calculation for determining the values of the parameters of the associated feature. For this purpose, identification of a subset of points may be required, from which initial estimates of these values can be computed.

NOTE 2 A Gaussian associated cylinder can be used for purposes of illustration: the first six points in a reference data set could be identified as a subset for initial-estimation purposes. For instance, the line joining the centres of the circles defined by the first three points and the second three points could be used as an approximation to the axis of the associated cylinder, and the radii of these circles could be used as approximations to the radius of the associated cylinder.

NOTE 3 Software under test which does not require initial estimates of the parameter values is more robust, being self-contained, and does not impose an operating procedure for measuring real features.

6 Test parameter values and converted test parameter values

Since different software suppliers may use different parametrizations, for the purposes of the test, the test parameter values produced by the software under test shall be modified, if necessary, by applying a conversion rule to produce converted test parameter values. The converted test parameter values so derived correspond to the same parametrization as the reference parameter values and can meaningfully be compared with them.

For this purpose, the software supplier shall provide full details of the test parametrization.

When necessary, the testing body shall implement and apply the appropriate conversion rule.

It is recommended that the software supplier provide test parameter values to adequate numerical resolution (see clause 8), in order that uncertainty may not be unnecessarily added in producing converted test parameter values.

Software under test may fail to produce results for some reference data sets.

NOTE Failure to produce results may be due to, for example:

a) the software under test indicating that the data set cannot be processed because it is beyond its domain of application (e.g. it contains too many data points or the data points are unsuitably distributed), or

b) lack of convergence of an iterative algorithm, or

c) a fatal error that has arisen during execution of the software (e.g. a floating-point overflow or an attempt to take the square root of a negative number).

7 Units

The units specified in Table 1 shall be used.

Table 1 — Units

	Reference data sets	Reference parameter values
Point coordinates	millimetres	—
Location parameters	—	millimetres
Size parameters	—	millimetres
Angle parameters	—	radians
Orientation parameters	—	(dimensionless) [a]
Submultiples, for example, micrometres and microradians, may be used on the test certificate for quoting differences between the converted test parameter values and corresponding reference parameter values and their uncertainties.		
[a] The orientation parameters are expressed as direction cosines.		

8 Numerical uncertainty

It is the responsibility of the testing body to evaluate any numerical uncertainty introduced by the finite number of digits used to transfer information and to represent numerical values computationally. This numerical uncertainty shall be included in the uncertainty statement reported in the test certificate (see clause 10).

NOTE 1 Information transferred includes point coordinates in reference data sets and reference parameter values (controlled by the testing body), as well as test parameter values (submitted by the software supplier).

NOTE 2 The computational representation affects the calculation of reference parameter values from reference data sets (in the case of reference software, see Figure 2) or reference data sets from reference parameter values (in the case of data generators, see Figure 3), in applying conversion rules and in calculating q values [see 9.3, d)].

NOTE 3 The numerical uncertainty also depends on how well a Gaussian associated feature is defined by a reference data set or, equivalently, the numerical condition of the fitting problem (a measure of the perturbation in the reference parameter values relative to a small perturbation in the coordinate values in a reference data set). The condition is influenced by the type of feature, and by the number and locations of points in the reference data set.

NOTE 4 The numerical uncertainty can be estimated by simulation if an analytical evaluation is not straightforward.

Depending on the manner in which the information is produced, the testing body may regard either the reference data sets or the reference parameter values as exact, provided the appropriate uncertainty is determined in the other.

9 Application of the test method

9.1 Principle

The principle underlying the method of test is that of comparing converted test parameter values with reference parameter values (see Figure 1). The converted test parameter values are obtained by applying the software under test to the reference data sets to obtain test parameter values, and by applying a conversion rule to these test parameter values. Each reference data set and the corresponding reference parameter values are regarded as a reference pair for testing purposes.

NOTE 1 The testing body will provide reference pairs using, for example, reference software or a data generator, as illustrated in Figures 2 and 3.

Different software under test may be intended, and regularly used, for different applications (e.g. for calculating complete or partial features), or with measurement points affected by small or large noise or form deviations or both. To tailor the test accordingly, four types of tests are possible, as summarized in Table 2; simplified tests are subsets of the corresponding regular tests, and are intended for software not designed for severe applications. The software supplier may choose the test or tests to which to submit the software under test; the test or tests chosen shall be reported in the test certificate.

NOTE 2 Since no partial features can be defined for lines in two and three dimensions and planes, tests are not available for these features in tests of the following types: partial feature, simplified test; partial feature, regular test.

A separate test shall be carried out for each feature and each test type (see Table 2).

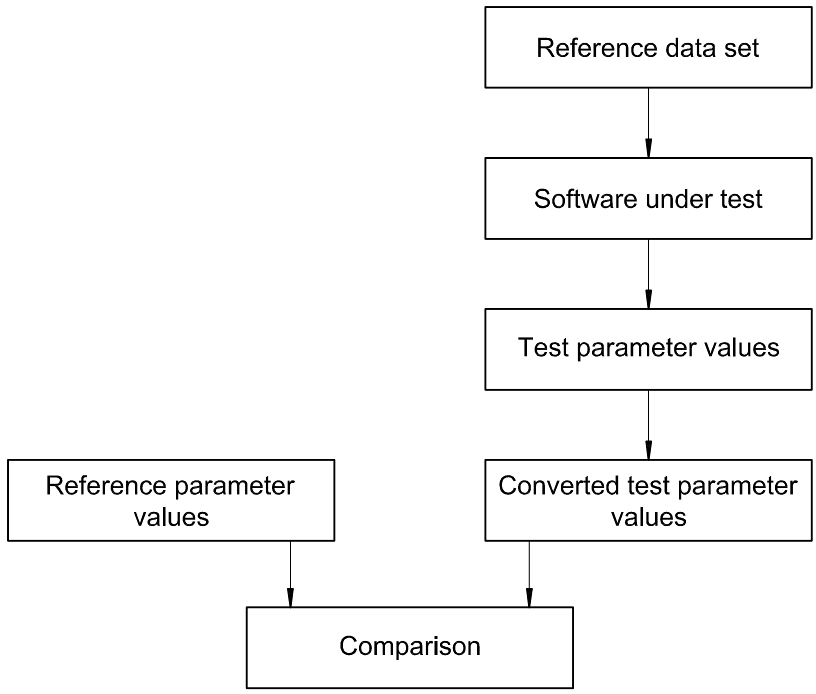

Figure 1 — Principle of test method

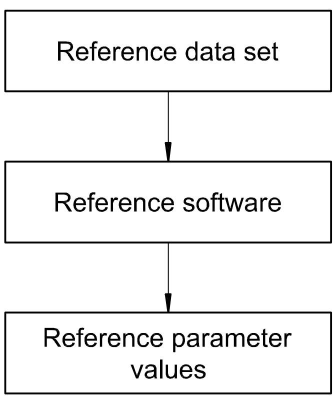

Figure 2 — Use of reference software for producing reference pairs

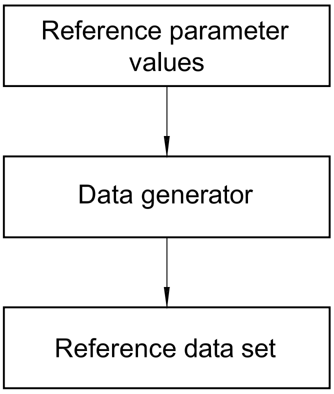

Figure 3 — Use of data generator for producing reference pairs

Table 2 — Types of tests

	Simplified test	**Regular test**
Full feature	Full feature, simplified test	Full feature, regular test
Partial feature	Partial feature, simplified test [a]	Partial feature, regular test [a]
[a] Not available for lines in two and three dimensions and planes.		

9.2 Basis for comparison

For any application of software under test for a specific feature to a reference data set for that feature, the basis for comparing converted test parameter values with reference parameter values is a performance value, p, for each class of parameter values defined as follows.

a) **Location parameters:** Euclidean distance between the location points (x_0, y_0) or (x_0, y_0, z_0) (see Table 3), defined by the converted test parameter values and the reference parameters, respectively.

b) **Orientation parameters:** Positive angle between the unit vectors (a, b) or (a, b, c) (see Table 3), defined by the converted test parameter values and the reference parameters, respectively.

Since very small angles are expected, when evaluating p particular care should be taken not to incur any significant numerical errors. If v and w are the two unit vectors, a numerically stable and recommended equation is:

$$p = 2 \arcsin\left(\frac{\|v - w\|}{2}\right)$$

c) **Size parameters:** Positive difference or differences of the corresponding size parameters r or r_1 or r_2 (see Table 3) within the sets of converted test parameter values and reference parameters, respectively. In the case of the torus, for which there are two size parameters, r_1 or r_2, the larger of the two positive differences is taken.

d) **Angle parameter:** Positive difference of the angle parameters, ψ (see Table 3), within the sets of converted test parameter values and the reference parameters, respectively.

The magnitude of the performance value is a measure of how well the converted test parameter values compare with the corresponding reference parameter values: the smaller the value, the better the degree of agreement.

9.3 Procedure

The testing body shall take the following steps for each feature and each test type.

a) Instruct the software supplier to apply the software under test to the reference data sets for that feature to obtain test parameter values; this step may alternatively be carried out by the testing body if the software supplier provides a controlled documented copy of the software under test.

b) Require the software supplier to state the measuring volume at which the test will be performed and the test results will be valid.

c) For each reference data set for that feature for which the software under test produced parameter values:

   1) if the test parametrization is different from the reference parametrization, apply a conversion rule to the test parameter values to produce converted test parameter values; otherwise, regard the test parameter values as the converted test parameter values;

   2) determine the performance values, p, as described in 9.2, for each class of parameter values relevant to the feature;

   3) unless the feature under consideration is a circle in two dimensions or a sphere or a cone, for which this step does not apply, re-compute the performance value for the orientation parameters after changing the signs of the orientation parameters within the set of converted test parameter values; repeat step 2) for these parameters, and take the most favourable result.

   NOTE 1 If the orientation parameters for a line or an axis are all changed in sign, the resulting parameters define the same line or axis (but pointing in the opposite direction). Hence, if orientation parameters are present, the test is based on comparing the vector defined by either the converted test parameter values corresponding to orientation with the corresponding reference parameter values, or such parameters, after all their signs are changed, with the corresponding reference parameter values.

   NOTE 2 The cone is the only feature for which the reversal of the direction of its axis is important, since the unit vector defining the orientation of the axis points towards the apex of the cone (see Table 3).

d) For each class of parameter values of the feature (location, orientation, size or angle, as appropriate), define an overall value, q, as the largest of the performance values, p, determined in step c), 2).

e) For each class of parameters of the feature (location, orientation, size or angle, as appropriate), report a record of the test result in the test certificate in the following forms:

   1) the corresponding value q and its numerical uncertainty;

   2) if the software under test failed to produce test parameter values for at least a reference data set, “FAIL IN n DATA SETS” (with n being the number of failed reference data sets), and the test result derived from the test parameter values actually produced.

10 Compliance with specification

The performance of the software under test is considered to have been verified if no “FAIL” is reported and none of the q values are greater than the corresponding maximum permissible errors, MPEq, taking into account the numerical uncertainty in accordance with ISO 14253-1. In the case where compliance with specification is numerical assessed by the testing body, the result of this assessment shall be part of the test certificate.

11 Test certificate

The test certificate issued by the testing body shall include the following.

a) Title (e.g. Test certificate).

b) Name and address of the software supplier, and the location or locations (if different) where:

   1) the test was carried out (physical location);

   2) the software under test was applied to the reference data sets.

c) Unique identification of the test certificate, such as the serial number, and of each page and total number of pages.

d) Name and address of the client, where appropriate.

e) Description and unambiguous identification (e.g. release number of software under test).

f) Identification of the hardware operating systems.

g) Date of application for the test, date of agreement to carry out test, and date or dates of test, as appropriate.

h) Statement to the effect that the test was carried out in accordance with this part of ISO 10360.

i) Description and unambiguous identification (e.g. the release number of the reference data sets).

j) Size of the measuring volume for which the test certificate is valid.

k) For each feature tested and for each test type:

   1) the type of feature (e.g. cylinder);

   2) the type of test (e.g. full feature, regular test);

   3) the test result (see 9.3), including the corresponding uncertainty statement;

   4) a statement of compliance with specified MPEq (only when the compliance assessment is made by the testing body);

   5) an acknowledgement of whether or not the software under test made use of an identified subset of points in the reference data sets to provide initial estimates of the parameter values.

l) Signature and title, or an equivalent authenticated identification of the person or persons accepting responsibility for the content of the test certificate, as well as the date of issue.

m) A statement to the effect that the results relate only to the software items tested and to the reference data sets used.

n) A statement to the effect that the test certificate is not to be reproduced except in full, without the written approval of the testing body.

Table 3 — Reference parametrizations for associated features

Associated feature	Parameters of the associated feature: Location mm	Orientation	Size mm	Angle rad	Description
Line (2D)	x_0, y_0	—	—	—	Centroid of the defining data set (lying on the associated line)
	—	a, b	—	—	Direction cosines of the associated line
Line (3D)	x_0, y_0, z_0	—	—	—	Centroid of the defining data set (lying on the associated line)
	—	a, b, c	—	—	Direction cosines of the associated line
Plane	x_0, y_0, z_0	—	—	—	Centroid of the defining data set (lying on the associated plane)
	—	a, b, c	—	—	Direction cosines of the normal to the associated plane
Circle (2D)	x_0, y_0	—	—	—	Centre of the associated circle
	—	—	r	—	Radius of the associated circle
Circle (3D)	x_0, y_0, z_0	—	—	—	Centre of the associated circle
	—	a, b, c	—	—	Direction cosines of the normal to the plane containing the associated circle
	—	—	r	—	Radius of the associated circle
Sphere	x_0, y_0, z_0	—	—	—	Centre of the associated sphere
	—	—	r	—	Radius of the associated sphere
Cylinder	x_0, y_0, z_0	—	—	—	Point on the axis of the associated cylinder closest to the centroid of the defining data set
	—	a, b, c	—	—	Direction cosines of the axis of the associated cylinder
	—	—	r	—	Radius of the associated cylinder
Cone	x_0, y_0, z_0	—	—	—	Point on the axis of the associated cone closest to the centroid of the defining data set
	—	a, b, c	—	—	Direction cosines of the axis of the associated cone, forming a unit vector pointing towards the apex of the cone.
	—	—	r	—	Radius of the associated cone at the point (x_0, y_0, z_0) measured normal to the axis of the cone
	—	—		ψ	Apex angle of the associated cone (equal to twice the angle between the generator of the cone and the axis)
Torus	x_0, y_0, z_0	—	—	—	Centre of the associated torus
		a, b, c	—	—	Direction cosines of the axis of the associated torus
	—	—	r_1	—	Radius of the circular section of the tube of the associated torus
	—	—	r_2	—	Mean radius of the ring of the associated torus

Annex A
(normative)

Procedure for generating reference data sets

A.1 General

This annex describes the procedure for generating the reference data sets, such that the homogeneity of different releases is ensured. It is therefore intended mainly for testing bodies and software developers.

The whole release of reference data sets shall be identified by a unique release number; the same release cannot be used to carry out more than one complete test, i.e. different test certificates shall report different reference data set release numbers, unless the test is a repetition of a previous one (see Note 1 to clause 4).

For each associated feature, four different types of reference data sets are possible (see Table A.1):

a) FI and PI reference data sets are used for

- full feature, and
- partial feature,

respectively, for regular tests (see Table 2) only;

b) FM and PM reference data sets are used for

- full feature, and
- partial feature,

respectively, for both simplified and regular tests.

Table A.1 — Types of reference data sets and their acronyms

	Mild (M)	**Intense (I)**
Full feature (F)	FM	FI
Partial feature (P)	PM [a]	PI [a]

[a] Not available for lines in two or three dimensions or for planes.

For each associated feature and reference data set type, 10 reference data sets are provided. They are named `feature.reference_data_set_type set_number`, where `feature` is a feature identifier to be selected in the list `{Line2D, Line3D, Plane, Circle2D, Circle3D, Sphere, Cylinder, Cone, Torus}`; `reference_data_set_type` is a reference data set type identifier to be selected from the list `{FM,FI,PM,PI}`; `set_number` is a one digit ordinal number of the reference data set in the range [0..9]. Examples of valid reference data set names are `circle3D.FI8` and `torus.PM0`.

NOTE For each associated feature, a simplified test involves 10 reference data sets (FM or PM), while a regular test involves 20 reference data sets (FM and FI, or PM and PI).

The points of the reference data sets are located in a measuring volume shaped as a rectangular parallelepiped; the reference system is oriented along the rectangular parallelepiped sides with origin in the central point of the rectangular parallelepiped. The rectangular parallelepiped size shall be specified by the software supplier and reported in the test certificate [see clause 11, j)].

Each reference data set shall be generated according to the following steps:

— a nominal extent of the associated feature is generated;

— form deviations are superimposed by deforming the nominal extent;

— the nominal extent is sampled to generate nominal points projected orthogonally onto the deformed extent;

— Gaussian zero-mean noise is added to each point to simulate probing and other errors.

The above steps are illustrated and specified in the following clauses.

A.2 Random generation

In this procedure, many values are required to be generated at random. The random generations shall be done according to the following rules.

a) Scalar value x specified within the interval $[a, b]$, $a \leqslant x \leqslant b$: take $x = a$ and $x = b$ for two data sets, respectively; for each of the others, generate a random value y uniformly distributed in the interval [0,1], and compute $x = a^{1-y} b^{y}$.

b) Scalar value x specified within two intervals $[a, b]$ and $[c, d]$, $a \leqslant x \leqslant b$ or $c \leqslant x \leqslant d$: divide the data sets into two groups of five; for either group, generate x in the interval $[a, b]$ and $[c, d]$, respectively, according to rule a).

c) Integer value n specified within the interval $[a, b]$, $a \leqslant n \leqslant b$: apply rule a) and round up to the nearest integer value.

d) 2D orientation (as represented by the unit vector $\boldsymbol{n}$): take $\boldsymbol{n} = (1,0)$ and $\boldsymbol{n} = (0,1)$ for two data sets, respectively; for each of the others, generate a random value φ uniformly distributed in the interval $[0,2\pi]$, and compute $\boldsymbol{n} = (\cos\varphi, \sin\varphi)$.

e) 3D orientation (as represented by the unit vector $\boldsymbol{n}$): take $\boldsymbol{n} = (1,0,0)$, $\boldsymbol{n} = (0,1,0)$ and $\boldsymbol{n} = (0,0,1)$ for three data sets, respectively; for each of the others, generate two random values, θ and z, uniformly distributed in the intervals $[0,2\pi]$ and $[-1,1]$, respectively, and compute $\boldsymbol{n} = (\cos\theta\sqrt{\left(1-z^2\right)}, \sin\theta\sqrt{\left(1-z^2\right)}, z)$.

f) Make the choice of the reference data sets where predefined values (extremes of intervals or coordinate directions) are taken according to rules a), b), c), d) and e) at random by generating an integer value uniformly distributed in the interval [0,9], to be used as the reference data set identifier.

g) Localization (as defined by a point chosen in the smallest convex region containing the extent): generate at random the values of the two or three coordinates uniformly distributed in the corresponding intervals of definition of the measuring volume; if the extent so localized is not completely contained within the measuring volume, generate the reference point again, and repeatedly, until the constraint is met.

h) Form deviations (see A.4): for each Fourier harmonic, generate two random values, x and φ, uniformly distributed in the intervals $[-1,1]$ and $[0,2\pi]$, respectively, and to be used as the amplitude in arbitrary units and the phase, respectively; after sampling (see A.5), calculate the resulting form deviations in all points and derive the maximum of their absolute values x_{max}; generate a random value κ uniformly distributed in the interval [–1,1], and re-scale all form deviations by the factor $\kappa\zeta$ / x_{max}, where ζ is the maximum form deviations specified in A.4.

i) Sampling (see A.5): for the line (2D and 3D), plane and circle (2D and 3D), generate a point uniformly distributed in each extent subset; for the other associated features, generate two random values of the surface coordinates of the point, uniformly distributed in their intervals of definition of the subset; in the case of cones not meeting the definition in the first footnote to Table A.5, the z coordinate is taken as $z = \sqrt{(c + z_{min}{}^2)}$ instead, where c is a random value uniformly distributed in the interval [$z_{min}{}^2$, $z_{max}{}^2$], the interval of definition of the extent subset z being [z_{min}, z_{max}].

j) Noise (see A.6): for each sampled point, generate a zero-mean random value, x, normally distributed with the standard deviation specified in Table A.6; define a unit vector, $\boldsymbol{u}$, normal to the nominal extent directed externally to the associated feature (2D circle, sphere, cylinder, cone, torus), or coincident with the orientation unit vector (a, b, c) (plane), or as obtained by rotating 90° clockwise the unit vector (a, b) (2D line), or generate $\boldsymbol{u}$ as a 2D orientation in the plane normal to the extent according to rule d) but overriding the constrained directions (3D line, 3D circle); take $x\boldsymbol{u}$ as a noise vector.

NOTE 1 The value x generated according to rule a) possesses the property of a uniformly distributed logarithm.

NOTE 2 The unit vector $\boldsymbol{n}$ generated according to rule e) is uniformly distributed over the unit sphere.

NOTE 3 The form deviations generated according to rule h) possess the property of a maximum form deviation uniformly distributed over the specified interval [0, ζ].

NOTE 4 The sampled point generated according to rule i) in the particular case of cones is uniformly distributed over the cone extent.

A.3 Generation of nominal extents

The orientations of the associated features from which the extents are taken shall be chosen at random, with the exception of 2D circles and spheres which have no orientation. Additional orientations are required for localizing each extents onto the corresponding associated feature; they shall be generated at random according to the following rules (see A.2):

a) (plane) orientation of a rectangle side: generate a 2D orientation in the plane;

b) (circle) orientation of the radial vector pointing to the central point of the arc: generate a 2D orientation in the circle plane;

c) (sphere) orientation of the radial vector to the point defined by the central values of (theta, phi) in their intervals (see 11.14 of ISO 10360-1:2000): generate a 3D orientation;

d) (cylinder and cone) orientation of the shortest vector from the axis to the point defined by the central values of (theta, z) in their intervals (see 11.14 of ISO 10360-1:2000): generate a 2D orientation in a plane normal to the axis;

e) (torus) orientation of the shortest vectors from the axis and from the ring to the point defined by the central values of (theta, phi) in their intervals (see 11.14 of ISO 10360-1:2000): generate two 2D orientations in a plane normal to the axis and in a plane containing it; this is equivalent to collapsing the torus to a sphere of radius r_1 by setting r_2 = 0, and to generating a 3D orientation.

The extent sizes, defined as the maximum distances between any pair of extent points, shall be generated at random in the intervals reported in Table A.2, specified in terms of percentages of the measuring volume size, which is defined as the shortest side of the rectangular parallelepiped.

Table A.2 — Percentages of the measuring volumes size covered by the extent sizes

Reference data set type	Range of percentage
FM, FI	[10 %, 90 %]
PM, PI	[1 %, 15 %] or [85 %, 99 %]

The localization of the extents shall be generated at random. The extents shall fulfil the further requirements listed in Table A.3.

Table A.3 — Further specifications of the extents

Associated feature	Reference data set type	Specifications [a]
Line (2D and 3D)	—	No further specification
Plane	—	(Greater) ratio ξ of length of the sides of a rectangle:
	FM	$1 \leqslant \xi \leqslant 10$
	FI	$1 \leqslant \xi \leqslant 100$
Circle (2D and 3D)		Centred angle α:
	FM, FI	$\pi\text{rad} \leqslant \alpha \leqslant 2\pi\text{rad}$
	PM, PI	$\frac{1}{8}\pi\text{rad} \leqslant \alpha \leqslant \pi\text{rad}$
Sphere	FM, FI	$\frac{1}{2}\pi\text{rad} \leqslant \theta \leqslant 2\pi\text{rad}$ $\frac{1}{2}\pi\text{rad} \leqslant \phi \leqslant \pi\text{rad}$
	PM, PI	$\frac{1}{8}\pi\text{rad} \leqslant \theta , \phi \leqslant \frac{1}{2}\pi\text{rad}$
Cylinder	—	Ratio ξ of the height to the diameter of the circular band:
	F	$\pi\text{rad} \leqslant \theta \leqslant 2\pi\text{rad}$
	P	$\frac{\pi}{4} \leqslant \theta \leqslant \pi$
	M	$\frac{1}{3} \leqslant \xi \leqslant 10$
	I	$\frac{1}{20} \leqslant \xi \leqslant \frac{1}{3}$ or $10 \leqslant \xi \leqslant 100$
Cone	—	Apex angle ψ, ratio ξ of the height to the largest diameter of the frustum [b]
	F	$\pi\text{rad} \leqslant \theta \leqslant 2\pi\text{rad}$
	P	$\frac{\pi}{4} \leqslant \theta \leqslant \pi$
	M	$\frac{1}{15}\pi\text{rad} \leqslant \psi \leqslant \frac{2}{3}\pi\text{rad}$ and $\frac{1}{4} \leqslant \xi \leqslant \frac{1}{2\tan\frac{\psi}{2}}$
	I	$\frac{1}{100}\pi\text{rad} \leqslant \psi \leqslant \frac{1}{15}\pi\text{rad}$ and $5 \leqslant \xi \leqslant \frac{1}{2\tan\frac{\psi}{2}}$ or $\frac{2}{3}\pi\text{rad} \leqslant \psi \leqslant \frac{9}{10}\pi\text{rad}$ and $\frac{1}{15} \leqslant \xi \leqslant \frac{1}{2\tan\frac{\psi}{2}}$
Torus	FM, FI	$\pi\text{rad} \leqslant \theta \leqslant 2\pi\text{rad}$ $\frac{1}{2}\pi\text{rad} \leqslant \phi \leqslant \frac{3}{2}\pi\text{rad}$
	PM, PI	$\frac{\pi}{2}\text{rad} \leqslant \theta \leqslant \pi\text{rad}$ $\frac{3}{4}\pi\text{rad} \leqslant \phi \leqslant \frac{5}{4}\pi\text{rad}$

[a] For the geometrical definitions and of the symbols used in the Table see the definition of extent in 11.14 of ISO 10360-1:2000.

[b] ψ is generated first, then ξ: for the generation of ξ, rule a) in clause A.2 is applied but overrides the extreme values of the interval.

A.4 Superimposition of form deviations

Each extent shall be deformed according to the rules listed in Table A.4. The maximum form deviation of the sampled points ζ [see A.2, h] is 10^{-4} for reference data set types (FM, PM), and 10^{-3} for reference data set types (FI, PI), respectively, multiplied by the extent size.

Table A.4 — Rules to add form deviations to nominal extents

Associated feature	Rule	Reference data set type	n
Line (2D)	Fourier harmonics up to order n are added to the segment;	FM	3
		FI	6
Line (3D)	Two orthogonal planes through the segment are chosen; in each plane the segment is deformed as a 2D line; the deformed segments are the projections on those planes of the resulting deformed 3D segment.	—	—
Plane	2D Fourier harmonics up to order n in either direction are added to the rectangle.	FM	3
		FI	6
Circle (2D)	The arc of circle is expressed in polar coordinates centred in the circle centre; the resulting segment is deformed as a 2D line with harmonics up to order n.	FM, PM	5
		FI, PI	8
Circle (3D)	In the plane of the circle, the arc of circle is perturbed as a 2D circle; then the circumscribed rectangle is deformed as a plane.	—	—
Sphere	The sphere is expressed in spherical coordinates centred in the sphere centre; the resulting rectangle is deformed as a plane with harmonics up to order n, constrained to assume a same value along each rectangle side corresponding to a pole [a].	FM, PM	5
		FI, PI	8
Cylinder	The cylinder is expressed in cylindrical coordinates aligned along the axis; the resulting rectangle is deformed as a plane with harmonics up to order n in either direction.	FM, PM	5
		FI, PI	8
Cone	The cone is expressed in cylindrical coordinates aligned along the axis; the resulting rectangle is deformed as a plane with harmonics up to order n in either direction. The resulting form deviation is taken normal to the cone rather than to the resulting rectangle.	FM, PM	5
		FI, PI	8
Torus	The generic torus point p is expressed as $p = r_1(\theta) + r_2(\theta,\phi)$, where θ is an angular coordinate about the axis, ϕ an angular coordinate about the ring r_1, and r_2 the shortest displacement of p from the ring; the resulting rectangle r_2 (θ, ϕ) is deformed as a plane with harmonics up to order n in either direction and the ring is deformed as a 3D circle.	FM, PM	5
		FI, PI	8

[a] The constraint guarantees the continuity of the deformed extent.

A.5 Sampling

Each nominal extent shall be divided into a number of line segments of equal length (extents in two dimensions) or patches of equal area (extents in three dimensions), as specified in Table A.5.

A nominal sampling point shall be generated at random in each subset and shall then be projected onto the deformed extent, normally to the nominal extent.

Table A.5 — Specifications for subdividing nominal extents into line segments or patches

Associated feature	Specifications	Parameter values
Line (2D) Line (3D)	The extent is divided into n segments of equal length.	$4 \leqslant n \leqslant 100$
Plane	The extent is divided into rectangles regularly disposed in a grid of n_x rows and n_y columns.	$2 \leqslant n_x \leqslant 10$ $2 \leqslant n_y \leqslant 10$
Circle (2D) Circle (3D)	The extent is divided into n arcs of equal length.	$5 \leqslant n \leqslant 100$
Sphere	The extent is divided into patches obtained by dividing the ranges of θ and z into n_θ and n_z, respectively, intervals of equal length, where θ and z are the angular and elevation coordinates of an associated cylindrical reference system (r, θ, z).	$3 \leqslant n_\theta \leqslant 10$ $2 \leqslant n_z \leqslant 10$
Cylinder	The extent is divided into patches obtained by dividing the ranges of θ and z into n_θ and n_z, respectively, intervals of equal length.	$3 \leqslant n_\theta \leqslant 10$ $3 \leqslant n_z \leqslant 10$
Cone	The extent is divided into patches obtained by dividing the range of θ into n_θ intervals of equal length, and the range of z (the origin being the vertex) into n_z intervals whose differences of the squares of the extremes are equal (i.e. $z^2_{i-1} - z^2_i$ = constant, $\forall i \leqslant n_\theta$) [a].	$3 \leqslant n_\theta \leqslant 10$ $3 \leqslant n_z \leqslant 10$
Torus	The extent is divided into patches obtained by dividing the ranges of θ and φ into n_θ and n_φ, respectively, intervals of equal length [b].	$4 \leqslant n_\theta \leqslant 10$ $3 \leqslant n_\varphi \leqslant 10$

[a] When the range of radii of the frustum is less than or equal to 1% of the mean radius (near-cylindrical cones or thin conic disks) the expression can be approximated by $z_{i-1} - z_i$ = constant, $\forall i \leqslant n_\theta$, thus resolving the ill-definition of the vertex elevation.

[b] The patches generated this way are not exactly of the same area; the resulting oversampling in the part of the torus closest to the axis is tolerated.

A.6 Superimposition of probing and accidental errors

Each point shall be perturbed by a noise random vector [see A.2, j)] with the standard deviations specified in Table A.6. The noise vectors of different points are assumed to be statistically independent of each other.

Table A.6 — Standard deviations of the noise components

Reference data set type	Noise standard deviation [μm]
FM, PM	2
FI, PI	10

Annex B
(informative)

Relation to the GPS matrix model

For full details about the GPS matrix model, see ISO/TR 14638.

B.1 Information about this part of ISO 10360 and its use

This part of ISO 10360 specifies the methods for estimating errors in software used for computing Gaussian associated features on a coordinate measuring machine (CMM). The tests of this part of ISO 10360 are:

— applicable to software for calculation of the line, plane, circle, sphere, cylinder, cone and torus;

— designed for Gaussian (least squares) assessment of data measured by a CMM;

— performed on the software alone and therefore independent of the CMM.

B.2 Position in the GPS matrix model

This part of ISO 10360 is a general GPS standard, which influences chain link 5 of the chains of standards on size, distance, radius, angle, form, orientation, location, run-out and datums in the general GPS matrix, as graphically illustrated in Figure B.1.

Fundamental GPS standards

Global GPS standards

General GPS standards						
Chain link number	1	2	3	4	5	6
Size						
Distance						
Radius						
Angle						
Form of line independent of datum						
Form of line dependent of datum						
Form of surface independent of datum						
Form of surface dependent of datum						
Orientation						
Location						
Circular run-out						
Total run-out						
Datums						
Roughness profile						
Waviness profile						
Primary profile						
Surface imperfections						
Edges						

Figure B.1

B.3 Related standards

The related standards are those of the chains of standards indicated in Figure B.1.

Bibliography

[1] ISO 1101:—[1], *Geometrical Product Specifications (GPS) — Geometrical tolerancing — Tolerances of form, orientation, location and run-out*

[2] ISO 5459:1981, *Technical drawings — Geometrical tolerancing — Datums and datum-systems for geometrical tolerances*

[3] ISO 10360-3:2000, *Geometrical product specifications (GPS) — Acceptance and reverification test for coordinate measuring machines (CMM) — Part 3: CMMs with the axis of a rotary table as the fourth axis*

[4] ISO 10360-4:2000, *Geometrical product specifications (GPS) — Acceptance and reverification test for coordinate measuring machines (CMM) — Part 4: CMMs used in scanning measuring mode*

[5] ISO 10360-5:2000, *Geometrical product specifications (GPS) — Acceptance and reverification test for coordinate measuring machines (CMM) — Part 5: CMMs using multiple-stylus probing systems*

[6] ISO/TR 14638:1995, *Geometrical Product Specifications (GPS) — Masterplan*

1) To be published (Revision ISO 1101:1983)

ICS 17.040.30
Price based on 19 pages