# INTERNATIONAL **STANDARD**

Second edition 2011-07-15

## **[Mechanical vibration and shock —](#page-4-0)  [Experimental determination of](#page-4-0)  [mechanical mobility —](#page-4-0)**

## Part 1: **Basic terms and definitions, and transducer specifications**

*[Vibrations et chocs mécaniques — Détermination expérimentale de la](#page-4-0)  [mobilité mécanique —](#page-4-0)* 

*Partie 1: Termes et définitions fondamentaux et spécification des transducteurs* 





## **COPYRIGHT PROTECTED DOCUMENT**

#### © ISO 2011

All rights reserved. Unless otherwise specified, no part of this publication may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying and microfilm, without permission in writing from either ISO at the address below or ISO's member body in the country of the requester. Provided by IHS under license with ISO NO RECITED DOCUMENT<br>
Provided by IHS under license with ISO No representation or networking permitted with ISO Not for Research<br>
ISO Not for Research ---,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,

ISO copyright office Case postale 56 · CH-1211 Geneva 20 Tel. + 41 22 749 01 11 Fax + 41 22 749 09 47 E-mail copyright@iso.org Web www.iso.org

Published in Switzerland

## **Contents**



## <span id="page-3-0"></span>**Foreword**

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 7626-1 was prepared by Technical Committee ISO/TC 108, *Mechanical vibration, shock and condition monitoring*.

This second edition cancels and replaces the first edition (ISO 7626-1:1986), which has been technically revised.

ISO 7626 consists of the following parts, under the general title *Mechanical vibration and shock — Experimental determination of mechanical mobility*:

- *Part 1: Basic terms and definitions, and transducer specifications*
- *Part 2: Measurements using single-point translation excitation with an attached vibration exciter*
- *Part 5: Measurements using impact excitation with an exciter which is not attached to the structure*  Copyright International Organization of The Channel Organization For Specifications<br>
— Part 2: Measurements using imple point translation excitation with an attached vibration exciter<br>
— Part 5: Measurements using impact e

## **[Mechanical vibration and shock — Experimental determination](#page-4-0)  [of mechanical mobility —](#page-4-0)**

## <span id="page-4-0"></span>Part 1: **Basic terms and definitions, and transducer specifications**

## <span id="page-4-1"></span>**1 Scope**

This part of ISO 7626 defines basic terms and specifies the calibration tests, environmental tests and physical measurements necessary to determine the suitability of impedance heads, force transducers and motion response transducers for use in measuring mechanical mobility. Primarily, it provides guidelines for the selection, calibration and evaluation of the transducers and instruments for their suitability in making mobility measurements. Procedures for carrying out mobility measurements in various circumstances are dealt with in subsequent parts of this International Standard.

This part of ISO 7626 is limited to information which is basic to various types of driving-point and transfer mobility, accelerance and dynamic compliance measurements. Measurements of the blocked impedance are not dealt with.

## <span id="page-4-2"></span>**2 Normative references**

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2041:2009, *Mechanical vibration,shock and condition monitoring — Vocabulary* 

ISO 5347 (all parts), *Methods for the calibration of vibration and shock pick-ups*

ISO 16063 (all parts), *Methods for the calibration of vibration and shock transducers*

<span id="page-4-3"></span>IEC 60263, *Scales and sizes for plotting frequency characteristics and polar diagrams* 

## <span id="page-5-0"></span>**3 Terms, definitions, and symbols**

## <span id="page-5-1"></span>**3.1 Terms and definitions**

For the purpose of this document, the terms and definitions given in ISO 2041 and the following apply.

NOTE As this part of ISO 7626 deals with mechanical mobility, the notes to the definitions below provide more detail than is given in ISO 2041.

#### **3.1.1**

#### **frequency-response function**

frequency-dependent ratio of the motion-response Fourier transform to the Fourier transform of the excitation force of a linear system

NOTE 1 Excitation can be harmonic, random or transient functions of time. The frequency-response function does not depend on the type of excitation function if the tested structure can be considered as a linear system in a certain range of the excitation or response. In such a case, the test results obtained with one type of excitation can be used for estimating the response of the system to any other type of excitation. Phasors and their equivalents for random and transient excitation are discussed in Annex B.

NOTE 2 Linearity of the system is a condition which, in practice, is met only approximately, depending on the type of system and on the magnitude of the input. Care has to be taken to avoid non-linear effects, particularly when applying impulse excitation. Structures which are known to be non-linear (e.g. certain riveted structures) should not be tested with impulse excitation and great care is required when using random excitation for testing such structures.

NOTE 3 Motion may be expressed in terms of velocity, acceleration or displacement; the corresponding frequencyresponse function designations are mobility (sometimes called mechanical admittance), accelerance (sometimes unfortunately called inertance; this term should be avoided because it is in conflict with the common definition of acoustic inertance and also contrary to the implication carried by the term inertance) and dynamic compliance (sometimes called receptance), respectively. These are summarized in Table 1. Each of these frequency-response functions is the phasor of the motion response at a point on a structure due to a unit force (or moment) excitation. The magnitude and the phase of these functions are frequency dependent. Typical magnitude graphs for accelerance and for dynamic compliance, corresponding to the mobility graph shown in Figure 1, are shown in Figures 2 and 3, respectively.

NOTE 4 Frequency response functions can be further differentiated as

- a) *driving point response function*, where the excitation and response are measured at the same location for the evaluation of the frequency-response function, e.g. the use of an impedance head for the measurements  $(i = j$  in the formulae in Table 1);
- b) *transfer response function*, where the excitation and response are not measured at the same location for the evaluation of the frequency-response function  $(i \neq j)$  in the formulae in Table 1).

NOTE 5 Adapted from ISO 2041:2009, 1.53.

#### **3.1.2 mobility**

## **mechanical mobility**

*Yij*

complex ratio of the velocity, taken at point *i* in the mechanical system, to the excitation force, taken at the same or another point in the system

NOTE 1 Mobility is the ratio of the complex velocity-response at point *i* to the complex excitation force at point *j* with all other measurement points on the structure allowed to respond freely without any constraints other than those constraints which represent the normal support of the structure in its intended application.

NOTE 2 The term "point" designates both a location and a direction.

NOTE 3 The velocity response can be either translational or rotational, and the excitation force can be either a rectilinear force or a moment. Copyright International Organization for Standardization Provided by IHS under License Prom IHS under License from IHS No reproduction or networking permitted without license from IHS Not for Resale -<br>
Provided by IHS unde

NOTE 4 If the velocity response measured is a translational one and if the excitation force applied is a rectilinear one, the units of the mobility term are m/(N·s) in the SI system. A typical graph is shown in Figure 1.

NOTE 5 Mechanical mobility is the matrix inverse of mechanical impedance.

NOTE 6 Adapted from ISO 2041:2009, 1.54.

#### **3.1.3**

#### **blocked impedance**

*Zij*

impedance at the input when all output degrees of freedom are connected to a load of infinite mechanical impedance

NOTE 1 Blocked impedance is the frequency-response function formed by the complex ratio of the blocking or drivingpoint force response at point *i* to the applied excitation velocity at point *j*, with all other measurement points on the structure blocked, i.e. constrained to have zero velocity.

NOTE 2 All forces and moments necessary to constrain fully all points of interest on the structure need to be measured in order to obtain a valid blocked impedance matrix. Blocked impedance measurements (see Reference [16]) are, therefore, seldom made and are not dealt with in the various parts of this International Standard.

NOTE 3 Any change in the number of measurement points or their location changes the blocked impedances at all measurement points.

NOTE 4 The primary usefulness of blocked impedance is in the mathematical modelling of a structure using lumped mass, stiffness and damping elements or finite element techniques. When combining or comparing such mathematical models with experimental mobility data, it is necessary to convert the analytical blocked impedance matrix into a mobility matrix, or vice versa, as discussed in Annex A.

NOTE 5 Adapted from ISO 2041:2009, 1.52.

#### **3.1.4 free impedance**

*Z*

ratio of the applied excitation complex force to the resulting complex velocity with all other connection points of the system free, i.e. having zero restraining forces

NOTE 1 Historically, often no distinction has been made between blocked impedance and free impedance. Caution should, therefore, be exercised in interpreting published data.

NOTE 2 Free impedance is the arithmetic reciprocal of a single element of the mobility matrix. While experimentally determined free impedances could be assembled into a matrix, this matrix would be quite different from the blocked impedance matrix resulting from mathematical modelling of the structure and, therefore, would not conform to the requirements for using mechanical impedance in an overall theoretical analysis of the system.

NOTE 3 Adapted from ISO 2041:2009, 1.51.

#### **3.1.5**

#### **frequency range of interest**

span between the lowest frequency to the highest frequency at which mobility data are to be obtained in a given test series



#### **Table 1 — Equivalent definitions to be used for various kinds of frequency-response functions related to mechanical mobility**



**Figure 1 — Typical graph of plotted mobility test results** 







**Figure 3 — Dynamic compliance graph corresponding to the mobility graph plotted in Figure 1** 

## <span id="page-11-0"></span>**3.2 Symbols**



## **4 Fundamentals and general relationships**

<span id="page-11-1"></span>Dynamic characteristics of structures can be determined as a function of frequency from mobility measurements or measurements of related frequency-response functions like accelerance and dynamic compliance (see Table 1). Accelerance and dynamic compliance differ from mobility (see 3.1.2) only in that the motion response is expressed in terms of acceleration or displacement, respectively, instead of in terms of velocity. In order to simplify the various parts of this International Standard, mostly only the term mobility is used. All test procedures and requirements specified in this International Standard are also applicable to the determination of accelerance and dynamic compliance.

Typical applications for mobility measurements are for:

- a) predicting the dynamic response of structures to known or assumed input excitation;
- b) determining the modal properties of a structure (natural frequencies, mode shapes and damping ratios);
- c) predicting the dynamic interaction of interconnected structures;
- d) checking the validity and improving the accuracy of mathematical models and structures;
- e) determining dynamic properties (i.e. the complex modulus of elasticity) of materials in pure or composite forms.

For some applications, a complete description of the dynamic characteristics may be required using measurements of translational forces and motions along three mutually perpendicular axes as well as measurements of moments and rotational motions about these three axes. This set of measurements results in a 6  $\times$  6 mobility matrix for each location of interest. For *N* locations on a structure, the system thus has an overall mobility matrix of size  $6N \times 6N$ . Copyright International Organization for Standardization  $\frac{1}{2}$ <br>
in a 6 × 6 mobility matrix for each location of interest. For *N* locations on a structure, the system thus has an<br>
overall mobility matrix of size  $6N \times$  For most practical applications, it is not necessary to know the entire  $6N \times 6N$  matrix. Often, it is sufficient to measure the driving-point mobility and a few transfer mobilities by exciting with a force at a single point in a single direction and measuring the translational response motions at key points on the structure. In other applications, only rotational mobilities may be of interest.

Since mechanical mobility is defined as the frequency-response function formed by the ratio of the phasor of the translational or rotational response velocity to the phasor of the applied force or moment excitation, when measuring with an accelerometer, conversion to velocity is required to obtain the mobility. Alternatively, accelerance, the ratio of acceleration to force, may be used to characterize a structure. In other cases, dynamic compliance, the ratio of displacement to force, may be used.

NOTE Historically, frequency-response functions of structures have often been expressed in terms of the reciprocal of one of the above-named dynamic characteristics. The arithmetic reciprocal of mechanical mobility has often been called mechanical impedance. This is misleading because the arithmetic reciprocal of mobility does not, in general, represent any of the elements of the impedance matrix of a structure. This point is elaborated upon in Annex A.

Mobility test data cannot be used directly as part of an impedance model of the structure. In order to achieve compatibility of the data and the model, the impedance matrix of the model shall be converted to mobility or vice versa (see A.3 for limitations).

Before carrying out mobility measurements, it is necessary to evaluate the characteristics of the force and motion response transducers to be used in order to ensure that accurate amplitude and phase information can be obtained over the entire frequency range of interest.

## <span id="page-12-0"></span>**5 Basic requirements for force and motion measurement transducers**

## <span id="page-12-1"></span>**5.1 General**

The basic characteristics of all measurement transducers which are important in acquiring adequate mobility data are as follows.

- a) Transducers shall have sufficient sensitivity and low noise in order to obtain a signal-to-noise ratio of the measurement chain which is adequate to cover the dynamic range of the mobility of the structure. Since lightly damped structures require a larger dynamic range than structures with considerable damping, transducer noise is of particular concern when testing lightly damped structures.
- b) If the frequency-response function of the measurement transducer is not compensated by suitable signal processing, the natural frequency of the transducer shall be far enough below or above the frequency range of interest that no unacceptable phase shift occurs.
- c) Transducer sensitivity shall be stable with time and have negligible direct current drift.
- d) Transducers shall be insensitive to extraneous environmental effects, such as temperature, humidity, magnetic fields, electrical fields, acoustical fields, strain and cross-axis inputs.
- e) Transducer mass and rotational inertia shall be small so as to avoid dynamic loading of the structure under test, or at least small enough so that a correction can be made for the loading.

Low susceptibility of the measurement system to the effects of electrical ground loops and other extraneous signals is also important.

## **5.2 Requirements for motion measurement transducers**

<span id="page-12-2"></span>**5.2.1** Although motion measurement transducers require the characteristics outlined in 5.1, certain of these characteristics are more important than others. Motion transducers used in mechanical mobility measurements are most commonly accelerometers; however, displacement or velocity transducers are sometimes used. The major characteristics to be considered in transducer selection are outlined in 5.2.2 to 5.2.5. **5.2.1** Although motion measurement transaucers require the characteristics outlined in 5.1, certain of the characteristics are more important than others. Motion transducers used in mechanical mobil measurements are most

**5.2.2** The motion transducer should be of low mass (or non-contacting) design so as to minimize structural loading of the structure under test.

**5.2.3** The attachment of the transducer to the structure under test should be stiff in the direction of the primary measurement axis of the transducer (see ISO 5348<sup>[2]</sup>).

**5.2.4** The attachment should have a sufficiently small contact area to prevent stiffening or damping of the structure by the transducer or its mounting fixture.

**5.2.5** When applying impulse excitation, zero drift of piezoelectric accelerometers due to the pyro-electric effect is likely to occur and this limits the accuracy of the measurement at low frequencies. Other types of motion transducers (e.g. piezoresistive, electrodynamic or some shear-type piezo-electronic accelerometers) can provide the solution to this problem.

## <span id="page-13-0"></span>**5.3 Requirements for force measurement transducers**

**5.3.1** Some of the characteristics outlined in 5.1 are more important than others in the selection of a force measurement transducer to be used for mechanical mobility measurements. Since compromises in design have to be made, the items outlined in 5.3.2 to 5.3.4 shall be considered as being of prime importance.

**5.3.2** The effective end mass (mass between the force-sensing element of the transducer and the structure) should be small enough to minimize extraneous inertial signals related to such mass (see 8.4 for further details).

**5.3.3** The stiffness of the force transducer and its components should be selected so that no resonances involving this stiffness occur within the frequency range of interest. As a compromise, the effect of such resonances on the signal from the force-sensing element should be compensated for by suitable signal processing.

**5.3.4** The static preload shall be adequate for the range of excitation forces required by the test application. Transducers with built-in preload are available to minimize this problem.

## <span id="page-13-1"></span>**5.4 Requirements for impedance heads and attachments to the structure under test**

**5.4.1** A device which combines an accelerometer and a force transducer in one assembly for the purpose of mobility measurement is called impedance head. The design is a compromise based on the characteristics outlined in 5.2 and 5.3. However, certain characteristics of prime importance, given in 5.4.2 to 5.4.5, should be borne in mind.

**5.4.2** The total compliance between the structure and the internal accelerometer should be small, because a large compliance causes errors in acceleration measurements.

NOTE The total compliance is the sum of the attachment compliance and the internal compliance of the impedance head. The attachment compliance includes the localized die effect compliance of the structure under test. The total compliance can be measured as described in Annex C.

**5.4.3** The effective end mass (mass between the force-sensing element of the transducer and the structure) should be small in relation to the free effective mass of the structure under test.

**5.4.4** The moment of inertia of the impedance head, relative to an axis in the plane of attachment, should be small enough to minimize structural loading due to rotational motion about that axis. Copyright International Organization for Standardization for Standardization Provided by IHSO 2011 – All rights and the structure or networking permitted by IHSO No reproduction Providing the Standardization Providing perm

NOTE Further quidelines for avoiding loading of the structure under test by the attachment of impedance heads are given in ISO 7626-2.

**5.4.5** In the design of an impedance head, care is required to avoid cross-sensitivity of the acceleration transducer to the applied force.

## <span id="page-14-0"></span>**6 Calibration**

## <span id="page-14-1"></span>**6.1 General**

Calibrations fall into three categories:

- a) operational calibration of the combined measurement and analysis system;
- b) basic transducer calibrations:
- c) supplementary transducer calibrations.

## <span id="page-14-2"></span>**6.2 Operational calibrations**

Operational calibrations of the combined measurement and analysis system shall be carried out at the beginning and end of each measurement series (and at intermediate times, as required). Detailed procedures are covered in the relevant parts of this International Standard pertaining to the various types of mobility measurements.

Combined system calibrations are easier to perform, more accurate and in wider use than the basic calibrations discussed in Clause 7. These system calibrations are achieved by driving, in free space, a known free mass while the acceleration and force channel gains are set at the values that are used in later measurements. The ratio output shall follow the appropriate mass line on the resulting mobility graph. If any difficulties are experienced in combined system calibrations, basic calibrations should be carried out.

The accuracy of the frequency scale of the response graph or other data output should always be checked during the operational calibration.

NOTE 1 An example of the resulting mobility graph, showing the effect of the impedance head attachment compliance, is shown in Annex C.

NOTE 2 It is usually unnecessary to perform a phase shift calibration provided that the accelerometers, force transducers and amplifiers are selected to have nearly flat response throughout the frequency range used for mobility measurements, and the accelerometer is of a low-damping design. However, it is good practice to display the phase angle between the force transducer and accelerometer outputs on a phase meter and record any deviations from the proper phase angle between the force transducer and accelerometer outputs while carrying out the operational calibration.

## **6.3 Basic and supplementary transducer calibrations**

The basic and supplementary calibrations (see Table 2) are intended for determining the suitability of the transducers for mobility measurements. Piezoelectric transducers are very frequently used. If other types of transducers are used, the procedures may have to be modified to determine the suitability of such transducers.

Transducers exhibiting changes in basic or supplementary calibrations should not be used if the changes are unacceptable, as indicated in the appropriate clauses in this part of ISO 7626.

<span id="page-14-3"></span>Transducers for use with specific amplifiers or signal conditioners should have the calibration performed under the conditions of intended use. For example, piezoelectric force transducers, impedance heads and accelerometers are intended for use with charge amplifiers or IEPE (integrated electronic piezoelectrics means a built-in charge amplifier) transducers with amplifiers with constant-current line drive capability and can be calibrated together or separately. The use of high-impedance amplifiers with piezoelectric sensors is no longer recommended. All transducers should be calibrated with their intended signal conditioning device in accordance with the manufacturer's specifications for electrical excitation, special terminating impedance, etc. phase angle between the force transducer and accelerometer ou<br> **6.3 Basic and supplementary calibrations** (see Table 2)<br>
transducers for mobility measurements. Piezoelectric tran<br>
transducers are used, the procedures may

When calibrating accelerometers, force transducers and impedance heads, special care should be taken to use mounting conditions similar to those specified by the manufacturer. Flatness of the mounting plate and proper torquing of the attaching screws is important. A thin film of oil, grease or wax between the transducer and the mounting surface may increase transducer coupling and rigidity for use at high frequencies. If special fixtures are used, every effort should be made to carry out force transducer calibrations using a mechanical arrangement closely resembling that to be used during the mobility measurement.





## <span id="page-15-0"></span>**7 Basic piezoelectric transducer calibrations**

## <span id="page-15-1"></span>**7.1 General**

All basic calibrations and tests listed in Table 2 shall be carried out by the manufacturer on each transducer and the results shall be recorded in the documentation supplied with the transducer. All basic calibrations should be repeated periodically by the user or by a commercial calibration laboratory, if the user does not have adequate calibration facilities.

The recommended time interval for repeating basic calibrations and tests is 2 years. In addition, calibrations such as sensitivity should be repeated more frequently, particularly if the transducer is subjected to conditions which may change its sensitivity.

Specific tests can be performed to check transducer malfunctions before and during any test in the meantime between the calibrations, e.g. bias voltage for IEPE (built-in charge amplifier) transducers.

## <span id="page-15-2"></span>**7.2 Sensitivity**

## **7.2.1 Accelerometer sensitivity**

Sensitivity of accelerometers and the accelerometer portion of impedance heads shall be determined by using the comparison method in accordance with the relevant part of ISO 5347 or ISO 16063. Calibration shall be carried out by mounting the accelerometer on a suitable vibration exciter equipped with a reference accelerometer previously calibrated by or referenced to an absolute calibration. The calibration amplitude should be within the range encountered in actual mobility measurements, generally between 1 m/s<sup>2</sup> and 100 m/s2.

The sensitivity calibration should be carried out at least at a single frequency, generally 80 Hz. A different frequency may be used, if 80 Hz is outside the operating frequency range of the transducer or if another frequency is more suitable for the particular transducer design or experimental application of the transducer.

Frequency-response calibration is recommended to ensure the right properties of the device under test.

NOTE Certain failures in an accelerometer can be detected by frequency-response measurement, e.g. broken or lost pre-tension of the piezoelectric element.

For accelerometers designed to be used with a charge amplifier, accelerometer sensitivity shall be expressed in  $pC/(m/s<sup>2</sup>)$ . For accelerometers designed to be used with a voltage amplifier and for accelerometers with a built-in charge amplifier or impedance transformer, accelerometer sensitivity shall be expressed in V/(m/s<sup>2</sup>).

Since the output signal of the charge or voltage amplifier associated with the accelerometer is always a voltage, the resultant acceleration channel sensitivity should always be expressed in V/(m/s2). In general, the acceleration channel sensitivity is determined from the knowledge of the separate sensitivities of the accelerometer and the amplifier; in cases where precise mobility measurements are required, the accelerometer and the amplifier to be used with the accelerometer should be calibrated together in order to obtain the acceleration channel sensitivity directly.

#### **7.2.2 Force transducer sensitivity**

Force transducers and the force section of impedance heads shall be calibrated using a mass-loading technique. Measurement technologies are described in the literature, e.g. in Reference [17].

The sensitivity calibration shall be carried out by mounting the force transducer on a suitable vibration exciter using the rated preloading torque recommended by the manufacturer. The force transducer shall be vibrated at a controlled acceleration amplitude in one axis only,  $a<sub>0</sub>$ , with only a reference accelerometer attached to its opposite face. The force transducer amplifier voltage output,  $U_0$ , and the applied acceleration,  $a_0$ , shall be measured. A loading mass, *m*, shall then be attached to the opposite face of the force transducer without changing the amplifier gain settings. The voltage output, *Um*, shall be measured with the vibration exciter input adjusted so that the applied acceleration, *a*, has exactly the same amplitude as before (i.e.  $a = a_0$ ). The sensitivity of the force channel,  $S_F$ , is then given by Equation (1):

$$
S_F = \frac{U_m - U_0}{(m + m_1 + m_2 + m_3)a - (m_1 + m_2 + m_3)a_0} \tag{1}
$$

where

- *m* is the loading mass;
- $m_1$  is the mass of the reference accelerometer;
- $m<sub>2</sub>$  is the effective mass of the bolt;
- $m<sub>3</sub>$  is the effective end mass of the force transducer.

All the masses are expressed in kilograms and the acceleration shall be expressed in metres per second squared. As  $(m_1 + m_2 + m_3) a_0 = (m_1 + m_2 + m_3) a$ , because  $a_0 = a$ , Equation (1) becomes

$$
S_F = \frac{U_m - U_0}{ma} \tag{2}
$$

#### **CAUTION — For force transducers which have not been pre-assembled, extreme care should be taken to ensure that the mounted preload does not change from application to application, since sensitivity and calibration depend on transducer preload.**

NOTE Using Equations (1) and (2) in a complex form also permits the phase response of the force transducer to deviate.

Equation (2) gives the sensitivity of the force channel (i.e. the transducer and amplifier combination) in volts per newton. The sensitivity of the force transducer alone can be deduced from Equation (2) and the sensitivity of the amplifier used. where<br>  $m$  is the loading mass;<br>  $m_1$  is the effective ends of the bolt;<br>  $m_2$  is the effective end mass of the bolt;<br>  $m_3$  is the effective end mass of the force transducer.<br>
All the masses are expressed in kilograms

For force transducers designed to be used with a charge amplifier, force transducer sensitivity shall be expressed in pC/N. For force transducers designed to be used with a voltage amplifier, force transducer sensitivity shall be expressed in V/N.

In cases where precise mobility measurements are required, the force transducer and the amplifier to be used with the force transducer should be calibrated together in order to obtain the total force channel sensitivity directly.

The method described is adequate only at frequencies below approximately one-fifth of the resonance frequency due to the effective mass of the system and the stiffness, *k*, of the force transducer. This resonance frequency of the mass-loaded transducer,  $f_m$ , can be estimated by means of Equation (3):

$$
f_m = \frac{1}{2\pi} \sqrt{\frac{k}{m + m_1 + m_2 + m_3}}
$$
(3)

#### **7.2.3 Impedance head sensitivity**

The sensitivity of an impedance head can be obtained from the individual calibrations of the accelerometer and the force transducer performed using the methods given in 7.2.1 and 7.2.2, respectively. Field calibration of an impedance head usually consists of driving two or three differently sized free masses. This ensures that the impedance head can measure known mobilities over a wide range.

## <span id="page-17-0"></span>**7.3 Frequency response**

#### **7.3.1 Accelerometer frequency response**

The frequency response calibration shall be performed by the comparison method in accordance with the relevant part of ISO 5347 or ISO 16063 at an acceleration amplitude chosen to provide a signal-to-noise ratio exceeding 10 (i.e.  $>20$  dB) and over the frequency range corresponding to the range of use of the accelerometer. Harmonic or wide-band excitation, applied with a suitable vibration exciter, may be used to measure the frequency responses. Wide-band excitation may be random or transient.

Harmonic excitation may be applied at discrete frequencies or by a continuous sweep. In the first case, there shall be enough discrete frequencies, for instance 10 frequencies per decade, to ensure that the transducer is free from local or internal resonances within the frequency range of interest. If the calibration is carried out at a limited number of discrete frequencies, addition of a frequency sweep is useful to ensure the same objective.

When a continuous sweep is used, the frequency should be swept slowly, while the amplitude of the excitation acceleration is maintained constant over the full frequency range by means of a feedback control system using a reference accelerometer.

The frequency response shall be flat within  $\pm 5$  % in the frequency range of interest. Care shall be taken so that transverse motion of the vibration exciter, combined with the cross-axis sensitivity of the transducer, does not cause measurement errors of the order of the specified  $\pm 5$  %.

#### **7.3.2 Force transducer frequency response**

Frequency response calibration shall be performed under the same conditions described for the accelerometer frequency response. However, the force transducer shall be mass loaded and vibrated at a constant amplitude of acceleration at each specified frequency. The voltage output of the force transducer amplifier,  $U_f$ , shall be measured at each of the frequencies,  $f$ . The frequency response deviations, expressed as a percentage, shall be determined from Formula (4): Harmonic excitation may be applied at discrete frequencies or by<br>shall be enough discrete frequencies, for instance 10 frequency range<br>timited number of discrete frequencies, addition of a frequency swe<br>When a continuous

$$
\left(\frac{U_f}{U_{\text{ref}}}-1\right)\times 100\,\%
$$

(4)

where the reference voltage,  $U_{\text{ref}}$ , is the output of the transducer amplifier obtained at the reference frequency, generally 80 Hz.

It is important that the mass load on the force transducer does not cause a resonance involving the loading mass and the transducer compliance. This can be checked using Equation (3). The frequency response deviation shall be less than  $\pm 5$  % of the response at the reference frequency over the frequency range of interest.

NOTE As in the sensitivity calibration of the force transducer, better accuracy is obtained by not adjusting the torque of the force transducer mounting bolt at any time during the frequency response calibration, if force transducers without built-in preload are used.

## <span id="page-18-0"></span>**7.4 Accelerometer transverse sensitivity**

Transverse-sensitivity calibrations shall be performed at a single frequency below 500 Hz in accordance with the relevant part of ISO 5347 or ISO 16063. The transducer shall be mounted on a vibration exciter perpendicular to the transducer sensing axis. Harmonic excitation shall be applied at a frequency where it is known that the vibration exciter motion in a plane perpendicular to the sensing axis is at least 100 times the motion in the direction of the sensing axis. For measurement of transverse-sensitivity ratios of much less than 1 %, the excitation requirements are more strict. Extreme care and skill are required to obtain valid values for such transverse sensitivities.

The transducer shall be mounted and rotated about its sensing axis through 360°, in increments of 45° or less, to determine the maximum transverse response. If the transverse-sensitivity ratio exceeds 5 % of the axial sensitivity or changes significantly from an earlier calibration, the transducer should be further evaluated by performing supplementary calibrations or it should be repaired, as required.

## <span id="page-18-1"></span>**7.5 Mass**

The mass specified is the total mass of the transducer, excluding mounting studs and cables which do not form an integral part of the transducer.

## **7.6 Dimensions**

Pertinent dimensions, including height, width, length, diameter and dimensions of any mounting holes or studs, shall be given. The dimensions shall be provided in the form of an outline drawing. Descriptions of connectors, cable sizes and cable types shall also be provided.

## **7.7 Electrical impedance**

## **7.7.1 Transducer resistance and capacitance**

The direct current resistance between the terminals of the transducer shall be measured by the manufacturer with a megohmmeter, using an applied voltage not exceeding 50 V.

<span id="page-18-3"></span><span id="page-18-2"></span>The capacitance shall be measured with an impedance bridge, using an excitation voltage within the operating frequency range of the transducer. If the capacitance changes with frequency, the measurement shall be carried out at a minimum of two frequencies, including the frequency at which the reference sensitivity has been determined. For transducers, the capacitance of which does not change significantly with frequency, the measurement shall usually be carried out at 1 000 Hz. Copyright International Organization for Conduction Formula Urbidation Provided in the formula Organization Conducts International Organization Provided by IHS under the terminal Organization Provided The direct current r

Since the capacitance of some piezoelectric materials changes with temperature and voltage, the capacitance measurement shall be performed at a room temperature of  $(23 \pm 3)$  °C and at an excitation voltage recommended by the transducer manufacturer. Variations due to temperature can be minimized by avoiding handling immediately before or during the capacitance measurements.

For some transducers containing internal electronic components, the above impedance measurement operation may produce inaccurate results or possibly cause damage. In these cases, insertion methods should be used.

Resistance and capacitance measurements should be repeated at suitable time intervals (see 7.1). Since these measurements are generally not very accurate, only significant variations, compared to previous calibrations, shall be taken into account. For instance, if resistance and/or capacitance has changed by more than 5 %, the transducer should be further evaluated by performing all basic and supplementary calibrations or it should be repaired, as required.

#### **7.7.2 Insulation resistance**

The direct current resistances between all the terminals of the transducer and its mounting surface shall be measured and supplied as data by the manufacturer.

If the transducer is not of an isolated type, the manufacturer shall indicate the type of mounting needed to achieve isolation when using the transducer.

## <span id="page-19-0"></span>**7.8 Polarity**

Polarity of the transducer, usually laid down by the manufacturer, shall be determined by a comparison method whether a positive or negative output voltage change is produced when the mechanical input is applied in a direction from the mounting surface towards the opposite end of the transducer and along the axis of maximum sensitivity of the transducer. This definition of polarity is applicable to most transducers which have the sensitivity axis perpendicular to the mounting surface.

## <span id="page-19-1"></span>**8 Supplementary calibrations**

## **8.1 General**

The supplementary calibrations and tests listed in Table 2 shall be carried out by the manufacturer on samples of the transducers for each type manufactured. Furthermore, some of the supplementary calibrations and tests should be carried out by the user to determine the condition of a transducer which exhibits peculiar operating characteristics or changes in performance.

## <span id="page-19-3"></span>**8.2 Linearity**

#### **8.2.1 General**

Linearity calibrations shall be carried out by harmonic excitation using the comparison method (for accelerometers, see the relevant part of ISO 5347 or ISO 16063).

#### **8.2.2 Accelerometer linearity**

<span id="page-19-2"></span>The amplitude linearity deviations shall be determined by measuring the ratio of the reference and test accelerometer outputs. This ratio shall be measured by the manufacturer at a minimum of three acceleration magnitudes in the upper decade of the design range of the accelerometer. It may be useful for users to check the linearity in their own range of use. The deviations at each acceleration shall be determined by computing the difference from the average ratio at all accelerations. This difference shall be expressed as a percentage of the average ratio. If the deviations at all accelerations are within  $\pm 2$  %, the accelerometer is suitable for mobility measurements. The supplementary calibrations and tests listed in Table 2 shamples of the transductors for each type manufactured. Furtherm and tests should be carried out by the user to determine the concorerating characteristics or ch

#### **8.2.3 Force transducer linearity**

The linearity calibration shall be carried out by vibrating the mass-loaded force transducer, generally at 80 Hz, at various acceleration magnitudes and measuring the voltage output of the force transducer amplifier, *Ux*, at each of the applied acceleration amplitudes,  $a_x$ , as determined by a standard accelerometer. The linearity deviations, expressed as a percentage, shall be determined from Formula (5):

$$
\left[\frac{U_x/a_x}{(U_x/a_x)_{\text{avg}}} - 1\right] \times 100\% \tag{5}
$$

where

$$
(U_x/a_x)_{\text{avg}} = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{U_x}{a_x}\right)_i
$$
 is the average of the ratio of force transducer amplifier output amplitude  
to the acceleration amplitude for all acceleration magnitudes.

A different frequency may be used if 80 Hz is outside the operating frequency range of the transducer or if another frequency is more suitable for the particular transducer design or application.

The applied force shall be computed as follows. At one of the applied accelerations, the applied force shall be computed by dividing the amplifier output,  $U_x$ , by the force transducer channel sensitivity, previously determined in 7.2.2. The force applied at every other calibration point shall be determined by multiplying the force just computed by the ratio of new applied acceleration to the acceleration used to compute the force above.

A force transducer, or the force transducer in an impedance head, is not suitable for mobility measurements if the linearity deviations exceed 2 %, within the rated range of operating forces.

## <span id="page-20-0"></span>**8.3 Effective end mass of force transducers and impedance heads**

The effective end mass of force transducers and impedance heads is the mass between the force-sensing elements and the specimen end of the transducer. This effective end mass should be included in the manufacturer's specifications. However, the user should be aware that the effective end mass is increased by the addition of the hardware used to attach the transducer to the structure under test and/or to preload the transducer. Thus, the total effective end mass is the sum of the manufacturer's effective end mass and the mass of the attaching hardware. computed by dividing the amplifier output,  $U_x$ , by the determined in 7.2.2. The force applied at every other callibration by the ratio of the valid or every other callibration A force that increase the financial or netwo

## <span id="page-20-1"></span>**8.4 Compliance of impedance heads**

The compliance of an impedance head is the compliance of that part of the assembly between the point of attachment to the structure and the internal accelerometer (see Reference [13]).

In addition to the compliance of the impedance head as provided by the manufacturer, the compliance of the attachment hardware shall also be considered when carrying out mobility measurements.

A procedure for experimentally determining the total compliance is given in Annex C.

## <span id="page-20-2"></span>**8.5 Supplementary calibrations necessitated by environmental and secondary effects**

#### **8.5.1 General**

Certain calibrations are necessitated by the fact that secondary and/or environmental conditions can cause transducer outputs which are not direct analogues of the transduced quantities. The supplementary calibrations described in 8.5.2 to 8.5.6 reflect the need to quantify such effects.

#### **8.5.2 Threshold in reference to the bandwidth of the transducer**

The threshold of a transducer may influence the accuracy of data especially in the lowest end of the measurement range. For a better comparison of data the threshold of transducers should be specified in a uniform way. The threshold in electrical output  $U_{\text{Th}}$  units of the transducer should be characterized by the measured spectral noise density *u*(*f*) integrated over the frequency range between the lower (*f* <sup>L</sup>) and the upper end  $(f_{\sf U})$  of the transducer, where the change of sensitivity is within  $\pm 5$  % of the reference sensitivity:

$$
U_{\text{Th}} = \sqrt{\int_{f_{\text{L}}}^{f_{\text{U}}} u^2(f) \, \mathrm{d}f}
$$
 (6)

Usually, the threshold is expressed in mechanical units of the transducer (for an accelerometer  $a_{\text{Th}}$ ) by using the transducer sensitivity *S* as a scaling factor.

$$
a_{\text{Th}} = \frac{U_{\text{Th}}}{S} \tag{7}
$$

Ground loops between several different transducers and the test structure may influence the threshold of a measurement chain, consisting of a transducer and an appropriate amplifier, negatively and should be prevented by case or ground isolated transducer design.

#### **8.5.3 Change in transducer sensitivity with temperature**

It is usually desirable to select transducers which have temperature sensitivity deviations not exceeding  $\pm 1$  % throughout the temperature range of intended use.

#### **8.5.4 Noise induced in transducer signals by temperature variations (transient temperature sensitivity)**

Variation in temperature generates transient temperature sensitivity as low-frequency noise in piezoelectric transducers and in any built-in transducer amplifiers that may be used. As the frequency range of use of piezoelectric transducers is generally much higher, it can happen that the noise and the measured signal interfere so that there are consequences on the results of the mobility measurements in the very low frequency part. ISO 5347-18[1] describes a method to get characteristic transducer data by immersion in a liquid with a sudden temperature jump.

Also, during low-frequency measurement, rapid variations in temperature can cause sufficient noise to saturate the transducer electronics, thus inducing wrong measurements.

## **8.5.5 Force transducer preload effects on sensitivity**

The effect of different mounting torques on the sensitivity of force transducers which have not been preassembled may be determined by repeating the sensitivity calibration when the force transducer is remounted using different torques.

## **8.5.6 Base strain effects**

The effect of base strain on the sensitivity of a transducer can be measured by use of a simple cantilever beam, clamped to a rigid support. A suitable beam design would have a first natural frequency of approximately 5 Hz. The transducer shall be mounted on the cantilever beam at a suitable location. The strain shall be measured by strain gauges bonded to the beam near the transducer mounting. For testing of base strain sensitivity, see the relevant part of ISO 5347 or ISO 16063.

Details of the procedure for determining the effect of base strain on the sensitivity of force transducers and impedance heads can be found in Reference [15].

## <span id="page-22-0"></span>**9 Presentation of data**

## <span id="page-22-1"></span>**9.1 General**

The interpretation of mobility test data can be greatly enhanced by the use of a suitable graphic display. A variety of display formats is available to suit various applications.

## <span id="page-22-2"></span>**9.2 Logarithmic plotting**

At frequencies well below the lowest resonance or anti-resonance frequency of a structure, the magnitude of any of the three structural frequency-response functions (see Table 1) follows an almost straight line on a plot of logarithmic magnitude versus logarithmic frequency. At frequencies well below the lowest resonance, the curve tends to be asymptotic to lines of constant compliance. Well below the lowest anti-resonance, the curve tends to be asymptotic to lines of constant mass. For mobility plots, compliance lines slope upwards at one magnitude decade per frequency decade and mass lines slope downwards at one magnitude decade per frequency decade, as shown in Figure 1.

If accelerance or dynamic compliance data are plotted, the slopes of the mass and compliance lines change accordingly, as shown in Figures 2 and 3.

When plotting the results of mobility measurements with a logarithmic frequency scale, the magnitude and frequency scales shall conform to IEC 60263. A recommended format for plotting mobility data, including lines for mass and dynamic compliance, is shown in Figure 4. All exponents for mobility, mass and compliance shall be specified by the user in this graph.

NOTE 1 In the system of SI units, lines for constant dynamic compliance, lines for constant mobility and lines for constant mass, in increments of one decade of magnitude, always intersect at frequencies of  $1/2\pi$ ,  $10/2\pi$ ,  $100/2\pi$ , etc. The frequency of 159 Hz, equivalent to 1 000/2 $\pi$ , is a convenient one for reference purposes since then mass, accelerance, mobility and compliance show the following multiples among each other:



NOTE 2 In accordance with IEC 60263, the ratio of the length of the scale for a decade of magnitude to the length of the scale for a decade of frequency is expected to be 2/5. The format in Figure 4 follows this rule.

NOTE 3 Appropriate exponents to use as the power of 10 for mobility, mass and dynamic compliance can be determined from the test results to suit the data to be plotted and then noted on the plot.

## <span id="page-22-3"></span>**9.3 Alternative plotting methods**

Instead of plotting the magnitude and phase of the mobility test results, it is sometimes advantageous to plot the real and imaginary parts as functions of frequency, as illustrated in Figure 5. It may also be desirable to plot the test results in polar coordinates, as illustrated in Figure 6 (Nyquist plot). The polar diagram has the advantage that the data may be enhanced by circle-fitting procedures. This can be important for extracting modal damping coefficients from the test data. Mass<br>
Acceleration 10 m/(Na<sup>2</sup>) 1nt/Na<sup>2</sup>) 1nt/Na<sup>2</sup>) 10<sup>-2</sup> m/(Na<sup>2</sup>) 10<sup>-2</sup> m/(Na<sup>2</sup>)<br>
Compliance 10 m/(Na<sup>2</sup>) 1nt<sup>2</sup> m/N<br>
NOTE 2 in accordance with Eig 00253, the ratio of the length of the scale of a decade of magnitu

NOTE 1 In Figure 6, the actual measured data points are connected by straight lines. These lines are not part of the measured data. They are shown as an aid to distinguish between data points belonging to one mode of the structure and those belonging to a second mode.

NOTE 2 The fact that the straight lines connecting the measured data points in Figure 6 do not approach a circle through the data points shows that not enough data points were measured near the resonance frequency to directly determine the true resonance frequency, peak response and damping of the modes. In order to identify these parameters properly, either more data points should be measured or a suitable circle-fitting procedure needs to be applied to the measured data points.







NOTE 2 Exponent ranges are user defined.

**Figure 4 — Diagram for plotting mobility graph** 



**Figure 5 — Graph of the real and imaginary parts of mobility corresponding to Figure 1** 



**Figure 6 — Polar diagram of the mobility results for the two lowest frequency modes of Figure 1** 

## **Annex A**

## (informative)

## <span id="page-26-0"></span>**Relationship between mechanical impedance, mobility and modal analysis**

## **A.1 General**

The dynamic response of a mechanical system basically depends on two parameters: the exciting forces and the dynamic characteristics of the system. If the magnitude of the exciting forces is within the range in which the system behaves essentially linearly, the system characteristics can be described in the frequency domain either by a mobility matrix [*Y*] or by an impedance matrix [*Z*], resulting in two equivalent relations between exciting forces and response velocities:

$$
\{\nu(\omega)\} = [Y]\{F(\omega)\}\tag{A.1}
$$

$$
\{F(\omega)\} = [Z]\{\nu(\omega)\}\tag{A.2}
$$

where

- ${F(\omega)}$  is the column vector of the external dynamic forces expressed as functions of frequency  $\omega$ , acting on the system at various points;
- $\{v(\omega)\}\;$  is the column vector of the corresponding response velocities at all points of interest.

NOTE The term point designates a location and the associated direction. The term coordinate is often used with the same meaning as point.

The matrix notation of Equations (A.1) and (A.2) is convenient because, in general, more than a single point on the structure has to be considered. If *N* is the number of points at which motion response is to be considered, plus any additional points at which external forces act (either exciting forces or reaction forces), the column vectors in Equations (A.1) and (A.2) are of order *N* and the matrices are of order  $N \times N$ . ( $y(\omega)$ ) is the column vector of the corresponding respeads and the associal American From Equation of Equations (A.1) and (A.2) is convertional Organization Considered, Dus any additional points at which external for the

From Equations (A.1) and (A.2), it follows that

$$
[Z] = [Y]^{-1}
$$
 (A.3)

It should be noted that the matrix inversion, expressed by Equation  $(A.3)$ , means that the elements  $Z_{ii}$  of the impedance matrix are not arithmetic reciprocals of the elements  $Y_{ii}$  of the mobility matrix, and vice versa.

## **A.2 Boundary conditions**

In mobility testing, a dynamic exciting force is supplied to the structure at one point at a time. Thus,

$$
F_k = 0 \qquad \text{for } k \neq j \tag{A.4}
$$

where

- *j* is the point of excitation;
- *k* denotes all other points of interest.

Equation (A.4) defines the boundary conditions for experimental determinations of mechanical mobility which easily can be achieved in experimental procedures (see Table 1). Given these boundary conditions, measurement of the velocity response at point *i* and of the force at point *j* yields the *ij*th element of the mobility matrix:

$$
Y_{ij} = \frac{\boldsymbol{v}_i}{\boldsymbol{F}_j} \bigg|_{\boldsymbol{F}_k = 0; \, k \neq j} \tag{A.5}
$$

This is the mathematical expression of the definition of mobility given in 3.1.2.

In contrast, the elements of the impedance matrix *Z*, as given in Equation (A.3), are

$$
Z_{ij} = F_i/v_j \tag{A.6}
$$

with the boundary conditions

$$
v_k = 0 \qquad \text{for } k \neq j \tag{A.7}
$$

Equations (A.6) and (A.7) are the mathematical expressions of the definition of blocked impedance given in 3.1.3. The boundary conditions of Equation (A.7) are very difficult or impossible to achieve in practical experimental procedures (see Table 1). Thus, it is generally not possible to determine the elements of the impedance matrix *Z* by experimental means.

The difference in the boundary conditions of Equations (A.4) and (A.7) is very important for the proper use of mobility and impedance data. It should be noted that this difference becomes irrelevant, however, in the special case in which a single point (and direction) on the structure (the driving point) is the only point of interest.

This special case is not uncommon in practice. This accounts for the often-used definition of impedance as being the ratio of the exciting force to the velocity response, without specifying the boundary conditions. Only in this special case, the impedance matrix and the mobility matrix have one single term, and, therefore, impedance is the arithmetic reciprocal of mobility. It should be noted that this special impedance is a free impedance as defined in 3.1.4. Thus, it is not an element of the general  $N \times N$  impedance matrix of the structure.

## **A.3 Comparing impedance and mobility data**

Experimental investigations of the dynamic characteristics of structures result in mobility type data. In mathematical modelling, however, it is generally easier to use mass and stiffness matrices. In the frequency domain, these result in blocked impedance data. When comparing mobility and blocked impedance data, it is necessary to convert one into the form of the other. In doing this, special care shall be exercised because blocked mechanical impedance is not an invariant characteristic of a structure, but it is dependent upon the number of degrees of freedom considered (see Reference [4]). The elements of an impedance matrix can, therefore, be compared with those of an inverted mobility matrix, only if all degrees of freedom of both matrices (points and directions) are identical. If the mathematical model (and its impedance matrix) has more degrees of freedom than the experimental mobility matrix, as is often the case in practice, it is necessary to convert impedance to mobility to allow comparison with the corresponding elements of an experimentally determined mobility matrix, rather than vice versa. This is so because mobility is free of artificial constraints, but impedance is not. Copyright International Organization Conditional Organization for Standardization Provided by IHS under the provided by IHS under later the mode

## **A.4 Modal analysis**

Modal analysis is a very useful tool for linking experimental analysis with mathematical modelling. It is particularly convenient for predicting the dynamic interaction of interconnected substructures.

When using experimental mobility data, modal analysis uses statistical methods to extract the modal parameters, including natural frequencies, damping and modal mass (or stiffness), within the frequency range of interest. Several different methods are available for identifying these parameters accurately and taking into account the effect of modes outside the frequency range of interest.

When using mathematical models, the modal parameters can be extracted from the computed mass, stiffness and damping matrices of the substructures by eigenvalue and eigenvector computation or other matrix reduction procedures. These procedures are often more efficient that the direct inversion of the entire impedance matrix.

## **Annex B**

## (informative)

## **Mobility as a frequency-response function**

## <span id="page-29-0"></span>**B.1 Harmonic excitation**

When a time function can be expressed as

$$
x(t) = A \cos(\omega t + \varphi) \tag{B.1}
$$

where

- *A* is the amplitude of the harmonic waveform;
- *ω* is the circular frequency;
- *t* is the time;
- $\varphi$  is an initial phase angle;

then the function can be represented on a complex plane as

$$
x(t) = A \exp[j(\omega t + \varphi)] \tag{B.2}
$$

where

$$
j=\sqrt{-1}
$$

 $x(t)$  is the projection of  $x(t)$  on the real axis.

The function  $x(t)$  can be thought of as a rotating vector centred at the origin of the complex plane. This rotating vector is commonly called a phasor<sup>[1](#page-4-3))</sup>. If the ratio of two phasors is formed at a given frequency, the result is a complex number, and is not a function of time. If a set of these complex numbers is formed for all frequencies of interest, the result is called a frequency-response function. If the two phasors were

$$
x_1(\omega, t) = A_1(\omega) \exp\{[(\omega t + \varphi_1(\omega)]\}
$$
  

$$
x_1(\omega, t) = A_1(\omega) \exp\{[(\omega t + \varphi_1(\omega)]\}
$$

$$
x_2(\omega, t) = A_2(\omega) \exp\{[(\omega t + \varphi_2(\omega))]\}
$$

then

$$
H(\omega) = \frac{x_1}{x_2} = B(\omega) \exp [j\theta(\omega)]
$$

where

l

$$
B(\omega) = A_1(\omega)/A_2(\omega)
$$

$$
\theta(\omega) = \varphi_1(\omega) - \varphi_2(\omega)
$$

 $(B.3)$ 

<sup>1)</sup> Some authors define the phasor as  $A \exp(i\varphi)$ . Both definitions are consistent with the use of the term phasor in this part of ISO 7626, which is only concerned with the ratio of phasors.  $H(\omega) = \frac{x_1}{x_2} = B(\omega) \exp [j\theta(\omega)]$ <br>
where<br>  $B(\omega) = A_1(\omega) A_2(\omega)$ <br>  $\theta(\omega) = \varphi_1(\omega) - \varphi_2(\omega)$ <br>
<br>
<br>
(1) Some authors define the phasor as  $A \exp(j\varphi)$ . Both definitions are c<br>
part of ISO 7626, which is only concerned with the rati

Both  $B(\omega)$  and  $\theta(\omega)$  are functions of the circular frequency,  $\omega$ . The frequency-response function,  $H(\omega)$ , can be expressed in polar form by the amplitude and phase,  $B(\omega)$  and  $\theta(\omega)$ , or in rectangular form as real and imaginary parts:

$$
H(\omega) = R(\omega) + j I(\omega) \tag{B.4}
$$

where

 $R(\omega) = B(\omega) \cos \theta(\omega)$ 

 $I(\omega) = B(\omega) \sin \theta(\omega)$ 

Mobility can be considered as a frequency-response function given by the ratio of the velocity and force phasors. Similar arguments can be made for the other mobility-like quantities.

## **B.2 Random excitation**

In random vibration with a stationary Gaussian distribution, a fundamental equation that relates the output and input of a linear bilateral system is

$$
G_{12}(\omega) = H(\omega) G_{22}(\omega) \tag{B.5}
$$

where

- $G_{12}(\omega)$  is the cross-spectral density between the input and output of the system;
- $G_{22}(\omega)$  is the auto-spectral density of the input;
- $H(\omega)$  is the frequency-response function of the system.

 $G_{12}$  is a complex function of frequency and  $G_{22}$  is a real function of frequency; hence,  $H(\omega)$  is a complex function of frequency. Mobility can be considered as a frequency-response function given by the ratio of the cross-spectral density between the velocity and force excitation to the auto-spectral density of the force excitation, the input. In actual practice, only estimates of the cross and auto spectra are available. Hence, mobilities can only be estimated using a random excitation. The estimation errors can be made less than other measurement errors and do not necessarily limit the accuracy of the measurements.

## **B.3 Transient excitation**

In transient vibration, the input and response of a linear system can be related through the expression

 $X_1(\omega) = H(\omega) X_2(\omega)$  (B.6)

where

 $X_1(\omega)$  is the Fourier transform of the output  $x_1(t)$ ;

- $X_2(\omega)$  is the Fourier transform of the input  $x_2(t)$ ;
- $H(\omega)$  is the frequency-response function of the system.

All of the quantities in Equation (B.6) are complex functions of frequency. The inverse Fourier transform of  $H(\omega)$  is the unit impulse response function,  $h(t)$ , of the system; that is, if a unit impulse is applied to the input, the system output is given by *h*(*t*). Mobility can be considered as a frequency-response function given by the ratio of the Fourier transforms of the time histories of the velocity response and of the input force.

In actual practice, the discrete Fourier transform (DFT) is used as an approximation of the continuous Fourier transform. Careful selection of sample rate and sample size can reduce the errors of this approximation to levels less than other measurement errors. The use of the DFT does not, therefore, necessarily limit the accuracy of the measurements.  $X_2(\omega)$  is the Fourier transform of the input  $x_2(t)$ ;<br>  $H(\omega)$  is the frequency-response function of the system<br>
All of the quantities in Equation (B.6) are complex function<br>  $H(\omega)$  is the unit impulse response function

## **Annex C**

## (informative)

## <span id="page-31-0"></span>**Determination of impedance head attachment compliance and damping**

Overall calibration and determination of the sensitivity of a mobility measurement set-up, including the attachment hardware between the vibration exciter and the structure under test, may be accomplished using the following test procedure. This procedure also provides data on the attachment compliance and the structural damping of the impedance head (see Reference [13]).

The test shall be carried out by mounting a large rigid block on a sufficiently compliant support so that the natural frequency of the block on the compliant support is 2 Hz or less. A cylindrical block of sufficient mass should be used to place the anti-resonance (response dip) frequency well within the frequency range of interest. The magnitude, in kilograms, of the appropriate mass of the block, *m*, may be estimated from

$$
m = 1/[(2\pi f)^2 C] \tag{C.1}
$$

where

- $f$  is a convenient frequency, in hertz, in the upper part of the frequency range over which mechanical mobility measurements are to be carried out;
- *C* is an estimate of the compliance, in metres per newton, of the impedance head and the attachment hardware between the force transducer and the structure under test.

In order to design the compliant support for the calibration block, Equation (C.2) yields the minimum required support compliance, *C*s, for the block of mass *m*:

$$
C_{\rm s} = 1/[(2\pi f_{\rm s})^2 \, m] \tag{C.2}
$$

where

- $f_{\rm s}$  is the natural frequency, in hertz, of the block on the compliant support (2 Hz or lower);
- *m* is the mass, in kilograms, of the block selected for the test.

The impedance head (or the force transducer) should be mounted colinear with the centre of gravity of the calibration block. The transducer manufacturer's specifications for mounting bolt torque should be closely adhered to. The mounting hardware should be as closely as possible identical to that to be used when measuring on the structure. The vibration exciter should be attached to the impedance head (or to the force transducer) in the manner to be used in the mobility measurement. The vibration exciter should be supplied with the same type of excitation waveform which is to be used during the mobility measurement. The range of frequencies to be used for the test with the calibration block should extend beyond the frequency range of interest at the high end.

NOTE 1 The mathematical model for this calibration test set-up is given in Reference [7].

The output signals from the force transducer and the motion response transducers should be processed in the same way as for the mobility measurement and should be documented (e.g. plotted as either a mobility or accelerance curve on suitable graph paper).

Figure C.1 shows a typical mobility data curve. The results shown in Figure C.1 were obtained with a calibration block having a mass of 5,5 kg and represent a rather typical attachment compliance of the impedance head of about  $8 \times 10^{-10}$  m/N. In the frequency range from 30 Hz to 1 100 Hz, the measured mobility curve closely approaches that of the mass of the block used for the calibration, plus the mass of the impedance head below the force transducer, combined with the mass of the attachment hardware. If the mass indicated by the low-frequency portion of the results of the mobility measurement does not equal the total calibration system mass, the mobility measuring system has failings which should be corrected. Inadequate sensitivity, incorrect calibration or electronic instrument problems are likely causes.

The anti-resonance (response dip) in the measured mobility curve at 2 400 Hz indicates that, at that frequency, the mobility of the calibration block has the same magnitude as that due to the attachment compliance of the impedance head. Since the two mobilities have essentially opposite phase, the total mobility magnitude tends towards zero at the anti-resonance frequency. It remains finite because of damping at the interface between the impedance head and the calibration block. This also accounts for the 0° phase angle at the anti-resonance.

At frequencies above the 2 400 Hz anti-resonance, the measured mobility curve approaches asymptotically the series combination of the compliance of the mounting hardware and that of the impedance head itself. It may be difficult to check the total compliance if the attachment has considerable damping. If the antiresonance dip is not very sharp, the mobility curve does not approach the effective compliance asymptote at high frequencies.

NOTE 2 The irregularity at 4 600 Hz in the measured mobility curve shown in Figure C.1 is the result of a transverse rocking mode of the impedance head about its attachment point.

If the overall calibration and sensitivity test is extended to sufficiently high frequencies, the mobility curve becomes controlled by the effective damping of the impedance head and its mounting hardware. The magnitude of the effective damping can seldom be computed. Thus, the test described in this annex is the only practical way to determine how much damping is present in the coupling of the force transducer to the structure to be tested.





## **Bibliography**

- <span id="page-33-0"></span>[1] ISO 5347-18, *Methods for the calibration of vibration and shock pick-ups — Part 18: Testing of transient temperature sensitivity*
- [2] ISO 5348, *Mechanical vibration and shock Mechanical mounting of accelerometers*
- [3] ISO 10846 (all parts), *Acoustics and vibration Laboratory measurement of vibro-acoustic transfer properties of resilient elements*
- [4] O'HARA, G.J. Mechanical impedance and mobility concepts. *J. Acoust. Soc. Am*. 1967, **41**, pp. 1180-1184
- [5] EWINS, D.J. Measurement and application of mechanical impedance data: Part 1 Introduction and ground rules; Part 2 — Measurement techniques; Part 3 — Interpretation and application of measured data. *J. Soc. Environ. Eng.* 1975, **14**(3), pp. 3-13; 1976, **15**(1), pp. 22-33; 1976, **15**(2), pp. 7-17 Copyright International Organization For Standardization For Standardization For Standardization (Provided By INST 100 The Copyright International International International International International International Inte
	- [6] SAINSBURY, M.G., EWINS, D.J. Vibration analysis of a damped machinery foundation structure using the dynamic stiffness coupling technique. *Trans. ASME — J. Eng. Ind*.1974, **96**, pp. 1000-1005
	- [7] HIXSON, E.J. Mechanical impedance. In: PIERSOL, A.G., PAEZ, T.L., editors. *Harris' shock and vibration handbook*, 6th edition, Chapter 10. New York, NY: McGraw-Hill, 2009
	- [8] RUBIN, S. Mechanical immittance- and transmission-matrix concepts, *J. Acoust. Soc. Am*. 1967, **41**, pp. 1171-1179
	- [9] EWINS, D.J. Whys and wherefores of modal testing. *J. Soc. Environ. Eng.*1979, **18**(3), pp. 3-15
	- [10] EWINS, D.J., GRIFFIN, J. A state-of-the-art assessment of mobility measurement techniques Results for the mid-range structures (30–3000 Hz). *J. Sound Vib.* 1981, **78**, pp. 197-222
	- [11] EWINS, D.J. State-of-the-art assessment of mobility measurements A summary of European results. *Shock Vib. Bull.* 1981, **51**, pp. 15-35
	- [12] EWINS, D.J. State-of-the-art assessment of mobility measurement techniques (SAMM) Summary of results. *J. Soc. Environ. Eng*. 1981, **20**, pp. 3-13
	- [13] BROWNJOHN, J.M.W., STEELE, G.H., CAWLEY, P., ADAMS, R.D. Errors in mechanical impedance data obtained with impedance heads. *J. Sound Vib.* 1980, **73**, pp. 461-468
	- [14] KLAASSEN, K.B. Piezoelectric accelerometers. In: REGTIEN, P.P.L,, editor, *Modern electronic measuring systems*, chap. III. Delft: Delft University Press, 1978
	- [15] BOUCHE, R.K. *Calibration of shock and vibration measuring transducers*. Shock and Vibration Information Center, U.S. Department of Defense, 1979. (Monograph SVM-11.)
	- [16] CANSDALE, R., GAUKROGER, D.R., SKINGLE, C.W. *A technique for measuring impedances of a spinning model rotor*. Farnborough: Royal Aircraft Establishment, 1971. (Technical Report TR 71092.)
	- [17] KUMME, R. A new calibration facility for dynamic forces up to 10 kN. In: *Proc. 17th IMEKO World Congress*, pp. 305-308, 2003
	- [18] ISO 7626-2, *Vibration and shock Experimental determination of mechanical mobility Part 2: Measurements using single-point translation excitation with an attached vibration exciter*

**ISO 7626-1:2011(E)** 

**ICS 17.160**  Price based on 30 pages

Copyright International Organization for Standardization  $\overline{V}$  vectors and the standardization of the standardization  $\overline{V}$  vectors and the standardization of the standardization  $\overline{V}$  vectors are the standardizat