TECHNICAL REPORT

ISO/TR 8550-3

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2007-06-01

Guidance on the selection and usage of acceptance sampling systems for inspection of discrete items in lots —

Part 3:
Sampling by variables

Lignes directrices pour la sélection d'un système, d'un programme ou d'un plan d'échantillonnage pour acceptation pour le contrôle d'unités discrètes en lots —

Partie 3: Échantillonnage par variables

Reference number
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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 8550-3 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

This first edition of ISO/TR 8550-3, together with ISO/TR 8550-1 and ISO/TR 8550-2, cancels and replaces ISO/TR 8550:1994.

ISO/TR 8550 consists of the following parts, under the general title *Guidance on the selection and usage of acceptance sampling systems for inspection of discrete items in lots*:

— *Part 1: Acceptance sampling*

— *Part 3: Sampling by variables*

The following part is under preparation:

— *Part 2: Sampling by attributes*

Introduction

This part of ISO/TR 8550 gives guidance on the selection of an acceptance sampling system for inspection by variables. It does this principally by reviewing the available systems specified by various standards and showing ways in which these can be compared in order to assess their suitability for an intended application. It is assumed that the choice has already been made to use sampling by variables in preference to sampling by attributes.

A corresponding guidance document on the selection of a generic acceptance sampling system, scheme or plan for inspection by attributes is given in ISO/TR 8550-2.

Guidance on the selection and usage of acceptance sampling systems for inspection of discrete items in lots —

Part 3:
Sampling by variables

1 Scope

The guidance in this part of ISO/TR 8550 is confined to acceptance sampling of products that are supplied in lots and that can be classified as consisting of discrete items (i.e. discrete articles of product). Each item in a lot can be identified and segregated from the other items in the lot and has an equal chance of being included in the sample. Each item of product is countable and has specific characteristics that are measurable on a continuous scale. Each characteristic has, at least to a good approximation, a normal distribution or a distribution that can be transformed so that it closely resembles a normal distribution.

Standards on acceptance sampling by variables are applicable to a wide variety of inspection situations. These include, but are not limited to, the following:

a) end items, such as complete products or sub-assemblies;

b) components and raw materials;

c) services;

d) materials in process;

e) supplies in storage;

f) maintenance operations;

g) data or records;

h) administrative procedures.

Although this part of ISO/TR 8550 is written principally in terms of manufacture and production, it should be interpreted liberally as it is applicable to the selection of sampling systems, schemes and plans for all types of product and processes as defined in ISO 9000.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition listed applies. For undated references, the latest edition of the referenced document (including any amendment) applies.

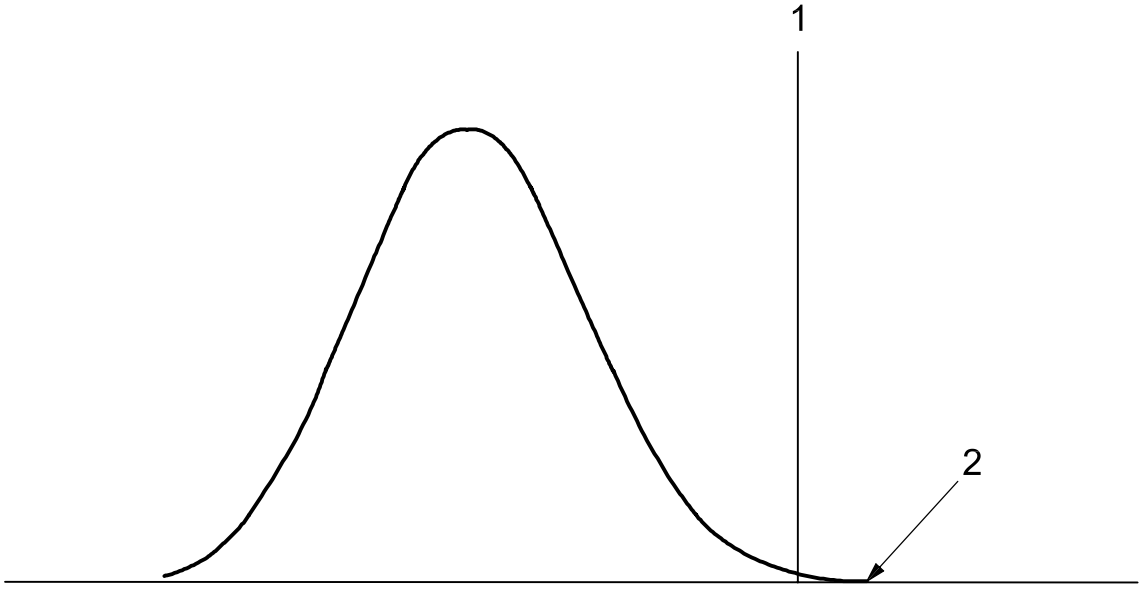
ISO/TR 8550-1:2007, *Guidance on the selection and usage of acceptance sampling systems for inspection of discrete items in lots — Part 1: Acceptance sampling*

3 Normality

3.1 Relationship between form of distribution of quality characteristic and percent nonconforming

A key aspect of sampling by variables is the form of the distributions of the quality characteristics. Consider a single quality characteristic. If it is normally distributed and if an upper specification limit is located at the mean plus two standard deviations, the percent nonconforming is about 2,5 %. If the specification limit is located at the mean plus three standard deviations, the percent nonconforming is about 0,1 %. However, if the distribution of the quality characteristic is not normal and has a large positive skewness, i.e. a long tail to the right, an upper specification limit located at the mean plus three standard deviations could conceivably yield a percent nonconforming approaching 10 % instead of about 0,1 % (see Figures 1 and 2).

Therefore, whenever a sampling plan for inspection by variables for percent nonconforming is to be employed, it is highly desirable to check any assumptions about the shape of the distribution, especially in the tails of the distribution. If the AQL is very small, for example 0,1 %, a study of several thousand items should be made, including a test of distributional form.

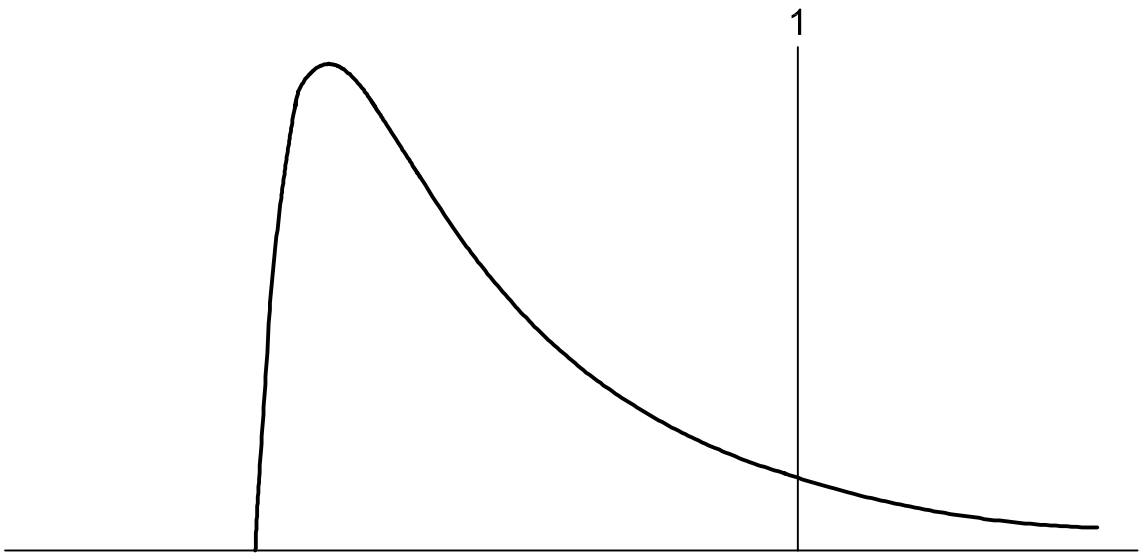


Key

1 upper specification limit

2 0,1 % above specification

Figure 1 — Normal distribution



Key

1 upper specification limit

Figure 2 — Distribution with large positive skewness

3.2 Identifying departure from normality

3.2.1 Subjective assessment

The degree to which a sample appears to have come from a normal distribution can be subjectively assessed by means of a normal probability plot. Such a plot is constructed in the following way. Once the random sample has been selected and the quality characteristic x has been measured for each item, the values x_1, x_2, . . ., x_n are arranged in ascending order $x_{[1]}$, $x_{[2]}$, . . ., $x_{[n]}$, such that $x_{[1]} \leqslant x_{[2]} \leqslant$, . . . $\leqslant x_{[n]}$. The points with

coordinates $\left\{x_{[i]}, \left(i - 3/8\right) / \left(n + 1/4\right)\right\}$ are then plotted on a sheet of normal probability paper for i = 1, 2, . . ., n. To facilitate this process, an A4 sheet of normal probability paper that can be freely photocopied is provided in Annex A.

Figure 3 shows the normal probability plot of a random sample of size 100 from a normal distribution. The graph paper is specially designed so that data from a normal distribution tend to lie close to a straight line. A straight line has been drawn by eye through the data, showing in this case that there are only minor departures from linearity.

When data originate from a normal distribution, departures of the probability plot from linearity are due solely to sampling fluctuations. Conversely, data from other types of distribution will tend to show departures from linearity of a characteristic type, helping in the determination of the family of distributions to which the data belong. Knowledge of this family can indicate the appropriate transformation to make to the data to bring these closer to normality.

Figures 4 to 7 show the density functions and examples of normal probability plots based on a random sample of size 100 for, respectively, a lognormal, Cauchy, Laplace, and exponential distribution, respectively. On the normal probability plot for Figures 4 to 6, a straight line has been drawn through the data points to aid the eye in identifying the characteristic differences.

For the lognormal distribution, there is a pronounced downward concavity.

The Cauchy distribution is almost indistinguishable from the normal distribution towards its centre, but the extra thickness of its tails results in the plot being relatively high for low values of x and relatively low for high values of x, the extremities of the plot being almost horizontal.

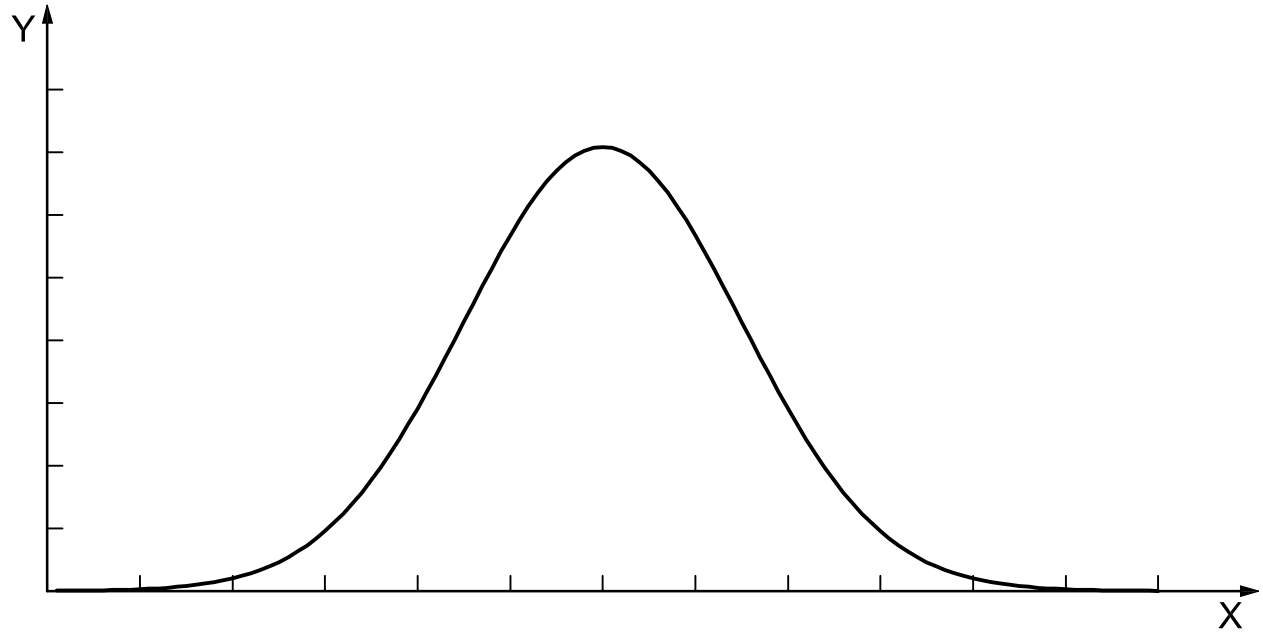
The Laplace distribution is similar, except that there is a shorter region in the normal probability plot where the distribution is indistinguishable from the normal distribution, and the extremities of the plot are far from horizontal.

The normal probability plot for the exponential distribution has a very characteristic shape, rising very steeply at the left and becoming almost horizontal towards the right.

These are a small selection from the many possible distributions from which data might have arisen. In some cases, e.g. the lognormal distribution, the distribution can be transformed exactly to normality without knowing its parameters (see 3.3.2 and 3.3.3). In other cases, approximate normality may be achieved, e.g. by using the fourth root transformation on exponentially distributed variables, as shown by Kittlitz[20]. In other cases, acceptance sampling by variables might not be possible without a method tailored to that family of distributions. If such a method does not exist, acceptance sampling by attributes might have to be used instead, the loss in efficiency being more than compensated for by the increase in integrity of the acceptance sampling results.

Figures 4 to 7 show normal probability plots for samples of size 100. Often there is not the luxury of such large samples. With small samples, it is less clear whether the departures from linearity of the normal probability plot are due to non-normality or merely to sampling fluctuations. In case of doubt, subjective assessment of departure from normality should be replaced by objective statistical tests, such as those discussed in 3.2.2.

Further information on tests for departure from normality is given in ISO 5479 and ISO 2854:1976, Clause 2.

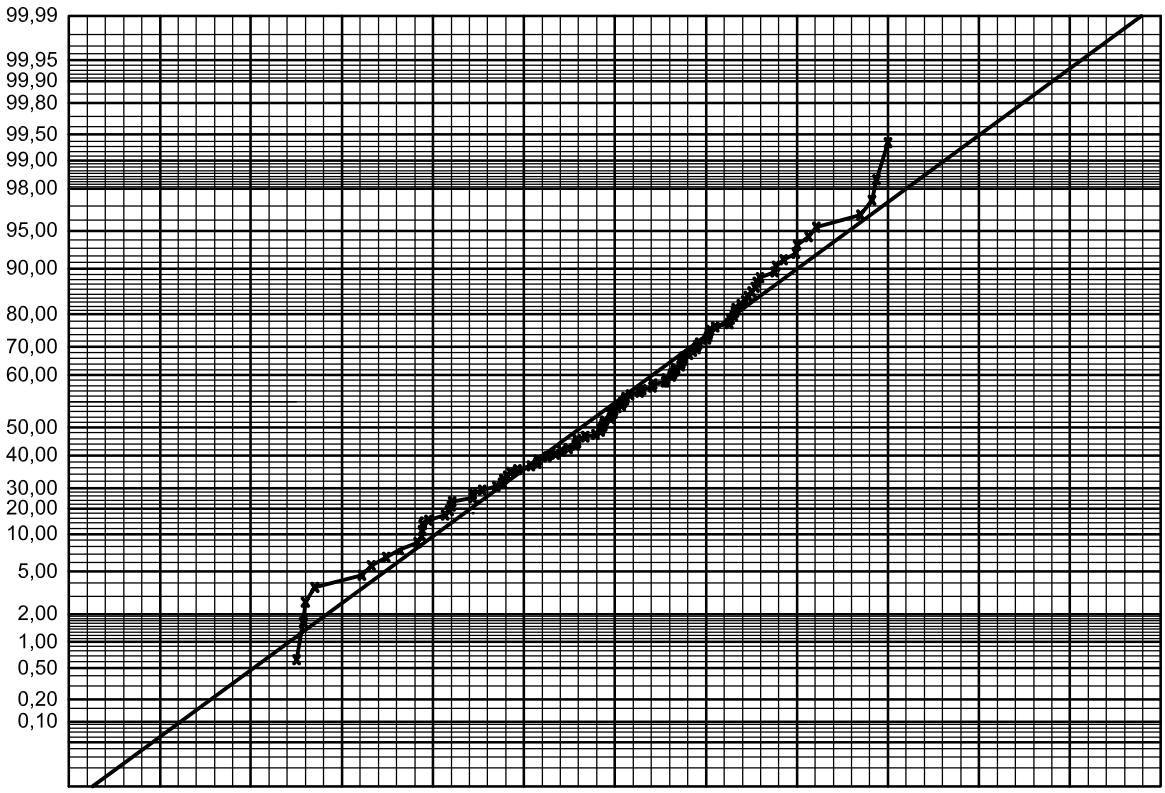


Key

X quality characteristic, x

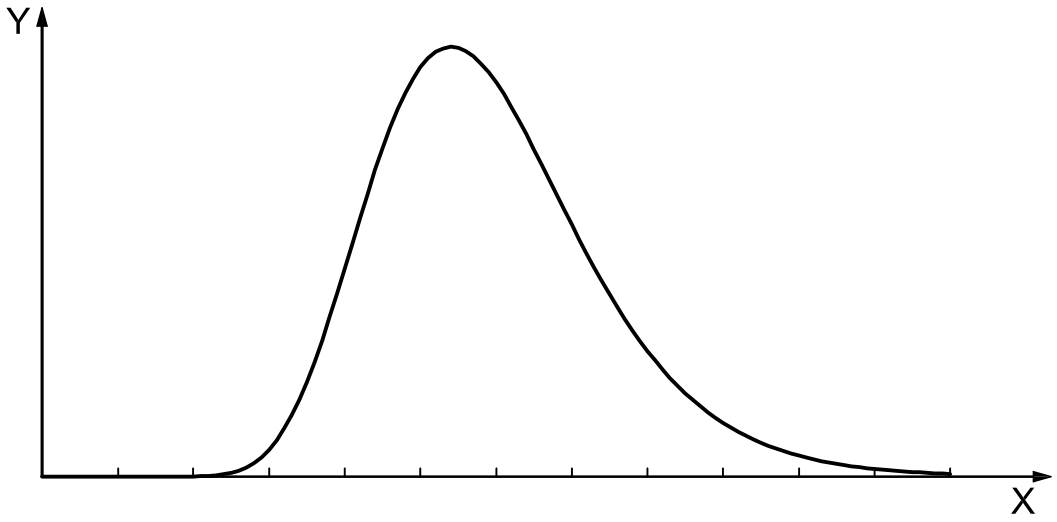
Y probability density of x

a) Normal distribution



b) Normal probability plot of a random sample of size 100 from a normal distribution

Figure 3 — Normal distribution and normal probability plot

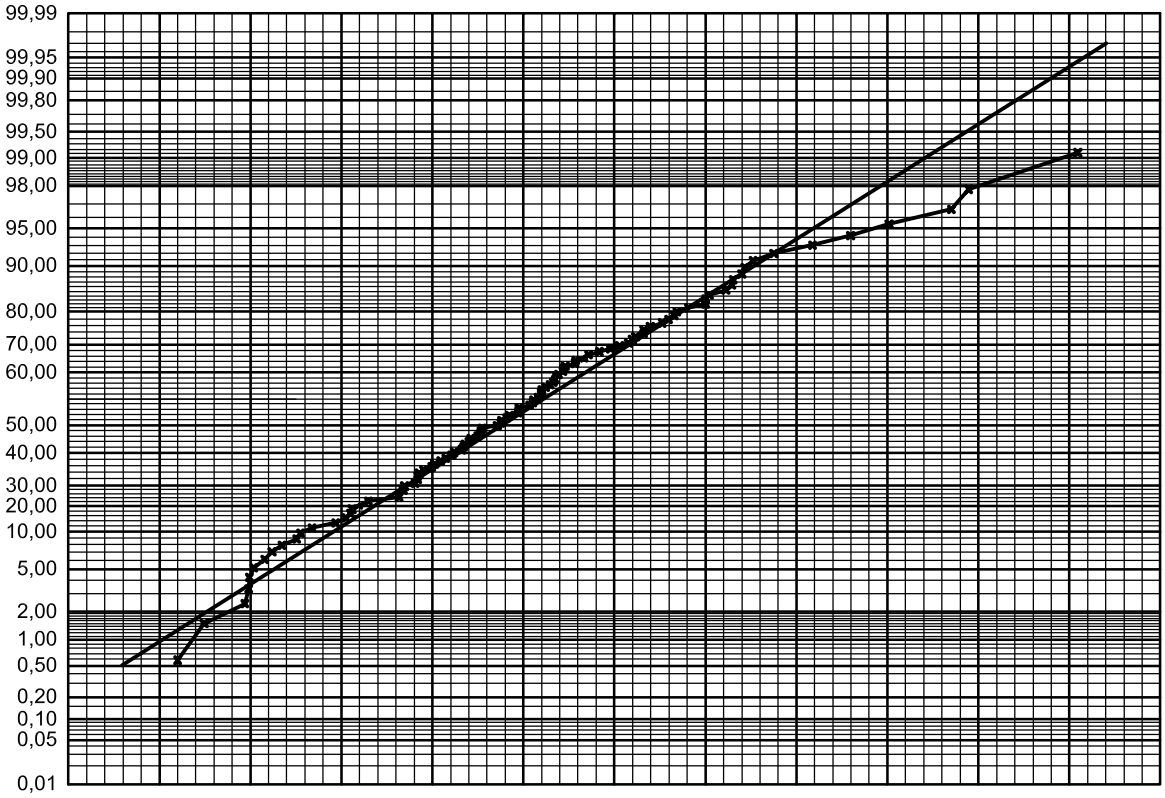


Key

X quality characteristic, x

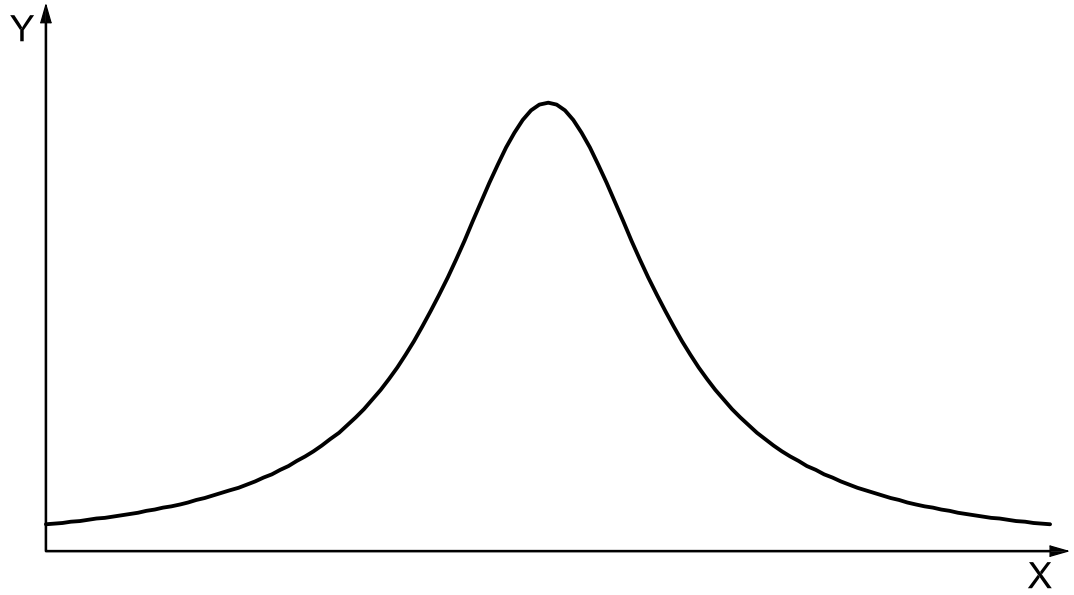
Y probability density of x

a) Lognormal distribution



b) Normal probability plot of a random sample of size 100 from a lognormal distribution

Figure 4 — Lognormal distribution and normal probability plot

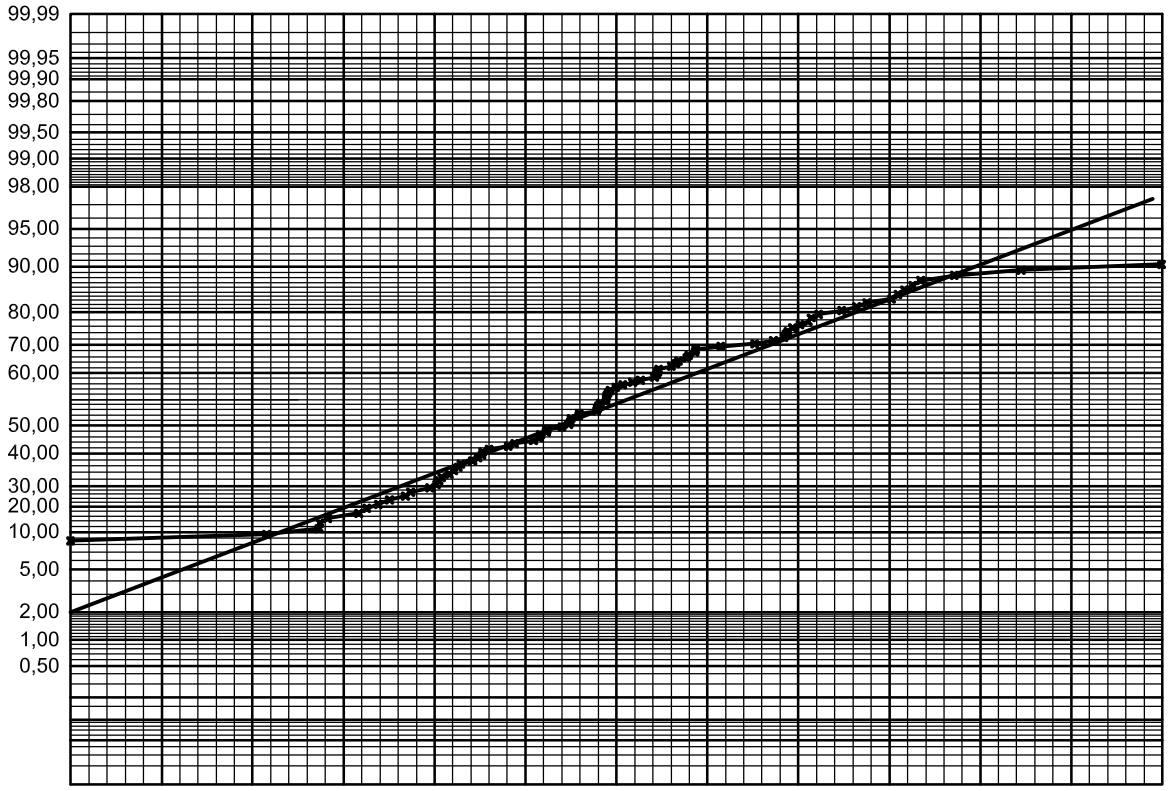


Key

X quality characteristic, x

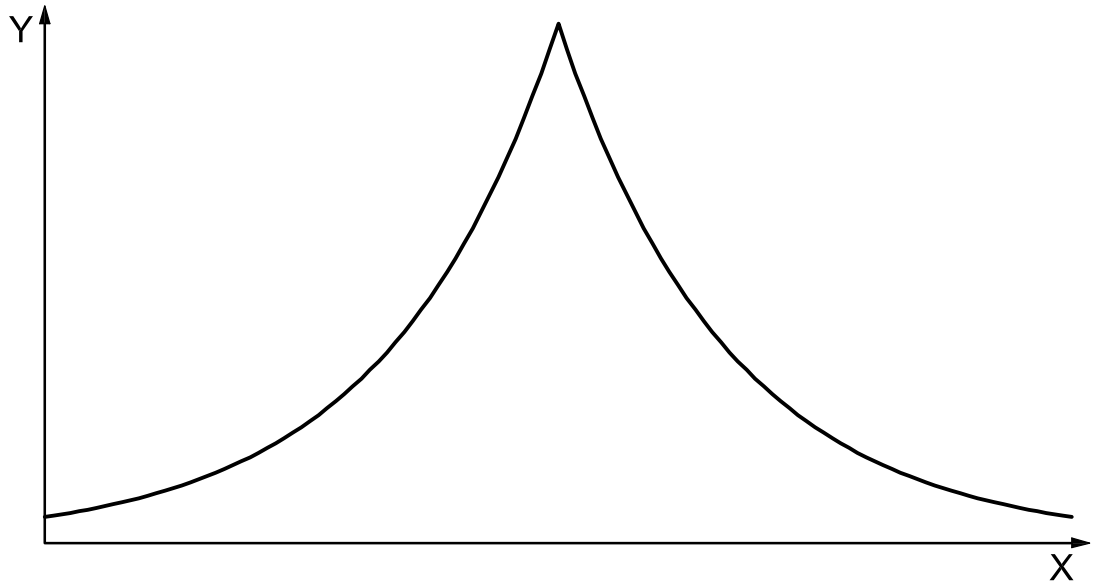
Y probability density of x

a) Cauchy distribution



b) Normal probability plot of a random sample of size 100 from a Cauchy distribution

Figure 5 — Cauchy distribution and normal probability plot

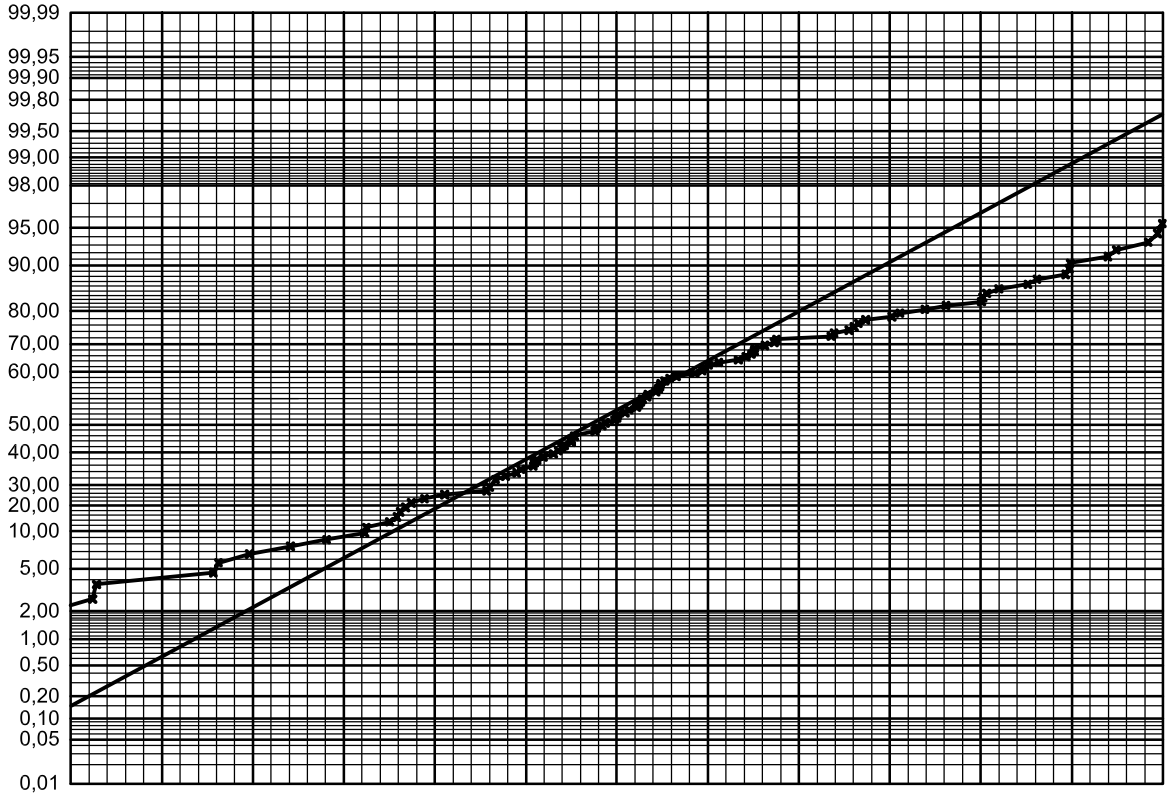


Key

X quality characteristic, x

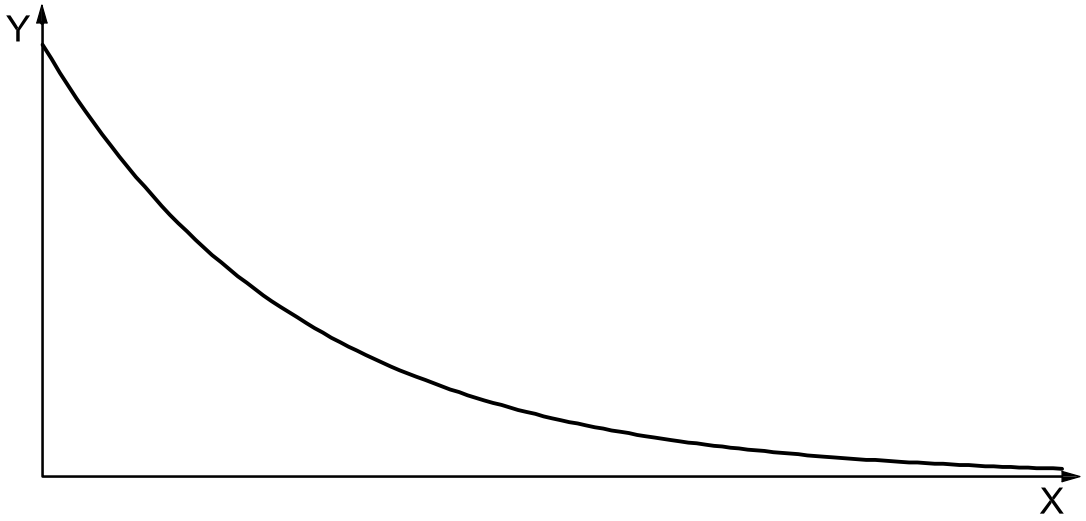
Y probability density of x

a) Laplace distribution



b) Normal probability plot of a random sample of size 100 from a Laplace distribution

Figure 6 — Laplace distribution and normal probability plot

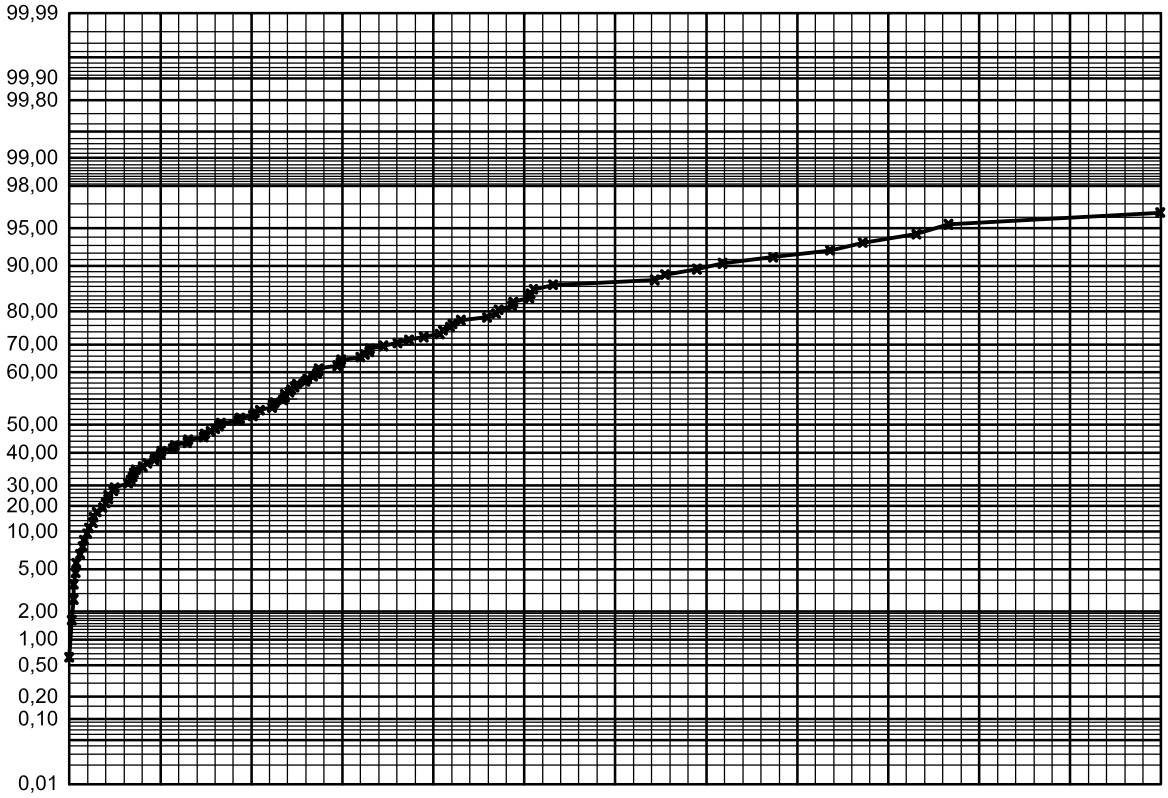


Key

X quality characteristic, x

Y probability density of x

a) Exponential distribution



b) Normal probability plot of a random sample of size 100 from an exponential distribution

Figure 7 — Exponential distribution and normal probability plot

3.2.2 Statistical tests for departure from normality

3.2.2.1 Directional versus omnibus tests

Statistical tests are used to determine the degree to which the available evidence fails to support a given null hypothesis, say H_0. The power of the test is the probability of rejecting the hypothesis H_0 in favour of the alternative hypothesis H_1 when the alternative hypothesis is true.

When testing for departures from normality, the null hypothesis H_0 is that the distribution is normal while the alternative hypothesis H_1 is that it is not normal. If the alternative hypothesis is more specific, stating the alternative family of distributions to which the distribution belongs, then the test is said to be directional. Otherwise, it is said to be an omnibus test.

In both cases, a statistic T is calculated from the sample evidence, and H_0 is rejected if the value of T lies in a so-called critical region. The critical region is chosen so that the probability of T falling in the critical region when H_0 is true is a small quantity, usually 5 %. For an omnibus test, the critical region simply consists of values of T that lie far away from the expected value of T under H_0. For a directional test, the critical region consists of the values of T for which the power is greatest.

In general, therefore, greater power is achieved by being as specific as can be justified about the alternative hypothesis, i.e. about the likely nature of the departure from the null hypothesis.

As might be expected, power is generally also increased by increasing the sample size on which T is based.

3.2.2.2 Directional tests

ISO 5479:1997 provides two directional tests. One of these is for skewness, the other for kurtosis (i.e. peakedness). A simultaneous bi-directional test for skewness and kurtosis is also provided. The skewness and kurtosis test statistics for n observations $x_1, x_2, \ldots, x_n$ are, respectively, the moment coefficients:

$$\sqrt{b_1} = m_3 / m_2^{3/2} \qquad \text{and} \qquad b_2 = m_4 / m_2^2,$$

where:

$$m_j = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^j \qquad \text{for} \qquad j = 2, 3 \text{ and } 4.$$

3.2.2.3 Omnibus tests

ISO 5479:1997 also provides two omnibus tests: the Shapiro-Wilk test and the Epps-Pulley test. The test statistic for the Shapiro-Wilk test is a linear function of the ordered observations. The Epps-Pulley test statistic is a little more complicated to implement as it involves a sum and a double sum of exponentiated quantities. A rule of thumb is given for deciding which of these to use in a given situation.

3.3 Transforming to normality

3.3.1 Normalization and variance stabilization

Much analysis of variance is invalidated if the quality characteristic under analysis is heteroscedastic, i.e. when its variance varies with its mean. A mathematical transformation of the characteristic that roughly equalizes the variance over all values of the mean is called a variance-stabilizing transformation. It is often the case that transforming such data to eliminate heteroscedasticity, i.e. to make the data homoscedastic, also has the effect of making the data more normal. In other words, variance-stabilizing transformations are often normalizing transformations.

A rough rule for determining the appropriate transformation is as follows. If the standard deviation σ of a process characteristic x can be expressed approximately as a function $h(\mu)$, where μ is the corresponding mean of the characteristic, then an approximate variance-stabilizing transformation of x is $g(x)$, where:

$$g(x) = \int^{x} \frac{\mathrm{d}t}{h(t)} . \tag{1}$$

Examples of the use of this method are given in 3.3.2 and 3.3.3.

If a test for departure from normality indicates that x is non-normal, the use of $y = g(x)$ should be considered instead of x.

3.3.2 The square root transformation

Where σ is a constant multiple of $\sqrt{\mu}$, i.e. $h(\mu) = c\sqrt{\mu}$, where c is a constant, then, from Expression (1):

$$g(x) = \int^{x} \frac{\mathrm{d}t}{c\sqrt{t}} = \frac{2}{c}\sqrt{x} .$$

As the coefficient $2/c$ has no effect on the stabilization of the variance, it can be ignored. An approximate variance-stabilizing transformation is therefore the square root transformation:

$$g(x) = \sqrt{x} .$$

The standard deviation of $g(x)$ is approximately $c/2$.

3.3.3 The logarithmic transformation

When σ is a constant multiple of μ, i.e. $h(\mu) = c\mu$, where c is a constant, then, from Expression (1):

$$g(x) = \int^{x} \frac{\mathrm{d}t}{ct} = \frac{1}{c}\ln(x) .$$

As the coefficient $1/c$ has no effect on the stabilization of the variance, it can be ignored. An approximate variance-stabilizing transformation is therefore the logarithmic transformation:

$$g(x) = \ln(x).$$

The standard deviation of $g(x)$ is approximately c.

This transformation might be appropriate when the quality characteristic is the sample variance based on a sample of size n, in which case $c = \sqrt{2/(n-1)}$.

3.3.4 The Box-Cox transformation

A general transformation, proposed by Box and Cox[19], is to set

$$g(x) = \frac{x^{\lambda} - 1}{\lambda}$$

where $\lambda > 0$.

Note that:

— setting λ equal to 1 simply relocates the existing distribution, leaving its original shape unchanged;

— setting λ equal to 0,5 is equivalent to first using the square root transformation and then relocating and rescaling the resulting distribution;

— letting λ tend to zero is equivalent to using the logarithmic transformation, i.e. equivalent to setting $g(x) = \ln(x)$.

However, λ is not limited to taking one of these particular values, so the value of λ in excess of zero that best normalizes the distribution can be found either by trial and error or by some optimization method applied to past data.

A more general version of the Box-Cox transformation is:

$$g(x) = \frac{(x+\lambda_2)^{\lambda_1} - 1}{\lambda_1} \text{ for } \lambda_1 > 0 \text{ and } \lambda_2 > -x_{[1]} .$$

This transformation effectively relocates the distribution of x by an amount λ_2 before applying the simpler Box-Cox power transformation. A consequence is that the more general transformation does not require all the data values to be positive. Because the more general transformation has two parameters, it allows a greater range of distribution families to be transformed to normality.

4 Types of control

4.1 Control of a single quality characteristic

4.1.1 General

Acceptance sampling by variables can become complicated when there are two or more quality characteristics, so the text first considers the case where only a single quality characteristic is being controlled. As the acceptance criterion for a single quality characteristic involves either $\bar{x}$ and s, or $\bar{x}$ and σ, it can always be represented diagrammatically as well as algebraically. A diagrammatic representation of an acceptance criterion is called an acceptance diagram.

Within the case of a single quality characteristic, there are several possible methods of control, which are described in 4.1.2 and 4.1.3.

4.1.2 Single specification limit

The simplest case of a single quality characteristic is where there is a single specification limit, i.e. where either an upper limit or a lower limit to values of the characteristic is specified, but not both. Control of such a characteristic by means of sampling by variables is relatively straightforward, requiring the sample mean to be within a specification and at least a given multiple, denoted by k, of the sample standard deviation (or process standard deviation, if known) away from the specification limit. When the acceptance criterion is expressed in terms of this factor k, the method is described as “Form k” (see 5.2).

4.1.3 Double specification limits

4.1.3.1 General

Rather more complicated is the case of a single quality characteristic with double specification limits, i.e. where both an upper limit and a lower limit to values of the characteristic are specified. In this case, there are three modes of control.

4.1.3.2 Combined control

Double specification limits are said to be under combined control when the fraction nonconforming beyond both limits belongs to the same class, to which a single AQL applies. By implication, nonconformity beyond either limit is of roughly the same degree of seriousness.

EXAMPLE 1 A weapon guidance system is to be tested against a moving target. Missing the target either to the left or to the right is equally unsatisfactory so combined control of both sides of the target might be appropriate in this case.

Form k is inadequate for combined control. Instead, Form p* is used, i.e. the lot is accepted only if an estimate of the process fraction nonconforming is less than a given value p*. In other words, p* is the maximum estimate of the process fraction nonconforming that is deemed to be acceptable for the given sample size and AQL.

4.1.3.3 Separate control

Double specification limits are said to be under separate control when the fraction nonconforming beyond the two limits belongs to different classes, to which different AQLs apply.

Again, by implication, nonconformity beyond each of the two limits is of a different degree of seriousness. The AQL for the class of greater seriousness will be smaller than the AQL for the other class.

EXAMPLE 2 In a given bottle-filling plant, overfilling leads to a marginal reduction in profit, whereas underfilling is much more serious as it could lead to weights and measures violations, financial penalties, bad publicity and loss of goodwill. The lower specification limit in this case should therefore have a much smaller AQL than the upper specification limit.

For separate control, a Form k acceptance criterion can be applied separately to each limit. The lot is accepted if both acceptance criteria are satisfied.

4.1.3.4 Complex control

Double specification limits are said to be under complex control when the fraction nonconforming beyond the limit of greater seriousness belongs to one class, to which a given AQL applies, and the combined fraction nonconforming beyond both limits belongs to another class, to which a larger AQL applies. This allows some trade-off between the fractions nonconforming at both ends of the distribution of values of the quality characteristic while still maintaining control of the fraction nonconforming at the more important end of the distribution that is of the greatest concern.

EXAMPLE 3 Wooden strips, supplied in batches and used in the construction of garden furniture, are specified to be between 86,5 cm and 86,7 cm in length. Strips that are too large can be shortened, but strips that are too short are unusable and have to be replaced, which is more time-consuming and can interfere with production. An AQL of 2,5 % is set for both limits combined, with another AQL on the lower limit of 0,65 %.

Complex control is a combination of combined control of both limits with control of just one of those limits. Form k is therefore again inadequate for this situation, so that Form p* has to be used.

4.2 Control of two or more quality characteristics

4.2.1 General

The number of possible combinations of control soon becomes vast as the number of quality characteristics increases. The discussion below is therefore confined to providing examples in the case of two quality characteristics, x and y.

4.2.2 Examples of the control of two independent quality characteristics

4.2.2.1 General

For most cases of two or more quality characteristics, it is necessary to use Form p*. For brevity, all the examples given for two variables are given in terms of Form p*.

In all cases, it is assumed that a single acceptance criterion is stipulated for each class of nonconformity, and that a lot is only acceptable if the criterion for each and every class is satisfied.

4.2.2.2 Notation

With two quality characteristics, some new notation is necessary. The two quality characteristics are denoted by x and y. The lower and upper specification limits on x are denoted by $L(x)$ and $U(x)$ respectively, and on y by $L(y)$ and $U(y)$. The process fraction nonconforming beyond each of these four limits is denoted by $p_L(x), p_U(x), p_L(y)$ and $p_U(y)$, respectively, and their estimates by $\hat{p}_L(x), \hat{p}_U(x), \hat{p}_L(y)$ and $\hat{p}_U(y)$.

Due to the independence of x and y, the total process fraction nonconforming in a class containing nonconformity at all four of these limits is given by:

$$p = 1-\left[1-p_L(x)-p_U(x)\right]\left[1-p_L(y)-p_U(y)\right] \tag{2}$$

and its estimate by:

$$\hat{p} = 1-\left[1-\hat{p}_L(x)-\hat{p}_U(x)\right]\left[1-\hat{p}_L(y)-\hat{p}_U(y)\right]. \tag{3}$$

The class of nonconformity is indicated by the appropriate subscript from A, B, etc. to p or $\hat{p}$. Expressions (2) and (3) may be used generally, with the elements not included in the class set to zero. The following examples demonstrate this.

Note that, if $p_L(x), p_U(x), p_L(y)$ and $p_U(y)$ are all very small, then $p \cong p_L(x)+p_U(x)+p_L(y)+p_U(y)$; similarly, if $\hat{p}_L(x), \hat{p}_U(x), \hat{p}_L(y)$ and $\hat{p}_U(y)$ are all very small, then $\hat{p} \cong \hat{p}_L(x)+\hat{p}_U(x)+\hat{p}_L(y)+\hat{p}_U(y)$.

4.2.2.3 Example of control of a single class (Example 4)

For a single class A consisting of $p_L(x), p_U(x)$ and $p_U(y)$, setting $p_L(y) = 0$ in Expression (2) gives:

$$p_{\mathrm{A}} = 1-\left[1-p_L(x)-p_U(x)\right]\left[1-p_U(y)\right].$$

Similarly, setting $\hat{p}_L(y) = 0$ in Expression (3), gives $\hat{p}_{\mathrm{A}} = 1-\left[1-\hat{p}_L(x)-\hat{p}_U(x)\right]\left[1-\hat{p}_U(y)\right]$.

$p_L(x)$ and $p_U(x)$ are under combined control. It is important to recognize this when the process standard deviation is presumed to be known (see 5.3.4.2).

4.2.2.4 Examples of control of two classes (Examples 5, 6 and 7)

4.2.2.4.1 Example 5

If class A consists of $p_U(x)$ and class B consists of $p_L(x), p_L(y)$ and $p_U(y)$, then, from Expression (3):

$$\hat{p}_{\mathrm{A}} = 1-\left[1-\hat{p}_U(x)-0\right]\left[1-0-0\right] = \hat{p}_U(x) \text{ and}$$
$$\hat{p}_{\mathrm{B}} = 1-\left[1-\hat{p}_L(x)-0\right]\left[1-\hat{p}_L(y)-\hat{p}_U(y)\right] = 1-\left[1-\hat{p}_L(x)\right]\left[1-\hat{p}_L(y)-\hat{p}_U(y)\right].$$

NOTE $p_L(x)$ and $p_U(x)$ are under separate control, while $p_L(y)$ and $p_U(y)$ are under combined control. Again, it is important to recognize this when the process standard deviations of x and y are presumed to be known (see 5.3.4.2).

4.2.2.4.2 Example 6

If class A consists of $p_L(x)$ and $p_L(y)$, while class B consists of $p_U(x)$ and $p_U(y)$, then:

$$\hat{p}_{\mathrm{A}} = 1-\left[1-\hat{p}_L(x)\right]\left[1-\hat{p}_L(y)\right] \text{ and } \hat{p}_{\mathrm{B}} = 1-\left[1-\hat{p}_U(x)\right]\left[1-\hat{p}_U(y)\right].$$

NOTE $p_L(x)$ and $p_U(x)$ are under separate control, and $p_L(x)$ and $p_U(y)$ are also under separate control. Again, this is important when the process standard deviations for x and y are presumed to be known (see 5.3.4.2).

4.2.2.4.3 Example 7

If class A consists of $p_U(x)$ while class B consists of $p_U(x)$, $p_L(y)$ and $p_U(y)$, then:

$$\hat{p}_{\mathrm{A}} = \hat{p}_U(x) \text{ and } \hat{p}_{\mathrm{B}} = 1-\left[1-\hat{p}_U(x)\right]\left[1-\hat{p}_L(y)-\hat{p}_U(y)\right].$$

NOTE $p_U(x)$, $p_L(y)$ and $p_U(y)$ are under complex control, while $p_L(y)$ and $p_U(y)$ are under combined control. Once again, this is important when the process standard deviations for x and y are presumed to be known (see 5.3.4.2).

4.2.2.5 Example of control of three classes (Example 8)

If class A consists of $p_L(x)$, class B consists of $p_U(y)$ and class C consists of $p_L(y)$, then:

$$\hat{p}_{\mathrm{A}} = \hat{p}_L(x) \text{ and } \hat{p}_{\mathrm{B}} = \hat{p}_U(y) \text{ and } \hat{p}_{\mathrm{C}} = \hat{p}_L(y).$$

NOTE $p_L(y)$ and $p_U(y)$ are under separate control, which is of significance if the process standard deviation of y is presumed to be known (see 5.3.4.2).

5 Forms of acceptance criteria

5.1 General

5.1.1 Target of acceptability test

ISO 2859-1:1999 subclause 8.3.3 explains that only Type B operating characteristic curves are relevant to sampling by variables. Thus, it is the process fraction nonconforming at the time that the lot was produced, rather than the lot fraction nonconforming, that is being assessed. For all the acceptance criteria, this assessed value is compared, either implicitly or explicitly, with an upper limit.

The acceptability constants referred to in this clause are given in ISO 3951-1 (Form k) and in ISO 3951-2 (Form k and Form p*).

5.1.2 The "s" method and the "σ" method

If a process standard deviation σ is unknown, it is estimated by the corresponding sample standard deviation s. Acceptance sampling procedures based on s are referred to collectively as the "s" method. Conversely, acceptance sampling procedures based on σ are referred to collectively as the "σ" method.

Under the "σ" method there is less uncertainty in the value of the quality statistic, which generally results in a lower sample size requirement, dramatically in the case of large lots.

NOTE The process standard deviation, although never known exactly, might on occasion be known accurately enough for practical purposes.

5.1.3 Maximum standard deviations

5.1.3.1 Maximum sample standard deviation, s_{max}

For the control of double specification limits under the "s" method, there is a limit to s above which it will be impossible for a lot to satisfy the acceptance criteria. This limit is called the maximum sample standard deviation s_{max}. The value of s_{max} will be different depending on whether separate, combined or complex control is in force, and whether inspection is normal, tightened or reduced. For technical reasons, it is optional under the "s" method to first test that s does not exceed the appropriate s_{max}. The advantage of doing so is that, when s exceeds s_{max} the lot can be non-accepted at once without carrying out any numerical calculations.

The international standards in the ISO 3951 series provide values of factors for s_{max} for combined control under normal, tightened and reduced inspection. Each of these is a two-dimensional table, with sample size code letter tabulated against AQL.

Tables of factors for s_{max} have not been provided for separate or complex control because three-dimensional tables would have been needed. For separate control, it would be necessary to tabulate the sample size code letter against both the AQL at the lower specification limit and the AQL at the upper specification limit. Similarly, for complex control it would be necessary to tabulate the sample size code letter against both the AQL for the single specification limit and the AQL for both specification limits combined. As each of these methods of control would require more than a dozen tables, and as an initial test of s against s_{max} is optional, such tables have not been provided in the standards.

5.1.3.2 Maximum process standard deviation, σ_{max}

For the control of double specification limits under the "σ" method, there will be a limit to σ above which it is impossible for a lot to satisfy the acceptance criteria. This limit is called the maximum process standard deviation σ_{max}. The value of σ_{max} will vary depending on whether separate, combined or complex control is applied. For technical reasons, it is necessary under the "σ" method to ascertain that σ does not exceed the appropriate σ_{max}. To avoid unnecessary calculations, this is normally done first.

Tables of σ_{max} are smaller and fewer than for s_{max}, because σ_{max} is

— independent of the sample size, and therefore independent of the sample size code letter, so that the table for combined control is one-dimensional and the tables for separate and complex control are two-dimensional, and

— determined under the worst case scenario of tightened inspection, so it is independent of inspection severity.

5.2 Form k procedures for single sampling plans

5.2.1 Applicability

Form k procedures are applicable to a single quality characteristic when control of a single specification limit or separate control of double specification limits is required. The value of the acceptability constant k depends on the lot size, the acceptance quality limit (AQL), the inspection level and whether or not the process standard deviation σ is known. Under single sampling, determination of a lot's acceptability is based on a single sample from the lot.

5.2.2 Form k with a single specification limit

In the case of a single specification limit, the lot is acceptable if the quality statistic Q is greater than or equal to the acceptability constant k. The quality statistic measures how far inside the specification limit the sample mean lies, as a multiple of the sample or process standard deviation, whichever is appropriate.

a) In the case of an upper specification limit U, the quality statistic is defined thus:

1) under the "s" method:

$$Q_U = \frac{U - \bar{x}}{s}; \tag{4}$$

2) under the "σ" method:

$$Q_U = \frac{U - \bar{x}}{\sigma}. \tag{5}$$

b) In the case of a single lower specification limit, the quality statistic is:

1) under the "s" method:

$$Q_L = \frac{\bar{x} - L}{s}; \tag{6}$$

2) under the "σ" method:

$$Q_L = \frac{\bar{x} - L}{\sigma}. \tag{7}$$

Regardless of which formula for Q is appropriate, the acceptance criterion necessitates that the sample mean be at least k standard deviations inside the specification limit, e.g.:

$$Q_U = \frac{U - \bar{x}}{s} \geqslant k, \tag{8}$$

which can alternatively be written thus:

$$\bar{x} \leqslant U - ks. \tag{9}$$

An acceptance diagram for a single, upper specification limit is shown in Figure 8.

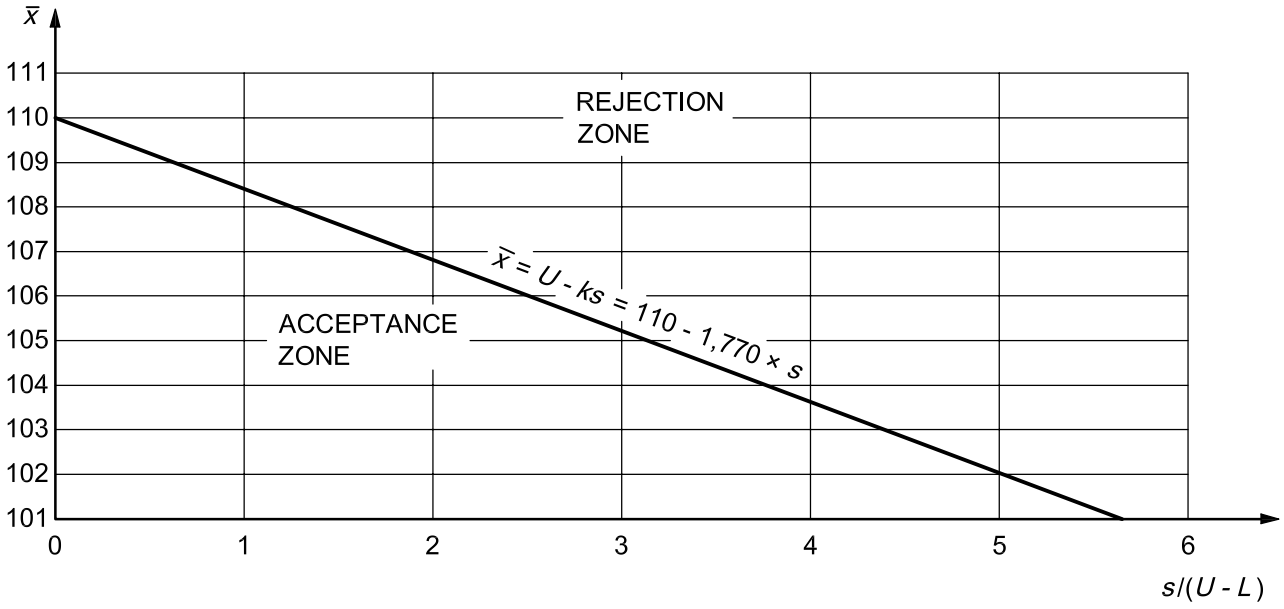


Figure 8 — Acceptance diagram for a single, upper specification limit of 110, sample size code letter G, "s" **method,** AQL = 1 %: n = 18, k = 2,770

5.2.3 Form k with separate control of double specification limits

In the case of separate control of double specification limits, the upper and lower limits belong to different classes with different AQLs that reflect their different degrees of seriousness. This in turn leads to different Form k acceptability constants that can be denoted by k_U and k_L, respectively. Denoting the corresponding quality statistics by Q_U and Q_L, the lot is considered to be acceptable only if both $Q_U \geqslant k_U$ and $Q_L \geqslant k_L$.

In the case of unknown process standard deviation (the "s" method), these two inequalities can be written:

$$L + k_L s \leqslant \bar{x} \leqslant U - k_U s \, .$$

It follows that:

$$L + k_L s \leqslant U - k_U s \, ,$$

from which it can be deduced that:

$$s \leqslant \frac{U - L}{k_U + k_L} \, . \tag{10}$$

In other words, if Inequality (10) is not satisfied, then it is not possible for inequalities $Q_U \geqslant k_U$ and $Q_L \geqslant k_L$ to be satisfied simultaneously. The right hand side of Inequality (10) is therefore $s_{\max}$ for separate double specification limits.

An acceptance diagram for the separate control of double specification limits under the "s" method is shown in Figure 9. By a simple transformation of the axes, the figure has been standardized to apply to any values of the upper and lower specification limits U and L.

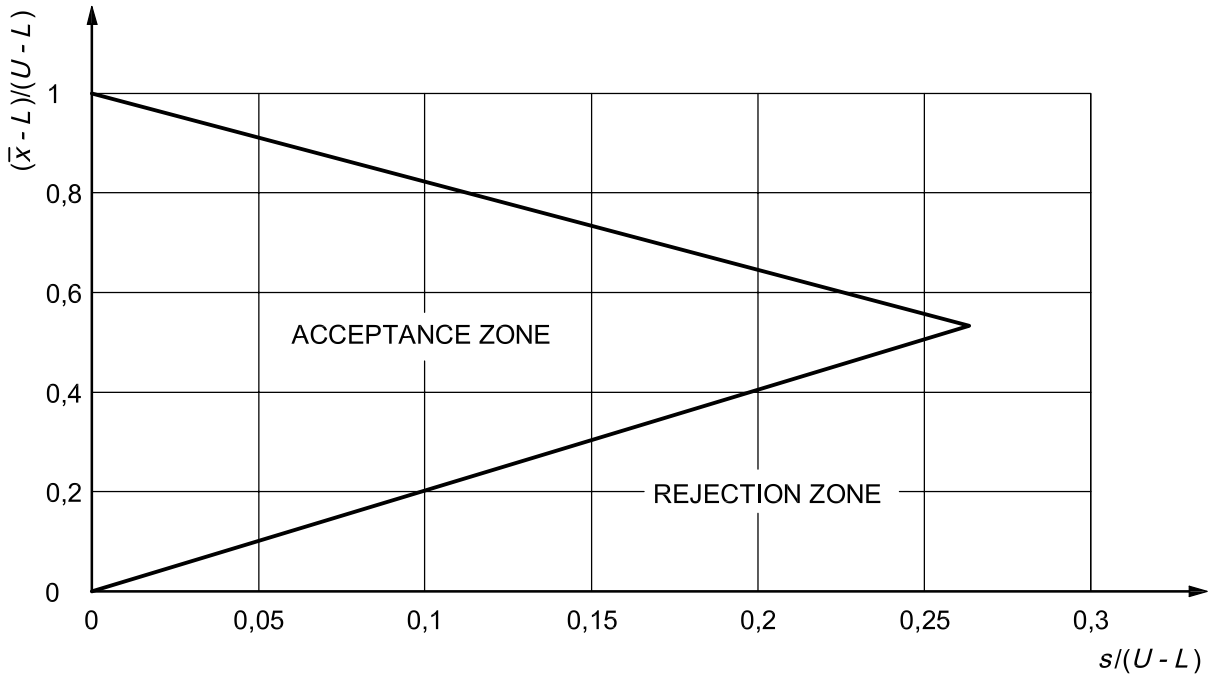


Figure 9 — Standardized acceptance diagram for separate control of double specification limits: "s" method, sample size code letter G, AQLs 0,40 % on lower limit and 1,0 % on upper limit

For the "σ" method, the method by which each value of $\sigma_{\max}$ was calculated is rather more complicated. First, the acceptance constants under tightened inspection were determined. The AQLs to which these acceptance constants corresponded under normal inspection were then found, say a_U and a_L. Finally, the value of $\sigma_{\max}$ was calculated as:

$$\sigma_{\max} = \frac{U - L}{K_{a_U} + K_{a_L}} \tag{11}$$

where K_p is the upper p-fractile of the standard normal distribution.

5.3 Form p^* procedures for single sampling plans

5.3.1 Applicability

Form p^* procedures involve accepting a lot only when an explicit estimate $\hat{p}$ of the process fraction nonconforming p does not exceed an upper limit p^*, i.e. the acceptance criterion is of the form:

Accept the lot if $\hat{p} \leqslant p^*$, otherwise do not accept. (12)

Unlike Form k procedures, Form p^* procedures are completely general. They can be applied to more than one quality characteristic at a time, and they encompass the combined and complex control of double specification limits. However, just like a Form k acceptability constant, the value of a Form p^* acceptability constant depends on the lot size, the acceptance quality limit (AQL), the inspection level, and whether or not the process standard deviation σ is known.

5.3.2 Form p^* for a single specification limit (single quality characteristic)

5.3.2.1 Suitability

Form k involves simpler calculations and is therefore much easier to use than Form p^* for a single quality characteristic and a single specification limit. However, the main advantage of using Form p^* under these circumstances is that an estimate of the process fraction nonconforming is obtained. This can be plotted on a control chart to provide an early warning of any deterioration in quality.

5.3.2.2 Acceptance criterion under the "s" method

The quality statistic Q is calculated in accordance with Expression (4) or Expression (6), as appropriate. The estimate of the process fraction nonconforming is then given by:

$$\hat{p} = B_{(n-2)/2}\left(\frac{1}{2} - \frac{1}{2}Q\frac{\sqrt{n}}{n-1}\right) \tag{13}$$

where:

n is the sample size;

$B_b(.)$ represents the symmetric beta distribution function with both parameters equal to b.

NOTE An approximation to this function that only requires tables of the normal distribution is provided in ISO 3951-2 and ISO 3951-3, in case the user does not have access to suitable software or beta function tables.

The acceptance criterion is obtained by substituting the expression for $\hat{p}$ from Expression (13) into Expression (12).

5.3.2.3 Acceptance criterion under the "σ" method

The quality statistic Q is calculated in accordance with Expression (5) or Expression (7), as appropriate. The estimate of the process fraction nonconforming is then given by:

$$\hat{p} = \Phi\left(-Q\sqrt{\frac{n}{n-1}}\right) \tag{14}$$

where $\Phi(.)$ denotes the standard normal distribution function.

The acceptance criterion is obtained by substituting the expression for $\hat{p}$ from Expression (14) into Expression (12).

5.3.3 Form p^* for separate control of double specification limits (single quality characteristic)

5.3.3.1 Suitability

Form k is also much easier to use than Form p^* for a single quality characteristic with separate control of double specification limits. Again, the advantage of using Form p^* under these circumstances is that an estimate of the process fraction nonconforming is obtained, which can be plotted on a control chart to provide an early warning of any deterioration in quality.

For separate double specification limits, each of the limits belongs to a different class to which a different AQL, and therefore a different Form p^* acceptability constant, applies. These acceptability constants are denoted by p_U^* at the upper limit and by p_L^* at the lower limit. The values of these constants are unaffected by whether the "s" method or the "σ" method is used, although the sample size is affected.

5.3.3.2 Acceptance criterion under the "s" method

Denote the quality statistic for the upper specification limit by Q_U, calculated in accordance with Expression (4). Similarly, denote the quality statistic for the lower specification limit by Q_L, calculated in accordance with Expression (6). Substitute Q_U in Expression (13) to obtain $\hat{p}_U$ and Q_L in Expression (13) to obtain $\hat{p}_L$ The lot is judged acceptable if, and only if, $\hat{p}_U \leqslant p_U^*$ and $\hat{p}_L \leqslant p_L^*$.

5.3.3.3 Acceptance criterion under the "σ" method

For separate control under the "σ" method, the applicable $\sigma_{\max}$ factor f for separate control with the given AQL requirements should be found, and the value of $\sigma_{\max}$ determined as $(U-L)f$. If the presumed value of σ exceeds $\sigma_{\max}$, then lots may be judged non-acceptable, without the need to draw any samples, until such time as the value of σ can be demonstrated to be less than or equal to σ.

If $\sigma \leqslant \sigma_{\max}$, sampling is potentially worthwhile. The quality statistic for the upper specification limit, Q_U, is calculated in accordance with Expression (5). Similarly, the quality statistic for the lower specification limit, Q_L, is calculated in accordance with Expression (7). The fractions nonconforming beyond the upper and lower specification limits are found by substituting Q_U in Expression (14) to obtain $\hat{p}_U$ and substituting Q_L in Expression (14) to obtain $\hat{p}_L$. The lot is judged acceptable if, and only if, $\hat{p}_U \leqslant p_U^*$ and $\hat{p}_L \leqslant p_L^*$.

5.3.4 Form p^* for combined control of double specification limits (single quality characteristic)

5.3.4.1 Acceptance criterion under the "s" method

The quality statistic Q_U for the upper specification limit is calculated in accordance with Expression (4). Similarly, the quality statistic Q_L for the lower specification limit is calculated in accordance with Expression (6). Q_U is substituted in Expression (13) to obtain $\hat{p}_U$ and Q_L in Expression (13) to obtain $\hat{p}_L$. The lot is judged acceptable if, and only if, the sum of these estimates is sufficiently small, i.e. if $\hat{p}_U + \hat{p}_L \leqslant p^*$.

Alternatively, an acceptance diagram can be used. Figure 10 shows the typical shape of a standardized "s" method acceptance diagram for the combined control of double specification limits. The main disadvantage of this approach is that no estimate of the process fraction nonconforming is produced. There is also, as with all diagrammatic methods, a small chance of sample points lying too close to the acceptance curve to determine whether they are inside or outside the acceptance zone.

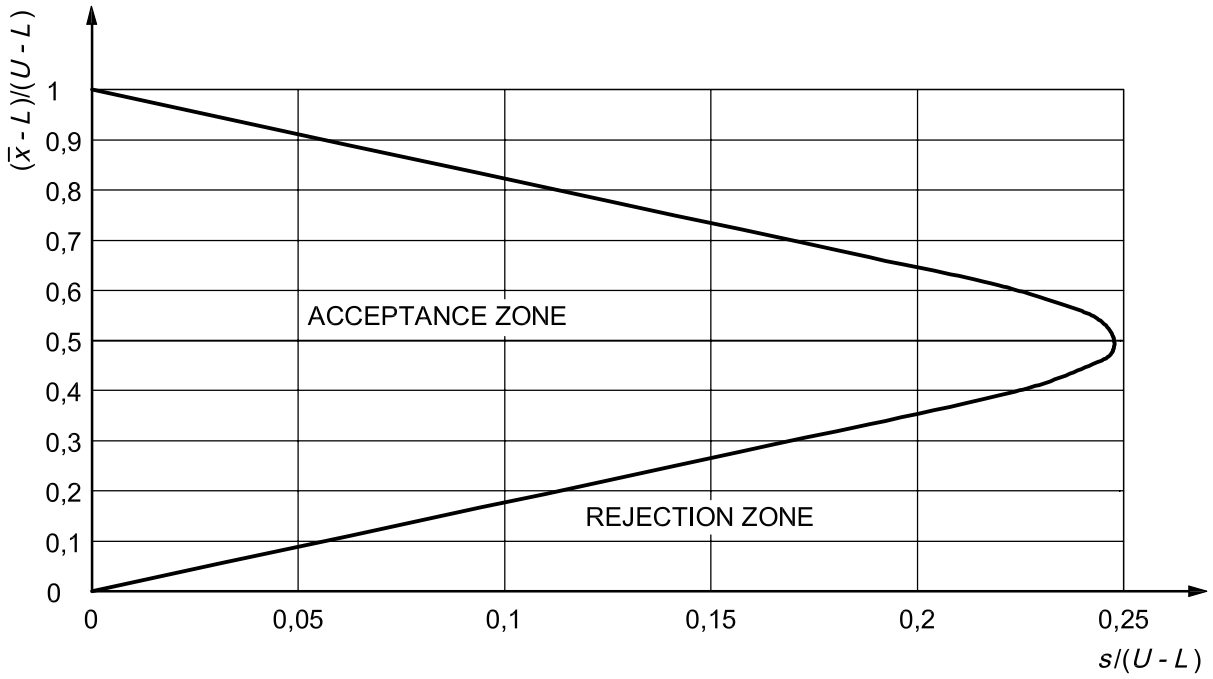


Figure 10 — Standardized acceptance diagram for combined control of double specification limits: "s" **method, sample size code letter G,** AQL = 1 %: n = 18, p^* = 0,0332 3

5.3.4.2 Acceptance criterion under the "σ" method

For combined control under the "σ" method, the first step is to look up the applicable $\sigma_{\max}$ factor f for combined control with the given AQL requirements, and to determine the value of $\sigma_{\max}$ as $(U - L)f$. If the presumed value of σ exceeds $\sigma_{\max}$, then lots may be judged non-acceptable without the need to draw any samples, until such time as the value of σ can be demonstrated to be less than or equal to $\sigma_{\max}$.

If $\sigma \leqslant \sigma_{\max}$, it is possible, though not certain, that the process is operating at a satisfactory level, so sampling is potentially worthwhile. The quality statistic Q_U for the upper specification limit is calculated in accordance with Expression (5). Similarly, the quality statistic Q_L for the lower specification limit is calculated in accordance with Expression (7). Q_U is substituted in Expression (14) to obtain $\hat{p}_U$ and Q_L in Expression (14) to obtain $\hat{p}_L$. The lot is judged acceptable only if $\hat{p}_U + \hat{p}_L \leqslant p^*$.

A standardized acceptance diagram is shown in Figure 11. Note that the upper and lower bounds to the acceptance zone are approximately straight lines. In fact, for all practical purposes, the upper and lower bounds can be drawn as the straight lines:

$$(\bar{x} - L)/(U - L) = k\sigma \text{ and } (\bar{x} - L)/(U - L) = 1 - k\sigma,$$

where k is the Form k acceptance constant corresponding to a single specification limit and the combined AQL.

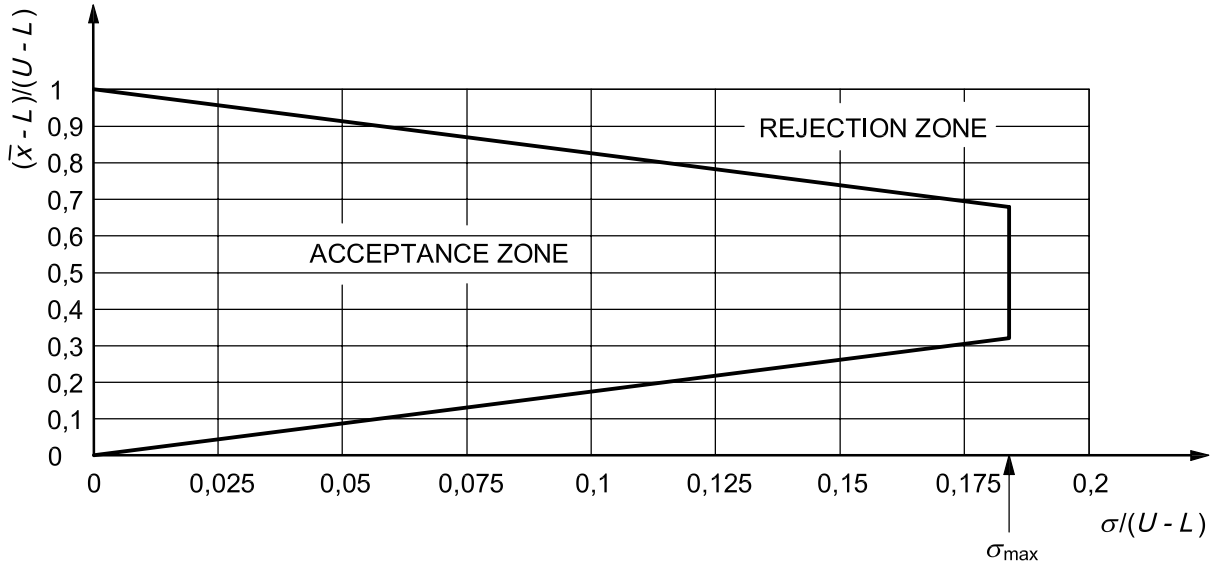


Figure 11 — Standardized acceptance diagram for combined control of double specification limits: sigma method, sample size code letter G, AQL = 1 %: n = 10, p^* = 0,0332 3

5.3.5 Form p^* for complex control of double specification limits (single quality characteristic)

5.3.5.1 General

The acceptability constant for both limits combined is denoted by p^* and the smaller acceptability constant for the limit of greater seriousness by p_1^*.

5.3.5.2 Acceptance criteria under the "s" method

The quality statistic Q_U for the upper specification limit is calculated in accordance with Expression (4). Similarly, the quality statistic Q_L for the lower specification limit is calculated in accordance with Expression (6). Q_U is substituted in Expression (13) to obtain $\hat{p}_U$ and Q_L in Expression (13) to obtain $\hat{p}_L$. $\hat{p}_1$ is set equal to $\hat{p}_U$ if the upper limit is the limit of greater seriousness; otherwise $\hat{p}_1$ is set equal to $\hat{p}_L$. The lot is judged acceptable only if $\hat{p}_U + \hat{p}_L \leqslant p^*$ and $\hat{p}_1 \leqslant p_1^*$.

A typical shape of the acceptance region for complex control when the process standard deviation is unknown is shown in Figure 12. This is for the case of sample size code letter G where there is an overall AQL of 1,5 % together with an AQL on the upper specification limit of 0,4 %. Note that it is the acceptance region for combined control minus the section above the line corresponding to control of the upper specification limit.

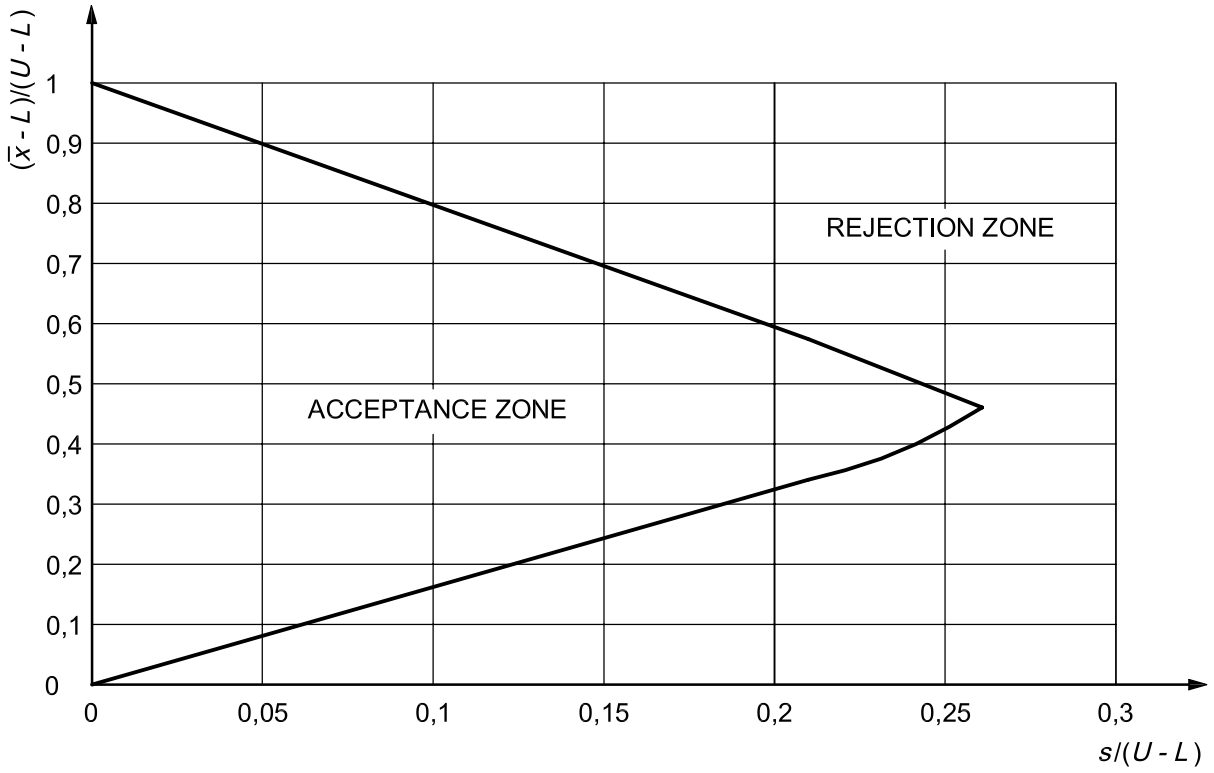


Figure 12 — Standardized acceptance diagram for combined control of double specification limits: sigma method, sample size code letter G, AQL = 1 %: n = 10, p^* = 0,0332 3

5.3.5.3 Acceptance criteria under the "σ" method

The first step is to look up the applicable $\sigma_{\max}$ factor f for complex control with the given AQL requirements, and determine the value of $\sigma_{\max}$ as $(U - L)f$. If the presumed value of σ exceeds $\sigma_{\max}$ then lots may be judged non-acceptable without the need to draw any samples, until such time as the value of σ can be demonstrated to be less than or equal to $\sigma_{\max}$

If $\sigma \leqslant \sigma_{\max}$, it is possible for lots to satisfy the acceptance criteria. A random sample is drawn from the lot. The quality statistic Q_U for the upper specification limit is calculated in accordance with Expression (5). Similarly, the quality statistic Q_L for the lower specification limit is calculated in accordance with Expression (7). Q_U is substituted in Expression (14) to obtain $\hat{p}_U$ and Q_L in Expression (14) to obtain $\hat{p}_L$. $\hat{p}_1$ is set equal to $\hat{p}_U$ if the upper limit is the limit of greater seriousness, otherwise it is set equal to $\hat{p}_L$. The lot is judged acceptable only if $\hat{p}_U + \hat{p}_L \leqslant p^*$ and $\hat{p}_1 \leqslant p_1^*$.

Characteristic shapes of the acceptance region for complex control when the process standard deviation is known are shown in Figure 13.

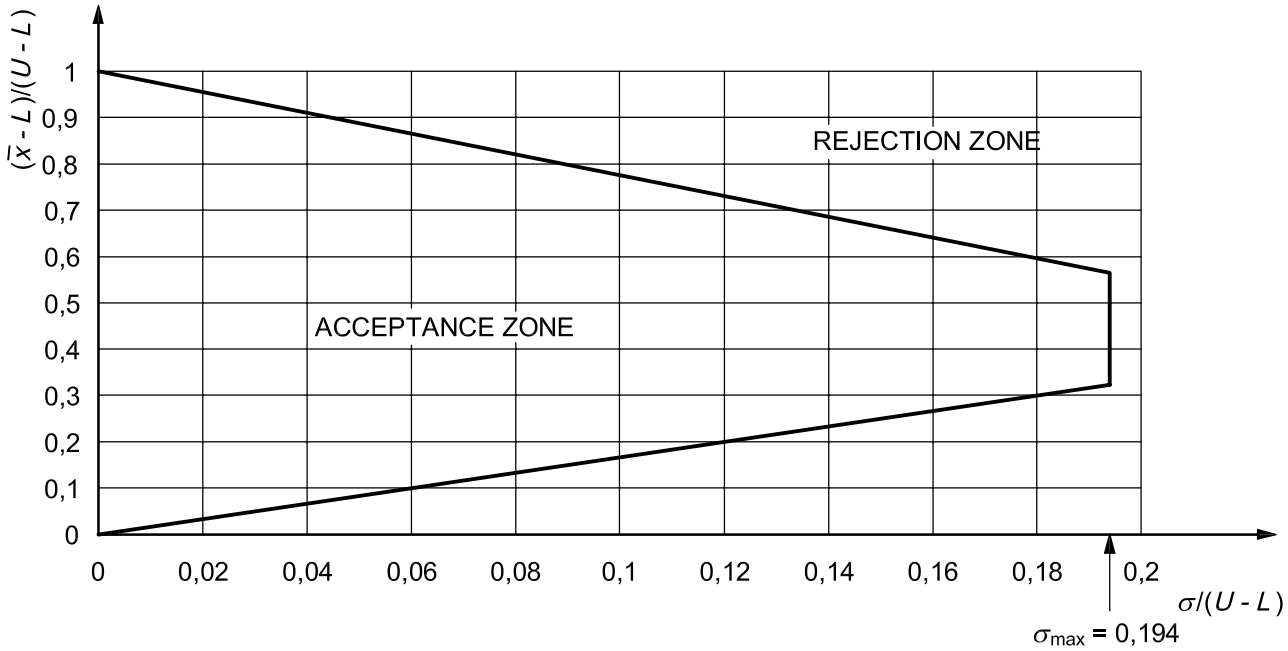


Figure 13 — Standardized acceptance diagrams for complex control of double specification limits: sigma method, sample size code letter M, combined AQL = 1,5 %, AQL 0,4 % **for upper limit**

5.3.6 Form *p** for two independent quality characteristics

5.3.6.1 Notation

If not obvious, the statistics and parameters for the two quality characteristics are distinguished in the following sub-clauses by attaching x or y in parentheses. Thus, the estimate of the process fraction nonconforming above the upper limit $U(x)$ for x is denoted by $\hat{p}_U(x)$ and the estimate of the process fraction nonconforming below the lower limit $L(x)$ for x is denoted by $\hat{p}_L(x)$. The estimate of the total fraction nonconforming in the case where x has double specification limits is denoted by $\hat{p}(x) = \hat{p}_L(x) + \hat{p}_U(x)$, otherwise $\hat{p}(x) = \hat{p}_L(x)$ in the case of a single, lower specification limit and $\hat{p}(x) = \hat{p}_U(x)$ in the case of a single, upper specification limit. The same applies for $U(y)$ and $L(y)$.

Where there is more than one class of nonconformity, the classes are indicated by the suffix A for the class of greatest seriousness, B for the next most serious class, etc. Thus, the respective estimated process fractions nonconforming are denoted by $\hat{p}_A$, $\hat{p}_B$, . . ., whereas the acceptability constants are denoted by p^*_A, p^*_B, … .

5.3.6.2 Acceptance criteria under the "*s*" method

5.3.6.2.1 Single class of nonconformity

Calculate $\hat{p}(x)$ and $\hat{p}(y)$, then $\hat{p} = 1 - \left[1 - \hat{p}(x)\right]\left[1 - \hat{p}(y)\right]$. Accept the lot only if $\hat{p} \leqslant p^*$.

5.3.6.2.2 Two classes of nonconformity

For two classes of nonconformity A and B, the quantities $\hat{p}_A$ and $\hat{p}_B$ are calculated, using in each case the formula:

$$\hat{p} = 1 - \left[1 - \hat{p}_L(x) - \hat{p}_U(x)\right]\left[1 - \hat{p}_L(y) - \hat{p}_U(y)\right],$$

except that only those elements that belong to each respective class are included. For example, if class A comprises the lower limit on x and class B comprises both limits on x and the lower limit on y, then:

$$\hat{p}_{\mathrm{A}} = 1 - \left[1 - \hat{p}_L(x)\right] = \hat{p}_L(x),$$

while

$$\hat{p}_{\mathrm{B}} = 1 - \left[1 - \hat{p}_L(x) - \hat{p}_U(x)\right]\left[1 - \hat{p}_L(y)\right].$$

The lot is only accepted if both $\hat{p}_{\mathrm{A}} \leqslant \hat{p}_{\mathrm{A}}^{*}$ and $\hat{p}_{\mathrm{B}} \leqslant \hat{p}_{\mathrm{B}}^{*}$.

5.3.6.2.3 Three or more classes of nonconformity

The generalization to three or more classes of nonconformity is now evident. The quantities $\hat{p}_{\mathrm{A}}$, $\hat{p}_{\mathrm{B}}$, $\hat{p}_{\mathrm{C}}, \ldots$ are calculated and the lot is accepted only if $\hat{p}_{\mathrm{A}} \leqslant \hat{p}_{\mathrm{A}}^{*}$ and $\hat{p}_{\mathrm{B}} \leqslant \hat{p}_{\mathrm{B}}^{*}$ and $\hat{p}_{\mathrm{C}} \leqslant \hat{p}_{\mathrm{C}}^{*} \ldots$

5.3.6.3 Acceptance criteria under the "σ" method

The procedure for the "σ" method are similar to those for the "s" method, except that:

— the formulae for $\hat{p}_L(x)$ and $\hat{p}_U(x)$ are tail areas under a normal distribution instead of under a beta distribution, and are therefore simpler to determine;

— wherever both limits of a quality characteristic are included in the same class, the value of σ for that characteristic has to be checked to see that it does not exceed the value of $\sigma_{\max}$ for combined control for that class;

— wherever one limit of a quality characteristic is included in one class and the other limit is included in another class, the value of σ for that characteristic has to be checked to see that it does not exceed the value of $\sigma_{\max}$ for separate control for that pair of classes;

— wherever both limits of a quality characteristic are included in one class and one of the limits is included in another class, the value of σ for that characteristic has to be checked to see that it does not exceed the value of $\sigma_{\max}$ for complex control for that pair of classes.

5.4 Double sampling plans

5.4.1 General

Double sampling plans achieve a reduction in the average amount of sampling and inspection by means of a two-stage sampling process, with both sample sizes considerably smaller than the corresponding single sample size. When quality is particularly good or particularly bad, the first sample generally furnishes results that are sufficiently unequivocal for a decision to be made without recourse to a second sample. For intermediate quality, a second sample is sometimes necessary to resolve doubt.

Much of the development work on double sampling plans assumes that both sample sizes are equal, to avoid unnecessary complexity. This is assumed to be the case in the remainder of this sub-clause. When both sample sizes are equal, they are typically about 60 % of the corresponding single sample size, so that average savings in sampling and inspection of up to about 40 % are possible.

Double sampling plans can have a number of disadvantages. When items take a long time to test, but can be inspected or tested simultaneously, replacing a single sampling plan by a double sampling plan can double the time needed to produce an accept or non-accept decision. This problem is made worse if time has to be booked in advance at the inspection facility.

Even worse is the case where items need to be transported a considerable distance to be tested. This raises a number of questions. Should both samples be transported to the inspection facility at the same time? Should

time for one or for both samples to be inspected be booked in advance, i.e. what are the costs of booking time that is subsequently not used? If the second sample is transported but not required, can it be transported back again and returned to the lot from which it was drawn, i.e. can it be assumed that the item is not adversely affected by its long journey? Does any delay caused by the use of double sampling cause a storage problem for the lots that are awaiting a disposition decision? Are the savings from the use of double sampling more than cancelled out by extra administrative and logistical costs?

The decision as to whether or not to replace single sampling plans by double sampling plans therefore depends on whether the potential savings from the reduction in the average amount of sampling and inspection outweighs the negative aspects of double sampling.

5.4.2 Form k double sampling plans

A Form k double sampling plan with equal first and second sample sizes has four parameters: n, k_a, k_r and k_c, the use of which, when the process standard deviation is unknown, is as follows. Suppose that there is a single, upper specification limit, U. A random sample of size n is drawn from the lot, and the value of the quality characteristic x is measured on each sampled item. The sample mean $\bar{x}_1$, the sample standard deviation s_1 and the quality statistic $Q_1 = (U - \bar{x}_1)/s_1$ are calculated. If $Q_1 \geqslant k_a$, the lot is immediately accepted. If $Q_1 \leqslant k_r$, the lot is immediately non-accepted.

If $k_r < Q_1 < k_a$, a second random sample of size n is drawn from the lot, and its mean $\bar{x}_2$ and standard deviation s_2 are calculated. The combined sample mean $\bar{x}_c = (\bar{x}_1 + \bar{x}_2)/2$ and the combined standard deviation $s_c = \sqrt{(s_1^2 + s_2^2)/2}$ are calculated, together with the combined quality statistic $Q_c = (U - \bar{x}_c)/s_c$. If $Q_c \geqslant k_c$, the lot is accepted; otherwise, the lot is non-accepted.

If the process standard deviation σ is presumed to be known, s_1 and s_c in the above expressions for Q_1 and Q_c are replaced by σ. The first sample standard deviation s_1 should still be calculated to verify that the value of σ has not changed.

If inspection is against a lower specification limit L, instead of against an upper specification limit, expressions of the form $U - \bar{x}$ are replaced by corresponding expressions of the form $\bar{x} - L$.

5.4.3 Form p^* double sampling plans

A Form p^* double sampling plan has four parameters: n, p_a^*, p_r^* and p_c^*, the meaning of which is as follows when the process standard deviation is unknown. A random sample of size n is drawn from the lot, and the value of the quality characteristic x is measured on each sampled item. The sample mean $\bar{x}_1$ and the sample standard deviation s_1 are calculated, together with an estimate $\hat{p}_1$ of the process fraction nonconforming. If $\hat{p}_1 \leqslant p_a^*$, the lot is immediately accepted. If $\hat{p}_1 \geqslant p_r^*$, the lot is immediately non-accepted.

If $p_a^* \leqslant \hat{p}_1 \leqslant p_r^*$, a second random sample of size n is drawn from the lot, and its mean $\bar{x}_2$ and standard deviation s_2 are calculated. The combined sample mean $\bar{x}_c = (\bar{x}_1 + \bar{x}_2)/2$ and the combined standard deviation $s_c = \sqrt{(s_1^2 + s_2^2)/2}$ are calculated, together with the combined estimate $\hat{p}_c$ of the process fraction nonconforming. If $\hat{p}_c \leqslant p_c^*$, the lot is accepted; otherwise, the lot is non-accepted.

When the process standard deviation σ is unknown, the estimates $\hat{p}_1$ and $\hat{p}_c$ are tail areas of a symmetrical beta distribution (see 5.3.2.2). For known σ, the estimates $\hat{p}_1$ and $\hat{p}_c$ are tail areas of a normal distribution (see 5.3.2.3). When σ is known, s_1 and s_c are not required for the purposes of calculating $\hat{p}_1$ and $\hat{p}_c$ However, as is the case with Form k, it is still advisable to calculate and use the first sample standard deviation, s_1, for monitoring purposes.

If inspection is against a lower specification limit L, instead of against an upper specification limit, expressions of the form $U - \bar{x}$ in $\hat{p}_1$ and $\hat{p}_c$ are replaced by corresponding expressions of the form $\bar{x} - L$.

Form p^* double sampling plans can be applied to multivariate situations in much the same way as Form p^* single sampling plans.

5.5 Multiple sampling plans

Multiple sampling plans for inspection by variables are conceptually simply an extension from two-stage plans to multi-stage plans. However, no such standardized plans exist or are currently in development.

5.6 Sequential sampling plans

5.6.1 General

Sequential sampling plans are the ultimate extension to multi-stage plans where a decision to accept, to non-accept or to continue sampling is made after each item in the sample has been selected and measured. Clearly, sequential sampling plans are inappropriate when a lot of time is required to test each item for conformity and when it is possible to test all the sampled items simultaneously. When sequential sampling plans are suitable, they provide, on average, the greatest economy of sampling and inspection effort.

At present, there are no International Standards on sequential sampling plans by variables for the case of unknown process standard deviation.

5.6.2 Curtailment

The disadvantage of sequential sampling plans is that sampling could go on almost indefinitely. (The chance of exceeding the single sample size is, in fact, much smaller for sequential sampling plans than for the corresponding double sampling plans.) However, to overcome this perceived drawback to sequential sampling plans, standardized sequential sampling plans are generally curtailed at a sample size n_t that is about 1,5 times the corresponding single sample size. Thus, an accept or non-accept decision is forced at sample size n_t if such a decision has not been reached earlier.

5.6.3 Form k sequential sampling plans

A Form k sequential sampling plan for inspection by variables has four parameters, namely: h_A, h_R, g and n_t. Items are selected one by one at random from the lot. The sample size after each item has been selected is denoted by n_{cum}, where $n_{cum} = 1, 2, \ldots, n_t$.

Consider first the case of a single specification limit on a single quality characteristic x, when the process standard deviation of the quality characteristic is σ. The "leeway" y is defined by $y = U - x$ in the case of an upper specification limit U, or as $y = x - L$ in the case of a lower specification limit L. The "cumulative leeway" is defined as:

$$Y_{n_{cum}} = \sum_{i=1}^{n_{cum}} y_i .$$

For $n_{cum} = 1, 2, \ldots, n_t - 1$, the lot is accepted without further sampling if:

$$Y_{n_{cum}} \geqslant A_{n_{cum}} = \left(g n_{cum} + h_A \right) \sigma ,$$

or non-accepted without further sampling if:

$$Y_{n_{cum}} \leqslant R_{n_{cum}} = \left(g n_{cum} + h_R \right) \sigma .$$

However, if:

$$R_{n_{cum}} < Y_{n_{cum}} < A_{n_{cum}} ,$$

another item is selected at random from the lot and measured. If the curtailment sample size n_t is reached, the lot is accepted if:

$$Y_{n_t} \geqslant A_{n_t} = \sigma g n_t$$

and non-accepted otherwise.

For simplicity A_{n_t} is sometimes abbreviated as A_t.

A typical acceptance chart for a single specification limit is shown in Figure 14.

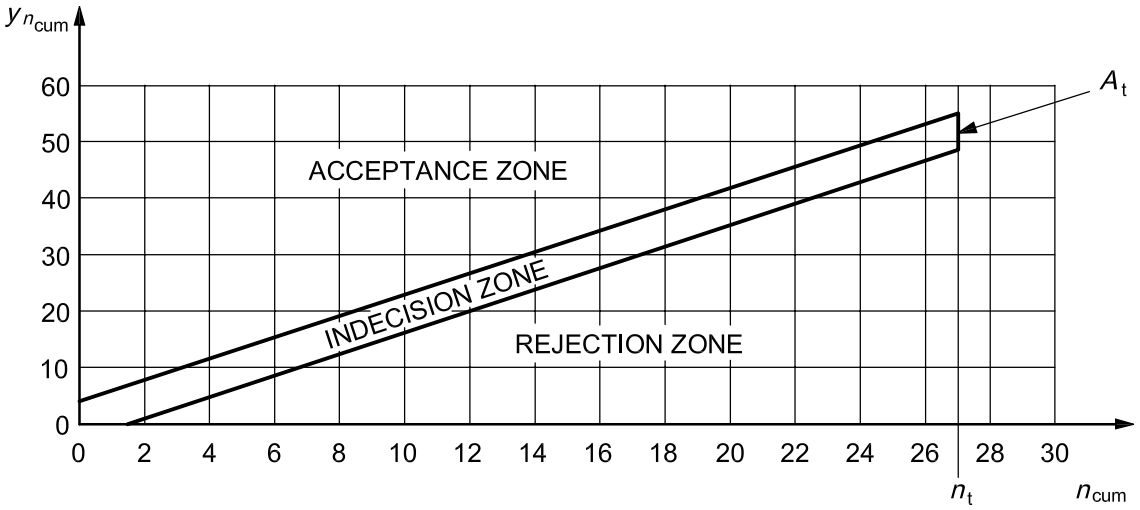


Figure 14 — Acceptance diagram for sequential sampling by variables for a single specification limit: sigma method, sample size code letter K, σ = 1,0, AQL = 1,0 %, h_A = 2,764, h_R = 3,895, g = 1,900, n_t = 27

5.6.4 Form p^* sequential sampling plans

Form p^* sequential sampling plans, even for a known process standard deviation, cannot be expressed as concisely as Form k plans. As a result, they have not been standardized. However, as with single sampling plans, a Form p^* plan can be derived from the corresponding Form k plan. The appropriate formulae for a known process standard deviation are:

$$p^*_{A,n_{cum}} = \Phi\left\{c\left(g + h_A / n_{cum}\right)\right\} \text{ for } n_{cum} = 1, 2, \ldots, n_t - 1;$$

$$p^*_{R,n_{cum}} = \Phi\left\{c\left(g - h_R / n_{cum}\right)\right\} \text{ for } n_{cum} = 1, 2, \ldots, n_t - 1;$$

$$p^*_{A,n_t} = \Phi(cg);$$

$$\hat{p}_{n_{cum}} = \Phi\left(cY_{n_{cum}} / \sigma_{n_{cum}}\right) \text{ for } n_{cum} = 1, 2, \ldots, n_t;$$

where $c = -\sqrt{\dfrac{n_{cum}}{n_{cum} - 1}}$ and $\Phi(.)$ represents the standard normal distribution function.

At sample size n_{cum}, where $n_{cum} < n_t$ the lot is accepted if $\hat{p}_{n_{cum}} \leqslant p^*_{A,n_{cum}}$ and non-accepted if $\hat{p}_{n_{cum}} \geqslant p^*_{R,n_{cum}}$; if $p^*_{A,n_{cum}} < \hat{p}_{n_{cum}} < p^*_{R,n_{cum}}$, another sample item is drawn. If sample size n_t is reached without an accept or a non-accept decision having been made, the lot is accepted if $\hat{p}_{n_t} \leqslant p^*_{A,n_t}$ and non-accepted otherwise.

6 International Standards for acceptance sampling of lots by variables

6.1 General

This clause describes the salient features of each of the International Standards on acceptance sampling by variables that are currently available or in development. These descriptions, together with the preceding information, should usually enable a user to select the International Standard on sampling by variables that suits a given purpose.

Should the comparisons between the various International Standard acceptance sampling systems by variables given here not be enough to allow a final selection of a sampling system, scheme or plan to be made in a particular situation, the user is advised to review the factors considered in ISO/TR 8550-1.

International Standards in the ISO 3951 series on acceptance sampling by variables are intended primarily for the inspection of a continuing series of lots from one source of sufficient duration to allow the switching rules to operate. Inspection carried out on an isolated lot using ISO 3951 standards will provide little evidence about the normality of the distribution(s) of the product characteristic(s) or about the standard deviation(s) of the process. ISO 3951 standards should therefore not be applied to the inspection of isolated lots.

6.2 ISO 3951-1: *Sampling procedures for inspection by variables — Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

ISO 3951-1 presents a sampling system indexed by lot size ranges, inspection levels and AQLs, and is complementary to ISO 2859-1. The two International Standards share a common philosophy and purpose. ISO 3951-1 is intended primarily for the inspection of a continuing series of lots from one source of sufficient duration to allow the switching rules to operate.

ISO 3951-1 provides single sampling plans, i.e. plans for which a decision on lot acceptability is based on a single sample. Unlike ISO 2859-1, it does not provide double or multiple sampling plans. Double sampling plans by variables are provided in ISO 3951-3.

ISO 3951-1 is only applicable where a single product characteristic, measurable on a continuous scale, is considered together with a single class of nonconformity. The product characteristic should be distributed in accordance with a normal distribution, a distribution closely approximating normality, or a distribution that can be transformed so that it closely approximates normality (see Clause 3).

A lot is judged unacceptable when the distribution of the product characteristic fails to indicate an average and variability that conforms to the prescribed sampling criteria for the given single or double specification limits. A choice is available between numerical and graphical acceptance criteria. Procedures are given both for the case where the process standard deviation is unknown and for the case where it is known.

Form k procedures are used throughout the standard, except in the case of sample sizes three and four, for which Form p^* procedures are provided.

6.3 ISO 3951-2: *Sampling procedures for inspection by variables — Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

ISO 3951-2 is a generalized version of ISO 3951-1. Like ISO 3951-1, ISO 3951-2 only provides single sampling plans, and is intended for the inspection of a continuing series of lots from one source of sufficient duration to allow the switching rules to operate.

Whereas ISO 3951-1 is applicable to a single quality characteristic measurable on a continuous scale, ISO 3951-2 is applicable to any number of such quality characteristics if they are independent, or at least nearly so. Each characteristic has to be distributed in accordance with a normal distribution, a distribution closely approximating normality, or a distribution that can be transformed so that it closely approximates normality (see Clause 3).

A lot is judged to be non-acceptable when the distributions of the product characteristics fail to indicate a process fraction nonconforming that meets the sampling criteria for the prescribed specification limits. All acceptance criteria are numerical. Procedures are given both for the case where the process standard deviation is unknown and also for the case where it is presumed to be known.

Both Form k and Form p* procedures are presented in the standard.

If there is more than one class of nonconformity, acceptance procedures have to be applied to each class separately. A lot is judged acceptable only if it conforms with the acceptance criteria for all classes.

6.4 ISO 3951-3: *Sampling procedures for inspection by variables — Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

ISO 3951-3 provides double sampling plans by variables (see 5.4). Because the procedures for double sampling plans are relatively complicated, the main text is confined to the case of a single quality characteristic. Procedures for two or more quality characteristics are provided in the annexes. Again, each characteristic should be measurable on a continuous scale and be distributed according to a normal distribution, a distribution closely approximating normality or a distribution that can be transformed so that it closely approximates normality (see Clause 3).

The purpose of these sampling plans is to reduce the average amount of sampling and inspection for a series of lots.

A lot is judged non-acceptable when the distributions of the product characteristics fail to indicate a process fraction nonconforming that meets the sampling criteria for the prescribed specification limits. All acceptance criteria are numerical. Procedures are given both for the case where the process standard deviation is unknown and for the case where it is known.

Both Form k and Form p* procedures are presented in the standard.

To keep the sampling plans as simple as possible, the first and second sample sizes of each plan have been kept equal. However, to provide a good match between the OC curves of these plans and those of the corresponding single sampling plans of ISO 2859-1, the sample sizes along rows of the master tables have been allowed to differ. Indeed, there is a marked tendency for sample sizes to be smaller for smaller AQLs, providing an additional incentive, if one were needed, for producers to improve their quality levels. This tendency is interrupted down the second and third (top right to bottom left) diagonals of the master tables for normal and tightened inspection, and down the second, third and fourth diagonals for reduced inspection. This is because the plans along these diagonals are matched to the optional plans of ISO 2859-1 that have fractional acceptance numbers, resulting in a subtly different shape of operating characteristic (OC) curve from the plans with integer acceptance numbers.

6.5 ISO 3951-4: *Sampling procedures for inspection by variables — Part 4: Procedures for assessment of declared quality levels* [1)]

This provides sampling plans and procedures by variables for assessing whether the quality level of a lot or process conforms to a declared value. Procedures for any number of quality characteristics are provided, with each characteristic being measurable on a continuous scale and distributed according to a normal distribution, a distribution closely approximating normality or a distribution that can be transformed so that it closely approximates normality (see Clause 3). The sampling plans have been devised to have a risk of less than 5 % of contradicting a correct declared quality level (DQL). Conversely, the risk is 10 % of failing to contradict an incorrect DQL, which is related to the limiting quality ratio. ISO 3951-4 provides sampling plans corresponding to three levels of discriminatory ability. Procedures are given both for the case where the process standard deviation is unknown and for the case where it is known.

1) Under development.

In contrast to other parts of ISO 3951, the procedures in ISO 3951-4 are not intended for the acceptance assessment of lots. In general, the balancing of the risks of reaching incorrect conclusions for assessment procedures differs from the balancing in the procedures for acceptance sampling.

ISO 3951-4 may be used for various forms of quality inspection in situations where objective evidence of conformity to some DQL is to be provided by means of inspection of a sample. The procedures are applicable to entities such as lots, process output, etc. that allow random samples of individual items to be selected.

ISO 3951-4 is intended for use when the quantity of interest is the fraction of nonconforming items in the process.

6.6 ISO 3951-5: *Sampling procedures for inspection by variables — Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)*

ISO 3951-5 presents a sampling system providing a wide range of sequential sampling plans for a series of lots, with plans indexed by lot size ranges, inspection levels and AQLs to supplement the systems in ISO 3951-1, ISO 3951-2 and ISO 3951-3, including switching rules. As with the plans provided in these other parts of ISO 3951, each product characteristic should be measurable on a continuous scale and be distributed according to a normal distribution, a distribution closely approximating normality or a distribution that can be transformed so that it closely approximates normality (see Clause 3).

The sampling procedures in ISO 3951-5 are based on a sequential assessment of inspection results and may be used to induce a supplier to supply lots of a quality with a high probability of acceptance while maintaining an upper limit for the risk to a consumer of accepting lots of poor quality.

The sampling plans are intended primarily for use in the inspection of a single quality characteristic in a continuing series of lots from the same production run.

6.7 ISO 8423: *Sequential sampling plans for inspection by variables for percent nonconforming (known standard deviation)*

ISO 8423 presents a sampling system providing a wide range of sequential sampling plans indexed in terms of the consumer's risk point (CRP) and the producer's risk point (PRP). Any product characteristic to which this standard is applied should be measurable on a continuous scale and be distributed according to a normal distribution, a distribution closely approximating normality or a distribution that can be transformed so that it closely approximates normality (see Clause 3). Because the plans are designed to test one quality level against another, the plans are suitable not only for acceptance sampling but also for hypothesis testing.

For the case of double specification limits, ISO 8423 provides procedures for combined control and separate control (see 4.1.3.2 and 4.1.3.3).

The sampling procedures in ISO 8423 are based on a sequential assessment of inspection results. When applied to a continuing series of lots, they may be used to induce a supplier - through the economic and psychological pressure of non-acceptance of lots of inferior quality - to supply lots of a quality with a high probability of acceptance while maintaining an upper limit for the risk to a consumer of accepting lots of poor quality.

The sampling plans are intended primarily for use in the inspection of a single quality characteristic.

7 Effect on the selection process of market and production conditions

Some of the ways in which the market and production conditions identified in ISO/TR 8550-1:2007, Clause **11** [2)], affect the choice of sampling systems, schemes or plans by attributes in differing inspection

2) In this clause and in the following tables, numbers in bold refer to subclause numbers in ISO/TR 8550-1:2007.

situations are summarized in Tables 1, 2 and 3. Tables 2 and 3 contain guidance notes, which are indexed by, and refer to, the market conditions (ISO/TR 8550-1:2007, **11.2**) and production conditions (ISO/TR 8550-1:2007, **11.3**). It is to be noted that any coexistence of various conditions can affect the selection. The inspection situation also has to be considered (see Table 1).

Figures 15 and 16 illustrate the selection procedure for sampling by variables respectively for the case of continuous production with a run length in excess of 10 lots and for the case where production is not continuous or the run length is 10 or fewer lots.

Table 1 — Guidance for selection of a candidate acceptance sampling system, scheme or plan for inspection by variables, based on the inspection situation

Example of inspection situations	Conditions that affect the choice of a sampling plan (see ISO/TR 8550-1:2007)	Applicable type of sampling plans	Specific acceptance sampling plans
Quality characteristic(s) not normally distributed	**11.3 p), 11.3 q), 11.3 g), 11.3 h)**		ISO 3951 schemes inappropriate, unless the characteristic(s) can be transformed to normality.
More than one quality characteristic		AQL	ISO 3951-2 or ISO 3951-3.
Expensive or critical items	**11.2 c), 11.2 d)**	100 % inspection AQL Sequential	No standard. ISO 3951-3. ISO 8423.
Final inspection	**11.2 a), 11.2 f), 11.3 a), 11.3 b), 11.3 j)**	AQL	ISO 3951-1 or ISO 3951-5 for a single quality characteristic and a single AQL. ISO 3951-2 otherwise.
History of received quality unknown	**11.2 c), 11.2 f), 11.2 h), 11.3 g), 11.3 m)**	LQ	ISO 2859-2.
Limiting average outgoing quality	**11.2 f), 11.3 n), 11.3 o)**	AOQL	Any standard in the series, but determine corresponding AQL from the OC curves and consider starting on tightened inspection.
Small lots with good quality needed	**11.2 c), 11.3 a), 11.3 j), 11.3 o)**	AQL	Preferable to use attributes (see ISO 2859-2).
History of received quality has been good	**11.2 g), 11.2 h)**	AQL	Any standard in the series, starting under normal inspection.
One-of-a-kind lots	**11.2 c), 11.2 d), 11.2 f), 11.3 i)**	One-off plans	Sampling by variables unsuitable.
NOTE See also Tables 2 and 3 and Figures 15 and 16 illustrating the selection process.			

Table 2 — Guidance for the selection of an acceptance sampling system, scheme or plan for sampling by variables, using existing market conditions

Market conditions (ISO/TR 8550-1:2007)	Indicated practice [a]	Guidance notes with ISO/TR 8550-1:2007 references
Production is continuous and feedback from the recipient can influence the supplier's quality or the warranty survives acceptance – **11.2 a)**, **11.2 b)**	ISO 3951-1, ISO 3951-2, ISO 3951-3, ISO 3951-5	Virtually any variables standards are recommended. The choice will depend on other factors. Long runs are implied (**8.1**).
The lot is isolated or it is one of a short sequence or the recipient cannot influence the supplier's quality or the warranty does not survive acceptance – **11.2 f)**, **11.2 g)**	ISO 2859-2	Sampling by variables is unsuitable. An attributes LQ plan is recommended (**8.5**).
A small fraction nonconforming can cause great loss – **11.2 c)**	ISO 2859-2, ISO 8422	This implies a numerically low value of CRQ (**8.5.2**) and adequate discrimination (**8.3, 8.4, 8.5, 8.6, 9.1**). Any variables standard may be suitable, with the AQL chosen by reference to the OC curve so that the CRQ is small enough. The choice between standards will depend on other factors.
A small fraction nonconforming cannot cause great loss; readily detected in processing – **11.2 b)**	All ISO 3951 schemes. Indirect inspection, grab sampling	Virtually all variables standards are recommended. The choice depends on other factors. Long runs are normally implied (**8.1**) and very likely category (**11.2 g**) also (**8, 9.2**), so moderate discrimination (or IL) (**8.3, 8.5, 8.6** and **9.1**).
Lot non-acceptance causes plant shut-down and economic loss – **11.2 d)**	Attributes AOQL plans, e.g. ISO 18414, ISO 2859-1	AOQL and rectifying inspection (**8.7**) could be helpful – ISO 2859 gives AOQL data. This situation will probably necessitate more administrative action, especially when sampling inspection yields a non-accept result. As there is no AOQL-indexed standard on sampling by variables, consider using ISO 18414. Also, Table 8 of ISO 2859-1:1999 relates the AOQL to the AQL for single sampling plans.
There are many other sources for the item – **11.2 i)**	See guidance note	Consider the other factors.
There is a history of received quality and the quality is consistently good – **11.2 e)**	All ISO 3951 schemes, ISO 2859-3, indirect inspection, grab sampling	Where reliance can be placed on the inspection at a source (indirect inspection by the producer), skip-lot or reduced inspection can be considered. Small sample sizes with moderate DR in a scheme, i.e. switching rules are an essential safeguard (**8.3, 8.4, 8.5** and **9.2**).
There is no history of received quality or there is a history and the received quality is poor – **11.2 j)**	Tightened inspection. All ISO 3951 schemes, ISO 8423, ISO 2859-2.	This implies the use of switching rules (**5**) and commencing with fairly stringent sampling conditions e.g. tightened inspection, only switching to less stringent (normal) when quality has been established which then implies continuous supply – long run (**8.1, 8.3, 8.4, 8.5, 8.7** and Clause **9**); ISO 2859 for short runs, for which sampling by variables is unsuitable.

[a] "Indicated practice" (sampling system or plan) simply means that the International Standards mentioned should be the first to be considered, though other factors might dictate, or lead to, a different selection. Practicality, risk and costs always have to be considered. Any coexistence of conditions may affect the selection. See also Table 3.

Table 3 — Guidance for the selection of an acceptance sampling system, scheme or plan for sampling by variables, using existing production conditions

Production conditions (ISO/TR 8550-1:2007)	Indicated practice [a]	Guidance notes with ISO/TR 8550-1:2007 references
There is a history of consistently good quality in production – **10.3 b)**	ISO 3951-1, ISO 3951-2, indirect inspection, grab sampling	Where reliance can be placed on the inspection at a source (indirect inspection by the producer), reduced inspection can be considered. Small sample sizes with moderate DR in a scheme, i.e. switching rules are an essential safeguard (**6, 8.1, 8.4.1, 9.2, 10.4**).
Production quality is very variable or poor – **11.3 k)**	Tightened inspection, ISO 3951-1	This implies the use of switching rules (Clause **6**) and commencing with fairly stringent sampling conditions e.g. tightened inspection, only switching to less stringent (normal) when quality has been established, which then implies continuous supply – long run (**8.1, 8.3, 8.4, 8.5, 8.7** and Clause **9**). Use the attributes standard ISO 2859-2 for short runs.
Random samples are easily chosen or the tests are rapid and inexpensive – **11.3 c), 11.3 d)**	ISO 3951-3	Look to means of benefiting from this situation by using sequential or double sampling, providing the administration does not become too complicated and costly (**9.3**).
Random sampling is difficult or expensive – **11.3 l)**	ISO 3951-1, ISO 3951-2, ISO 3951-3, ISO 3951-5	Implies that the whole sample be taken initially, which leads to single sampling. Double sampling can be used with rules for sub-sample selection and sequential with curtailment (**9.3**). Administration problems can arise.
Tests are long or expensive – **11.3 m)**	ISO 3951-1, ISO 8423	Implies small samples with greater lot-by-lot risk (sampling plans with smaller discrimination ratios) but with long-term control of quality (**8.3, 8.4, 8.5, 8.6** and Clause **8**).
The shape and width of the distribution is of importance – **10.3 g)**	ISO 3951-5, ISO 8423, ISO 5479	Give particular attention to the choice of PR, CR, PRQ and CRQ (**8.4**).
The shape and width of the distribution(s) is(are) not important – **11.3 p)**	See guidance note.	Consider the other factors.
The distribution(s) is(are) known to be normal – **11.3 h)**	ISO 3951-1, ISO 3951-2, ISO 3951-3, ISO 3951-4, ISO 5479	This refers to the characteristics in question, e.g. length distributed normally, but inspection may be by variables or attributes. The conformity criteria will be determined from knowledge of the distribution (**8.2, 8.3, 8.4, 8.5, 8.6**).
The distribution is unknown or known not to be normal – **11.3 q)**	ISO 5479 See guidance note.	Sampling by variables can be used if information on the distribution can be obtained in the long run, and the distribution can be transformed to normality, but there are added risks for short runs, or at the start of a continuous supply. Sampling by attributes is recommended. Virtually any attributes standard applies, the choice depending on other factors. If inspection is by measurement, consider conversion to attributes.
The lot is a one-of-a-kind lot – **11.3 i)**	See 6.1 and guidance note.	See **10.3**. Use of sampling by variables for isolated lots is not recommended.
Tests are rapid **11.3 d),11.3 e), 11.3 f)**	ISO 3951-5, ISO 8423	Sequential sampling is particularly appropriate when inspection is rapid, costly and destructive.
Inspection of an item is costly or destructive – **11.3 e), 11.3 f)**	ISO 3951-3, ISO 3951-5	Minimum inspection implies small samples (including sampling by variables): double or sequential sampling are prime candidates (**9.3**).
Inspection of an item is inexpensive or non-destructive – **11.3 n), 11.3 o)**	See guidance note.	Consider the other factors.

[a] "Indicated practice" (sampling system or plan) simply means that the International Standards mentioned should be the first to be considered, though other factors may dictate, or lead to, a different selection. Practically, risks and costs always have to be considered. Any coexistence of conditions may affect the selection. See also Table 1.

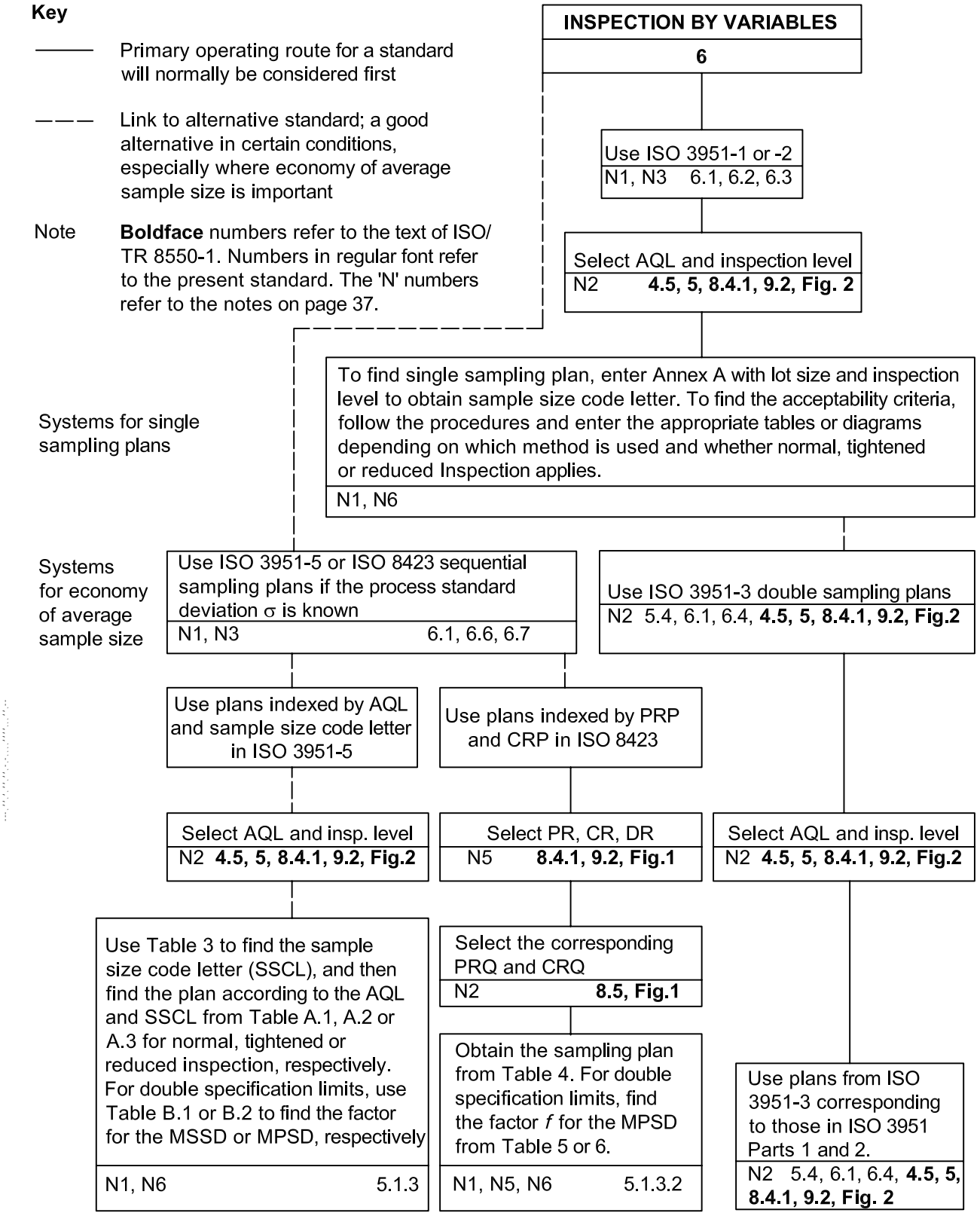


Figure 15 — Illustration of the selection procedure for inspection by variables when production is continuous and run length exceeds 10 lots on original inspection

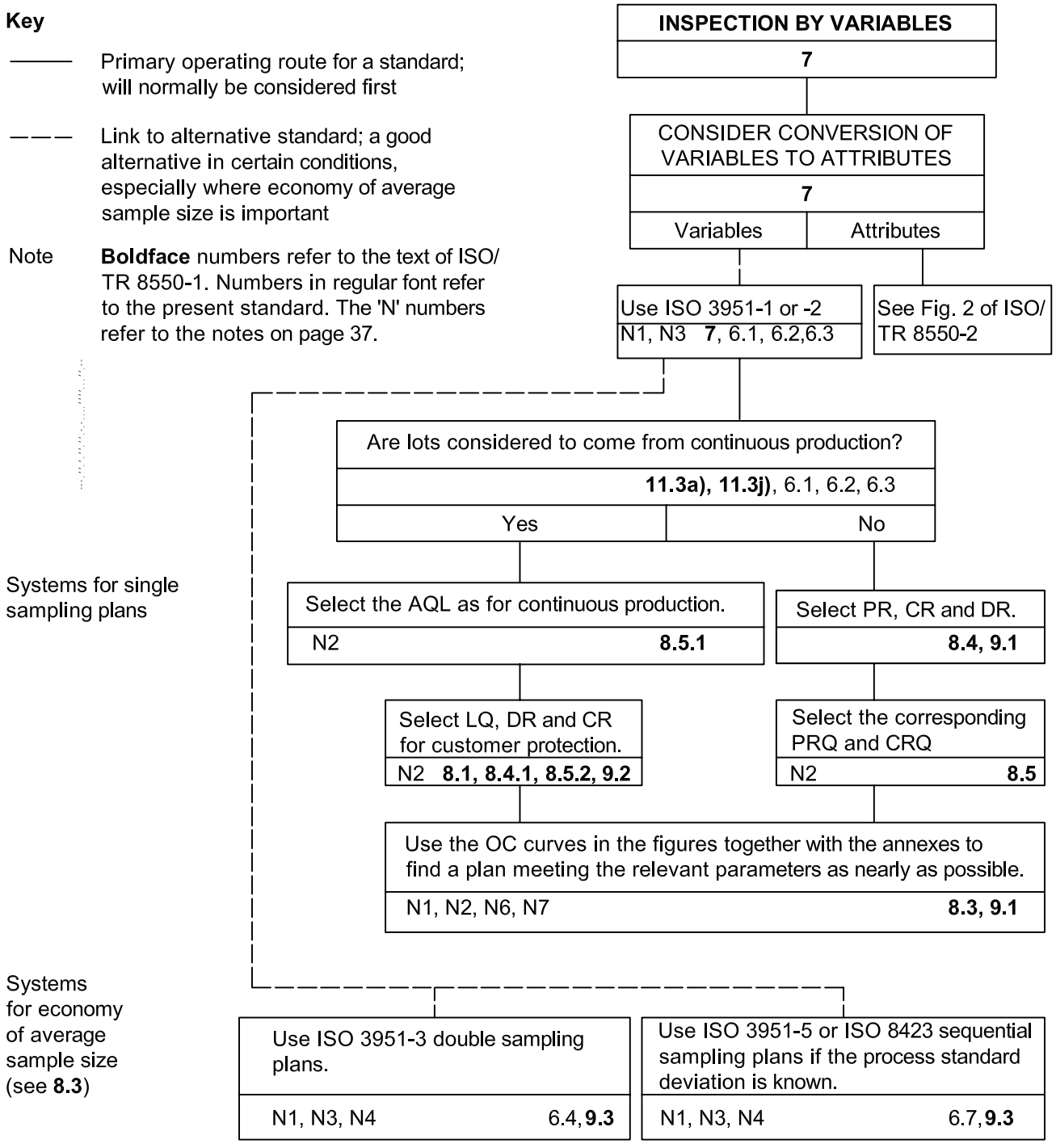


Figure 16 — Illustration of the selection procedure for inspection by variables when production is not continuous or run length is 10 lots or fewer on original inspection

Notes to Figures 15 and 16

The following notes are common to Figures 15 and 16. References to the notes are made in the figures by means of "N" numbers in the bottom left-hand corners of many of the boxes in the figures.

N1 Further detailed instructions for selecting the appropriate sampling plan are given in the relevant standard.

N2 The acceptance quality limit (AQL), producer's risk quality (PRQ), limiting quality (LQ), consumer's risk quality (CRQ), inspection level (IL) or discrimination ratio (DR) may be prescribed, e.g. by contract. If this is not the case, the appropriate parameters need to be determined before a sampling plan can be selected from the relevant standard.

N3 When using ISO 3951-1, ISO 3951-2 or ISO 3951-3, it will be necessary first to select from the "s" method and the "σ" method for obtaining a sampling plan, as indicated in the standard.

Sequential sampling using ISO 3951-5 or ISO 8423 has a requirement that the standard deviation can be considered constant and taken to be σ.

N4 This is subject to certain provisions regarding the lot size to sample size ratio and curtailment of inspection. The fact that the standard deviation will not be known exactly introduces added sampling risk.

N5 The basic sampling plan table (Table 4 in ISO 8423:—[3)]) is based on a producer's risk of 5 % and a consumer's risk of 10 %.

N6 If the sampling plan derived at the first attempt is unacceptable for any reason, e.g. the sample size is too large, it is necessary first to make sure that the selection has been made correctly.

If the plan is still unacceptable, then the "quality levels" and "sampling risks" need to be reconsidered by all parties concerned to reach an understanding and to agree upon revised parameters for selecting the sampling system/plan.

N7 Switching rules is not applicable but any suitable plan can be chosen from the tables of normal or tightened inspection sampling plans.

3) To be published. (Revision of ISO 8423:1991)

Annex A
(normative)

Normal probability paper

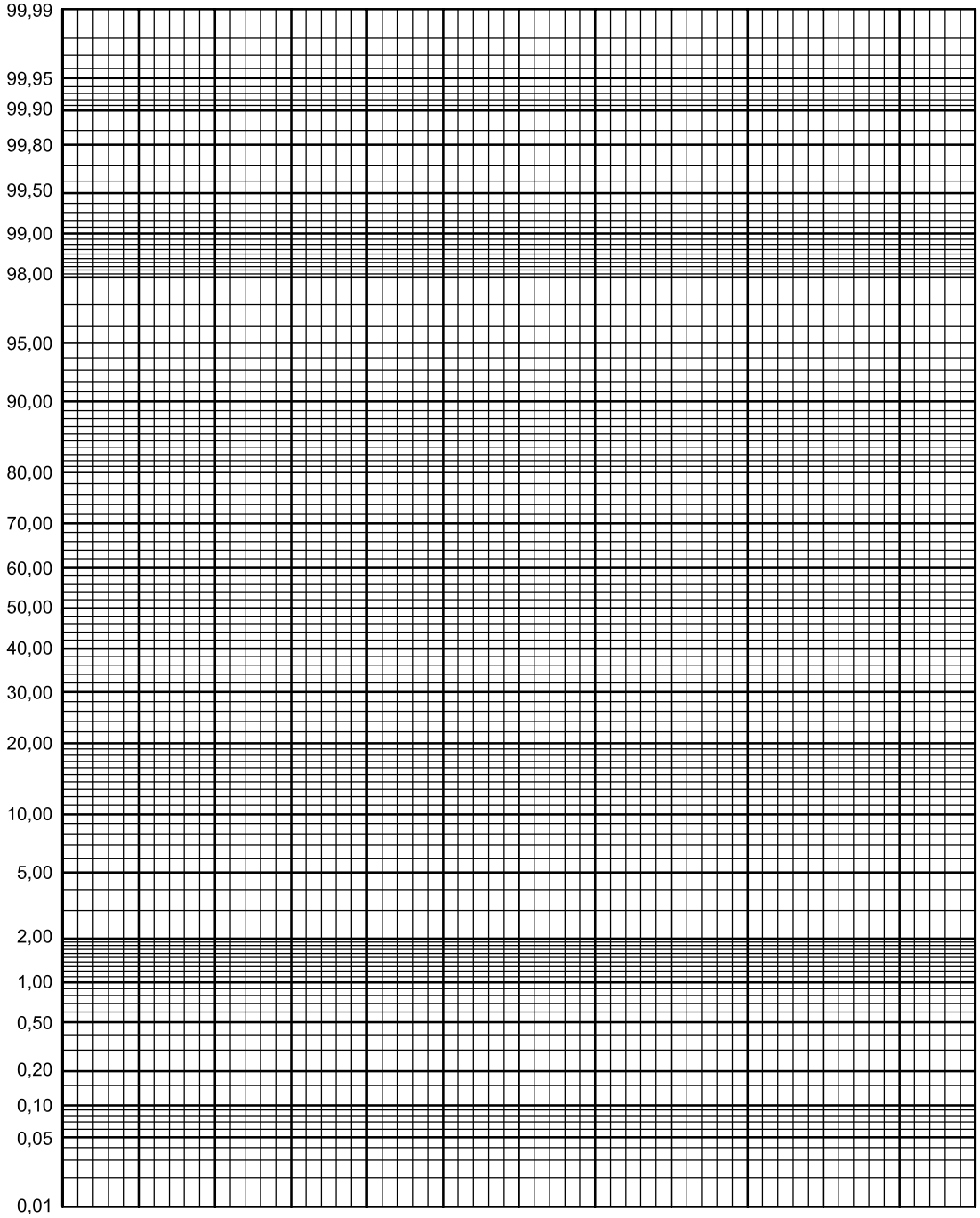


Figure A.1 — Normal probability paper

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