TECHNICAL
REPORT

ISO/TR
10688

First edition
2015-11-15

Paper, board and pulps — Basic terms and equations for optical properties

Papiers, cartons et pâtes — Équations et termes de base pour propriétés optiques

Reference number
ISO/TR 10688:2015(E)

International Organization for Standardization

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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The committee responsible for this document is ISO/TC 6, *Paper, board and pulps*.

Introduction

International Standards published by ISO/TC 6 for the determination of optical properties include a lot of definitions and formulae which are used to perform optical measurements and calculations on papers and boards.

It is very valuable for the pulp and paper industry utilizing these International Standards to have access to a single document which gathers together all the various formulae required for the calculation of these optical properties. This Technical Report is based on a SCAN-test document first published in 1994 and revised in 2003.

This Technical Report includes not only formulae but also the values of various constants which appear in these formulae. It is particularly valuable to have the various formulae but also these constants standardized and gathered into a single document when new software programs are being developed either by an instrument manufacturer or in an independent laboratory to ensure that exactly the same expressions are used for such calculations in all the laboratories worldwide when measurements are made in accordance with the ISO/TC 6 standards.

Paper, board and pulps — Basic terms and equations for optical properties

1 Scope

This Technical Report provides a summary of the formulae used for determining the optical properties of pulp, paper and board. This Technical Report is to be used in conjunction with the particular International Standards for the determination of the desired optical properties.

This Technical Report provides the information necessary for those involved in development of software for computation of optical properties in accordance with current ISO standards.

2 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

2.1 Brightness

2.1.1
ISO brightness, R457
diffuse blue reflectance factor, UV level C
intrinsic diffuse radiance (reflectance) factor measured with a reflectometer having the characteristics described in ISO 2469, equipped with a filter or corresponding function having an effective wavelength of 457 nm and a half bandwidth of 44 nm, and adjusted so that the UV content of the irradiation incident upon the test piece corresponds to that of the CIE illuminant C

Note 1 to entry: The filter function is described more fully by the weighting function factors given in ISO 2470-1, Annex A.

[SOURCE: ISO 2470-1:2009, 3.4, modified]

2.1.2
D65 brightness, $R457_{D65}$
diffuse blue reflectance factor, UV level D65
intrinsic diffuse radiance (reflectance) factor measured with a reflectometer having the characteristics described in ISO 2469, equipped with a filter or corresponding function having an effective wavelength of 457 nm and a half-peak bandwidth of 44 nm, and adjusted so that the UV content of the irradiation incident upon the test piece corresponds to that of the CIE standard illuminant D65

Note 1 to entry: The filter function is described more fully by the weighting function factors given in ISO 2470-2, Annex A and Table A.1.

[SOURCE: ISO 2470-2:2008, 3.4, modified]

2.2
CIE colour matching functions
$R(\lambda)$
functions in the CIE 1931 standard colorimetric system describing the tristimulus values X, Y, Z for monochromatic colour stimuli of equal radiance and where the wavelength λ is a variable

2.3
CIE colour matching functions

$\bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$

functions in the CIE 1964 standard colorimetric system describing the tristimulus values X_{10}, Y_{10}, Z_{10} for monochromatic colour stimuli of equal radiance and where the wavelength λ is a variable

2.4
chromaticity coordinates

ratio of each of a set of three tristimulus values to their sum

Note 1 to entry: As the sum of the three chromaticity coordinates is equal to one, two of them are sufficient to define a chromaticity.

Note 2 to entry: In the CIE standard colorimetric systems, the chromaticity coordinates are represented by the symbols x, y, z and x_{10}, y_{10}, z_{10}.

[SOURCE: CIE S 017/E:2011 ILV, 17-145]

2.5
CIELAB colour space

three-dimensional approximately uniform colour space, produced by plotting in rectangular coordinates L^*, a*, b^* quantities defined by the formulae given in 3.7

Note 1 to entry: The quantity L^* is a measure of the lightness of the test piece, where $L^* = 0$ corresponds to black and $L^* = 100$ is defined by the perfect reflecting diffuser. Visually, the quantities a^* and b^* represent respectively the red-green and yellow-blue axes in colour space, such that

— $+a^*$ is a measure of the degree of redness of the test piece,

— $-a^*$ is a measure of the degree of greenness of the test piece,

— $+b^*$ is a measure of the degree of yellowness of the test piece, and

— $-b^*$ is a measure of the degree of blueness of the test piece.

If both a^* and b^* are equal to zero, the test piece is grey.

[SOURCE: ISO 5631-3:2014, 3.6, modified]

2.5.1
CIELAB colour (C/2°)

(L^*, a^*, b^*)

L^*, a^* and b^* values of the sample according to the CIELAB 1976 system, evaluated according to the CIE 1931 (2°) standard colorimetric observer and the CIE illuminant C

2.5.2
CIELAB colour (D65/10°)

(L^*, a^*, b^*)

L^*, a^* and b^* values of the sample according to the CIELAB 1976 system, evaluated according to the CIE 1964 (10°) standard colorimetric observer and the CIE standard illuminant D65

2.5.3
CIELAB colour (D50/2°)

(L^*, a^*, b^*)

L^*, a^* and b^* values of the sample according to the CIELAB 1976 system, evaluated according to the CIE 1931 (2°) standard colorimetric observer and the CIE illuminant D50

2.5.4
CIELAB colour difference

ΔE_{ab}^{*}

distance in the CIELAB colour space between two colour stimuli

2.6
effective residual ink concentration
ERIC number

ratio of the light absorption coefficient of pulp or paper containing ink to the light absorption coefficient of the ink itself, both being determined at a wavelength of 950 nm

Note 1 to entry: The ERIC number is dimensionless.

[SOURCE: ISO 22754:2008, 3.6]

2.7
fluorescence component
$F_{B,S}$ or $F_{W,S}$ for specified CIE illuminant

S

fluorescence component is used as a measure of the extent to which the brightness (F_B) or whiteness (F_W) of the material is affected by emission from added fluorescent whitening agent (FWA) when the light source of the specified CIE illuminant (S = C or D65) emits UV radiation

Note 1 to entry: Relevant standards: ISO 2470-1, ISO 2470-2, ISO 11475, ISO 11476.

Note 2 to entry: Examples of codification:

— $F_{B\ C}$: fluorescence component calculated for C/2° brightness measurement;

— $F_{W,\ D65}$: fluorescence component calculated for D65/10° whiteness measurement.

2.8
fluorescent whitening agent
FWA

fluorescing materials absorbing ultraviolet light and converting it into visible blue light

Note 1 to entry: The fluorescing light adds to light reflected by the pulp in the blue range and compensates for the absorbed share of the light.

Note 2 to entry: The absorption maximum of the usual FWAs is around 360 nm in the UV range and their maximum of emission is in the blue range of the visible light at approximately 440 nm. By this fact, the yellow tint of the bleached pulp is compensated and seen by the human eye as white.

Note 3 to entry: FWAs in paper can only be effective when they are exposed to a light source with an adequate component of UV light. Light emitted by incandescent lamps and some LEDs have practically no UV component, ie radiation with a wavelength of less than 400 nm. FWAs are not sufficiently activated by such light sources. Daylight does contain an adequate UV component although the intensity of the light and the relative contribution of the UV component depends on the time of day, time of the year, geographical standpoint, weather conditions, etc.

Note 4 to entry: This term is often equivalent to *optical brightening agent (OBA)* (2.25).

[SOURCE: Bayer Blankophor — fluorescent whitening agents for the paper industry]

2.9
gloss

<of a surface> mode of appearance by which reflected highlights of objects are perceived as superimposed on the surface due to the directionally selective properties of that surface

[SOURCE: CIE S 017/E:2011 ILV, 17-500]

2.10
illuminant

radiation with a relative spectral power distribution defined over the wavelength range that influences object colour perception

Note 1 to entry: In everyday English, this term is not restricted to this sense, but is also used for any kind of light falling on a body or scene.

[SOURCE: CIE S 017/E:2011 ILV, 17-554]

2.11
light-scattering coefficient

s

fraction of the spectral radiant flux diffusely incident on a differential layer within a material that is reflected when the flux passes through the layer, divided by the thickness of the layer

Note 1 to entry: The flux referred to is a radiant flux across the differential layer.

Note 2 to entry: It is assumed that no reflection occurs at the boundaries of the material.

Note 3 to entry: In a two-flux system, the scattering coefficient is equal to the net transfer of flux from the stronger flux to the weaker flux in a differential layer within a material divided by the product of the thickness of the layer and the difference between the fluxes.

[SOURCE: ISO 9416:2009, 3.7]

2.12
light-scattering coefficient by reflectance factor measurements

s_{V}

<Kubelka-Munk method> coefficient calculated by application of the Kubelka-Munk equations to luminance factor data weighted with respect to the CIE illuminant C, obtained in an instrument having a specified geometry and calibrated in a specified manner, on the basis of grammage

Note 1 to entry: s_v is expressed in square metres per kilogram (m^2/kg).

2.13
light scattering coefficient at 950 nm by reflectance factor measurements

s_{950}

<Kubelka-Munk method> coefficient calculated by application of the Kubelka-Munk equations to reflectance factor data obtained at a wavelength of 950 nm in an instrument having a specified geometry and calibrated in a specified manner and taking into consideration the grammage

Note 1 to entry: Units: m^2/kg.

Note 2 to entry: The relevant equations are given in ISO 22754, Clause 9.

[SOURCE: ISO 22754:2008, 3.4, modified]

2.14
light-absorption coefficient

k

fraction of the spectral radiant flux diffusely incident on a differential layer within a material that is absorbed when the flux passes through the layer, divided by the thickness of the layer

Note 1 to entry: The flux referred to is a radiant flux across the differential layer.

[SOURCE: ISO 9416:2009, 3.6]

2.15
light-absorption coefficient by reflectance factor measurements

k_v

<Kubelka-Munk method> coefficient calculated by application of the Kubelka-Munk equations to luminance factor data weighted with respect to the CIE illuminant C, obtained in an instrument having a specified geometry and calibrated in a specified manner, on the basis of grammage

Note 1 to entry: k_v is expressed in square metres per kilogram (m^2/kg).

Note 2 to entry: *light-scattering coefficient* (2.11) and *light-absorption coefficient* (2.14) are strictly applicable to monochromatic light but, for the purpose of this International Standard, the relevant light absorption and scattering coefficients apply to broad-band radiation. In research work, s_v and k_v can and should be determined at the relevant wavelength for the study concerned. As general descriptions of a given paper, they are defined here in relation to the $V(\lambda)$ function and the CIE illuminant C.

[SOURCE: ISO 9416:2009, 3.9, modified]

2.16
light absorption coefficient at 950 nm by reflectance factor measurements

k_{950}

<Kubelka-Munk method> coefficient calculated by application of the Kubelka-Munk equations to reflectance factor data obtained at a wavelength of 950 nm in an instrument having a geometry according to ISO 2469 and having been calibrated as specified in ISO 2470-1 and ISO 11475 and taking into consideration the grammage

Note 1 to entry: Units: m^2/kg.

Note 2 to entry: The relevant equations are given in ISO 22754, Clause 9.

[SOURCE: ISO 22754:2008, 3.5]

2.17
luminance factor (C), R_y

luminous reflectance factor, Y(C/2°)-value

reflectance factor defined with reference to the spectral luminous efficiency function $V(\lambda)$ and the CIE illuminant C

Note 1 to entry: The visual efficiency function describes the sensitivity of the eye to light so that the luminance factor corresponds to the attribute of visual perception of the reflecting surface.

Note 2 to entry: For computational purposes, the $V(\lambda)$ function is identical to the CIE 1931 colour matching function $\bar{y}(\lambda)$.

Note 3 to entry: The luminance factor (C) is also known as the Y(C/2°)-value. In previous editions of ISO 9416 and ISO 2471, it was referred to as the luminous reflectance factor.

Note 4 to entry: The CIE term “luminance factor” is a more general term since it does not specify the illuminant or observer condition.

2.18
single-sheet luminance factor (C)

$R_{y,0}$

luminance factor (C) of a single sheet of paper with a black cavity as backing

[SOURCE: ISO 9416:2009, 3.3, modified]

2.19
intrinsic luminance factor (C)
$R_{y,\infty}$

luminance factor (C) of a layer or pad of material thick enough to be opaque, i.e. such that increasing the thickness of the pad by doubling the number of sheets results in no change in the measured reflectance factor

[SOURCE: ISO 9416:2009, 3.4, modified]

2.20
spectral luminous efficiency
$V(\lambda)$ **for photopic vision**

ratio of the radiant flux at wavelength λ_{m} to that at wavelength λ, such that both produce equally intense luminous sensations under specified photometric conditions, and λ_{m} is chosen so that the maximum value of this ratio is equal to one

Note 1 to entry: Function describing the sensitivity to light of the human eye at different wavelengths.

Note 2 to entry: $\lambda_{\mathrm{m}} = 555\,\mathrm{nm}$.

Note 3 to entry: For computational purposes, the $V(\lambda)$ function is identical with the $\bar{y}(\lambda)$ function for the CIE 1931 (2°) standard observer.

[SOURCE: CIE S 017/E:2011, 17-1222]

2.21
metameric colour stimuli

spectrally different colour stimuli that have the same tristimulus values in a specified colorimetric system

Note 1 to entry: Equivalent term: “metamers”.

Note 2 to entry: The corresponding property is called “metamerism”.

[SOURCE: CIE S 017/E:2011: ILV, 17-768 and 17-769]

2.22
metamerism index

degree of colour mismatch, calculated in the form of a colour difference, caused by substituting a test illuminant (observer) of different relative spectral composition (responsivity) for the reference illuminant (observer)

Note 1 to entry: The colour difference is evaluated using a CIE colour difference formula and it should be clearly stated which formula has been used.

[SOURCE: CIE S 017/E: 2011: ILV, 17-770]

2.23
diffuse reflectance factor
diffuse radiance factor

for the purpose of this Technical Report, only diffuse reflectance factor is considered.

Note 1 to entry: For non-fluorescent materials, the diffuse radiance factor, *β*, is simply the diffuse reflectance factor, *R*.

2.23.1

diffuse reflectance factor

R

ratio of the radiation (reflectance) reflected and emitted from a body to that reflected from the perfect reflecting diffuser under the same conditions of diffuse illumination and normal detection

Note 1 to entry: The ratio is often expressed as a percentage.

Note 2 to entry: In the context of ISO 2469 and related standards, the irradiation is diffuse and the direction of detection is perpendicular to the surface of the specimen (d:0 geometry).

A gloss trap ensures that there is negligible irradiation from directions close to the direction of detection.

Note 3 to entry: This term is often expressed simply as reflectance factor.

[SOURCE: ISO 2469:2014, 3.5, modified]

2.23.2

intrinsic diffuse reflectance factor

R_∞

diffuse reflectance factor of a layer or pad of material thick enough to be opaque, i.e. such that increasing the thickness of the pad by doubling the number of sheets results in no change in the measured diffuse reflectance factor

Note 1 to entry: The diffuse reflectance factor of a single non-opaque sheet is dependent on the background and is not a material property.

[SOURCE: ISO 2469:2014, 3.6, modified]

2.23.3

spectral diffuse reflectance factor

$R(\lambda)$

$R(\lambda_\mathrm{i})$

reflectance factor expressed as a function of wavelength

Note 1 to entry: In this Technical Report, a wavelength-specific reflectance factor is indicated as $R(\lambda)$. In computational contexts, the reflectance factor variable is denoted $R(\lambda_\mathrm{i})$.

2.24

opacity

ratio of the single-sheet luminous reflectance factor (C), $R_{\mathrm{y},0}$, to the intrinsic luminous reflectance factor, $R_{\mathrm{y},\infty}$, of the same sample

Note 1 to entry: The single-sheet reflectance factor is defined as the reflectance factor of a single sheet of paper with a black cavity as backing.

2.25

optical brightening agent

OBA

see *fluorescent whitening agent* (2.8)

Note 1 to entry: This term is often equivalent to fluorescent whitening agent (FWA).

2.26

source

object that produces light or other radiant flux

[SOURCE: CIE S 017/E:2011, 17.1202]

2.27
CIE tint, T_W and $T_{W,10}$

green/red tint
measure of the deviation from CIE whiteness of the test material towards the green or red region

Note 1 to entry: The deviation is expressed as CIE tint units.

Note 2 to entry: A positive value of T_W or $T_{W,10}$ indicates a greenish tint and a negative value indicates a reddish tint.

Note 3 to entry: T_W refers to C/2° conditions and $T_{W,10}$ refers to D65/10° conditions.

2.28
transmittance

τ
ratio of the transmitted radiant or luminous flux to the incident flux under given conditions (CIE Publ 17.4 - 845-04-59)

[SOURCE: ISO 22891:2013, 3.6]

2.29
regular transmittance

τ_r
ratio of the regularly transmitted part of the (whole) transmitted flux to the incident flux (CIE S 0117/E:2011 ILV, 17-1079)

[SOURCE: ISO 22891:2013, 3.7]

2.30
diffuse transmittance

τ_d
ratio of the diffusely transmitted part of the (whole) transmitted flux to the incident flux (CIE S 017/E: 2011 ILV, 17-308)

Note 1 to entry: $\tau = \tau_r + \tau_d$

[SOURCE: ISO 22891:2013, 3.8]

2.31
transmittance from luminous reflectance factor measurement

T_y
transmittance obtained by measurement of reflectance factors and subsequent calculation as defined in ISO 22891

2.32
tristimulus values

X, Y, Z
X_{10}, Y_{10}, Z_{10}
amounts of the three reference colour stimuli, in a given chromatic system, required to match the stimulus considered

Note 1 to entry: Depending on the observer conditions, the tristimulus values are represented differently.

Note 2 to entry: In ISO 5631-1, the CIE 1931 (2°) standard observer and the CIE illuminant C are used to define the trichromatic system.

Note 3 to entry: In ISO 5631-2, the CIE 1964 (10°) standard observer and the CIE standard illuminant D65 are used to define the trichromatic system.

Note 4 to entry: In ISO 5631-3, the CIE 1931 (2°) standard observer and the CIE illuminant D50 are used to define the trichromatic system.

Note 5 to entry: The tristimulus values (X, Y, Z) without subscript are used for the CIE 1931 (2°) standard observer. The subscript 10 is used for the CIE 1964 (10°) standard observer

[SOURCE: ISO 5631-3:2014, 3.5, modified]

2.33
dominant wavelength

λ_d

<of a colour stimulus> wavelength of the monochromatic stimulus that, when additively mixed in suitable proportions with the specified achromatic stimulus, matches the colour stimulus considered in the CIE 1931 x, y chromaticity diagram

Note 1 to entry: Unit: nm.

Note 2 to entry: In the case of purple stimuli, the dominant wavelength is replaced by the complementary wavelength

[SOURCE: eILV of the Jan 2014 CIE website, 17.345]

2.34
CIE-whiteness

W

measure of CIE whiteness derived from the CIE tristimulus values determined under the conditions specified in relevant International Standard (ISO 11475, ISO 11476)

Note 1 to entry: The CIE whiteness is expressed in CIE whiteness units.

Note 2 to entry: The observer and illuminant should be specified as a suffix, e.g. $W_{\mathrm{D65}/10}$.

Note 3 to entry: If the sample is fluorescent, the UV content in the radiation falling on the sample should correspond to UV(D65) or UV(C) conditions, respectively, to meet the requirements of ISO 11475 or ISO 11476.

Note 4 to entry: For paper and board and for outdoor daylight conditions, $W_{\mathrm{D65}/10}$, i.e. D65/10° is used, and for indoor illumination conditions, $W_{\mathrm{C}/2}$, i.e. C/2°, is used.

2.35
yellowness index

YI

measure of the extent to which a nearly achromatic stimulus deviates in yellowness from an achromatic stimulus having the same Y-value

3 Calculations

3.1 Colour appearance, tristimulus values

It is important to note that colour is a feature of human perception and cannot therefore be measured. Colour can, however, be described in terms of different attributes (lightness, redness, saturation, etc.). Psychometric scales and systems of alphanumerical notations have been developed purely on the basis of perceptual judgements to provide a means of locating colours in a three-dimensional colour space.

Physically, it is only possible to measure the intensity of the radiation which is the stimulus giving rise to the perception of colour. For pulp, paper and board, it is assumed that the sample is illuminated diffusely and that the viewing direction is normal to the surface of the sample. The stimulus, i.e. the spectral radiation flux $\Phi_{\mathrm{refl}}(\lambda)$, reflected from the sample in the normal direction is proportional to the product of the incident spectral radiation flux $\Phi_{\mathrm{in}}(\lambda)$ falling upon the surface and the spectral reflectance factor $R(\lambda)$ of the surface.

$$\Phi_{\mathrm{refl}}(\lambda) = r \cdot R(\lambda) \cdot \Phi_{\mathrm{in}}(\lambda) \tag{1}$$

where

r is the ratio of the solid angle of the detection cone to the solid angle of the hemisphere (2π steradians).

In order to establish some connection with the visual experience, it is essential to realize that any colour percept may be produced by the mixing of three reference colour stimuli in a generalized colour matching experiment. A requirement is that the colour of none of the reference stimuli can be produced by mixing the other two. The values to which the fluxes of the reference stimuli have to be adjusted to make a perfect colour match of a test stimulus are described by the tristimulus values for the test stimulus. In the standard colorimetric system of the Commission Internationale d'Eclairage (CIE), the test stimuli are represented by the symbols X, Y and Z or X_{10}, Y_{10} and Z_{10}. These are related to colour matching experiments where the fields of view were 2° and 10°, respectively. The tristimulus values for a specific test stimulus are evaluated for the CIE 1931 standard colorimetric observer for the case of the 2° field of view and the tristimulus values are denoted X, Y, Z . Correspondingly, the CIE 1964 standard colorimetric observer is used in the case of the 10° field of view and the tristimulus values are denoted X_{10}, Y_{10}, Z_{10}.

The tristimulus values X, Y, Z for an object, which is reflective and may also be luminescent and which is illuminated by a chosen illuminant, $S_{\lambda}(\lambda)$, are defined as:

$$\begin{aligned} X &= k \cdot \int E_{\lambda}(\lambda).\bar{x}(\lambda) \cdot d\lambda \\ Y &= k \cdot \int E_{\lambda}(\lambda).\bar{y}(\lambda) \cdot d\lambda \\ Z &= k \cdot \int E_{\lambda}(\lambda).\bar{z}(\lambda) \cdot d\lambda \end{aligned} \tag{2}$$

where

$$k = \frac{100}{\int S_{\lambda}(\lambda).\bar{y}(\lambda) \cdot d\lambda} \tag{3}$$

$$E_{\lambda}\left(\lambda\right) = S_{\lambda}\left(\lambda\right) \cdot R(\lambda) \tag{4}$$

and

$E_{\lambda}\left(\lambda\right)$	is the spectral concentration (distribution) of the colour stimulus due to the illuminated object;
$R(\lambda)$	is the spectral reflectance factor (or spectral radiance factor) of the object (on a scale from zero to unity for the perfect reflecting diffuser);
$S_{\lambda}(\lambda)$	is the relative spectral power distribution of the chosen illuminant;
$\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$	are the three colour matching functions of the CIE 1931 standard colorimetric observer.

Note that the factor 100 in Formula (3) is not a conversion to percentage units. It defines the scale for the tristimulus values.

In general, if the material fluoresces, the relative spectral power distribution of the light source of the instrument and $S(\lambda)$ need to be a close match. However, in special cases, reliable measurements can be still be made if this match is not perfect, although this is only approximately true. One such case is when the only fluorescent component is a fluorescent whitening agent (FWA). Here, the most important requirement is that the light source matches the desired illuminant with regard to the relative power available in the excitation band (mainly UV radiation) compared with the emission band (blue end of the visible spectrum) of the FWA.

Since $S(\lambda)$ and the colour matching functions $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ are defined for only a limited number of wavelengths λ_i, the tristimulus values are obtained by summation instead of integration. Equidistant λ_i -values are assumed. The wavelength pitch (measurement interval) is denoted $\Delta\lambda$. For the CIE 1931 standard colorimetric observer, Formula (5) modifies to:

$$\begin{aligned} X &= k \cdot \sum_i R(\lambda_i).S_\lambda(\lambda_i).\bar{x}(\lambda_i) \cdot \Delta\lambda \\ Y &= k \cdot \sum_i R(\lambda_i).S_\lambda(\lambda_i).\bar{y}(\lambda_i) \cdot \Delta\lambda \\ Z &= k \cdot \sum_i R(\lambda_i).S_\lambda(\lambda_i).\bar{z}(\lambda_i) \cdot \Delta\lambda \end{aligned} \tag{5}$$

where

$$k = \frac{100}{\sum_i S(\lambda_i).\bar{y}(\lambda_i) \cdot \Delta\lambda} \tag{6}$$

In the corresponding calculation of X_{10}, Y_{10}, Z_{10} for the CIE 1964 standard colorimetric observer (10° observer), the colour matching functions $\bar{x}(\lambda_i), \bar{y}(\lambda_i)$ and $\bar{z}(\lambda_i)$ in Formulae (5) to (6) are replaced by $\bar{x}_{10}(\lambda_i), \bar{y}_{10}(\lambda_i)$ and $\bar{z}_{10}(\lambda_i)$.

The bandwidth of the spectrophotometer is the wavelength interval surrounding the nominal wavelength of the monochromator within which the sensitivity of the instrument exceeds half the sensitivity at the nominal wavelength. In the ISO standards, the bandwidth is considered to be equal to the wavelength pitch $\Delta\lambda$ and the sensitivity is considered to be a triangular function of wavelength.

For $\Delta\lambda$ -values up to 5 nm, the CIE tables for $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ and $\bar{x}_{10}(\lambda_i), \bar{y}_{10}(\lambda_i), \bar{z}_{10}(\lambda_i)$ given in S014-1 should be used, together with the tables for the appropriate illuminant given in S014-2 or CIE publ. 15: 2004. The tables are provided for $\Delta\lambda$ -values of 1 nm and 5 nm. For other $\Delta\lambda$ -values below 5 nm, a proper table of interpolated values has to be calculated.

For $\Delta\lambda$ -values exceeding 5 nm, the bandwidth of the spectrophotometer affects the shape of the $R(\lambda)$ curve such that steep slopes cannot be rendered with sufficient precision. To mitigate this influence to some extent, American Society for Testing and Materials (ASTM) has prepared tables of special tristimulus weighting factors. To calculate the tristimulus values X, Y, Z and X_{10}, Y_{10}, Z_{10} for $\Delta\lambda$ -values of 10 nm and 20 nm, Formulae (7) and (8) should be used:

$$\begin{aligned} X_2 &= \sum_i R(\lambda_i).W_X(\lambda_i) \\ Y_2 &= \sum_i R(\lambda_i).W_Y(\lambda_i) \\ Z_2 &= \sum_i R(\lambda_i).W_Z(\lambda_i) \end{aligned} \qquad (7)$$

and

$$\begin{aligned} X_{10} &= \sum_i R(\lambda_i).W_{10,X}(\lambda_i) \\ Y_{10} &= \sum_i R(\lambda_i).W_{10,Y}(\lambda_i) \\ Z_{10} &= \sum_i R(\lambda_i).W_{10,Z}(\lambda_i) \end{aligned} \qquad (8)$$

Where W_X, W_Y, W_Z and $W_{10,X}$, $W_{10,Y}$, $W_{10,Z}$ are called tristimulus weighting factors and the values tabled in ASTM E308:08 for different combinations of standard observers and standard and recommended illuminants should be used. Note that $R(\lambda_i)$ is on a scale from zero to unity for the perfect reflecting diffuser.

As an alternative to the determination from spectrophotometric data, the tristimulus values can be determined from measurements with an abridged filter instrument such as the Zeiss Elrepho, where the combination of the spectral properties of the lamps, the optics, the filters and the detector are chosen by the manufacturer to give reflectance factors which are weighted averages over three broad wavelength bands. The measured values are designated R_x, R_y and R_z and are on a scale from zero to unity for the perfect reflecting diffuser, although they are usually reported as percentages. Thereafter, X, Y and Z are calculated as follows:

C-illuminant, 2° observer

$$\begin{aligned} X &= 78,321R_x + 19,753R_Z \\ Y &= 100R_Y \\ Z &= 118,232R_Z \end{aligned} \qquad (9)$$

D65-illuminant, 10° observer

$$\begin{aligned} X_{10} &= 76,841R_x + 17,970R_Z \\ Y_{10} &= 100R_Y \\ Z_{10} &= 107,304R_Z \end{aligned} \qquad (10)$$

The inverse relationships are:

C-illuminant, 2° observer

$$\begin{aligned} R_X &= \frac{X - 0,16707Z}{78,321} \\ R_Y &= \frac{Y}{100} \\ R_Z &= \frac{Z}{118,232} \end{aligned} \qquad (11)$$

D65-illuminant, 10° observer

$$
\begin{aligned}
R_X &= \frac{X_{10} - 0{,}16747 Z_{10}}{76{,}841} \\
R_Y &= \frac{Y_{10}}{100} \\
R_Z &= \frac{Z_{10}}{107{,}304}
\end{aligned}
\tag{12}
$$

3.2 Reflectance factor, *Y*-value, opacity, transmittance

3.2.1 Reflectance factor, *R*

In ISO 2469 and all other standards or methods to which this Technical Report refers, it is specified that the illumination shall be diffuse and that the direction of detection shall be perpendicular to the surface of the specimen (d:0 geometry). There is no or considerably reduced irradiation from directions close to the direction of detection, since a so-called gloss-trap is also specified.

It is a primary requirement of all optical measurements relating to pulp, paper and board that the reflectance factor measurements be traceable to the perfect reflecting diffuser, which is the absolute standard. ISO 2469 specifies that a calibration procedure shall be adopted in which an IR3 reference standard supplied by an Authorized Laboratory (according to ISO 4094) is used. The values assigned to this Technical Report shall be traceable to the perfect reflecting diffuser via an IR2 reference standard supplied by a Standardizing Laboratory.

Although reflectance factors are usually reported as percentages, it should be noted that a reflectance factor of, e.g. 80 % means that $R = 0{,}80$.

3.2.2 Luminous reflectance factor, R_{y}

This term is analogous to the CIE term “luminance factor”, except that the CIE term is more general and does not refer to a specific illuminant condition.

The CIE definition refers to the $V(\lambda)$ function which corresponds to the spectral sensitivity to light of the normal eye. This function is mathematically the same as the $\bar{y}(\lambda)$ function for the CIE 1931 (2°) observer. To provide a value which takes into account the appearance of the material in daylight, the reflectance is thus weighted with respect to the energy distribution of the C-illuminant and the spectral sensitivity to light of the normal eye.

$$
R_{\mathrm{y}} = k \cdot \sum_i R(\lambda_{\mathrm{i}}).S_{\mathrm{C}}(\lambda_{\mathrm{i}}).V(\lambda_{\mathrm{i}}) \cdot \Delta\lambda \tag{13}
$$

where

$$
k = \frac{1}{\sum_i S_{\mathrm{C}}(\lambda_{\mathrm{i}}).V(\lambda_{\mathrm{i}}) \cdot \Delta\lambda} \tag{14}
$$

$S_{\mathrm{C}}(\lambda_{\mathrm{i}})$ is the relative spectral power distribution of the C-illuminant;

$V(\lambda_{\mathrm{i}})$ is the visual efficiency function which is identical to the CIE 1931 colour matching function, $\bar{y}(\lambda)$.

When the measurement is made with a single sheet of the material over a black cavity, the luminous reflectance factor is designated $R_{\mathrm{y},0}$. The intrinsic luminous reflectance factor is designated $R_{\mathrm{y},\infty}$.

3.2.3 *Y*-value (C/2°)

This should be interpreted as the tristimulus value *Y* of a layer of material of such a thickness that there is no change in the value when the thickness is doubled. CIE illuminant C and the CIE 1931 (2°) standard colorimetric observer are used in the determination of the *Y*-value.

The *Y*-value is related to the intrinsic luminous reflectance factor as $Y = 100\ R_{\mathrm{y},\infty}$.

3.2.4 Opacity

The opacity is calculated as:

$$Opacity = \frac{R_{\mathrm{y},0}}{R_{\mathrm{y},\infty}} \tag{15}$$

However, opacity is usually expressed as a percentage, i.e.:

$$Opacity = \frac{R_{\mathrm{y},0}}{R_{\mathrm{y},\infty}} \times 100\,\% \tag{16}$$

Note that the use of $R_{\mathrm{y},0}$ and $R_{\mathrm{y},\infty}$ means that the two luminous reflectance factors involved are weighted with respect to the C-illuminant and the $\bar{y}(\lambda)$ function for the CIE 1931 (2°) observer.

3.2.5 Transmittance from luminous reflectance factor measurements

The (luminous) transmittance is calculated from the luminous reflectance (luminance) factors of the test piece over a white and back backing and from the luminous reflectance factor of the white backing. If it is apparent from the context that luminous values are required, for clarity, the subscript *y* should be omitted. Thus, the transmittance is calculated from R_{y}, $R_{\mathrm{y},0}$ and $R_{\mathrm{y,w}}$ according to Formula (17):

$$T = \sqrt{\left(\frac{1}{R_{\mathrm{y,w}}} - R_{\mathrm{y},0}\right) \cdot \left(R_{\mathrm{y}} - R_{\mathrm{y},0}\right)} \tag{17}$$

where

R_{y} is the luminance factor of a single sheet over a white backing with the luminance factor $R_{\mathrm{y,w}}$.

All luminance factors are on a scale from zero to unity for the perfect reflecting diffuser.

Another formula using $R_{\mathrm{y},0}$ and $R_{\mathrm{y},\infty}$ may be encountered but it is not conforming to the ISO 22891 standard:

$$T = \sqrt{\left(\frac{1}{R_{\mathrm{y},\infty}} - R_{\mathrm{y},0}\right) \cdot \left(R_{\mathrm{y},\infty} - R_{\mathrm{y},0}\right)} \tag{18}$$

3.3 Brightness

Intrinsic diffuse radiance (reflectance) factor measured with a reflectometer having the characteristics described in ISO 2469, equipped with a filter or corresponding function having an effective wavelength of 457 nm and a width at half height of 44 nm and adjusted so that the UV-content of the illumination incident upon the test piece corresponds to that of the CIE illuminant C (ISO 2470-1) or CIE standard illuminant D65 (ISO 2470-2).

Brightness should be calculated via Formula (19) from reflectance factor data according to a special set of weighting factors, $F(\lambda_\mathrm{i})$ given in Table 1.

$$Brightness = \frac{\sum_{i} R\left(\lambda_\mathrm{i}\right) \cdot F\left(\lambda_\mathrm{i}\right)}{\sum_{i} F\left(\lambda_\mathrm{i}\right)} \qquad (19)$$

If the sample is non-fluorescent, ISO Brightness and D65 Brightness are equal.

Table 1 — Weighting factors for the calculation of brightness

Wavelength λ_i nm	$F(\lambda_\mathrm{i})$ for $\Delta\lambda$ = 10 nm arbitrary units	$F(\lambda_\mathrm{i})$ for $\Delta\lambda$ = 20 nm arbitrary units
380	0,0	0,0
390	0,0	
400	1,0	1,0
410	6,7	
420	18,2	18,2
430	34,5	
440	57,6	57,6
450	82,5	
460	100,0	100,0
470	88,7	
480	53,1	53,1
490	20,3	
500	5,6	5,6
510	0,3	
520	0,0	0,0
Sum	468,5	235,5

NOTE In ISO 2470-1 and ISO 2470-2, values of $F(\lambda_\mathrm{i})$ for $\Delta\lambda$ = 5 nm are also given.

3.4 Light-scattering and light-absorption coefficients

3.4.1 Conditions for the Kubelka-Munk theory

The Kubelka-Munk theory is based on a number of assumptions. These may be summarized as follows.[25]

a) The medium (sample) is modeled as a plane layer of finite thickness, but infinite width and length (infinite sheet approximation), so there are no boundary effects.

b) A perfectly diffuse and homogeneous illumination incident on the surface.

c) The only interactions of light with the medium are scattering and absorption; polarization and spontaneous emission (fluorescence) are ignored.

d) The medium is considered isotropic and homogeneous, containing optical heterogeneities (small compared to the thickness of the layer) able to disperse light.

e) No external or internal surface reflections occur.

f) Parameters and are constant whatever the thickness of layer is.

It is recommended that these assumptions are not violated in such a way that the results are significantly affected. One example is that the irradiation need not be monochromatic provided that the sample is non-fluorescent. If the material contains a fluorescent whitening agent (FWA), it is a requirement of ISO 9416 that a 420 nm cut-off filter be placed in the light beam to ensure that the fluorescence effect is eliminated.

Two other recommendations are: (a) the difference between the reflectance factors of the sample measured under different conditions, which are used in the formulae below, should be sufficiently large; (b) the reflectance factor of the sample should be sufficiently high.

3.4.2 Calculations of *s* and *k*

In the Kubelka-Munk theory, *s* and *k* are calculated from reflectance data. However, in common practice, the reflectance factor is used instead of the reflectance. This is often an acceptable approximation.

The spectral light-scattering coefficient is calculated as:[26]

$$s(\lambda) = \frac{R_\infty(\lambda)}{w\left[1 - R_\infty^2(\lambda)\right]} \ln \frac{R_\infty(\lambda)\left[1 - R_0(\lambda) \cdot R_\infty(\lambda)\right]}{R_\infty(\lambda) - R_0(\lambda)} \quad (20)$$

In Formula (20), all reflectance factors are on a scale from zero to unity for the perfect reflecting diffuser.

where

$s(\lambda)$ is the light-scattering coefficient, in square metre per kilogram;

W is the grammage of the conditioned sheet, in kilogram per square metre (Caution: Grammage is usually reported in gram per square metre);

$R_0(\lambda)$ is the spectral reflectance factor (black backing);

$R_\infty(\lambda)$ is the intrinsic spectral reflectance factor.

The spectral light-absorption coefficient is calculated as:

$$k(\lambda) = \frac{s(\lambda) \cdot \left[1 - R_\infty(\lambda)\right]^2}{2R_\infty(\lambda)} \quad (21)$$

where

$k(\lambda)$ is the light-absorption coefficient, in square metre per kilogram.

The light-scattering and light-absorption coefficients are, as indicated in these formulae, dependent on the wavelength. When non-spectral values are desired, the calculations are usually based on the luminance factors $R_{\mathrm{V},0}$ and $R_{\mathrm{V},\infty}$, which means that the data are averaged over the visible spectrum using a product of a daylight irradiation spectrum (the C-illuminant) and the colour matching function $\bar{y}(\lambda)$ for the CIE 1931 (2°) observer as weighting function. The luminance factor-based light-scattering and light-absorption coefficients, s_V and k_V, respectively, are calculated as:

$$s_\mathrm{y} = \frac{R_{\mathrm{y},\infty}}{w\left[1 - R_{\mathrm{y},\infty}^2\right]} \ln \frac{R_{\mathrm{y},\infty}\left[1 - R_{\mathrm{y},0} \cdot R_{\mathrm{y},\infty}\right]}{R_{\mathrm{y},\infty} - R_{\mathrm{y},0}} \quad (22)$$

$$k_{\mathrm{y}} = \frac{s_{\mathrm{y}} \cdot \left[1 - R_{\mathrm{y},\infty}\right]^2}{2R_{\mathrm{y},\infty}} \tag{23}$$

If only a single sheet of paper is available, no value for R_∞ is available in Formulae (22) to (25). However, a value for R_∞ can be determined if measurements are made over two different backings, preferably black and white. R_∞ is then calculated according to Formula (24):

$$R_\infty = a - \left(a^2 - 1\right)^{1/2} \tag{24}$$

where

$$a = \frac{1}{2} \cdot \frac{\left(R_{\mathrm{gw}} - R_{\mathrm{gs}}\right) \cdot \left(1 + R_{\mathrm{w}} \cdot R_{\mathrm{s}}\right) - \left(R_{\mathrm{w}} - R_{\mathrm{s}}\right) \cdot \left(1 + R_{\mathrm{gw}} \cdot R_{\mathrm{gs}}\right)}{R_{\mathrm{s}} \cdot R_{\mathrm{gw}} - R_{\mathrm{w}} \cdot R_{\mathrm{gs}}} \tag{25}$$

and

R_{gs}	is the reflectance factor of the black backing;
R_{gw}	is the reflectance factor of the white backing;
R_{s}	is the reflectance factor of a single sheet against the backing R_{gs};
R_{w}	is the reflectance factor of a single sheet against the backing R_{gw};
R_{gs}, R_{gw}, R_{s} and R_{w}	may be spectral reflectance factors or luminance factors.

In Formulae (22) to (25), the difference between the reflectance factors measured under different conditions should be sufficiently large, depending on the accuracy required and on the noise level of the instrument. A satisfactory accuracy is usually obtained when the opacity of the sheet does not exceed 95 %. To lower the opacity of a dark pulp, the sheets should be made to a lower grammage than is usual. Very thin sheets are, however, sometimes too inhomogeneous, and in addition, the requirement of the Kubelka-Munk theory that there shall be multiple reflections within the material may not be met. For mechanical pulp, the lower limit of grammage is about 30 g/m^2 and for bleached chemical pulps, the lower limit is about 50g/m^2.

3.5 Chromaticity coordinates

The chromaticity coordinates *x*, *y* and *z* for the CIE 1931 Standard Observer are defined as:

$$\begin{aligned} x &= X/(X + Y + Z) \\ y &= Y/(X + Y + Z) \\ z &= Z/(X + Y + Z) \end{aligned} \tag{26}$$

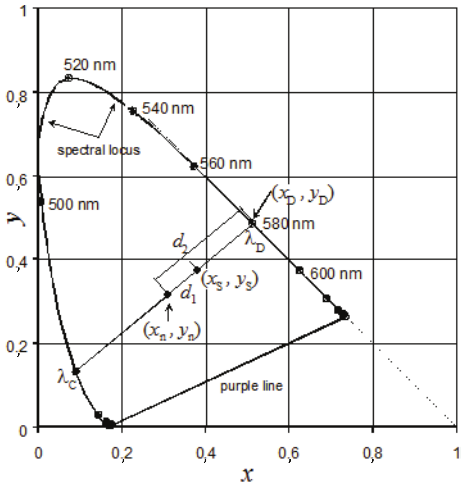
Correspondingly the chromaticity coordinates x_{10}, y_{10} and z_{10} for the CIE 1964 Standard Observer are defined as:

$$\begin{aligned} x_{10} &= X_{10}/(X_{10} + Y_{10} + Z_{10}) \\ y_{10} &= Y_{10}/(X_{10} + Y_{10} + Z_{10}) \\ z_{10} &= Z_{10}/(X_{10} + Y_{10} + Z_{10}) \end{aligned} \tag{27}$$

3.6 Dominant wavelength, λ_D

In Figure 1, the loci of the sample stimulus and an achromatic stimulus have been plotted in the CIE *xy* chromaticity diagram. Since additive colour mixings are represented as straight lines in this diagram, the definition means that the locus of the dominant wavelength is found if a straight line from the achromatic locus $\left(x_n, y_n\right)$ through the stimulus locus $\left(x_s, y_s\right)$ is extended to the spectral locus $\left(x_D, y_D\right)$. The wavelength having the coordinates $\left(x_D, y_D\right)$ is λ_D.

Stimuli with a purplish appearance have no dominant wavelength. For these samples, the complementary wavelength λ_C may be used. λ_C is defined by the point of intersection of the spectral locus and the line used to determine the dominant wavelength but extended in the opposite direction. In order to obtain accurate values of λ_D (and λ_C), the chromaticity coordinates of the stimulus should differ significantly from the chromaticity coordinates of the achromatic stimulus.



Key

$\left(x_n, y_n\right)$	loci of CIE illuminant C
$\left(x_s, y_s\right)$	sample
λ_D	dominant wavelength of the sample at the coordinates $\left(x_D, y_D\right)$
λ_C	complementary wavelength

Figure 1 — CIE *xy* chromaticity diagram

3.7 CIELAB colour space coordinates

In the CIELAB system, the coordinate system is chosen so that $a^* = 0$ and $b^* = 0$ represent achromatic stimuli. If at the same time $L^* = 100$, then the stimulus corresponds to the perfect reflecting diffuser under the illuminant chosen. To achieve this, the tristimulus values for the perfect reflecting diffuser under the chosen illuminant should be incorporated into the formulae. These values are denoted X_n,Y_n, Z_n and $X_{n,10}$, $Y_{n,10}$, $Z_{n,10}$ for the respective standard observers. Formulae (28) to (30) are given for the 2° observer. The corresponding formulae for the 10° observer are obtained by replacing X, Y, Z, X_n, Y_n and Z_n by X_{10}, Y_{10}, Z_{10}, $X_{n,10}$, $Y_{n,10}$ and $Z_{n,10}$.

The CIE lightness L^* is calculated as:

$$L^* = 116 \cdot \left(Y/Y_\mathrm{n}\right)^{1/3} - 16 \;\; if \;\; Y/Y_\mathrm{n} > \left(24/116\right)^3 \tag{28}$$

$$L^* = 903{,}3 \cdot \left(Y/Y_\mathrm{n}\right) \;\; if \;\; Y/Y_\mathrm{n} \le \left(24/116\right)^3$$

The a^* and b^* values of the CIELAB system are calculated as:

$$a^* = 500 \cdot \left[f\left(X/X_\mathrm{n}\right) - f\left(Y/Y_\mathrm{n}\right)\right] \tag{29}$$

$$b^* = 200 \cdot \left[f\left(Y/Y_\mathrm{n}\right) - f\left(Z/Z_\mathrm{n}\right)\right]$$

where

$$f\left(\xi\right) = \xi^{1/3} \;\; if \;\; \xi > \left(24/116\right)^3 \tag{30}$$

$$f\left(\xi\right) = 7{,}787 \cdot \xi \;\; if \;\; \xi \le \left(24/116\right)^3$$

where

ξ is X/X_n, Y/Y_n or Z/Z_n

The required values of X_n, Y_n, Z_n and $X_{\mathrm{n},10}$, $Y_{\mathrm{n},10}$, $Z_{\mathrm{n},10}$ the tristimulus values for the perfect reflecting diffuser, for different illuminants are:

A-illuminant:	X_n = 109,850	$X_{\mathrm{n},10}$ = 111,144
	Y_n = 100	$Y_{\mathrm{n},10}$ = 100
	Z_n = 35,585	$Z_{\mathrm{n},10}$ = 35,200
C-illuminant:	X_n = 98,074	$X_{\mathrm{n},10}$ = 97,285
	Y_n = 100	$Y_{\mathrm{n},10}$ = 100
	Z_n = 118,232	$Z_{\mathrm{n},10}$ = 116,145
D65-illuminant:	X_n = 95,047	$X_{\mathrm{n},10}$ = 94,811
	Y_n = 100	$Y_{\mathrm{n},10}$ = 100
	Z_n = 108,883	$Z_{\mathrm{n},10}$ = 107,304

[SOURCE: CIE 15:2004, 8.2 and Table T.3]

The chromatic information in the a^* and b^* coordinates may also be expressed in polar coordinates C^*_ab and h_ab where the CIE 1976 *a, b* chroma C^*_ab is a correlate of chromaticness and is analogous to the excitation purity, and the CIE 1976 *a, b* hue-angle h_{ab} is a correlate of hue and is analogous to the dominant wavelength.

C^*_{ab} and h_{ab} are calculated as:

$$C^*_{\mathrm{ab}} = \sqrt{\left(a^*\right)^2 + \left(b^*\right)^2} \tag{31}$$

$$h_{\mathrm{ab}} = \arctan\left(\frac{b^*}{a^*}\right) \cdot \frac{180°}{\Pi} + \phi \text{ if } a^* \neq 0 \tag{32}$$

$h_{\mathrm{ab}} = 90°$ if $a^* = 0$ and $b^* > 0$

$h_{\mathrm{ab}} = 270°$ if $a^* = 0$ and $b^* < 0$

h_{ab} undefined if $a^* = 0$ and $b^* = 0$

where

$\phi = 0°$ if $b^* \geq 0$ and $a^* > 0$

$\phi = 180°$ if $b^* \geq 0$ and $a^* < 0$

$\phi = 360°$ if $b^* < 0$ and $a^* > 0$

$\phi = 180°$ if $b^* < 0$ and $a^* < 0$

Since the value of the arctangent function corresponds to an angle in radians, the factor $180°/\pi$ has been inserted to convert to degrees which is the unit in which h_{ab} is reported. The chosen values for Φ mean that h_{ab} has its zero value at the a^*-axis and that h_{ab} increases in the counter-clockwise direction. Mathematically, h_{ab} is undefined when $C^*_{\mathrm{ab}} = 0$. It is, however, recommended to report h_{ab} as undefined when C^*_{ab} is so small that the stimulus cannot be perceptually distinguished from an achromatic stimulus having the same L^*-value.

3.8 Colour differences in CIELAB colour space, ΔE^*_{ab}

The CIELAB colour space is constructed to be approximately uniform with respect to colour differences. However, it has to be noted that it is an approximation and allows neither for large colour differences nor exact comparisons of colour differences in different parts of the colour space. It is recommended to use ΔE^*_{ab} for applications in which the CIELAB system can be considered to be sufficiently uniform.

The colour difference ΔE^*_{ab} between two stimuli $\left(L_1^*, a_1^*, b_1^*\right)$ and $\left(L_2^*, a_2^*, b_2^*\right)$ is defined as

$$\Delta E^*_{\mathrm{ab}} = \sqrt{\left(\Delta L^*\right)^2 + \left(\Delta a^*\right)^2 + \left(\Delta b^*\right)^2} \tag{33}$$

where

$$\Delta L^* = L_2^* - L_1^*$$

$$\Delta a^* = a_2^* - a_1^*$$

$$\Delta b^* = b_2^* - b_1^*$$

3.9 CIE-whiteness and Tint

3.9.1 CIE tint, T_w

The CIE tint value (also known as green/red tint) is calculated according to:

$$T_{\mathrm{w},10} = 900 \cdot \left(x_{\mathrm{n},10} - x_{10}\right) - 650 \cdot \left(y_{\mathrm{n},10} - y_{10}\right) \tag{34}$$

for the CIE 1964 Standard Observer, or

$$T_\mathrm{w} = 1\,000 \cdot \left(x_\mathrm{n} - x\right) - 650 \cdot \left(y_\mathrm{n} - y\right) \tag{35}$$

for the CIE 1931 Standard Observer

where the values for $x_{\mathrm{n},10}$, $y_{\mathrm{n},10}$, x_n and y_n are given in Table 2.

A positive T_w -value indicates a greenish tint and a negative value a reddish tint. The primary purpose of the tint value calculation is not to provide a colorimetric parameter but to provide a limit for the applicability of the whiteness equations. The material is considered to be white only if the tint value lies within the limits given by:

$$-4 < T_\mathrm{w} < 2 \tag{36}$$

3.9.2 CIE-whiteness, *W*

The CIE-whiteness value *W* is defined as:

$$W_{10} = Y_{10} + 800 \cdot \left(x_{\mathrm{n},10} - x_{10}\right) + 1\,700 \cdot \left(y_{\mathrm{n},10} - y_{10}\right) \tag{37}$$

for the 10° observer and

$$W = Y + 800 \cdot \left(x_\mathrm{n} - x\right) + 1\,700 \cdot \left(y_\mathrm{n} - y\right) \tag{38}$$

for the 2° observer,

where $x_{n,10}$, $y_{n,10}$, and x_n, y_n are the chromaticity coordinates for the perfect reflecting diffuser under the illumination considered and for the 10° observer and the 2° observer, respectively. Values are given in Table 2.

The material is considered to be white provided that:

W lies within the limits given by:

$$40 < W_{10} < \left(5 \cdot Y_{10} - 280\right) \tag{39}$$

or

$$40 < W \ < \left(5 \cdot Y - 280\right) \tag{40}$$

and

$$-4 < T_w < 2 \tag{41}$$

Table 2 — Data for $x_{n,10}$, $y_{n,10}$, and x_n, y_n for CIE illuminant C and CIE standard illuminant D65[21]

Illuminant C	x_n =0,31006 $x_{n,10}$ = 0,31039	y_n =0,31616 $y_{n,10}$ = 0,31905
Standard Illuminant D65	x_n = 0,31272 $x_{n,10}$ = 0,31381	y_n = 0,32903 $y_{n,10}$ = 0,33098

3.10 Fluorescence component

The contribution of the fluorescent whitening agent to the brightness or whiteness is calculated from differences of measurements performed with UV illuminant adjustment and with a filter with a cut-off wavelength of 420 nm placed in the light beam (B_0 or W_0).

It is calculated as the difference between the whiteness or brightness measured using a source of light having a UV-content corresponding to the chosen CIE illuminant, $S_i(\lambda)$, where I = C or D65 and the whiteness or brightness measured with a source without radiation in the excitation band. Thus,

$$F_{B,S} = B_S - B_0 \quad \text{or} \quad F_{w,S} = W_S - W_0 \tag{42}$$

where

B_S is the brightness value under specified CIE illuminant, S(λ) conditions (usually C or D65);

W_S is the whiteness value under specified CIE illuminant, S(λ) conditions (usually C or D65);

B_0 is the brightness value when the fluorescent effect has been fully eliminated;

W_0 is the whiteness value when the fluorescent effect has been fully eliminated.

It is not sufficient merely to eliminate the UV-component of the illumination since the added fluorescent whitening agent is to some extent excited by visible light. The instrument should be equipped with a sharp cut-off, UV-absorbing filter having a transmittance not exceeding 5,0 % at and below a wavelength of 410 nm and not exceeding 50 % at a wavelength of 420 nm. The cut-off filter should have characteristics such that a reliable reflectance factor value is obtained at 420 nm. The reflectance factor value obtained at 420 nm should then be considered for computational purposes to be the value which applies at all lower wavelengths at which it is not possible to make any measurements.

3.11 Metamerism index

A measure of the metamerism for two specimens is the colour difference between the two metameric specimens caused by substituting an illuminant, “special metamerism index: change in illuminant”, and by substituting an observer, “special metamerism index: change in observer”. The colour difference is evaluated using a CIE colour difference formula.

It is recommended that for two specimens whose corresponding tristimulus values are identical with respect to a reference illuminant and observer, the metamerism index, M, be set equal to the colour difference between the two specimens computed for the test illuminant or for the test observer.

The metamerism index is given by:

$$M = \Delta E^{*}_{\mathrm{ab}} \qquad (43)$$

where $\Delta E^{*}_{\mathrm{ab}}$ for the two samples is calculated using Formula (34).

3.12 Yellowness index

The yellowness index is calculated as:

$$YI = 100 \cdot \left(R_{\mathrm{X}} - R_{\mathrm{Z}}\right) \Big/ R_{\mathrm{Y}} \qquad (44)$$

For the C/2° illuminant-observer condition, the yellowness index is therefore calculated as:

$$YI = \frac{1{,}277 \cdot X - 1{,}059 \cdot Z}{Y} \cdot 100 \qquad (45)$$

and for the D65/10° illuminant-observer condition, the yellowness index is calculated as:

$$YI = \frac{1{,}301 \cdot X - 1{,}149 \cdot Z}{Y} \cdot 100 \qquad (46)$$

Bibliography

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ICS 85.040; 85.060

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