TECHNICAL REPORT

# ISO/TR 10771-2

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# Hydraulic fluid power — Fatigue pressure testing of metal pressure-containing envelopes —

## Part 2: Rating methods

*Transmissions hydrauliques — Essais de fatigue des enveloppes métalliques sous pression —*

*Partie 2: Méthodes de classement*

Reference number
ISO/TR 10771-2:2008(E)

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# Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 10771-2 was prepared by Technical Committee ISO/TC 131, *Fluid power systems*, Subcommittee SC 8, *Product testing*.

ISO/TR 10771 consists of the following parts, under the general title *Hydraulic fluid power — Fatigue pressure testing of metal pressure-containing envelopes*:

— *Part 1: Test method*

— *Part 2: Rating methods*

# Introduction

In hydraulic fluid power systems, power is transmitted and controlled under pressure within a closed circuit. It is important for the manufacturer and user of hydraulic components to have information on their global reliability because of the importance of the fatigue failure mode and the relationship with their functional safety and service life. This part of ISO 10771 provides a method for fatigue-testing in order to verify the rating of a pressure-containing envelope.

During operation, components in a system can be subjected to loads that arise from:

- internal pressure;
- external forces;
- inertia and gravitational effects;
- impact or shock;
- temperature changes or gradients.

The nature of these loads can vary from a single static application to continuously varying amplitudes, repetitive loadings and even shocks. It is important to know how well a component can withstand these loads, but this part of ISO 10771 addresses only the loads due to internal pressure.

There are several International Standards already in existence for pressure rating of individual components (e.g. for determining maximum allowable rated pressure) and this part of ISO 10771 is not intended to replace them. Instead, a method of fatigue verification is provided.

This part of ISO 10771 describes a universal verification test to give credibility to the many in-house and other methods of determining the pressure rating of the components. Credibility is based upon the fundamental nature of metal fatigue with its statistical treatment and a mathematical theory of statistical verification. Nevertheless, it is necessary to have design knowledge of the component and its representative specimens to maximize accuracy of the verification method. The use of this test method can reduce the risk of fatigue failure for a hydraulic component regardless of sample size.

In order to rate components in accordance with this part of ISO 10771, it is necessary to propose a rating for the component, select test specimens and select a test pressure. A fatigue test is then conducted in accordance with ISO 10771-1. If the test is successful, the proposed rating is verified for the family of components represented by the sample.

This part of ISO 10771 is based on ANSI/(NFPA) T 2.6.1, a standard which was developed and has been used in the United States for over 25 years and has been adopted for use in Japan as JSME S006-1985. If sufficient experience is gained in other parts of the world, and additional data on materials are obtained, this part of ISO 10771 might be re-drafted as an International Standard in the future.

It should be noted that the test factors in Annex A are based on material data obtained from sources originating in the USA. One of the objectives in issuing this part of ISO 10771 is to obtain material data from other countries. The test factors are based only on the material properties and not on any tolerances of the elements in the pressure-containing envelope.

Annex C describes a possible method for accelerating testing. The example shows how material property data can be used to determine an acceleration factor and shows that they have to be carefully chosen. Another objective of this part of ISO 10771 is to seek additional data as described in Annex C. Contributors are asked to submit any available data to the secretary of ISO TC 131/SC 8.

# Hydraulic fluid power — Fatigue pressure testing of metal pressure-containing envelopes —

# Part 2: Rating methods

## 1 Scope

This part of ISO 10771 specifies a test method for fatigue rating of the pressure-containing envelopes of components used in hydraulic fluid power systems, as tested under steady internal cyclic pressure loads in accordance with ISO 10771-1.

This part of ISO 10771 is only applicable to components whose failure mode is the fatigue of any element in the pressure-containing envelope, and that:

— are manufactured from metals;

— are operated at temperatures that exclude creep and low-temperature embrittlement;

— are only subjected to pressure-induced stresses;

— are not subjected to loss of strength due to corrosion or other chemical action;

— can include gaskets, seals and other non-metallic components; however, these are not considered part of the pressure-containing envelope being tested (see note 3 of 5.5 of ISO 10771-1:2002).

This part of ISO 10771 does not apply to piping as defined in ISO 4413 (i.e. connectors, hose, tubing, pipe).

NOTE See ISO 19879, ISO 6803 and ISO 6605 for methods of fatigue testing of tube connectors, hoses and hose assemblies.

This part of ISO 10771 establishes a general rating method that can be applied to many hydraulic fluid power components. In addition, EN 14359 has been developed for accumulators.

## 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 5598, *Fluid power systems and components — Vocabulary*

ISO 10771-1:2002, *Hydraulic fluid power — Fatigue pressure testing of metal pressure-containing envelopes — Part 1: Test method*

## 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 5598, ISO 10771-1 and the following apply.

**3.1**
**rated fatigue pressure**
$P_{\mathrm{RF}}$
maximum pressure that a component pressure-containing envelope, selected at random, has been verified to sustain for the rated cycle life without failure, with a known probability

**3.2**
**assurance level**
probability that the fatigue strength of a randomly selected test specimen exceeds its rated fatigue pressure

**3.3**
**verification level**
probability that the fatigue strength of a randomly selected test specimen is not less than its cyclic test pressure

**3.4**
**coefficient of variation**
$k_{\mathrm{o}}$
standard deviation of the fatigue strength distribution of a material at a given fatigue life, divided by its mean

NOTE Adapted from ISO 3534-1:2006 [1].

**3.5**
**variability factor**
$K_{\mathrm{V}}$
ratio of cyclic test pressure to rated fatigue pressure

**3.6**
**element**
part of a component; for example, tie rods on a cylinder, end caps on a valve, bolts on a pump housing

## 4 Selection of material factors

**4.1** Select a coefficient of variation, $k_{\mathrm{o}}$, for each type of material in the pressure-containing envelope. The $k_{\mathrm{o}}$ factor should be obtained from fatigue tests on coupons for the particular temper of material used in the pressure-containing envelope. The fatigue test method used to obtain this data should be in accordance with a recognized national or International Standard.

**4.2** As an alternative to testing the specific material, coefficients described in Annex A can be used for the $k_{\mathrm{o}}$ factor.

## 5 Determination of cyclic test pressure

**5.1** Select an assurance level for the fatigue pressure rating. A nominal value is 90 %.

**5.2** Select a verification level for the fatigue pressure rating. A nominal value is 90 %.

NOTE See Annex D for a tutorial that describes these terms.

**5.3** Select a number of component specimens to be tested, then determine the number of element specimens that will be tested in the components.

NOTE The verification is independent of sample size because the test pressure compensates for different quantities.

**5.4** Determine the variability factor, $K_V$, for each element in the component using Table 1 and the procedure described in the example given in Annex B. Use the largest $K_V$ factor so obtained, for the calculations described in the example.

**5.5** Propose a rated fatigue pressure for the pressure-containing envelope of the component.

**5.6** Calculate the cyclic test pressure, $P_{CT}$, using Equation (1):

$$P_{CT} = K_V \times P_{RF} \tag{1}$$

where

$K_V$ is the variability factor;

$P_{RF}$ is the rated fatigue pressure of the component pressure-containing envelope.

**Table 1 — Variability factor, $K_V$ (at a verification level of 90 %)**

| Assurance level | No. of specimens | Material coefficient of variation, $k_o$ [b] | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $n$ [a] | 0 | 0,02 | 0,04 | 0,06 | 0,08 | 0,10 | 0,12 | 0,14 | 0,16 | 0,18 | 0,20 | 0,22 | 0,24 | 0,26 | 0,28 | 0,30 |
| 99,9 % | 1 | 1,00 | 1,09 | 1,20 | 1,32 | 1,46 | 1,63 | 1,83 | 2,08 | 2,38 | 2,77 | 3,29 | — | — | — | — | — |
| | 2 | 1,00 | 1,08 | 1,16 | 1,26 | 1,38 | 1,52 | 1,68 | 1,88 | 2,13 | 2,45 | 2,87 | — | — | — | — | — |
| | 3 | 1,00 | 1,07 | 1,15 | 1,23 | 1,34 | 1,46 | 1,61 | 1,78 | 2,01 | 2,29 | 2,66 | 3,18 | — | — | — | — |
| | 4 | 1,00 | 1,06 | 1,13 | 1,22 | 1,31 | 1,42 | 1,56 | 1,72 | 1,93 | 2,19 | 2,54 | 3,02 | — | — | — | — |
| | 5 | 1,00 | 1,06 | 1,13 | 1,20 | 1,29 | 1,40 | 1,53 | 1,68 | 1,87 | 2,12 | 2,44 | 2,89 | — | — | — | — |
| 99 % | 1 | 1,00 | 1,08 | 1,16 | 1,25 | 1,35 | 1,47 | 1,60 | 1,75 | 1,92 | 2,12 | 2,35 | 2,63 | 2,96 | — | — | — |
| | 2 | 1,00 | 1,06 | 1,12 | 1,20 | 1,28 | 1,37 | 1,47 | 1,58 | 1,72 | 1,87 | 2,05 | 2,26 | 2,52 | 2,85 | — | — |
| | 3 | 1,00 | 1,05 | 1,11 | 1,17 | 1,24 | 1,32 | 1,40 | 1,50 | 1,62 | 1,75 | 1,90 | 2,09 | 2,31 | 2,59 | 2,94 | — |
| | 4 | 1,00 | 1,05 | 1,10 | 1,15 | 1,21 | 1,28 | 1,36 | 1,45 | 1,55 | 1,67 | 1,81 | 1,98 | 2,18 | 2,43 | 2,74 | 3,16 |
| | 5 | 1,00 | 1,04 | 1,09 | 1,14 | 1,20 | 1,26 | 1,33 | 1,41 | 1,51 | 1,62 | 1,75 | 1,90 | 2,08 | 2,31 | 2,60 | 2,98 |
| 90 % | 1 | 1,00 | 1,05 | 1,11 | 1,17 | 1,23 | 1,29 | 1,36 | 1,44 | 1,52 | 1,60 | 1,69 | 1,79 | 1,89 | 2,00 | 2,12 | 2,25 |
| | 2 | 1,00 | 1,04 | 1,07 | 1,11 | 1,16 | 1,20 | 1,25 | 1,30 | 1,35 | 1,41 | 1,47 | 1,54 | 1,61 | 1,69 | 1,77 | 1,86 |
| | 3 | 1,00 | 1,03 | 1,06 | 1,09 | 1,12 | 1,16 | 1,19 | 1,23 | 1,28 | 1,32 | 1,37 | 1,42 | 1,48 | 1,54 | 1,60 | 1,67 |
| | 4 | 1,00 | 1,02 | 1,05 | 1,07 | 1,10 | 1,13 | 1,16 | 1,19 | 1,23 | 1,26 | 1,30 | 1,34 | 1,39 | 1,44 | 1,49 | 1,55 |
| | 5 | 1,00 | 1,02 | 1,04 | 1,06 | 1,08 | 1,11 | 1,13 | 1,16 | 1,19 | 1,22 | 1,25 | 1,29 | 1,33 | 1,37 | 1,41 | 1,46 |

[a] Test twice the number of specimens if a 99 % verification level is chosen.

[b] Use an interpolation of $k_o$ values between those tabulated here, or calculate $K_V$ from Equation (D.14) in Annex D.

## 6 Conduct of fatigue test

**6.1** Determine the number of cycles, between $1 \times 10^5$ and $1 \times 10^7$, for which the component will be rated.

**6.2** Subject the test specimens to a fatigue pressure test in accordance with ISO 10771-1 for the number of cycles determined in 6.1, using the $P_{CT}$ calculated from Equation (1).

**6.3** The fatigue pressure test is successful if all of the element specimens selected in 5.3 do not fail as described in ISO 10771-1:2002, Clause 8.

## 7 Rating by similarity

It is permitted to extend a verified $P_{RF}$ to other components of similar shape if it can be shown that differences between those components and the components tested do not result in any reduction of their fatigue strength capabilities. Examples of this are components that have smaller ports or different axial lengths but are otherwise identical in geometry to the component tested.

## 8 Rating declaration

The $P_{RF}$ proposed in 5.5 will be verified if the requirements of 6.3 are met. A code should be applied to the component to declare its rating as:

$P_{RF}$ = $P_{RF}$ (in megapascals)/assurance level/verification level/$K_V$ in the test/number of test cycles

EXAMPLE The rated fatigue pressure (12,5 MPa) of a component's pressure-containing envelope that was tested at an assurance level of 99 %, a verification level of 90 %, a $K_V$ of 1,36 for $2 \times 10^6$ cycles, would be declared as:

$P_{RF}$ = 12,5 MPa/ 99 %/ 90 %/ 1,36/ $2 \times 10^6$ cycles

## 9 Identification statement (reference to this part of ISO 10771)

Use the following statement in test reports, catalogues and sales literature when complying with this part of ISO 10771:

"Method for fatigue pressure rating conforms to ISO TR 10771-2:2008, *Hydraulic fluid power — Fatigue pressure testing of metal pressure-containing envelopes — Part 2: Rating methods*".

# Annex A
(informative)

# Material factor database

## A.1 Values of coefficient of variation, $k_o$, for commonly used metals

Table A.1 tabulates data calculated from the sources listed in the bibliography.

**Table A.1 — Values of coefficient of variation, $k_o$, for commonly used metals**

| Metal | | $k_o$ |
|---|---|---|
| Steel | Alloy, low | 0,14 |
| | Carbon, plain | 0,08 |
| | Nickel | 0,10 |
| | Stainless | 0,09 |
| | Tool | 0,10 |
| Iron | | 0,14 |
| Nonferrous | Aluminium (except unalloyed) | 0,13 |
| | Unalloyed aluminium | 0,23 |
| | Cobalt | 0,13 |
| | Copper | 0,09 |
| | Magnesium | 0,17 |
| | Monel[1)] | 0,27 |
| | Titanium | 0,12 |

## A.2 Procedures used to establish values of coefficient of variation, $k_o$, for the metals listed in Table A.1

**A.2.1** Values of $k_o$ were calculated from fatigue test data on test coupons that were published in the references cited in the bibliography. The types of data taken from these references were one of the following:

a) Means, $\mu$, and standard deviations, $\sigma$, of normal distributions;

b) parameters of Weibull distributions;

c) raw data points on $S$-$N$ curves. From these data, individual coefficients of variation, $k_o$, were calculated at $10^6$ cycles for:

1) normal distributions; $k_o$ equals the standard deviation divided by the mean;

1) This is an example of a suitable product available commercially. This information is given for the convenience of users of this document and does not constitute an endorsement by ISO of this product.

2) Weibull distributions; $k_o$ were calculated from a formula given in Reference [12]. The formula includes a gamma function, the value of which was selected as a constant at 0,89 because its variations were generally less than ± 2 % in the range of interest (a few data points went to a difference of ± 4 %);

3) $S$-$N$ curves; the references had either included limit bands (assumed to be two sigma from the mean) or actual standard deviation points. These were then used to calculate $k_o$ in the same manner as a normal distribution.

**A.2.2** The resulting $k_o$ values (shown as individual values in Table A.2 to Table A.13) include a mix of notched and unnotched specimens, several different tempers, plus different methods of testing (e.g. axial, rotating beam). However, only those tested at room temperature were used. No attempt was made to segregate these data. It is reasoned that the components to be tested will have a variety of tempers and notches, so an application of these published data to components can only be justified if the data are treated statistically at a conservative value.

**A.2.3** Therefore, the values given in Table A.1 were derived by assuming that all $k_o$ data for a particular metal group are part of a normal distribution, and a value that is greater than 90 % of this distribution was selected. This ensures that the selection is substantially conservative. However, this part of ISO 10771 allows the use of a more accurate $k_o$ value, which is representative of the specific alloy and temper of the elements being tested, if sufficient testing is performed to obtain those data, as described in 4.1. This approach will likely yield a value that would be more advantageous for a particular application, but less than the conservative values presented in Table A.1.

**A.2.4** Table A.2 to Table A.13 describe all of the $k_o$ calculations made from the data obtained from Reference [10], Reference [11], and Reference [13] to Reference [17]. Most of the data are based on the strength distribution at $10^6$ cycles, but some data are at the endurance limit and these are identified in each table, if applicable.

**Table A.2 — Summary of $k_o$ calculations for iron**

| Type | Reference | Number of distributions | $k_o$ values [a] |
|---|---|---|---|
| Armco[1)] | [11] | 1 | 0,0220 |
| Pearlitic (grey) | [15];[17] | 8 | 0,0402; 0,042[a]; 0,044[a]; 0,0652; 0,126[a]; 0,146[a]; 0,137[a] |
| Ferritic (malleable) | [15];[17] | 7 | 0,055[a]; 0,0596; 0,0649; 0,065[a]; 0,075[a]; 0,109[a] |
| Nodular | [15] | 10 | 0,029[a]; 0,040[a]; 0,049[a]; 0,065[a]; 0,086[a]; 0,094[a]; 0,095[a]; 0,098[a]; 0,173[a]; 0,185[a] |
| Fe; 5,5 % Mo; 2,5 % Cr; 0,5 % C | [13] | 2 | 0,0286; 0,0477 |
| Summary of all data above | [13];[15];[11];[17] | 28 | ($k_o$) 90 % = 0,1335; ($\mu$ = 0,0771; $\sigma$ = 0,0440)<br>Conclusion: $k_o$ value selected = 0,14 |

[a] Data from reference [15] are at endurance limit.

**Table A.3 — Summary of $k_o$ calculations for aluminium**

| Alloy | Reference | Number of distributions | $k_o$ values [a] |
|---|---|---|---|
| Duraluminum1) | [13] | 1 | 0,0720 |
| 356 | [15] | 2 | 0,038; 0,042 |
| 355 | [14] | 1 | 0,0766 |
| 1100 | [14] | 2 | 0,1742; 0,2377 |
| 2014 | [14];[15] | 13 | 0,017; 0,0400; 0,0527; 0,0534; 0,0541; 0,0556; 0,0702; 0,0732; 0,1152; 0,1164; 0,1215; 0,1386; 0,1400 |
| 2024 | [14];[15] | 14 | 0,026; 0,0498; 0,0542; 0,0561; 0,0613; 0,0708; 0,0717; 0,0765; 0,0825; 0,0974; 0,1039; 0,1190; 0,1404; 0,1840 |
| 2025 | [14] | 3 | 0,0347; 0,0549; 0,0947 |
| 2026 | [14] | 2 | 0,0507; 0,0834 |
| 2219 | [14] | 2 | 0,0701; 0,0705 |
| 5052 | [14] | 2 | 0,0845; 0,0914 |
| 5056 | [14] | 1 | 0,0947 |
| 5086 | [14] | 1 | 0,0640 |
| 5154 | [14] | 1 | 0,0662 |
| 5456 | [14];[15] | 2 | 0,012; 0,0708 |
| 6061 | [14];[15] | 4 | 0,018; 0,027; 0,0478; 0,087 |
| 7039 | [14] | 1 | 0,1405 |
| 7075 | [14];[15] | 8 | 0,040; 0,0505; 0,059; 0,0615; 0,0689; 0,0906; 0,1686; 0,2157 |
| 7076 | [10] | 2 | 0,0413[a]; 0,0593[a] |
| 7079 | [14] | 3 | 0,0560; 0,0942; 0,1486 |
| 7178 | [14] | 2 | 0,0484; 0,0881 |
| R303 | [14] | 1 | 0,0434 |
| 5 Mg Al | [14] | 2 | 0,0934; 0,1302 |
| 7,5 Zn 2,5 Mg Al | [14] | 1 | 0,0570 |
| Summary of all data above | [13];[14];[15];[10] | 71 | $(k_o)$ 90 % = 0,1390; $\mu = 0{,}0811$; $\sigma = 0{,}0452$ |
| Remove the 1100 data | | 69 | $(k_o)$ 90 % = 0,1288; $\mu = 0{,}0775$; $\sigma = 0{,}0400$<br>Conclusion: select $k_o$ = 0,13 for alloyed aluminium;<br>select $k_o$ = 0,23 for unalloyed aluminium |

[a] Data from reference [10] are at endurance limit.

**Table A.4 — Summary of $k_o$ calculations for low alloy steels (containing silicon at less than 1 % and 1 or more of the following: nickel - less than 4 %; chrome - less than 2 %; molybdenum - less than 0,5 %)**

| Alloy | Reference | Number of distributions | $k_o$ values [a] |
|---|---|---|---|
| 2340 | [13] | 6 | 0,0190; 0,0296; 0,0311; 0,0374; 0,0622 0,0696 |
| 3140 | [13] | 4 | 0,0145; 0,0283; 0,0435; 0,0919 |
| 4140 | [13] | 2 | 0,0650; 0,1102 |
| 4330 | [10] | 6 | 0,0313; 0,0372; 0,0498; 0,0644; 0,1063; 0,1129 |
| 4340 | [13] | 14 | 0,0525; 0,0819; 0,1023; 0,1037; 0,1219; 0,1285; 0,1301; 0,1335; 0,1428; 0,1438; 0,1476; 0,1484; 0,1600; 0,2035 |
| 4340 | [11] | 4 | 0,0253; 0,0301; 0,0321; 0,0627 |
| 4340 | [10] | 10 | 0,0497; 0,0509; 0,0607; 0,0608; 0,0640; 0,0795; 0,0822; 0,0833; 0,0880; 0,0966 |
| 4350 | [10] | 8 | 0,0759; 0,0837; 0,0878; 0,0895; 0,1025; 0,1055; 0,1209; 0,1213 |
| AMS 5727 | [13] | 3 | 0,0341; 0,0385; 0,1002 |
| Summary of all data above | [13];[11];[10] | 57 | ($k_o$) 90 % = 0,1348; $\mu = 0{,}0812$; $\sigma = 0{,}0418$<br>Conclusion: $k_o$ value selected = 0,14 |

[a] Data from reference [10] are at endurance limit.

**Table A.5 — Summary of $k_o$ calculations for cobalt**

| Metal and alloy | Reference | Number of distributions | $k_o$ values |
|---|---|---|---|
| Stellite 31[1)] | [14] | 3 | 0,0771; 0,1202; 0,1472 |
| S-816 (AMS5765) | [13];[14] | 4 | 0,0448; 0,0456; 0,0730; 0,0777 |
| S-816 (AMS5534) | [13] | 1 | 0,0646 |
| Summary of all data above | [13];[14] | 8 | ($k_o$) 90 % = 0,1269; $\mu = 0{,}0813$; $\sigma = 0{,}0355$<br>Conclusion: $k_o$ value selected = 0,13 |

**Table A.6 — Summary of $k_o$ calculations for copper**

| Metal and alloy | Reference | Number of distributions | $k_o$ values [a] |
|---|---|---|---|
| 100 % Cu | [14] | 2 | 0,0522; 0,0804 |
| 70/30 Brass | [14] | 2 | 0,0153; 0,0735 |
| Cu-7,3 Al BRNZ | [14] | 2 | 0,0496; 0,1037 |
| Al Ni BRNZ | [10] | 1 | 0,0938[a] |
| Beryllium | [10] | 1 | 0,0740[a] |
| Copper casting alloys | [16] | 8 | 0,0153; 0,0173; 0,0181; 0,0185; 0,0200; 0,0211; 0,0234; 0,0514 |
| Summary of all data above | [14];[16];[10] | 16 | $(k_o)$ 90 % = 0,0853; $\mu$ = 0,0455; $\sigma$ = 0,0311<br>Conclusion: $k_o$ value selected = 0,09 |

[a] Data from reference [10] are at endurance limit.

**Table A.7 — Summary of $k_o$ calculations for magnesium**

| Metal and alloy | Reference | Number of distributions | $k_o$ values |
|---|---|---|---|
| AZ31A | [14] | 1 | 0,0331 |
| AZ31B | [14] | 1 | 0,0714 |
| AZ61A | [14] | 3 | 0,0854; 0,1705; 0,1735 |
| AZ80A-F | [14] | 1 | 0,0457 |
| AZ81 | [14] | 2 | 0,0458; 0,0761 |
| ZK60A | [14] | 1 | 0,1053 |
| 2,5 Al Mg | [14] | 3 | 0,1152; 0,1444; 0,1702 |
| Summary of all data above | [14] | 12 | $(k_o)$ 90 % = 0,1694; $\mu$ = 0,1031; $\sigma$ = 0,0517<br>Conclusion: $k_o$ value selected = 0,17 |

**Table A.8 — Summary of $k_o$ calculations for plain carbon steel**

| Group and alloy | Reference | Number of distributions | $k_o$ values |
|---|---|---|---|
| 1045 | [13] | 3 | 0,0273; 0,0581; 0,0682 |
| 1050 | [11] | 1 | 0,0171 |
| Summary of all data above | [13];[11] | 4 | $(k_o)$ 90 % = 0,0739; $\mu$ = 0,0427; $\sigma$ = 0,0244<br>Conclusion: $k_o$ value selected = 0,08 |

**Table A.9 — Summary of $k_o$ calculations for stainless steel**

| Metal and alloy | Reference | Number of distributions | $k_o$ values |
|---|---|---|---|
| 321 | [13] | 3 | 0,0439; 0,0606; 0,0755 |
| A-286 | [13] | 2 | 0,0958; 0,1303 |
| 347 | [13] | 4 | 0,0302; 0,0491; 0,0802; 0,1162 |
| Multimet N-155[1)] | [13] | 14 | 0,0163; 0,0180; 0,0230; 0,0313; 0,0313; 0,0315; 0,0325; 0,0367; 0,0381; 0,0407; 0,0427; 0,0544; 0,0547; 0,0574 |
| PH 15-7 | [13] | 2 | 0,0676; 0,0936 |
| 17-7 PH | [13] | 4 | 0,0135; 0,0145; 0,0168; 0,0505; |
| 403 | [13] | 2 | 0,0160; 0,0381 |
| Summary of all data above | [13] | 31 | $(k_o)$ 90 % = 0,0868; $\mu$ = 0,0484; $\sigma$ = 0,0300<br>Conclusion: $k_o$ value selected = 0,09 |

**Table A.10 — Summary of $k_o$ calculations for tool steel**

| Metal and alloy | Reference | Number of distributions | $k_o$ values[a] |
|---|---|---|---|
| Tricent[1)] | [10] | 3 | 0,0350[a];0,0483[a]; 0,0513[a] |
| Ferrovac[1)] | [10] | 1 | 0,0909[a] |
| H-23 | [10] | 1 | 0,0648[a] |
| M-10 | [13];[10] | 6 | 0,0672; 0,0704[a]; 0,0707; 0,0714[a]; 0,1032; 0,1087 |
| D (6AC) | [13] | 4 | 0,0400; 0,0546; 0,0572; 0,1178; 0,0258; 0,0266 |
| H-11 | [13] | 19 | 0,0307; 0,0325; 0,0399; 0,0444; 0,0544; 0,0555; 0,0633; 0,0635; 0,0690; 0,0726; 0,0760; 0,0766; 0,0798; 0,0843; 0,0923; 0,1195; 0,1219 |
| Thermold J[1)] | [13] | 2 | 0,0251; 0,0724 |
| Timken 16-25-6[1)] | [13] | 1 | 0,0231 |
| Summary of all data above | [13];[10] | 37 | $(k_o)$ 90 % = 0,0999; $\mu$ = 0,0649; $\sigma$ = 0,0273<br>Conclusion: $k_o$ value selected = 0,10 |

[a] Data from reference [10] are at endurance limit.

**Table A.11 — Summary of $k_o$ calculations for titanium**

| Alloy | Reference | Number of distributions | $k_o$ values |
|---|---|---|---|
| Ti-140 (AMS 493) | [13] | 1 | 0,0956 |
| Ti-6Al-4V | [13];[14] | 13 | 0,0385; 0,0480; 0,0514; 0,0514; 0,0556; 0,0557; 0,0616; 0,0741; 0,0792; 0,0841; 0,0882; 0,1086; 0,1226 |
| Ti-A55 | [14] | 8 | 0,0272; 0,0507; 0,0637; 0,0794; 0,0837; 0,1003; 0,1033; 0,1515 |
| Ti-75A | [14] | 6 | 0,0298; 0,0380; 0,0413; 0,0508; 0,0621; 0,0626 |
| Ti-150A | [14] | 2 | 0,0707; 0,0882 |
| Ti-0,2 ($0_2$) | [14] | 3 | 0,0422; 0,0461; 0,0906 |
| Ti-0,2C | [14] | 1 | 0,0691 |
| Ti-4Al-3Mo-1V | [14] | 5 | 0,0961; 0,0979; 0,1338; 0,1450; 0,2041 |
| Ti-5Al-2,5Sn-0,07($N_2$) | [14] | 2 | 0,0581; 0,0697 |
| Ti-5Al-2,5Sn-0,2($0_2$) | [14] | 1 | 0.07760 |
| Ti-6Al | [14] | 4 | 0,0442; 0,0472; 0,0480; 0,0821 |
| Summary of all data above | [13];[14] | 46 | $(k_o)$ 90 % = 0,1203 ; $\mu$ = 0,0754; $\sigma$ = 0,0350<br>Conclusion: $k_o$ value selected = 0,12 |

**Table A.12 — Summary of $k_o$ calculations for nickel steel (nickel content at least 40 %)**

| Alloy | Reference | Number of distributions | $k_o$ values |
|---|---|---|---|
| GMR-235 | [13] | 1 | 0,0301 |
| Udimet 500[1)] | [13] | 1 | 0,0642 |
| Hastelloy C[1)] | [14] | 1 | 0,0753 |
| Hasteloy R235 | [14] | 2 | 0,0592; 0,1166 |
| Incoloy 901[1)] (AMS 5560A) | [14] | 1 | 0,0570 |
| Inconel 718[1)] | [14] | 3 | 0,0705; 0,0933; 0,0952 |
| Waspaloy[1)] | [14] | 1 | 0,0714 |
| Rene-41 (AMS 5713)[1)] | [14] | 3 | 0,0336; 0,0741; 0,0876 |
| 6 Mo Waspaloy[1)] | [14] | 2 | 0,0327; 0,0741 |
| Summary of all data above | [13];[14] | 15 | $(k_o)$ 90 % = 0,1003; $\mu$ = 0,0690; $\sigma$ = 0,0244<br>Conclusion: $k_o$ value selected = 0,10 |

**Table A.13 — Summary of $k_o$ calculations for Monel**

| Alloy | Reference | Number of distributions | $k_o$ value |
|---|---|---|---|
| Monel | [14] | 1 | 0,2643<br>Conclusion: $k_o$ value selected = 0,27 |

# Annex B
(normative)

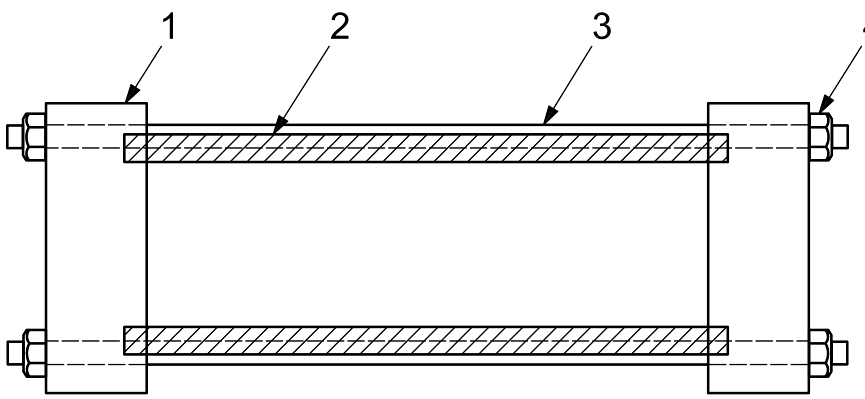
# Calculation of variability factor $K_V$

## B.1 General

Annex B provides an example of how to calculate variability factor $K_V$.

## B.2 Method

Consider a simple pressure-containing envelope consisting of a cylindrical tube with a square head on each end, held together by tie rods and nuts in each of the square's four corners (see Figure B.1).




**Key**

1 square head
2 cylinder tube
3 tie rod
4 tie rod nuts

**Figure B.1 — Pressure-containing envelope**

The test operator has chosen an assurance level of 90 % and a verification level of 99 %. Information needed to obtain $K_V$ values from Table 1 is shown in Table B.1.

**Table B.1 — Other information needed to calculate $K_V$**

| Element | Material | $k_o$ | Number of actual test specimens | Number of specimens chosen from Table 1 | $K_v$ |
|---|---|---|---|---|---|
| Cylinder tube | Aluminium | 0,13 | 2 | 1 | 1,40 |
| End head | Magnesium | 0,17 | 4 | 2 | 1,38 |
| Tie rod | Steel | 0,08 | 8 | 4 | 1,10 |
| Tie rod nut | Steel | 0,08 | 16 | See example | 1,05 |

The value of $K_V$ for the last row in Table B.1 is calculated as described in the example that follows. Equation (D.6) in Annex D is used, and is shown here as Equation (B.1).

$$K_V = \frac{1 + k_o Z_4}{1 - k_o Z_2} \tag{B.1}$$

EXAMPLE The value of $K_V$ for the tie rod nut was calculated as:

For an assurance level of 90 %, tail area $A_2 = 0{,}10$; then, $Z_2 = 1{,}282$.

For a verification level of 99 %, tail area $A_1 = 0{,}01$, $n = 16$.

$(A_1 + A_4) = A_1{}^{1/n} = (0{,}01)^{1/16} = 0{,}7499$, and $Z_4 = -\,0{,}674$

$$K_V = \frac{1 + (0{,}08)(-0{,}674)}{1 - (0{,}08)(1{,}282)} = 1{,}054 \text{ which is rounded to } 1{,}05$$

NOTE 1 $Z_4 = 0$ at $(A_1 + A_4) = 0{,}5$ and is negative above 0,5.

In the case described above, the highest value of $K_V$ for any of the elements is 1,40, therefore, this is the value of $K_V$ that is used to calculate the $P_{CT}$.

NOTE 2 The highest $K_V$ value does not always correspond to the highest $k_o$ value.

NOTE 3 When a verification level of 99 % is chosen, the number of test specimens is always an even number.

# Annex C
(informative)

# Proposal for an acceleration factor

## C.1 General

It is proposed that a product can be rated for $10^7$ cycles, but tested for only $10^6$ cycles if it is tested at a higher pressure.

If material data for a fatigue strength distribution at both the $10^6$ and $10^7$ lives are available, then an acceleration factor can be determined. This acceleration factor would be a simple ratio of the fatigue strength at $10^6$ cycles, to that at $10^7$ cycles. The acceleration factor would be applied to the test pressure in order to raise the stress level in the test samples.

Data at the characteristic life are often available for Weibull distributions, and ratios of the characteristic strengths between those two levels can be used to calculate the acceleration factor. Likewise, data at the median life are often available for Normal distributions, and ratios of the median strengths between those two levels can be used for the acceleration factor.

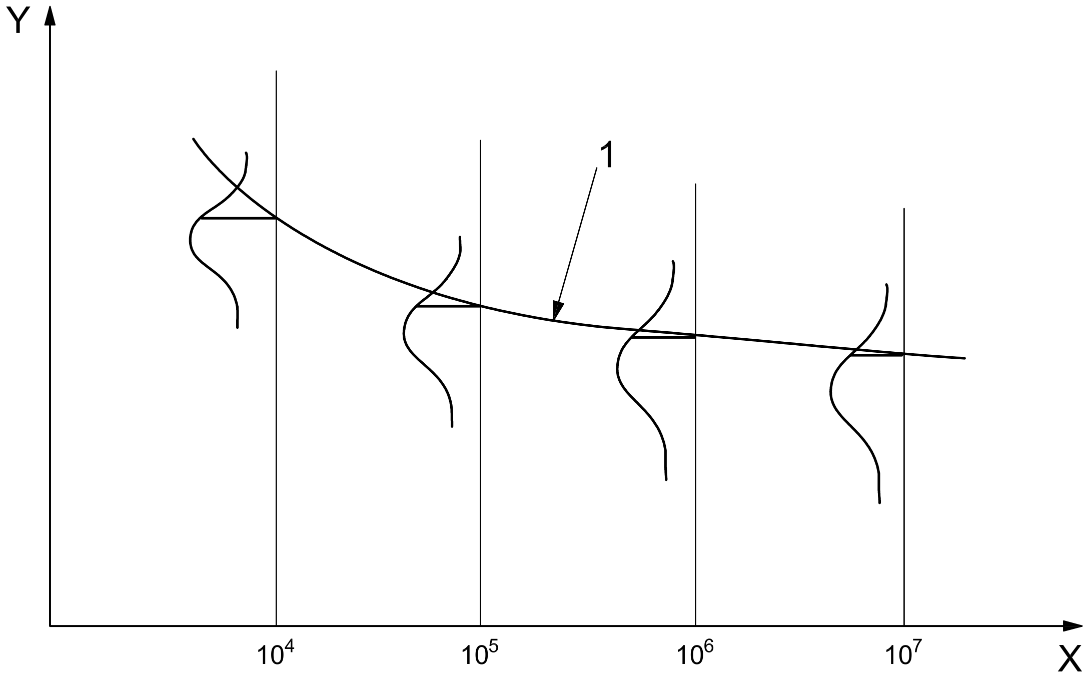
For example, an AISI 4140 steel with a Weibull distribution at $10^6$ cycles has a characteristic fatigue strength of 530 MPa; and the distribution at $10^7$ cycles has a characteristic fatigue strength of 486 MPa. The ratio of these fatigue strengths is 1.09. Therefore, the test pressure would be raised by this factor if a $10^7$ cycle rating was desired with a $10^6$ cycle test. This factor would be in addition to any other factors imposed on the test pressure.

Considerable judgment would be required in proposing a pressure rating when using acceleration factors, because the probabilities of failure during the test are greater.

## C.2 Extrapolating data to $10^7$ cycles

The number of fatigue strength distributions available at $10^7$ cycles is not very abundant. Therefore, a method to extrapolate data to $10^7$ cycles is proposed if data are available at three lower levels, but spaced well apart, such as $10^4$, $10^5$ and $10^6$ cycles.

Consider an $S$-$N$ curve as shown in Figure C.1:




**Key**

X life, expressed in number of cycles

Y applied stress from testing; and material strength from failures

1 $S = CN^{-D} + E$, where $S$ is strength, $N$ is life in cycles; $C$, $D$ and $E$ are coefficients

**Figure C.1 — *S*-*N* curve**

At each level of life, there is a fatigue strength distribution. A constant probability curve joins each point of the fatigue strength from each of the distributions, and is suggested to have an equation of the form shown in Equation (C.1).

$$S = CN^{-D} + E \quad \text{(C.1)}$$

Since Equation (C.1) has three unknown coefficients, it is theoretically possible to determine them from three sets of data points. Therefore let these data points be the pairs:

$S_1, N_1$ @$10^4$ $S_2, N_2$ @$10^5$ $S_3, N_3$ @$10^6$

Then, knowing the coefficients, the corresponding value of $S$ can be projected to $N = 10^7$ cycles. This value can then be used to determine the acceleration factor discussed in the previous section.

To begin, re-arrange Equation (C.1) as:

$$(S - E) = CN^{-D} \text{ and } \ln(S - E) = \ln C - D \ln N \quad \text{(C.2)}$$

Inserting data points into Equation (C.2) gives:

$$\ln(S_1 - E) = \ln C - D \ln N_1$$

$$\ln(S_2 - E) = \ln C - D \ln N_2$$

$$\ln(S_3 - E) = \ln C - D \ln N_3$$

Subtracting:

$$\ln(S_1 - E) - \ln(S_2 - E) = D \ln N_2 - D \ln N_1 = D(\ln N_2 - \ln N_1) \tag{C.3}$$

Similarly:

$$\ln(S_1 - E) - \ln(S_3 - E) = D \ln N_3 - D \ln N_1 = D(\ln N_3 - \ln N_1) \tag{C.4}$$

Dividing Equation (C.3) by Equation (C.4):

$$\frac{\ln(S_1 - E) - \ln(S_2 - E)}{\ln(S_1 - E) - \ln(S_3 - E)} = \frac{D(\ln N_2 - \ln N_1)}{D(\ln N_3 - \ln N_1)} = \frac{\ln N_2 - \ln N_1}{\ln N_3 - \ln N_1} = C_1 \tag{C.5}$$

It is observed that the right-hand side of Equation (C.5) is a constant.

Continuing from Equation (C.5):

$$\ln(S_1 - E) - \ln(S_2 - E) = C_1 \ln(S_1 - E) - C_1 \ln(S_3 - E)$$

$$\ln\left(\frac{S_1 - E}{S_2 - E}\right) = C_1 \ln\left(\frac{S_1 - E}{S_3 - E}\right) = \ln\left(\frac{S_1 - E}{S_3 - E}\right)^{C_1} \tag{C.6}$$

Equating the logs of both sides, and expanding using the binomial theorem to 3 terms gives Equation (C.7).

$$\frac{S_1 - E}{S_2 - E} = \left(\frac{S_1 - E}{S_3 - E}\right)^{C_1} = \frac{S_1^{C_1} - C_1 S_1^{(C_1-1)} E + \frac{1}{2} C_1 (C_1 - 1) S_1^{(C_1-2)} E^2 + \cdots}{S_3^{C_1} - C_1 S_3^{(C_1-1)} E + \frac{1}{2} C_1 (C_1 - 1) S_1^{(C_1-2)} E^2 + \cdots} \tag{C.7}$$

It is observed that the coefficients of the binomial can be replaced by constants:

$$C_2 = C_1 S_1^{(C_1-1)} \qquad C_3 = \frac{1}{2} C_1 (C_1 - 1) S_1^{(C_1-2)} \qquad C_4 = C_1 S_3^{(C_1-1)} \qquad C_5 = \frac{1}{2} C_1 (C_1 - 1) S_1^{(C_1-2)}$$

Resulting in Equation (C.8):

$$\frac{S_1 - E}{S_2 - E} = \frac{S_1^{C_1} - C_2 E + C_3 E^2}{S_3^{C_1} - C_4 E + C_5 E^2} \tag{C.8}$$

Cross multiplying and expanding Equation (C.8):

$$(S_1 - E)\left(S_3^{C_1} - C_4 E + C_5 E^2\right) = (S_2 - E)\left(S_1^{C_1} - C_2 E + C_3 E^2\right)$$

$$S_1 S_3^{C_1} - S_1 C_4 E + S_1 C_5 E^2 - S_3^{C_1} E + C_4 E^2 - C_5 E^3 = S_2 S_1^{C_1} - S_2 C_2 E + S_2 C_3 E^2 - S_1^{C_1} E + C_2 E^2 - C_3 E^3$$

Combining terms results in a cubic equation:

$$(C_3 - C_5) E^3 + \left[(C_4 - C_2) + (S_1 C_5 - S_2 C_3)\right] E^2 + \left(S_2 C_2 - S_1 C_4 + S_1^{C_1} - S_3^{C_1}\right) E + \left(S_1 S_3^{C_1} - S_2 S_1^{C_1}\right) = 0$$

Dividing by the first coefficient gives Equation (C.9):

$$E^3 + \frac{\left(C_4 - C_2 + S_1 C_5 - S_2 C_3\right)}{\left(C_3 - C_5\right)} E^2 + \frac{\left(S_2 C_2 - S_1 C_4 + S_1^{C_1} - S_3^{C_1}\right)}{\left(C_3 - C_5\right)} E + \frac{\left(S_1 S_3^{C_1} - S_2 S_1^{C_1}\right)}{\left(C_3 - C_5\right)} = 0 \tag{C.9}$$

Substituting for the coefficients:

$$p = \frac{\left(C_4 - C_2 + S_1 C_5 - S_2 C_3\right)}{\left(C_3 - C_5\right)} \quad q = \frac{\left(S_2 C_2 - S_1 C_4 + S_1^{C_1} - S_3^{C_1}\right)}{\left(C_3 - C_5\right)} \quad r = \frac{\left(S_1 S_3^{C_1} - S_2 S_1^{C_1}\right)}{\left(C_3 - C_5\right)}$$

This results in the classic textbook cubic Equation (C.10):

$$E^3 + pE^2 + qE + r = 0 \tag{C.10}$$

which can be solved with software (also available on the Internet [18]). There are three roots to Equation (C.10), and one of them should be selected for continued use. This choice is made by examining the values of the three roots and selecting the one that is a real number, and less than the value of $S_3$ (it might even be negative). It can also be necessary to try more than one root and examine the results.

Returning to Equation (C.1), and substituting the chosen value of *E*:

$$S = CN^{-D} + E \text{ and } (S - E) = CN^{-D}$$

Inserting data points and dividing gives:

$$\left(\frac{S_1 - E}{S_2 - E}\right) = \frac{CN_1^{-D}}{CN_2^{-D}} = \left(\frac{N_1}{N_2}\right)^{-D} \text{ and } \ln\left(\frac{S_1 - E}{S_2 - E}\right) = -D \ln\left(\frac{N_1}{N_2}\right)$$

which can be solved for *D*:

$$-\frac{\ln\left(\dfrac{S_1 - E}{S_2 - E}\right)}{\ln\left(\dfrac{N_1}{N_2}\right)} = D \tag{C.11}$$

Finally, substituting known values into Equation (C.1) again gives:

$$S_1 - E = CN_1^{-D} \text{ and}$$

$$\frac{S_1 - E}{N_1^{-D}} = C \tag{C.12}$$

Thus, all three coefficients for Equation (C.1) are now known, and one of the fatigue strength values (characteristic or mean) at $10^7$ cycles can be determined using Equation (C.13):

$$S_4 = CN_4^{-D} + E \tag{C.13}$$

## C.3 Examples

### C.3.1 General

Weibull fatigue strength distributions for five materials (both smooth and notched specimens) were found for cycle lives at $10^4$, $10^5$, $10^6$ and $10^7$ cycles. The equations in Clause C.2 were used to calculate the fatigue strengths at $10^7$ cycles, and then compared to the published value at $10^7$ cycles. From this, the acceleration factors were calculated. Results are summarized in Table C.1 and Table C.2.

**Table C.1 — Smooth specimen examples**

| Material | | 4140 steel | H11 steel | 4340 steel | Alloy steel | 2024 Aluminium |
|---|---|---|---|---|---|---|
| Characteristic life | Published at $10^7$ | 485,5 MPa | 673,1 MPa | 620,7 MPa | 1079,3 MPa | 186,2 MPa |
| | Calculated at $10^7$ | 487,2 MPa | 684,0 MPa | 609,3 MPa | 1082,9 MPa | 186,8 MPa |
| | Error | +0,36 % | +1,61 % | −1,83 % | +0,33 % | −0,04 % |
| | Acceleration factor published | 1,09 | 1,12 | 1,13 | 1,08 | 1,22 |
| | Acceleration factor calculated | 1,09 | 1,10 | 1,15 | 1,08 | 1,21 |
| | Error | −0,36 % | −1,59 % | +1,86 % | −0,33 % | −0,28 % |

**Table C.2 — Notched specimen examples**

| Material | | 4140 steel | H11 steel | 4340 steel | Alloy steel | 2024 Aluminium |
|---|---|---|---|---|---|---|
| Characteristic life | Published at $10^7$ | 219,3 MPa | 675,9 MPa | 87,6 MPa | 497,2 MPa | 92,3 MPa |
| | Calculated at $10^7$ | 219,7 MPa | 677,4 MPa | 80,6 MPa | 503,1 MPa | 92,2 MPa |
| | Error | +0,17 % | +0,22 % | −1,83 % | +1,17 % | −0,19 % |
| | Acceleration factor published | 1,18 | 1,16 | 1,81 | 1,05 | 1,41 |
| | Acceleration factor calculated | 1,18 | 1,16 | 1,97 | 1,04 | 1,41 |
| | Error | −0,17 % | −0,22 % | +8,70 % | −1,15 % | +0,19 % |

The materials used in this comparison were all with three-parameter Weibull distributions and the data sources were Reference [13] and Reference [14].

### C.3.2 Observations

The following observations were made.

a) The accuracy of projecting the characteristic and mean life to $10^7$ cycles was very good for all the example materials, except the 4340 notched steel. However, data for this material are questionable because the notched values for minimum life were very low, resulting in only one real root to its cubic equation and that was larger than the minimum life at $10^6$ cycles.

b) The acceleration factor from projected life had good accuracy compared to the acceleration factor calculated directly from published data.

c) There is a significant difference in the acceleration factor between smooth and notched specimens, for some of the materials, for example:

   1) the pressure increase would double for notched specimens over smooth ones in the 4140 steel. (For example, if the test pressure for a $10^6$ cycle test were 10 MPa [2)], the smooth specimen data would result in a 0,9 MPa increase in test pressure, to 10,9 MPa, for rating at $10^6$ MPa. If notched data were used, the increase would be 1,8 MPa, resulting in a test pressure of 11,8 MPa);

   2) the pressure increase is about 33 % more for notched specimens than smooth ones in the H11 steel;

   3) the pressure increase is actually less for notched specimens than smooth samples in the alloy steel;

   4) the acceleration factor is unreasonably high for notched specimens in aluminium and the 4340 steel.

### C.3.3 Proposal

The examples justify the accuracy of the analysis technique, but selection of material data for use in a standard should be explored further. The sources cited for the data had some questionable values (as noted for the 4340 steel), but they had values from a Weibull distribution and not just endurance values. Therefore, new data should be sought with Normal, Weibull, or other fatigue distribution data.

With enough data, a proposal could be made for a table of acceleration factors to be used in an International Standard.

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2) 1 bar = 0,1 MPa = 0,1 N/mm$^2$ = 105 N/m$^2$.

# Annex D
(informative)

# Basis of fatigue pressure rating[3)] [9]

## D.1 Basis of pressure rating

### D.1.1 Abstract

A theoretical development of the NFPA verification method for fatigue pressure rating fluid power components is presented in Annex D. In addition, a method is shown for generating a fatigue strength distribution at a desired life point, using raw data from a fatigue test. Finally, examples are given for using the methods developed and discussion offered for this technique.

### D.1.2 Introduction

Fluid power products are designed to operate in a pressurized condition, which necessitates some claim by its manufacturer about pressure-containing ability. There are many standards which describe, in various degrees of detail, just how a fluid power product might be designed for a given pressure rating. It has generally been found, however, that the ingenuity of designers tends to outpace the rate at which design rules can be standardized. The philosophy of this part of ISO 10771 on pressure rating has recognized this fact and suggests that manufacturers begin with their own methods for assigning a fatigue pressure rating. Then apply this part of ISO 10771, with its methods of determining material strength variability and usage of statistical tools to substantiate the proposed fatigue pressure rating. That this method employs fundamental concepts, lends itself to a universal application for all fluid power products, if made and used within the limits of its scope (i.e. metal only, environment, temperature, etc.).

The purpose of this part of ISO 10771 then is to define a method by which the claimed pressure rating can be verified on a fatigue basis.

Annex D is an analytical development of the basis theory for verification. It is offered as an academic tutorial, as well as a guide for those wishing to calculate their own test factors.

This paper is divided into three subject matters: Statistical Analysis Theory (Clause D.2), Fatigue Distribution Data (Clause D.3) and a Data Calculation Example (Clause D.4). Clause D.2 assumes that fatigue strength distribution data for the material are available, and develops a verification theory. Clause D.3 describes how raw data from a fatigue test are transformed into the statistical distribution for use in the verification theory.

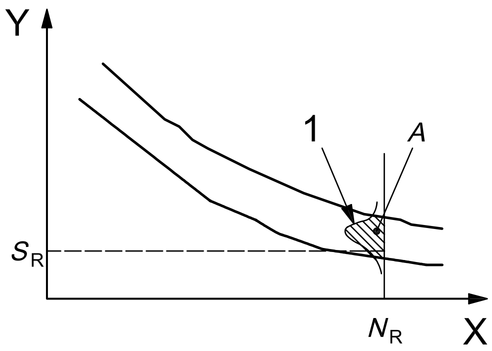
Clause D.4 uses data extracted from several industry interlaboratory tests in a single example of the theoretical results.

3) Annex D is reproduced from the Proceedings of the 43rd National Conference on Fluid Power, Chicago, Illinois, USA, October, 1988.

## D.2 Statistical analysis theory

### D.2.1 Objective

Fatigue data necessary for this analysis are a statistical distribution of the material strength at some rated life, as shown in Figure D.1.




**Key**

X life, expressed in number of cycles

Y fatigue strength

1 strength distribution at rated fatigue life

$S_R$ rated strength

$N_R$ rated life

$A$ area under curve above $S_R$

**Figure D.1 — Fatigue strength at rated life**

From the parameters of this distribution, it is possible to define some level near the lower end for rating purposes. The objective of verification, then, is to demonstrate by test that a population of products belongs to this distribution.

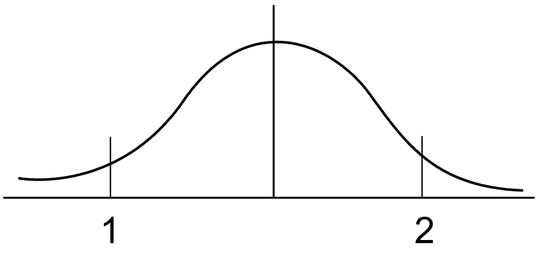
One way to make this demonstration is to test several products to destruction. From the data, a sample distribution curve can be made and statistically compared to the population distribution. If the sample size is sufficiently large, the results could even be used as a new population and the rating obtained in a direct manner. These are both accurate methods for verification, but costly.

This part of ISO 10771 proposes a method of non-failure testing (verification testing), using a few sacrificial specimens. By incorporating conservative principles, the verification process can be greatly simplified with an assurance of product integrity.

### D.2.2 Planned conservatism

The area below the population strength distribution curve, above the rated strength ($A$ in Figure D.1) represents the probability of surviving $N_R$ cycles of operation *at the rated strength*.

However, if a few specimens were tested at the rated strength and survived, they could not yield statistical confidence about that rated strength. It would only suggest that the sample strength level was above the level of testing conducted. A conservative test, however would be one conducted in the upper tail as depicted in Figure D.2.




**Key**

1 rated load

2 test level

**Figure D.2 — Conservative test level**

Now the survivors of such a test level would offer more confidence for some rated load located in the left-hand tail.

Surviving a fatigue test in the upper tail, however, is very unlikely. But if an artificial shift of the distribution curve is made to a lower strength region, as shown in Figure D.3, then the test level becomes relocated into a left-hand tail where passing is more probable. The new curve is defined as a rated strength distribution and the rating level is correspondingly lowered.

Conservatism in such a rating scheme is now apparent:

a) the selected test level gives high probability ($1 - A_1$) that survivors are not representative of a strength distribution lower than the new, shifted rated one;

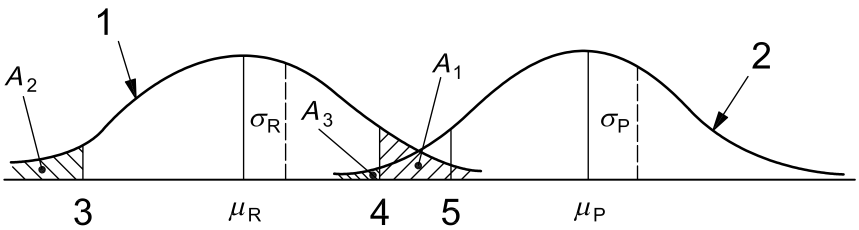
b) the rating level gives high probability ($1 - A_2$) that product strength will exceed this rating level;

c) there is high probability ($1 - A_3$) that the product is strong enough to pass the test. Using the tail areas so described, the following terms are defined:

— ($1 - A_1$) is the verification level;

— ($1 - A_2$) is the assurance level;

— ($1 - A_3$) is the test success level.




**Key**

1 rated strength distribution

2 population strength distribution

3 rating level

4 test level

5 rated strength ($S_R$ in Figure D.1.)

**Figure D.3 — Shifted curve to a rated position**

An examination of Figure D.3 reveals that some relationship between the two distributions should be established otherwise the shape of the rated strength distribution might be arbitrary. Therefore select Equation (D.1):

$$\frac{\sigma_R}{\mu_R} = \frac{\sigma_P}{\mu_P} = k_o \tag{D.1}$$

where

$k_o$ is the coefficient of variation;

$\mu_R$ is the median of the rated strength distribution;

$\mu_P$ is the median of the population strength distribution;

$\sigma_R$ is the rated standard deviation;

$\sigma_P$ is the population standard deviation.

The amount of shift between the two distributions will be examined later.

### D.2.3 Number of test specimens

When random specimens are subjected to a verification test, they will either survive or fatigue before $N_R$ cycles. Since there are only these two possibilities, a binomial distribution can be used to describe the probability of success (achieving fatigue) in a set of specimens:

where

$$P(y) = C_y^n p^y q^{(n-y)} \text{ where } C_y^n = \frac{n!}{y!(n-y)!} \tag{D.2}$$

and:

$P(y)$ is the probability that from a group of $n$ specimens, $y$ units will fatigue at or before $N_R$ cycles;

$p$ is the probability of an individual specimen becoming fatigued at or before $N_R$ cycles;

$q$ is the probability of an individual specimen surviving.

Since the basis of this verification test is that all specimens survive: $y = \text{o}$

Then:

$$C_n^y = 1$$

$$p^y = 1$$

$$q = 1 - p$$

and

$$P(0) = (1-p)^n \tag{D.3}$$

This means that the probability of no failures in a test group is related to the fatigue probability of a typical unit and the number of units in the test group. Assuming the probability of fatigue is related to strength, and Equation (D.3) is applied to the rated strength distribution of Figure D.3, then the area under this curve to the left of the test level represents the proportion of units weaker than the imposed load and is the probability of failure in a typical unit.

$p_1 = 1 - A_1$ (the verification level)

For a random specimen, the probability of passing the verification test would be:

$$P(0) = (1 - p_1)^n = (1 - 1 + A_1)^1 = A_1 \quad \text{(D.4)}$$

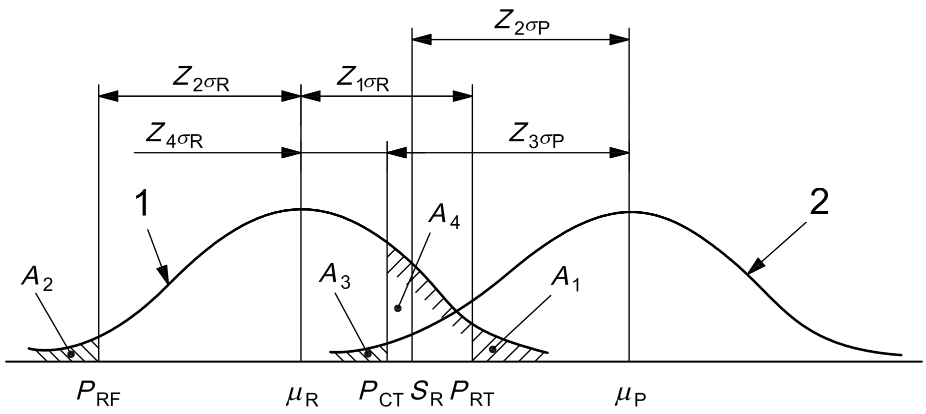
which is the right-hand tail area of the rated strength distribution.

If the number of specimens is increased but the probability of passing the verification test is held constant:

$$P(0) = (1 - p_n)^n = A_1 \quad \text{(D.5)}$$

$$p_n = 1 - (A_1)^{1/n} \quad \text{(D.6)}$$

From this, it is observed that the probability of failure in an individual unit of multiple specimens, $p_n$, is smaller than $p_1$. This means that the tail area, $A_1$, has to be larger for multiple specimens than for a single specimen, if the verification level value is maintained. Such a case can be depicted by returning the rated strength distribution curve back slightly towards the population curve, but maintaining area, $A_3$, constant, as shown in Figure D.4.




**Key**

1 rated strength distribution

2 population strength distribution

**Figure D.4 — Shifted curve for multiple specimens**

where

$P_{CT}$ is the cyclic test pressure for a multi-specimen test;

$P_{RT}$ is the rated test pressure for a single specimen;

$P_{RF}$ is the rated fatigue pressure;

$A_4$ is the expanded tail area, $A_1$;

$Z_i$ is the number of standard deviations for a normal probability distribution.

Other factors are the same as previously defined.

If the overall probability for passing the verification test is held constant again, Equation (D.3) yields:

$$P(0) = (1 - p)^n$$

$$A_1 = \{1 - [1 - (A_1 + A_4)]\}^n$$

$$A_1^{1/n} = A_1 + A_4 \tag{D.7}$$

Using the value of ($A_1 + A_4$) in a normal probability distribution table, a corresponding value of $Z_4$ can be determined which will be used in further calculations.

Note that for $N = 1$, $A_4 = 0$, $Z_4 = Z_1$ ($Z_4 \neq 0$) and the area under the curve becomes ($1 - A_1$), the verification level. Thus, the verification level becomes a fixed parameter in the rating system.

### D.2.4 Rating selection level

The degree to which the rated strength distribution is displaced from the population strength distribution can be now judged from a ratio:

$$F_R = \frac{S_R}{P_{RF}} = \frac{\mu_p - Z_2\sigma_p}{\mu_R - Z_2\sigma_R} = \frac{\mu_p(1 - Z_2 k_o)}{\mu_R(1 - Z_2 k_o)} = \frac{\mu_p}{\mu_R} \tag{D.8}$$

where

$F_R$ is the rating ratio.

The degree of conservation thus introduced is similar to the "safety factor" concept commonly used in static stress analysis. The rating ratio is dependent upon a probability selected to pass the verification test and can be evaluated from an examination of Figure D.4.

$$F_R = \frac{\mu_p}{\mu_R} = \frac{\mu_R + Z_4\sigma_R + Z_3\sigma_p}{\mu_R} \tag{D.9}$$

$$= 1 + Z_4 k_o + Z_3 \frac{\sigma_p}{\mu_R} \tag{D.10}$$

From Equation (D.1):

$$k_o = \frac{\sigma_p}{\mu_p} = \frac{\sigma_R}{\mu_R}$$

and from Equation (D.9):

$$F_R = \frac{\mu_p}{\mu_R}$$

$$\sigma_p = \frac{\mu_p}{\mu_R}\sigma_R = F_R \sigma_R$$

Substituting into Equation (D.10):

$$F_R = 1 + Z_4 k_o + Z_3 \frac{F_R \sigma_R}{\mu_R} = 1 + Z_4 k_o + Z_3 k_o F_R$$

$$F_R (1 - Z_3 k_o) = 1 + Z_4 k_o$$

$$F_R = \frac{1 + Z_4 k_o}{1 - Z_3 k_o} \tag{D.11}$$

Factor $Z_3$ is obtained from a normal probability distribution table, based upon the one-sided tail area, $A_3$. From Figure D.4, it is seen that the complementary area represents the probability of passing the verification test. Thus the degree of conservation introduced into this rating process can be measured from the parameters chosen.

### D.2.5 Calculation of $K_v$ factor

From all of the factors now determined, a relationship can be calculated between $P_{CT}$ and $P_{RF}$. From Figure D.4:

$$P_{RF} + Z_2 \sigma_R + Z_4 \sigma_R = P_{CT} \tag{D.12}$$

$$\sigma_R = \frac{P_{CT} - P_{RF}}{Z_2 + Z_4}$$

Also from Figure D.4:

$$P_{RF} + Z_2 \sigma_R = \mu_R = \frac{\sigma_R}{k_o} \tag{D.13}$$

$$P_{RF} = \sigma_R \left( \frac{1}{k_o} - Z_2 \right)$$

$$\frac{P_{RF}}{\frac{1}{k_o} - Z_2} = \sigma_R$$

Equating the two $\sigma_R$ expressions:

$$\frac{P_{CT} - P_{RF}}{Z_2 + Z_4} = \frac{P_{RF}}{\frac{1}{k_o} - Z_2}$$

$$\frac{P_{CT}}{P_{RF}} - 1 = \frac{Z_2 + Z_4}{\frac{1}{k_o} - Z_2}$$

$$\frac{P_{CT}}{P_{RF}} = \frac{1 - k_o Z_2}{1 - k_o Z_2} + \frac{k_o Z_2 + k_o Z_4}{1 - k_o Z_2}$$

$$\frac{P_{CT}}{P_{RF}} = \frac{1 + k_o Z_4}{1 - k_o Z_2} = Z_V \tag{D.14}$$

Constant $K_v$ is thus determined as the ratio between $P_{CT}$ and $P_{RF}$. The $K_v$ constant is a function of material property $k_o$ plus a combination of sample size, verification level and assurance level.

## D.3 Fatigue distribution data

### D.3.1 Objective

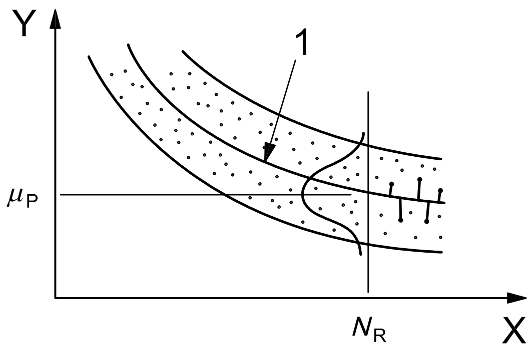
From Clause D.2, it is seen that the verification procedure requires a statistical distribution of fatigue strength at some desired life. Clause D.3 describes how a distribution, as shown in Figure D.1 can be generated from the raw data of a fatigue test.

It is important to point out that collecting the raw data is not a simple matter. It is important to follow careful controls, as described by the rules of statistical experiments, in order to obtain samplings from a population.

### D.3.2 Generating a best fit curve

Consider the raw data plotted on an $S$-$N$ curve, as shown in Figure D.5

NOTE This is a linear plot, not logarithmic on either axis.




**Key**

X life, expressed in number of cycles

Y applied stress from testing; and material strength from failures, $S$

1 best fit curve to the data

**Figure D.5 — *S*-*N* curve with data points and bounds**

Consider that a best fit line passes through the data, in the form:

$$S = C_o N^{-D} + E_o \text{ where } N > N_{min}$$

for values of $N$ greater than some minimum $N$.

This qualification is necessary to overcome the difficulty of large values of $S$ satisfying the model equation at low values of $N$ but not representative of the actual fatigue strength. Thus, the range of applicability is limited.

Now a linear proportionality exists between pressure, $P$, and stress, $S$, such that:

$$P = CN^{-D} + E \quad \text{(D.15)}$$

let

$$(P - E)^2 = C^2 N^{-2D}$$

and

$$\ln (P - E)^2 = 2 \ln C - 2 D \ln N \quad \text{(D.16)}$$

During the conduct of a fatigue test, the independent variable is actually the vertical axis Y ($S$ or $P$). However, if a linear regression analysis is applied with $P$ as the independent variable, the variance of each data point from the best fit line is parallel to the $N$ axis. Those data points below the asymptotic portion of the best fit line will then not make an intersection, resulting in an undefined variance. More likely, however, the best fit line will become distorted from the results, pushing the asymptotic portion below the data points so that the horizontally directed variance makes an intersection. This results in an untrue representation of the best fit line.

Therefore, an arbitrary choice is made to allow the $N$ axis to become the independent variable, at least for the purpose of obtaining a best fit line in the data field and also for values above the $N$ minimum. Now the variances are taken in a vertical direction, parallel to the Y ($S$ or $P$) axis and will intersect the line in the region of interest. But yet another difficulty has to be overcome in performing the regression analysis. If Equation (D.15) were not squared, the left-hand expression of Equation (D.16) would be ln ($P$–$E$). Since $E$ is the asymptotic limit of the curve, those data points below the curve would have values of $P$ less than $E$. When used in a variance calculation, ($P$–$E$) would be zero or negative and the logarithm is undefined. Therefore, the squared function is chosen to overcome this difficulty.

Then, from Equation (D.16) let:

$y = \ln (P - E)^2$ ; $x = \ln N$ ; $A = 2 \ln C$ ; $B = -2D$

then

$$y = A + B_x \tag{D.17}$$

Performing a least squares technique:

$$\overline{x} = \frac{\sum_1^k x_i}{k} \quad \overline{y} = \frac{\sum_1^k y_i}{k} \tag{D.18}$$

$$S_{Sx} = \sum_1^k x_i^2 - \frac{\left(\sum_1^k x_i\right)^2}{k} \tag{D.19}$$

$$S_{Sy} = \sum_1^k y_i^2 - \frac{\left(\sum_1^k y_i\right)^2}{k} \tag{D.20}$$

$$S_{Sxy} = \sum_1^k x_i y_i - \frac{\left(\sum_1^k x_i\right)\left(\sum_1^k y_i\right)}{k} \tag{D.21}$$

where

$k$ is the number of data points.

From these, the following parameters may be determined:

$$B = \frac{S_{Sxy}}{S_{Sx}} \quad D = \frac{-B}{2} = \frac{-S_{Sxy}}{2}$$

$$A = \overline{y} - B\overline{x} \quad C = \mathrm{e}^{A/2}$$

A measure of how well the data fit the resulting equation is judged from the coefficient of determination:

$$r^2 = \frac{(S_{\mathrm{S}xy})^2}{(S_{\mathrm{S}x})(S_{\mathrm{S}y})} \tag{D.22}$$

Now the preceding analysis includes a parameter, $E$, which has not been calculated. Unless arbitrary values are initially assumed, calculation of $Y_\mathrm{i}$ values cannot proceed. If arbitrary values are indeed assumed, the effect will be observed in the value of $r^2$. Therefore, the best assumption to make for $E$ would be one that results in the maximum value of $r^2$ or:

$$\frac{\mathrm{d}(r^2)}{\mathrm{d}E} = 0 \tag{D.23}$$

from

$$r^2 = \frac{(S_{\mathrm{S}xy})^2}{(S_{\mathrm{S}x})(S_{\mathrm{S}y})}$$

$$\frac{\mathrm{d}(r^2)}{\mathrm{d}E} = 0 = \frac{(S_{\mathrm{S}x})(S_{\mathrm{S}y})2(S_{\mathrm{S}xy})\frac{\mathrm{d}}{\mathrm{d}E}(S_{\mathrm{S}xy}) - (S_{\mathrm{S}xy})^2(S_{\mathrm{S}x})\frac{\mathrm{d}}{\mathrm{d}E}(S_{\mathrm{S}y}) - (S_{\mathrm{S}xy})^2(S_{\mathrm{S}y})\frac{\mathrm{d}}{\mathrm{d}E}(\cancelto{0}{S_{\mathrm{S}x}})}{(S_{\mathrm{S}x})^2(S_{\mathrm{S}y})^2} \tag{D.24}$$

Since $S_{\mathrm{S}x}$ is not a function of $E$, it is constant relative to $E$

and $\frac{\mathrm{d}S_{\mathrm{S}x}}{\mathrm{d}E} = 0$

rearranging

$$2(\cancel{S_{\mathrm{S}x}})(S_{\mathrm{S}y})(\cancel{S_{\mathrm{S}xy}})\frac{\mathrm{d}}{\mathrm{d}E}(S_{\mathrm{S}xy}) = (S_{\mathrm{S}xy})^{\cancel{2}}(\cancel{S_{\mathrm{S}x}})\frac{\mathrm{d}}{\mathrm{d}E}(S_{\mathrm{S}y})$$

$$2(S_{\mathrm{S}y})\frac{\mathrm{d}}{\mathrm{d}E}(S_{\mathrm{S}xy}) = (S_{\mathrm{S}xy})\frac{\mathrm{d}}{\mathrm{d}E}(S_{\mathrm{S}y}) \tag{D.25}$$

Evaluating the differentials individually (again noting that only $y_\mathrm{i}$ is a function of $E$):

$$\frac{\mathrm{d}}{\mathrm{d}E}(S_{\mathrm{S}xy}) = \frac{\mathrm{d}}{\mathrm{d}E}\left[\sum x_i y_i - \frac{\sum x_i \sum y_i}{k}\right] = \left[\sum\left(x_i\frac{\mathrm{d}y_i}{\mathrm{d}E}\right) - \frac{1}{\mathrm{k}}\left(\sum x_i\right)\left(\sum\frac{\mathrm{d}y_i}{\mathrm{d}E}\right)\right] \tag{D.26}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}E}(S_{\mathrm{S}y}) = \frac{\mathrm{d}}{\mathrm{d}E}\left[\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{k}\right] = \left[2\left(\sum y_i\frac{\mathrm{d}y_i}{\mathrm{d}E}\right) - \frac{2}{k}\left(\sum y_i\right)\left(\sum\frac{\mathrm{d}y_i}{\mathrm{d}E}\right)\right] \tag{D.27}$$

Consider the equation for $y_i$ in two forms:

$y_i = \ln\,(P_i - E)^2$ $\qquad$ $y_i = \ln\,(P_i - E)^2$

$y_i = 2\ln\,(P_i - E)$ $\qquad$ $y_i = 2\ln\,(P_i - E)$

$$e^{\frac{1}{2}y_i} = P_i - E \quad \frac{dy_i}{dE} = \frac{-2}{P_i - E}$$

Substituting the left-hand equation into the right-hand equation yields Equation (D.28):

$$\frac{dy_i}{dE} = -2e^{-\frac{1}{2}y_i} \tag{D.28}$$

Substituting this last expression into the two differential Equations (D.26) and (D.27) yields:

$$\frac{d}{dE}(S_{Sxy}) = \left[ -2\left( \sum x_i e^{-\frac{1}{2}y_i} \right) + \frac{2}{k}\left(\sum x_i\right)\left( \sum e^{-\frac{1}{2}y_i} \right) \right]$$

$$\frac{d}{dE}(S_{Sy}) = \left[ -4\left( \sum y_i e^{-\frac{1}{2}y_i} \right) + \frac{4}{k}\left(\sum y_i\right)\left( \sum e^{-\frac{1}{2}y_i} \right) \right]$$

Now these differentials are substituted back into Equation (D.25), yielding Equation (D.29):

$$\begin{gathered} \cancel{2}\left[S_{Sy}\right]\left[ -\cancel{2}(\sum x_i e^{-\frac{1}{2}y_i}) + \frac{\cancel{2}}{k}(\sum x_i)(\sum e^{-\frac{1}{2}y_i}) \right] = \\ \left[S_{Sxy}\right]\left[ -\cancel{4}(\sum y_i e^{-\frac{1}{2}y_i}) + \frac{\cancel{4}}{k}(\sum y_i)(\sum e^{-\frac{1}{2}y_i}) \right] \end{gathered} \tag{D.29}$$

Substituting back other terms into Equation (D.29) yields Equation (D.30):

$$\begin{gathered} Z = \left[S_{\mathrm{Sy}}\right]\left[ \overline{\mathrm{x}}(\sum e^{-\frac{1}{2}y_i}) - \sum x_i e^{-\frac{1}{2}y_i} \right] \\ -\left[S_{\mathrm{Sxy}}\right]\left[ \overline{\mathrm{y}}(\sum e^{-\frac{1}{2}y_i}) - \sum y_i e^{-\frac{1}{2}y_i} \right] = 0 \end{gathered} \tag{D.30}$$

Equation (D.30) now has to be evaluated with trials on $E$ to find a value that results In $Z = 0$. That value of $E$ is then used in the preceding least squares technique [Equation (D.17) to Equation (D.21)] to determine parameters $C$ and $D$. Those results then yield the highest possible value of $r^2$ to be obtained from the set of data.

It should be noted that division by $S_{Sx}$ and $S_{Sxy}$ occurred in developing Equation (D.25), necessitating that their values not be zero. This is impossible for $S_{Sx}$ if there is more than one data point, and if they are not equal to each other. If $S_{Sxy}$ were zero, then B = 0 and the best fit curve $S$-$N$ would be constant, an obvious incongruity to the facts. Thus, division by these factors is considered valid.

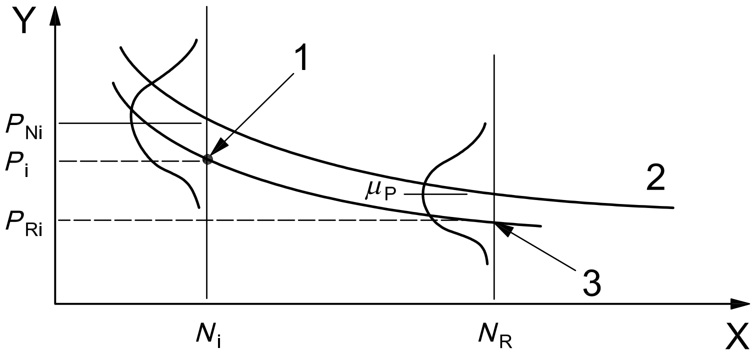
It is possible that more than one solution might occur for Equation (D.30), so those values of $E$ have to be tried for best fits and the results judged for acceptance.

After a value for $E$ has been selected, constants B and A can be determined from the least squares technique; then constants C and D are derived, which define the best fit curve (Equation D.15). Values of $\mu_P$ and $\sigma_P$ at $N_R$ are then determined by translation as explained D.3.3.

### D.3.3 Translation of individual data points

A return to Figure D.5 shows the individual data points clustered about the best fit curve. Each of these data points lies on an individual "probability" curve, proportional to the best fit describing a family of curves for all of the data. Thus, the best fit curve represents a 50 % probability of the data and the other curves, likewise, describe other probabilities.

This is shown in more detail in Figure D.6. A typical data point at $P_{\mathrm{i}}$, $N_{\mathrm{i}}$ is on the individual probability curve that intersects the $N_{\mathrm{R}}$ rated life at the "translated point" of the $P_{\mathrm{Ri}}$ value.




**Key**

X life, expressed in number of cycles

Y fatigue pressure

1 arbitrary data point

2 best fit curve $P = CN^{-\mathrm{D}} + E$

3 translated point

**Figure D.6 — Translation of data points to rated life**

The collection of such translated points at $N_{\mathrm{R}}$ provides a statistical distribution of strength as shown. In fact, a strength distribution exists at each value of life, including the one shown in Figure D.6 passing through the data point. A relationship exists between these distributions, at least in the mid-to-long life regimes, which is that their coefficient of variation is constant. Thus a proportionality can be written between the means, data points and translated points as:

$$\frac{\mu_{\rho}}{P_{\mathrm{Ri}}} = \frac{P_{\mathrm{Ni}}}{P_{\mathrm{i}}}$$

$$P_{\mathrm{Ri}} = P_{\mathrm{i}}\left(\frac{\mu\rho}{P_{\mathrm{Ni}}}\right)$$

$$P_{\mathrm{Ri}} = P_{\mathrm{i}}\left[\frac{CN_{\mathrm{R}}{}^{-\mathrm{D}} + E}{CN_{\mathrm{i}}{}^{-\mathrm{D}} + E}\right] \tag{D.31}$$

Thus the translated value of each data point can be obtained from the parameters of the best fit curve. The collection of values can now be used to determine:

$$\mu_\rho = \frac{1}{k}\sum_{1}^{k} P_{\text{ri}} \tag{D.32}$$

$$\sigma_\rho = \sqrt{\frac{1}{k-1}\left[\sum P_{\text{ri}}^2 - \frac{1}{k}\left(\sum P_{\text{ri}}\right)^2\right.} \tag{D.33}$$

These are the values necessary for determining $k_{\text{o}}$ in Equation (D.1).

## D.4 Data calculation example

### D.4.1 Objective

Clause D.2 and Clause D.3 give a theoretical description of the basis for fatigue pressure verification. However, certain techniques are necessary to actually compute the theories, therefore an example is given in Clause 4. Comparisons to a simplified two-parameter model of the best fit curve are also demonstrated.

### D.4.2 Interlaboratory test

#### D.4.2.1 General

Prior to the development of this part of ISO 10771, several interlaboratory tests were conducted on an aluminium, die cast filter bowl. Testing was conducted at several pressure levels during the course of these tests, and selected data are taken for this example. In addition, data from the two independent series of simultaneous testing at low pressures are also included because they generated high fatigue lives.

The consequence of such a selection of data does not follow the rules of a controlled experiment. Hence, the results cannot be used as representative of any population. Nevertheless, it serves the purpose of an example with some physical association to a real product. The reader is thus cautioned in this regard.

The selected data for this example are given in Clause D.5, and Figure D.7 is an $S$-$N$ curve showing the data point locations.

Note that all suspensions are treated as failures at the point of suspension, an assumption that is admissible for this example problem.

#### D.4.2.2 Evaluation of Equation (D.30)

The initial value to be determined is the parameter $E$. From an examination of Equation (D.15) and Figure D.5, it is seen that this would be the asymptotic limit of $P$ as $N$ becomes very large. Therefore, an expected value would be $E$ greater than the lowest value of a tested pressure (450 psi[4)]).

Since this is a new procedure, and its accuracy is unknown, the sample problem was laboriously calculated in 1 psi increments from $P = 0$ to $P = 840$ psi. The stream of data is not all presented but sample results are extracted and shown in Table D.1 to demonstrate the numerical trend of Equation (D.30).

4) psi = pound-force per square inch: 1 lbf/in$^2$ = 6,894 kPa.

**Table D.1 — Sample calculations**

| *E* | *Z* | *E* | *Z* |
|---|---|---|---|
| 0 | −79,85 | 604 | 55 388 |
| 40 | −89,37 | 605 | 55 503 |
| 80 | −94,36 | 610 | 59 910 |
| 120 | −86,04 | 630 | 146 060 |
| 160 | −47,05 | 660 | 4 626 196 |
| 177 | −4,42 | 670 | 1 079 822 |
| 178 | −1,48 | 671 | 1 023 421 |
| 179 | 1,43 | 672 | 1 001 936 |
| 180 | 4,52 | 673 | 1 013 144 |
| 200 | 85,26 | 674 | 1 060 522 |
| 300 | 2 513,30 | 675 | 1 155 846 |
| 400 | 81 564 | 682 | 2 372 747 |
| 430 | 2 002 083 | 711 | 3 117,882 |
| 452 | 20 826159 | 720 | 741 587 |
| 480 | 124 837 | 750 | 31 512 |
| 490 | 19 689 | 782 | 9 066 |
| 495 | 3 978 | 783 | 9 046 |
| 496 | 2 942 | 784 | 9 040 |
| 497 | 2 557 | 785 | 9 046 |
| 498 | 2 808 | 786 | 9 062 |
| 499 | 3 687 | 800 | 9 835 |
| 500 | 5 190 | 808 | 10 105 |
| 520 | 204 886 | 832 | 2 687 |
| 548 | 22 011 754 | 833 | 1 761 |
| 560 | 1 961 607 | 834 | 742 |
| 600 | 57 758 | 835 | −378 |
| 601 | 56 717 | 836 | −1 610 |
| 602 | 55 983 | 837 | −2 964 |
| 603 | 55 544 | 840 | −7 898 |

NOTE Note that values from Equation (D.30) change signs in two places (see Table D.1); one below the expected value and a second that is quite high in value. However, there are several inflection points that might be of interest so a least squares technique was applied to all of these and best fit curves calculated (see Table D.2). The coefficient of determination, $r^2$, now becomes the judging criterion.

**Table D.2 — Best fit curve calculations**

| $E$ | $r^2$ | $D$ | $C$ |
|---|---|---|---|
| 0 | 0,7816 | 0,1319 | 4 501,7 |
| 80 | 0,7833 | 0,1494 | 5 055,2 |
| 178 | 0,7844 | 0,1790 | 6 349,0 |
| 452 | 0,5867 | 0,7300 | 2 987 675 |
| 497 | 0,7993 | 0,4159 | 57 588,2 |
| 548 | 0,3999 | 0,5444 | 202 854,9 |
| 604 | 0,4321 | 0,2409 | 4 768,1 |
| 660 | 0,0465 | 0,1149 | 683,2 |
| 672 | 0,0391 | 0,0970 | 536,4 |
| 711 | 0,0016 | 0,0240 | 177,4 |
| 784 | 0,0427 | -0,0542 | 89,8 |
| 835 | 0,2060 | -0,1372 | 28,5 |

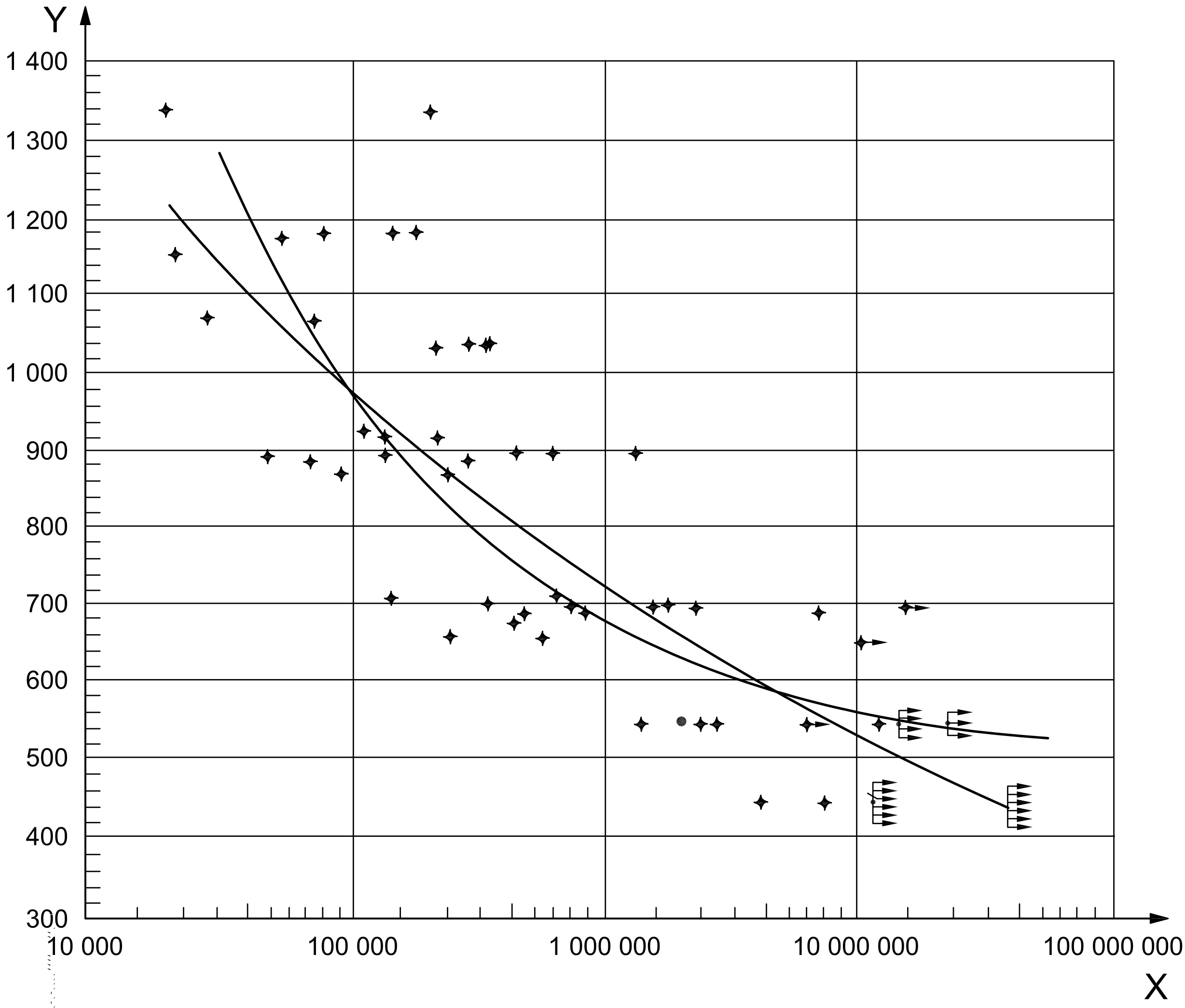
Several observations can now be made:

a) The best value of $r^2$ did not occur where Equation (D.30) passed through zero, but did occur at a low inflection point ($E = 497$). It is also at a level in the region of the expected value.

b) As the value of $E$ increases, the logarithm D becomes negative. This changes the shape of the best fit curve to one that is convex through the data field, with a subsequently improving trend in $r^2$ values.

c) A very good fit can be obtained with $E = 0$, which is only a two-parameter curve and much easier to compute. If this assumption had been made at the beginning, calculation of Equation (D.30) would be unnecessary.

d) The best fit curves for $E = 0$ and E = 497 are shown plotted in Figure D.7.




**Key**

X life, expressed in number of cycles

Y pressure, expressed in psig

1 best fit curve at $E = 0$

2 best fit curve at $E = 497$

**Figure D.7 — *S*-*N* curve**

## D.4.3 Translation of data points

A computer program can also be used to perform the data translation calculations, once a value of $E$ has been determined. The results of mean and standard deviation for a 1 million-cycle rated fatigue life are shown in Table D.3.

**Table D.3 — Translated results at 1 million cycles**

| $E$ | $r^2$ | $\mu_p$ psi | $\sigma_p$ psi | $k_o$ |
|---|---|---|---|---|
| 0 | 0,7816 | 736,1 | 118,1 | 0,1605 |
| 178 | 0,7844 | 725,1 | 116,8 | 0,1611 |
| 497 | 0,7993 | 677,3 | 124,9 | 0,1845 |
| 604 | 0,4321 | 714,0 | 174,9 | 0,2450 |

Some differences now appear among the mean and standard deviations when the value of $E$ varies. This affects the value of $k_o$, the coefficient of variation from Equation (1). The significance of these differences will become apparent when $K_v$ calculations are performed.

### D.4.4 Calculation of $K_v$ factors

A selection has first to be made for the assurance level and, for demonstration purposes, multiple choices will be used. Values of $Z_2$ are given in Table D.4.

**Table D.4 — Values of $Z_2$**

| Let $(1-A_2) =$ | 0,999 | 0,99 | 0,90 | For assurance level |
|---|---|---|---|---|
| And $A_2 =$ | 0,001 | 0,01 | 0,10 | |
| With $Z_2 =$ | 3,090 | 2,327 | 1,282 | |

Values of $Z_2$ are the number of standard deviations in a normal probability distribution for the one-sided tail area $A_2$ and are taken from standard tables.

Next, consider the number of test units for verification testing at quantities of 1 to 5. Using Equation (D.7), calculations are made for the factor $(A_1 + A_4)$ and converted into $Z_4$ values using the same normal probability distribution tables. Results are shown in Table D.5.

As a side point, note that the values of $Z_4$ at $(1 - A_1) = 0{,}90$ for $n = 1$ and $n = 2$, are equal to those at $(1 - A_1) = 0{,}99$ for $n = 2$ and $n = 4$, respectively. The values of $Z_2$ at the 0,99 verification level will always be the same as those at the 0,90 verification level if the number of samples is doubled. This can be proved from Equation (D.7):

$$A_1^{1/n} = A_1 + A_4$$

at $(1 - A_1) = 0{,}90$ and $n = t$;

$$A_1^{1/n} = (0{,}1)^{1/t}$$

at $(1 - A_1) = 0{,}99$ and $n = 2t$;

$$(A_1)^{1/n} = (0{,}01)^{1/2t} = (0{,}1)^{1/t} \quad \text{(D.34)}$$

Thus, the two expressions always give the same value for $(A_1 + A_4)$, with the same corresponding $Z_4$ value.

This side point can be used to reduce the number of combinations necessary for calculating $K_v$ factors. Consider only the 90 % verification level as a basis and double the number of samples for a 99 % verification level.

A table of $K_v$ factors can now be determined from Equation (D.14) using $Z_2$ values from Table D.4, $Z_4$ values from Table D.5 for the 90 % verification level and $k_o$ values from Table D.3. This is shown in Table D.6.

**Table D.5 — Values of "$(A_1 + A_4)/Z_4$" from Equation (D.7)**

| | Number of test units | | | | |
|---|---|---|---|---|---|
| Level | $n = 1$ | $n = 2$ | $n = 3$ | $n = 4$ | $n = 5$ |
| $(1-A_1) = 0{,}90$ | 0,100/1,282 | 0,3162/0,478 | 0,4642/0,090 | 0,5623/-0,157 | 0,6310/-0,334 |
| $(1-A_1) = 0{,}99$ | 0,010/2,327 | 0,1000/1,282 | 0,2154/0,788 | 0,3162/0,478 | 0,3981/0,258 |

**Table D.6 — Values of $K_V$ from Equation (D.14)**

| Verification level = 0,90 | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | Number of test units | | | | |
| Level | $E$ | $k_o$ | 1 | 2 | 3 | 4 | 5 |
| 0,999 | 0 | 0,1605 | 2,392 | 2,136 | 2,013 | 1,934 | 1,878 |
| | 497 | 0,1845 | 2,876 | 2,531 | 2,365 | 2,259 | 2,183 |
| 0,99 | 0 | 0,1605 | 1,925 | 1,719 | 1,619 | 1,556 | 1,511 |
| | 497 | 0,1845 | 2,167 | 1,907 | 1,781 | 1,702 | 1,644 |
| 0,90 | 0 | 0,1605 | 1,518 | 1,356 | 1,277 | 1,227 | 1,192 |
| | 497 | 0,1845 | 1,620 | 1,425 | 1,332 | 1,272 | 1,229 |

Table D.6 used only two examples of the value of $E$ to demonstrate the differences in $K_V$ values. The two values of $E$ correspond with the two curves shown in Figure D.7 and compare the two-parameter model ($E = 0$) with the best fitting three-parameter model ($E = 497$). The resulting differences in the coefficient of variation, $k_o$, influence the $K_V$ factors by approximately 18 % at the 99,9 % assurance level, approximately 10 % at the 99 % assurance level, and approximately 4 % at the 90 % assurance level. There is not much change in these percentages with the number of test units.

## D.4.5 Conclusions

It can be concluded that the two-parameter model, with its considerable simplification of the calculations, is adequate for general purpose applications at the 90 % assurance level, especially in view of all the other probabilities occurring. Use of the three-parameter model is justified at the higher assurance levels, especially in view of the calculation ease offered from a computer.

Comparison of these $K_V$ values to the ones published for aluminium in the 1974 edition of Reference [7] show these to be slightly higher, i.e. about 14 % for the 90 % assurance level, using the two-parameter model (which was employed in Reference [7]). The differences are much greater (about 88 %) if the 99,9 % assurance levels are compared and the three-parameter model is employed.

However, several facts have to be remembered in making this comparison:

— Coupon data are used in Reference [7], whereas a component was used in this example.

— The component example was an aluminium die casting.

— The example data were not from a controlled test.

— The example data treated run outs as failures.

Nevertheless, the value to be gained from this example is the demonstration of the calculation method, an evaluation between the use of two- and three-parameter models, plus some preliminary comparisons to existing $K_V$ factors (but only in a broad sense).

Additional means of evaluation for possible future study include application of the rating selection factor to assist users in determining a cyclic test pressure, $P_{CT}$, rating at several levels of life expectancy and their relationship to one another and also the differences one might expect between coupon data versus component element data.

## D.5 Raw data points for sample problem

The raw data are given in Table D.7.

**Table D.7 — Raw data points for sample problem**

| **Pressure** psi | **Cycles** × $10^3$ | **Pressure** psi | **Cycles** × $10^3$ |
|---|---|---|---|
| 1341 | 19,2 | 700 | 15 000 [a] |
| 1341 | 202,0 | 655 | 10 316 [a] |
| | | 693 | 6 933 |
| 1186 | 179,89 | 700 | 2 240 |
| 1186 | 145,7 | 700 | 1 518 |
| 1185 | 78,0 | 704 | 1 740 |
| 1178 | 53,8 | 662 | 562 |
| 1155 | 21,0 | 694 | 829 |
| | | 700 | 729 |
| 1072 | 28,0 | 715 | 651 |
| 1070 | 72,6 | 680 | 440 × |
| 1042 | 350,4 | 705 | 348 |
| 1042 | 292,0 | 713 | 147 |
| 1040 | 334,0 | 662 | 248 |
| 1035 | 217,0 | 693 | 482 |
| | | | |
| 900 | 1 320,0 | | |
| 902 | 624,0 | | |
| 903 | 446,0 | 550 | 21 900 [b] |
| 893 | 291,0 | 550 | 14 125 [c] |
| 871 | 242,0 | 550 | 11 800 |
| 920 | 221,0 | 550 | 6 666 [a] |
| 897 | 137,0 | 550 | 2 725 |
| 913 | 141,0 | 550 | 2 350 |
| 930 | 113,0 | 550 | 1 380 |
| 873 | 93,0 | | |
| 890 | 71,0 | 450 | 38 000 [d] |
| 895 | 48,0 | 450 | 11 233 [d] |
| | | 450 | 7 255 |
| | | 450 | 4 061,1 |

[a] One suspension sample.
[b] Three suspension samples.
[c] Four suspension samples.
[d] Six suspension samples.

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