TECHNICAL REPORT

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Guidance on the selection and usage of acceptance sampling systems for inspection of discrete items in lots —

Part 1:
Acceptance sampling

Lignes directrices pour la sélection d'un système, d'un programme ou d'un plan d'échantillonnage pour acceptation pour le contrôle d'unités discrètes en lots —

Partie 1: Lignes directrices générales pour l'échantillonnage pour acceptation

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 8550-1 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

This first edition of ISO/TR 8550-1, together with ISO/TR 8550-2 and ISO/TR 8550-3, cancels and replaces ISO/TR 8550:1994.

ISO/TR 8550 consists of the following parts, under the general title *Guidance on the selection and usage of acceptance sampling systems for inspection of discrete items in lots*:

— *Part 1: Acceptance sampling*

— *Part 3: Sampling by variables*

The following part is under preparation:

— *Part 2: Sampling by attributes*

Introduction

This part of ISO/TR 8550 gives guidance on the selection of an appropriate acceptance sampling scheme for the inspection of discrete items submitted in lots from the schemes described in various national and international standards.

There are many situations where products (materials, parts, components, assemblies and systems) are transferred from one organization to another, where the organizations may be different companies or parts of a single company or even different shops within a plant. In these situations both the supplier and the customer may use acceptance sampling procedures to satisfy themselves that the product is of acceptable quality. Suppliers will be seeking to maintain a reputation for good quality and to reduce the likelihood of claims under warranty, but without incurring unnecessary production and supply costs. On the other hand, customers will require adequate evidence, at minimum cost to themselves, that the product they receive conforms to specifications. Compared with, say, 100 % inspection, suitable sampling methods will often be beneficial in achieving these aims. Sometimes acceptance sampling methods are the only practical procedure, especially when the tests for conformance are destructive.

Several types of sampling systems, schemes and plans are available for these purposes. They are presented in a number of ISO Standards that explain how they are to be used. However, it is often difficult to decide on the most appropriate procedure for use in a particular situation. The purpose of this part of ISO/TR 8550 is to assist in that decision.

The choice of sampling system, scheme or plan depends on a number of conditions and on the prevailing circumstances. In any supply situation, the first essential is that the supplier and the customer understand, and have agreed upon, the requirements and the basis for release and acceptance of the product, including any acceptance sampling methods to be used.

Lots that are non-acceptable cause difficulties for both supplier and customer. The supplier incurs additional costs in rework, scrap, increased inspection, damage to reputation and possibly loss of sales. Delays in delivery and re-inspection costs are a burden to the customer. For these reasons, it is usually considered essential for the supplier to provide lots that have a very high probability of being accepted, i.e. 95 % or more. The supplier has to ensure that quality control of the production or delivery process provides lots of a quality sufficient to meet this objective. A basic principle of some acceptance sampling inspection schemes is to promote the production of lots of acceptable quality. The primary purpose of these schemes is not to discriminate between acceptable and non-acceptable lots, i.e. to sort, but to keep production under control to yield an acceptable process average quality. Although all acceptance sampling plans are discriminatory to some degree, the process average quality (expressed in terms of percent nonconforming or number of nonconformities) should not be greater than half the acceptance quality limit in order to ensure a very high probability of acceptance.

The primary purpose of the ISO/TR 8550 series is to give guidance on the selection of an acceptance sampling system, scheme or plan. It does this principally by reviewing the available systems specified by various standards and showing ways in which these can be compared in order to assess their suitability for an intended application. The guide also indicates how prior knowledge of the manufacturing or service delivery process and quality performance might influence the choice of sampling system, scheme or plan, and likewise how the particular needs of the customer affect selection. Some specific circumstances encountered in practice are described and the method of choosing a plan is explained. Some checklists or pointers and tables are provided to assist users in selecting an appropriate system, scheme or plan for their purposes. Charts are included to illustrate the procedures to be followed in the selection process.

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

Guidance on the selection and usage of acceptance sampling systems for inspection of discrete items in lots —

Part 1:
Acceptance sampling

1 Scope

This part of ISO/TR 8550 gives general guidance on the selection of an acceptance sampling system, scheme or plan. It does this principally in the context of standards that either already exist or are presently under development. (For more detailed information about specific acceptance sampling systems, see ISO/TR 8550-2 for sampling by attributes or ISO/TR 8550-3 for sampling by variables.)

The guidance in this part of ISO/TR 8550 is confined to acceptance sampling of products that are supplied in lots and that can be classified as consisting of discrete items (i.e. discrete articles of product). It is assumed that each item in a lot can be identified and segregated from the other items in the lot and has an equal chance of being included in the sample. Each item of product is countable and has specific characteristics that are measurable or classifiable as being conforming or nonconforming (to a given product specification).

Standards on acceptance sampling are typically generic, as a result of which they can be applied to a wide variety of inspection situations. These include, but are not limited to, the following:

a) end items, such as complete products or sub-assemblies;

b) components and raw materials;

c) services;

d) materials in process;

e) supplies in storage;

f) maintenance operations;

g) data or records;

h) administrative procedures.

Although this part of ISO/TR 8550 is written principally in terms of manufacture and production, this should be interpreted liberally, as it is applicable to the selection of sampling systems, schemes and plans for all types of products and processes as defined in ISO 9000.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition listed applies. For undated references, the latest edition of the referenced document (including any amendment) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 9000, *Quality management systems — Fundamentals and vocabulary*

3 Terms and definitions

For the purposes of this part of ISO/TR 8550, the terms and definitions given in ISO 3534-1, ISO 3534-2 and ISO 9000 apply.

4 Abuses and uses of acceptance sampling

4.1 Abuses of acceptance sampling

Acceptance sampling has become unpopular since the early 1980s. Some of the reasons for this (although certainly not all) are well founded, so it is important to be able to distinguish those situations where acceptance sampling should not be used from those where it may be appropriate.

The chief arguments used against the use of acceptance sampling are as follows.

a) When quality is generally very high, the sample sizes needed to detect a slip in quality are uneconomically large.

b) Quality cannot be inspected into a product.

c) It is far better to establish a robust design and to implement comprehensive process controls than to try to find and eliminate nonconforming items after manufacture.

d) Most acceptance sampling standards are indexed in terms of acceptable quality level (AQL). Once an AQL has been established and quality has been brought sufficiently below the AQL to achieve high probabilities of lot acceptance, there is no incentive for the producer to try continuously to improve quality.

e) Quoting an AQL is tantamount to granting a licence to produce defects.

f) The only acceptable quality level is zero defects.

These arguments are examined in turn in the following subclauses.

4.2 Example 1

The following simplified example, devised by Baillie [18], demonstrates how the optimum sampling plan can vary according to the quality level against which it is desired to guard. A certain item is produced in lots of size 10 000, with a unit production cost of £10,00. The selling price per item is £a in accepted lots and at a discounted price of £0,50 in lots non-accepted by the acceptance procedure. Testing is destructive, and the cost of testing each item is £1,00. The downstream cost (e.g. warranty cost plus loss of goodwill) of a nonconforming item in an accepted lot is £10 000, but zero in non-accepted lots sold at a discount. Historical data indicate that the process fraction nonconforming is p for 99 % of lots, but that it unaccountably and randomly slips to 100p for 1 % of the lots. A single sampling plan by attributes is to be used, i.e. a random sample of size n is to be selected from each lot, and the lot is to be considered acceptable if the sample contains no more than Ac nonconforming items. What is the optimal sampling plan, i.e. the plan that maximizes the profit per item sold?

Mathematical details are provided in Annex A for information. Table 1 shows the optimal sampling plan for a range of values of the process quality level p. The results are instructive.

Table 1 — Optimal sampling plans for Example 1

Usual quality level, in fraction nonconforming	Quality level after slippage, in fraction nonconforming	Optimal plan		Selling price per item, a (£)	Average profit per item sold (£)
		Sample size n	Acceptance number, Ac		
0,001 00	0,100	104	2	20,25	0,022
0,000 50	0,050	139	1	15,40	0,091
0,000 30	0,030	197	1	13,60	0,211
0,000 20	0,020	249	1	12,75	0,280
0,000 10	0,010	141	0	12,00	0,378
0,000 09	0,009	137	0	11,95	0,436
0,000 08	0,008	129	0	11,90	0,499
0,000 07	0,007	113	0	11.85	0,570
0,000 06	0,006	86	0	11,75	0,603
0,000 05	0,005	34	0	11,70	0,710
0,000 04	0,004	Accept without sampling		11,60	0,804
0,000 03	0,003	Accept without sampling		11,50	0,903
0,000 02	0,002	Accept without sampling		11,35	0,952
0,000 01	0,001	Accept without sampling		11,20	1,001

Not surprisingly, it is found that improvements in the quality level allow the selling price to be decreased while at the same time increasing the profit per item sold. At first, improvements in quality levels necessitate larger sample sizes in order to be able to provide the necessary discrimination between the two quality levels. As quality levels improve, the optimal acceptance number Ac reduces and there comes a point when the sample size that is required also begins to reduce until, eventually, it becomes uneconomical to sample at all. This final state is called “indirect inspection” as the inspection has effectively been transferred from the producer to the consumer; nonconforming items are so rare that it is more economical not to sample and inspect but to reimburse consumers on the infrequent occasions that they invoke the warranty. Thus 4.1a) is seen to be misleading for, when quality levels reach a sufficiently high level, acceptance sampling simply becomes an unnecessary overhead rather than requiring uneconomically large sample sizes.

4.3 Inspecting quality into a product

Inspection makes little difference to the outgoing quality if the incoming quality is more or less constant unless the sample size is a large proportion of the lot size, in which case the inspection process is a large overhead. Either way, it is not a particularly sensible approach to improving quality levels.

4.4 Design and control

The advantages of establishing a robust design and a comprehensive process control system are many. The robust design places the least possible demands on the manufacturing process and the process control system tends to prevent process parameters from straying too far from their target values, so process variation and waste is kept low and output quality is kept high. Moreover, the design and the control system of the production process can be reviewed and improved in the light of experience to provide continual quality improvement.

4.5 AQLs

The initials AQL used to stand for Acceptable Quality Level, although in reality the AQL is simply an index to a sampling plan. Standards tried to make this clear by explaining that the level was acceptable for the purposes

of acceptance sampling (rather than in an absolute sense). Indeed, lot quality levels typically have to be better than half the AQL to have a very high chance of being accepted.

During the late 20th century, many companies came to realize that the only way to survive in a global marketplace was to strive endlessly for improved levels of quality. The notion that any level of quality other than zero defects (see 4.7) was acceptable began to be scorned. In order to clarify the situation, the meaning of the initials AQL was changed in international standards to Acceptance Quality Limit, which more accurately describes its function. Unfortunately, the damage was already done, for many organizations no longer entertain the use of standards indexed by AQL.

The argument that AQLs provide no incentive for the producer to continuously improve quality once it has been improved to a level that is better than the AQL is not a strong one; in many medium or long-term agreements between supplier and customer, a progressive reduction in the AQL could easily be agreed upon and written into the contract. Moreover, a producer intent on staying in business is already striving for better levels of quality in order to maintain or improve his place in the national or global market.

4.6 A licence to produce defects?

It is untrue that an AQL provides a licence for the producer to provide defects. Most AQL-indexed standards expressly caution that the designation of an AQL does not imply that the supplier has the right knowingly to supply any nonconforming items of product.

4.7 The zero defects philosophy

Crosby [19] introduced the idea that quality can be free, i.e. the extra resources used to improve quality would often be more than compensated for by the reduction in rework or scrap or loss of goodwill. Unfortunately, the corresponding idea that the producer should strive for a perfect process that produces no nonconforming items inevitably often became misconstrued to stipulate that acceptance sampling plans should always have an acceptance number of zero, i.e. that the plans should lead to lot non-acceptance if one or more nonconforming items are found in the sample. Example 1 shows this not to be an inevitable corollary. An acceptance number of zero is seen to be optimal only over a certain range of quality levels; at lower quality levels, acceptance numbers of 1 or more are optimal, while at higher quality levels, it is best not to sample at all.

4.8 The use of acceptance sampling

For many mature production processes, quality levels will have become so close to perfection that it is a needless waste of resources to implement acceptance sampling procedures. The design will have been refined such that there are no difficulties in the production process due to any of the process parameters being difficult to achieve or maintain, and safeguards will have been built into the process control system wherever necessary.

It can be seen from Table 1 that acceptance sampling became redundant at a quality level somewhere in the range 0,000 1 to 0,000 2 nonconforming. One of the variables in Example 1 was the 1 % of lots that slip to the worse quality level. If this percentage could be substantially reduced, then acceptance sampling would become redundant at quality levels in the good lots worse than 0,000 2 nonconforming. Thus a two-pronged attack on internal variation and on external, “special causes” of variation in the production process, together with repeated reviews of the product design, ultimately lead to acceptance sampling becoming unnecessary for many products.

However, what about the early stages while the process and its controls are being refined? Example 1 demonstrates that appropriate use of acceptance sampling can play a key part in maximizing profitability during this interim period.

Some processes never run long enough to become mature. This is particularly true for some defence industries. There is not much point in continuing to build an offensive weapon of a given specification once an effective defence to it has been devised and is widely available. Specifications are therefore frequently modified, which can make it difficult to achieve a robust design or efficient process controls. Sometimes the

materials used in the production of armaments are so new that they have properties and limitations that are not completely understood. Sometimes it is in the assembly of individually sound components into complex items where it might be necessary to use acceptance sampling to maintain quality; it will be too late once the items are being used in anger. Sometimes what might seem to be very high levels of nonconformity may be acceptable. For example, an over-the-shoulder anti-tank weapon system would be more than acceptable even if it had only a 50 % chance of destroying a tank costing one thousand times as much, although this translates into 50 % nonconforming. Acceptance sampling may be applied periodically to munitions held in storage over many years, to check that they have not degraded to an intolerable level. In the computer industry, a process yield as low as 50 % when etching the latest and fastest computer chips may be considered acceptable. Acceptance sampling might even be used as a tool by which to verify statistical process control results.

In summary, acceptance sampling has a legitimate part to play in the quality assurance of many products.

5 Acceptance sampling plans, schemes and systems

An acceptance sampling plan is a set of rules by which a lot is to be inspected and its acceptability determined. The plan stipulates the number of items (units) in the sample, to be drawn randomly from a lot for inspection against the product specification. The lot is then sentenced as “acceptable” or “non-acceptable” according to how the inspection results compare with the criteria of the acceptance sampling plan.

Sometimes, when a long series of lots is being inspected, a sampling procedure might call for a shift from one sampling plan to another, depending on the current and previous sample results. Sampling procedures that call for switching from one sampling plan to another, and possibly back again, are called sampling schemes. A sampling scheme might also call for discontinuation of inspection if product quality appears to remain poor. The customer may then shift to another supplier, if available, or initiate 100 % screening until the supplier can improve the production process sufficiently to produce acceptable product.

In the case of destructive testing, the customer may cease to accept product until the supplier has demonstrated to his satisfaction that the production problems that were giving rise to the previous low quality have been overcome.

A collection of sampling plans and related sampling schemes constitute a sampling system. The system is generally indexed in some way, e.g. by lot size, inspection level and acceptance quality limit (e.g. ISO 2859-1).

The standards reviewed in ISO/TR 8550-2 and ISO/TR 8550-3 present plans for single, double, multiple or sequential sampling. Procedures for skip-lot sampling for inspection by attributes are given in ISO 2859-3. A comparison of the various sampling methods and the principles on which they are based assists in assessing their suitability for a particular application and enables an appropriate selection to be made.

6 Practical and economic advantages of using standard sampling plans

To those concerned with the writing of specifications, it is of benefit that statistically sound sampling procedures be provided. Because there are economies of scale for larger lots, most sampling schemes presented in the standards reviewed in ISO/TR 8550-2 and ISO/TR 8550-3 relate sample size to lot size. Apart from providing control over the methods of selection of the sample, these standards should normally be invoked because they specify requirements that control the treatment of nonconformities found during inspection and the treatment of lots resubmitted after initial non-acceptance. Furthermore, most of these sampling systems contain built-in switching rules (e.g. from ‘normal’ to ‘tightened’ or to ‘reduced’ inspection) to adjust the sampling plan in the event of deterioration or improvement in quality. Use of these basic reference standards can save much time often wasted in subjective discussion, and reduce the large areas of discretion often contained in non-standard sampling schemes that have only limited value, particularly for international trade.

Sampling involves risk and, quite naturally, all parties concerned attempt to minimize their share. Theoretically, these risks are functions of the sampling plan and the quality level agreed upon, without relation to the industry or the product. In practice, these risks are reduced by controlling the production process and improving the level of quality.

These risks cannot be eliminated, but they can be precisely calculated and economically assessed by the use of modern statistical techniques. Consequently, it is of benefit to all parties that statistically sound acceptance criteria be specified in product/process specifications and that, wherever possible, the generally applicable basic reference standards on sampling, such as the ISO 2859 and ISO 3951 series, be utilized.

In general, when arriving at the optimum performance of an acceptance sampling plan or scheme, the costs of preventing nonconformities should be balanced against the probabilities of failure in service. Subject to various assumptions being made with regard to the sample size to lot size ratio (n/N) and to the appropriate distribution theory, it is a relatively straightforward matter to formulate sampling plans from statistical theory. Note that, while existing standards on sampling by variables are only applicable to product characteristics that have normal distributions, standards on sampling by attributes are not dependent on the distributional shape of the product characteristics.

Development of generic acceptance sampling standards is a more difficult matter. There are undeniable advantages in having relatively few standard schemes, as this leads to greater uniformity of action and simplifies the administrative procedures across organizational and national boundaries. However, for these to be adopted for general use by industry worldwide, sampling standards have to be practical and flexible enough to take account of the many and varied situations met in practice. The established AQL-indexed procedures given in the ISO 2859 and ISO 3951 series, and in corresponding international standards, have served industry well in the past, and are continuing to be developed to fulfil current and future needs.

The motivation for acceptance sampling is primarily economic: inspection of a sample from a lot is the (usually small) price paid to achieve desirable quality in the accepted lots. This quality is achieved by two pressures:

1) the purely statistical pressure of different probabilities of acceptance of good and bad quality lots;

2) when sequences of lots are purchased, the commercial pressure of frequent non-acceptance of lots and the switch to tightened inspection or discontinuation of inspection when quality is poor.

The problem associated with acceptance sampling inspection relates to defining unambiguously the criteria used to judge discrete individual items supplied in quantity, the criterion for acceptance of the lot, the quality level expected from the manufacturing process, the discrimination afforded by the sampling plans and the rules to be followed when a lot is not accepted. Above all, however, it is necessary to design the sampling scheme so that it can be invoked easily in a purchasing contract. The sampling plans in the sets of related standards discussed in ISO/TR 8550-2 and ISO/TR 8550-3 enable this to be done efficiently.

The parties should agree on the following:

a) the specification to which the discrete items of product are to conform; this is necessary because, in all dealings between the parties, there has to be agreement on what constitutes a conforming item and what constitutes a nonconforming item;

b) whether the acceptance of the product is to be determined by the acceptance of individual items or collectively by the acceptance of inspection lots of items (acceptance of individual items precludes sampling).

When the acceptance is to be on a lot basis, the agreement between supplier and recipient needs to include

— the criteria for item conformance,

— the criteria for lot acceptance,

— the criteria for non-acceptance of the lot, and

— the acceptance sampling system, scheme or plan to be used.

The latter should be based on risk factors that are mutually acceptable to both producer and customer.

Having agreed on the acceptance sampling system, scheme or plan to use, the supplier knows, for various quality levels, the probability that his supply lots will be accepted. Likewise, the customer understands the protection provided by the sampling system, scheme or plan against acceptance of poor quality product.

Current standards present plans for single, double, multiple, sequential and skip-lot sampling. A comparison of the various sampling methods and the principles on which they are based will assist in assessing their suitability for a particular application and enable an appropriate selection to be made.

7 Attributes versus variables

Acceptance sampling standards generally describe procedures for inspection by attributes or for inspection by variables; a key decision to make is which of these to use.

If certain assumptions are true, the variables method has the advantage of generally requiring a smaller sample size than the attributes method to attain a given degree of protection against incorrect decisions. In addition, it provides more information on whether quality is being adversely affected by process mean, process variability, or both.

The attributes method has the advantage that it is more robust in the sense that it is not subject to assumptions of distributional shape, and that it is simpler to use. The larger sample sizes and consequential increased costs associated with using attribute sampling methods might be justifiable for these reasons. Furthermore, an attribute scheme might be understood and accepted more readily by inspection personnel. To avoid the assumption of normality and the attendant inability or difficulty in checking for this with “short runs” or lots of an “isolated” nature, sampling by attributes is recommended even to the extent of converting measurements to attributes.

When the quality characteristics are known to be normally distributed, at least to a good approximation, sampling by variables has a substantial advantage when inspection is expensive, e.g. when testing is destructive. Often, a simple mathematical transformation, such as taking the logarithm or square root, will convert a set of measurements from a non-normal to a normal, or near-normal, distribution.

Table 2 gives a comparison of the sample sizes for inspection by attributes and by variables for certain lot size ranges when using single sampling plans at inspection level II (see 8.6.1) under normal inspection. Similar advantages exist when comparing inspection by variables and by attributes in double and sequential sampling.

Table 2 — Comparison of sample sizes in inspection by attributes and by variables

Lot sizes	**Single sample sizes under normal inspection**		
	Inspection by attributes (ISO 2859-1)	**Inspection by variables** (ISO 3951-1)	
		Unknown process standard deviation	**Known process standard deviation**
16 to 25	5	4	3
91 to 150	20	13	8
281 to 500	50	25	12
1 201 to 3 200	125	50	18
35 001 to 150 000	500	125	32

8 Further considerations influencing a selection

8.1 Long and short production runs

Most acceptance sampling standards are intended for use primarily on a continuing series of lots of sufficient duration to allow the switching rules to be applied. This implies a “long” production run.

The principal exception is ISO 2859-2, which comprises limiting quality (LQ) plans that can be used when the switching rules of ISO 2859-1 are not applicable. These are primarily intended for use with single lots or lots of an “isolated nature”. By implication, this embraces a “short” series of inspection lots - or a “short” production run.

ISO 2859-5 and ISO 3951-5 provide sequential plans that match other standards in their respective series, and in many cases are thus similarly applicable to long or short runs.

In order for a production run to qualify as “long”, one criterion is clearly that the switching rules have a reasonable chance of coming into effect if “the quality is unsatisfactory”. It is equally clear that this alone raises a number of supplementary issues (as indicated by the quotation marks) depending on the requirements and circumstances prevailing in each case considered. It is impossible to stipulate simply and precisely what constitutes a short run (number of lots) in the context of sampling inspection.

In the absence of any other guide or evidence on which to base a judgement, anything up to 10 consecutive inspection lots should be considered as a “short run”, and the plans in ISO 2859-2 should be used. However, lots should not be subdivided arbitrarily in order to create a “long run”. It is generally preferable to have large homogeneous lots because they allow a smaller sample size to lot size ratio, and provide better representation by the sample, sharper discrimination and more economical inspection.

In a long production run, there is continuity and stability, so production settles down to a long-term stable process average. Nevertheless, the quality of individual lots varies about this process average. On the other hand, at the start of production, after a significant break or change in production, or for a short production run, the lot quality might well be somewhat different and more variable, even markedly so. The practical factor to consider is whether there is evidence that a stable process average has been established and still exists.

8.2 Nonconformity and nonconforming item

8.2.1 Failure to conform

8.2.1.1 General

Any failure to conform to a specified product characteristic, dimension, attribute or performance requirement represents nonconformity. A nonconforming item might have one or more nonconformities.

The failure of a ballpoint pen to write, for example, is a nonconformity; the pen is nonconforming. However, the same pen might have failed to conform in a number of other ways, e.g. colour, dimensions, etc. A pen that exhibited several nonconformities would still be counted as one nonconforming item.

The qualification “nonconformity” does not necessarily imply that the unit of product cannot be used for the purpose intended. For example, a brick with one of its dimensions outside the prescribed tolerance interval, though nonconforming, can still be used for building.

The distinction between nonconformity and nonconforming item is of no importance if the items have no more than one nonconformity, but becomes essential when multiple nonconformities can occur.

The quality of a given quantity of product may be expressed either as “percent nonconforming” or as the “number of nonconformities per hundred items”, but these are only interchangeable when items can have no more than one nonconformity.

Under sampling by attributes, sampling plans are available for either percent nonconforming or the number of nonconformities per hundred items.

8.2.1.2 Example 2

In counting pinholes in metal foil, the number of pinholes per square metre might be of interest. Here we would count all the pinholes in each square metre (item) examined and then express the quality in pinholes per 100 m^2.

8.2.1.3 Example 3

Suppose that a lot consists of 500 articles. Of these, 480 conform and are acceptable, 15 have one nonconformity each, four have two nonconformities each, and one has three nonconformities.

The lot percent nonconforming is given by the formula:

$$\begin{aligned}\text{Percent nonconforming} &= \frac{\text{number of nonconforming items}}{\text{total number of items}} \times 100 \\ &= \frac{20}{500} \times 100 \\ &= 4\,;\end{aligned}$$

that is, the lot is 4 % nonconforming.

The number of nonconformities per hundred items in the lot is given by the formula:

$$\begin{aligned}\text{Nonconformities per 100 items} &= \frac{\text{number of nonconformities}}{\text{total number of items}} \times 100 \\ &= \frac{26}{500} \times 100 \\ &= 5{,}2\,;\end{aligned}$$

that is, the lot has 5,2 nonconformities per hundred items.

8.2.1.4 Comments on Examples 2 and 3

Hence, under sampling by attributes, whether percent nonconforming or nonconformities per hundred items is to be used is a matter for individual consideration in each particular case. The important thing is that it has to be considered, specified, and agreed upon beforehand, not left until a sample has been inspected and then considered.

Under sampling by variables, sampling plans are only available for percent nonconforming, so there is no choice to be made. However, different quality characteristics might belong to different classes (see 8.2.3), in which case they are treated separately.

8.2.1.5 Factors to be taken into account

Factors to be taken into account in deciding whether to use percent nonconforming or nonconformities per hundred items under sampling by attributes are as follows.

a) Under inspection for percent nonconforming it is assumed that, if an item contains one or more nonconformities, the item is nonconforming and is not acceptable.

   It also presupposes that the number of different ways in which an item can be nonconforming is limited and known, e.g. there are only 5 ways in which each particular item could be nonconforming [see also item b)].

b) Under inspection for nonconformities, every nonconformity found is counted. Three nonconformities found in one item count as three, and are given the same weight as three items each having one nonconformity.

A special case arises when a nonconformity can occur an unknown and almost unlimited number of times in items, e.g. surface blemishes or pinholes can occur in any number and it is not known how many times they do not occur, so percent nonconforming for this feature is meaningless. In such cases, nonconformities per hundred items should be used (see Example 2).

NOTE Percent nonconforming under sampling inspection by attributes implies a binomial distribution; for nonconformities per hundred items, a Poisson distribution is appropriate.

c) Two properties are dependent if nonconformities in an item arise, in part or wholly, through some common cause, or if one property affects the other. Detailed knowledge of the production process is thus needed to decide whether properties are independent. In statistical terms, if two characteristics, say length and diameter, are independent, it means that if all the units produced were taken and sorted into two groups according to whether the length was nonconforming or not, then the percent nonconforming for diameter would be found to be essentially the same in each of these two groups; or, alternatively, if they were sorted into two groups according to whether the diameter was nonconforming or not, then the percent nonconforming for length would be essentially the same in the two groups. It can be shown mathematically that these two procedures are equivalent.

   If two nonconformities are not independent, then they are said to be related, or dependent. It should be agreed that the occurrence of both in one item is to count as only one nonconformity, not as two. Occasionally the correlation between two related nonconformities is low. Under these conditions, the two may be considered independent. Inspection for percent nonconforming avoids this difficulty.

d) If the percentage of nonconformities in the lot is less than 2,5 %, then the probability distributions of nonconforming items and nonconformities will be almost identical. In the range 2,5 % to 10 % some difference will be apparent, a nonconformities per hundred items plan being rather more severe than the equivalent percent nonconforming plan.

e) At an inspection station, and where admissible, it might be simpler and better practice to use one method rather than to change frequently from one method to the other, e.g. nonconforming items rather than nonconformities per hundred items.

f) From the point of view of keeping records that will be useful for improving quality, nonconformities per hundred items might be preferable as the records will then automatically contain information on all nonconformities, whereas some nonconformities might escape the record if the percent nonconforming approach is adopted.

8.2.2 Nomenclature

The discussion in the remainder of this guide is in terms of inspection for nonconforming items. When appropriate, it may be read in terms of inspection for nonconformities, by replacing “nonconforming items” by “nonconformities”, and by replacing “percent nonconforming” by “nonconformities per hundred items”.

8.2.3 Classification of nonconformities

The discussion so far has assumed that, if an article can be nonconforming in more than one way, the different possible nonconformities are all of equal importance. It is then possible to sentence by counting the nonconforming items. For example, suppose that there are three dimensions to be checked and, in a sample, three articles are nonconforming in the first dimension alone, three articles in the second dimension alone, one article in the third dimension alone, and one article in both the first and second dimensions. This gives a total of eight nonconforming items, which is the number to compare with the acceptance and rejection numbers.

The procedure of adding nonconforming items of different types is reasonable only if the nonconformities are of equal, or nearly equal, importance. Where this is not so, it is necessary to classify the possible nonconformities into groups so that nonconformities in different groups are of different orders of importance but all nonconformities within a group are of approximately the same order of importance. Different AQLs are then used for the different groups.

For many purposes, two groups are sufficient, namely major nonconformities of class A, which are of greatest concern, and nonconformities of class B, which are of lower concern. Sometimes it is necessary to introduce further classes or sub-classes within these classes. The most important class of all contains the critical nonconformities, which adversely affect usage or, in the extreme, render the articles hazardous or potentially hazardous.

Critical nonconformities are a special case and are discussed in more detail in 8.2.4. For the moment, the discussion is restricted to the major and minor classes. These classes refer to the relative importance of different nonconformities within any given product, and as products themselves vary in importance, the classes do not correspond to any absolute quality levels. It follows that there is no one AQL that normally goes with any particular class.

The classification of nonconformities should be done properly. It is clear that care has to be taken not to "under-classify" (for example, to classify as a class B nonconformity a feature that should be in class A), as this will lead to the allowance of more nonconformities of this class in the sampling plan for the feature concerned than is really required. However, it is also very important not to "over-classify".

When the system of classification of nonconformities is adopted, it is necessary to allocate a different AQL to each class to ensure that the more important, class A, nonconformities are more tightly controlled than the class B nonconformities.

Under sampling by attributes, if an article has two or more nonconformities and the nonconformities come within different classes, it counts as a nonconforming item of the more serious class. However, if inspection is in terms of nonconformities rather than in terms of nonconforming items, each nonconformity in the sample is counted in its appropriate class.

It is possible that, at any one time, different classes can be under different inspection severities, e.g. class A might be under normal inspection while class B is under tightened inspection. A lot is sentenced as acceptable if and only if the acceptance criteria for all classes are satisfied.

8.2.4 Critical nonconformities

8.2.4.1 General

By definition, critical nonconformities present a hazard and/or adversely affect usage or safety. These nonconformities form a special category. It is impossible to choose any value of percent nonconforming for these nonconformities, however small, and say, "... this percentage of critical nonconformities is tolerable."

Where non-destructive inspection is involved, the solution generally adopted is to require that critical characteristics are to be inspected using a sample size equal to the lot size and an acceptance number of zero. This is 100 % inspection, but it should be noted that it is not the traditional 100 % sorting. There is no attempt here to sort the articles into the conforming and the nonconforming but an attempt is made to check that there are no bad ones. If a critical nonconformity is found, this does not merely mean that it is put into a different box and the inspection continues; it means that the whole lot is not accepted (although non-acceptance does not necessarily mean scrapping). Whenever possible, it should also mean that production is stopped while a thorough investigation takes place to attempt to discover how the nonconformity arose and to devise methods to prevent another occurrence. The reason for this procedure is to try to prevent the production of items with serious nonconformities and to avoid giving the producer the impression that it does not matter too much if some of these are produced, as the inspector will sort them out. Even the best inspector might occasionally fail to notice nonconformity, so it is only by preventing critical nonconformities from being made that it can be ensured that none get through to the customer.

If it is thought that any particular critical nonconformity does not warrant this procedure, then serious consideration should be given to having it reclassified as a major nonconformity. Critical nonconformities really have to be critical; then no amount of effort is too great.

Where the only possible inspection for critical nonconformities is destructive, the search for ways of preventing them from ever arising at all is even more important. In this case, we cannot have a sample that is 100 % of

the lot, and it is necessary to decide what size of sample should be taken. This can be done for sampling by attributes using a simple formula relating:

a) the number of nonconformities/nonconforming items for which, if they were present, we would wish to be almost certain of finding at least one nonconformity/nonconforming item in the sample;

b) the lot size;

c) the sample size;

d) the risk we are prepared to take of failing to find a nonconformity/nonconforming item.

The sample size, n, is obtained using the following formula and then rounding up to the nearest integer [1]:

$$n = \left(N - \frac{d}{2} \right) \left(1 - \beta^{1/(d+1)} \right) \quad (1)$$

where

N is the lot size;

β is the specified probability of failing to find at least one critical nonconformity;

d is the maximum number of critically nonconforming items 'allowed' in the lot.

NOTE If p is the maximum fraction nonconforming specified for the lot, then $d = Np$ rounded down to the nearest integer [2].

The lot is acceptable if no critical nonconformities are found in the sample.

8.2.4.2 Example 4

Suppose that there is a lot of 3 454 items. A probability, β, of 0,001 and a maximum percentage of 0,2 % critically nonconforming items are stipulated.

Then $p = 0{,}2/100 = 0{,}002$ and $Np = 3\,454 \times 0{,}002 = 6{,}908$, which is rounded down to give $d = 6$.

NOTE The rounding here is down because rounding up would result in $d = 7$, i.e. a percent nonconforming of $100 \times 7/3\,454 = 0{,}2027$ %, which is in excess of the 0,2 % stipulated.

Thus, $(N - d/2)(1 - \beta^{1/(d+1)}) = (3\,454 - 3)(1 - 0{,}001^{1/7}) = 3\,451 \times 0{,}627\,24 = 2\,164{,}61$, which is rounded up to give $n = 2\,165$.

The sampling plan is:

— sample size $n = 2\,165$;

— acceptance number Ac = 0 nonconforming items;

— rejection number Re = 1 nonconforming item.

---

1) This approximation is accurate enough for most practical purposes in acceptance sampling. In rare cases it will give a result that is one unit larger than necessary.

2) Only small values of percent nonconforming should be considered tolerable, as the nonconformities are critical.

NOTE The very large sample size is due to the requirement of high confidence in a low fraction of critically nonconforming items.

To find the lot size, N, needed to yield a specified number of items, L, after destruction of the sample of n items under test, assuming no nonconforming items are found, then for given values of the probability β and the number of nonconforming items in the lot, the lot size is:

$$N = (L - d/2)/\beta^{1/(d+1)} + d/2$$

rounded upwards to an integer.

8.2.4.3 Example 5

If 1 500 items are required after testing the sample, using $\beta = 0{,}001$ and $d = 6$ as in Example 4, then L is 1 500 and the lot size is (1 500 − 6/2)/0,0011/7 + 6/2 = 1497/0,372 76 + 3 = 4 018,99, which is rounded up to give $N = 4\ 019$.

(It follows that $n = N - L = 4\ 019 - 1\ 500 = 2\ 519$. This value of n is also obtained using Equation (1) with a lot size of 4 019.)

8.2.4.4 Comments

If the initial calculation yields an impracticable sample or lot size, then the risk (probability) and/or the possible number of nonconformities/nonconforming items in the lot need(s) to be reassessed and new criteria established.

An alternative sampling plan for critical nonconformities, where the critical characteristic is something that can be measured rather than a pure attribute, is to sample with a safety margin. Thus, if the minimum allowable breaking load for some component is 2 000 kg, it might be possible, instead of agreeing that the limit is 2 000 kg and the nonconformity is critical, to agree that the limit is 2 500 kg and the nonconformity is major. Just where the limits should be set, and what sampling plan is allowable, depends upon some past knowledge of the amount of variability observed in the strength of the components in question. When this approach is possible, it can give much more satisfactory results for all concerned than does 100 % inspection. In this case, there is the possibility of sampling by variables (ISO 3951), which will allow over-stress testing and yield information on the average and the variability of the characteristic.

8.3 The operating characteristic (OC) curve

8.3.1 General

The operating characteristic curve is a curve that shows what any particular sampling plan can be expected to do in terms of accepting and not accepting lots; that is, it is a sort of “efficiency curve”. An OC curve refers to a particular sampling plan. Each possible plan has its own curve.

8.3.2 OC curves for sampling by attributes

In acceptance sampling by attributes, there are two “types” of OC curve, known as Type A and Type B. Taking first the general case of a long production run with a stable process average quality (100p % nonconforming, where p lies in the range 0 to 1), the quality of the lots taken from the run will vary about this process average in accordance with a binomial distribution. For each variation in lot quality, the corresponding ordinate of the OC curve gives the proportion of lots (of that particular quality) that, on average, will be accepted by the sampling plan on which the OC curve is based. The OC curve in this case is said to be of Type B and describes how a user would view the operating characteristic of a sampling plan in respect of a steady supply of product from a given source.

In the case of isolated or individual lots, the OC curve is really a series of distinct points at quality levels 0, 1/N, 2/N, . . . rather than a curve. Because of the isolated nature of the lot or lots, it might not seem reasonable to interpret the ordinates of the OC curve as long-run average proportions of accepted lots. However, such an

interpretation is possible if we imagine a fictitious process producing a series of identical lots, i.e. lots that are all of exactly the same size and quality (100p % nonconforming). The ordinate of the OC curve is then, again, the proportion of those identical lots that will be accepted by the given sampling plan. However, in this case we are not sampling from a process with random variations in quality but from a finite number of items making up one lot. The ordinates of the OC curve indicate probabilities of acceptance (rather than average proportions of lots accepted), which are given by the hypergeometric distribution and depend on the lot size. The OC curve is said to be of Type A and describes how a user would view the operating characteristic in the case of isolated or individual lots.

Although the two types of OC curves are determined by different probability distributions, the Type B curve serves both purposes. This is because it can be taken as a good approximation to the Type A curve when the lot size is sufficiently large, say, 10 or more times the size of the sample, although it should be kept in mind that the quality is that of the isolated lot and not that of the production. If the sample size comprises a greater proportion of the lot and acceptance numbers are positive integers (as opposed to zero), the Type B curve (as an approximation to Type A) gives a pessimistic indication of the producer's and customer's risks, i.e. it errs "on the safe side". For large lots, the Type A and Type B curves are virtually identical. Thus, for practical purposes, Type B curves can be used for both types of sampling without significant error in most cases. The OC curves for acceptance sampling plans for percent nonconforming given in ISO 2859 and ISO 3951 are of Type B.

ISO 2859-1 presents operating characteristic curves of sampling inspection plans for percent nonconforming and for nonconformities per hundred items. These operating characteristic (OC) curves show the average percentage of lots accepted as an ordinate plotted against the percent nonconforming or the number of nonconformities per hundred items in the process quality as the abscissa. For percent nonconforming, they have been calculated based on the binomial distribution when the single sample size is equal to or less than 80. For nonconformities per 100 items, the Poisson distribution is appropriate and has been used when calculating the OC curves for these sampling plans.

The Poisson distribution is based on the assumption that nonconformities occur independently with constant expectation. This assumption holds in many cases. Any substantial departure from this assumption yields distributions with greater variance than that of the Poisson distribution. In these cases, the consumer's protection is somewhat better than that indicated by the operating characteristic curves.

In ISO 2859-2, the tables for Procedure A (i.e. for lots in isolation) are based on random sampling from finite lots for both the customer's and the producer's risk. However, for Procedure B, the tables are based on random sampling from a finite lot for the consumer's risk at the LQ, but on random sampling from a process for the producer's risk and the OC curves. The operating characteristic curve understates the probability of acceptance where it is indicated to be greater than 0,90 and overstates the probability of acceptance where it is indicated to be less than 0,90.

8.3.3 OC curves for sampling by variables

Standards for sampling by variables are based on the assumption that the quality characteristics are normally distributed, or have distributions that can be transformed to normality. This assumption will be unverifiable for isolated lots or short runs. Besides, the measurements of characteristics on a lot of finite size can never be considered to represent a true normal distribution. On the other hand, it is quite possible that the production process at the time the lot was being produced could have been producing items whose quality characteristics were normally distributed or transformable to normality. For these reasons, only Type B OC curves apply to sampling by variables.

For sampling by variables, the operating characteristic curves are matched to those for the attribute sampling plans for similar lot sizes and quality levels. Whereas the Type B OC curves for sampling by attributes involve the binomial distribution, those for sampling by variables involve:

a) the non-central t-distribution for cases where the process standard deviations are unknown;

b) the normal distribution in cases where the process standard deviations are known.

The acceptability decisions are based on an assessment of the percent nonconforming determined from the means and standard deviations of the measurements of the product characteristics on all items in the sample.

The OC curves for sampling by variables show the average percentage of lots accepted, but do not show probabilities of acceptance of particular lots. For a particular lot, it may happen that a rejected lot may be free of nonconforming items. Conversely, an individual lot with a given high fraction of nonconforming items may have a smaller actual probability of non-acceptance than is shown by the OC curve for the whole process.

8.4 Sampling risks

8.4.1 Risks when sampling: producer's risk and consumer's risk

Because samples constitute only a small part of the whole of an inspection lot or consignment, sampling involves risks for both the producer and the consumer. Occasionally a "good" lot might not be accepted because the sample inspected, though randomly selected, does not reflect the true quality of the lot. The risk of this happening is known as the "producer's risk" (PR). Conversely, a "poor quality" lot might pass inspection because of the limited data available in the sample. This eventuality is known as the "consumer's risk" (CR).

Subclause 8.3 stated that the risks associated with sampling can be calculated and assessed. Using its operating characteristic curve, for each sampling plan it is possible to read off the proportion of lots that will be accepted for a given input (or process) quality, i.e. the probability of acceptance for a stated quality level.

The producer would require a high probability of acceptance if the quality were good, while the customer would want a low probability of acceptance if the quality were poor. Conventionally, these probabilities have been set at 0,95 and 0,10 respectively. This gives a PR of non-acceptance of 0,05, or 5 %, and a CR of accepting poor quality of 0,10, or 10 %. It is becoming increasingly common practice to make both the PR and the CR equal to 5 %. For predetermined PR and CR percentages, the corresponding producer's risk quality (PRQ) and consumer's risk quality (CRQ) can be read from the OC curve (see Figure 1). Conversely, for a given OC curve the AQL and the limiting quality level (LQL) determine the PR and the CR respectively (see Figure 2). Alternatively, the sampling plan and its OC curve can be specially designed to "fit" the pre-selected producer's risk point (AQL, 1,0 − PR)[3] and consumer's risk point (LQL, CR).

An examination of the OC curves for single sampling plans indexed by AQL under normal inspection, for example sampling by attributes as specified in ISO 2859-1, will show that at the designated AQL the probability of acceptance varies from approximately 0,87 to 0,99 (i.e. the PR varies from 13 % to 1 %). This is a feature of these AQL sampling plans and, accordingly, of any plans designed to have characteristics "matching" those of these AQL-indexed single sampling plans. The term "AQL" should not be used without reference to one or another of the standards in the ISO 2859 or ISO 3951 series, or equivalent standards. The OC charts and the tables in these standards also show the effect of moving to tightened inspection: the PR increases for the same AQL whereas the CR decreases for the same LQ.

When a sampling system is operated, the switching rules are an important factor in considering the risks due to sampling. For example, the OC curves in ISO 2859-1 show what to expect under normal inspection. They show that for all the sampling plans specified in that standard, the percentage of lots likely to be accepted if the process quality is running at twice the AQL is less than 80 %. Before long, such an acceptance rate will lead to a switch to tightened inspection. The rate of acceptance at the AQL under tightened inspection will be only of the order of 80 % and at twice the AQL it will drop to approximately 50 %, and much less in a number of cases. These low acceptance rates under tightened inspection should prompt investigation into the cause of the inferior quality. The rule for discontinuation of sampling inspection ultimately makes this investigation a necessity. The remedial action taken will result in a return to the previous quality level or, as happens often, to an improved quality.

CAUTION — Although OC curves are a very useful concept, not only in risk analysis, in practice lots in a series are rarely, if ever, identical and operating processes are rarely strictly random. While the curves indicate what to expect under the stated conditions, they cannot accurately describe what

3) Alternatively expressed as (AQL, 100 % - PR[%]).

happens in a period when conditions are constantly changing. Therefore, one has to be wary of making dogmatic assertions.

8.4.2 Methods for reducing the risks

Risks in sampling inspection, in both the acceptance of bad lots and the non-acceptance of good lots, are unavoidable, but these risks should be tolerable if the AQL and inspection level have been well chosen.

If either the producer or the customer considers in a particular instance that the risk they are taking is too high, it is recommended to check that the AQL and the inspection level have been well chosen.

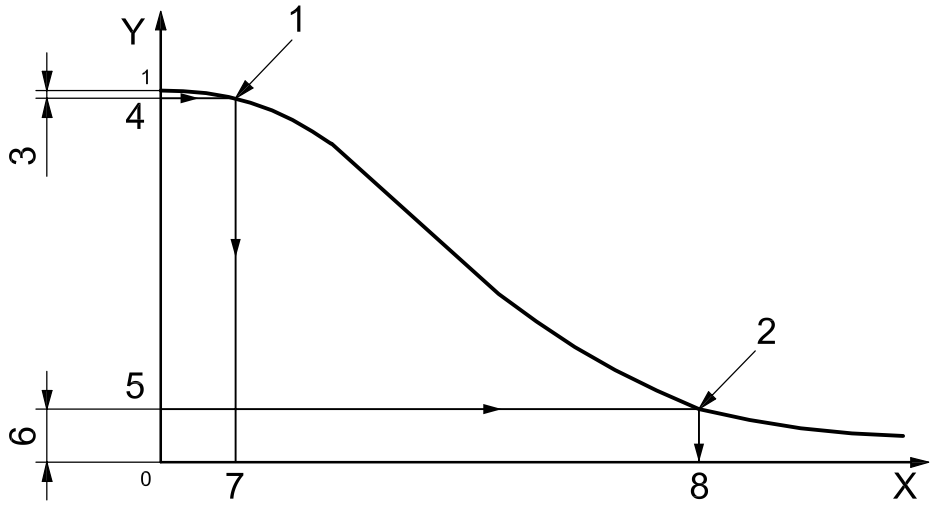
The producer will be interested in reducing risks when quality is better than the AQL - they are not entitled to any reduction of risk otherwise. The customer will be particularly interested in the risks when quality is worse than the AQL as, if quality is better than the AQL, they are getting the quality required.

Methods of reducing the risks for both parties are provided in ISO/TR 8550-2 for sampling by attributes and in ISO/TR 8550-3 for sampling by variables. In summary, they are, where possible:

a) to improve the quality of production;

b) to increase the lot size;

c) in the case of sampling by attributes where the acceptance number is zero, to increase the acceptance number to 1 while retaining the same AQL.

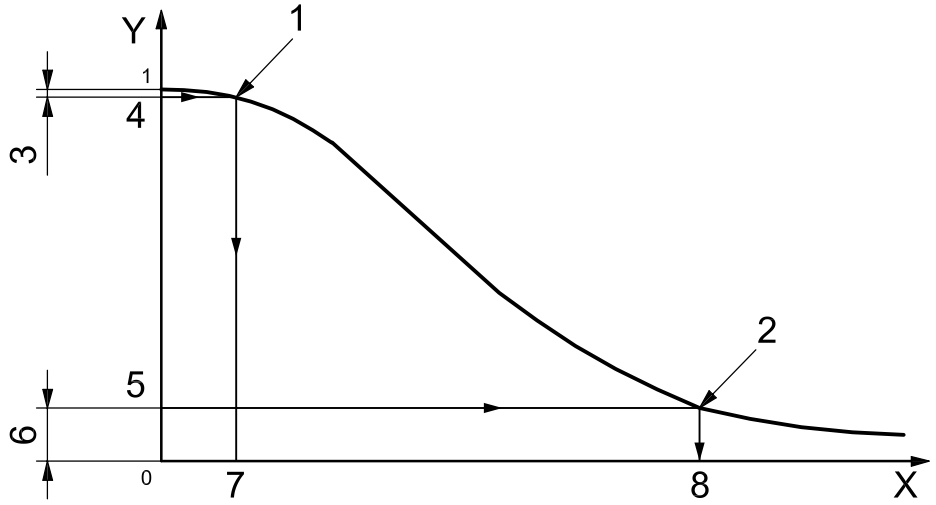


Key

X process quality (decreasing)
Y probability of acceptance

1 producer's risk point (PRP)
2 consumer's risk point (CRP)
3 producer's risk (PR)
4 high probability of acceptance
5 low probability of acceptance
6 consumer's risk (CR)
7 producer's risk quality (PRQ)
8 consumer's risk quality (CRQ)

Figure 1 — Operating characteristic curve defined by producer's risk (PR) and consumer's risk (CR)



Key

X process quality (decreasing)
Y probability of acceptance

1 producer's risk point (PRP)
2 consumer's risk point (CRP)
3 producer's risk (PR)
4 high probability of acceptance
5 low probability of acceptance
6 consumer's risk (CR)
7 acceptance quality limit (AQL)
8 limiting quality level (LQ)

Figure 2 — Operating characteristic curve defined by acceptance quality limit (AQL) and limiting quality (LQ)

8.5 Selecting the AQL, PRQ, LQ and CRQ values

8.5.1 The AQL and PRQ

8.5.1.1 Meaning of AQL and PRQ

For the purpose of this part of ISO/TR 8550, the AQL and the PRQ can be deemed synonymous. They are both indices of what quality can be tolerated for the purposes of sampling inspection, the difference being that the PRQ is associated with a specified small PR whereas the AQL denotes a quality level for which the (unspecified) PR will be small.

8.5.1.2 Setting an AQL

In setting an AQL, it has to be remembered that the AQL provides an indication of the quality that is required in production. The supplier is being asked to produce lots of an average quality better than the AQL. On the one hand, this quality has to be reasonably attainable, whilst on the other hand, it has to be a reasonable quality from the customer's point of view. Frequently, this will mean a compromise between the quality the customer would like and the quality he can afford, for the tighter the requirement the more difficult it might be for the production to meet it, and the more expensive might be the inspection to ensure that it is met.

The primary consideration has to be the customer's stipulation but it is necessary to make sure that the customer is being realistic and is not demanding better quality than is really needed. It is necessary to take into account both how the items in question are to be used and the consequences of a failure. If the items are to be available in large numbers and the failure is simply a failure to assemble so that the nonconforming item

can be put aside and another used in its place, a relatively generous AQL might be tolerable. If, on the other hand, a failure is going to cause a failure to function of an expensive and important piece of equipment at a time and place where a replacement of the nonconforming item cannot be made, a tighter AQL will be required.

It is also necessary to consider how many components will be contained in the eventual equipment. If, for example, it is decided that the quality of a piece of equipment containing three different but equally important independent components should be not more than 10 % nonconforming, then each of the three components could be 3,45 % nonconforming and the requirement would be met. If, however, the machinery were to contain ten components, these would have to be no worse than 1,04 % nonconforming on average.

It should be remembered that even if components are sampled to a single AQL, they should come from a source or sources with a smaller process average to increase the probability of lot acceptance.

If the components conform or fail to conform independently, it follows from the multiplication law of probabilities that the overall fraction conforming is:

$$1-\frac{X}{100}=\left(1-\frac{x}{100}\right)^{k},$$

from which it follows that the overall fraction nonconforming is:

$$\frac{X}{100}=1-\left(1-\frac{x}{100}\right)^{k},$$

where

k is the number of components in the assembly;

X is the percent nonconforming of the assembly;

x is the percent nonconforming of each of the components.

An AQL of X for the assembly thus translates into an AQL of x for the individual components.

NOTE 1 The value of X, however, does not take into account nonconformities that might arise through a faulty assembly process.

NOTE 2 The AQL is the limit of a satisfactory process average. It is expected that the supplier's quality will be better than the AQL.

The above formula applies when the components are, to all intents and purposes, identical. More generally, if the percent nonconforming of the k components are respectively x_1, x_2, . . . x_k, then, if the components conform or fail to conform independently, it follows that:

$$\frac{X}{100}=1-\left(1-\frac{x_1}{100}\right)\left(1-\frac{x_2}{100}\right)\ldots\left(1-\frac{x_k}{100}\right).$$

Here, there will be different requirements for the different components, contributing to the overall requirement X. In apportioning X into the quality levels x_1, x_2, . . . x_k, the product designer should always be consulted for advice, if possible.

In these circumstances, the producer would probably wish to choose a suitable AQL for each component and then calculate what quality can be expected in the overall equipment, whereas the consumer would wish to stipulate an AQL for the overall equipment and then calculate what has to be the quality for the components. In general, the second of these approaches is probably the more reasonable in that it is the performance of the overall equipment that matters, but it is also the more expensive approach because it usually leads to smaller AQLs. However, it has to be accepted that good quality in a complicated article is inevitably more expensive than equally good quality in a simple one.

The question of what quality level can reasonably be expected, at the price the customer is prepared to pay and with the methods of production envisaged, can often be answered by an examination of what quality level has been produced and tolerated in the past. When the article is new, i.e. there has been no past production, there will often be other similar articles from which relevant information can be obtained. Calculations of the past process average might be particularly helpful. This idea of looking at the quality obtained in the past should not be taken as meaning that past quality levels are always good enough. The production cost of a nonconforming item is nearly equal to that of a conforming item, so that a reduction of percent nonconforming frequently results in a reduction of production cost.

Although it is a common belief that better quality costs more, the reverse is often true. Effective process control can provide extremely good quality, even to the extent of parts per million. Quality achieved in this manner is much less costly than the poorer quality that results when process control is non-existent. The main reason for this is that there is virtually no scrap, rework or returned material.

Attempting to achieve excellent quality by inspection and sorting is much more expensive and, due to the inefficiency of inspection, not highly effective. Therefore, it becomes necessary that attempts to achieve the quality of fractional AQL, or parts per million, be accompanied by a review of process control procedures and the design of the product, rather than attempting to achieve such results by inspection.

8.5.1.3 The effect of sampling variability

Merely setting an AQL does not give the customer a guarantee that lots of a poorer quality will not be accepted. In the first place, the AQL refers to the average. Some lots might be worse than the AQL while the average is better than the AQL. Secondly, if the average quality being offered is a little worse than the AQL, a number of lots will probably be accepted before a switch to tightened inspection is called for, and even after the switch, there is likely still to be some acceptance. In general, however, it can be expected that the customer will receive a product with an average that is better than the AQL, as sampling schemes have the built-in economic incentive that a producer cannot afford to have more than a small proportion of lots not accepted. He will therefore take steps to improve the quality of production if this proportion is exceeded.

It might be thought that this is not very satisfactory from the customer's point of view, relying as it does upon what is likely to happen rather than upon what will certainly happen. However, in practice, most producers take steps to see that their process average does not exceed the AQL, if only because relatively frequent lot non-acceptance makes life difficult. In any case, the customer's protection depends upon the lower end of the OC curve as well as upon the upper end with which the AQL is concerned, and this lower end can be adjusted by considering the LQ values of any suggested sampling plan.

If it is decided that this approach is not adequate for a particular product and more positive customer protection is needed, then this can always be attained by stipulating a smaller AQL, bearing in mind that this could increase the cost of the product. Usually, however, a proper approach to improved quality and less nonconformity will result in lower rather than higher costs. Even if there is extra cost, it will often be worthwhile in terms of keeping or expanding the customer base.

8.5.1.4 AQL as an index

It is not necessary that the AQL be the primary choice from which all else is derived. It is always possible, when circumstances so require, to enter sampling tables "through the back door", choosing a plan by some other criterion and then finding the AQL specified to get the desired result. In this case, the AQL is a convenient index to enable the standard tables to be used, and this permits the producer to determine the quality at which he has to manufacture to get most lots accepted.

If such a "back-door" method is used, the primary choice might either be a low point on the curve, where this is thought particularly important, or some economic criterion. Probably the simplest economic criterion that has been suggested is to make an estimate of the break-even point, i.e. the lot quality such that, if the lot were accepted, the cost of the damage done by the accepted nonconforming items would be the same as the cost of failing to accept the lot.

8.5.1.5 Break-even quality

If this break-even, or indifference, quality can be estimated, it would be sensible to choose a sampling plan such that, at this quality, 50 % of lots are expected to be accepted. This is not because a 50 % chance of acceptance with this quality is particularly wanted. By definition, if this particular quality is offered, it is not of much interest what the inspection plan does with it. Rather, it is because such a plan ensures a greater than 50 % chance of acceptance for better quality than break-even quality, and a greater than 50 % chance of non-acceptance for quality worse than break-even quality.

8.5.1.6 Trading off the AQL and the tolerances

Sometimes it is suggested that there could be a "trade-off" between AQL and tolerances. For example, a dimension is specified with a tolerance of $\pm$ 0,6 mm and an AQL of 0,1 % selected. If the measurement of this dimension is approximately normally distributed, a tolerance of $\pm$ 0,3 mm with an AQL of 10 % will give much the same result. This is based on the relative areas in the tails of the distribution against the measurement scale. Adoption of this modification has the merit that 10 % nonconforming is much easier to detect, leading to a big reduction in sample size. However, there are penalties associated with these extremes.

For example, using ISO 2859-1 and a moderate inspection level, an AQL of 0,1 % implies that the lot size has to exceed 10 000, whereas for an AQL of 10 % the lot size can range from as low as 30. A reduction in tolerance imposes unreal pressure on the inspection function as inspection will be required to record as nonconforming dimensions actually conforming to the stipulated dimensional tolerance. Worse still is the situation in which a lot has been sentenced as "non-acceptable" and has to be screened when, in fact, most of the product is within specifications. Under these circumstances, it would be tempting, but wrong, to reassess the offending sample measurements based on the product specification tolerance. These uses of compressed or narrowed "tolerance" limits for gauging conformity are more appropriate for process control or internal acceptance practices. They are not appropriate for specifications.

8.5.1.7 The need for realism

This example indicates the need for setting realistic specifications and quality levels for sampling purposes, which brings us back to the nonconformity classification and the question of just how serious are departures from conformity. The specifications should be realistic and the quality levels should be consistent and properly reflect the nonconformity classification and its relative importance.

Among the most common bases for setting the AQL are the following:

a) **Historical data**: Past data are used to estimate the process average. The AQL is then set at, or close to, the estimate obtained.

b) **Empirical judgement**: The AQL is set relative to a known satisfactory level for a similar item.

c) **Engineering judgement**: The AQL is based on "engineering" estimates of the quality requirements for function, performance, life, interchangeability, etc.

d) **Experimental**: AQLs are set tentatively and subsequently adjusted by performance and experience.

e) **Minimum total cost**: The AQL is based on an analysis of the "cost of quality" versus the "cost of not having quality".

f) **Knowledge of the product and of the supplier**: The AQL is based on "experience".

These bases may be used singly or in combination.

8.5.1.8 Preferred AQLs

Finally, having taken all of these factors into account, it is desirable to choose one of the AQL values given in the tables to be used if possible, as the tables are otherwise inapplicable and a special sampling plan or

scheme would have to be designed. The preferred AQL values approximately form a geometric progression with a common ratio of about 1,6, so it should be a rare occurrence to find none of them suitable.

8.5.2 The LQ and CRQ

In analogy with the AQL and the PRQ, the LQ and the CRQ can be considered equivalent indices whose stipulated values express, for sampling purposes, a level of "objectionable" quality that has only a small chance of acceptance.

The process of setting these quality levels is similar to that for the AQL, except that we are now considering an intolerable level that, if it were to exist, would cause operational problems, additional costs, etc. The levels will be associated with the chosen level of consumer's risk and should be decided accordingly.

8.6 Inspection level (IL) - Sample size/lot size relation

8.6.1 The inspection level

The inspection level is an index of the relative amount of inspection used in a sampling scheme, and relates the sample size to the lot size and hence to the discrimination afforded between "good" and "poor" quality. For example, ISO 2859-1 provides seven inspection levels, whereas ISO 3951-1 provides five.

Having set the AQL (or PRQ), the "inspection level" is determined in ISO 2859-1:1999, Table 1, or ISO 3951-1:2005, Table 1, or, more generally, by considering what quality should have only a small chance of acceptance if an occasional lot of that quality is submitted for inspection, i.e. the LQ or the CRQ. The sampling plan OC curves, in the basic acceptance sampling standards (ISO 2859-1 or ISO 3951-1), are then surveyed to ascertain which sampling plan most closely meets the joint requirement, and its sample size code letter is noted. Table 1 of the applicable standard (ISO 2859-1 or ISO 3951-1) gives the range of lot sizes appropriate to this code letter for different inspection levels and indicates the preferred lot size range to be used. Inspection level II is considered to be appropriate for many applications.

NOTE 1 ISO 2859-5:2005 and ISO 3951-5:2006 provide sequential sampling plans corresponding to the basic acceptance sampling standards ISO 2859-1 and ISO 3951-1 respectively, in which inspection levels apply. On the other hand, ISO 8422:2006 (see ISO/TR 8550-2) and ISO 8423:— [4] (see ISO/TR 8550-3) provide sequential sampling plans indexed by PRP and CRP where, therefore, inspection level does not apply.

NOTE 2 It is always possible to deal with "special" cases by assigning a constant code letter irrespective of the lot size, for example if a definite OC curve were required. Although the design of "special plans" for a particular case is outside the scope of this guide, it should be possible to avoid this recourse in the vast majority of cases.

NOTE 3 It is possible that a "low" inspection level, such as S-1, might have to be used for economic reasons or because the tests are destructive. In these cases, the discrimination might suffer. However, if records are kept for a continuing series of lots the cumulative sample might show that the CR is more acceptable.

NOTE 4 Procedure A in ISO 2859-2 does not index the sample plans by inspection level because this procedure is for use with isolated lots for which both the producer and the consumer are concerned with the LQ.

If no OC curve can be found that meets the required AQL and LQ values, it is necessary to establish whether the requirement is essential. If the CRQ (or LQ) is not relaxed, the only course is to tighten the AQL, with a consequent increase in producer's risk.

If, having established a sample size code letter, it is found that none of the inspection levels includes that code letter for the expected lot sizes, it will be necessary to review and amend the requirement or, failing this, to specify the sample size code letter without an inspection level. If the latter results in a sample size exceeding the lot size, then 100 % inspection is called for.

4) To be published. (Revision of ISO 8423:1991)

8.6.2 Comment on sample size/lot size relation

There is no simple mathematical relationship between the sample size and the lot size. The reason most sampling schemes relate sample size to lot size is the accepted principle that more evidence should be obtained in order to determine the acceptability of larger lots, as more is at stake. Provided the sample size is small compared with the lot size, for practical purposes the lot size can be ignored in developing the OC curve for a sampling plan. The choice of a sampling plan and the sample size depends on the homogeneity of the lot. Larger samples are needed when there is a lack of homogeneity. AQL plans require that lots be homogeneous. LQ plans require larger samples. For inspection with AQL-indexed schemes, lots should be formed under essentially similar conditions and should not be “mixed”. Although the effect of lot size is greater for small lots, the absolute sample size is more important than its proportion of the lot.

From the point of view of sampling inspection, there is an advantage in large lots, provided homogeneity is maintained, as it is then possible and economical to take a large sample while maintaining a large lot-to-sample ratio, thereby achieving better discrimination. Furthermore, for a given efficiency, the sample size will not increase as rapidly as the lot size and will not increase at all after a certain lot size. However, there are a number of reasons for limiting the lot size:

a) the formation of larger lots might result in the inclusion of widely varying quality;

b) the production or supply rate might be too low to permit the formation of large lots;

c) storage and handling might preclude the formation of large lots;

d) accessibility for drawing random samples is difficult with large lots;

e) the economic consequence of non-acceptance of a large lot in terms of scrap, rework and further inspection might be large.

8.7 Rectifying inspection for lot-by-lot sampling – AOQL

When all items in a lot have the same probability of being nonconforming, a lot acceptance sampling plan has no direct effect on the quality of the lots that are being inspected except for the replacement of the nonconforming items found in the sample. If 3 % of the product is nonconforming prior to submission to sampling inspection, approximately 3 % will be nonconforming after sampling inspection. The primary effects of a sampling plan on product quality are indirect and stem from the effects that the non-acceptance of lots has on subsequent production, e.g. the attitude of the operators and the steps taken by management to improve the process.

Rectifying inspection for lot-by-lot sampling is a combination of sampling inspection and 100 % inspection (or “screening”) that seeks directly to control the quality of product that passes through inspection. All lots are sample inspected but lots that are not accepted under sampling inspection may be inspected 100 %. Any nonconforming items that are found are discarded and may be replaced by conforming items. The average outgoing quality (AOQ) that results from this rectifying inspection will be better than the incoming quality. It is emphasized that the AOQ is an *average* over many lots and does not pertain to individual lots.

When incoming quality is perfect, no rectification will be necessary and the outgoing quality will be perfect. When incoming quality is very poor, screening will be applied to all lots and therefore the outgoing quality will theoretically again be perfect, albeit at a huge cost in inspection. As the incoming quality level, measured in percent nonconforming, rises from 0 % to 100 %, the AOQ rises from zero, reaches an upper limit, and then decreases to zero again. The average outgoing quality limit (AOQL) is this upper limit of the AOQ.

While rectifying inspection undoubtedly puts some pressure on the supplier to improve the quality (assuming he is carrying out the inspection), the requirements for assessing the effect are somewhat idealistic. The calculation of the AOQ assumes 100 % perfect sorting and, in some cases, the replacement of nonconforming product with good product (new or reworked). As perfection is unlikely in practice, the results of the screening process and the calculation should be viewed with some caution, as the calculated result could be quite optimistic.

A major problem with rectifying inspection is that the supplier might be inclined to rely on the rectification, especially if it is carried out by the purchaser, and might not diligently attempt to improve the process quality.

Another problem associated with this kind of inspection is the pressure and cost imposed on the inspection department. Whenever a lot is sentenced as unacceptable, the increased workload can be considerable and so demanding in manpower and equipment resources that "standards" and throughput can be difficult to maintain. This might adversely affect the 100 % screening efficiency, and very likely affect the quality of "original" inspection. This situation could create a conscious or subconscious temptation to cut corners, with a consequent degeneration of quality. However, the AOQL can be a useful concept in assessing the effect of rectifying inspection on outgoing quality, as well as providing an incentive to produce good quality in the first place.

Not all AOQL-indexed systems suffer from this disadvantage. ISO 18414 is an AOQL-indexed acceptance sampling system for inspection by attributes that only requires 100 % inspection of a non-acceptable lot when the lot is the first in the series or when the previous lot is non-acceptable. Thus, by only submitting a large lot when the previous lot has been accepted, 100 % inspection of large lots can be avoided.

The AOQL can also be used as a basis for comparing and selecting a sampling plan for an application. Where AOQL factors are not given in a standard, and where 100 % inspection and replacement of nonconforming items with conforming items is mandatory for all non-accepted lots, the AOQL can be calculated from AOQ values using the approximation:

$$\mathrm{AOQ} = P_{\mathrm{a}}(p) \times 100p,$$

where

p is the fraction nonconforming;

$P_{\mathrm{a}}(p)$ is the probability of acceptance at quality level p.

From this formula, the AOQ, and subsequently the AOQL, can be obtained in percent nonconforming. In fact, this approximation could be a better indication than the more detailed calculation when rectification is not applied efficiently.

9 Making a comparison of the methods for sampling inspection

9.1 Use of OC curves for comparing sampling plans

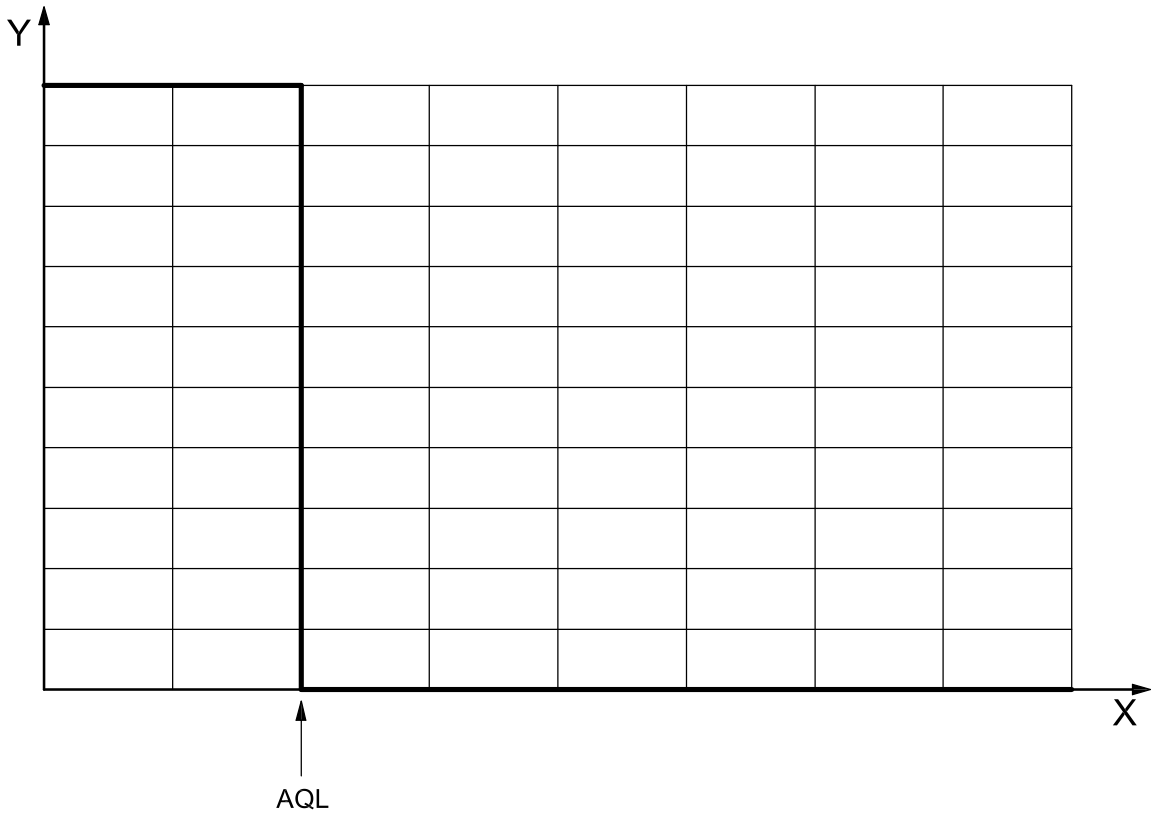
OC curves provide an excellent basis for making direct comparisons between sampling plans because each sampling plan has its own OC curve. Alternatively, some comparison can be made using factors derived from the OC curve, e.g. the AOQL (see 8.7), the discrimination ratio (see 9.2) or the average sample size (see 9.3.2).

9.2 The discrimination ratio (DR)

A perfect sampling inspection plan would give absolute discrimination at a percent nonconforming equal to the acceptance quality limit. The OC curve would be a straight vertical line at the AQL. For qualities of a lower percent nonconforming than the AQL, there would be a region of total acceptance, and for qualities of a higher nonconforming percent than the AQL, there would be a region of total non-acceptance. Figure 3 shows an ideal OC curve. This ideal is unattainable with much less than perfect 100 % inspection.

Once a method of *sampling* inspection is introduced, the ability to discriminate absolutely between "acceptable" and "unacceptable" quality is lost. The penalty for not inspecting every item is reflected in the slope of the OC curve, in particular the absolute value of the slope of that part of the curve lying between the PRP and the CRP, which is known as the discrimination ratio (DR). The more vertical is this slope, the greater is the discrimination of the plan. Thus, the comparison of the slope over this part of the OC curves provides a direct comparison of the effectiveness of the sampling plans in terms of discrimination.

When OC curves are on different pages in a publication, or to a different scale, it is not so easy to compare them directly and the following numerical alternative can then be useful. This method utilizes the slope of the straight line joining the PRP and the CRP. The straight line is a satisfactory approximation of that part of the OC curve.



Key

X process percent nonconforming

Y percentage of lots expected to be accepted

Figure 3 — Ideal operating characteristic curve

The factor (CRQ)/(PRQ) gives this slope as a numerical index of the discrimination ratio. The larger this number, the less discriminatory is the sampling plan. The index can be used for plan selection as well as for comparison, or for any other purpose based on the OC curves.

However, sampling plans with the same discrimination ratio are not necessarily equivalent. For example, using ISO 2859-1, the OC curve for the plan with AQL 1 % and code letter K has a similar discrimination ratio to the curve for the plan with AQL 4 % and code letter M, but the CR and PR points are quite different. In the most frequent application of this method, curves with similar PR points and quality level are compared for their degree of discrimination against poor quality, i.e. the variation in consumer's risk.

9.3 Comparison of single, double, multiple and sequential sampling

9.3.1 Equivalent plans

9.3.1.1 Characteristics to be taken into account

For most single sampling plans, it is possible to find a double, multiple or sequential sampling plan with an operating characteristic curve close to that of the single sampling plan. In such cases, there is no reason to

choose between single, double, multiple or sequential sampling because of differences between the operating characteristic curves. Neither is there reason to prefer one to another for all possible situations. The balance of advantages and disadvantages sometimes favours one or sometimes another of the sampling procedures. The characteristics that should be taken into account are as follows.

a) **Complexity**. Single sampling is the easiest to describe and administer. Double sampling requires more administration to arrange for the second sample to be made available when required. Multiple and sequential sampling are obviously even more complicated. Sometimes the attraction of simplicity is the major consideration in the selection of the sampling plan. On the other hand, there will be apparently marginal cases when the psychological attraction of being able to take a second sample will favour double sampling plans.

b) **Variability in the amount of sampling inspection**. In single sampling, the sample size is fixed and the amount of inspection effort required to reach a decision is known in advance. For the other types of sampling, the number of items tested varies according to the results from the early samples. It is possible to calculate an average amount of sampling inspection and the average cost of inspection for any given input quality. This varies with the quality, being least for both very good and very poor quality. In addition to the uncertainty associated with the unknown input quality, but also even when the input quality is known, there is the uncertainty due to the variation of the amount of sampling inspection in this average. This uncertainty can lead to problems in arranging for sufficient resources to be made available for the inspection required. If insufficient resources are available, the result is delayed. In the contrary case, there will be inefficient use of resources. In some situations, the variable inspection load will often be considered a small price to pay for the significant reduction in the average total inspection cost.

c) **Difficulty of drawing sample items**. Sometimes it is easy to draw a second sample and drawing two samples is no more trouble than drawing one sample of the combined size. At other times however, the situation arises where the drawing of sample items forms a large part of the inspection task and here, having disturbed the lot to draw one sample, it is hardly feasible to disturb it again to draw another sample. In these cases, single sampling is usually the best plan. There is, of course, the alternative possibility of drawing a sample of the maximum size that could be needed and then inspecting according to the pre-selected double, multiple or sequential plan. This might provide little cost saving compared with the single plan due to problems in returning uninspected items to the lot.

d) **Duration of test**. If a test is of long duration and it is possible to apply it to a number of items simultaneously, it is usually better to do so rather than to risk finding, after testing the first sample, that the result is inconclusive and a second sample, or even more, is needed, therefore at least doubling the time taken. This is another case where single sampling is usually the best, provided that the whole of the single sample size can be tested at once. However, if only one or two articles can be tested at one time, multiple or sequential sampling might be preferable. (See Examples 6 and 7 in 9.3.1.2 and 9.3.1.3)

e) **Multiple nonconformities**. The more complicated the product in terms of the number of possible nonconformities and the number of classes of nonconformities, the more complicated double or multiple sampling becomes. Efficient use of labour and inspection equipment is difficult if the first sample has to be inspected for all features, a second sample only for some features, and possibly a third sample only for some of those features. In general, it can be said that a complicated inspection favours a simple sampling plan, whereas a simpler inspection might merit a more complicated sampling plan.

The operating characteristic curve (for the single sampling plan with sample size 200, acceptance number 3, and rejection number 4) and the equivalent double and multiple plans are shown in Figure 4. The match is not exact, but is good enough for most practical purposes. The equivalent sequential plan is also matched to the single sampling plan OC curve but is not shown in order to avoid overcrowding. The operating characteristic curves of the sequential and single sampling plans are virtually indistinguishable from each other.

9.3.1.2 Example 6

Tinned meat is to be tested for keeping qualities by storing a number of tins for 3 weeks under certain atmospheric conditions.

To achieve a desired OC curve, the choice might perhaps lie between a single sampling plan with a sample of 80 tins, a double sampling plan with samples each of 50 tins, and a five-stage multiple sampling plan with samples each of 20 tins. If single sampling is used, the answer will be available 3 weeks after the test is started; under double sampling, the result might be available in 3 weeks, but might require 6 weeks instead; under multiple sampling, nearly 5 months might be required.

Single sampling will probably be chosen in such circumstances.

9.3.1.3 Example 7

A destructive inspection is to be performed. All the articles in the lot are available at the testing station and the testing apparatus can take only one article at a time. As the principal cost of the test is the destruction of the article, it is desirable to destroy as few as possible consistent with the desired OC curve.

As the articles in the sample have to be tested one at a time, the use of sequential rather than single sampling will probably save time as well as cut down the average sample size. This would be well worth considering.

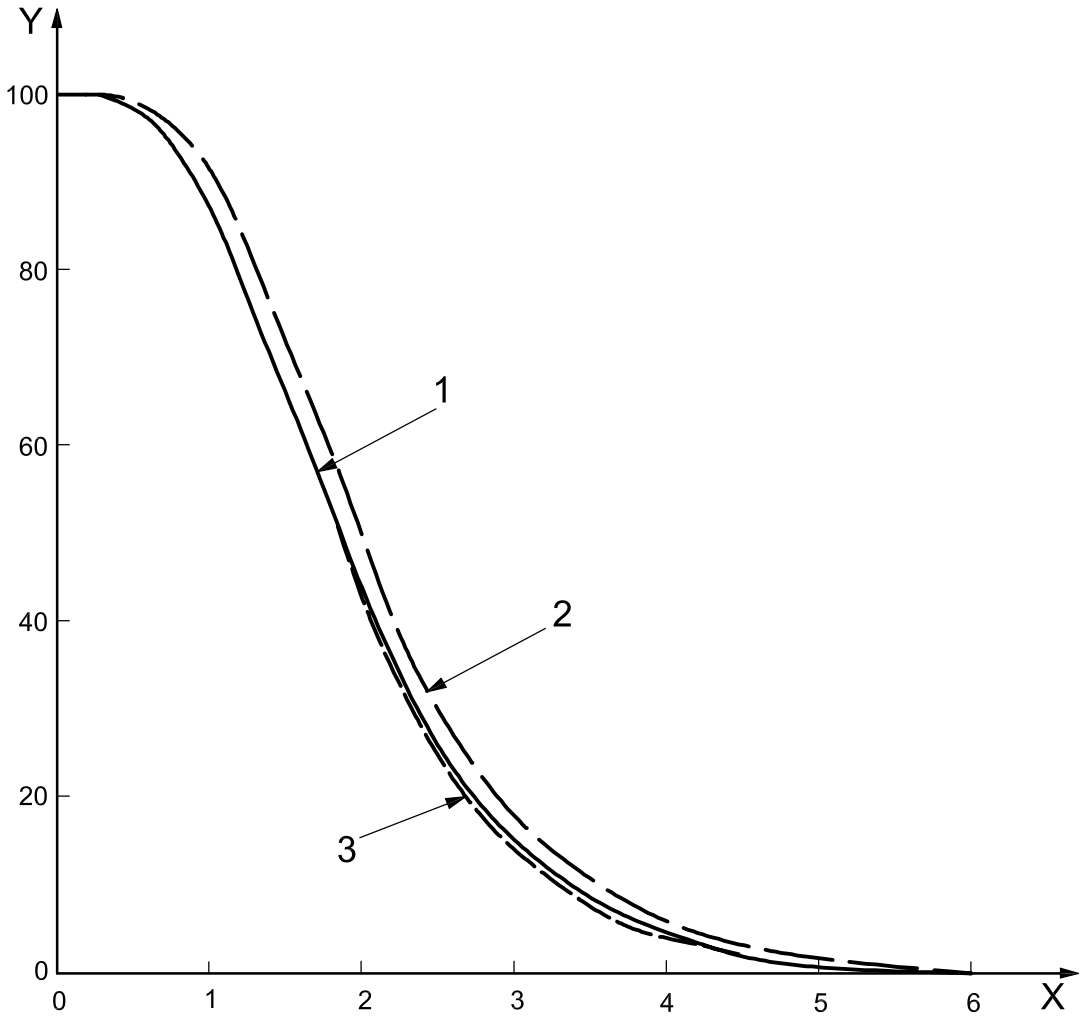
9.3.2 Average sample size

In order to compare single, double, multiple and sequential sampling, it is helpful to consider the average sample size that would be needed in a long run of sampling at different average levels of product quality. The average sample size curve is indicative of the relative efficiency of the different sampling systems. Such curves indicate the number of items to be examined on average before arriving at a decision to accept or reject at a range of product quality levels. Figure 5 shows the average sample sizes for the set of equivalent single, double, multiple and sequential sampling plans given in Table 3 and featured in Figure 4.

Table 3 — Equivalent sampling plans by attributes for code letter L, AQL 0,65 %

Type of sampling plan	Sample size(s)	Acceptance/rejection numbers	
		Ac	Re
Single	Sample size n = 200	3	4
Double	First sample size n = 125	1	3
	Combined 1st and 2nd sample size n = 250	4	5
Multiple	First sample n = 50	# [a]	3
	Cumulative sample size n = 100	0	3
	Cumulative sample size n = 150	1	4
	Cumulative sample size n = 200	2	5
	Cumulative sample size n = 250	4	5
Sequential	h_A = 1,383, h_R = 1,582, g = 0,0161, n_t = 315 (see ISO 2859-5).		

[a] Acceptance is not permitted at this sample size.



NOTE The OC curve of the sequential plan is virtually indistinguishable from that of the single plan.

Key

X quality of submitted product in percent nonconforming, $100p$

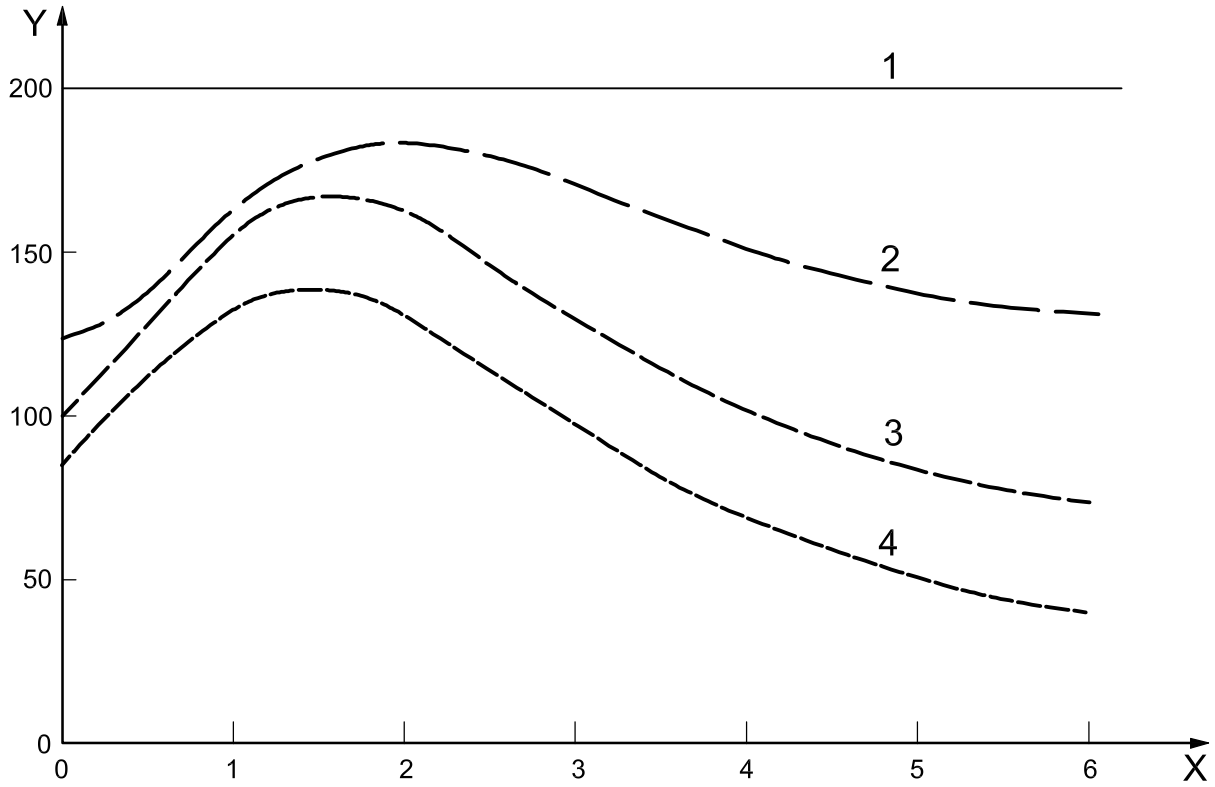
Y percentage of lots expected to be accepted, P_a

1 single and sequential

2 double

3 multiple

Figure 4 — Comparison of operating characteristic curves for single, double, multiple and sequential sampling plans



Key

X quality of submitted product in percent nonconforming, $100p$

Y average sample number

1 single

2 double

3 multiple

4 sequential

Figure 5 — Comparison of average sample sizes for single, double, multiple and sequential sampling

On average, the number of items to be examined before reaching a decision is largest when single sampling is used. The greatest reduction in sample size when using double, multiple or sequential sampling occurs when lots are of very good quality or very bad quality.

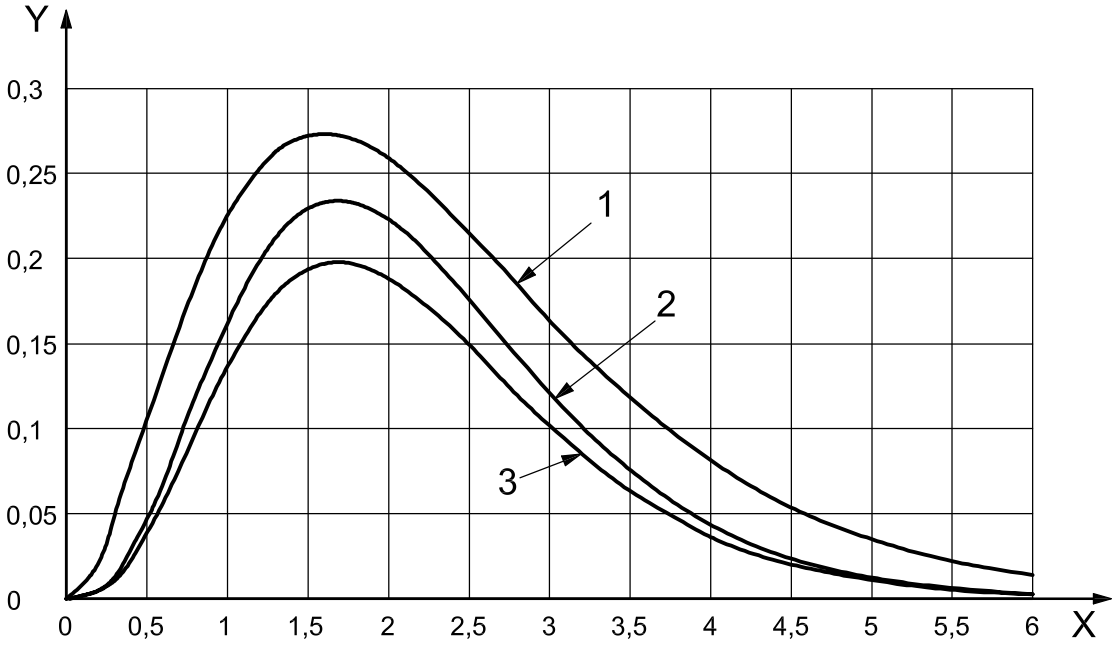
For good or bad quality, the average saving in inspection can be substantial, but the actual number of items to be inspected for a particular lot when using a double, multiple or sequential sampling plan might exceed that for the corresponding single plan. This is most likely to occur when quality is at an intermediate value, e.g. two or three times the AQL.

It is for these reasons that single sampling may be preferred in some instances, for example, when the test duration is long and all items can be tested at the same time. On the other hand, when the tests can only be done one at a time or are destructive, double, multiple or sequential sampling can offer a substantial advantage (see Examples 4 and 5 in 8.2.4).

For double and multiple plans, there is an upper limit to the number of items to be inspected. For sequential plans, there is no such limit unless a curtailment rule is invoked to restrict the potential number of items inspected. ISO 2859-5 and ISO 3951-5 provide for curtailment of sample size at about 1,5 times the corresponding single sample size.

Double, multiple and sequential sampling offer the opportunity for significant savings in sample size, but they require more administrative control. When apparatus for semi-automatic use is available, automated sequential sampling offers an opportunity for increased efficiency and economy, particularly when destructive tests are performed.

One deterrent to the wider implementation of double, multiple and sequential sampling plans is the fear that the single sample size might be exceeded. The probability of this happening might not be as high as suspected. Figure 6 shows the probabilities as a function of the incoming quality level for the acceptance plans in Table 3. It can be seen that, particularly for intermediate quality levels, there is a considerable reduction in the probability of the sample size for single sampling being exceeded as one moves from double to multiple to sequential sampling.



Key

X process percent nonconforming

Y probability of the single sampling sample size being exceeded

1 double

2 multiple

3 sequential

Figure 6 — Comparison of the probabilities of the single sampling sample size being exceeded for the double, multiple and sequential sampling plans

10 Other methods sometimes adopted in practice

10.1 100 % inspection

Many modern plants incorporate automatic methods of product quality checking on each item produced. Similar practices are adopted to remove nonconforming items from lots accepted by sampling. However, some independent monitoring might be necessary to insure against malfunctions of the test system that result in the continued production and “acceptance” of nonconforming product. In some cases, the actual process control, or some other part of the quality system, can provide this monitoring.

10.2 Grab samples

The practice of grab sampling, a technique of sampling according to no fixed plan, is generally deprecated because it provides no statistical control and is thus unreliable, uneconomical and ineffective for acceptance and control purposes. However, it is sometimes used in a “feed forward” system where a nonconforming item causes no real inconvenience at a subsequent stage. For these reasons, this practice is normally confined to “in-house” operations.

A grab sample of between 2 and 20 items (see ISO 2859-1 sampling plans with code letters A through F at inspection level II) can be used to establish that a shipment contains items similar to those ordered, particularly when all the items in the small grab sample conform to the item description and specification.

A grab sample has none of the statistical characteristics of a proper random sample and a sampling plan with acceptance criteria. Grab sampling should only be used when

a) the history of the producer and the item have been good, or

b) conditions are such that the recipient cannot contribute to controlling or correcting the supplier’s process and the nonconformity is of such a nature that it is easily recognized and nonconforming items replaced in subsequent assembly or test.

Grab sampling is inappropriate for expensive items whose warranty does not survive lot acceptance.

A grab sample that is found to contain any nonconforming items is a signal to revert to the use of a sampling plan from a suitable acceptance sampling standard, and to follow the normal decision process that such a sample indicates.

10.3 One-of-a-kind lots

An unusual situation exists when one specially produced, unique or “one-off” lot fails to be accepted by the sampling plan. Resubmission of the same lot, which after screening and/or correction contains many more nonconforming items than desired, or submission of a substitute lot, is likely to lead to an eventual acceptance due to cumulative sampling risk. Sampling plans have been designed to reduce the probability of the acceptance of a lot containing an excessive number of nonconformities, whether the lot is submitted once or resubmitted several times. This procedure changes the relationship between sample size and acceptance number for each successive resubmission of a rejected lot and limits the number of resubmissions allowed. The main problem with such sampling plans in general is the large amount of sampling and inspection required.

10.4 Audit sampling

Where a supplier has a quality system operating, e.g. to an ISO 9000 series standard, sampling techniques may be used during the review or assessment of the production or quality system. (See ISO 2859-4.) Here the assessor is investigating whether the records or status of some equipment, materials in process or materials in storage are properly identified and at an acceptable level of quality. This can be viewed as sampling an isolated lot from a continuing process, the plan being chosen for small producer’s risk at the desired level (e.g. 1 % in error) with whatever discrimination ratio, consumer’s risk and quality level are necessary. The available resources and effort involved also have to be considered and these might influence the other considerations.

Non-acceptance by the sampling plan would be taken as a “prima facie” indication of non-conformity and evidence that the area of operations responsible for the non-conformity needed review, further investigation or correction.

This application of sampling is not to be taken as an independent verification of quality.

11 Relevance of market and production conditions

11.1 General

The task of selecting a suitable sampling system, scheme or plan is influenced by market and production conditions. In addition, the economics of the sampling system (as exemplified in 4.2), the resources of the inspection organization and other aspects need to be considered. Therefore, the selection process becomes complex and rarely is there one method of acceptance sampling that fits all situations, even though different situations might appear to be similar.

In ISO/TR 8550-2 and ISO/TR 8550-3, tables and figures are provided for sampling by attributes and sampling by variables, respectively, to illustrate the process for selecting a sampling system, scheme or plan. The tables provide “candidate” sampling systems, schemes and plans to fit given inspection situations, production conditions and market conditions.

These tables should be reviewed and as many “candidate plans” as fit the situation selected. These candidates should then be reviewed by means of the figures so that the user finally arrives at a system, scheme or plan that is most feasible and economic for the situation.

11.2 Market conditions

Many conditions have to be considered to select the most suitable sampling system, scheme or plan. Not least are the conditions that exist in the industrial market where products are transferred from producer/supplier to customer/user, e.g. conditions where the following apply:

a) the customer can influence the vendor to improve his quality (percent conforming);

b) the warranty of items survives acceptance;

c) the acceptance of a small number of nonconforming items can cause great loss, or represents a serious hazard;

d) plant shutdowns and economic loss result when a lot fails acceptance, thus affecting supply to the customer (single sourcing);

e) the history of the product supplied by the vendor is very good.

On the other hand, the following converse conditions can exist:

— the purchaser cannot influence the quality of product produced;

— the vendor’s responsibility ceases upon acceptance of the unit or the lot;

— nonconforming units have little economic or other effect and/or they are readily recognized and removed during assembly;

— there are many other readily available sources of equivalent product;

— there is no history of received quality or there is a history of poor quality.

11.3 Production conditions

In addition to the market conditions, there are production conditions that affect the acceptance sampling situation. These include the following:

a) the lot is one of a long continuous series of lots;

b) there is a history of consistently good quality in production;

c) the selection of a random sample is simple: all items are readily and equally available for choice and selection;

d) the test of an item, or a number of tests of several characteristics of an item, is/are rapid;

e) the inspection of an item is costly;

f) the inspection of an item is destructive;

g) there is a need to know the shape, the location and the extent of the dispersion of the distribution(s) of the characteristic(s);

h) the distribution(s) of the characteristic(s) to be tested is(are) known to be normal (or can be converted to normality);

i) the lot is a “one-of-a-kind” lot and is the only lot made to this specification (also known as a “one-off” lot).

Conditions opposite to those given above can also exist:

— the lot is isolated or is one from a short production run;

— production quality is very variable and/or poor;

— random sampling is difficult or expensive;

— the test(s) on the item take(s) a long time to perform;

— item inspection is inexpensive;

— item inspection is non-destructive;

— the shape and the dispersion of the distribution(s) of the characteristic(s) are not important;

— the distribution(s) of the characteristic(s) is(are) unknown or known not to be normal.

The effect that these considerations have on the selection process is dealt with in more detail in ISO/TR 8550-2 and ISO/TR 8550-3.

12 The final selection — Realism

A final selection should be based on both the requirements of the situation and the resources of the inspection organization.

When selecting a sampling system, scheme or plan, compromises are almost inevitable in achieving practicality, economical use of resources and acceptable risks. The selection process might thus become iterative. Previous experience can greatly assist in deciding what is the most advantageous of the suggested “candidate” plans.

If the candidate sampling plan derived at the first attempt is impracticable for any reason, e.g. if the sample size is too large, then a check has to be made first to see that the selection has been made correctly.

If the plan is still impracticable, then the “quality levels” and “sampling risks” need to be considered by all parties concerned to reach an understanding and to agree upon revised parameters that will lead to the selection of an acceptable sampling system and plan.

Annex A
(informative)

Example of a simple model for profit maximization under destructive inspection by attributes

A.1 The problem

A certain item is produced in lots of size N items, with a unit production cost of c. Lots are classified by the producer as acceptable or sub-standard. The sale price per item is a for lots classified as acceptable and s for lots classified as sub-standard. Typically, s is very much less than the unit production cost c. Every nonconforming item sold in a lot that is classified as acceptable results in a downstream cost d to the producer, e.g. in warranty costs or loss of goodwill. The corresponding cost in lots that are sold as sub-standard is zero.

Lot quality is the fraction, p, of nonconforming items in the lot. Historical data indicates that p is at the low value p_0 for a high fraction f_0 of lots, but that it unaccountably and apparently randomly slips to a higher value p_1 for the remaining fraction $f_1 = 1 - f_0$ of lots. Testing of items, to determine whether they conform to requirements, is destructive, at an additional cost of t per item tested.

The producer has the option of relying on historical evidence alone and classifying all lots as acceptable, or combining historical evidence with some sample data from each lot in order to determine the acceptability of individual lots. The producer's objective is to maximize the profit per item sold.

A.2 Method of solution

It is necessary first to determine the conditions under which the best course of action is to sample. If these conditions are satisfied, the optimal sampling plan and the resulting average profit per item sold are then determined. Finally, this profit is compared to the average profit from not sampling, in order to determine the best policy. For simplicity, single sampling by attributes is considered, i.e. plans with sample size n and acceptance number Ac, so that determination of the optimal plan reduces to finding the optimal values of n and Ac.

A.3 Average profit per item sold as a function of the parameters of the sampling plan

The average profit from assessing a lot as acceptable based on a sample of size n when $p = p_0$ is:

$$P(n) = (N - n)a - N \cdot c - n \cdot t - (N - n)d \cdot p_0 \quad \text{(A.1)}$$

The average profit from assessing a lot as acceptable based on a sample of size n when $p = p_1$ is:

$$Q(n) = (N - n)a - N \cdot c - n \cdot t - (N - n)d \cdot p_1 \quad \text{(A.2)}$$

The average profit from assessing a lot as sub-standard, regardless of p, is:

$$R(n) = (N - n)s - N \cdot c - n \cdot t \quad \text{(A.3)}$$

The probability of passing the attributes acceptance test is denoted by $b_0(n,\mathrm{Ac})$ when $p = p_0$ and by $b_1(n,\mathrm{Ac})$ when $p = p_1$, noting that lot acceptance with no sampling implies that both $b_0(0,0)$ and $b_1(0,0)$ are equal to 1. The average total profit per lot is:

$$\begin{aligned} T(n,\mathrm{Ac}) &= f_0 \cdot b_0(n,\mathrm{Ac}) \cdot P(n) + f_1 \cdot b_1(n,\mathrm{Ac}) \cdot Q(n) \\ &\quad + \left\{ f_0 \cdot \left[1 - b_0(n,\mathrm{Ac})\right] + f_1 \cdot \left[1 - b_1(n,\mathrm{Ac})\right] \right\} R(n) \end{aligned} \tag{A.4}$$

Substituting in Equation (A.4) from Equations (A.1), (A.2) and (A.3), rearranging and dividing throughout by $(N-n)$, it is found that the average profit per item sold is:

$$\begin{aligned} U(n,\mathrm{Ac}) &= s - c + f_0 \cdot b_0(n,\mathrm{Ac}) \cdot \left(a - s - d \cdot p_0\right) \\ &\quad + f_1 \cdot b_1(n,\mathrm{Ac}) \cdot \left(a - s - d \cdot p_1\right) - n(c+t)/(N-n). \end{aligned} \tag{A.5}$$

A.4 The solution when $a - s \geqslant dp_1$

If $a - s - dp_1 \geqslant 0$, it follows that $a - s - dp_0 > 0$ and the maximum of $U(n,\mathrm{Ac})$ with respect to n and Ac is obtained by making both $b_0(n,\mathrm{Ac})$ and $b_1(n,\mathrm{Ac})$ as large as possible, i.e. by setting n and Ac to zero. Thus, when $a - s - dp_1 \geqslant 0$, the maximum average profit per item sold is:

$$\begin{aligned} U(0,0) &= s - c + f_0 \cdot (a - s - d \cdot p_0) + f_1 \cdot \left(a - s - d \cdot p_1\right) \\ &= a - c - d \cdot (f_0 \cdot p_0 + f_1 \cdot p_1). \end{aligned}$$

This is achieved by judging all lots to be acceptable without any sampling whatsoever. However, if $U(0,0)$ is not positive, it is best not to be in business!

A.5 The solution when $a - s \leqslant dp_0$

If $a - s - dp_0 \leqslant 0$, it follows that $a - s - dp_1 < 0$, so $U(n,\mathrm{Ac}) < s - c < 0$ for all n and Ac, in which case it would also be best not to be in business.

A.6 The solution when $dp_0 < a - s < dp_1$

In the intermediate range between the two extremes of A.4 and A.5, $a - s - dp_1 < 0$ but $a - s - dp_0 > 0$, i.e.:

$$(a-s)/p_1 < d < (a-s)/p_0 . \tag{A.6}$$

Within this range, it is not immediately clear whether the optimum policy is to sample or to accept the lot without sampling. Let $g_0 = f_0 \cdot \left[(a-s) - d \cdot p_0\right]$ and $g_1 = f_1 \cdot \left[d \cdot p_1 - (a-s)\right]$. Then Equation (A.5) becomes:

$$U(n,\mathrm{Ac}) = (s-c) + g_0 \cdot b_0(n,\mathrm{Ac}) - g_1 \cdot b_1(n,\mathrm{Ac}) - n(c+t)/(N-n) . \tag{A.7}$$

For a given sample size n, only the second and third terms on the right hand side of Equation (A.7) are affected by changing the acceptance number. For fixed n, decreasing the acceptance number from k to k–1 is advantageous if and only if:

$$g_0 \binom{n}{k} p_0^k (1-p_0)^{n-k} < g_1 \binom{n}{k} p_1^k (1-p_1)^{n-k}$$

which, on rearrangement, becomes:

$$k > \frac{\ln(g_0/g_1) - n \cdot \ln\{(1-p_1)/(1-p_0)\}}{\ln(p_1/p_0) - \ln\{(1-p_1)/(1-p_0)\}}$$

where "ln" denotes a Napierian logarithm. It follows that for any given sample size n, the optimal acceptance number is:

$$k_0(n) = \max\left(0, \left[\frac{\ln(g_0/g_1) - n \cdot \ln\{(1-p_1)/(1-p_0)\}}{\ln(p_1/p_0) - \ln\{(1-p_1)/(1-p_0)\}}\right]\right) \quad \text{(A.8)}$$

where $[x]$ represents the integral part of x. Thus, the optimal plan over all sample sizes might be found by a search for the maximum of Equation (A.7) as:

$$\max_n U(n, k_0(n)) \quad \text{(A.9)}$$

A.7 Remark concerning accept-zero plans

It is worth observing from Equation (A.8) that the optimal acceptance number will be zero whenever

$$\ln(g_0/g_1) - n \cdot \ln\{(1-p_1)/(1-p_0)\} < \ln(p_1/p_0) - \ln[(1-p_1)/(1-p_0)], \quad \text{(A.10)}$$

i.e. whenever

$$\frac{g_0}{g_1} < \frac{p_1}{p_0}\left(\frac{1-p_1}{1-p_0}\right)^{n-1}. \quad \text{(A.11)}$$

This is often the case when p_0 is very small.

A.8 Example

Suppose that the unit production cost is c = £10,00, the unit selling price in sub-standard lots is s = £0,50, the additional cost to destructively test an item is t = £1,00, the downstream cost of a nonconforming unit in an accepted batch is d = £10 000, the lot size is N = 10 000, and the probability that the lot is at quality level p_0 is f_0 = 0,99. Table A.1 shows the optimal inspection policy changing and the average profit per item sold increasing as the quality levels are improved, even though the unit selling price in acceptable lots is decreased.

Table A.1 — Optimal sampling plans for example of A.8

Usual quality level, p_0	**Quality level**, p_1, **after slippage**	**Optimal single sampling plan by attributes**		**Item selling price**, a (£)	**Average profit per item sold** (£)
		n	Ac		
0,001 00	0,100	104	2	20,25	0,022
0,000 50	0,050	139	1	15,40	0,091
0,000 30	0,030	197	1	13,60	0,211
0,000 20	0,020	249	1	12,75	0,280
0,000 10	0,010	141	0	12,00	0,378
0,000 09	0,009	137	0	11,95	0,436
0,000 08	0,008	129	0	11,90	0,499
0,000 07	0,007	113	0	11,85	0,570
0,000 06	0,006	86	0	11,75	0,603
0,000 05	0,005	34	0	11,70	0,710
0,000 04	0,004	Accept without sampling		11,60	0,804
0,000 03	0,003	Accept without sampling		11,50	0,903
0,000 02	0,002	Accept without sampling		11,35	0,952
0,000 01	0,001	Accept without sampling		11,20	1,001

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