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Representation of results of particle size analysis —

Part 5:
Methods of calculation relating to particle size analyses using logarithmic normal probability distribution

Représentation de données obtenues par analyse granulométrique —

Partie 5: Méthodes de calcul relatif à l'analyse granulométrique à l'aide de la distribution de probabilité logarithmique normale

Reference number
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Contents

Page

Foreword

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ISO 9276-5 was prepared by Technical Committee ISO/TC 24, *Sieves, sieving and other sizing methods*, Subcommittee SC 4, *Sizing by methods other than sieving*.

ISO 9276 consists of the following parts, under the general title *Representation of results of particle size analysis*:

— *Part 1: Graphical representation*

— *Part 2: Calculation of average particle sizes/diameters and moments from particle size distributions*

— *Part 4: Characterization of a classification process*

— *Part 5: Methods of calculation relating to particle size analyses using logarithmic normal probability distribution*

Further parts are under preparation:

— *Part 3: Fitting of an experimental cumulative curve to a reference model*

— *Part 6: Descriptive and quantitative representation of particle shape and morphology*

Introduction

Many cumulative particle size distributions, $Q_r(x)$, may be plotted on special graph paper which allow the cumulative size distribution to be represented as a straight line. Scales on the ordinate and the abscissa are generated from various mathematical formulae. In this part of ISO 9276, it is assumed that the cumulative particle size distribution follows a logarithmic normal probability distribution.

In this part of ISO 9276, the size, x, of a particle represents the diameter of a sphere. Depending on the situation, the particle size, x, may also represent the equivalent diameter of a particle of some other shape.

Representation of results of particle size analysis —

Part 5: Methods of calculation relating to particle size analyses using logarithmic normal probability distribution

1 Scope

The main objective of this part of ISO 9276 is to provide the background for the representation of a cumulative particle size distribution which follows a logarithmic normal probability distribution, as a means by which calculations performed using particle size distribution functions may be unequivocally checked. The design of logarithmic normal probability graph paper is explained, as well as the calculation of moments, median diameters, average diameters and volume-specific surface area. Logarithmic normal probability distributions are often suitable for the representation of cumulative particle size distributions of any dimensionality. Their particular advantage lies in the fact that cumulative distributions, such as number-, length-, area-, volume- or mass-distributions, are represented by parallel lines, all of whose locations may be determined from a knowledge of the location of any one.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 9276-1, *Representation of results of particle size analysis — Part 1: Graphical representation*

ISO 9276-2:2001, *Representation of results of particle size analysis — Part 2: Calculation of average particle sizes/diameters and moments from particle size distributions*

3 Symbols

For the purposes of this part of ISO 9276, the following symbols apply.

c	cumulative percentage
e = 2,718 28...	base of natural logarithms
k	power of x in a moment
$M_{k,r}$	complete kth moment of a density distribution of dimensionality r
p	dimensionality (type of quantity) of a distribution, $p = 0$: number, $p = 1$: length, $p = 2$: area, $p = 3$: volume or mass
$q_r(x)$	density distribution of dimensionality r
$Q_r(x)$	cumulative distribution of dimensionality r

r — dimensionality (type of quantity) of a distribution,
$r = 0$: number, $r = 1$: length, $r = 2$: area, $r = 3$: volume or mass

s — standard deviation of the density distribution

s_g — geometric standard deviation, exponential function of the standard deviation

S_V — volume-specific surface area

x — particle size, diameter of a sphere

x_{min} — particle size below which there are no particles in a given size distribution

x_{max} — particle size above which there are no particles in a given size distribution

$x_{84,r}$ — particle size at which $Q_r = 0{,}84$

$x_{50,r}$ — median particle size of a cumulative distribution of dimensionality r

$x_{16,r}$ — particle size at which $Q_r = 0{,}16$

$\bar{x}_{k,r}$ — average particle size based on the kth moment of a distribution of dimensionality r

z — dimensionless variable proportional to the logarithm of x (see Equation 3)

ξ — integration variable based on x (see Equation 11)

ζ — integration variable based on z (see Equation 2)

Subscripts of different sense are separated by a comma in this and all other parts of ISO 9276.

4 Logarithmic normal probability function

Normal probability density distributions are described in terms of a dimensionless variable z:

$$q^*_r(z) = \frac{1}{\sqrt{2\pi}} e^{-0{,}5z^2} \tag{1}$$

The cumulative normal probability distribution is represented by:

$$Q^*_r(z) = \int_{-\infty}^{z} q^*_r(\zeta)\, d\zeta = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-0{,}5\zeta^2}\, d\zeta \tag{2}$$

A sample table of values for $Q^*_r(z)$ as a function of z is given in Table A.1.

The logarithmic normal probability distribution is a formulation in which z is defined as a logarithm of x scaled by two parameters, the mean size $x_{50,r}$ and either the dimensionless standard deviation, s, or the geometric standard deviation, s_g, that characterize the distribution:

$$z = \frac{1}{s}\ln\left[\frac{x}{x_{50,r}}\right] = \frac{1}{\ln s_g}\ln\left[\frac{x}{x_{50,r}}\right] = \frac{1}{\log s_g}\log\left[\frac{x}{x_{50,r}}\right] \tag{3}$$

which is equivalent to

$$x = x_{50,r}\ \mathrm{e}^{s\,z} \tag{4}$$

According to Equation 3, the standard deviation, s, is linked with the geometric standard deviation, s_{g}, by:

$$s = \ln s_{\mathrm{g}} \text{ or } s_{\mathrm{g}} = \mathrm{e}^{s} \tag{5}$$

Although Equation 1 has no explicit dependences on r, the dimensionality of the density distribution is involved through the relationship of z to $x_{50,r}$ in Equation 3. The value of $x_{50,r}$ for a specific size distribution may be determined from experimental data according to ISO 9276-1. The standard deviation of a logarithmic normal probability distribution may be calculated from the values of the cumulative distribution at certain characteristic values of z:

either at $z = 1$, for which

$$Q^*_r\ (z = 1) = 0{,}84 \text{ and } s = \ln\left[\frac{x_{84,r}}{x_{50,r}}\right] \tag{6}$$

or at $z = -1$, for which

$$Q^*_r\ (z = -1) = 0{,}16 \text{ and } s = \ln\left[\frac{x_{50,r}}{x_{16,r}}\right] \tag{7}$$

Throughout this part of ISO 9276, the values 0,84 and 0,16 (and their representation as percentages 84 and 16) are used in place of the more precise values 0,841 34 and 0,158 65.

Logarithmic probability graph presentation: Useful information about the nature of a particle size distribution may be obtained by plotting the cumulative distribution on special graph paper, on which the abscissa (representing particle size) is marked with an exponential scale and the ordinate (representing cumulative distribution) is marked with a scale of $Q^*_r(z)$ values (see Annex A). Preprinted paper marked with these scales is available. Graphical representation is now more often displayed as a specific graphical screen created by software in a computer. Experimental values of each cumulative fraction (expressed in terms of number, length, area or volume) of undersize particles, $Q_r(x)$, (that is, of particles smaller than x) are plotted at the size corresponding to the upper size limit of the particles in that cumulative fraction. A logarithmic normal probability distribution gives a straight line in Figure 1.

To fulfil the condition of normalization, the cumulative fraction smaller than or equal to the particle having the largest size in the sample must be unity, that is, $Q_r(x_{\mathrm{max}})$ must be equal to 1. If this is so, then

$$q^*_r(z)\,\mathrm{d}z = q_r(x)\,\mathrm{d}x \tag{8}$$

NOTE The superscript* is used to distinguish the distributions defined in terms of the dimensionless integration variable z, such as $q^*_r(z)$, from those defined in terms of the size x, such as $q_r(x)$. This is because z, the integration variable, is related to the particle size x, as shown in Equation 3.

$$q_r(x) = q^*_r(z)\frac{\mathrm{d}z}{\mathrm{d}x} = q^*_r(z)\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{1}{s}\ln\left[\frac{x}{x_{50,r}}\right]\right\} = \frac{1}{x\,s}\,q^*_r(z) \tag{9}$$

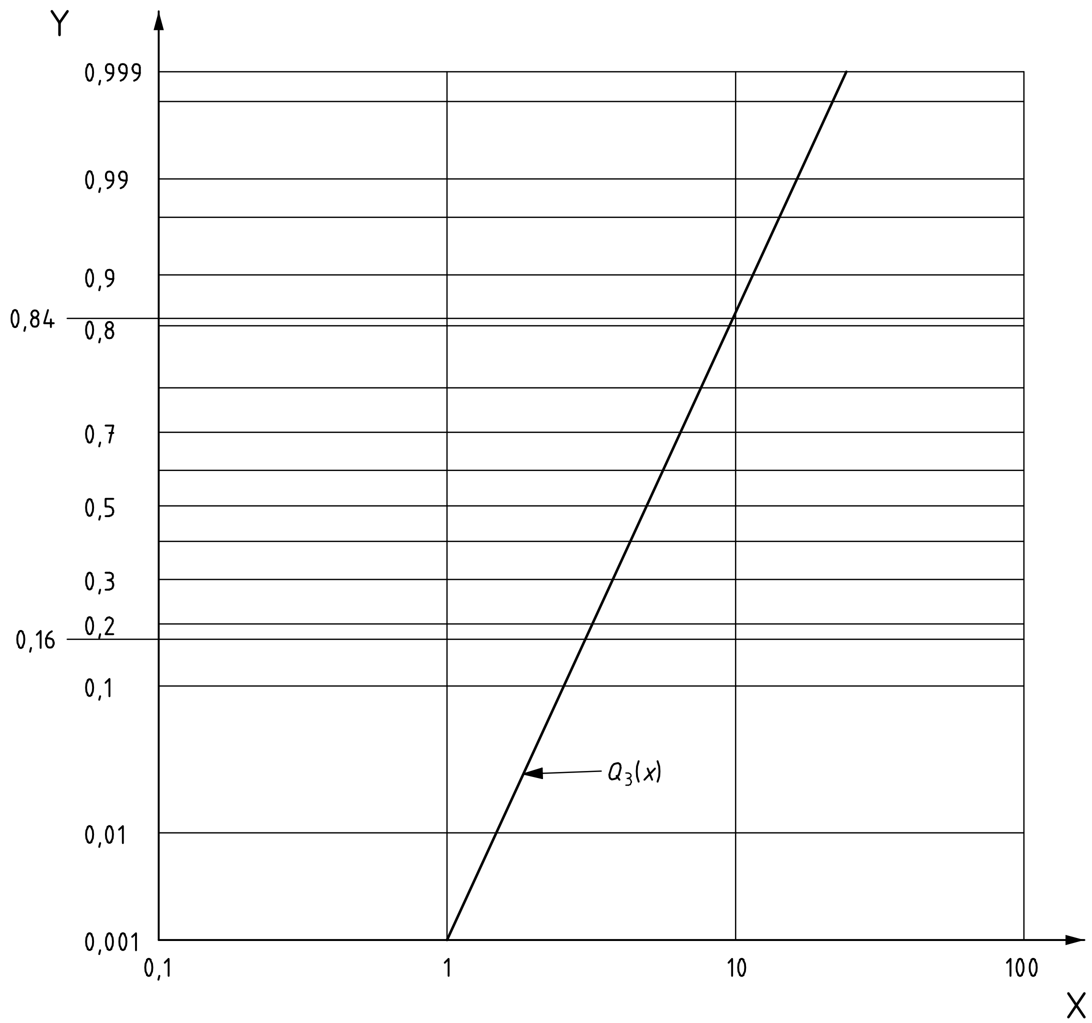
or, using Equation 1,

$$q_r(x) = \frac{1}{x\,s\,\sqrt{2\pi}}\,\mathrm{e}^{-0{,}5\,z^2} \tag{10}$$

and, parallel to Equation 2,

$$Q_r(x) = \int_{x_{\min}}^{x} q_r(\xi)\,\mathrm{d}\xi \tag{11}$$

EXAMPLE A logarithmic normal probability distribution of volume ($r = 3$), with a median size of $x_{50,3} = 5$ µm and a standard deviation of $s = 0{,}5$, has $x_{16,3} = 3{,}0$ µm and $x_{84,3} = 8{,}2$ µm (see ISO 9276-2:2001, Annex A). Figure 1 shows a plot of the cumulative volume distribution, $Q_3(x)$, on logarithmic probability graph paper.



Key

X particle size, x, µm

Y cumulative distribution, Q

Figure 1 — Plot of a logarithmic normal probability distribution on logarithmic probability graph paper

5 Special values of a logarithmic normal probability distribution

5.1 Complete kth moments

The *complete* kth *moment* of a logarithmic normal probability distribution, $q_r(x)$, is

$$M_{k,r} = x_{50,r}{}^{k}\ \mathrm{e}^{0{,}5\,k^2 s^2} = \mathrm{e}^{k \ln x_{50,r} + 0{,}5\,k^2 s^2} \tag{12}$$

with $k = 2$ and $r = 3$:

$$M_{2,3} = x_{50,3}{}^{2}\ \mathrm{e}^{2 s^2} = \mathrm{e}^{2 \ln x_{50,3} + 2 s^2} \tag{13}$$

5.2 Average particle sizes

A series of average particle sizes, $\bar{x}$, of a logarithmic normal probability distribution, $q_r(x)$, can be calculated from the kth root of the kth moment (or from the $x_{50,r}$ and s) of that distribution using Equation 14:

$$\bar{x}_{k,r} = \sqrt[k]{M_{k,r}} = x_{50,r}\ \mathrm{e}^{0{,}5\,k\,s^2} \tag{14}$$

For a logarithmic normal probability distribution, the median is the same as the geometric mean and the average size in one dimension, r, may be calculated from the parameters describing the distribution in a different dimensionality, p, using:

$$\bar{x}_{k,r} = x_{50,p}\ \mathrm{e}^{(0{,}5\,k + r - p)\,s^2} \tag{15}$$

or

$$\ln \bar{x}_{k,r} = \ln x_{50,r} + 0{,}5\,k\,s^2 = \ln x_{50,p} + (0{,}5\,k + r - p)\,s^2 \tag{16}$$

EXAMPLE The first several moments ($k = 1$, 2 or 3) of the arithmetic average particle size ($r = 0$) for a logarithmic normal probability distribution may be computed from the parameters for any of the dimensionalities ($p = 0$, 1, 2 or 3) using:

$$\bar{x}_{1,0} = x_{50,0}\ \mathrm{e}^{0{,}5\,s^2} = x_{50,1}\ \mathrm{e}^{-0{,}5\,s^2} = x_{50,2}\ \mathrm{e}^{-1{,}5\,s^2} = x_{50,3}\ \mathrm{e}^{-2{,}5\,s^2} \tag{17}$$

$$\bar{x}_{2,0} = x_{50,0}\ \mathrm{e}^{s^2} = x_{50,1} = x_{50,2}\ \mathrm{e}^{-s^2} = x_{50,3}\ \mathrm{e}^{-2\,s^2} \tag{18}$$

$$\bar{x}_{3,0} = x_{50,0}\ \mathrm{e}^{1{,}5\,s^2} = x_{50,1}\ \mathrm{e}^{0{,}5\,s^2} = x_{50,2}\ \mathrm{e}^{-0{,}5\,s^2} = x_{50,3}\ \mathrm{e}^{-1{,}5\,s^2} \tag{19}$$

EXAMPLE The first moment ($k = 1$) weighted average particle size for the different dimensionalities ($r = 0$, 1, 2, or 3) of a logarithmic normal probability distribution may be computed from the parameters for any of the dimensionalities ($p = 0$, 1, 2 or 3) using:

$$\bar{x}_{1,0} = x_{50,0}\ \mathrm{e}^{0{,}5\,s^2} = x_{50,1}\ \mathrm{e}^{-0{,}5\,s^2} = x_{50,2}\ \mathrm{e}^{-1{,}5\,s^2} = x_{50,3}\ \mathrm{e}^{-2{,}5\,s^2} \tag{17}$$

$$\bar{x}_{1,1} = x_{50,0}\ \mathrm{e}^{1{,}5\,s^2} = x_{50,1}\ \mathrm{e}^{0{,}5\,s^2} = x_{50,2}\ \mathrm{e}^{-0{,}5\,s^2} = x_{50,3}\ \mathrm{e}^{-1{,}5\,s^2} \tag{20}$$

$$\bar{x}_{1,2} = x_{50,0}\ \mathrm{e}^{2{,}5\,s^2} = x_{50,1}\ \mathrm{e}^{1{,}5\,s^2} = x_{50,2}\ \mathrm{e}^{0{,}5\,s^2} = x_{50,3}\ \mathrm{e}^{-0{,}5\,s^2} \tag{21}$$

$$\bar{x}_{1,3} = x_{50,0}\ \mathrm{e}^{3{,}5\,s^2} = x_{50,1}\ \mathrm{e}^{2{,}5\,s^2} = x_{50,2}\ \mathrm{e}^{1{,}5\,s^2} = x_{50,3}\ \mathrm{e}^{0{,}5\,s^2} \tag{22}$$

5.3 Median particle sizes

A unique feature of the logarithmic normal probability distribution is that lines representing the cumulative distributions of number, length, area and volume (or mass) for a given size distribution on logarithmic probability graph paper have the same slope and are shifted horizontally from one another, so that there is a simple relationship between the median size for the number distribution, $x_{50,0}$, the median size for the diameter distribution, $x_{50,1}$, the median size for the area distribution, $x_{50,2}$, and the median size for the volume (or mass) distribution, $x_{50,3}$. The general form of the relationship is

$$x_{50,r} = x_{50,p}\, \mathrm{e}^{(r-p)s^2} \tag{23}$$

or its equivalent

$$\ln x_{50,r} = \ln x_{50,p} + (r-p)\, s^2 \tag{24}$$

EXAMPLE The x_{50} point for a cumulative distribution of dimensionality $r = 3$ is related to the x_{50} points for cumulative distributions of other dimensionalities by:

$$\ln x_{50,3} = \ln x_{50,0} + 3\, s^2 = \ln x_{50,1} + 2\, s^2 = \ln x_{50,2} + s^2 \tag{25}$$

The same relationship, expressed by Equation 25, holds for the comparable points (x_{16}, x_{84}, etc.) at all other cumulative distribution values, so that a general formula can be given as:

$$\ln x_{c,r} = \ln x_{c,p} + (r-p)\, s^2 \tag{26}$$

where c is any value from 0 to 100. The consequence of this relationship means that the lines representing all the different cumulative distributions are parallel to one another. See Figure 2.

5.4 Horizontal shifts between plotted distribution values

5.4.1 Linear abscissa

If the particle cumulative data is plotted on probability graph paper when the abscissa is marked with a scale linear in z (not shown in Figure 2), the cumulative distributions of different dimensionalities for a logarithmic normal probability distribution are related by:

$$Q^*_r(z) = Q^*_p\left[z - (r-p)\, s\right] \tag{27}$$

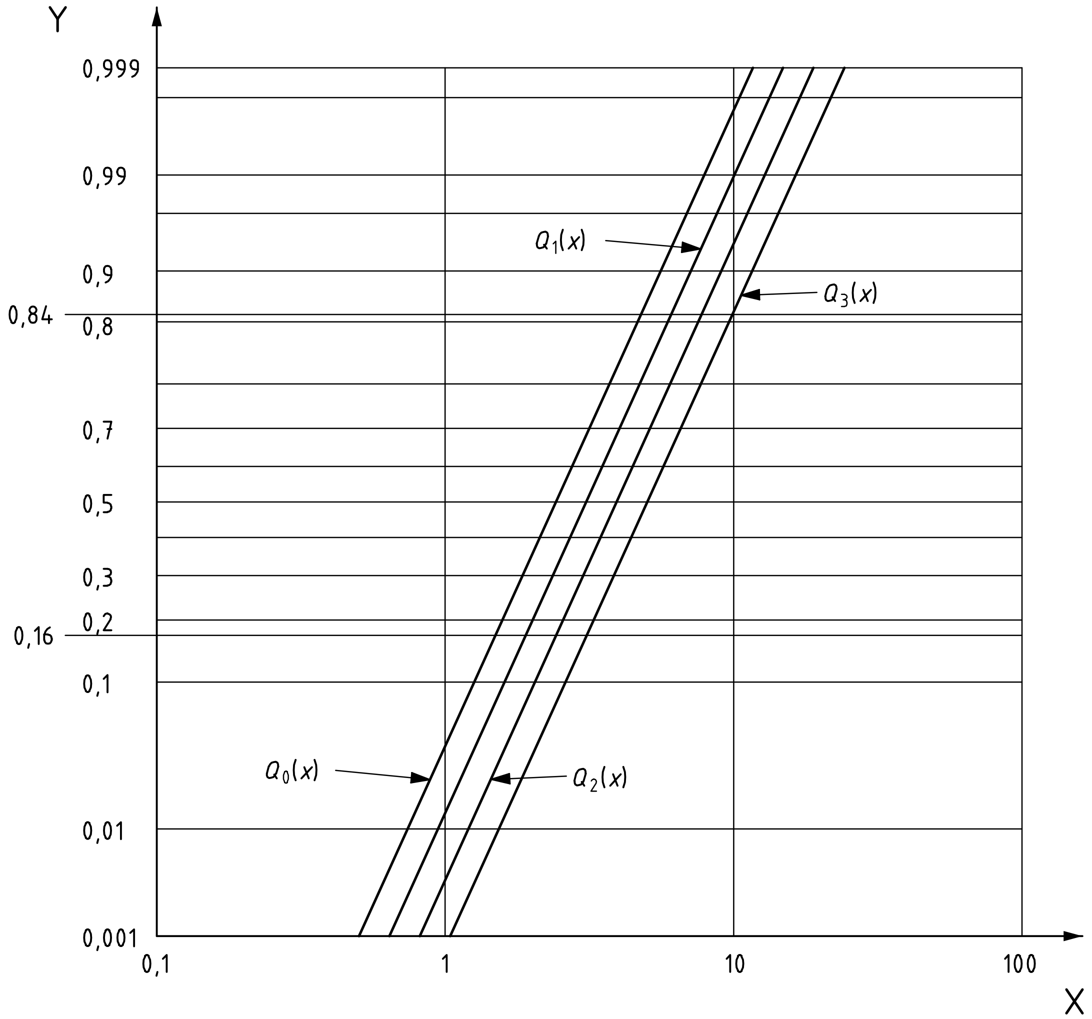
so that the cumulative distribution of dimensionality, r, will coincide with the cumulative distribution for dimensionality, p, when shifted by a distance $(r-p)\, s$.

EXAMPLE If $r = 3$ and $p = 2$, the volume distribution curve, $Q^*_3(z)$, is obtained from the area distribution curve, $Q^*_2(z)$, by shifting the latter towards coarser sizes (right) by one standard deviation.

$$Q^*_3(z) = Q^*_2\,(z - s) \tag{28}$$

EXAMPLE The number distribution curve $Q^*_0(z)$, $p = 0$, is obtained from the volume distribution curve $Q^*_3(z)$, $r = 3$, by shifting the latter toward finer sizes (left) by three standard deviations:

$$Q^*_0(z) = Q^*_3(z + 3\, s) \tag{29}$$



Key

X particle size, x, µm

Y cumulative distributions of number, length, area and volume (or mass), Q

Figure 2 — Cumulative distributions of number, length, area and volume (or mass) for a logarithmic normal probability distribution, plotted on logarithmic probability graph paper

5.4.2 Logarithmic abscissa

If the particle cumulative data is plotted on logarithmic normal probability graph paper, when the abscissa is marked with a logarithmic scale for x, the cumulative distributions of different dimensionalities for a logarithmic normal probability distribution are related by

$$Q_r\left\{\ln\left[\frac{x}{x_{50,r}}\right]\right\} = Q_p\left\{\ln\left[\frac{x}{x_{50,p}}\right] - (r-p)\,s^2\right\} \tag{30}$$

With this abscissa scale, the shift from the cumulative distribution or one dimensionality to another becomes $(r-p)\,s^2$. This corresponds to the shift of the median sizes as given in Equations 25 and 26.

Figure 2 shows the cumulative distributions of number, length, area and volume (or mass) for a logarithmic normal probability distribution on logarithmic probability graph paper. These lines represent the same distribution as that shown in Figure 1, so the shift from $Q_3(x)$ to $Q_2(x)$ in Figure 2 may be computed from Equation 30 as $-(3-2)\,0{,}5^2 = -0{,}25$ units on the (natural) logarithmic scale. Since $x_{50,3} = 5$ µm, the median value for $Q_2(x)$ occurs at $x_{50,2} = x_{50,3}\,e^{-0{,}25} = 3{,}9$ µm.

5.5 Volume-specific surface area (Sauter diameter)

The volume-specific surface area of spheres can be calculated from the weighted average size of an area distribution, the so-called Sauter diameter, as:

$$S_V = \frac{6}{\bar{x}_{1,2}} \tag{31}$$

Introducing Equation 21 in the denominator yields:

$$S_V = \frac{6}{x_{50,p}} \mathrm{e}^{(p-2{,}5)\,s^2} \tag{32}$$

Thus, the Sauter diameter may be obtained from the median and standard deviation of any of the four dimensionalities of size distribution ($p = 0$, 1, 2 or 3) using:

$$S_V = \frac{6}{x_{50,0}} \mathrm{e}^{-2{,}5\ s^2} = \frac{6}{x_{50,1}} \mathrm{e}^{-1{,}5\ s^2} = \frac{6}{x_{50,2}} \mathrm{e}^{-0{,}5\ s^2} = \frac{6}{x_{50,3}} \mathrm{e}^{+0{,}5\ s^2} \tag{33}$$

Annex A
(informative)

Cumulative distribution values of a normal probability distribution

Numerical values of the normal cumulative distribution, $Q^*_r(z)$, as a function of z, may be obtained by numerical integration of Equation 2 from minus infinity to z. Equation A.1, taken from Equation 26.2.18 on page 932 in Reference [4] is a series approximation that generates values of $Q^*_r(z)$ accurate to 0,000 25, sufficient for constructing the ordinates of logarithmic normal graph coordinates on computer screens.

$$Q^*_r(z) = 1 - \frac{0{,}5}{(1 + c_1 z + c_2 z^2 + c_3 z^3 + c_4 z^4)^4} \qquad \text{(A.1)}$$

where $c_1 = 0{,}196\ 854$, $c_2 = 0{,}115\ 194$, $c_3 = 0{,}000\ 344$, $c_4 = 0{,}019\ 527$

Table A.1 gives a list of $Q^*_r(z)$ values as a function of z, rounded to a number of significant figures suitable for use with most particle size distributions. Tables with less rounding are available in many books covering statistical procedures. For example, $Q^*_r(z)$ is given to fifteen significant figures in the second column in Table 26.1 (pages 966-972) in Reference [4].

Values of z corresponding to a given value of $Q^*_r(z)$ may be determined from tables of z and $Q^*_r(z)$, or computed from Equation A.1 by successive approximation. For a given value of Q, start with a trial z, calculate the corresponding value of $Q^*_r(z)$, compare that with the given value of $Q^*_r(z)$, and recalculate using a z likely to produce a $Q^*_r(z)$ that is closer to the given value of $Q^*_r(z)$ (repeat until the difference between the values of $Q^*_r(z)$ is within the required tolerance).

Table A.2 gives a list of z values as a function of $Q^*_r(z)$, rounded to a number of significant figures suitable for use with most particle size distributions.

Table A.1 — $Q^*_r(z)$ as a function of z

z	$Q^*_r(z)$	z	$Q^*_r(z)$
3,00	1,00	0,00	0,50
2,90	1,00	–0,10	0,46
2,80	1,00	–0,20	0,42
2,70	1,00	–0,30	0,38
2,60	1,00	–0,40	0,34
2,50	0,99	–0,50	0,31
2,40	0,99	–0,60	0,27
2,30	0,99	–0,70	0,24
2,20	0,99	–0,80	0,21
2,10	0,98	–0,90	0,18
2,00	0,98	–1,00	0,16
1,90	0,97	–1,10	0,14
1,80	0,96	–1,20	0,12
1,70	0,96	–1,30	0,10
1,60	0,95	–1,40	0,08
1,50	0,93	–1,50	0,07
1,40	0,92	–1,60	0,05
1,30	0,90	–1,70	0,04
1,20	0,88	–1,80	0,04
1,10	0,86	–1,90	0,03
1,00	0,84	–2,00	0,02
0,90	0,82	–2,10	0,02
0,80	0,79	–2,20	0,01
0,70	0,76	–2,30	0,01
0,60	0,73	–2,40	0,01
0,50	0,69	–2,50	0,01
0,40	0,66	–2,60	0,00
0,30	0,62	–2,70	0,00
0,20	0,58	–2,80	0,00
0,10	0,54	–2,90	0,00
0,00	0,50	–3,00	0,00

Table A.2 — z as a function of $Q^*_r(z)$

$Q^*_r(z)$	z	$Q^*_r(z)$	z
0,98	2,05	0,50	0,00
0,96	1,75	0,48	−0,05
0,94	1,56	0,46	−0,10
0,92	1,42	0,44	−0,15
0,90	1,28	0,42	−0,20
0,88	1,18	0,40	−0,26
0,86	1,08	0,38	−0,31
0,84	1,00	0,36	−0,36
0,82	0,92	0,34	−0,42
0,80	0,84	0,32	−0,47
0,78	0,78	0,30	−0,53
0,76	0,71	0,28	−0,59
0,74	0,65	0,26	−0,65
0,72	0,59	0,24	−0,71
0,70	0,53	0,22	−0,78
0,68	0,47	0,20	−0,84
0,66	0,42	0,18	−0,92
0,64	0,36	0,16	−1,00
0,62	0,31	0,14	−1,08
0,60	0,26	0,12	−1,18
0,58	0,20	0,10	−1,28
0,56	0,15	0,08	−1,42
0,54	0,10	0,06	−1,56
0,52	0,05	0,04	−1,75
0,50	0,00	0,02	−2,05

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