INTERNATIONAL STANDARD

ISO 8258

First edition
1991-12-15

Shewhart control charts

Cartes de contrôle de Shewhart

Reference number
ISO 8258:1991(E)

Contents

International Organization for Standardization
Case Postale 56 • CH-1211 Genève 20 • Switzerland

Printed in Switzerland

Annex

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 8258 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*.

Annex A of this International Standard is for information only.

Introduction

The traditional approach to manufacturing is to depend on production to make the product and on quality control to inspect the final product and screen out items not meeting specifications. This strategy of detection is often wasteful and uneconomical because it involves after-the-event inspection when the wasteful production has already occurred. Instead, it is much more effective to institute a strategy of prevention to avoid waste by not producing unusable output in the first place. This can be accomplished by gathering process information and analysing it so that action can be taken on the process itself.

The control chart as a graphical means of applying the statistical principles of significance to the control of a production process was first proposed by Dr. Walter Shewhart in 1924[1]. Control chart theory recognizes two kinds of variability. The first kind is random variability due to "chance causes" (also known as "common causes"). This is due to the wide variety of causes that are consistently present and not readily identifiable, each of which constitutes a very small component of the total variability but none of which contributes any significant amount. Nevertheless, the sum of the contributions of all of these unidentifiable random causes is measurable and is assumed to be inherent to the process. The elimination or correction of common causes requires a management decision to allocate resources to improve the process and system.

The second kind of variability represents a real change in the process. Such a change can be attributed to some identifiable causes that are not an inherent part of the process and which can, at least theoretically, be eliminated. These identifiable causes are referred to as "assignable causes" or "special causes" of variation. They may be attributable to the lack of uniformity in material, a broken tool, workmanship or procedures or to the irregular performance of manufacturing or testing equipment.

Control charts aid in the detection of unnatural patterns of variation in data resulting from repetitive processes and provide criteria for detecting a lack of statistical control. A process is in statistical control when the variability results only from random causes. Once this acceptable level of variation is determined, any deviation from that level is assumed to be the result of assignable causes which should be identified and eliminated or reduced.

The object of statistical process control is to serve to establish and maintain a process at an acceptable and stable level so as to ensure conformity of products and services to specified requirements. The major statistical tool used to do this is the control chart, which is a graphical method of presenting and comparing information based on a sequence of samples representing the current state of a process against limits established after consideration of inherent process variability. The control chart method helps first to evaluate whether or not a process has attained, or continues in, a state of statistical control at the proper specified level and then to obtain and maintain control and a high degree of uniformity in important product or service characteristics by keeping a continuous record of quality of the product while production is in progress. The use of a control chart and its careful analysis leads to a better understanding and improvement of the process.

Shewhart control charts

1 Scope

This International Standard establishes a guide to the use and understanding of the Shewhart control chart approach to the methods for statistical control of a process.

This International Standard is limited to the treatment of statistical process control methods using only the Shewhart system of charts. Some supplementary material that is consistent with the Shewhart approach, such as the use of warning limits, analysis of trend patterns and process capability is briefly introduced. There are, however, several other types of control chart procedures, a general description of which can be found in ISO 7870.

2 Symbols

n Subgroup size; the number of sample observations per subgroup

k Number of subgroups

X Value of measured quality characteristic (individual values are expressed as $(X_1, X_2, X_3, ...)$. Sometimes the symbol Y is used instead of X.

$\overline{X}$ (X bar) Subgroup average value:

$$\overline{X} = \frac{\sum X_i}{n}$$

$\overline{\overline{X}}$ (X double bar) Average value of the subgroup averages

μ True process mean value

Me Median value of a subgroup. For a set of n numbers $X_1, X_2, ...X_n$ arranged in ascending or descending order of magnitude, the median is the middle number of the set if n is odd and the mean of the two middle numbers if n is even

$\overline{Me}$ Average value of the subgroup medians

R Subgroup range: difference between the largest and smallest observations of a subgroup.

NOTE 1 In the case of charts for individuals, R represents the moving range, which is the absolute value of the difference between two successive values $|X_1 - X_2|$, $|X_2 - X_3|$, etc.

$\overline{R}$ Average value of the R values for all subgroups

s Sample standard deviation:

$$s = \sqrt{\frac{\sum(X_i - \overline{X})^2}{n-1}}$$

$\overline{s}$ Average value of the subgroup sample standard deviations

σ True within-subgroup process standard deviation value

$\hat{\sigma}$ Estimated within-subgroup process standard deviation value

p Proportion or fraction of nonconforming units in a subgroup:

p = number of nonconforming units in a subgroup/subgroup size

$\overline{p}$ Average value of the proportion or fraction nonconforming:

$\overline{p}$ = number of nonconforming units in all subgroups/total number of inspected units

np Number of nonconforming units in a subgroup

c Number of nonconformities in a subgroup

$\overline{c}$ Average value of the c values for all subgroups

u Number of nonconformities per unit in a subgroup

$\bar{u}$ Average value of the u values:

$\bar{u}$ = number of nonconformities in all units/total number of inspected units

3 Nature of Shewhart control charts

A shewhart control chart requires data obtained by sampling the process at approximately regular intervals. The intervals may be defined in terms of time (for example hourly) or quantity (every lot). Usually, each subgroup consists of the same product or service with the same measurable units and the same subgroup size. From each subgroup, one or more subgroup characteristics are derived, such as the subgroup average, $\bar{X}$, and the subgroup range, R, or the standard deviation, s. A Shewhart control chart is a graph of the values of a given subgroup characteristic versus the subgroup number. It consists of a central line (CL) located at a reference value of the plotted characteristic. In evaluating whether or not a state of statistical control exists, the reference value is usually the average of the data being considered. For process control, the reference value is usually the long-term value of the characteristic as stated in the product specifications or a nominal value of the characteristic being plotted based on past experience with the process or from implied product or service target values. The control chart has two statistically determined control limits, one on either side of the central line, which are called the upper control limit (UCL) and the lower control limit (LCL) (see figure 1.

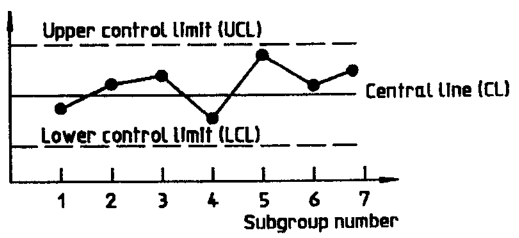

Figure 1 — Outline of a control chart

The control limits on the Shewhart charts are at a distance of 3σ on each side of the central line, where σ is the population within-subgroup standard deviation of the statistic being plotted. The within-subgroup variability is used as a measure of the random variation. Sample standard deviations or appropriate multiples of sample ranges are computed to give an estimate of σ. This measure of σ does not include subgroup-to-subgroup variation but only the within-subgroup components. The 3σ limits indicate that approximately 99,7 % of the subgroup values will be included within the control limits, provided the process is in statistical control. Interpreted another way, there is approximately a 0,3 % risk, or an average of three times in a thousand, of a plotted point being outside of either the upper or lower control limit when the process is in control. The word "approximately" is used because deviations from underlying assumptions such as the distributional form of the data will affect the probability values.

It should be noted that some practitioners prefer to use the factor 3,09 instead of 3 to provide a nominal probability value of 0,2 % or an average of one spurious observation per thousand, but Shewhart selected 3 so as not to lead to attempts to consider exact probabilities. Similarly, some practitioners use actual probability values for the charts based on non-normal distributions such as for ranges and fraction nonconforming. Again, the Shewhart control chart used $\pm 3\sigma$ limits, instead of the probabilistic limits, in view of the emphasis on empirical interpretation.

The possibility that a violation of the limits is really a chance event rather than a real signal is considered so small that when a point appears outside of the limits, action should be taken. Since action is required at this point, the 3σ control limits are sometimes called the "action limits".

Many times, it is advantageous to mark 2σ limits on the chart also. Then, any sample value falling beyond the 2σ limits can serve as a warning of an impending out-of-control situation. As such, the 2σ control limits are sometimes called "warning limits".

Two types of error are possible when control charts are applied. The first is referred to as a type 1 error, which occurs when the process involved remains in control but a point falls outside the control limits due to chance. As a result, it is incorrectly concluded that the process is out of control, and a cost is then incurred in an attempt to find the cause of a nonexistent problem.

The second error is referred to as a type 2 error. It occurs when the process involved is out of control but the point generated falls within the control limits due to chance. In this case, it is incorrectly concluded that the process is in statistical control, and there is a cost associated with failing to detect an increase in nonconforming output. The risk of a type 2 error, however, is a function of three things: the width of the control limits, the degree to which the process is out of control, and the sample size. The nature of these items is such that only a generalization can be made about the size of the risk of a type 2 error.

The Shewhart system takes into account only the type 1 error and the size of this error is 0,3 % with 3σ limits. Since it is generally impractical to make a meaningful estimate of the cost of a type 2 error in a given situation, and since it is convenient to arbitrarily select a small subgroup size, such as 4 or 5, it is appropriate and feasible to use 3σ limits and focus attention on controlling and improving the performance of the process itself.

When a process is in statistical control, the control chart provides a method for continuously testing a statistical null hypothesis that the process has not changed and remains in statistical control. Since the specific departures of the process characteristic from the target value that may be of concern are not usually defined in advance, along with the risk of a type 2 error, and the sample size is not calculated to satisfy appropriate risk levels, the Shewhart control chart should not be considered in the sense of a test of hypothesis. (See ISO 7966 and ISO 7870). Shewhart emphasized the empirical usefulness of the control chart for recognizing departures from an "in-control" process and de-emphasized probabilistic interpretation. Some users do examine operating characteristic curves as a means of making a hypothesis test interpretation.

When a plotted value falls outside of either control limit or a series of values reflect unusual patterns such as discussed in clause 7, the state of statistical control can no longer be accepted. When this occurs, an investigation is initiated to locate the assignable cause and the process may be stopped or adjusted. Once the assignable cause is determined and eliminated, the process is ready to continue. As discussed above, for a type 1 error, on rare occasions, no assignable cause can be found and it must be concluded that the point outside the limits represents the occurrence of a very rare event, a random cause which has resulted in a value outside of the control limits even through the process is in control.

When control charts are first set up for a process, it frequently occurs that the process will be found to be out of control. Control limits calculated from data from an out-of-control process would lead to erroneous conclusions because they would be too far apart. Consequently, it is always necessary to bring an out-of-control process into control before permanent control chart parameters are established. The procedure for establishing control charts for a process will be discussed in the following clauses.

4 Types of control charts

Shewhart control charts are basically of two types: variables control charts and attributes control charts. For each of the control charts, there are two distinct situations:

a) when no standard values are given, and

b) when standard values are given.

The standard values are some specified requirement or target values.

4.1 Control charts where no standard values are given

The purpose here is to discover whether observed values of the plotted characteristics, such as $\overline{X}, R$ or any other statistic, vary among themselves by an amount greater than that which should be attributed to chance alone. Control charts based entirely on the data collected from samples are used for detecting those variations caused other than by chance.

4.2 Control charts with respect to given standard values

The purpose here is to identify whether the observed values of $\overline{X}$, etc., for several subgroups of n observations each, differ from the respective standard values X_0 (or μ_0), etc. by amounts greater than that expected to be due to chance causes only. The difference between charts with standards given and those where no standards are given is the additional requirement concerning the location of the centre and variation of the process. The specified values may be based on experience obtained by using control charts with no prior information or specified standard values. They may also be based on economic values established upon consideration of the need for service and cost of production or be nominal values designated by the product specifications.

Preferably, the specified values should be determined through an investigation of preliminary data that is supposed to be typical of all future data. The standard values should be compatible with the inherent process variability for effective functioning of the control charts. Control charts based on such standard values are used particularly during manufacture to control processes and to maintain product uniformity at the desired level.

4.3 Types of variables and attributes control charts

The following control charts are considered.

a) Variables control charts:

1) average $(\overline{X})$ chart and range (R) or standard deviation (s) chart;

2) charts for individuals (X) and moving range (R);

3) median (Me) chart and range (R) chart.

b) Attributes control charts:

1) fraction nonconforming (p) chart or number of nonconforming units (np) chart;

2) number of nonconformities (c) chart or nonconformities per unit (u) chart.

5 Variables control charts

Variables data represent observations obtained by measuring and recording the numerical magnitude of a characteristic for each of the units in the subgroup under consideration. Examples of variables measurements are length in metres, resistance in ohms, noise in decibels, etc. Variables charts — and especially their most customary forms, the $\bar{X}$ and R charts — represent the classic application of control charting to process control.

Control charts for variables are particularly useful for several reasons.

a) Most processes and their output have characteristics that are measurable, so the potential applicability is broad.

b) A measurement value contains more information than a simple yes—no statement.

c) The performance of a process can be analysed without regard to the specification. The charts start with the process itself and give an independent picture of what the process can do. Afterwards, the process may or may not be compared with the specification.

d) Although obtaining one piece of measured data is generally more costly than obtaining one piece of go/no go data, the subgroup sizes for variables are almost always much smaller than those for attributes, and so are more efficient. This helps to reduce the total inspection cost in some cases and to shorten the time gap between the production of parts and corrective action.

A normal (Gaussian) distribution is assumed for within-sample variability for all variables control chart applications considered in this International Standard and departures from this assumption will affect the performance of the charts. The factors for computing control limits were derived using the assumption of normality. Since most control limits are used as empirical guides in making decisions, reasonably small departures from normality should not cause concern. In any case, because of the central limit theorem, averages tend to be normally distributed even when individual observations are not; this makes it reasonable to assume normality for $\bar{X}$ charts, even for sample sizes as small as 4 or 5 for evaluating control. When dealing with individual observations for capability study purposes, the true form of the distribution is important. Periodic checks on the continuing validity of such assumptions are advisable, particularly for ensuring that only data from a single population are being used. It should be noted that the distribution of the ranges and standard deviations are not normal, although approximate normality was assumed in the estimation of the constants for the calculation of control limits, which is satisfactory for an empirical decision procedure.

5.1 Mean ($\bar{X}$) chart and range (R) or standard deviations (s) chart

Variables charts can describe process data in terms of both spread (piece-to-piece variability) and location (process average). Because of this, control charts for variables are almost always prepared and analysed in pairs — one chart for location and another for spread. The most commonly used pair is the $\bar{X}$ and R charts. Table 1 and table 2 give the control limit formulae and the factors for variables control charts respectively.

Table 1 — Control limit formulae for Shewhart variables control charts

Statistic	No standard values given		Standard values given	
	Central line	UCL and LCL	Central line	UCL and LCL
$\bar{X}$	$\bar{\bar{X}}$	$\bar{\bar{X}} \pm A_2\bar{R}$ or $\bar{\bar{X}} \pm A_3\bar{s}$	X_0 or μ	$X_0 \pm A\sigma_0$
R	$\bar{R}$	$D_3\bar{R}$, $D_4\bar{R}$	R_0 or $d_2\sigma_0$	$D_1\sigma_0$, $D_2\sigma_0$
s	$\bar{s}$	$B_3\bar{s}$, $B_4\bar{s}$	s_0 or $c_4\sigma_0$	$B_5\sigma_0$, $B_6\sigma_0$

NOTE — X_0, R_0, s_0, μ and σ_0 are given standard values.

Table 2 — Factors for computing control chart lines

Observations in subgroup	Factors for control limits											Factors for central line			
n	A	A_2	A_3	B_3	B_4	B_5	B_6	D_1	D_2	D_3	D_4	C_4	$1/C_4$	d_2	$1/d_2$
2	2,121	1,880	2,659	0,000	3,267	0,000	2,606	0,000	3,686	0,000	3,267	0,797 9	1,253 3	1,128	0,886 5
3	1,732	1,023	1,954	0,000	2,568	0,000	2,276	0,000	4,358	0,000	2,574	0,886 2	1,128 4	1,693	0,590 7
4	1,500	0,729	1,628	0,000	2,266	0,000	2,088	0,000	4,698	0,000	2,282	0,921 3	1,085 4	2,059	0,485 7
5	1,342	0,577	1,427	0,000	2,089	0,000	1,964	0,000	4,918	0,000	2,114	0,940 0	1,063 8	2,326	0,429 9
6	1,225	0,483	1,287	0,030	1,970	0,029	1,874	0,000	5,078	0,000	2,004	0,951 5	1,051 0	2,534	0,394 6
7	1,134	0,419	1,182	0,118	1,882	0,113	1,806	0,204	5,204	0,076	1,924	0,959 4	1,042 3	2,704	0,369 8
8	1,061	0,373	1,099	0,185	1,815	0,179	1,751	0,388	5,306	0,136	1,864	0,965 0	1,036 3	2,847	0,351 2
9	1,000	0,337	1,032	0,239	1,761	0,232	1,707	0,547	5,393	0,184	1,816	0,969 3	1,031 7	2,970	0,336 7
10	0,949	0,308	0,975	0,284	1,716	0,276	1,669	0,687	5,469	0,223	1,777	0,972 7	1,028 1	3,078	0,324 9
11	0,905	0,285	0,927	0,321	1,679	0,313	1,637	0,811	5,535	0,256	1,744	0,975 4	1,025 2	3,173	0,315 2
12	0,866	0,266	0,886	0,354	1,646	0,346	1,610	0,922	5,594	0,283	1,717	0,977 6	1,022 9	3,258	0,306 9
13	0,832	0,249	0,850	0,382	1,618	0,374	1,585	1,025	5,647	0,307	1,693	0,979 4	1,021 0	3,336	0,299 8
14	0,802	0,235	0,817	0,406	1,594	0,399	1,563	1,118	5,696	0,328	1,672	0,981 0	1,019 4	3,407	0,293 5
15	0,775	0,223	0,789	0,428	1,572	0,421	1,544	1,203	5,741	0,347	1,653	0,982 3	1,018 0	3,472	0,288 0
16	0,750	0,212	0,763	0,448	1,552	0,440	1,526	1,282	5,782	0,363	1,637	0,983 5	1,016 8	3,532	0,283 1
17	0,728	0,203	0,739	0,466	1,534	0,458	1,511	1,356	5,820	0,378	1,622	0,984 5	1,015 7	3,588	0,278 7
18	0,707	0,194	0,718	0,482	1,518	0,475	1,496	1,424	5,856	0,391	1,608	0,985 4	1,014 8	3,640	0,274 7
19	0,688	0,187	0,698	0,497	1,503	0,490	1,483	1,487	5,891	0,403	1,597	0,986 2	1,014 0	3,689	0,271 1
20	0,671	0,180	0,680	0,510	1,490	0,504	1,470	1,549	5,921	0,415	1,585	0,986 9	1,013 3	3,735	0,267 7
21	0,655	0,173	0,663	0,523	1,477	0,516	1,459	1,605	5,951	0,425	1,575	0,987 6	1,012 6	3,778	0,264 7
22	0,640	0,167	0,647	0,534	1,466	0,528	1,448	1,659	5,979	0,434	1,566	0,988 2	1,011 9	3,819	0,261 8
23	0,626	0,162	0,633	0,545	1,455	0,539	1,438	1,710	6,006	0,443	1,557	0,988 7	1,011 4	3,858	0,259 2
24	0,612	0,157	0,619	0,555	1,445	0,549	1,429	1,759	6,031	0,451	1,548	0,989 2	1,010 9	3,895	0,256 7
25	0,600	0,153	0,606	0,565	1,435	0,559	1,420	1,806	6,056	0,459	1,541	0,989 6	1,010 5	3,931	0,254 4

Source: ASTM, Philadelphia, PA, USA.

5.2 Control chart for individuals (X)

In some process control situations, it is either impossible or impractical to take rational subgroups. The time or cost required to measure a single observation is so great that repeat observations cannot be considered. This would typically occur when the measurements are expensive (e.g. in a destructive test) or when the output at any time is relatively homogeneous. In other situations there is only one possible value, e.g. an instrument reading or a property of a batch of input material. In such situations, it is necessary for process control to be based on individual readings.

In the case of charts for individuals, since there are no rational subgroups to provide an estimate of within-batch variability, control limits are based on a variation measure obtained from moving ranges of, often, two observations. A moving range is the absolute difference between successive pairs of measurements in a series; i.e. the difference between the first and second measurements, then between the second and third, and so on. From the moving ranges, the average moving range $\bar{R}$ is calculated and used for the construction of control charts. Also, from the entire data, the overall average $\bar{\bar{X}}$ is calculated. Table 3 gives the control limit formulae for control charts for individuals.

Some caution should be exercised with respect to control charts for individuals.

a) The charts for individuals are not as sensitive to process changes as are the $\bar{X}$ and R charts.

b) Care shall be taken in the interpretation of charts for individuals if the process distribution is not normal.

c) Charts for individuals do not isolate the piece-to-piece repeatability of the process and, therefore, it may be better in some applications to use a conventional $\bar{X}$ and R chart with small subgroup sample sizes (2 to 4) even if this requires a larger period between subgroups.

Table 3 — Control limit formulae for control charts for individuals

Statistic	No standard values given		Standard values given	
	Central line	UCL and LCL	Central line	UCL and LCL
Individual, X	$\overline{X}$	$\overline{X} \pm E_2\overline{R}$	X_0 or μ	$X_0 \pm 3\sigma_0$
Moving range, R	$\overline{R}$	$D_4\overline{R}$, $D_3\overline{R}$	R_0 or $d_2\sigma_0$	$D_2\sigma_0$, $D_1\sigma_0$

NOTES

1 X_0, R_0, μ and σ_0 are given standard values.

2 $\overline{R}$ denotes the average moving range of $n = 2$ observations.

3 The values of the factors d_2, D_1, D_2, D_3, D_4 and, indirectly, E_2 ($= 3/d_2$) can be obtained from table 2 for $n = 2$

5.3 Control charts for medians (Me)

Median charts are alternatives to $\overline{X}$ and R charts for the control of a process with measured data; they yield similar conclusions and have several specific advantages. They are easy to use and do not require as many calculations. This can increase shop floor acceptance of the control chart approach. Since individual values (as well as medians) are plotted, the median chart shows the spread of process output and gives an ongoing picture of the process variation.

Control limits for median charts are calculated in two ways: by using the median of the subgroup medians and the median of the ranges; or by using the average of the subgroup medians and the average of the ranges. Only the latter approach, which is easier and more convenient, is considered in this International Standard.

The control limits are calculated as follows.

5.3.1 Median chart

Central line $= \overline{Me} =$ average of the subgroup medians

$$UCL_{Me} = \overline{Me} + A_4\overline{R}$$

$$LCL_{Me} = \overline{Me} - A_4\overline{R}$$

The range chart is constructed in the same way as for the case of the $\overline{X}$ and R charts in 5.1.

The values of the constant A_4 are given in table 4.

Table 4 — Values of A_4

n	2	3	4	5	6	7	8	9	10
A_4	1,88	1,19	0,80	0,69	0,55	0,51	0,43	0,41	0,36

It should be noted that the median chart with 3σ limits gives a slower response to out-of-control conditions than an $\overline{X}$ chart.

5.3.2 Range chart

Central line $= \overline{R} =$ average value of the R values for all subgroups

$$UCL_R = D_4\overline{R}$$

$$LCL_R = D_3\overline{R}$$

The values of the constants D_3 and D_4 are given in table 2.

6 Control procedure and interpretation for variables control charts

The Shewhart system of charts stipulates that if the process piece-to-piece variability and the process average were to remain constant at their present levels (as estimated by $\overline{R}$ and $\overline{\overline{X}}$ respectively), the individual subgroup ranges (R) and averages ($\overline{X}$) would vary by chance alone and they would seldom go beyond the control limits. Likewise, there would be no obvious trends or patterns in the data, beyond what would probably occur due to chance.

The $\overline{X}$ chart shows where the process average is centred and indicates the stability of the process. The $\overline{X}$ chart reveals undesirable variations between subgroups as far as their average is concerned. The R chart reveals any undesirable variation within subgroups and is an indicator of the magnitude of the variability of the process under study. It is a measure of process consistency or uniformity. The R chart stays in control if the within-subgroup variations are essentially the same. This happens only if all the samples receive the same treatment. If the R chart does not remain in control, or if its level rises, it may indicate that either different subgroups are being subjected to different treatments or several different cause-effect systems are operating on the process.

$\overline{X}$ charts can also be affected by out-of-control conditions on the R chart. Since the ability to interpret either the subgroup ranges or the subgroup averages depends on the estimate of piece-to-piece variability, the R chart is analysed first. The following control procedure should be followed.

6.1 Gather and analyse data, calculating averages and ranges.

6.2 Plot the R chart first. Check the data points against the control limits for points out of control or for unusual patterns or trends. For each indication of an assignable cause in the range data, conduct an analysis of the operation of the process to determine the cause; correct that condition and prevent in from recurring.

6.3 Exclude all subgroups affected by an identified assignable cause; then recalculate and plot the new average range $(\overline{R})$ and control limits. Confirm that all range points show statistical control when compared to the new limits, repeating the identification/correction/recalculation sequence if necessary.

6.4 If any subgroups are dropped from the R chart because of identified assignable causes, they shall also be excluded from the $\overline{X}$ chart. The revised $\overline{R}$ and $\overline{\overline{X}}$ values shall be used to recalculate the trial control limits for averages, $\overline{\overline{X}} \pm A_2\overline{R}$.

NOTE 2 The exclusion of subgroups representing out-of-control conditions is not just "throwing away bad data". Rather, by excluding the points affected by known assignable causes, we have a better estimate of the background level of variation due to chance causes. This, in turn, gives the most appropriate basis for the control limits used to detect most efficiently future occurrences of assignable causes of variation.

6.5 When the ranges are in statistical control, the process spread (the within-subgroup variation) is considered to be stable. The averages can then be analysed to see if the process location is changing with time.

6.6 Now plot the $\overline{X}$ chart and check the data points against the control limits for points out of control or for unusual patterns or trends. As for the R chart, analyse any out-of-control condition and take corrective and preventive action. Exclude any out-of-control points for which assignable causes have been found; recalculate and plot the new process average $(\overline{\overline{X}})$ and control limits. Confirm that all data points show statistical control when compared to the new limits, repeating the identification/-correction/recalculation sequence if necessary.

6.7 When the initial data to establish control limit reference values are consistently contained within the trial limits, extend the limits to cover future periods. These limits shall be used for ongoing control of the process, with the responsible individuals (operator and/or supervisor) responding to signs of out-of-control conditions on either the $\overline{X}$ or R chart with prompt action.

7 Pattern tests for assignable causes of variation

A set of eight supplementary tests used for interpreting patterns in Shewart charts is schematically presented in figure 2. For a more complete discussion of these tests, see [2] and [3].

Although this can be taken as a basic set of tests, analysts should be alert to any unique patterns of points that might indicate the influence of special causes in their process. These tests should therefore be viewed as simply practical rules for action whenever the presence of assignable causes is indicated. An indication of any of the conditions stipulated in these tests is an indication of the presence of assignable causes of variation that must be diagnosed and corrected.

The upper and lower control limits are set at a distance of 3σ above and below the central line. For the purpose of applying the tests, the control chart is equally divided into six zones, each zone being 1σ wide. These are labelled A, B, C, C, B, A with zones C placed symmetrically about the central line. These tests are applicable to $\overline{X}$ charts and to individual (X) charts. A normal distribution is assumed.

8 Process control and process capability

The function of a process control system is to provide a statistical signal when assignable causes of variation are present. The systematic elimination of assignable causes of excessive variation, through continuous determined efforts, brings the process into a state of statistical control. Once the process is operating in statistical control, its performance is predictable and its capability to meet the specifications can be assessed.

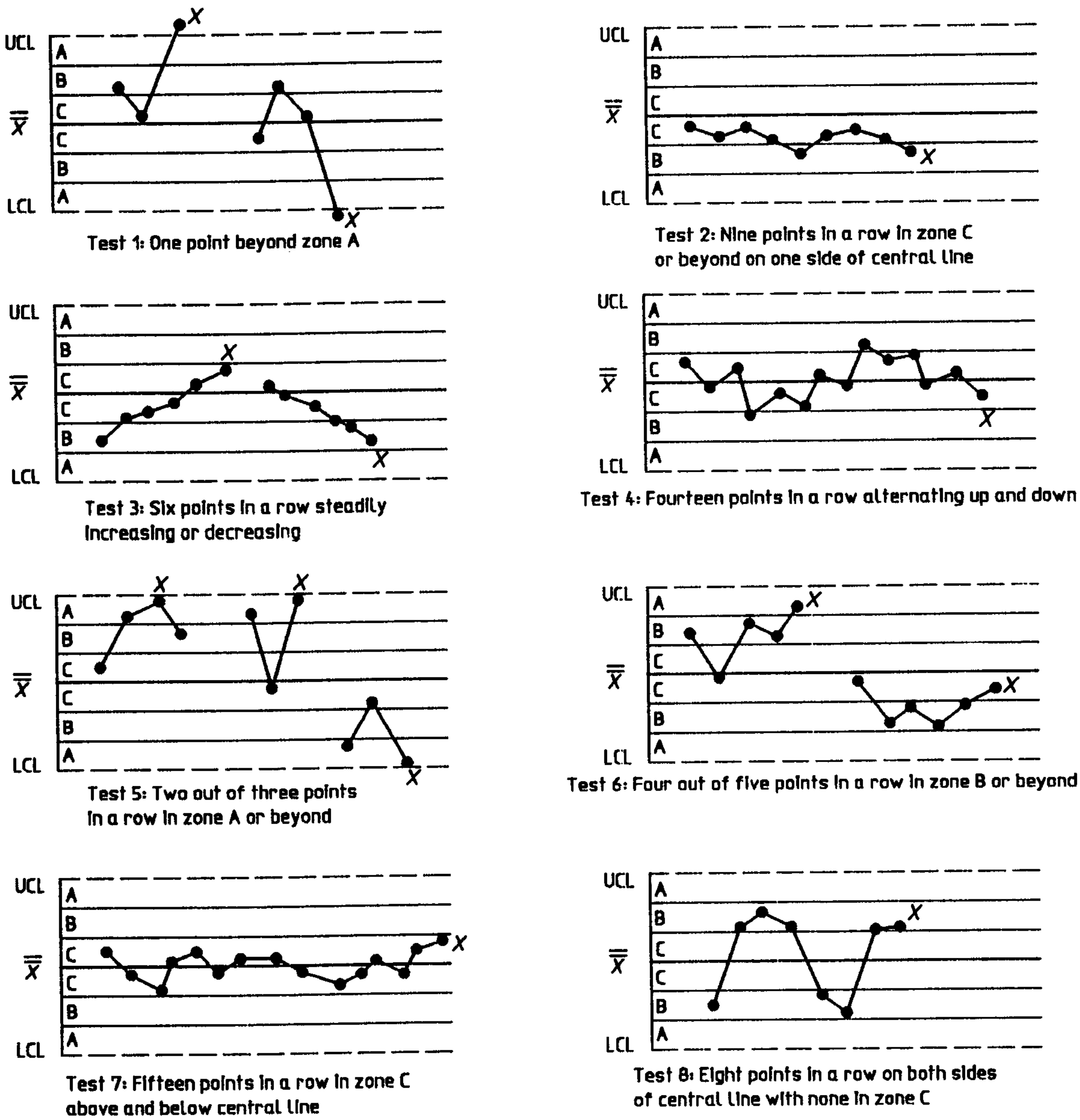

Figure 2 — Tests for assignable causes

Process capability is determined by the total variation that comes from common causes — the minimum variation that can be achieved after all assignable causes have been eliminated. Process capability represents the performance of the process itself, as demonstrated when the process is being operated in a state of statistical control. As such, the process must first be brought into statistical control before its capability can be assessed. Thus, the assessment of process capability begins after control issues in both the $\overline{\overline{X}}$ and R charts have been resolved; that is, special causes have been identified, analysed, corrected and prevented from recurring and the ongoing control charts reflect a process that has remained in statistical control preferably for at least the past 25 subgroups. In general, the distribution of the process output is compared with the engineering specifications, to see whether these specifications can consistently be met.

Process capability is generally measured in terms of a process capability index PCI (or c_p) as follows:

$$\text{PCI} = \frac{\text{tolerance specified}}{\text{process spread}} = \frac{\text{UTL} - \text{LTL}}{6\hat{\sigma}}$$

where

UTL is the upper tolerance limit;

LTL is the lower tolerance limit;

$\hat{\sigma}$ is estimated from the average within-subgroup variability and is given by $\bar{s}/c_4$ or $\bar{R}/d_2$.

A PCI value of less than 1 indicates that the process is not capable, while PCI = 1 implies that the process is only just capable. In practice, a PCI value of 1,33 is generally taken as the minimum acceptable value because there is always some sampling variation and no process is ever fully in statistical control.

It must, however, be noted that the PCI measures only the relationship of the limits to the process spread; the location or the centring of the process is not considered. It would be possible to have any percentage of values outside the specification limits with a high PCI value. For this reason, it is important to consider the scaled distance between the process average and the closest specification limit. Further discussion of this topic is beyond the scope of this International Standard.

In view of the above discussion, a procedure, as schematically presented in figure 3, can be used as a guide to illustrate key steps leading towards process control and improvement.

9 Attributes control charts

Attributes data represent observations obtained by noting the presence or absence of some characteristic (or attribute) in each of the units in the subgroup under consideration, then counting how many units do or do not possess the attribute, or how many such events occur in the unit, group or area. Attributes data are generally rapid and inexpensive to obtain and often do not require specialized collection skills. Table 5 gives control limit formulae for attributes control charts.

In the case of control charts for variables, it is common practice to maintain a pair of control charts, one for the control of the average and the other for the control of the dispersion. This is necessary because the underlying distribution in the control charts for variables is the normal distribution, which depends on these two parameters. However, in the case of control charts for attributes, a single chart will suffice since the assumed distribution has only one independent parameter, the average level. The p and np charts are based on the binomial distribution, while the c and u charts are based on the Poisson distribution.

Computations for these charts are similar except in cases where the variability in subgroup size affects the situation. When the subgroup size is constant, the same set of control limits can be used for each subgroup. However, if the number of items inspected in each subgroup varies, separate control limits have to be computed for each subgroup. np and c charts may thus be reasonably used with a constant sample size, whereas p and u charts could be used in either situation.

Where the sample size varies from sample to sample, separate control limits are calculated for each sample. The smaller the subgroup size, the wider the control bands, and vice versa. If the subgroup size does not vary appreciably, then a single set of control limits based on the average subgroup size can be used. For practical purposes, this holds well for situations in which the subgroup size is within ± 25 % of the target subgroup size.

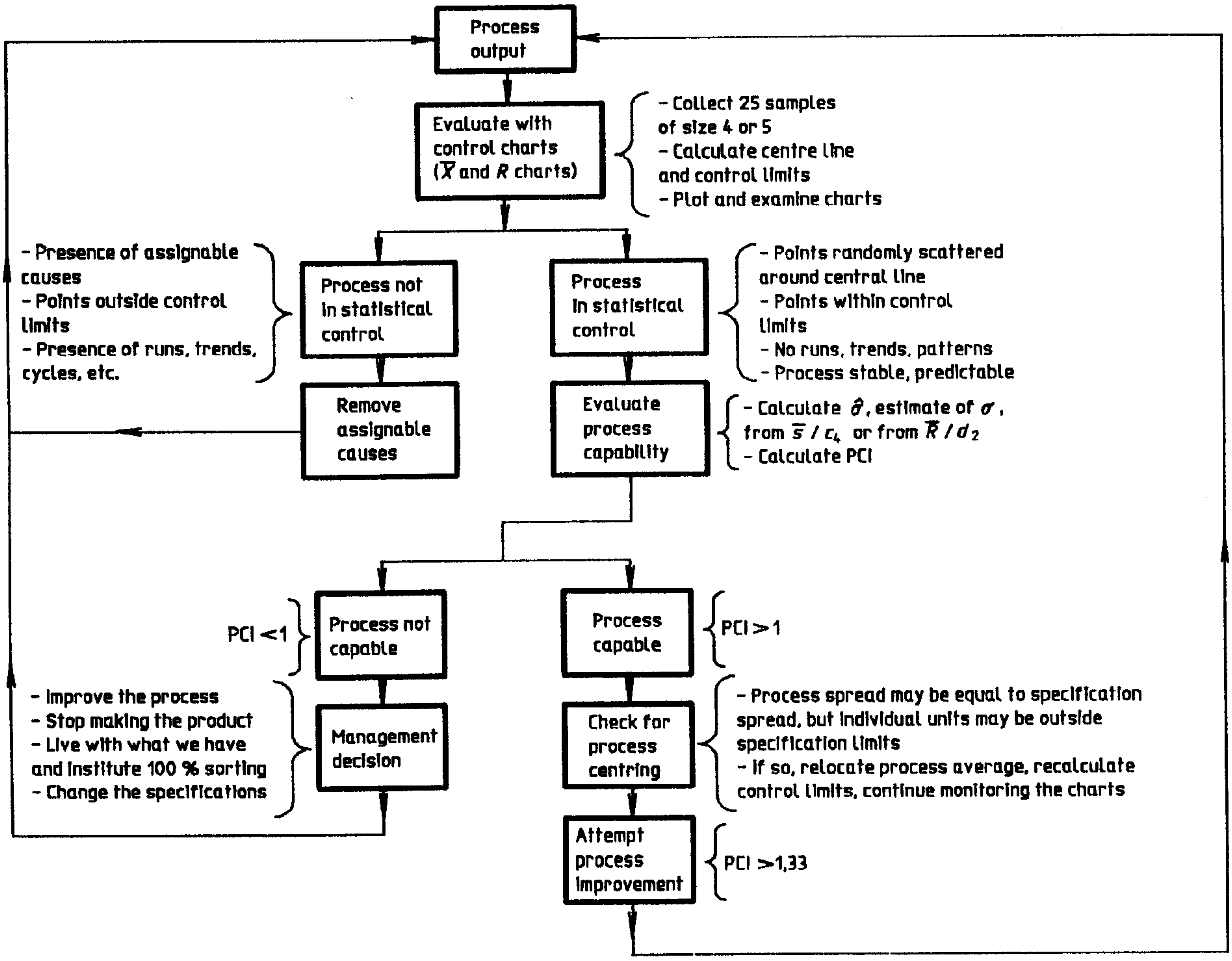

Figure 3 — Strategy for process improvement

Table 5 — Control limit formulae for Shewhart attributes control charts

Statistic	No standard values given		Standard values given	
	Central line	3σ control limits	Central line	3σ control limits
p	$\overline{p}$	$\overline{p} \pm \sqrt{\overline{p}(1-\overline{p})/n}$	p_0	$p_0 \pm 3\sqrt{p_0(1-p_0)/n}$
np	$n\overline{p}$	$n\overline{p} \pm 3\sqrt{n\overline{p}(1-\overline{p})}$	np_0	$np_0 \pm 3\sqrt{np_0(1-p_0)}$
c	$\overline{c}$	$\overline{c} \pm 3\sqrt{\overline{c}}$	c_0	$c_0 \pm 3\sqrt{c_0}$
u	$\overline{u}$	$\overline{u} \pm 3\sqrt{\overline{u}/n}$	u_0	$u_0 \pm 3\sqrt{u_0/n}$
NOTE — p_0, np_0, c_0 and u_0 are given standard values.				

An alternative procedure for situations in which the sample size varies greatly is the use of a standardized variate. For example, instead of plotting p, plot the standardized value

$$Z = \frac{p - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

or

$$Z = \frac{p - \bar{p}}{\sqrt{\bar{p}(1 - \bar{p})/n}}$$

according to whether the standard value for p is specified or not. The central line as well as the control limits become constant, independent of subgroup size, and are given as

central line = 0

UCL = 3

LCL = −3

The p chart is used to determine the average percentage of nonconforming items submitted over a period of time. It brings to the attention of process personnel and management any changes in this average. The process is judged to be in statistical control in the same way as is done for the $\bar{X}$ and R charts. If all the sample points fall within the trial control limits without exhibiting any indication of an assignable cause, the process is said to be in control. In such a case, the average fraction nonconforming, $\bar{p}$, is taken as the standard value for the fraction nonconforming, p_0.

10 Preliminary considerations before starting a control chart

10.1 Choice of quality characteristics

Select the quality characteristics for the control programme. Characteristics affecting the performance of the product or service should normally be the object of first attention. These may be aspects of the service offered or features of the material used, of the component parts of the product, as well as of the finished product delivered to the purchaser. Statistical methods of control should be primarily introduced where the control chart will assist in furnishing information about a process in a timely manner so that the process can be corrected and a better product or service produced. Product or service quality characteristics should be selected to have a decisive effect on product or service quality and to ensure the stability of processes.

10.2 Analysis of the production process

A detailed analysis of the production process should be made to determine

a) the kind and location of causes that may give rise to irregularities;

b) the effect of the imposition of specifications;

c) the method and location of inspection;

d) all other pertinent factors that may affect the production process.

Analysis should also be performed to determine the stability of production processes, the accuracy of production and testing equipment, the quality of products or services produced, and the patterns of correlation between the types and causes of nonconformities. The conditions of production operations and product quality are required to make arrangements to adjust the production process and equipment, if needed, as well as to devise plans for the statistical control of production processes. This will help pinpoint the most optimal place to establish controls and identify quickly any irregularities in the performance of the production process to allow for prompt corrective action.

10.3 Choice of rational subgroups

At the basis of control charts is Shewhart's central idea of the division of observations into what are called "rational subgroups"; that is the classification of the observations under consideration into subgroups, within which the variations may be considered to be due to chance causes only, but between which any difference may be due to assignable causes which the control chart is intended to detect.

This depends on some technical knowledge and familiarity with the production conditions and the conditions under which the data were taken. By identifying each subgroup with a time or a source, specific causes of trouble may be more readily traced and corrected, if advantageous. Inspection and test records given in the order in which the observations were taken provide a basis for subgrouping with respect to time. This is commonly useful in manufacturing where it is important to maintain the production cause system constant with time.

It should always be remembered that analysis will be greatly facilitated if, when planning for the collection of data, care is taken to select the samples so that the data from each subgroup can be properly treated as a separate rational subgroup and that the subgroups are identified in such a way as to make this possible. Also, insofar as possible, the subgroup size, n, should be kept constant to avoid tedium in calculations and interpretation. However, it should be noted that the principles of Shewhart charts can equally be applied to situations where n varies.

10.4 Frequency and size of samples

No general rules may be laid down for the frequency of subgroups or the subgroup size. The frequency may depend upon the cost of taking and analysing samples and the size of the subgroup may depend upon practical considerations. For instance, large subgroups taken at less frequent intervals may detect a small shift in the process average more accurately, but small subgroups taken at more frequent intervals will detect a large shift more quickly. Often, the subgroup size is taken to be 4 or 5, while the sampling frequency is generally high in the beginning and low once a state of statistical control is reached. Normally, 20 to 25 subgroups of size 4 or 5 are considered adequate for providing preliminary estimates.

It is worth noting that sampling frequency, statistical control and process capability need to be considered together. The reasoning is as follows. The value of the average range $\overline{R}$ is often used to estimate σ. The number of sources of variation increases as the time interval between samples within a subgroup increases. Therefore, spreading out the samples within a subgroup over time will increase $\overline{R}$, increase the estimate of σ, widen the control limits and will thus appear to decrease the process capability index. Conversely, it is possible to increase process capability by consecutive piece sampling, giving a small $\overline{R}$ and σ estimate, but statistical control will be difficult to attain.

10.5 Preliminary data collection

After having decided upon the quality characteristic which is to be controlled and the frequency and size of the subgroup to be taken, some initial inspection data or measurements have to be collected and analysed for the purpose of providing preliminary control chart values that are needed in determining the central line and control limits to be drawn on the chart. The preliminary data may be collected subgroup by subgroup until 20 to 25 subgroups have been obtained from a continuous run of the production process. Care shall be exercised that, during the course of this initial data collection, the process is not unduly influenced intermittently by extraneous factors such as change in the feed of raw material, operations, machine settings, etc. In other words, the process should exhibit a state of stability during the period when preliminary data are being gathered.

11 Steps in the construction of control charts

The steps involved in the construction of the $\overline{X}$ chart and the R chart, for the case when no standard values are given, are described in 11.1 to 11.5. They are described in the form of an example in 12.2. In the construction of other control charts, the same basic steps shall be followed but the constants for the computations are different (see table 1 and table 2). A general format of a standard control chart form is shown in figure 4. Modifications to this form can be made in concert with the particular requirements of a process control situation.

11.1 If the preliminary data were not taken in subgroups according to a prescribed plan, break up the total set of observed values into sequential subgroups, according to the criteria for rational subgroups as discussed in 10.3. The subgroups must be of the same structure and size. The items of any one subgroup should have what is believed to be some important common factor, for example units produced during the same short interval of time or units coming from one of several distinct sources or locations. The different subgroups should represent possible or suspect differences in the process that produced them, for example different intervals of time or different sources or locations.

11.2 For each subgroup, calculate the average, $\overline{X}$, and the range, R.

11.3 Compute the grand average of all the observed values, $\overline{\overline{X}}$, and the average range, $\overline{R}$.

11.4 On a suitable form or graph paper, lay out an $\overline{X}$ chart and an R chart. The vertical scale on the left is used for $\overline{X}$ and for R and the horizontal scale for the subgroup number. Plot the computed values for $\overline{X}$ on the chart for averages and plot the computed values for R on the chart for ranges.

11.5 On these respective charts, draw solid horizontal lines to represent $\overline{\overline{X}}$ and $\overline{R}$.

11.6 Place the control limits on these charts. On the $\overline{X}$ chart, draw two horizontal dashed lines at $\overline{\overline{X}} \pm A_2\overline{R}$ and, on the R chart, draw two horizontal dashed lines at $D_3\overline{R}$ and $D_4\overline{R}$, where A_2, D_3 and D_4 are based on n, the number of observations in a subgroup, and are given in table 2. The LCL on the R chart is not needed whenever n is less than 7 since the ensuing value of D_3 is considered zero.

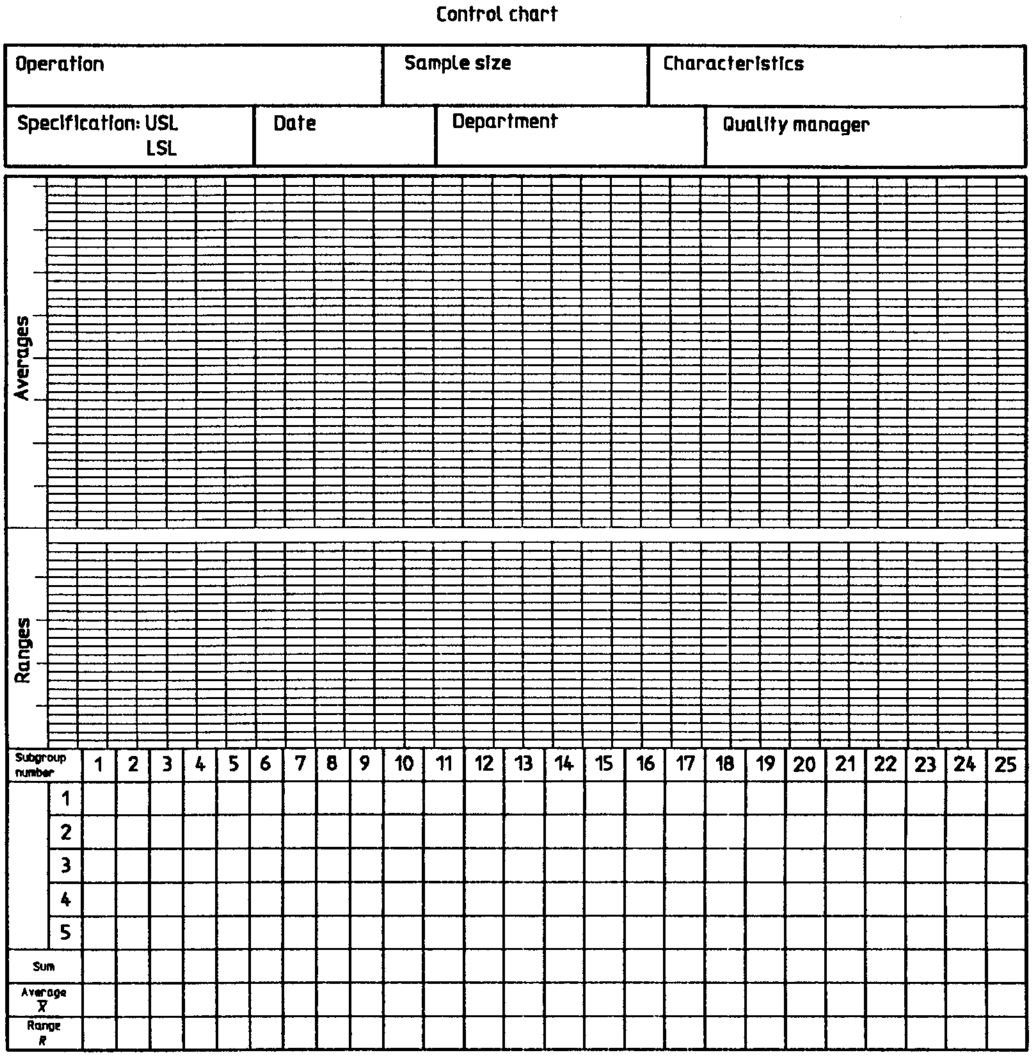

Figure 4 — General format of a control chart

12 Illustrative examples: Variables control charts

12.1 $\bar{X}$ chart and R chart: Standard values given

The production manager of a tea importer wishes to control his packaging process such that the mean weight of the packages is 100,6 g. The assumed process standard deviation is 1,4 g, based on similar packing processes.

Since the standard values are given ($X_0 = 100{,}6$, $\sigma_0 = 1{,}4$), the control charts for the mean and range can be immediately constructed using the formulae given in table 1 and the factors A, d_2, D_2 and D_1 given in table 2 for $n = 5$.

$\overline{X}$ chart

Central line $= X_0$
$= 100,6$ g

UCL $= X_0 + A\sigma_0$
$= 100,6 + (1,342 \times 1,4)$
$= 102,5$ g

LCL $= X_0 - A\sigma_0$
$= 100,6 - (1,342 \times 1,4)$
$= 98,7$ g

***R* chart**

Central line $= d_2\sigma_0$
$= 2,326 \times 1,4$
$= 3,3$ g

UCL $= D_2\sigma_0$
$= 4,918 \times 1,4$
$= 6,9$ g

LCL $= D_1\sigma_0$
$= 0 \times 1,4$ (since n is less than 7, LCL will not be shown)

Twenty five samples of size 5 are now selected; their subgroup average and range values are calculated (see table 6) and plotted with the control limits calculated above (see figure 5).

The charts, shown in figure 5, indicate that the process is out of control at the desired level because there is a sequence of 13 points below the central line in the $\overline{X}$ chart and 16 points above the central line in the R chart. The cause of such a long sequence of low values of the mean should be investigated and eliminated.

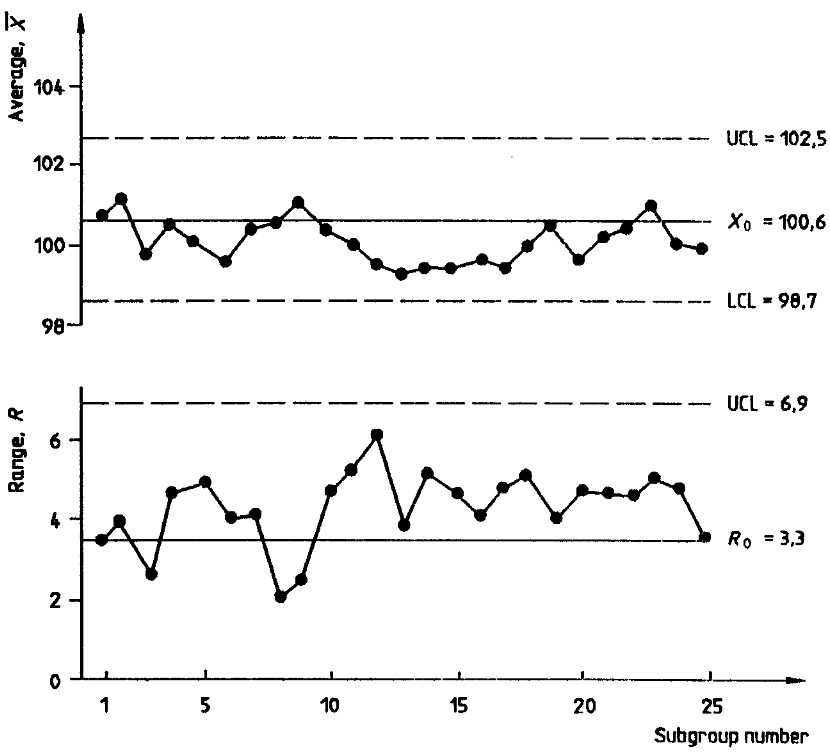

Figure 5 — Average and range chart for data given in table 6

Table 6 — Tea packing process

Subgroup No.	Subgroup average $\overline{X}$	Subgroup range R
1	100,6	3,4
2	101,3	4,0
3	99,6	2,2
4	100,5	4,5
5	99,9	4,8
6	99,5	3,8
7	100,4	4,1
8	100,5	1,7
9	101,1	2,2
10	100,3	4,6
11	100,1	5,0
12	99,6	6,1
13	99,2	3,5
14	99,4	5,1
15	99,4	4,5
16	99,6	4,1
17	99,3	4,7
18	99,9	5,0
19	100,5	3,9
20	99,5	4,7
21	100,1	4,6
22	100,4	4,4
23	101,1	4,9
24	99,9	4,7
25	99,7	3,4

12.2 $\overline{X}$ chart and R chart: No standard values given

In table 7, measurements of the outside radius of a plug are given. Four measurements are taken every half hour for a total of 20 samples. The subgroup averages and ranges are also given in table 7. The specified tolerances are 0,219 dm and 0,125 dm. The objective is to evaluate the process performance and to control the process with respect to its location and spread so that the process would meet the specifications.

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{k}$$

$$= \frac{3,8480}{20} = 0,1924$$

$$\overline{R} = \frac{\sum R}{k}$$

$$= \frac{0,5734}{20} = 0,0287$$

The first step is to plot an R chart and evaluate its state of control.

R **chart**

Central line $= \overline{R}$
$= 0,028\,7$

UCL $= D_4\overline{R}$
$= 2,282 \times 0,028\,7$
$= 0,065\,5$

LCL $= D_3\overline{R}$
$= 0 \times 0,028\,7$ (since n is less than 7, LCL is not shown)

The values of multiplying factors D_3 and D_4 are taken from table 2 for $n = 4$. Since the R values in table 7 are within the R chart control limits, the R chart indicates a state of statistical control. The $\overline{R}$ value can now be used to calculate $\overline{X}$ chart control limits.

$\overline{X}$ **chart**

Central line $= \overline{\overline{X}}$
$= 0,192\,4$

UCL $= \overline{\overline{X}} + A_2\overline{R}$
$= 0,192\,4 + (0,729 \times 0,028\,7)$
$= 0,213\,3$

LCL $= \overline{\overline{X}} - A_2\overline{R}$
$= 0,192\,4 - (0,729 \times 0,028\,7)$
$= 0,171\,5$

The value of the factor A_2 is taken from table 2 for $n = 4$. The control charts for $\overline{X}$ and R are plotted in figure 6. An examination of the $\overline{X}$ chart reveals that the last three points are out of control. It indicates that some assignable causes of variation may be operating. If the limits had been calculated from previous data, action would have been called for at point 18.

At this point, suitable remedial action is taken to eliminate the assignable causes and prevent their re-occurrence. The charting procedure is continued by establishing revised control limits by discarding the out-of-control points, i.e. the values for sample numbers 18, 19 and 20. The values of $\overline{\overline{X}}$, $\overline{R}$ and control chart lines are recalculated as follows:

$$\text{Revised } \overline{\overline{X}} = \frac{\sum \overline{X}}{k}$$

$$= \frac{3,3454}{17} = 0,1968$$

$$\text{Revised } \overline{R} = \frac{\sum R}{k}$$

$$= \frac{0,5272}{17} = 0,0310$$

Revised $\overline{X}$ chart

Central line $= \overline{\overline{X}}$
$= 0,196\,8$

UCL $= \overline{\overline{X}} + A_2\overline{R}$
$= 0,196\,8 + (0,729 \times 0,031\,0)$
$= 0,219\,4$

LCL $= \overline{\overline{X}} - A_2\overline{R}$
$= 0,196\,8 - (0,729 \times 0,031\,0)$
$= 0,174\,2$

Revised R chart

Central line $= \overline{R}$
$= 0,031\,0$

UCL $= D_4\overline{R}$
$= 2,282 \times 0,031\,0$
$= 0,070\,7$

LCL $= D_3\overline{R}$
$= 0 \times 0,031\,0$ (since n is less than 7, LCL is not shown).

The revised control charts are plotted in figure 7.

Table 7 — Production data on the outside radius of a plug

Subgroup No.	Radius				Mean	Range
	X_1	X_2	X_3	X_4	$\overline{X}$	R
1	0,189 8	0,172 9	0,206 7	0,189 8	0,189 8	0,033 8
2	0,201 2	0,191 3	0,187 8	0,192 1	0,193 1	0,013 4
3	0,221 7	0,219 2	0,207 8	0,198 0	0,211 7	0,023 7
4	0,183 2	0,181 2	0,196 3	0,180 0	0,185 2	0,016 3
5	0,169 2	0,226 3	0,206 6	0,209 1	0,203 3	0,057 1
6	0,162 1	0,183 2	0,191 4	0,178 3	0,178 8	0,029 3
7	0,200 1	0,192 7	0,216 9	0,208 2	0,204 5	0,024 2
8	0,240 1	0,182 5	0,191 0	0,226 4	0,210 0	0,057 6
9	0,199 6	0,198 0	0,207 6	0,202 3	0,201 9	0,009 6
10	0,178 3	0,171 5	0,182 9	0,196 1	0,182 2	0,024 6
11	0,216 6	0,174 8	0,196 0	0,192 3	0,194 9	0,041 8
12	0,192 4	0,198 4	0,237 7	0,200 3	0,207 2	0,045 3
13	0,176 8	0,198 6	0,224 1	0,202 2	0,200 4	0,047 3
14	0,192 3	0,187 6	0,190 3	0,198 6	0,192 2	0,011 0
15	0,192 4	0,199 6	0,212 0	0,216 0	0,205 0	0,023 6
16	0,172 0	0,194 0	0,211 6	0,232 0	0,204 9	0,060 0
17	0,182 4	0,179 0	0,187 6	0,182 1	0,182 8	0,008 6
18	0,181 2	0,158 5	0,169 9	0,168 0	0,169 4	0,022 7
19	0,170 0	0,156 7	0,169 4	0,170 2	0,166 6	0,013 5
20	0,169 8	0,166 4	0,170 0	0,160 0	0,166 6	0,010 0

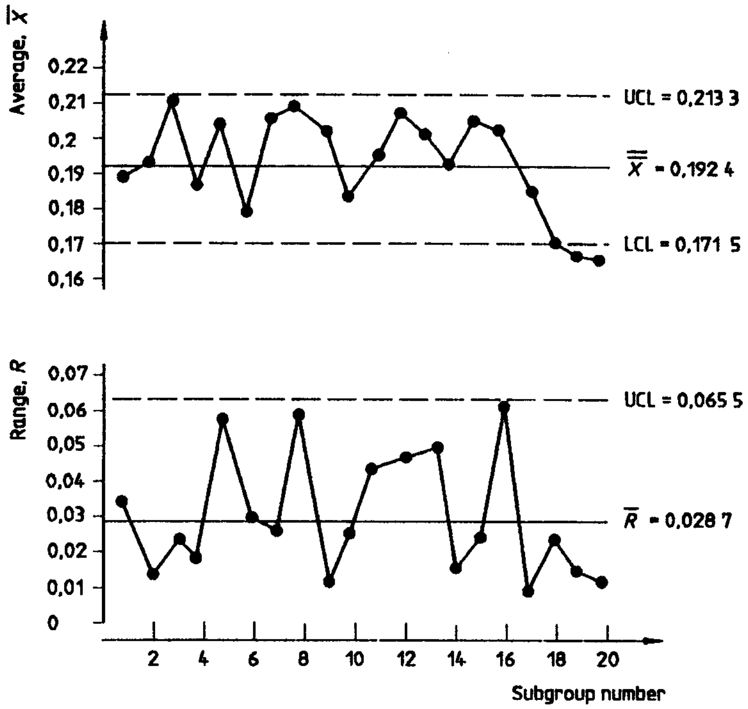

Figure 6 — Average and range charts for data given in table 7

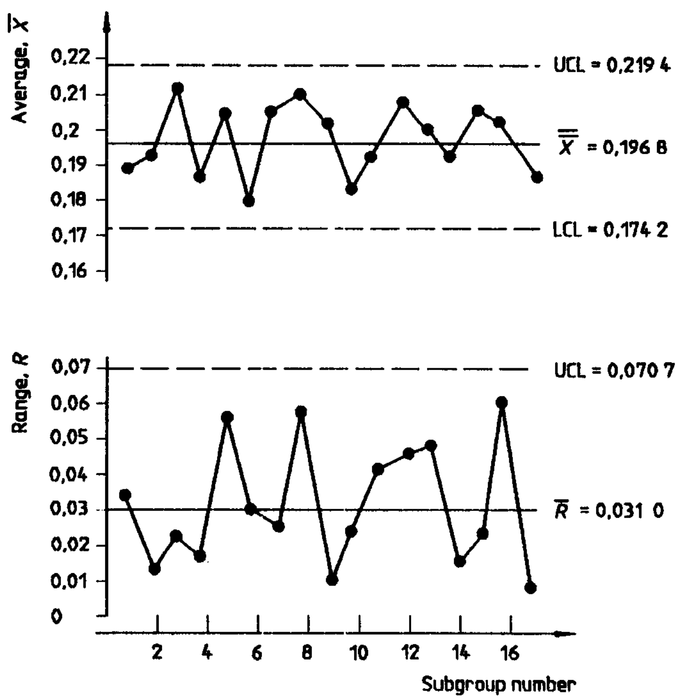

Figure 7 — Revised $\overline{X}$ and R charts for data given in table 7

With the process exhibiting a state of statistical control with respect to the revised control limits, process capability can be evaluated.

Calculate

$$\mathrm{PCI} = \frac{\text{tolerance specified}}{\text{process spread}} = \frac{\mathrm{UTL} - \mathrm{LTL}}{6\hat{\sigma}}$$

where $\hat{\sigma}$ is estimated by $\overline{R}/d_2$ = 0,031 0/2,059 = 0,015 1.

The value of the constant d_2 is taken from table 2 for $n = 4$.

Thus

$$\mathrm{PCI} = \frac{0{,}2190 - 0{,}1250}{6 \times 0{,}0151}$$

$$= \frac{0{,}0940}{0{,}0910} = 1{,}0330$$

Since PCI is greater than 1, the process can be considered capable. However, on close examination, it can be seen that the process is not centred properly with respect to the specification and therefore about 11,8 % of the individuals will be outside the upper specification limit. Therefore, before permanent control chart parameters are established, attempts should be made to centre the process properly while maintaining a state of statistical control.

12.3 Control chart for individuals, X, and moving range, R: No standard values given

Table 8 gives the results of laboratory analysis of "percent moisture" of samples from 10 successive lots of skim milk powder. A sample of skim milk powder, representing a lot, is analysed in the laboratory for such various characteristics as fat, moisture, acidity, solubility index, sedimentation, bacteria and whey protein. It was intended to control the percentage of moisture below 4 % for this process. The sampling variation within a single lot was found to be negligible, so it was decided to take only one observation per lot and to set control limits on the basis of the moving range of successive lots.

$$\overline{X} = \frac{2{,}9 + 3{,}2 + \ldots + 3{,}5}{10}$$

$$= \frac{34{,}5}{10} = 3{,}45\%$$

$$\overline{R} = \frac{0{,}3 + 0{,}4 + \ldots + 0{,}1}{9}$$

$$= \frac{3{,}4}{9} = 0{,}38\%$$

Control chart lines for moving ranges, R

Central line $= \overline{R}$
$= 0{,}38$

UCL $= D_4\overline{R}$
$= 3{,}267 \times 0{,}38$
$= 1{,}24$

LCL $= D_3\overline{R}$
$= 0 \times 0{,}38$ (since n is less than 7, LCL is not shown)

The values of the factors D_3 and D_4 are obtained from table 2 for $n = 2$. Since the range chart exhibits a state of statistical control, the plotting of the control chart for individuals can be carried out.

Control chart lines for individuals, X

Central line $= \overline{X} = 3{,}45$

UCL $= \overline{X} + E_2\overline{R}$
$= 3{,}45 + (2{,}66 \times 0{,}38)$
$= 4{,}46$

LCL $= \overline{X} - E_2\overline{R}$
$= 3{,}45 - (2{,}66 \times 0{,}38)$
$= 2{,}44$

The formulae for control limits and the value of the factor E_2 are given in table 3 and table 4. The control charts are plotted in figure 8. The control charts indicate that the process is in statistical control.

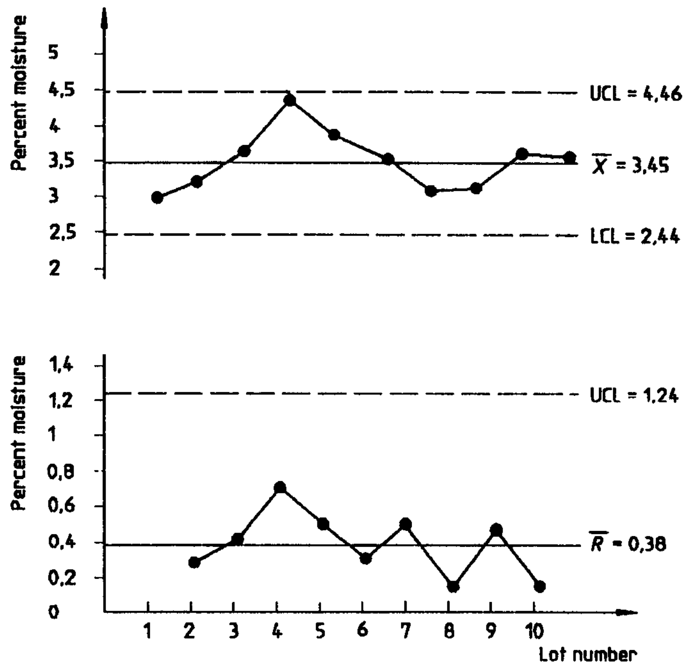

Figure 8 — Control charts for individuals, X, for data given in table 8

Table 8 — Percent moisture for 10 successive samples of skim milk powder

Lot No.	1	2	3	4	5	6	7	8	9	10
X: % moisture	2,9	3,2	3,6	4,3	3,8	3,5	3,0	3,1	3,6	3,5
R: moving range		0,3	0,4	0,7	0,5	0,3	0,5	0,1	0,5	0,1

12.4 Median chart: No standard values given

A machine is manufacturing electronic discs with specified thickness between 0,007 cm and 0,016 cm. Samples of size 5 are drawn every half hour and their thickness in centimetres is recorded as shown in table 9. It was decided to install a median chart for the purpose of controlling the quality. The values of medians and ranges are also shown in table 9.

Calculate the average of subgroup medians and ranges as follows:

$\overline{Me}$ = average of subgroup medians

$$= \frac{12 + 10 + 12 + ... + 11}{15}$$

$$= \frac{172}{15} = 11,47$$

$\overline{R}$ = average range

$$= \frac{6 + 5 + 7 + ... + 7}{15}$$

$$= \frac{86}{15} = 5,73$$

The range chart is calculated as follows:

***R* chart**

Central line $= \overline{R}$
$= 5,73$

UCL $= D_4\overline{R}$
$= 2,114 \times 5,73$
$= 12,11$

LCL $= D_3\overline{R}$
$= 0 \times 5,73$ (since n is less than 7, LCL is not shown)

The value of the constants D_3 and D_4 are taken from table 2 for $n = 5$. Since the range chart exhibits a state of control, the median chart lines can be calculated.

Median control chart

Central line $= \overline{Me}$
$= 11,47$

UCL_{Me} $= \overline{Me} + A_4\overline{R}$
$= 11,47 + (0,69 \times 5,73)$
$= 15,42$

LCL_{Me} $= \overline{Me} - A_4\overline{R}$
$= 11,47 - (0,69 \times 5,73)$
$= 7,52$

The value of A_4 is taken from table 4 for $n = 5$. The graphs are plotted in figure 9. As is evident from the chart, the process is exhibiting a state of statistical control.

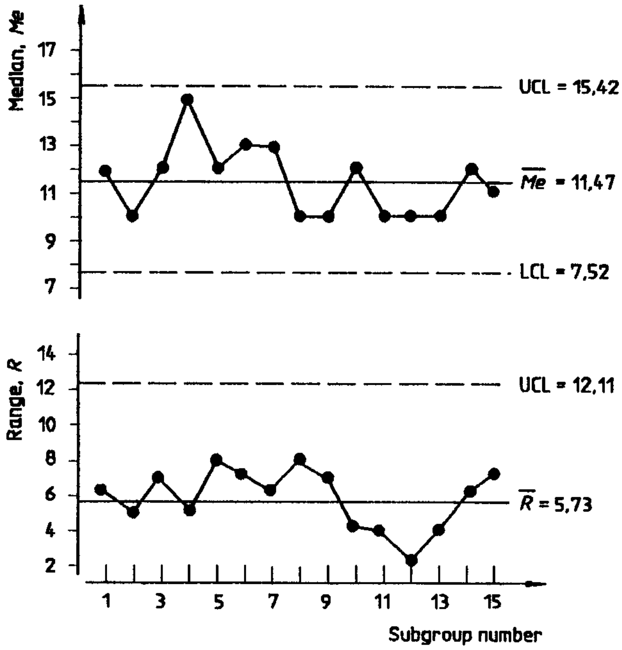

Figure 9 — Median chart and range chart for data given in table 9

Table 9 — Control data for thickness of mica discs

Values in units of 0,001 cm

Subgroup No.	Thickness					Median	Range
	X_1	X_2	X_3	X_4	X_5	Me	R
1	14	8	12	12	8	12	6
2	11	10	13	8	10	10	5
3	11	12	16	14	9	12	7
4	16	12	17	15	13	15	5
5	15	12	14	10	7	12	8
6	13	8	15	15	8	13	7
7	14	12	13	10	16	13	6
8	11	10	8	16	10	10	8
9	14	10	12	9	7	10	7
10	12	10	12	14	10	12	4
11	10	12	8	10	12	10	4
12	10	10	8	8	10	10	2
13	8	12	10	8	10	10	4
14	13	8	11	14	12	12	6
15	7	8	14	13	11	11	7

13 Illustrative examples: Attributes control charts

13.1 p chart and np chart: No standard values given

The data in table 10 give the number of nonconforming units per hour regarding malfunctions found by 100 % inspection on small switches with automatic inspection devices. The switches are produced in an automatic assembly line. Since the malfunction is serious, the percent nonconforming is used to identify when the assembly line is out of control. A p chart is prepared by gathering data of 25 groups as the preliminary data (see table 10).

The central line and the control limits are calculated below and plotted in figure 10.

p chart

Central line $= \bar{p}$

$$= \frac{8 + 14 + \dots + 14}{4000 \times 25}$$

$$= \frac{269}{100000} = 0{,}0027 = 0{,}27\%$$

$$\text{UCL} = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$$

$$= 0{,}0027 + 3\sqrt{0{,}0027(1-0{,}0027)/4000}$$

$$= 0{,}0052 = 0{,}52\%$$

$$\text{LCL} = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n}$$

$$= 0{,}0027 - 3\sqrt{0{,}0027(1-0{,}0027)/4000}$$

$$= 0{,}0002 = 0{,}02\%$$

The chart indicates that the quality of switches is in statistical control, although the percent nonconforming may be too large. These control limits may now be used for future subgroups until such time that the process is altered or that the process goes out of statistical control. Note that since the process is in statistical control, it is unlikely that any improvement can be made without a process change. Simply telling people to be "more careful" is not sufficient.

If an improvement is made, then different control limits will have to be computed for future subgroups to reflect the altered process performance. If the process has been improved (smaller p value), use the new limits, but if the process has deteriorated (higher p value), search for additional assignable causes.

Note that an np chart would have been equally appropriate for this data since all sample sizes are equal. The calculations for the np chart are given as follows and the chart is plotted in figure 11.

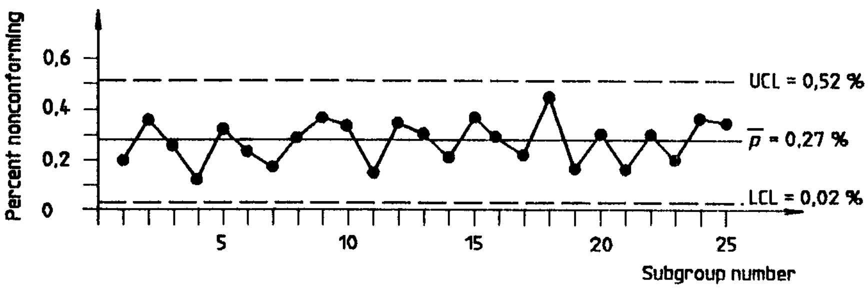

Figure 10 — p chart for data given in table 10

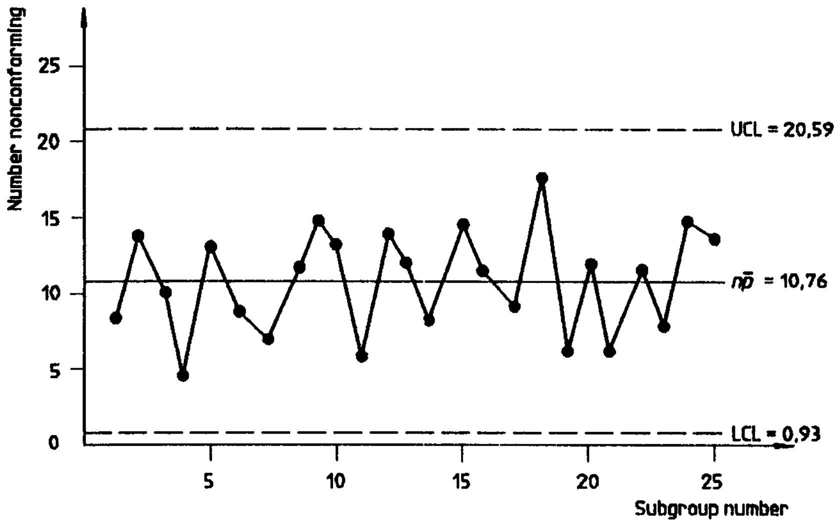

Figure 11 — *np* chart for data given in table 10

Table 10 — Preliminary data: Switches

Subgroup No.	Number of switches inspected n	Number of nonconforming switches np	Percent nonconforming p
1	4 000	8	0,200
2	4 000	14	0,350
3	4 000	10	0,250
4	4 000	4	0,100
5	4 000	13	0,325
6	4 000	9	0,225
7	4 000	7	0,175
8	4 000	11	0,275
9	4 000	15	0,375
10	4 000	13	0,325
11	4 000	5	0,125
12	4 000	14	0,350
13	4 000	12	0,300
14	4 000	8	0,200
15	4 000	15	0,375
16	4 000	11	0,275
17	4 000	9	0,225
18	4 000	18	0,450
19	4 000	6	0,150
20	4 000	12	0,300
21	4 000	6	0,150
22	4 000	12	0,300
23	4 000	8	0,200
24	4 000	15	0,375
25	4 000	14	0,350
Total	100 000	269	

np **chart**

$$\text{Central line} = n\bar{p}$$

$$= \frac{8 + 14 + \dots + 14}{25} = 10{,}76$$

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= 10{,}76 + 3\sqrt{10{,}76(1-0{,}0027)} = 20{,}59$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= 10{,}76 - 3\sqrt{10{,}76(1-0{,}0027)} = 0{,}93$$

13.2 *p* chart: No standard values given

In a manufacturing company producing radio transistors, it was decided to install a fraction nonconforming *p* chart. Data were collected and analysed for a period of 1 month. From each day's production a random sample was collected at the end of the day and examined for the number of nonconforming items. The data are shown in table 11.

The values of the fraction nonconforming calculated for each subgroup is also given in table 11. The average fraction nonconforming for the month is calculated as follows:

$$\bar{p} = \frac{\text{total number nonconforming}}{\text{total number inspected}}$$

$$= \frac{233}{3893} = 0{,}060$$

Since subgroup sizes are different, the UCL and LCL values are calculated for each subgroup separately from

$$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

where n is the size of the subgroup.

These values are also given in table 11. It can be seen that plotting the UCL and LCL values for each subgroup is a time-consuming task. However, it can be observed from table 11 that the fractions nonconforming for subgroup numbers 17 and 26 are falling outside their corresponding upper control limits. These two subgroups are eliminated from the data as they are shown to be subject to variations different from those affecting the other subgroups. To include them in the computations would result in an overstated process average and control limits which would not reflect the true random variations. The reasons for these high values should be sought so that corrective action may be taken to prevent future occurrences. A revised average fraction nonconforming is calculated from the remaining 24 subgroup values:

$$\bar{p} = \frac{195}{3596} = 0{,}054$$

Calculating the revised UCL and LCL values for each subgroup, using the revised $\bar{p}$ value, would reveal that all the fractions nonconforming are within their corresponding control limits. Hence, this revised value of $\bar{p}$ is taken as the standard fraction nonconforming for the purpose of installation of control charts. Thus, $p_0 = 0{,}054$.

As remarked above, the plotting of upper control limits for each subgroup of varying sizes is a time-consuming and tedious process. However, since the subgroup sizes do not vary widely from the average subgroup size, which comes out to be 150, the revised *p* chart (using $p_0 = 0{,}054$) can be plotted with an upper control limit using a subgroup size of $n = 150$, as the average subgroup size.

Thus, the revised *p* chart lines are calculated as follows:

Revised *p* chart

$$\text{Central line} = p_0 = 0{,}054$$

$$\text{UCL} = p_0 + 3\sqrt{\frac{p_0(1-p_0)}{n}}$$

$$= 0{,}054 + 3\sqrt{\frac{0{,}054 \times 0{,}946}{150}} = 0{,}109$$

$$\text{LCL} = p_0 - 3\sqrt{\frac{p_0(1-p_0)}{n}}$$

$$= 0{,}054 - 3\sqrt{\frac{0{,}054 \times 0{,}946}{150}}$$

(since negative values are not possible, the lower limit is not shown)

The revised *p* chart is plotted in figure 12. The process is exhibiting a state of statistical control.

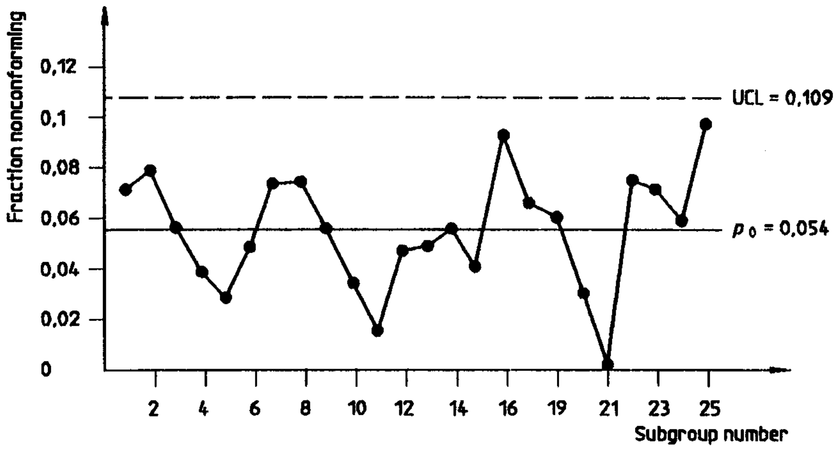

Figure 12 — Revised p chart for data given in table 11

Table 11 — Radio transistors: p chart (initial data)

Subgroup No.	Number inspected	Number nonconforming np	Fraction nonconforming p	UCL	LCL
1	158	11	0,070	0,117	0,003
2	140	11	0,079	0,120	0,000
3	140	8	0,057	0,120	0,000
4	155	6	0,039	0,177	0,003
5	160	4	0,025	0,116	0,004
6	144	7	0,049	0,119	0,001
7	139	10	0,072	0,120	0,000
8	151	11	0,073	0,118	0,002
9	163	9	0,055	0,116	0,004
10	148	5	0,034	0,119	0,001
11	150	2	0,013	0,118	0,002
12	153	7	0,046	0,118	0,002
13	149	7	0,047	0,118	0,002
14	145	8	0,055	0,119	0,001
15	160	6	0,038	0,116	0,004
16	165	15	0,091	0,115	0,005
17	136	18	0,132	0,121	0,000
18	153	10	0,065	0,118	0,002
19	150	9	0,060	0,118	0,002
20	148	5	0,034	0,119	0,001
21	135	0	0,000	0,121	0,000
22	165	12	0,073	0,115	0,005
23	143	10	0,070	0,120	0,000
24	138	8	0,058	0,121	0,000
25	144	14	0,097	0,119	0,001
26	161	20	0,124	0,116	0,004
Total	3 893	233			

13.3 *c* chart: No standard values given

A manufacturer of video tape wishes to control the number of spot nonconformities in video tape. Video tape is manufactured in lengths of 4 000 m. The following data give the number of spot nonconformities found by examining successively the surface of 20 hoops of video tape, each being 350 m long, from a certain production process in which one end of the video tape is investigated.

In order to control this process, it is intended to apply a *c* chart plotting the number of spot nonconformities. The data for 20 hoops, given in table 12, are taken as the preliminary data to prepare a *c* chart.

The central line and control limits are calculated below and plotted in figure 13.

***c* chart**

$$\text{Central line} = \bar{c} = \frac{7 + 1 + \dots + 6}{20}$$

$$= \frac{68}{20} = 3{,}4$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$$

$$= 3{,}4 + 3\sqrt{3{,}4} = 8{,}9$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$

$$= 3{,}4 - 3\sqrt{3{,}4}$$

(since negative values are not possible, the lower limit is not shown)

The preliminary data indicate that the process is in a state of statistical control.

13.4 Number of nonconformities per unit: *u* chart

In a tyre manufacturing plant, 15 tyres were inspected every half hour and the total number of nonconformities and number of nonconformities per unit were recorded. It was decided to install a *u* chart for the number of nonconformities per unit to study the state of control of the process. The data are shown in table 13.

The average of the *u* values is calculated from table 13 as follows.

Divide the total number of nonconformities (from the row of *c* values) by the total number of units inspected (i.e. 14 × 15):

$$\bar{u} = \frac{\sum c}{\sum n}$$

$$= \frac{55}{14 \times 15} = 0{,}26$$

***u* chart**

$$\text{Central line} = \bar{u} = 0{,}26$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n}$$

$$= 0{,}26 + 3\sqrt{0{,}26/15} = 0{,}65$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n}$$

$$= 0{,}26 - 3\sqrt{0{,}26/15}$$

(since negative values are not possible, the lower limit is not shown)

The data and control lines are plotted in figure 14.

The chart indicates that the process is in a state of statistical control.

Note that, since subgroup sizes are constant, a *c* chart could have been used instead.

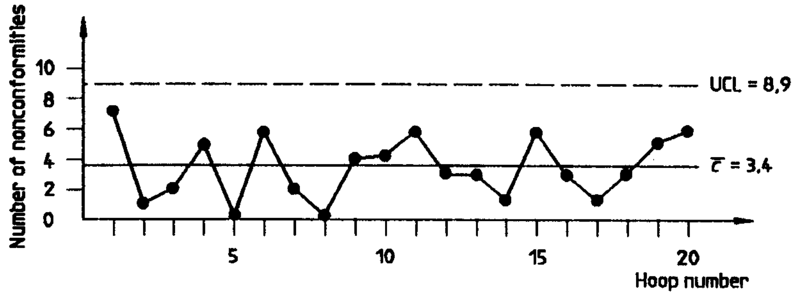

Figure 13 — *c* chart for data given in table 12

Table 12 — Preliminary data: Video tape

Hoop No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
Number of spot non-conformi-ties	7	1	2	5	0	6	2	0	4	4	6	3	3	3	1	6	3	1	5	6	68

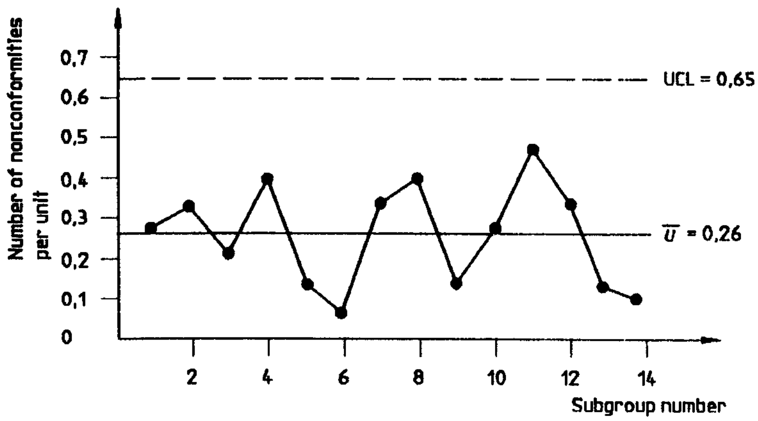

Figure 14 — *u* chart for data given in table 13

Table 13 — Tyre manufacturing plant: Number of nonconformities per unit (units inspected per subgroup, $n = 15$)

Subgroup No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total
c : Number of nonconformities	4	5	3	6	2	1	5	6	2	4	7	5	2	3	55
u : Number of nonconformities per unit	0,27	0,33	0,20	0,40	0,13	0,07	0,33	0,40	0,13	0,27	0,47	0,33	0,13	0,20	

Annex A
(informative)

Bibliography

[1] ISO 7870:—[1], *Control charts — General guide and introduction.*

[2] ISO 7873:—[1], *Control charts for arithmetic average with warning limits.*

[3] ISO 7966:—[1], *Acceptance control charts.*

[4] SHEWHART, W.A., *Economic Control of Quality of Manufactured Product.* D. Van Nostrand, Co., New York, 1931, 501 pp.

[5] NELSON, L.S., The Shewhart Control Chart — Tests for Special Causes. *Journal of Quality Technology*, **16**, No.4, October 1984, pp. 237-239.

[6] NELSON, L.S., Interpreting Shewhart $\overline{X}$ Control Charts. *Journal of Quality Technology*, **17**, No.2, April 1985, pp. 114-116.

1) To be published.

UDC 658.562.012.7:519.233

Descriptors: statistical analysis, quality control, process control, charts, specifications.

Price based on 29 pages