INTERNATIONAL STANDARD

ISO 10300-3

Second edition
2014-04-01

Calculation of load capacity of bevel gears —

Part 3:
Calculation of tooth root strength

Calcul de la capacité de charge des engrenages coniques —

Partie 3: Calcul de la résistance du pied de dent

Reference number
ISO 10300-3:2014(E)

Contents

Page

Foreword

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The committee responsible for this document is ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

This second edition cancels and replaces the first edition (ISO 10300-3:2001), which has been technically revised.

ISO 10300 consists of the following parts, under the general title *Calculation of load capacity of bevel gears*:

— *Part 1: Introduction and general influence factors*

— *Part 2: Calculation of surface durability (pitting)*

— *Part 3: Calculation of tooth root strength*

Introduction

When ISO 10300:2001 (all parts, withdrawn) became due for (its first) revision, the opportunity was taken to include hypoid gears, since previously the series only allowed for calculating the load capacity of bevel gears without offset axes. The former structure is retained, i.e. three parts of the ISO 10300 series, together with ISO 6336-5, and it is intended to establish general principles and procedures for rating of bevel gears. Moreover, ISO 10300 (all parts) is designed to facilitate the application of future knowledge and developments, as well as the exchange of information gained from experience.

In view of the decision for ISO 10300 (all parts) to cover hypoid gears also, it was agreed to include a separate clause: “Gear tooth rating formulae — Method B2” in this part of ISO 10300, while the former methods B and B1 were combined into one method, i.e. method B1. So, it became necessary to present a new, clearer structure of the three parts, which is illustrated in ISO 10300-1:2014, Figure 1. Note, ISO 10300 (all parts) gives no preferences in terms of when to use method B1 and when method B2.

Failure of gear teeth by breakage can be brought about in many ways; severe instantaneous overloads, excessive pitting, case crushing and bending fatigue are a few. The strength ratings determined by the formulae in this part of ISO 10300 are based on cantilever projection theory modified to consider the following:

— compressive stress at the tooth roots caused by the radial component of the tooth load;

— non-uniform moment distribution of the load, resulting from the inclined contact lines on the teeth of spiral bevel gears;

— stress concentration at the tooth root fillet;

— load sharing between adjacent contacting teeth;

— lack of smoothness due to a low contact ratio.

The formulae are used to determine a load rating, which prevents tooth root fracture during the design life of the bevel gear. Nevertheless, if there is insufficient material under the teeth (in the rim), a fracture can occur from the root through the rim of the gear blank or to the bore (a type of failure not covered by this part of ISO 10300). Moreover, it is possible that special applications require additional blank material to support the load.

Surface distress (pitting or wear) can limit the strength rating, either due to stress concentration around large sharp cornered pits, or due to wear steps on the tooth surface. Neither of these effects is considered in this part of ISO 10300.

In most cases, the maximum tensile stress at the tooth root (arising from bending at the root when the load is applied to the tooth flank) can be used as a determinant criterion for the assessment of the tooth root strength. If the permissible stress number is exceeded, the teeth can break.

When calculating the tooth root stresses of straight bevel gears, this part of ISO 10300 starts from the assumption that the load is applied at the tooth tip of the virtual cylindrical gear. The load is subsequently converted to the outer point of single tooth contact. The procedure thus corresponds to method C for the tooth root stress of cylindrical gears (see ISO 6336-3[1]).

For spiral bevel and hypoid gears with a high face contact ratio of $\varepsilon_{v\beta} > 1$ (method B1) or with a modified contact ratio of $\varepsilon_{v\gamma} > 2$ (method B2), the midpoint in the zone of action is regarded as the critical point of load application.

The breakage of a tooth generally means the end of a gear’s life. It is often the case that all gear teeth are destroyed as a consequence of the breakage of a single tooth. A safety factor, S_F, against tooth breakage higher than the safety factor against damage due to pitting is, therefore, generally to be preferred (see ISO 10300-1).

Calculation of load capacity of bevel gears —

Part 3:
Calculation of tooth root strength

1 Scope

This part of ISO 10300 specifies the fundamental formulae for use in the tooth root stress calculation of straight and helical (skew), Zerol and spiral bevel gears including hypoid gears, with a minimum rim thickness under the root of 3,5 m_{mn}. All load influences on tooth root stress are included, insofar as they are the result of load transmitted by the gearing and able to be evaluated quantitatively. Stresses, such as those caused by the shrink fitting of gear rims, which are superposed on stresses due to tooth loading, are intended to be considered in the calculation of the tooth root stress, σ_F, or the permissible tooth root stress σ_{FP}. This part of ISO 10300 is not applicable in the assessment of the so-called flank breakage, a tooth internal fatigue fracture (TIFF).

The formulae in this part of ISO 10300 are based on virtual cylindrical gears and restricted to bevel gears whose virtual cylindrical gears have transverse contact ratios of $\varepsilon_{v\alpha} < 2$. The results are valid within the range of the applied factors as specified in ISO 10300-1 (see also ISO 6336-3[1]). Additionally, the given relationships are valid for bevel gears, of which the sum of profile shift coefficients of pinion and wheel is zero (see ISO 23509).

This part of ISO 10300 does not apply to stress levels above those permitted for 10^3 cycles, as stresses in that range could exceed the elastic limit of the gear tooth.

Warning — The user is cautioned that when the formulae are used for large average mean spiral angles $(\beta_{m1} + \beta_{m2})/2 > 45°$, for effective pressure angles $\alpha_e > 30°$ and/or for large face widths $b > 13\ m_{mn}$, the calculated results of ISO 10300 (all parts) should be confirmed by experience.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable to its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 1122-1, *Vocabulary of gear terms — Part 1: Definitions related to geometry*

ISO 6336-5, *Calculation of load capacity of spur and helical gears — Part 5: Strength and quality of materials*

ISO 10300-1:2014, *Calculation of load capacity of bevel gears — Part 1: Introduction and general influence factors*

ISO 10300-2:2014, *Calculation of load capacity of bevel gears — Part 2: Calculation of surface durability (pitting)*

ISO 23509:2006, *Bevel and hypoid gear geometry*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 1122-1 and ISO 23509 (geometrical gear terms) and the following apply.

3.1
tooth root breakage
failure of gear teeth at the tooth root by static or dynamic overload

3.2
nominal tooth root stress
σ_{F0}
bending stress in the critical section of the tooth root calculated for the critical point of load application for error-free gears loaded by a constant nominal torque

3.3
tooth root stress
σ_F
determinant bending stress in the critical section of the tooth root calculated for the critical point of load application including the load factors which consider static and dynamic loads and load distribution

3.4
nominal stress number
$\sigma_{F,lim}$
maximum tooth root stress of standardized test gears and determined at standardized operating conditions as specified in ISO 6336-5

3.5
allowable stress number
σ_{FE}
maximum bending stress of the un-notched test piece under the assumption that the material is fully elastic

3.6
permissible tooth root stress
σ_{FP}
maximum tooth root stress of the evaluated gear set including all influence factors

4 Symbols, units and abbreviated terms

For the purposes of this document, the symbols and units given in Table 1 and Table 2 of ISO 10300-1:2014, as well as the abbreviated terms given in Table 1 of ISO 10300-2:2014, apply (see ISO 6336-5).

5 General rating procedure

There are two main methods for determining tooth bending strength of bevel and hypoid gears: method B1 and method B2. They are provided in Clauses 6 and 7, while Clause 8 contains those influence factors which are equal for both methods. With method B1, the same set of formulae may be used for bevel and hypoid gears; method B2 partly has different sets of formulae for bevel gears and for hypoid gears (see 7.4.3 for general aspects).

With both methods, the capability of a gear tooth to resist tooth root stresses shall be determined by the comparison of the following stress values:

— **tooth root stress** σ_F, based on the geometry of the tooth, the accuracy of its manufacture, the rigidity of the gear blanks, bearings and housing, and the operating torque, expressed by the tooth root stress formula (see 6.1 and 7.1);

— **permissible tooth root stress** σ_{FP}, based on the bending stress number, $\sigma_{F,lim}$, of a standard test gear and the effect of the operating conditions under which the gears operate, expressed by the permissible tooth root stress formula (see 6.2 and 7.2).

NOTE In respect of the permissible tooth root stress, reference is made to a stress "number", a designation adopted because pure stress, as determined by laboratory testing, is not calculated by the formulae in this part of ISO 10300. Instead, an arbitrary value is calculated and used in this part of ISO 10300, with accompanying changes to the allowable stress number in order to maintain consistency for design comparison.

The ratio of the permissible root stress and the calculated root stress is the safety factor S_F. The value of the minimum safety factor for tooth root stress, $S_{F,min}$, should be ≥ 1,3 for spiral bevel gears. For straight bevel gears, or where $\beta_m \leq 5°$, $S_{F,min}$ should be ≥ 1,5.

It is recommended that the gear designer and customer agree on the value of the minimum safety factor.

Tooth breakage usually ends transmission service life. The destruction of all gears in a transmission can be a consequence of the breakage of one tooth, then, the drive train between input and output shafts is interrupted. Therefore, the chosen value of the safety factor, S_F, against tooth breakage should be carefully chosen to fulfil the application requirements (see ISO 10300-1 for general comments on the choice of safety factor).

6 Gear tooth rating formulae — Method B1

6.1 Tooth root stress formula

The calculation of the tooth root stress is based on the maximum bending stress at the tooth root. It is determined separately for pinion (suffix 1) and wheel (suffix 2); in the case of hypoid gears, additionally for drive flank (suffix D) and coast flank (suffix C):

$$\sigma_{F\text{-}B1} = \sigma_{F0\text{-}B1} K_A K_v K_{F\beta} K_{F\alpha} < \sigma_{FP\text{-}B1} \tag{1}$$

with the load factors K_A, K_v, $K_{F\beta}$, $K_{F\alpha}$ as specified in ISO 10300-1.

The nominal tooth root stress is defined as the maximum bending stress at the tooth root (30° tangent to the root fillet):

$$\sigma_{\text{F0-B1}} = \frac{F_{\text{vmt}}}{b_{\text{v}}\, m_{\text{mn}}} Y_{\text{Fa}} Y_{\text{Sa}} Y_{\varepsilon} Y_{\text{BS}}\, Y_{\text{LS}} \tag{2}$$

where

F_{vmt} is the nominal tangential force of the virtual cylindrical gear which should be in accordance with Formula (2) of ISO 10300-1:2014;

b_{v} is the face width of the virtual cylindrical gear calculated for the active flank, drive or coast side, as specified in ISO 10300-1:2014, Annex A;

Y_{Fa} is the tooth form factor (see 6.4.1), which accounts for the influence of the tooth form on the nominal bending stress at the tooth root for load application at the tooth tip;

Y_{Sa} is the stress correction factor (see 6.4.2), which accounts for the stress increasing notch effect in the root fillet, as well as for the radial component of the tooth load and the fact that the stress condition in the critical root section is complex, but not for the influence of the bending moment arm;

Y_{ε} is the contact ratio factor (see 6.4.3), which accounts for the conversion of the root stress determined for the load application at the tooth tip to the determinant position;

Y_{BS} is the bevel spiral angle factor, which accounts for smaller values for l_{bm} compared to the total face width, b_{v}, and the inclined lines of contact (see 6.4.4);

Y_{LS} is the load sharing factor, which accounts for load distribution between two or more pairs of teeth (see 6.4.5).

The determinant position of load application is:

a) the outer point of single tooth contact, if $\varepsilon_{\text{v}\beta} = 0$;

b) the midpoint of the zone of action, if $\varepsilon_{\text{v}\beta} \geq 1$;

c) interpolation between a) and b), if $0 < \varepsilon_{\text{v}\beta} < 1$.

6.2 Permissible tooth root stress

The permissible tooth root stress, σ_{FP}, shall be calculated separately for pinion and wheel. The values should preferably be evaluated on the basis of the strength of a standard test gear instead of a prismatic specimen, which deviates too much with respect to similarity in geometry, course of movement and manufacture.

$$\sigma_{\text{FP-B1}} = \sigma_{\text{FE}}\, Y_{\text{NT}}\, Y_{\delta,\text{relT-B1}}\, Y_{\text{R,relT-B1}}\, Y_{\text{X}} \tag{3}$$

$$\sigma_{FP\text{-}B1} = \sigma_{F,lim}\, Y_{ST}\, Y_{NT}\, Y_{\delta,relT\text{-}B1}\, Y_{R,relT\text{-}B1}\, Y_X \quad (4)$$

where

σ_{FE}	is the allowable stress number (tooth root);
σ_{FE}	$= \sigma_{F,lim1,2}\, Y_{ST}$, is the basic bending strength of the un-notched specimen under the assumption that the material (including heat treatment) is fully elastic;
$\sigma_{F,lim}$	is the nominal stress number (bending) of the test gear, which accounts for material, heat treatment, and surface influence at test gear dimensions as specified in ISO 6336-5;
Y_{ST}	is the stress correction factor for the dimensions of the standard test gear, Y_{ST} = 2,0;
$Y_{\delta,relT\text{-}B1}$	is the relative notch sensitivity factor for the permissible stress number, related to the conditions at the standard test gear (see 6.5.2), $Y_{\delta,relT} = Y_\delta / Y_{\delta T}$ accounts for the notch sensitivity of the material;
$Y_{R,relT\text{-}B1}$	is the relative surface condition factor (see 6.5.1), $Y_{R,relT} = Y_R / Y_{RT}$ accounts for the surface condition at the root fillet, related to the conditions at the test gear;
Y_X	is the size factor for tooth root strength, which accounts for the influence of the module on the tooth root strength (see 8.1);
Y_{NT}	is the life factor, which accounts for the influence of required numbers of cycles of operation (see 8.2).

6.3 Calculated safety factor

The evaluated tooth root stress, σ_F, shall be $\leq \sigma_{FP}$, which is the permissible tooth root stress. The calculated safety factor against tooth breakage shall be determined separately for pinion and wheel:

$$S_{F\text{-}B1} = \frac{\sigma_{FP\text{-}B1}}{\sigma_{F\text{-}B1}} > S_{F,min} \quad (5)$$

NOTE This is the calculated safety factor with respect to the transmitted torque.

Considerations in reference to the safety factors and the risk (probability) of failure are given in ISO 10300-1:2014, 5.2.

6.4 Tooth root stress factors

6.4.1 Tooth form factor, Y_{Fa}

6.4.1.1 General

The tooth form factor, Y_{Fa}, accounts for the influence of the tooth form on the nominal tooth root stress in the case of load application at the tooth tip. It is determined separately for pinion and wheel. In doing so, the possibility to manufacture bevel and hypoid gears with different pressure angles at drive and coast side shall be considered (see Figure 1).

In the case of gears with tip and root relief, the actual bending moment arm is slightly smaller, but this should be neglected and the calculation is on the safe side.

Bevel gears without offset generally have octoid teeth and tip and root relief. However, deviations from an involute profile are small, especially in view of the tooth root cord and bending moment arm, and thus both, tip and root relief, may be neglected when calculating the tooth form factor.

The distance between the contact points of the 30° tangents at the root fillets of the tooth profile of the virtual cylindrical gear is taken as the critical section for calculation (see Figure 1).

By method B1 of ISO 10300, the tooth form factor, Y_{Fa}, and stress correction factor, Y_{Sa}, are determined for the nominal gear without deviations. The slight reduction in tooth thickness for backlash between teeth may be neglected for the load capacity calculation. However, the size reduction shall be taken into account when the outer tooth thickness allowance $A_{sne} > 0{,}05\ m_{mn}$.

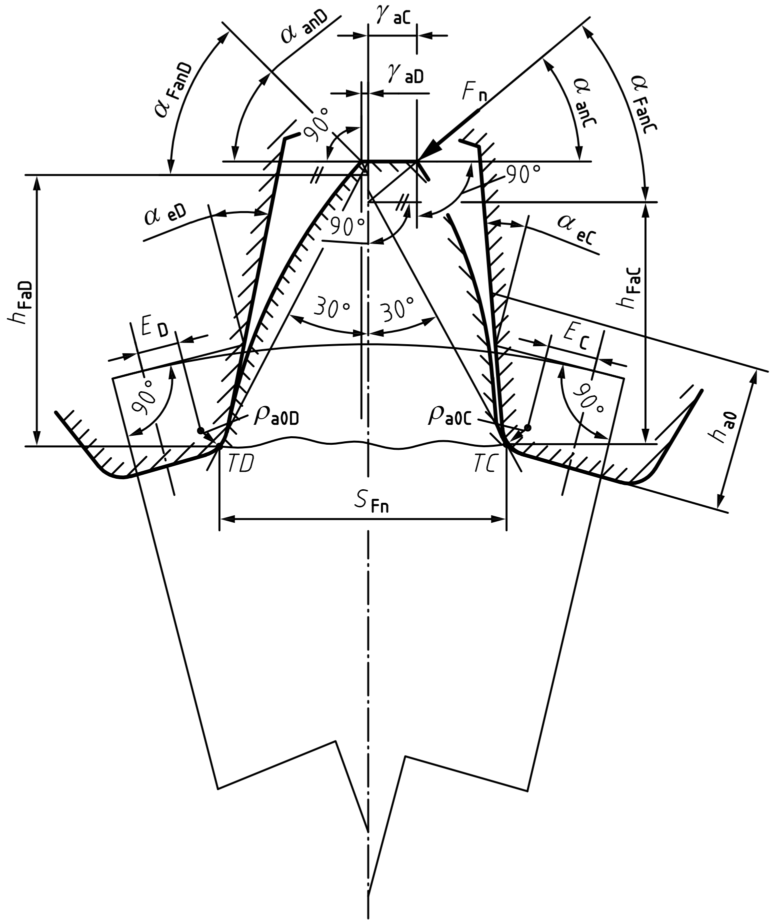


Figure 1 — Tooth root chordal thickness s_{Fn} and bending moment arm h_{Fa} in normal section for load application, F_n, at the tooth tip of the virtual cylindrical gear

6.4.1.2 Tooth form factor for generated gears

6.4.1.2.1 General

The tooth form factor, Y_{Fa}, of generated bevel gears is calculated with parameters of the active flank of the virtual cylindrical gear in normal section which includes the corresponding effective pressure angle α_{eD} or α_{eC} (see Annex A of ISO 10300-1:2014). However, the direction of the normal force, F_n, in relation to the tangential force, F_{vmt}, is given by the generated pressure angle α_{nD} or α_{nC}.

Attention — The tooth form factor, Y_F, and its parameters shall be determined for the pinion (suffix 1) and the wheel (suffix 2) separately:

$$Y_{\mathrm{FaD,C}} = \frac{6\dfrac{h_{\mathrm{FaD,C}}}{m_{\mathrm{mn}}}\cos\alpha_{\mathrm{FanD,C}}}{\left(\dfrac{s_{\mathrm{Fn}}}{m_{\mathrm{mn}}}\right)^2\cos\alpha_{\mathrm{nD,C}}} \quad (6)$$

where

$h_{\mathrm{FaD,C}}$ and $\alpha_{\mathrm{FanD,C}}$ see 6.4.1.2.5;

$\alpha_{\mathrm{n}} = \alpha_{\mathrm{nD}}$ = generated pressure angle for drive side (specified in ISO 23509);

$\alpha_{\mathrm{n}} = \alpha_{\mathrm{nC}}$ = generated pressure angle for coast side (specified in ISO 23509).

See Figure 1 for the explanation of parameters; see ISO 6336-3[1] for more information about the tooth form factor.

6.4.1.2.2 Auxiliary quantities

For the calculation of the tooth root chord, s_{Fn}, and the bending moment arm, h_{Fa}, firstly, the auxiliary quantities *E, G, H* and ϑ shall be determined.

The parameter, *E*, is calculated for the magnitudes of the active flank. For generated hypoid gears, the effective pressure angle $\alpha_{\mathrm{e}} = \alpha_{\mathrm{eD}}$ for the drive side and $\alpha_{\mathrm{e}} = \alpha_{\mathrm{eC}}$ (see ISO 23509) for the coast side, respectively, are used in Formula (7). Note, the cutter edge radii ρ_{a0D} and ρ_{a0C} as well as the protuberance, $s_{\mathrm{prD,C}}$, might also be different, but not h_{a0}, which is the tool addendum:

$$E_{\mathrm{D,C}} = \left(\frac{\pi}{4} - x_{\mathrm{sm}}\right)m_{\mathrm{mn}} - h_{\mathrm{a0}}\tan\alpha_{\mathrm{eD,C}} - \frac{\rho_{\mathrm{a0D,C}}\left(1-\sin\alpha_{\mathrm{eD,C}}\right) - s_{\mathrm{prD,C}}}{\cos\alpha_{\mathrm{eD,C}}} \quad (7)$$

$$G_{\mathrm{D,C}} = \frac{\rho_{\mathrm{a0D,C}}}{m_{\mathrm{mn}}} - \frac{h_{\mathrm{a0}}}{m_{\mathrm{mn}}} + x_{\mathrm{hm}} \quad (8)$$

$$H_{\mathrm{D,C}} = \frac{2}{z_{\mathrm{vnD,C}}}\left(\frac{\pi}{2} - \frac{E_{\mathrm{D,C}}}{m_{\mathrm{mn}}}\right) - \frac{\pi}{3} \quad (9)$$

$$\vartheta_{\mathrm{D,C}} = \frac{2G_{\mathrm{D,C}}}{z_{\mathrm{vnD,C}}}\tan\vartheta_{\mathrm{D,C}} - H_{\mathrm{D,C}} \quad (10)$$

For the solution of the transcendent Formula (10), $\vartheta = \pi/6$ may be inserted as the initial value. A suggested value for the difference ($\vartheta_{\mathrm{new}} - \vartheta$) is 0,000 001. In most cases, the calculation already converges after a few iterations.

6.4.1.2.3 Tooth root chordal thickness, s_{Fn}

The tooth root chords s_{FnD} and s_{FnC} are calculated for pinion and wheel, each with the corresponding geometry data for the drive flank and the coast flank:

$$s_{\mathrm{FnD,C}} = m_{\mathrm{mn}} z_{\mathrm{vnD,C}} \sin\left(\frac{\pi}{3} - \vartheta_{\mathrm{D,C}}\right) + m_{\mathrm{mn}}\sqrt{3}\left(\frac{G_{\mathrm{D,C}}}{\cos\vartheta_{\mathrm{D,C}}} - \frac{\rho_{\mathrm{a0D,C}}}{m_{\mathrm{mn}}}\right) \quad (11)$$

Then, the respective tooth root chord s_{Fn} for pinion or wheel results in:

$$s_{\mathrm{Fn}} = 0{,}5 s_{\mathrm{FnD}} + 0{,}5 s_{\mathrm{FnC}} \quad (12)$$

6.4.1.2.4 Fillet radius, ρ_F, at contact point of 30° tangent

The fillet radius, ρ_F, is calculated with the corresponding geometry data for the drive flank and the coast flank:

$$\rho_{F\,D,C} = \rho_{a0D,C} + \frac{2G_{D,C}^2 m_{mn}}{\cos\vartheta_{D,C}\left(z_{vnD,C}\cos^2\vartheta_{D,C} - 2G_{D,C}\right)} \tag{13}$$

6.4.1.2.5 Bending moment arm, h_{Fa}

The bending moment arm, h_{Fa}, is calculated with geometry data referring to the drive flank and to the coast flank:

$$h_{FaD,C} = \frac{m_{mn}}{2}\left[\left(\cos\gamma_{aD,C} - \sin\gamma_{aD,C}\tan\alpha_{FanD,C}\right)\frac{d_{vanD,C}}{m_{mn}} - z_{vnD,C}\cos\left(\frac{\pi}{3} - \vartheta_{D,C}\right) - \frac{G_{D,C}}{\cos\vartheta_{D,C}} + \frac{\rho_{a0D,C}}{m_{mn}}\right] \tag{14}$$

where

$$\alpha_{FanD,C} = \alpha_{anD,C} - \gamma_{aD,C} \tag{15}$$

$$\alpha_{anD,C} = \arccos\left(\frac{d_{vbnD,C}}{d_{vanD,C}}\right) \tag{16}$$

$$\gamma_{aD,C} = \frac{1}{z_{vnD,C}}\left[\frac{\pi}{2} + 2\left(x_{hm}\tan\alpha_{eD,C} + x_{sm}\right)\right] + \mathrm{inv}\,\alpha_{eD,C} - \mathrm{inv}\,\alpha_{anD,C} \tag{17}$$

where

$\alpha_e = \alpha_{eD}$ = generated pressure angle for drive side;

$\alpha_e = \alpha_{eC}$= effective pressure angle of coast side (specified in ISO 23509).

Data of the virtual cylindrical gears (pinion and wheel) in normal section, d_{van}, d_{vbn} and z_{vn}, are specified in ISO 10300-1:2014, A.3. Dimensions at the basic rack profile of the tooth are shown in Figure 2.

At the design stage, the tooth form factor Y_{Fa} for bevel gears without offset may be calculated for a basic rack profile of the tool with the following data α_n = 20°, h_{a0}/m_{mn} = 1,25, and ρ_{a0}/m_{mn} = 0,25. Diagrams for this and other basic rack profiles are given in ISO 6336-3.[1]

6.4.1.3 Tooth form factor for non-generated gears

The tooth form factor, Y_{Fa}, for non-generated bevel gears should be considered separately. In this case of form cutting the slot profile of the wheel is identical to the tool profile and so the tooth form factor can directly be determined (see Figure 2).

The tooth form factor of the pinion, which is manufactured by a specific generating process, may be approximated by the formulae according to 6.4.1.2. Hypoid gears are calculated with geometry data for drive side and coast side.

Tooth root thickness of the wheel (suffix 2):

$$s_{\mathrm{FnD,C}} = \pi\ m_{\mathrm{mn}} - 2E_{\mathrm{D,C}} - 2\rho_{\mathrm{a0D,C}} \cos 30° \tag{18}$$

where

$$E_{\mathrm{D,C}} = \left(\frac{\pi}{4} - x_{\mathrm{sm}}\right) m_{\mathrm{mn}} - h_{\mathrm{a0}} \tan\alpha_{\mathrm{nD,C}} - \frac{\rho_{\mathrm{a0D,C}}\left(1 - \sin\alpha_{\mathrm{nD,C}}\right) - s_{\mathrm{prD,C}}}{\cos\alpha_{\mathrm{nD,C}}} \tag{19}$$

The tooth root chord s_{Fn} is then calculated by:

$$s_{\mathrm{Fn}} = 0{,}5 s_{\mathrm{FnD}} + 0{,}5 s_{\mathrm{FnC}} \tag{20}$$

Fillet radius at contact point of 30° tangent:

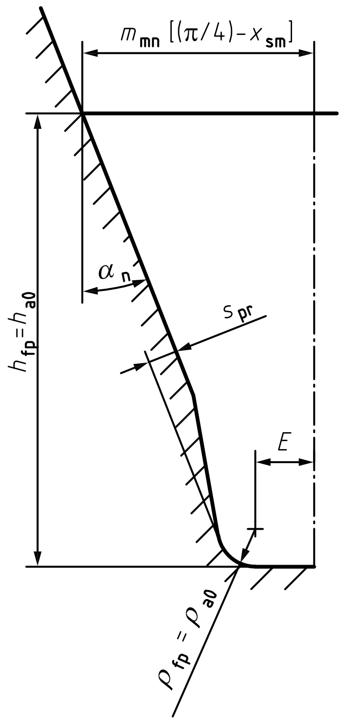
$$\rho_{\mathrm{FD,C}} = \rho_{\mathrm{a0D,C}} \tag{21}$$

Bending moment arm:

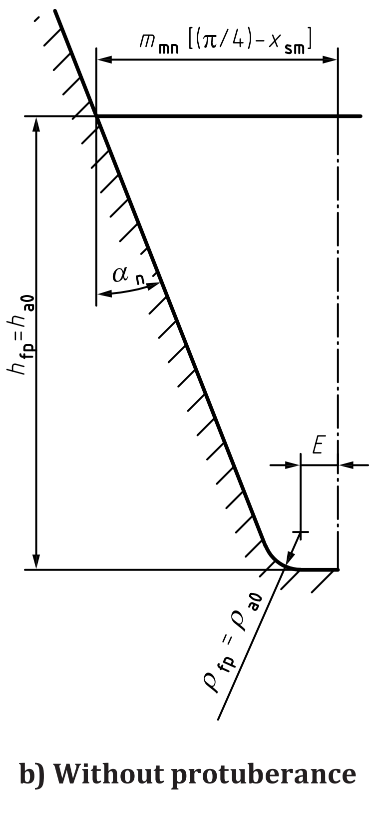
$$h_{\mathrm{FaD,C}} = h_{\mathrm{a0}} - \frac{\rho_{\mathrm{a0D,C}}}{2} + m_{\mathrm{mn}} - \left(\frac{\pi}{4} + x_{\mathrm{sm}} - \tan\alpha_{\mathrm{nD,C}}\right) m_{\mathrm{mn}} \tan\alpha_{\mathrm{nD,C}} \tag{22}$$

Tooth form factor of the wheel according to Formula (6) with $\alpha_{\mathrm{FanD,C}} = \alpha_{\mathrm{nD,C}}$:

$$Y_{\mathrm{FaD,C}} = \frac{6 \dfrac{h_{\mathrm{FaD,C}}}{m_{\mathrm{mn}}}}{\left(\dfrac{s_{\mathrm{Fn}}}{m_{\mathrm{mn}}}\right)^2} \tag{23}$$



a) With protuberance



b) Without protuberance

Figure 2 — Dimensions at the basic rack profile of the tooth

6.4.2 Stress correction factor, Y_{Sa}

The stress correction factor, Y_{Sa}, accounts for the stress increasing notch effect in the root fillet as well as for other stress components which arise beside the tooth root stress (see ISO 6336-3[1] for additional information).

$$Y_{SaD,C} = \left(1{,}2 + 0{,}13\ L_{aD,C}\right)\ q_{sD,C}^{\left(\frac{1}{1{,}21 + 2{,}3/L_{aD,C}}\right)} \tag{24}$$

$$L_{aD,C} = \frac{s_{Fn}}{h_{FaD,C}} \tag{25}$$

$$q_{sD,C} = \frac{s_{Fn}}{2\rho_{FD,C}} \tag{26}$$

where

s_{Fn} is calculated for generated or non-generated gears according to Formula (12) or Formula (20);

h_{Fa} is calculated for generated or non-generated gears according to Formula (14) or Formula (22);

ρ_F is calculated for generated or non-generated gears is according to Formula (13) or Formula (21).

The range of validity of Formula (26) is $1 \le q_s < 8$ (see ISO 6336-3[1] for the influence of grinding notches).

6.4.3 Contact ratio factor, Y_ε

The contact ratio factor, Y_ε, converts the load application at the tooth tip, where the tooth form factor, Y_{Fa}, and stress correction factor, Y_{Sa}, apply, to the determinant point of load application.

There are three ranges for $\varepsilon_{v\beta}$ to calculate Y_ε:

a) for $\varepsilon_{v\beta} = 0$:

$$Y_\varepsilon = 0{,}25 + \frac{0{,}75}{\varepsilon_{v\alpha}} \ge 0{,}625 \tag{27a}$$

b) for $0 < \varepsilon_{v\beta} \le 1$:

$$Y_\varepsilon = 0{,}25 + \frac{0{,}75}{\varepsilon_{v\alpha}} - \varepsilon_{v\beta}\left(\frac{0{,}75}{\varepsilon_{v\alpha}} - 0{,}375\right) \ge 0{,}625 \tag{27b}$$

c) for $\varepsilon_{v\beta} > 1$:

$$Y_\varepsilon = 0{,}625 \tag{27c}$$

6.4.4 Bevel spiral angle factor, Y_{BS}

The bevel spiral angle factor, Y_{BS}, accounts for the non-uniform distribution of the tooth root stress along the face width. The stress distribution depends on the inclination of the contact lines due to the spiral angle. With an increasing spiral angle the inclination angle also increases till the contact lines are limited by tip and root of the teeth. Thus, the face width is not completely used to carry the load. This leads to a higher stress maximum in the tooth root in the middle of the face width (see Figure 3), where a tooth developed into a plane is replaced by a cantilever beam.

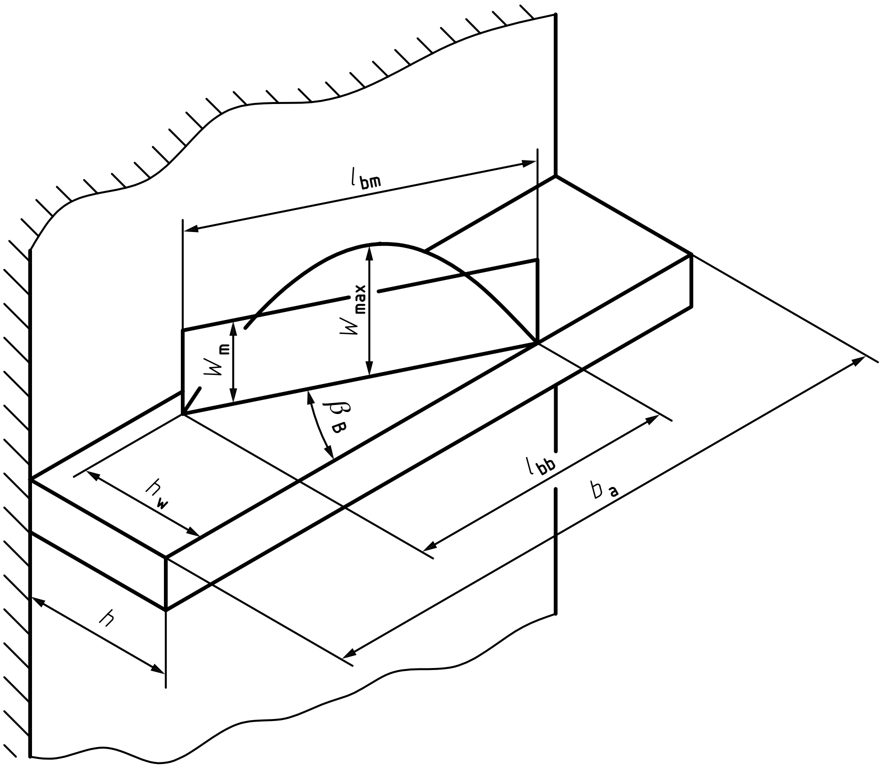


Figure 3 — Definition of geometric parameters of tooth model

Y_{BS} is given by the following empirical formulae [i.e. Formula (28) to Formula (31)]:

$$Y_{BS} = \frac{a_{BS}}{c_{BS}} \left(\frac{l_{bb}}{b_a} - 1{,}05 \cdot b_{BS} \right)^2 + 1 \tag{28}$$

$$a_{BS} = -0{,}018\,2 \left(\frac{b_a}{h} \right)^2 + 0{,}473\,6 \left(\frac{b_a}{h} \right) - 0{,}32 \tag{29}$$

$$b_{BS} = -0{,}003\,2 \left(\frac{b_a}{h} \right)^2 + 0{,}052\,6 \left(\frac{b_a}{h} \right) + 0{,}712 \tag{30}$$

$$c_{BS} = -0{,}005\,0 \left(\frac{b_a}{h} \right)^2 + 0{,}085\,0 \left(\frac{b_a}{h} \right) + 0{,}54 \tag{31}$$

with auxiliary values a_{BS}, b_{BS}, c_{BS}

The developed length of one tooth as face width of the calculation model:

$$b_a = b_v / \cos \beta_v \tag{32}$$

Part of the model's face width covered by the contact line:

$$l_{bb} = l_{bm} \frac{\cos \beta_{vb}}{\cos \beta_v} \tag{33}$$

Average tooth depth:

$$h = \left(h_{m1} + h_{m2} \right) / 2 \tag{34}$$

with mean whole tooth depth, h_m, as specified in ISO 23509.

6.4.5 Load sharing factor, Y_{LS}

The load sharing factor, Y_{LS}, for bending accounts for load sharing between two or more pairs of teeth:

$$Y_{LS} = Z_{LS}^2 \tag{35}$$

with load sharing factor, Z_{LS}, as specified in 6.4.2 of ISO 10300-2:2014.

6.5 Permissible tooth root stress factors

6.5.1 Relative surface condition factor, $Y_{R,relT-B1}$

The tooth root strength depends on the surface condition at the root predominantly on the roughness in the root fillet. The surface condition factor, $Y_{R,relT}$, accounts for this dependence related to standard test gear conditions with Rz = 10 µm (see ISO 6336-3[1] for general remarks) and is determined separately for pinion (suffix 1) and wheel (suffix 2). If no surface condition factors determined according to method A are available, method B described in 6.5.1 shall be used.

Warning — This method is only valid if there are no scratches or similar defects deeper than 2 *Rz*.

The relative surface condition factor, $Y_{R,RelT}$, determined by tests with test specimens, may be taken from Figure 4 as a function of roughness Rz and material.

For calculation, Formulae (36) to (41) shall be used depending on two ranges of roughness.

Range Rz < 1 µm:

a) For through hardened and case hardened steels:

$$Y_{R,relT} = 1{,}12 \tag{36}$$

b) For non-hardened steels:

$$Y_{R,relT} = 1{,}07 \tag{37}$$

c) For grey cast iron, nitrided and nitro carburized steels:

$$Y_{R,relT} = 1{,}025 \tag{38}$$

Range 1 µm ≤ Rz ≤ 40 µm:

a) For through hardened and case hardened steels:

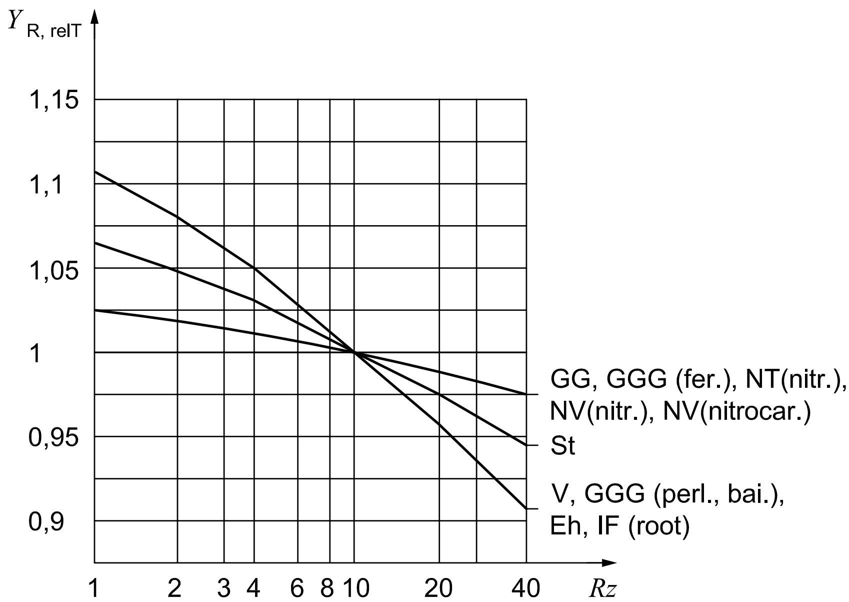
$$Y_{R,relT} = \frac{Y_R}{Y_{RT}} = 1{,}674 - 0{,}529\ (Rz+1)^{1/10} \tag{39}$$

b) For non-hardened steels:

$$Y_{R,relT} = \frac{Y_R}{Y_{RT}} = 5{,}306 - 4{,}203\ (Rz+1)^{1/100} \tag{40}$$

c) For grey cast iron, nitrided and nitro carburized

$$Y_{R,relT} = \frac{Y_R}{Y_{RT}} = 4{,}299 - 3{,}259\ (Rz+1)^{1/200} \tag{41}$$



Key

Rz surface roughness (µm)

$Y_{R,RelT}$ surface condition factor (–)

Figure 4 — Surface condition factor, $Y_{R,relT}$, for permissible stress number relative to standard test gear dimensions

6.5.2 Relative notch sensitivity factor, $Y_{\delta,relT\text{-}B1}$

The dynamic notch sensitivity factor, Y_δ, indicates the amount by which the theoretical stress peak exceeds the permissible stress number in the case of fatigue breakage. It is a function of the material and relative stress drop. It is possible to calculate the notch sensitivity factor on the basis of strength values determined at un-notched or notched specimens, or at test gears. If more exact test results (method A) are not available, method B described in 6.5.2 shall be used.

The calculation of permissible tooth root stresses of bevel gears is based on bending strength values determined for both, bevel and cylindrical test gears. Therefore, the relative notch sensitivity factor, $Y_{d,relT}$, is the ratio between the sensitivity factor of the gear to be calculated and the sensitivity factor of the standard test gear. $Y_{\delta,\mathrm{relT}} = Y_\delta / Y_{\delta\mathrm{T}}$ may be taken directly from Figure 5 as a function of q_s (see 6.4.2) of the gear to be calculated and of the material.

In order to calculate the relative notch sensitivity factor, $Y_{\delta,\mathrm{relT}}$ according to method B1, Formulae (42) and (43), representing the curves in Figure 5, shall be used:

$$Y_{\delta,\mathrm{rel\,T}\,1,2} = \frac{1+\sqrt{\rho' \chi^{\mathrm{X}}_{1,2}}}{1+\sqrt{\rho' \chi^{\mathrm{X}}_{\mathrm{T}}}} \tag{42}$$

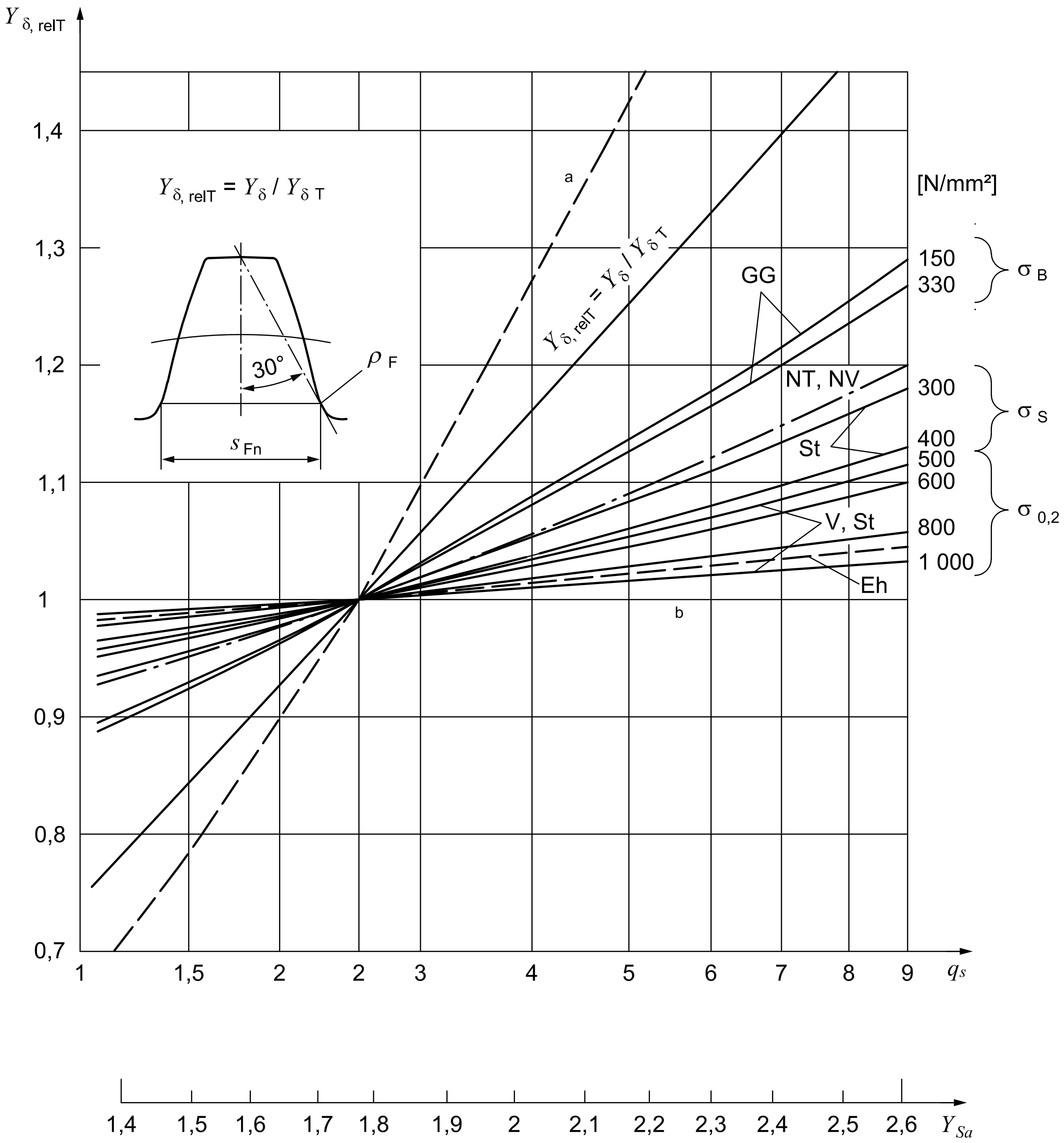
$$\chi^{\mathrm{X}}_{1,2} = \frac{1}{5}\left(1+2\, q_{\mathrm{s}1,2}\right) \tag{43}$$

where

ρ'	shall be taken from Table 1 as a function of the material;
$\chi^{\mathrm{X}}_{1,2}$	is applicable to module $m_{\mathrm{mn}} = 5$, with the size influence accounted for by Y_{X} (see 8.1);
$\chi^{\mathrm{X}}_{\mathrm{T}} = 1,2$	is calculated with $q_{\mathrm{sT}} = 2,5$ according to Formula (43).

Table 1 — Slip layer thickness ρ'

No.	Material		Slip layer thickness ρ'
1	GG	$\sigma_{\mathrm{B}} = 150\ \mathrm{N/mm^2}$	0,3124
2	GG, GGG (ferr.)	$\sigma_{\mathrm{B}} = 300\ \mathrm{N/mm^2}$	0,3095
3	NT (nitr.), NV (nitr.), NV (nitrocar.)	for all hardnesses	0,1005
4	St	$\sigma_{\mathrm{S}} = 300\ \mathrm{N/mm^2}$	0,0833
5	St	$\sigma_{\mathrm{S}} = 400\ \mathrm{N/mm^2}$	0,0445
6	V, GTS, GGG (perl., bain.)	$\sigma_{0,2} = 500\ \mathrm{N/mm^2}$	0,0281
7	V, GTS, GGG (perl., bain.)	$\sigma_{0,2} = 600\ \mathrm{N/mm^2}$	0,0194
8	V, GTS, GGG (perl., bain.)	$\sigma_{0,2} = 800\ \mathrm{N/mm^2}$	0,0064
9	V, GTS, GGG (perl., bain.)	$\sigma_{0,2} = 1000\ \mathrm{N/mm^2}$	0,0014
10	Eh, IF (root)	for all hardnesses	0,0030



Key

q_s notch parameter (–)

Y_{Sa} stress correction factor (–)

$Y_{\delta,\mathrm{relT}}$ relative notch sensitivity factor (–)

a Complete insensitivity to notches.

b Complete sensitivity to notches.

Figure 5 — Relative notch sensitivity factor with respect to standard test gear dimensions

7 Gear tooth rating formulae — Method B2

7.1 Tooth root stress formula

The tooth root stress is determined separately for pinion (suffix 1) and wheel (suffix 2):

$$\sigma_{\text{F-B2}} = \sigma_{\text{F0-B2}} K_{\text{A}} K_{\text{v}} K_{\text{F}\beta} K_{\text{F}\alpha} < \sigma_{\text{FP-B2}} \tag{44}$$

with load factors K_{A}, K_{v}, $K_{\text{F}\beta}$ and $K_{\text{F}\alpha}$, as specified in ISO 10300-1.

The tooth root stress $\sigma_{\text{F0-B2}}$ is defined as the maximum tensile stress arising at the tooth root due to the nominal torque when an error-free gear is loaded.

When applying method B2, the combined geometry factor Y_{P} replaces the factors Y_{Fa}, Y_{Sa}, Y_{ε}, Y_{BS} and Y_{LS} of method B1 in the tooth root stress equation:

$$\sigma_{\text{F0-B2}} = \frac{F_{\text{mt1,2}}}{b_{1,2} m_{\text{mn}}} Y_{\text{P1,2}} \tag{45}$$

The value of Y_{P} is determined by Formula (46):

$$Y_{\text{P1,2}} = \frac{Y_{\text{A1,2}}}{Y_{\text{J1,2}}} \frac{m_{\text{mt1,2}} \cdot m_{\text{mn}}}{m_{\text{et2}}^2} \tag{46}$$

Substitution in Formula (45):

$$\sigma_{\text{F0-B2}} = \frac{F_{\text{mt1,2}}}{b_{1,2}} \cdot \frac{m_{\text{mt1,2}}}{m_{\text{et2}}^2} \cdot \frac{Y_{\text{A1,2}}}{Y_{\text{J1,2}}} \tag{47}$$

where

F_{mt} is the nominal tangential force of bevel gears in accordance with 6.1 of ISO 10300-1:2014;

Y_{A} is the root stress adjustment factor for method B2 (see 7.4.7);

Y_{J} is the bending strength geometry factor for method B2 (see 7.4.3).

The bending strength geometry factor, Y_{J}, evaluates the shape of the tooth, the position at which the most damaging load is applied, the stress concentration due to the geometric shape of the root fillet, the sharing of load between adjacent pairs of teeth, the tooth thickness balance between the wheel and mating pinion, the effective face width due to lengthwise crowning of the teeth, and the buttressing effect of an extended face width on one member of the pair. Both the tangential (bending) and radial (compressive) components of the tooth load are included.

7.2 Permissible tooth root stress

The permissible tooth root stress, σ_{FP}, is determined separately for pinion and wheel. It should be calculated on the basis of the strength determined at an actual gear. In this way, the reference value for geometrical similarity, course of movement and manufacture lies within the field of application:

$$\sigma_{\text{FP-B2}} = \sigma_{\text{FE}} Y_{\text{NT}} Y_{\delta,\text{relT-B2}} Y_{\text{R,relT-B2}} Y_{\text{X}} \tag{48}$$

$$\sigma_{\text{FP-B2}} = \sigma_{\text{F,lim}}\ Y_{\text{ST}}\ Y_{\text{NT}}\ Y_{\delta,\text{relT-B2}}\ Y_{\text{R,relT-B2}}\ Y_{\text{X}} \tag{49}$$

where

σ_{FE} is the allowable stress number (bending);

σ_{FE} $= \sigma_{\text{F,lim1,2}}\ Y_{\text{ST}}$, the basic bending strength of the un-notched specimen under the assumption that the material (including heat treatment) is fully elastic;

$\sigma_{\text{F,lim}}$ is the nominal stress number (bending) of the standard test gear, which accounts for material, heat treatment and surface influence at test gear dimensions, as specified in ISO 6336-5;

Y_{ST} is the stress correction factor for the dimensions of the standard test gear, $Y_{\text{ST}} = 2,0$;

$Y_{\delta,\text{relT-B2}}$ is the relative notch sensitivity factor (see 7.5.2) for the bending stress number related to the conditions at the standard test gear ($Y_{\delta,\text{relT}} = Y_\delta / Y_{\delta\text{T}}$ accounts for the notch sensitivity of the material);

$Y_{\text{R,relT-B2}}$ is the relative surface condition factor (see 7.5.1) ($Y_{\text{R,relT}} = Y_{\text{R}} / Y_{\text{RT}}$ accounts for the surface condition at the root fillet, related to the conditions at the test gear);

Y_{X} is the size factor for tooth root strength (see 8.1), which accounts for the influence of the module on the tooth root strength;

Y_{NT} is the life factor, which accounts for the influence of required numbers of cycles of operation (see 8.2).

7.3 Calculated safety factor

The determined tooth root stress, σ_{F}, shall be $\leq \sigma_{\text{FP}}$, which is the permissible tooth root stress. The calculated safety factor against tooth breakage shall be determined separately for pinion and wheel, on the basis of the bending stress number determined for the standard test gear:

$$S_{\text{F-B2}} = \frac{\sigma_{\text{FP-B2}}}{\sigma_{\text{F-B2}}} > S_{\text{F min}} \tag{50}$$

NOTE This is the calculated safety factor with respect to the transmitted torque.

Considerations in reference to the safety factors and the risk (probability) of failure are given in ISO 10300-1:2014, 5.2.

7.4 Tooth root stress factors

7.4.1 General

To calculate the bending strength geometry factor, Y_{J}, the formulae in 7.4.3 should be used. Because of the complexity of the calculation, computerization is recommended.

ANSI/AGMA 2003-C10[3] contains graphs for the bevel geometry factor, Y_{J}, for straight, Zerol and spiral bevel gears for a series of gear designs, based on the smaller of the face width to be chosen $b = 0,3R_{\text{e}}$ or $b = 10m_{\text{et}}$. Corresponding graphs for hypoid gears can be found in AGMA 932-A05.[4] These may be used whenever the tooth proportions and thickness, face widths, tool edge radii, pressure and spiral angles of the design, and driving with the concave side, correspond to those in the graphs.

7.4.2 Stress parabola according to Lewis

The basis for method B2 is the Lewis formula applied to a virtual cylindrical gear, which has been defined in transverse section as specified in Annex B of ISO 10300-1:2014, with the following additions and modifications:

— the tooth strength is considered in the normal section rather than in the transverse section;

— the position of the point of load application is determined by taking into account theoretical lines of contact, tooth bearing modifications and experimental evidence;

— the amount of load carried by one tooth is estimated based on tooth bearing modification and contact ratio;

— the radial component of the normal load is considered;

— a stress concentration factor based on experimental data are applied;

— the concept of effective face width is used.

Bending stress shall be calculated assuming the tooth shaped beam is simulated by a parabola tangent to the tooth profile at the most highly stressed section. Figure 6 shows a layout for the cases of: a) no load sharing and b) load sharing.

7.4.3 Basic formula of geometry factor, Y_J

The parameters for calculating the geometry factor, Y_J are the same for bevel and hypoid gears. However, the calculation procedures are different. See 7.4.4 for bevel gears without hypoid offset or 7.4.5 for hypoid gears.

The bevel geometry factor, Y_J, is calculated using Formula (51):

$$Y_{J1,2} = \frac{Y_{1,2}}{Y_{f1,2} \cdot \varepsilon_N \cdot Y_i} \cdot \frac{r_{my01,2}}{r_{mpt1,2}} \cdot \frac{b_{ce1,2}}{b_{1,2}} \cdot \frac{m_{mt1,2}}{m_{et2}} \tag{51}$$

where

$Y_{1,2}$ is the tooth form factor of pinion and wheel (see 7.4.4.4 for bevel gears and 7.4.5.4 for hypoid gears);

ε_N is the load sharing ratio (see 7.4.4.3 and 7.4.5.2, respectively);

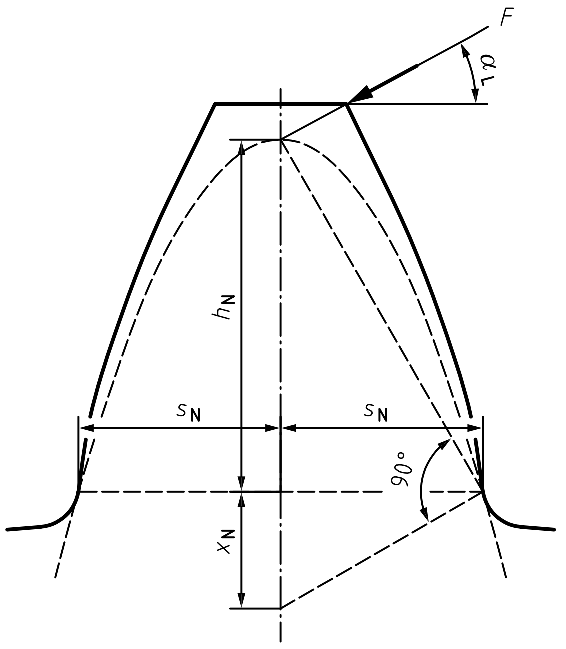
$r_{my01,2}$ is the mean transverse radius to point of load application for pinion or wheel, in millimetres (see 7.4.4.2 and 7.4.5.5);

$r_{mpt1,2}$ is the mean transverse pitch radius, in millimetres (see ISO 23509);

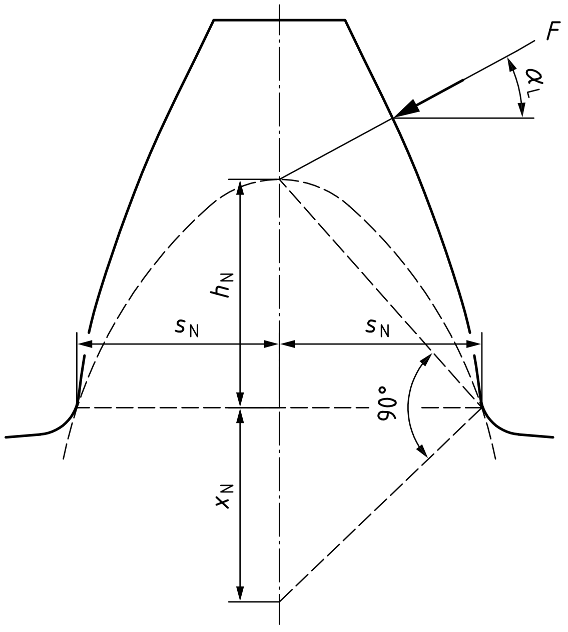
$Y_{f1,2}$ is the stress concentration and correction factor (see 7.4.6.2);

Y_i is the inertia factor for gears with a low contact ratio (see 7.4.6.3);

$b_{ce1,2}$ is the calculated effective face width of pinion or wheel, in millimetres (see 7.4.6.4).



a) No load sharing



b) Load sharing

Figure 6 — Stress parabola according to Lewis

7.4.4 Geometry factor, Y_J, for bevel gears (for hypoid gears, see 7.4.5)

7.4.4.1 Point of load application for maximum tooth root stress, y_3

For most straight, Zerol and spiral bevel gears, the maximum tooth root stress occurs at the equivalent of the highest point of single tooth contact when the modified contact ratio is ≤ 2. When the modified contact ratio is >2, it is assumed that the contact line passes through the centre of the path of action. For statically loaded straight bevel and Zerol bevel gears, such as those used in automotive differentials, the load is applied at the tip of the tooth. In any case, the position is measured along the path of action from its centre, and is designated by y_J. Its distance from the beginning of the path of action is designated by y_3.

So, there are three cases for the determination of y_J:

when $\varepsilon_{v\gamma} \le 2{,}0$:

$$y_J = \frac{\pi \ m_{mn} \cos\alpha_a}{m_{et2}} - \frac{g_\eta}{2} \tag{52}$$

for α_a see ISO 10300-1:—, Formula (B.14), and for g_η see ISO 10300-2:2014, Formula (34)

when $\varepsilon_{v\gamma} > 2{,}0$:

$$y_J = 0 \tag{53}$$

for statically loaded straight bevel and Zerol bevel gears (tip loading):

$$y_J = \frac{g_\eta}{2} \tag{54}$$

where

$$g_\eta^2 = g_{v\alpha n}^2 \cos^4\beta_{vb} + b_v{}^2 \sin^2\beta_{vb} \tag{55}$$

The determination of the distance, y_3, depends on the type of bevel gears:

for straight bevel and Zerol bevel gears:

$$y_3 = \frac{g_{v\alpha n}}{2} + \frac{g_{v\alpha n}^2 y_J}{g_\eta^2} \tag{56}$$

for spiral bevel pinions:

$$y_{31} = \frac{g_{v\alpha n}}{2} + \frac{g_{v\alpha n}^2 \cdot y_J \cdot \cos^2\beta_{vb} + b_v \cdot g_{v\alpha n} \cdot g_J \cdot k' \cdot \sin\beta_{vb}}{g_\eta^2} \tag{57}$$

for spiral bevel wheels:

$$y_{32} = \frac{g_{v\alpha n}}{2} + \frac{g_{v\alpha n}^2 \cdot y_J \cdot \cos^2\beta_{vb} - b_v \cdot g_{v\alpha n} \cdot g_J \cdot k' \cdot \sin\beta_{vb}}{g_\eta^2} \tag{58}$$

where

$$g_J = \sqrt{g_\eta^2 - 4 y_J^2} \tag{59}$$

k' is the contact shift factor (see ISO 10300-1:2014, B.5)

7.4.4.2 Transverse radius to point of load application, $r_{my0\ 1,2}$

Since the point of load application does not usually lie in the mean section of the tooth, the actual radius is determined using Formulae (60) to (67). The distance from mean section to point of load application, $x_{oo1,2}$, measured in the lengthwise direction along the tooth, is calculated depending on the type of gear:

a) for straight and Zerol bevel gears:

$$x_{oo1,2} = \frac{g_{v\alpha n}\ g_J\, k'}{g_\eta^2} \tag{60}$$

b) for spiral bevel pinion:

$$x_{oo1,2} = \frac{b_v\ g_{v\alpha n}\ g_J k' \cos^2\beta_{vb} m_{et2} - b_v^2\ y_J \sin\beta_{vb} m_{et2}}{g_\eta^2} \tag{61}$$

c) for spiral bevel wheel:

$$x_{oo2} = \frac{b_v\ g_{v\alpha n}\, g_J\ k' \cos^2\beta_{vb} m_{et2} + b_v^2\ y_J \sin\beta_{vb} m_{et2}}{g_\eta^2} \tag{62}$$

The normal pressure angle at point of load application, $\alpha_{L1,2}$, for pinion and wheel is derived from:

$$\tan\alpha_{L1,2} = \frac{y_{3\ 1,2} + a_{vn} \cdot \sin\alpha_n - \sqrt{r_{va1,2}^2 - r_{vbn1,2}^2}}{r_{vbn1,2}} \tag{63}$$

The rotation angle, $\xi_{h1,2}$, used in bending strength calculations for pinion and wheel, $\xi_{h1,2}$, is:

$$\xi_{h1,2} = \left(\frac{s_{mn1,2}}{2r_{vn1,2}} - \mathrm{inv}\alpha_{L1,2} + \mathrm{inv}\alpha_n \right) \tag{64}$$

Relative distance from pitch circle to the pinion point of load application and the wheel tooth centreline is:

$$\Delta r_{y0\ \ 1,2} = \frac{r_{vbn1,2}}{\cos\alpha_{h1,2}} - r_{vn1,2} \tag{65}$$

where

$$\alpha_{h1,2} = \alpha_{L1,2} - \xi_{h1,2} \tag{66}$$

Mean transverse radius to point of load application, in millimetres:

$$r_{my0\,1,2} = r_{mpt1,2} \left(\frac{R_m + x_{oo1,2}}{R_m} \right) + \Delta r_{y01,2} m_{et2} \tag{67}$$

7.4.4.3 Load sharing ratio, ε_N

The load sharing ratio, ε_N, is used to calculate the proportion of the total load carried on the tooth being analysed. It is given by the following formulae (i.e. Formulae (68), (69) and (70):

$$\begin{aligned} g'^3_J = g_J^3 &+ \sum_{k=1}^{k=x} \sqrt{\left[g_J^2 - 4k\ \frac{\pi\ m_{mn} \cos\alpha_a}{m_{et2}} \left(k\ \frac{\pi\ m_{mn} \cos\alpha_a}{m_{et2}} + 2 y_J \right) \right]^3} \\ &+ \sum_{k=1}^{k=y} \sqrt{\left[g_J^2 - 4k\ \frac{\pi\ m_{mn} \cos\alpha_a}{m_{et2}} \left(k\ \frac{\pi\ m_{mn} \cos\alpha_a}{m_{et2}} - 2 y_J \right) \right]^3} \end{aligned} \tag{68}$$

In Formula (68), k is a positive integer, which has successive values from 1 to x or y, generating all real terms (positive values under the radical) in each series. Imaginary terms (negative values under the radical) shall be ignored. For most designs, x and y are not greater than 2.

The load sharing ratio is:

$$\varepsilon_{\mathrm{N}} = \frac{g_{\mathrm{J}}^{3}}{g'^{3}_{\mathrm{J}}} \tag{69}$$

For statically loaded straight and Zerol bevel gears:

$$\varepsilon_{\mathrm{N}} = 1{,}0 \tag{70}$$

7.4.4.4 Tooth form factor, $Y_{1,2}$

The tooth form factor incorporates both the radial and tangential components of the normal load. Since this factor defines the weakest section, its value shall be determined by iteration for pinion and wheel separately.

$$g_{01,2} = 0{,}5 s_{\mathrm{vmn}1,2} + h_{\mathrm{vfm}1,2} \tan\alpha_{\mathrm{n}} + \rho_{\mathrm{va}01,2}\left(\frac{1-\sin\alpha_{\mathrm{n}}}{\cos\alpha_{\mathrm{n}}}\right) \tag{71}$$

$$g_{\mathrm{yb}1,2} = h_{\mathrm{vfm}1,2} - \rho_{\mathrm{va}01,2} \tag{72}$$

$$g_{\mathrm{f}01,2(1)} = g_{01,2} + g_{\mathrm{yb}1,2} \tag{73}$$

Start of iteration with $g_{\mathrm{f}01,2(1)}$ as initial value:

$$\xi_{1,2} = \frac{g_{\mathrm{f}0\,1,2}}{r_{\mathrm{vn}1,2}} \tag{74}$$

$$g_{\mathrm{xb}1,2} = g_{\mathrm{f}01,2} - g_{01,2} \tag{75}$$

$$g_{\mathrm{za}1,2} = g_{\mathrm{yb}1,2}\cos\xi_{1,2} - g_{\mathrm{xb}1,2}\sin\xi_{1,2} \tag{76}$$

$$g_{\mathrm{zb}1,2} = g_{\mathrm{yb}1,2}\sin\xi_{1,2} + g_{\mathrm{xb}1,2}\cos\xi_{1,2} \tag{77}$$

$$\tan\tau_{1,2} = \frac{g_{\mathrm{za}1,2}}{g_{\mathrm{zb}1,2}} \tag{78}$$

$$s_{\mathrm{N}1,2} = r_{\mathrm{vn}1,2}\sin\xi_{1,2} - \rho_{\mathrm{va}01,2}\cos\tau_{1,2} - g_{\mathrm{zb}1,2} \tag{79}$$

$$h_{\mathrm{N}1,2} = \Delta r_{\mathrm{y}01,2} + r_{\mathrm{vn}1,2}\left(1-\cos\xi_{1,2}\right) + \rho_{\mathrm{va}01,2}\sin\tau_{1,2} + g_{\mathrm{za}1,2} \tag{80}$$

Change $g_{\mathrm{f}01,2}$ until

$$\frac{s_{\mathrm{N}1,2}\cot\tau_{1,2}}{h_{\mathrm{N}1,2}} = 2{,}0 \pm 0{,}001 \tag{81}$$

For the second trial, make $g_{\mathrm{f}01,2(2)} = g_{\mathrm{f}01,2(1)} + 0{,}005 m_{\mathrm{et}2}$, for the third and subsequent trials, interpolate.

End of iteration.

Tooth strength factor:

$$x_{N1,2} = \frac{s_{N1,2}^{2}}{h_{N1,2}} \tag{82}$$

Tooth form factor:

$$Y_{1,2} = \frac{2}{3}\left[\frac{1}{\left(\frac{1}{x_{N1,2}} - \frac{\tan\alpha_{h1,2}}{3\,s_{N1,2}}\right)}\right] \tag{83}$$

Figure 6 illustrates the parameters h_N, s_N and x_N.

7.4.5 Geometry factor, Y_J, for hypoid gears

7.4.5.1 Initial formulae

Tooth surface points are calculated using the function $e^{\theta \tan\alpha_f}$ to approximate the tooth surfaces.

Drive flank pressure angle in wheel root coordinates:

$$\alpha_{Dnf} = \alpha_{nD} - \theta_{f2}\sin\beta_{m2} \tag{84}$$

where θ_{f2} is the dedendum angle of the wheel (given in ISO 23509:2006, Table C.5).

Coast flank pressure angle in wheel root coordinates:

$$\alpha_{Cnf} = \alpha_{nC} + \theta_{f2}\sin\beta_{m2} \tag{85}$$

Average pressure angle unbalance:

$$\Delta\alpha_1 = \frac{(\alpha_{Dnf} - \alpha_{Cnf})}{2,0} \tag{86}$$

Limit pressure angle in wheel root coordinates:

$$\alpha_f = \alpha_{lim} - \theta_{f2}\sin\beta_{m2} \tag{87}$$

Relative distance from blade edge to centreline:

$$g_{rb} = \frac{\left(h_{fm2}\tan\frac{\alpha_{nD}+\alpha_{nC}}{2,0} + \frac{W_{m2}}{2,0}\right)\cos\frac{\alpha_{nD}+\alpha_{nC}}{2,0}}{m_{et2}} \tag{88}$$

where W_{m2} is wheel mean slot width (see ISO 23509:2006, Figure 16).

Intermediate value:

$$\eta_{\mathrm{D}} = \tan\alpha_{\mathrm{Dnf}} \left(\frac{g_{\mathrm{rb}}}{\sin\alpha_{\mathrm{Dnf}}} - h_{\mathrm{vfm2}} \right) \tag{89}$$

Intermediate value:

$$\eta_{\mathrm{C}} = \tan\alpha_{\mathrm{Cnf}} \left(\frac{g_{\mathrm{rb}}}{\sin\alpha_{\mathrm{Cnf}}} - h_{\mathrm{vfm2}} \right) \tag{90}$$

Intermediate angle:

$$\tan\beta_{\mathrm{a}} = \frac{\dfrac{W_{\mathrm{m2}}}{2{,}0 m_{\mathrm{et2}}} - \rho_{\mathrm{va02}} \left(\sec\dfrac{\alpha_{\mathrm{nD}} + \alpha_{\mathrm{nC}}}{2{,}0} - \tan\dfrac{\alpha_{\mathrm{nD}} + \alpha_{\mathrm{nC}}}{2{,}0} \right)}{h_{\mathrm{vfm2}} - \rho_{\mathrm{va02}}} \tag{91}$$

Intermediate angle:

$$\left(\beta_{\mathrm{D}} - \Delta\alpha \right) = \beta_{\mathrm{a}} - \Delta\alpha_1 \tag{92}$$

Intermediate angle:

$$\left(\beta_{\mathrm{C}} - \Delta\alpha \right) = -\beta_{\mathrm{a}} - \Delta\alpha_1 \tag{93}$$

Intermediate value:

$$g_1 = \frac{h_{\mathrm{vfm2}} - \rho_{\mathrm{va02}}}{\cos\beta_{\mathrm{a}}} \tag{94}$$

Wheel angle between centreline and fillet point on drive side:

$$\tan \Delta\theta_{\mathrm{D}} = \frac{g_1 \sin\left(\beta_{\mathrm{D}} - \Delta\alpha \right)}{r_{\mathrm{vn2}} - g_1 \cos\left(\beta_{\mathrm{D}} - \Delta\alpha \right)} \tag{95}$$

Wheel angle between centreline and fillet point on coast side:

$$\tan \Delta\theta_{\mathrm{C}} = \frac{g_1 \sin\left(\beta_{\mathrm{C}} - \Delta\alpha \right)}{r_{\mathrm{vn2}} - g_1 \cos\left(\beta_{\mathrm{C}} - \Delta\alpha \right)} \tag{96}$$

Wheel angle between fillet points:

$$\Delta\theta_2 = \frac{\theta_{\mathrm{v2}} + \Delta\theta_{\mathrm{D}} + \Delta\theta_{\mathrm{C}}}{2{,}0} \tag{97}$$

where θ_{v2} is angular pitch of virtual cylindrical wheel (see ISO 10300-1:2014, B.10).

Vertical distance from pitch circle to fillet point:

$$y_1 = r_{\mathrm{vn2}} - \frac{\left[r_{\mathrm{vn2}} - g_1 \cos\left(\beta_{\mathrm{D}} - \alpha_{\mathrm{f}}\right)\right] \cos\left(\Delta\theta_2 - \Delta\theta_{\mathrm{D}}\right)}{\cos\Delta\theta_{\mathrm{D}}} \tag{98}$$

Horizontal distance from centreline to fillet point:

$$x_1 = \frac{\left[r_{\mathrm{vn2}} - g_1 \cos\left(\beta_{\mathrm{D}} - \alpha_{\mathrm{f}}\right)\right] \sin\left(\Delta\theta_2 - \Delta\theta_{\mathrm{D}}\right)}{\cos\Delta\theta_{\mathrm{D}}} \tag{99}$$

Generated pressure angle of wheel at fillet point [required for Formula (167)]:

$$\alpha_{\mathrm{LN2}} = \alpha_{\mathrm{Dnf}} - \Delta\theta_2 \tag{100}$$

Distance from centreline to tool critical drive side fillet point:

$$\mu_{\mathrm{1D}} = \eta_{\mathrm{D}} + \tan\alpha_{\mathrm{Dnf}}\left(h_{\mathrm{vfm1}} + h_{\mathrm{vfm2}}\right) + \rho_{\mathrm{va01}}\left(\sec\alpha_{\mathrm{Dnf}} - \tan\alpha_{\mathrm{Dnf}}\right) \tag{101}$$

Distance from centreline to tool critical coast side fillet point:

$$\mu_{\mathrm{1C}} = \eta_{\mathrm{C}} + \tan\alpha_{\mathrm{Cnf}}\left(h_{\mathrm{vfm1}} + h_{\mathrm{vfm2}}\right) - \rho_{\mathrm{va01}}\left(\sec\alpha_{\mathrm{Cnf}} - \tan\alpha_{\mathrm{Cnf}}\right) \tag{102}$$

Wheel angle between centreline and critical pinion drive side fillet point:

$$\tan\theta_{\mathrm{DLS}} = \frac{\mu_{\mathrm{1D}}}{r_{\mathrm{vn2}} + h_{\mathrm{vfm1}}} \tag{103}$$

Wheel angle between centreline and critical pinion coast side fillet point:

$$\tan\theta_{\mathrm{CLS}} = \frac{\mu_{\mathrm{1C}}}{r_{\mathrm{vn2}} + h_{\mathrm{vfm1}}} \tag{104}$$

Radius from tool centre to critical pinion drive side fillet point:

$$R_{\mathrm{DL2}} = \frac{r_{\mathrm{vn2}} + h_{\mathrm{vfm1}}}{\cos\theta_{\mathrm{DLS}}} \tag{105}$$

Radius from tool centre to critical pinion coast side fillet point:

$$R_{\mathrm{CL2}} = \frac{r_{\mathrm{vn2}} + h_{\mathrm{vfm1}}}{\cos\theta_{\mathrm{CLS}}} \tag{106}$$

Wheel angle from centreline to pinion tip on drive side, θ_{D1}:

For start of iteration, assume $\theta_{\mathrm{D1}} = \theta_{\mathrm{v2}}$.

$$\Delta r_1 = r_{\mathrm{vn2}}\left(e^{\theta_{\mathrm{D1}}\tan\alpha_{\mathrm{f}}} - 1{,}0\right) \tag{107}$$

$$h_1 = \left(r_{\mathrm{vn2}} + \Delta r_1\right)\sin\left(\alpha_{\mathrm{Dnf}}\theta_{\mathrm{D1}}\right) - \left(r_{\mathrm{vn2}}\sin\alpha_{\mathrm{Dnf}} - g_{\mathrm{rb}}\right) \tag{108}$$

$$h_{\mathrm{1o}} = \sqrt{r_{\mathrm{vn1}}^2 - \left(r_{\mathrm{vn1}} - \Delta r_1\right)^2 \cos^2\left(\alpha_{\mathrm{vDnf}} + \theta_{\mathrm{D1}}\right)} - \left(r_{\mathrm{vn1}} - \Delta r_1\right)\sin\left(\alpha_{\mathrm{Dnf}} + \theta_{\mathrm{D1}}\right) \tag{109}$$

Change θ_{D1} until $h_{\mathrm{1o}} = h_1$ which is the end of iteration.

Wheel angle from centreline to tooth surface at critical fillet point on drive side, θ_{D2o}:

For the start of iteration, assume $\theta_{\mathrm{D2o}} = 1/2\theta_{\mathrm{v2}}$.

$$\mu_{\mathrm{1Do}} = r_{\mathrm{vn2}} e^{\theta_{\mathrm{D2o}} \tan\alpha_{\mathrm{f}}} \sin\theta_{\mathrm{D2o}} \tag{110}$$

Change θ_{D2o} until $\mu_{\mathrm{1Do}} = \mu_{\mathrm{1D}}$ which is the end of iteration.

Wheel angle from centreline to tooth surface at critical fillet point on coast side, θ_{C2o}:

For start of iteration, assume $\theta_{\mathrm{C2o}} = -1/2\theta_{\mathrm{v2}}$.

$$\mu_{\mathrm{1Co}} = r_{\mathrm{vn2}} e^{\theta_{\mathrm{C2o}} \tan\alpha_{\mathrm{f}}} \sin\theta_{\mathrm{C2o}} \tag{111}$$

Change θ_{C2o} until $\mu_{\mathrm{1Co}} = \mu_{\mathrm{1C}}$ which is the end of iteration.

Pinion angle from centreline to tooth surface at critical drive side fillet point, θ_{D1o}:

Formula (112) shall be solved for θ_{D1o}.

$$r_{\mathrm{vn2}}\left(e^{\theta_{\mathrm{D2o}} \tan\alpha_{\mathrm{f}}} - 1{,}0\right) = r_{\mathrm{vn1}}\left(1{,}0 - e^{\theta_{\mathrm{D1o}} \tan\alpha_{\mathrm{f}}}\right) \tag{112}$$

Pinion angle from centreline to tooth surface at critical coast side fillet point, θ_{C1o}:

Formula (113) shall be solved for θ_{C1o}.

$$r_{\mathrm{vn2}}\left(e^{\theta_{\mathrm{C2o}} \tan\alpha_{\mathrm{f}}} - 1{,}0\right) = r_{\mathrm{vn1}}\left(1{,}0 - e^{\theta_{\mathrm{C1o}} \tan\alpha_{\mathrm{f}}}\right) \tag{113}$$

Wheel difference angle between tool and surface at drive side fillet point:

$$\Delta\theta_{\mathrm{D2o}} = \theta_{\mathrm{DLS}} - \theta_{\mathrm{D2o}} \tag{114}$$

Wheel difference angle between tool and surface at coast side fillet point:

$$\Delta\theta_{\mathrm{C2o}} = \theta_{\mathrm{CLS}} - \theta_{\mathrm{C2o}} \tag{115}$$

Pinion difference angle between tool and surface at drive side fillet point:

$$\tan\Delta\theta_{\mathrm{D1o}} = -\frac{R_{\mathrm{DL2}} \sin\Delta\theta_{\mathrm{D2o}}}{r_{\mathrm{vn2}} + r_{\mathrm{vn1}} - R_{\mathrm{DL2}} \cos\Delta\theta_{\mathrm{D2o}}} \tag{116}$$

Pinion difference angle between tool and surface at coast side fillet point:

$$\tan\Delta\theta_{\mathrm{C1o}} = -\frac{R_{\mathrm{CL2}} \sin\Delta\theta_{\mathrm{C2o}}}{r_{\mathrm{vn2}} + r_{\mathrm{vn1}} - R_{\mathrm{CL2}} \cos\Delta\theta_{\mathrm{C2o}}} \tag{117}$$

Pinion angle unbalance between fillet points:

$$\Delta\theta_1 = \frac{\theta_{\mathrm{D1o}} + \theta_{\mathrm{C1o}} + \Delta\theta_{\mathrm{D1o}} + \Delta\theta_{\mathrm{C1o}}}{2{,}0} \tag{118}$$

Pinion angle from centreline to pinion tip, θ_{Do}:

Formula (119) shall be solved for θ_{Do}.

$$\Delta r_1 = r_{\mathrm{vn1}}\left(1{,}0 - e^{\theta_{\mathrm{Do}} \tan\alpha_{\mathrm{f}}}\right) \tag{119}$$

Wheel angle from centreline to tooth surface at pitch point on drive side, θ_{D}:

For start of iteration, assume $\theta_D = -1/3\,\theta_{v2}$.

$$\Delta r = r_{vn2}\left(e^{\theta_D \tan\alpha_f} - 1{,}0\right) \tag{120}$$

$$h = \left(r_{vn2} + \Delta r\right)\,\sin\left(\alpha_{Dnf} + \theta_D\right) - \left(r_{vn2}\sin\alpha_{Dnf} - g_{rb}\right) \tag{121}$$

Change θ_D until h = 0,0 which is the end of iteration

The wheel angle from centreline to fillet point on drive flank, θ_{D2}, is evaluated by iteration. The initial value, θ_{D2}, should be determined depending on the amount of $\left(r_{vn2} + \Delta r\right)$:

a) $\left(r_{vn2} + \Delta r\right) > r_{va2}$

$$\theta_{D2} = 0{,}8\,\theta_D \tag{122a}$$

b) $\left(r_{vn2} + \Delta r\right) = r_{va2}$

$$\theta_{D2} = 1{,}0\,\theta_D \tag{122b}$$

c) $\left(r_{vn2} + \Delta r\right) < r_{va2}$

$$\theta_{D2} = 1{,}2\theta_D \tag{122c}$$

Start of iteration:

$$\Delta r_2 = r_{vn2}\left(e^{\theta_{D2} \tan\alpha_f} - 1{,}0\right) \tag{123}$$

$$h_2 = \left(r_{vn2} + \Delta r_2\right)\sin\left(\alpha_{Dnf} + \theta_{D2}\right) - \left(r_{vn2}\sin\alpha_{Dnf} - g_{rb}\right) \tag{124}$$

$$h_{2o} = \pm\sqrt{r_{va1}^2 - \left(r_{vn1} + \Delta r_2\right)^2 \cos^2\left(\alpha_{Dnf} + \theta_{D2}\right)} + \left(r_{vn1} + \Delta r_2\right)\sin\left(\alpha_{Dnf} + \theta_{D2}\right) \tag{125}$$

with +sign, if $\left(r_{vn2}\sin\alpha_{Dnf} - g_{rb}\right) < 0{,}0$ or –sign, if $\left(r_{vn2}\sin\alpha_{Dnf} - g_{rb}\right) \geq 0{,}0$

Change θ_{D2} until h_2 = h_{2o} which is the end of iteration.

7.4.5.2 Load sharing ratio, ε_N, for hypoid gears

Load sharing ratio for hypoid gears:

$$\varepsilon_N = 1{,}0 \tag{126}$$

7.4.5.3 Tooth strength factor, $x_{N1,2}$, for hypoid gears

Length of action from pinion tip to pitch circle in normal section:

$$g_{v\alpha1} = \sqrt{h_1^2 + (\Delta r_1 - \Delta r)^2 - 2{,}0 h_1 (\Delta r_1 - \Delta r) \sin(\alpha_{Dnf} + \theta_{D1})} \quad (127)$$

Length of action from wheel tip to pitch circle in normal section:

$$g_{v\alpha2} = \sqrt{h_2^2 + (\Delta r_2 - \Delta r)^2 - 2{,}0 h_2 (\Delta r_2 - \Delta r) \sin(\alpha_{Dnf} + \theta_{D2})} \quad (128)$$

Length of action in normal section:

$$g_{v\alpha n} = g_{v\alpha1} + g_{v\alpha2} \quad (129)$$

Profile contact ratio in mean normal section:

$$\varepsilon_{v\alpha n} = g_{v\alpha n} / p_{mn} \quad (130)$$

Modified contact ratio for hypoid gears:

$$\varepsilon_{v\gamma} = \sqrt{\varepsilon_{v\alpha n}^2 + \varepsilon_{v\beta}^2} \quad (131)$$

The profile load sharing factor is

a) when $\varepsilon_{v\gamma} < 2{,}0$:

$$\varepsilon_f = 1{,}0 - 0{,}5\varepsilon_{v\gamma} \quad (132a)$$

b) when $\varepsilon_{v\gamma} \geq 2{,}0$:

$$\varepsilon_f = 0{,}0 \quad (132b)$$

The lengthwise load sharing factor is

a) when $\varepsilon_{v\gamma} < 2{,}0$:

$$\varepsilon_b = 2{,}0\sqrt{\varepsilon_{v\gamma} - 1{,}0} \quad (133a)$$

b) when $\varepsilon_{v\gamma} \geq 2{,}0$:

$$\varepsilon_{\mathrm{b}} = \varepsilon_{\mathrm{v}\gamma} \tag{133b}$$

Length of action from pinion tip to point of load application:

$$g_{\mathrm{v\alpha 3}} = \left| \frac{p_{\mathrm{mn}} \varepsilon_{\mathrm{v\alpha n}}^{2}}{\varepsilon_{\mathrm{v}\gamma}^{2}} \left(\frac{0{,}5 \varepsilon_{\mathrm{v}\gamma}^{2}}{\varepsilon_{\mathrm{v\alpha n}}} - \frac{\varepsilon_{\mathrm{v}\beta} \varepsilon_{\mathrm{b}}\, k'}{\varepsilon_{\mathrm{v}\alpha}} + \varepsilon_{\mathrm{f}} \right) - g_{\mathrm{v\alpha 1}} \right| \tag{134}$$

Length of action from wheel tip to point of load application:

$$g_{\mathrm{v\alpha 4}} = \left| \frac{p_{\mathrm{mn}} \varepsilon_{\mathrm{v\alpha n}}^{2}}{\varepsilon_{\mathrm{v}\gamma}^{2}} \left(\frac{0{,}5 \varepsilon_{\mathrm{v}\gamma}^{2}}{\varepsilon_{\mathrm{v\alpha n}}} + \frac{\varepsilon_{\mathrm{v}\beta} \varepsilon_{\mathrm{b}}\, k'}{\varepsilon_{\mathrm{v}\alpha}} + \varepsilon_{\mathrm{f}} \right) - g_{\mathrm{v\alpha 2}} \right| \tag{135}$$

Length of action to point of load application:

$$g_{\mathrm{J1,2}} = g_{\mathrm{v\alpha n}} - g_{\mathrm{v\alpha 3,4}} \tag{136}$$

Wheel angle from pinion tip to point of load application, θ_{D3}:

For start of iteration, assume $\theta_{\mathrm{D3}} = -1/2\theta_{\mathrm{v2}}$.

$$\Delta r_{3} = r_{\mathrm{vn2}} \left(e^{\theta_{\mathrm{D3}} \tan \alpha_{\mathrm{f}}} - 1{,}0 \right) \tag{137}$$

$$h_{3} = \left(r_{\mathrm{vn2}} + \Delta r_{3} \right) \sin\left(\alpha_{\mathrm{vDnf}} + \theta_{\mathrm{D3}} \right) - \left(r_{\mathrm{vn2}} \sin \alpha_{\mathrm{vDnf}} - g_{\mathrm{rb}} \right) \tag{138}$$

$$h_{3\mathrm{o}} = \sqrt{g_{\mathrm{v\alpha 3}}^{2} - \left(\Delta r_{3} - \Delta r \right)^{2} \cos^{2}\left(\alpha_{\mathrm{vDnf}} + \theta_{\mathrm{D3}} \right)} - \left| \Delta r_{3} - \Delta r \right| \sin\left(\alpha_{\mathrm{vDnf}} + \theta_{\mathrm{D3}} \right) \tag{139}$$

Change θ_{D3} until $h_{3} = h_{3\mathrm{o}}$ which is the end of iteration.

Pinion angle from wheel tip to point of load application, θ_{D4}:

For start of iteration, assume $\theta_{\mathrm{D4}} = 1/3\theta_{\mathrm{v2}}$.

$$\Delta r_{4} = r_{\mathrm{vn2}} \left(e^{\theta_{\mathrm{D4}} \tan \alpha_{\mathrm{f}}} - 1{,}0 \right) \tag{140}$$

$$h_{4} = \left(r_{\mathrm{vn2}} + \Delta r_{4} \right) \sin\left(\alpha_{\mathrm{vDnf}} + \theta_{\mathrm{D4}} \right) - \left(r_{\mathrm{vn2}} \sin \alpha_{\mathrm{vDnf}} - g_{\mathrm{rb}} \right) \tag{141}$$

$$h_{4\mathrm{o}} = \sqrt{g_{\mathrm{v\alpha 4}}^{2} - \left(\Delta r_{4} - \Delta r \right)^{2} \cos^{2}\left(\alpha_{\mathrm{vDnf}} + \theta_{\mathrm{D4}} \right)} - \left| \Delta r_{4} - \Delta r \right| \sin\left(\alpha_{\mathrm{vDnf}} + \theta_{\mathrm{D4}} \right) \tag{142}$$

Change θ_{D4} until $h_{4} = h_{4\mathrm{o}}$ which is the end of iteration.

Distance from pitch circle to point of load application, Δr_{LN2}:

$$\Delta r_{\mathrm{LN2}} = \frac{\left(r_{\mathrm{vn2}} + \Delta r_{3} \right) \cos\left(\alpha_{\mathrm{vDnf}} + \theta_{\mathrm{D3}} \right)}{\cos \alpha_{\mathrm{LN2}}} - r_{\mathrm{vn2}} \tag{143}$$

Angle between centreline and line from point of load application and fillet point on wheel, $\alpha_{2\mathrm{oo}}$:

For start of iteration, assume $\alpha_{2\mathrm{oo}} = 2{,}0\alpha_{\mathrm{Dnf}}$.

Horizontal distance from centreline to critical fillet point:

$$s_{N2} = x_1 - \rho_{va02} \cos\alpha_{2oo} \tag{144}$$

Vertical distance from pitch circle to critical fillet point:

$$y_2 = y_1 + \rho_{va02} \sin\alpha_{2oo} \tag{145}$$

Wheel load height at weakest section:

$$h_{N2} = y_2 + \Delta r_{LN2} \tag{146}$$

Auxiliary value:

$$h_{N2o} = \frac{s_{N2}}{2{,}0 \tan\alpha_{2oo}} \tag{147}$$

Change α_{2oo} until $h_{N2} = h_{N2o}$ which is the end of iteration.

At this stage, the wheel tooth strength factor is calculated by Formula (148):

$$x_{N2} = \frac{s_{N2}^2}{h_{N2}} \tag{148}$$

For the pinion angle from pitch point to point of load application, θ_{D5}:

Formula (149) shall be solved for θ_{D5}.

$$\Delta r_4 = r_{vn1}\left(1{,}0 - e^{\theta_{D5}\tan\alpha_f}\right) \tag{149}$$

Pinion pressure angle at point of load application:

$$\alpha_{LN1} = \alpha_{vnf} + \theta_{D4} - \theta_{D5} + \Delta\theta_1 \tag{150}$$

Pinion radial distance to point of load application:

$$r_{41o} = \frac{\left(r_{vn1} - \Delta r_4\right)\cos\left(\alpha_{Dnf} + \theta_{D4}\right)}{\cos\alpha_{LN1}} \tag{151}$$

Start of iteration comprising Formula (152) to Formula (165):

$\alpha_{Do} = \alpha_{nD}$ should be used as initial value.

Wheel angle between centreline and pinion fillet, θ_{D2oo}:

For an enclosed iteration, assume $\theta_{D2oo} = 1/2\theta_{v2}$.

$$\Delta r_5 = r_{vn2} e^{\theta_{D2oo}\tan\alpha_f} \tag{152}$$

$$\mu_{D2} = \frac{r_{vn2} + h_{vfm1} - \rho_{va01} - \Delta r_5 \cos\theta_{D2oo}}{\tan\alpha_{Do}} \tag{153}$$

$$\mu_D = \Delta r_5 \sin\theta_{D2oo} \tag{154}$$

Change θ_{D2oo} until $\mu_D = \mu_{D1} + \mu_{D2}$ which is the end of the enclosed iteration.

For the pinion angle between centreline and pinion fillet solve Formula (155) for θ_{D1oo}:

$$r_{\mathrm{vn2}}\left(e^{\theta_{\mathrm{D2oo}}\tan\alpha_{\mathrm{f}}}-1{,}0\right)=r_{\mathrm{vn1}}\left(1{,}0-e^{\theta_{\mathrm{D1oo}}\tan\alpha_{\mathrm{f}}}\right) \tag{155}$$

Wheel rotation through path of action:

$$\tan\theta_{\mathrm{L2o}}=\frac{\mu_{\mathrm{D1}}-\rho_{\mathrm{va01}}\cos\alpha_{\mathrm{Do}}}{r_{\mathrm{vn2}}+h_{\mathrm{vfm1}}-\rho_{\mathrm{va01}}+\rho_{\mathrm{va01}}\sin\alpha_{\mathrm{Do}}} \tag{156}$$

Wheel angle difference between path of action and tooth surface at pinion fillet:

$$\Delta\theta_{\mathrm{D2oo}}=\theta_{\mathrm{L2o}}-\theta_{\mathrm{D2oo}} \tag{157}$$

Wheel radius to pinion fillet point:

$$r_{\mathrm{L2o}}=\frac{r_{\mathrm{vn2}}+h_{\mathrm{vfm1}}-\rho_{\mathrm{va01}}+\rho_{\mathrm{va01}}\sin\alpha_{\mathrm{Do}}}{\cos\theta_{\mathrm{L2o}}} \tag{158}$$

Pinion angle to fillet point:

$$\tan\Delta\theta_{\mathrm{D1oo}}=-\frac{r_{\mathrm{L2o}}\sin\Delta\theta_{\mathrm{D2oo}}}{r_{\mathrm{vn2}}+r_{\mathrm{vn1}}-r_{\mathrm{L2o}}\cos\Delta\theta_{\mathrm{D2oo}}} \tag{159}$$

Pinion radius to fillet point:

$$r_{\mathrm{L1o}}=\frac{r_{\mathrm{vn2}}+r_{\mathrm{vn1}}-r_{\mathrm{L2o}}\cos\Delta\theta_{\mathrm{D2oo}}}{\cos\Delta\theta_{\mathrm{D1oo}}} \tag{160}$$

Pinion angle from centreline to fillet point:

$$\left(\Delta\theta_1-\theta_{\mathrm{L1o}}\right)=\Delta\theta_1-\theta_{\mathrm{D1oo}}-\Delta\theta_{\mathrm{D1oo}} \tag{161}$$

Angle between centreline and line from point of load application and fillet point on pinion, α_1

$$\alpha_1=\alpha_{\mathrm{Do}}-\theta_{\mathrm{D2oo}}+\theta_{\mathrm{D1oo}} \tag{162}$$

Horizontal distance to critical fillet point:

$$s_{\mathrm{N1}}=r_{\mathrm{L1o}}\sin\left(\Delta\theta_1-\theta_{\mathrm{L1o}}\right) \tag{163}$$

Pinion load height at weakest section:

$$h_{\mathrm{N1}}=r_{\mathrm{41o}}-r_{\mathrm{L1o}}\cos\left(\Delta\theta_1-\theta_{\mathrm{L1o}}\right) \tag{164}$$

Auxiliary value:

$$h_{\mathrm{N1o}}=\frac{s_{\mathrm{N1}}}{2{,}0\tan\alpha_1} \tag{165}$$

Change α_{Do} until $h_{\mathrm{N1}}=h_{\mathrm{N1o}}$

End of iteration.

At this stage, the pinion tooth strength factor is calculated by:

$$x_{\mathrm{N1}}=\frac{s_{\mathrm{N1}}^2}{h_{\mathrm{N1}}} \tag{166}$$

7.4.5.4 Tooth form factor, $Y_{1,2}$, for hypoid gears

Calculations shall be carried out for pinion and wheel:

$$Y_{1,2} = \frac{2}{3}\left[\frac{1}{\left(\frac{1}{x_{N1,2}} - \frac{\tan\alpha_{LN1,2}}{3\, s_{N1,2}}\right)}\right] \tag{167}$$

7.4.5.5 Transverse radius to point of load application, $r_{my0\ 1,2}$

Mean face width of pinion and wheel:

$$b_{k1,2} = \frac{b_{1,2}\varepsilon_{v\alpha n}\varepsilon_{v\beta}}{\varepsilon_{v\gamma}^2} \tag{168}$$

Contact shift due to load for pinion and wheel:

$$x_{oo1} = k' b_{k1} - \frac{b_1\varepsilon_f\varepsilon_{v\beta}}{\varepsilon_{v\gamma}^2} \tag{169}$$

$$x_{oo2} = k' b_{k2} + \frac{b_2\varepsilon_f\varepsilon_{v\beta}}{\varepsilon_{v\gamma}^2} \tag{170}$$

where k' is the contact shift factor as specified in ISO 10300-1:2014, B.5.

Transverse radius to point of load application for pinion and wheel:

$$r_{my01} = \frac{r_{mpt1}\left(x_{oo1} + R_{m2}\right)}{R_{m2}} + \left(r_{410} - r_{vn1}\right) m_{et2} \tag{171}$$

$$r_{my02} = \frac{r_{mpt2}\left(x_{oo2} + R_{m2}\right)}{R_{m2}} + \Delta r_{LN2}\ m_{et2} \tag{172}$$

7.4.6 Additional tooth strength parameters (for bevel and hypoid gears)

7.4.6.1 Tooth fillet radius at root diameter, r_{mf}

The minimum tooth fillet radius occurs at the point where the fillet becomes tangent to the root circles, and is given by Formula (173).

Relative fillet radius at root of tooth:

$$r_{mf1,2} = \frac{\left(h_{vfm1,2} - \rho_{va01,2}\right)^2}{r_{vn1,2} + h_{vfm1,2} - \rho_{va01,2}} + \rho_{va01,2} \tag{173}$$

7.4.6.2 Stress concentration and correction factor, Y_f

The stress concentration and stress correction factor, Y_f, depends on the following:

a) tooth geometry;

b) location of the load;

c) plasticity effects;

d) residual stress effects;

e) material composition effects;

f) surface finish resulting from gear production and subsequent service;

g) Hertzian stress effects;

h) size effects;

i) end of tooth effects.

The following stress concentration and stress correction factors, derived by Dolan and Broghamer, consider a) and b) only:

$$Y_{f1,2} = L + \left(\frac{2s_{N1,2}}{r_{mf1,2}}\right)^{M} \left(\frac{2s_{N1,2}}{h_{N1,2}}\right)^{O} \tag{174}$$

where

$$L = 0{,}325\,454\,5 - 0{,}007\,272\,7\,\alpha_n \tag{175}$$

$$M = 0{,}331\,818\,2 - 0{,}009\,090\,9\,\alpha_n \tag{176}$$

$$O = 0{,}268\,181\,8 + 0{,}009\,090\,9\,\alpha_n \tag{177}$$

α_n is the actual pressure angle, in degrees.

Other factors from c) to i) may be included under those covered by Formula (48). Usually, d) and e) are included in the allowable stress number σ_{FE}, while h) is included in size factor Y_X and i) in calculated effective face width, b_{ce} (see 7.4.6.4).

7.4.6.3 Inertia factor, Y_i

The inertia factor, Y_i, allows for the lack of smoothness of the tooth action in dynamically loaded gears with a relatively low contact ratio, and is given as follows:

a) when $\varepsilon_{v\gamma} < 2{,}0$:

$$Y_i = 2{,}0/\varepsilon_{v\gamma} \tag{178a}$$

b) when $\varepsilon_{v\gamma} \geq 2{,}0$:

$$Y_i = 1{,}0 \tag{178b}$$

For statically loaded gears, such as those in vehicle drive axle differential gears, Y_i equals 1,0 even when $\varepsilon_{v\gamma}$ is less than 2,0.

7.4.6.4 Calculated effective face width, b_{ce}

This quantity evaluates the effectiveness of the tooth in distributing the load over the root cross section, as the instantaneous contact line frequently does not extend over the entire face width. Formulae (179) to (184) are used to determine the value of the effective face width:

$$g_{K1,2} = \frac{b_{1,2}\, g_{v\alpha n}\, g_{J1,2}\, \cos^2 \beta_{vb}}{g_\eta^2} \qquad (179)$$

where

g_K is the projected length of the instantaneous contact line in the tooth lengthwise direction;

g_J is the length of action from mean point to point of load application, [see Formula (59) for bevel gears without offset or Formula (136) for hypoid gears].

For the toe increment:

$$\Delta b'_{i1,2} = \frac{b_{1,2} - g_{K1,2}}{2\cos \beta_{m1,2}} - \frac{x_{oo1,2}}{\cos \beta_{m1,2}} \qquad (180)$$

For the heel increment:

$$\Delta b'_{e1,2} = \frac{b_{1,2} - g_{K1,2}}{2\cos \beta_{m1,2}} + \frac{x_{oo1,2}}{\cos \beta_{m1,2}} \qquad (181)$$

with $x_{oo1,2}$ as calculated in 7.4.4.2.

The toe and heel increment are used to select the correct $\Delta b_{i1,2}$ and $\Delta b_{e1,2}$ values for Formula (183):

a) when $\Delta b'_{i1,2}$ and $\Delta b'_{e1,2}$ are both positive:

$$\Delta b_{i1,2} = \Delta b'_{i1,2} \qquad (182a)$$

b) when $\Delta b'_{i1,2}$ is positive and $\Delta b'_{e1,2}$ is negative:

$$\Delta b_{i1,2} = (b_{1,2} - g_{K\,1,2}) / \cos \beta_{m1,2} \qquad (182b)$$

c) when $\Delta b'_{i1,2}$ is negative and $\Delta b'_{e1,2}$ is positive:

$$\Delta b_{i1,2} = 0 \qquad (182c)$$

d) when $\Delta b'_{e1,2}$ and $\Delta b'_{i1,2}$ are both positive:

$$\Delta b_{e1,2} = \Delta b'_{e1,2} \qquad (182d)$$

e) when $\Delta b'_{e1,2}$ is positive and $\Delta b'_{i1,2}$ is negative:

$$\Delta b_{e1,2} = (b_{1,2} - g_{K\,1,2}) / \cos \beta_{m1,2} \qquad (182e)$$

f) when $\Delta b'_{e1,2}$ is negative and $\Delta b'_{i1,2}$ is positive:

$$\Delta b_{e1,2} = 0 \tag{182f}$$

The calculated effective face width is determined by Formula (183):

$$b_{ce1,2} = 25{,}4\ h_{N1,2} \cos\beta_{m1,2} \left[\arctan\left(\frac{\Delta b_{i1,2}}{25{,}4\ h_{Na1,2}} \right) + \arctan\left(\frac{\Delta b_{e1,2}}{25{,}4\ h_{Na1,2}} \right) \right] + g_{K1,2} \tag{183}$$

7.4.7 Root stress adjustment factor, Y_A

The root stress adjustment factor, Y_A, adjusts the calculation results of method B2 so that it is possible to use the nominal stress numbers of ISO 6336-5. The equation for Y_A is based on carburized case hardened steel. Good quality material is defined by MQ grade in ISO 6336-5 as that meeting the requirements of experienced manufacturers at moderate cost with an allowable stress range of 425 N/mm^2 to 500 N/mm^2. The equivalent in ANSI/AGMA 2003-C10[3] is 280 N/mm^2 to 480 N/mm^2.

Using average values for carburized case hardened steel:

$$Y_A = 1{,}075 \tag{184}$$

For other specific materials and qualities, Y_A should be calculated in the same manner.

7.5 Permissible tooth root stress factors

7.5.1 Relative surface condition factor, $Y_{R,RelT\text{-}B2}$

For gears with a roughness height of $Rz \leq 16$ µm at the root, it may generally be assumed that:

$$Y_{R,relT} = 1{,}0 \tag{185}$$

The reduction of the allowable stress number is small in the range 10 µm < $Rz \leq 16$ µm. In the case of Rz < 10 µm, the calculation according to Formula (166) is on the safe side.

7.5.2 Relative notch sensitivity factor, $Y_{\delta,relT\text{-}B2}$

In the case of gears with $q_s \geq 1{,}5$ the relative notch sensitivity factor is set as:

$$Y_{\delta,RelT} = 1{,}0 \tag{186a}$$

The reduction of the permissible tooth root stress expected in case of $q_s < 1{,}5$ is accounted for by:

$$Y_{\delta,RelT} = 0{,}95 \tag{186b}$$

8 Factors for permissible tooth root stress common for method B1 and method B2

8.1 Size factor, Y_X

8.1.1 General

The size factor, Y_X, accounts for the decrease in strength with increasing size (size effect) and is determined for pinion and wheel, respectively.

The main factors of influence for Y_X are the following:

— tooth size;

— diameter of the part;

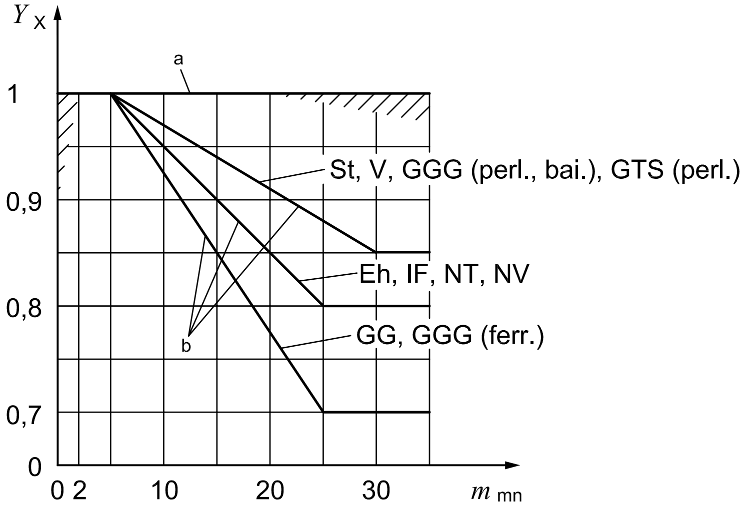
— ratio of tooth size to diameter;

— area of stress pattern;

— material and heat treatment;

— ratio of case depth to tooth thickness.

If no test values or other proven experience are available, Y_X may be approximated according to Figure 7 as a function of the mean normal module m_{mn} and the material. For the abbreviated terms, see ISO 10300-2:2014, Table 1.



Key

m_{mn} mean normal module (mm)

Y_X size factor for tooth root stress

a Static stress (all materials).

b Reference stress.

Figure 7 — Size factor, Y_X, for permissible tooth root stress

When Y_X is calculated, Formula (188) to Formula (190), which approximately represent the slope of the curves in Figure 7, may be used.

8.1.2 Formulae

8.1.2.1 Structural and through hardened steels, spheroidal cast iron, perlitic malleable cast iron

$$Y_X = 1{,}03 - 0{,}006\, m_{mn} \qquad (187)$$

with $0{,}85 \le Y_X \le 1{,}0$

8.1.2.2 Case, flame, induction hardened steels, nitrided or nitro carburized steels

$$Y_X = 1{,}05 - 0{,}01\, m_{mn} \tag{188}$$

with $0{,}80 \le Y_X \le 1{,}0$

8.1.2.3 Grey cast iron and spheroidal cast iron (ferrit structure)

$$Y_X = 1{,}075 - 0{,}015\, m_{mn} \tag{189}$$

with $0{,}70 \le Y_X \le 1{,}0$.

8.2 Life factor, Y_{NT}

8.2.1 General

The life factor, Y_{NT}, accounts for higher tooth root stresses, which may be tolerable for a limited life (number of load cycles), compared with the allowable stress at 3×10^6 cycles. It shall be determined for pinion and wheel respectively.

The principal influence factors for Y_{NT} are the following:

— material and heat treatment (see ISO 6336-5);

— number of load cycles N_L (service life);

— failure criteria;

— smoothness of operation;

— gear material cleanliness;

— material ductility and fracture toughness;

— residual stress.

The number of load cycles N_L is defined as the number of mesh contacts under load. The allowable stress numbers are established for 3×10^6 tooth load cycles at 99 % reliability.

Where justified by experience, a value of unity for Y_{NT} may be used beyond 3×10^6 cycles. In this case optimum conditions for material quality and manufacturing, together with an appropriate safety factor, should be considered.

8.2.2 Method A

The S-N or damage curve derived from facsimiles of the actual gear is determinant for the establishment of limited life. In this case, the factors $Y_{\delta,RelT}$, $Y_{R,RelT}$ and Y_X are, in effect, already included in the S-N/damage curves, and therefore, 1,0 is to be substituted for each in the calculation of permissible stress.

8.2.3 Method B

8.2.3.1 General

For this method, the life factor Y_{NT} of the standard reference test gear is used for the calculation of permissible stress for limited life. Unlike for method A, the factors $Y_{\delta,RelT}$, $Y_{R,RelT}$ and Y_X shall be considered.

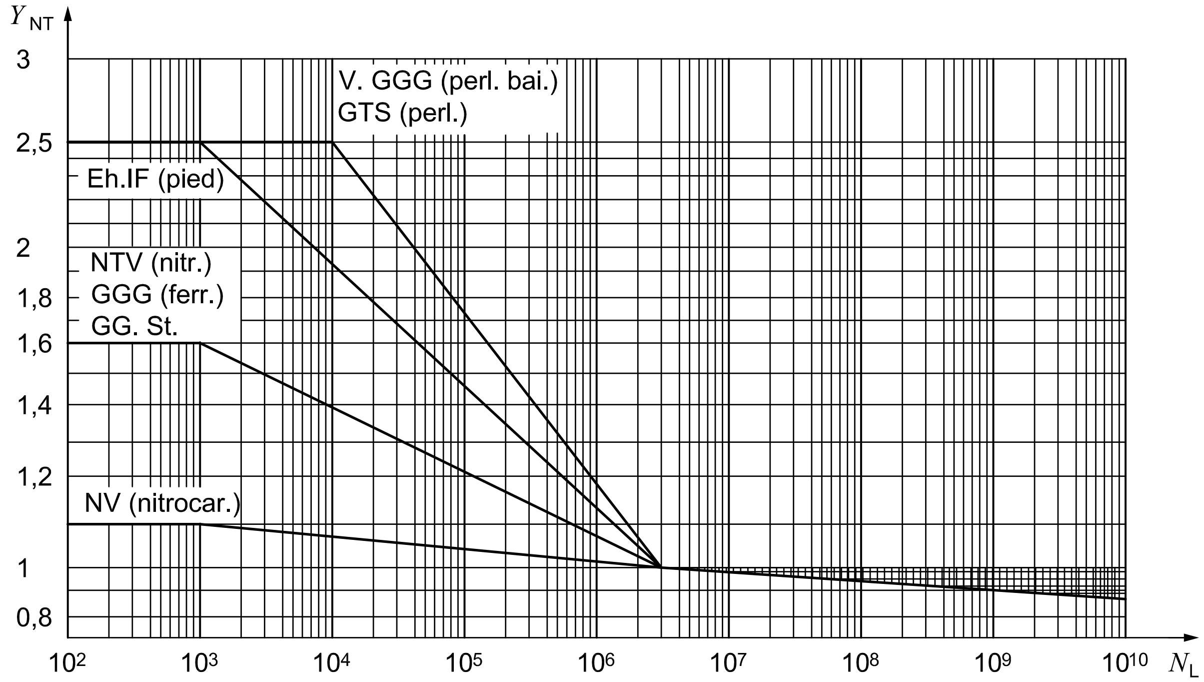
8.2.3.2 Graphed values

Graphed values of Y_{NT} may be read from Figure 8 for the static, limited life and endurance strength, as a function of material and heat treatment. Values from a large number of tests are presented as typical damage or crack initiation curves for surface hardened and nitride hardened steels, or curves of yield stress for structural and through hardened steels.

8.2.3.3 Determination by calculation

The life factor, Y_{NT}, for static and endurance strengths shall be taken from Table 2. Y_{NT} for limited life stress is determined by means of interpolation between the values for endurance and static strength limits. (The evaluation of Y_{NT} is described in ISO 6336-6.[2])

Warning — Stress levels above those permissible for 10^3 cycles should be avoided, since they might exceed the elastic limit of the gear tooth.



Key

N_L number of load cycles (–)

Y_{NT} life factor (bending) (–)

Figure 8 — Life factor, Y_{NT} (standard reference test gears)

Table 2 — Life factor, Y_{NT}, for static and endurance stress

Material[a]	Number of load cycles, N_L	Life factor, Y_{NT}
V, GGG (perl. bai.), GTS (perl.)	$N_L \le 10^4$, static	2,5
	$N_L = 3 \times 10^6$, endurance	1,0
	$N_L = 10^{10}$, endurance Optimum conditions; material, manufacturing and experience	0,85 1,0
Eh, IF (root)	$N_L \le 10^3$, static	2,5
	$N_L = 3 \times 10^6$, endurance	1,0
	$N_L = 10^{10}$, endurance Optimum conditions; material, manufacturing and experience	0,85 1,0
St, NTV (nitr.), GG, GGG (ferr.)	$N_L \le 10^3$, static	1,6
	$N_L = 3 \times 10^6$, endurance	1,0
	$N_L = 10^{10}$, endurance Optimum conditions; material, manufacturing and experience	0,85 1,0
NV (nitrocar.)	$N_L \le 10^3$, static	1,1
	$N_L = 3 \times 10^6$, endurance	1,0
	$N_L = 10^{10}$, endurance Optimum conditions; material, manufacturing and experience	0,85 1,0

[a] For descriptions of the material abbreviated terms, see Table 1.

Bibliography

[1] ISO 6336-3, *Calculation of load capacity of spur and helical gears — Part 3: Calculation of tooth bending strength*

[2] ISO 6336-6, *Calculation of load capacity of spur and helical gears — Part 6: Calculation of service life under variable load*

[3] ANSI/AGMA 2003–C10, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel, and Spiral Bevel Gear Teeth*

[4] AGMA 932–A05, *Rating the Pitting Resistance and Bending Strength of Hypoid Gears*

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