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Geometrical product specifications (GPS) — Inspection by measurement of workpieces and measuring equipment —

Part 6:
Generalized decision rules for the acceptance and rejection of instruments and workpieces

Spécification géométrique des produits (GPS) — Vérification par la mesure des pièces et des équipements de mesure —

Partie 6: Règles de décision générales pour l'acceptation ou le rejet d'instruments et de pièces

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 14253-6 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO 14253 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Inspection by measurement of workpieces and measuring equipment*:

- *Part 1: Decision rules for proving conformance or non-conformance with specifications*
- *Part 2: Guidance for the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification*
- *Part 3: Guidelines for achieving agreements on measurement uncertainty statements* [Technical Specification]
- *Part 4: Background on functional limits and specification limits in decision rules* [Technical Specification]
- *Part 6: Generalized decision rules for the acceptance and rejection of instruments and workpieces* [Technical Report]

The following part is under preparation:

- *Part 5: Uncertainty in testing indicating measuring instruments*

Introduction

This part of ISO 14253 is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO 14638). It influences the measurement, measurement equipment and calibration chain links of the chains of standards in the general GPS matrix.

The ISO/GPS Masterplan given in ISO 14638 gives an overview of the ISO/GPS system of which this document is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this document, unless otherwise indicated.

For more detailed information on the relation of this part of ISO 14253 to other standards and to the GPS matrix model, see Annex A.

This document is based on the ISO 14253-1 concept of a decision rule, and expands the terminology beyond the default rule (stringent acceptance with a 100 % expanded uncertainty guard band) to allow the communication of other possible rules that can be adapted for different industrial needs.

This document follows the guidance provided in ISO/IEC Guide 98-4. Decision rules determine where the gauging limits are set and do not affect the workpiece tolerance; they address the (always present) uncertainty in measurement and explicitly state how this uncertainty will impact acceptance or rejection decisions.

The selection of the decision rule typically involves the designer, who can provide information on the function relative to the dimensional specification, the metrologist, who can provide information on the accuracy of the dimensional measurements, and management, who can provide information on the economic consequences of various acceptance or rejection scenarios.

The selection of a decision rule is only one element of a manufacturing effort, other activities that also affect the number or conforming (or nonconforming) workpieces include the specification of tolerances, the selection of the manufacturing process, and the selection of the measurement process; all of these issues are interconnected and should be considered together.

Both parties (the manufacturer and the customer) should discuss and agree on the decision rule as it affects the economics of the product.

Geometrical product specifications (GPS) — Inspection by measurement of workpieces and measuring equipment —

Part 6:
Generalized decision rules for the acceptance and rejection of instruments and workpieces

1 Scope

This part of ISO 14253 expands the scope of decision rules to industrial situations where the default rule of ISO 14253-1 might not be economically optimal.

NOTE 1 ISO 14253-1 provides a default decision rule having a very high probability that a measured value resulting in product acceptance also yields a product with the corresponding measurand conforming to specifications.

NOTE 2 Changing the decision rule from the default case to a more task-specific case requires agreement between the two parties.

This part of ISO 14253 does not address how to determine the cost of correct decisions (accepting conforming workpieces or rejecting nonconforming workpieces) or incorrect decisions (rejecting conforming workpieces or accepting nonconforming workpieces) as this is a business concern. However, the terminology and requirements to communicate and implement the particular decision rules desired by an organization are provided along with examples to guide the reader.

NOTE 3 The decision rules in this part of ISO 14253 pertain to a single metrological characteristic under consideration. Unless otherwise stated, all probability distributions discussed in this document are Gaussian and centrally located and the cost functions are simple step functions; however, the principles of this document can be applied to any probability distribution function or cost function.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 14253-1:1998, *Geometrical Product Specifications (GPS) — Inspection by measurement of workpieces and measuring equipment — Part 1: Decision rules for proving conformance or non-conformance with specifications*

ISO 14978:2006, *Geometrical product specifications (GPS) — General concepts and requirements for GPS measuring equipment*

ISO 17450-2:2012, *Geometrical product specifications (GPS) — General concepts — Part 2: Basic tenets, specifications, operators and uncertainties*

ISO 21747:2006, *Statistical methods — Process performance and capability statistics for measured quality characteristics*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purpose of this International document, the terms and definitions given in ISO 14253-1, ISO 14978, ISO 17450-2, ISO/IEC Guide 98-3, ISO/IEC Guide 99 and the following apply.

3.1
acceptance limit
upper or lower bound of permissible measured quantity values

NOTE 1 For workpieces, the acceptance limits are often called the gauging limits.

NOTE 2 In the case of a simple-acceptance decision rule, the acceptance limits equal the specification limits.

3.2
acceptance zone
acceptance interval
interval of permissible measured quantity values

NOTE 1 Unless otherwise stated in the specification, the acceptance limits belong to the acceptance interval.

NOTE 2 In ISO 14253-1, the stringent acceptance zone defined by the default decision rule is (loosely speaking) called the conformance zone because of the high probability that a measurement result in this zone corresponds to a conforming product.

NOTE 3 A measured value that is in the acceptance zone does not necessarily correspond to a (truly) conforming characteristic due to measurement uncertainty.

3.3
binary decision rule
decision rule with only two possible outcomes, either acceptance or rejection

3.4
conforming
having the quality that its true value lies within or on the boundary of the tolerance zone or specification zone

NOTE In this part of ISO 14253, it is assumed that the true value of the measurand is essentially unique.

3.5
consumer's risk
probability that a particular accepted item is nonconforming

NOTE In ISO/IEC Guide 98-4, this is called "specific consumer's risk".

3.6
decision rule
documented rule that describes how measurement uncertainty will be allocated with regard to accepting or rejecting a product according to its specification and the result of a measurement

3.7
guard band
interval between a tolerance limit and a corresponding acceptance limit

NOTE In this part of ISO 14253, the term "tolerance limit" is synonymous with "specification limit".

3.8
measurement capability index

C_m

tolerance divided by a multiple of the standard measurement uncertainty associated with the measured value of a property of an item

NOTE 1 In this part of ISO 14253, the multiple is taken to be 4; hence, in the case of measuring a characteristic for conformance to a two-sided tolerance zone of width T, $C_m = T/4u_m$, where u_m is the standard uncertainty associated with the measurement of the characteristic.

NOTE 2 In this part of ISO 14253, the term “tolerance limit” is synonymous with “specification limit”.

3.9
nonconforming

having the quality that its true value lies outside the boundary of the tolerance zone or specification zone

NOTE In this part of ISO 14253, it is assumed that the true value of the measurand is essentially unique.

3.10
process distribution

probability distribution characterizing reasonable belief in values of a characteristic resulting from a manufacturing process

NOTE The form of this distribution can be inferred from a frequency distribution (usually displayed in a histogram) of measured characteristics from a large sample of items.

3.11
process capability index

C_p

index describing process capability in relation to a specified tolerance

NOTE 1 This definition is specific to this part of ISO 14253 and is a special case of the more general definition given in ISO 21747.

NOTE 2 In this part of ISO 14253, the process distribution is centred in the middle of the tolerance (i.e. specification) zone, and the index is the ratio of the zone width to 6 standard deviations of the production distribution.

3.12
producer’s risk

probability that a particular rejected item is conforming

NOTE In ISO/IEC Guide 98-4, this is called “specific producer’s risk”.

3.13
relaxed acceptance

situation when the acceptance zone is increased, and partially outside, the specification limit by the amount of a guard band

NOTE 1 Relaxed acceptance should be used with caution as it increases the size of the acceptance zone and hence decreases the probability that an accepted product is a conforming product.

NOTE 2 Relaxed acceptance and stringent rejection occur together in a binary decision rule.

NOTE 3 The magnitude (in mm) of the relaxed guard band should be specified instead of % U to avoid poor metrology (large U) increasing the number of acceptable workpieces.

NOTE 4 See Figure 2 as an example of relaxed acceptance.

3.14

relaxed rejection

situation when the rejection zone is increased, and partially inside, the specification limit by the amount of a guard band

NOTE 1 Relaxed rejection increases the size of the rejection zone and hence decreases the probability that a rejected product is a nonconforming product.

NOTE 2 Stringent acceptance and relaxed rejection occur together in a binary decision rule.

NOTE 3 See Figure 1 as an example of relaxed rejection.

3.15

rejection zone
rejection interval

interval of non-permissible measured quantity values

NOTE In ISO 14253-1, the stringent rejection zone defined by the default decision rule is (loosely speaking) called the nonconformance zone because of the high probability that a measurement result in this zone corresponds to a nonconforming product.

3.16

simple acceptance

acceptance criterion where the specification zone equals the acceptance zone

NOTE A common binary decision rule combines simple acceptance with simple rejection.

3.17

simple rejection

rejection criterion rule where the rejection zone equals everything outside of the specification zone

NOTE A common binary decision rule combines simple acceptance with simple rejection.

3.18

stringent acceptance
guarded acceptance

situation when the acceptance zone is decreased, and completely inside, the specification limit by the amount of a guard band

NOTE 1 Stringent acceptance decreases the size of the acceptance zone and hence increases the probability that an accepted product is a conforming product.

NOTE 2 Stringent acceptance and relaxed rejection occur together in a binary decision rule.

NOTE 3 The default rule ISO 14253-1 is an example of stringent acceptance with a 100 % U guard band.

NOTE 4 See Figure 1 as an example of stringent acceptance.

3.19

stringent rejection

situation when the rejection zone is decreased, and completely outside, the specification limit by the amount of a guard band

NOTE 1 Stringent rejection decreases the size of the rejection zone and hence increases the probability that a rejected product is a nonconforming product.

NOTE 2 Relaxed acceptance and stringent rejection occur together in a binary decision rule.

NOTE 3 See Figure 2 as an example of stringent rejection.

3.20

transition zone

range of values of a characteristic that is neither in the acceptance zone nor rejection zone

NOTE 1 There may be more than one transition zone; each should be separately labelled.

NOTE 2 A binary decision rule does not have a transition zone.

3.21

uncertainty interval

<of a measurement> interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand

NOTE 1 The width of the uncertainty interval is typically twice the expanded uncertainty.

NOTE 2 The uncertainty interval is also known as the coverage interval [ISO/IEC Guide 99:2007, 2.36].

NOTE 3 The uncertainty interval for the mean of repeated measurements may decrease with increasing numbers of measurements.

4 General

ISO 14253-1 raised the awareness of the metrology community to the importance of uncertainty in decision rules for the acceptance and rejection decisions of products. This part of ISO 14253 expands the scope of applications to include cases where the default rule of ISO 14253-1 might not be the optimal choice. The procedure and terminology follow that of recent developments in risk analysis.

While the ISO 14253-1 default rule provides a high probability that an accepted product actually conforms to specifications, in some less critical applications, the economic optimal decision rule may be less stringent. For example, consider a workpiece production distribution where the true values of the measurand form a Gaussian distribution such that six standard deviations of the distribution lie within the specification zone (C_p = 1). Then using the ISO 14253-1 default rule with a measurement system having a measurement capability index of four (C_m = 4) will have only a 0,000 02 probability of accepting a nonconforming product. Hence, in this case, using the ISO 14253-1 default rule yields an acceptance decision that almost certainly results in accepting a conforming product.

In contrast, if a simple-acceptance decision rule is used in this example, which allows acceptance up to (and including) the specification limits, then there is a 0,000 74 probability of accepting a nonconforming product – a factor of more than 30 times as large as the default case. In safety-critical situations or situations with very high consequences for defective products, the ISO 14253-1 default rule is often economically justified as it provides very high assurance that an accepted product is actually conforming and hence reduces costly mistakes. The price of this high assurance is that a significant fraction of conforming products will not be accepted; in the above example, the ISO 14253-1 default rule rejects 3,3 % of conforming production, in contrast to rejecting 0,3 % of conforming product for the simple acceptance rule.

For less critical products where the economic cost of accepting a nonconforming product is less significant, a decision rule that accepts more product may be economically optimal. This is often cost driven because the requirement of accepting a low number of nonconforming products usually necessitates rejecting many times as many conforming products. The factors associated with the cost of accepting a nonconforming product are many; they include replacement of the product, increased warranty costs, company reputational damage, and potential legal actions (lawsuits). In particular, where safety-critical factors might result in the injury or loss of human life, such an outcome may be very expensive and justify the costs associated with unintentionally rejecting conforming products to increase the probability that the products that are accepted are conforming. The financial risk of accepting a nonconforming product in this situation is usually calculated as the sum of all expected (probability × cost) value outcomes; for a case involving human safety, the costs may be very high. Ultimately, the choice of the decision rule is a business decision that is based on all of the relevant costs associated with the product. Additionally, it should be clear (but beyond the scope of this part of ISO 14253) that many other factors can be optimized. In particular, changing the manufacturing or the measurement process may be more economical than changing the decision rule. In practice, all of these factors should be considered and optimized; in this document, only the decision-rule factor is considered.

Given a specified process capability (C_p), measurement capability (C_m), and a particular decision rule, the outcome matrix is completely determined (e.g. see Table 2). Hence, these three quantities determine the four probabilities associated with accepting or rejecting either conforming or nonconforming products. Once the output matrix probabilities are known, the economic outcome is computed by multiplying the output matrix probabilities by the associated costs (e.g. see Table 1) to determine the net profit of that particular scenario.

NOTE If the production or measurement probability distribution function (pdf) is not Gaussian and centrally located, then the actual pdf is needed to compute the output matrix probabilities.

5 Decision rules

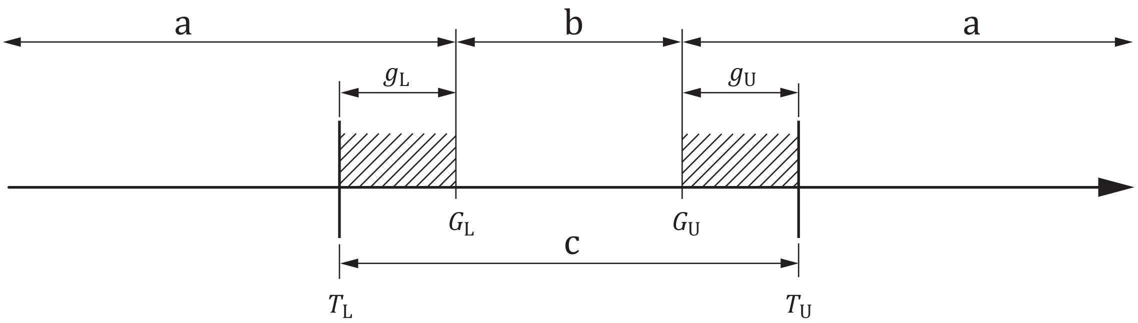
5.1 Guard bands

When considering the economic implications of decision rules, there exists a continuum of possibilities ranging from very stringent (conservative) acceptance criteria to very relaxed acceptance rules. To discuss this continuum, the concept of the guard band is introduced. The guard band, g, offsets the measurement acceptance limits, also known as the gauging limits, from the specification (i.e. tolerance) limits, see Figure 1. For convenience, this offset is often expressed as a percentage of the associated expanded uncertainty associated with the measurement result. It is important to note that the calculation of measurement uncertainty is a technical activity depending on the metrology system whereas the calculation of the guard band is a business activity depending on the economics of the measurement.

NOTE For purposes of acceptance and rejection, a guard band could include an uncorrected bias term in addition to the measurement uncertainty. This inclusion is not allowed for the measurement uncertainty statement in a calibration report but can be included in acceptance / rejection decisions, because the assignment of a value and uncertainty to the measurand is not passed on to any subsequent measurement activities.

5.2 Acceptance zones

The situation where increasing the guard band increases the probability that the accepted product is conforming to specifications is called “stringent acceptance”; see Figure 1. The default ISO 14253-1 decision rule is an example of stringent acceptance with the guard band equal to 100 % of the expanded uncertainty. It is worth noting the terminology attempts to convey the concept that stringent acceptance *reduces* the size of the acceptance interval while *increasing* the confidence that the accepted product is conforming to specifications.



Key

a relaxed rejection zone

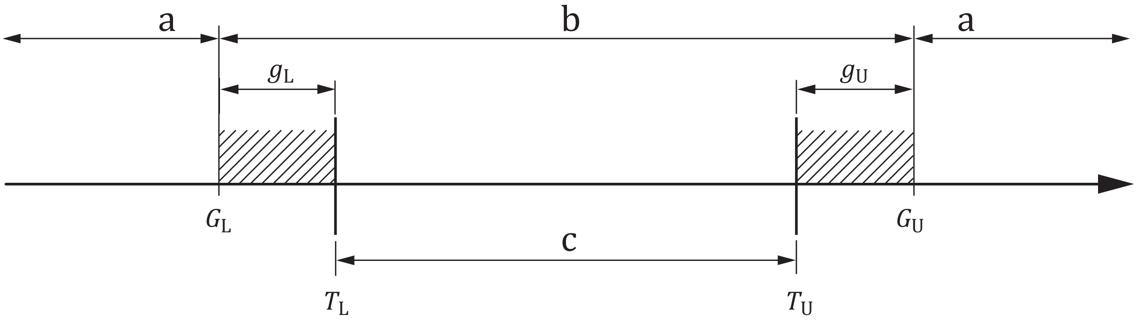
b stringent acceptance zone

c specification zone

NOTE A stringent acceptance zone for measured workpieces, defined by the upper (G_U) and lower (G_L) gauging limits that lie inside the tolerance limits T_U and T_L which define the specification (tolerance) zone. Also shown are two relaxed rejection zones. The offsets between the tolerance limits and the gauging limits are the guard bands g_U and g_L. A stringent-acceptance decision rule reduces the probability of accepting a nonconforming workpiece.

Figure 1 — Stringent acceptance zone for measured workpieces

While many guard bands are designed to produce stringent acceptance, in some situations, the opposite effect is desired. In order to increase the quantity of accepted product, the guard band shown in Figure 2 could be used; this is known as “relaxed acceptance”. This situation might occur if a product specification has been assigned a value that is beyond the state of the art in metrology. In such a situation, a stringent acceptance guard band could result in no acceptance zone and thus zero product would be accepted. Hence, in order to accept a reasonable fraction of the product, it may be necessary to use relaxed acceptance. Similarly, if the cost of accepting a nonconforming product is approximately that of the production cost, then relaxed acceptance increases profits by accepting more product. Note that the terminology of relaxed acceptance conveys an *increase* in the size of the acceptance interval and a *decrease* in the confidence that an accepted product is conforming to specifications.



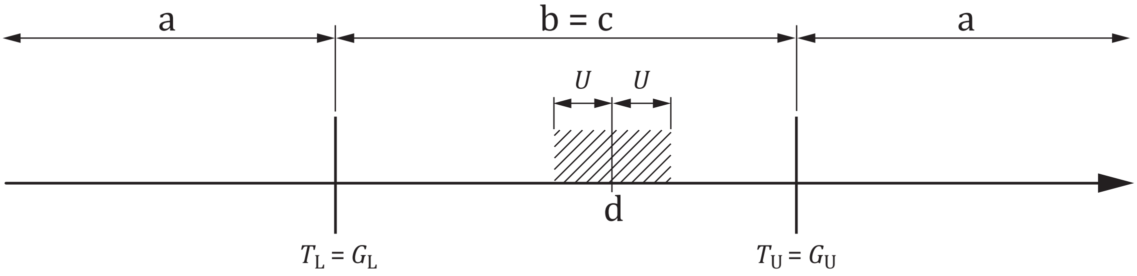
Key

a stringent rejection zone
b relaxed acceptance zone
c specification zone

NOTE A relaxed acceptance zone for measured workpieces, defined by the upper (G_U) and lower (G_L) gauging limits that lie outside the tolerance limits. Also shown are two stringent rejection zones. The offsets between the tolerance limits and the gauging limits are the guard bands g_U and g_L.

Figure 2 — Relaxed acceptance zone for measured workpieces

Historically, the most common acceptance criterion is to accept product with a measurement result up to, and including, the specification limits. This criterion (with zero guard bands) is known as simple acceptance; see Figure 3. While stringent and relaxed acceptance address the allocation of measurement uncertainty through guard bands, simple acceptance addresses it by limiting the magnitude of the measurement uncertainty relative to the specification zone. This is achieved using the measurement capability index which is the ratio of the specification zone to the uncertainty interval; see Figure 3. A common application is 4:1 simple acceptance where the width of the uncertainty interval (of width $2 \times U$) is one-fourth that of the specification zone, i.e. $C_m = 4$.



Key

a simple rejection zone
b simple acceptance zone
c specification zone
d measurement result

NOTE The measurement uncertainty interval is of width $2 \times U$, where U is the expanded uncertainty, and the uncertainty interval is no larger than one-fourth the product's specification zone. The measurement result shown verifies product acceptance.

Figure 3 — Simple acceptance and rejection using a 4:1 ratio

5.3 Rejection zones

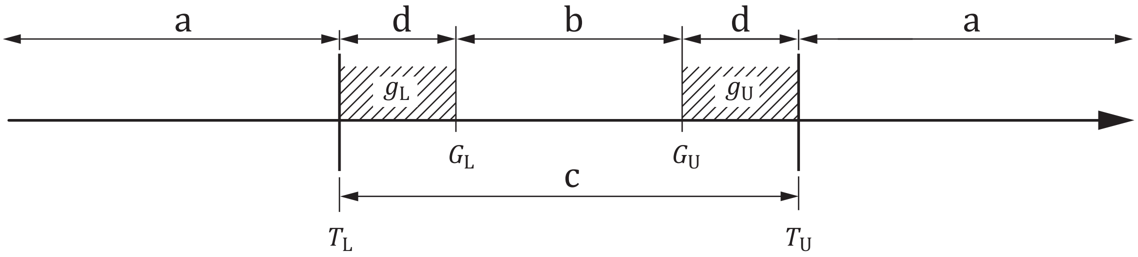
For binary decision rules, rejection zones are the counterpart to acceptance zones. Hence, in the case of simple acceptance, the **simple rejection** zone extends for all measured values beyond the specification limits; see Figure 3.

A **relaxed rejection** zone extends into the specification region and is the counterpart to stringent acceptance; see Figure 1. Note that the term “relaxed rejection” implies both an *increase* in the extent of the zone as it now extends into the specification zone and a *decrease* in the confidence that a rejected component is nonconforming to specifications.

A **stringent rejection** zone begins somewhat beyond the specification limits and is the counterpart to relaxed acceptance; see Figure 2. Note that the term “stringent rejection” implies both a *decrease* in the extent of the rejection zone compared to simple rejection and an *increase* in confidence that a rejected component is nonconforming to specifications.

5.4 Transition zones

In some advanced situations, additional alternatives to acceptance or rejection may be desirable. These can be implemented by the use of transition zones that lie in between the acceptance and rejection zones. The location and decision outcome of any transition zones must be documented in the decision rule. Figure 4 presents an example of stringent acceptance, simple rejection, and a transition zone. An example of the outcome for a measurement result in the transition zone is to designate the product as a lower class, e.g. class 2, and sell at a reduced price and warranty.



Key

a simple rejection zone
b stringent acceptance zone
c specification zone
d transition zone

NOTE A stringent acceptance zone defined by the upper (G_U) and lower (G_L) gauging limits that lie inside the tolerance limits. Also shown are two simple rejection zones and two transition zones. The offsets between the tolerance limits and the gauging limits are the guard bands g_U and g_L.

Figure 4 — Stringent acceptance, simple rejection, and transition zone defined by gauging limits

5.5 Decision rule requirements

A complete decision rule must have four elements: (1) the region of each zone clearly identified; (2) the outcome corresponding to each zone is clearly assigned (e.g. reject product); (3) a policy for addressing repeated measurements; (4) a policy for addressing data rejection, i.e. “outliers”.

For example, if a measurement results lies just inside the rejection zone, a common practice is to repeat the measurement. If the second result now lies in the acceptance zone, a decision must be made either to accept or reject the product. One reasonable policy for repeated measurements is to average the two measurement results and base the decision outcome upon which zone the mean result falls. This policy for repeated measurements needs to be specific so that, in the case of this example, if the second measurement lies just within the acceptance zone but the average is still in the rejection zone, the operator does not continue making measurements (and then computing the average value) until the desired outcome is achieved.

Similarly, a decision rule must have a policy for addressing “outliers”, i.e. rejected measurement results. Measurement results cannot be rejected just because they produce undesirable decision outcomes. One reasonable policy is to require a documented cause in order to reject data, e.g. measurement result was rejected due to vibrations created by a passing truck.

6 Examples of decision rules

6.1 General

In the examples below, the consumer’s and producer’s risk are calculated using techniques found in the bibliography. The examples demonstrate the advantage of working smarter by making appropriate decisions and illustrate the use of decision-rule terminology to communicate the consequences of the inspection process. For brevity, the issue of repeated measurements and outliers will be omitted in these examples.

6.2 Process capability index = 2/3 and measurement capability index = 2

6.2.1 General

A production facility manufactures, inspects, and installs a high accuracy workpiece into an assembly. Due to the very small tolerances specified on the drawing, the production process has C_p = 2/3, where $C_p = T/(6u_p)$, and T is the component tolerance and u_p is one standard deviation of the production distribution. Suppose also that the small tolerances result in a measurement capability index of 2, i.e. C_m = 2, where $C_m = T/(4u_m)$, and u_m is one standard uncertainty associated with the measurement value. (This is sometimes referred to as a 2:1 gauging ratio.) It is noteworthy to point out that for these small tolerances the C_m = 2 value might be achieved through the arithmetic average of many measurements which independently have C_m < 2, but through averaging the uncertainty components arising from random effects yields the C_m = 2 value for the final (averaged) result. The manufacturer and customer of this product have discussed this issue and agreed to change from the default decision rule and select a decision rule based on an economic analysis of the situation.

6.2.2 Cost model

The cost model for this example is given in Table 1. The net profit (selling price minus all costs) of accepting a component conforming to specifications is $ 0,5. (The monetary units are arbitrary, e.g. they might be thousands of Euros; only the relative values of profits and costs are required in this problem.) The net loss of rejecting a component is $ 1; it does not matter if the component is conforming or nonconforming, in either case the decision outcome (rejection leading to a discarded workpiece) is the same; hence, the loss is the same – the production cost. Presented are six different cases of accepting a nonconforming component. These cases range from no negative impact from accepting a nonconforming component other than the replacement cost (case A), to very significant impact (case F), where the cost of accepting a nonconforming component is 50 times the production cost; this last case might be a consequence of having to rebuild an entire complex assembly due to a defective workpiece, or due to a safety-critical component that has potential litigation costs if it fails. The net profit margin per workpiece of the entire process is the profit obtained from accepting a conforming product multiplied by its associated probability minus the net loss from the other three outcomes multiplied by their respective probabilities; this is shown in the last six rows of Table 2 (multiplied by 1 000).

Table 1 — The profit matrix showing the profit or loss for each decision

Decision outcome	Conforming to specification	Nonconforming to specification
Accepted	+0,5	Case A: −1 Case B: −2 Case C: −5 Case D: −10 Case E: −20 Case F: −50
Rejected	-1	-1

6.2.3 Decision rule outcomes

Table 2 presents six different decision rules ranging from no inspection – which is equivalent to 100 % acceptance (i.e. relaxed rejection with an infinitely wide guard band) – to stringent acceptance with a guard band equal to 100 % of the expanded measurement uncertainty. This latter rule is the ISO 14253-1 default rule. The first four rows of Table 2 show, for each rule, the fraction of workpieces that are accepted and conforming, accepted and nonconforming, rejected and conforming, and rejected and nonconforming. The economic consequences of the cost functions (given in Table 1) are also shown for each decision rule.

Table 2 — The outcome matrix

Decision/True Value	100 % *U* **Stringent Acceptance**	75 % *U* **Stringent Acceptance**	25 % *U* **Stringent Acceptance**	0 % *U* **Simple Acceptance**	25 % *U* **Relaxed Acceptance**	75 % *U* **Relaxed Acceptance**	100 % *U* **Relaxed Acceptance**	**No Inspection**
Accept/ Conforming	0,628 6	0,735 3	0,875 8	0.914 0	0.936 1	0.952 1	0,953 8	0,954 5
Accept/ Nonconforming	0,000 3	0,001 1	0,006 6	0,012 4	0,019 7	0,034 0	0,038 9	0,045 5
Reject/ Conforming	0,325 9	0,219 2	0,078 7	0,040 5	0,018 4	0,002 4	0,000 7	0,000 0
Reject/ Nonconforming	0,045 2	0,044 4	0,038 9	0,033 1	0,025 8	0,011 5	0,006 6	0,000 0
Net Profit per 1 000 pieces	100 % *U* **Stringent Acceptance**	75 % *U* **Stringent Acceptance**	25 % *U* **Stringent Acceptance**	0 % *U* **Simple Acceptance**	25 % *U* **Relaxed Acceptance**	75 % *U* **Relaxed Acceptance**	100 % *U* **Relaxed Acceptance**	**No Inspection**
Case A: cost = $1	-57,14	103,01	313,74	370,96	404,20	428,19	430,77	**431,75**
Case B: cost = $2	-57,48	101,90	307,09	358,57	384,51	**394,23**	391,91	386,25
Case C: cost = $5	-58,48	98,58	287,14	321,40	**325,42**	292,33	275,32	249,75
Case D: cost = $10	-60,16	93,04	253,90	**259,46**	226,95	122,51	81,01	22,25
Case E: cost = $20	-63,51	81,96	**187,41**	135,57	30,00	-217,14	-307,62	-432,76
Case F: cost = $50	-73,56	**48,72**	-12,06	-236,09	-560,84	-1 236,09	-1 473,49	-1 797,76

The outcome matrix shows the outcome of accepting or rejecting either a conforming or nonconforming component for various decision rules for the case of C_p = 2/3 and C_m = 2. The net profits for each of the rules and cases are also shown; the most profitable outcome for each case is shown in bold font.

Case A: This is the limiting case where the cost of accepting a nonconforming component is just the cost of its replacement, i.e. the production cost, which is $ 1 in this example. It should be obvious that in this case that the “no inspection” rule is the economically optimal decision because there is no penalty other than the replacement cost of accepting a nonconforming component.

Cases B and C: As the cost of accepting a nonconforming component begins to increase, the economics favours a relaxed-acceptance decision rule because this rule rejects some of the nonconforming components which would otherwise result in a cost.

Case D: In this example, when the cost of accepting a nonconforming component is 10 times the production cost, the economics now favour a decision rule of simple acceptance. For this cost structure, stringent acceptance rejects too many conforming components thereby reducing revenue and relaxed acceptance accepts too many nonconforming components thereby excessively increasing costs.

Cases E and F: As the cost of accepting a nonconforming component becomes relatively large, the economics favours a stringent-acceptance decision rule because this rule rejects a large percentage of nonconforming components which would otherwise result in a significant cost if (due to the measurement uncertainty) they were accepted.

For case F, where the cost of accepting a nonconforming component is very high, a 75 % *U* stringent acceptance is the best selection. This demonstrates the value of having a decision rule with a very high probability that all accepted components are conforming to specifications. For example, a high cost might be associated with a safety-critical component that has large litigation costs should the component fail in service (see 6.2.1).

6.3 Process capability index = 1 and measurement capability index = 4

Consider the previous example where improved production and measurement technology results in C_p = 1 and C_m = 4; the cost structure remains the same as in Table 1. Table 3 presents the outcomes for the six different decision rules for each of the different cost structures.

Table 3 — The outcome matrix

Decision/ True Value	100 % *U* **Stringent Accept-ance**	75 % *U* **Stringent Accept-ance**	25 % *U* **Stringent Accept-ance**	0 % *U* **Simple Accept-ance**	25 % *U* **Relaxed Accept-ance**	75 % *U* **Relaxed Accept-ance**	100 % *U* **Relaxed Accept-ance**	**No Inspec-tion**
Accept/ Conforming	0,964 8	0,977 5	0,991 2	0,994 3	0,996 0	0,997 1	0,997 3	0,997 3
Accept/ Nonconforming	0,000 0	0 000 1	0,000 4	0,000 7	0,001 2	0,002 0	0,002 3	0,002 7
Reject/ Conforming	0,032 5	0 019 8	0,006 1	0,003 0	0,001 3	0,000 2	0,000 0	0,000 0
Reject/ Nonconforming	0,002 7	0,002 6	0,002 3	0,002 0	0,001 5	0,000 7	0,000 4	0,000 0
Net Profit per 1 000 pieces	100 % *U* **Stringent Accept-ance**	75 % *U* **Stringent Accept-ance**	25 % *U* **Stringent Accept-ance**	0 % *U* **Simple Accept-ance**	25 % *U* **Relaxed Accept-ance**	75 % *U* **Relaxed Accept-ance**	100 % *U* **Relaxed Accept-ance**	**No Inspec-tion**
Case A: cost = $1	447,26	466,19	486,73	491,44	493,99	495,71	495,89	**495,95**
Case B: cost = $2	447,24	466,13	486,33	490,70	492,82	**493,70**	493,59	493,25
Case C: cost = $5	447,18	465,93	485,14	488,49	**489,31**	487,67	486,70	485,15
Case D: cost = $10	447,08	465,60	483,16	**484,81**	483,46	477,63	475,21	471,65
Case E: cost = $20	446,88	464,93	**479,19**	477,43	471,77	457,54	452,24	444,65
Case F: cost = $50	**446,27**	462,94	467,.29	455,32	436,70	397,26	383,32	363,66

The outcome matrix shows the outcome of accepting or rejecting either a conforming or nonconforming component for various decision rules for the case of C_p = 1 and C_m = 4. The net profits for each of the rules and cases are also shown; the most profitable outcome for each case is shown in bold font.

While the trend in decision rules is the same as the previous example, namely toward stringent acceptance as the cost of accepting a nonconforming product increases, the profitability of the different rules is much more uniform. This is a result of the process capability index, $C_\text{p} = 1$, and hence there are far fewer nonconforming products produced. Additionally, the measurement capability index $C_\text{m} = 4$ improves the decision-making process resulting in still fewer incorrect inspection decisions.

6.4 Measurements without production distributions

The prior two examples had the benefit that the production distribution of the workpiece was known before the inspection measurements occurred. Indeed, even with $C_\text{p} = 2/3$, it is known that the average workpiece had a 95 % probability of being in conformance with specifications prior to inspection. The purpose of inspection was to further reduce the probability of accepting a nonconforming workpiece – and the cost of this was primarily the rejection of conforming workpieces.

Consider an example where it is assumed that no prior information is known about the workpiece and that only the tolerance, measurement result, and measurement uncertainty is known. (In the case of testing instruments, the analogous quantities are the MPE specification, the test result, and the test uncertainty.) In this example, the guard bands needed to ensure that no more than a specified level of nonconforming workpieces are accepted will be much larger than the prior examples because it is not known that most workpieces are conforming. By establishing guard bands, a known level of confidence can be assigned to each individual measurement result. Assuming the manufacturer and customer of this product have discussed this issue and agreed to change from the default decision rule and select a decision rule based on an economic analysis of the situation, then Table 4 provides the relationship between the level of confidence and the associated stringent-acceptance guard band assuming a Gaussian distribution for the measurement uncertainty.

Table 4 — Relationship between the level of confidence and the associated stringent acceptance guard band

Conformance probability	0,80	0,85	0,90	0,95	0,977	0,99	0,999
Guard Band (in % *U*)	42 %	52 %	64 %	82 %	100 %	116 %	155 %
NOTE The relationship between the level of confidence of accepting a conforming workpiece for a measurement value at the gauging limits and the corresponding guard band is expressed as a percentage of the expanded measurement uncertainty.							

Consider the situation where a single workpiece is required to complete a special purpose assembly and this workpiece is purchased “as is” with no additional information provided, i.e. the production distribution is unknown. Suppose also that because of contractual time constraints the assembly must be completed immediately so that there is no additional time to purchase additional workpieces. At this point in time both the workpiece and the nearly completed assembly are “sunk costs”, i.e. the funds have been spent manufacturing them and they will be worthless unless the assembly is completed and sold. A conforming workpiece will complete the assembly which can be sold for price *P*, but a nonconforming workpiece will create a loss of *L*, with $L >> P$ since the nonconforming workpiece will not only damage the entire assembly but also create legal liabilities. Let *c* be the probability that the workpiece is conforming to specifications then (1 - *c*) is the probability that the workpiece is nonconforming.

A reasonable decision rule would require $c \times P - (1 - c) \times L > 0$, i.e. that a profit can be expected; hence, $c > \frac{L}{P + L}$. Suppose in this example $\frac{L}{P} = 43$, then a minimum value of *c* is 0,977 and by using Table 4, this corresponds to a 100 % *U* guard band. Thus, a decision rule of stringent acceptance with at least 100 % *U* guard band would be appropriate. Furthermore, it can be shown that $c = \frac{L}{P + L}$ yields the maximum profit and hence provides the optimal guard band when the production distribution is unknown.

Annex A
(informative)

Relation to the GPS matrix model

A.1 General

For full details about the GPS matrix model, see ISO 14638.

The ISO/GPS Masterplan given in ISO 14638 gives an overview of the ISO/GPS system of which this document is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this document, unless otherwise indicated.

A.2 Information about the document and its use

This part of ISO 14253 expands the scope of decision rules to industrial situations where the default rule of ISO 14253-1 might not be economically optimal.

A.3 Position in the GPS matrix model

This part of ISO 14253 is a general GPS standard, which influences the measurement, measurement equipment and calibration chain links of the chains of standards in the general GPS matrix, as illustrated in Table A.1.

Table A.1 — Fundamental and general ISO GPS standards matrix

	Chain links						
	Symbols and indications	**Tolerance zones and parameters**	**Feature characteristics**	**Comparison and compliance**	**Measurement**	**Measurement equipment**	**Calibration**
	(Former chain link 1)	(Former chain link 2)	(Former chain link 3)		(Former chain link 4)	(Former chain link 5)	(Former chain link 6)
Size					•	•	•
Distance					•	•	•
Radius					•	•	•
Angle					•	•	•
Form					•	•	•
Orientation					•	•	•
Location					•	•	•
Run-out					•	•	•
Profile surface texture					•	•	•
Areal surface texture					•	•	•
Surface imperfections					•	•	•
Edges					•	•	•

A.4 Related International Standards

The related standards are those of the chains of standards indicated in Table A.1.

Bibliography

[1] ISO/IEC Guide 98-4, *Uncertainty of measurement — Part 4: Role of measurement uncertainty in conformity assessment*

[2] ISO 8015, *Geometrical product specifications (GPS) — Fundamentals — Concepts, principles and rules*

[3] ASME B89.7.3.1 – 2001, *Guidelines for decision rules: Considering measurement uncertainty in determining conformance to specifications*

[4] ASME B89.7.4.1 – 2005, *Measurement Uncertainty and Conformance Testing: Risk Analysis*

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