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Liquid hydrocarbons — Dynamic measurement — Statistical control of volumetric metering systems

Hydrocarbures liquides — Mesurage dynamique — Contrôle statistique des systèmes de mesurage volumétrique

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Contents

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Foreword

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Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 4124 was prepared by Technical Committee ISO/TC 28, *Petroleum products and lubricants*, Subcommittee SC 2, *Dynamic petroleum measurement*.

Annexes A, B, C, D, E and F of this International Standard are for information only.

Liquid hydrocarbons — Dynamic measurement — Statistical control of volumetric metering systems

Section 1: General

1.1 Scope

In dynamic measuring systems the performance of meters for liquid hydrocarbons will vary with changes in flow conditions, viz. flowrate, viscosity, temperature, pressure, density of product, and with mechanical wear.

This International Standard has been prepared as a guide for establishing and monitoring the performance of such meters, using appropriate statistical control procedures for both central and on-line proving. These procedures may be applied to measurements made by any type of volumetric or mass metering system.

The procedures to be followed for collecting data, on which the control limits are based, are described. An alternative method for establishing the reliability of these data is described in ISO 7278-3.

Methods are described for calculating the warning and action control limits for the charts covering the selected performance characteristics, the application of these control charts to subsequent routine measurements, and their interpretation. Worked examples are given in the appropriate central and on-line proving sections.

1.2 Definitions

For the purposes of this International Standard, the following definitions apply.

1.2.1 proving; proof; calibration: Determination of the meter performance via the relationship between the volume of liquid actually passing through a meter and the reference volume of the pipe prover.

1.2.2 *K*-factor: Relationship between the number of pulses (N) generated by the meter during the proving run and the volume of liquid (V) displaced by the sphere or piston in the pipe prover between detectors.

Normally, $K = N/V$; it is recommended that this value be corrected by the pulse interpolation technique described in ISO 7278-3.

1.2.3 meter factor: Ratio of the actual volume passed through a meter, as derived from the pipe prover, to the volume indicated by the meter totalizer.

1.3 Symbols and units

1.3.1 General symbols

h_1	high liquid level in tank	metres
h_2	low liquid level in tank	metres
E_h	gauging error	millimetres
E_m	meter volumetric error	percent
E_t	temperature error	degrees Celsius
K	K-factor	pulses per unit volume
ΔK	change in K-factor	pulses per unit volume
MF	meter factor	dimensionless
MF_m	mean meter factor	dimensionless
MF_{max}	maximum meter factor in a set of measurements	dimensionless
MF_{min}	minimum meter factor in a set of measurements	dimensionless
N	number of pulses generated by meter during proving run	dimensionless
p	pressure at line conditions	kilopascals (1 bar = 100 kPa)
p_0	pressure at standard conditions (101,325 kPa)	kilopascals
t	temperature at line conditions	degrees Celsius
t_0	temperature at standard conditions (15 °C or 20 °C)	degrees Celsius
T_1	elapsed time	seconds
Q	volume rate of flow	cubic metres per hour
V_p	reference volume of pipe prover at standard conditions (15 °C or 20 °C and 101,325 kPa)	litres or cubic metres
ν	kinematic viscosity of the fluid	millimetres squared per second [centistoke (cSt)]

1.3.2 Statistical symbols

X	true value of quantity
μ	mean value
σ	standard deviation
x	value of measurement
$\bar{x}$	mean of a set of measurements
n	number of repeated measurements
m	number of quantities
s	estimate of standard deviation
w	range of a set of measurements
$\bar{w}$	mean of a set of ranges
t	value of Student's t-distribution
r	estimate of repeatability
Φ	degrees of freedom

1.4 Central proving

With the method of central proving, the performance of a meter is established at a testing station by proving the meter over its entire operating range of flowrate, viscosity, temperature and oil density used in service.

Meter performance charts are then prepared from the proving data, and are used to establish the relationship between the meter factor and flowrate or flow and viscosity.

Any large deviation in meter performance on site can be detected by secondary control procedures, which monitor the output of two meters in series or in parallel. Long-term deviations in meter factors can be established by statistical control charts. The latter method can also be used in on-line proving.

1.5 On-line proving

With the method of on-line proving, the meter is proved under operating conditions with a portable or fixed installation pipe prover. Where significant changes in flowrate, viscosity, temperature or density occur, the meter can be reproved.

Any marked deviation or abnormal trend in meter factor can be monitored by use of statistical control charts.

By statistical analysis it is possible to establish whether the deviations are due to changes in flow conditions, random error or some other assignable cause.

Section 2: Statistical measurements

2.1 Principles of statistical measurement

2.1.1 Introduction

Measurements taken via central or on-line meter proving provide information on the random variability of the parameters of hydrocarbon flow through the meter (for example meter factor, flowrate, temperature, Reynolds number). Using this information, it is possible to assign a level of probability to a deviation observed in practice, and thereby differentiate between a normal or "allowable" deviation and one that has been caused by an external and systematic influence, such as meter component wear.

The true value of the meter characteristic in question, and its range of variability, can be represented diagrammatically on a control chart (see 2.2.5). This will indicate the deviation (warning limit) which should be taken as an early indication of malfunction, and the deviation (action limit) at which it is almost certain that meter failure has occurred. It is standard practice to assign a probability of 95 % to warning limits, and 99 % to action limits. This means, for example, that there is only a 1 % chance that a measurement falling outside the action limits did so as a result of normal variation when the process is under statistical control. Once a control chart is established, the measurements from subsequent meter provings can be entered periodically onto the control chart, from which it is possible to monitor trends in meter performance over a period of time.

In order to establish control through this means, reliable estimates should be obtained of the statistics to be used. The initial period in which data is collected, and against which the performance of the meter is to be monitored, is called the "learning period". This should be long enough to provide a reliable assessment of the true value of the meter characteristic in question.

Before considering the steps to be followed in the creation, use and maintenance of control charts, it is first necessary to understand the statistical treatment which is to be applied.

2.1.2 Distribution of measurements

The measurement of any physical quantity, be it direct (for example temperature by thermometer) or indirect (for example meter factor) is always subject to error. The error is sometimes systematic and assignable to a definite cause, for example a large change in temperature may result in a large change in meter factor. If that is not the case, however, data scatter can be regarded as random, and is thus amenable to statistical treatment.

Random errors often vary in magnitude with the quantity being measured (in which case they are expressed as percentages) or with some other external factor. The error in K-factor, for example, will change in magnitude according to the flowrate (see performance chart in figure 1). For this reason it is vital that operating conditions are controlled while measurements are being taken (see 2.2.2). In practice, the distribution of errors approximates a Gaussian (normal) distribution, and this is fully defined if its two parameters are known. The parameters in this case are mean value, represented by μ, and standard deviation, represented by σ. The Gaussian distribution is described in more detail in annex C.

Each of the parameters of a distribution of measurements is assumed to have a true value, and is represented algebraically by a Greek or capital Roman letter. Estimates of the parameters, or statistics, are represented algebraically by small Roman letters. When necessary these will be qualified algebraically by the use of brackets. For example the standard deviation estimate of a measurement x will be shown as $s(x)$ (see 2.1.4). The statistics which are of primary interest are mean, standard deviation, range of a set of measurements, and uncertainty.

2.1.3 Estimate of true quantity

Given a set of measurement x_i, for $i = 1$ to n, the estimate of the true quantity which is most likely to be correct is the mean $\bar{x}$ (termed "x bar") of the set of measurements, where

$$\bar{x} = \frac{1}{n}\ (x_1 + x_2 + \dots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \dots (2.1)$$

As n tends to infinity, so the estimate x will tend towards the true value μ, provided there are no systematic errors.

2.1.4 Estimate of standard deviation

The standard deviation $\sigma(x)$ is a measure of the random error of a single measurement x. The usual unbiased estimate of $\sigma(x)$ is $s(x)$, where:

$$s(x) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}x_i^2 - \frac{n}{(n-1)}x^2} \qquad \dots (2.2)$$

Another estimate is given by:

$$s(x) = \frac{\overline{w}}{D(n)} \qquad \dots (2.3)$$

where

$\overline{w}$ is the mean range difference between the maximum and minimum values of x, using a number of sets of n measurements;

$D(n)$ is a conversion factor (see annex A).

This estimate becomes less reliable as the number of ranges on which it was based becomes smaller, and should only be regarded as a rough check when based on a single range.

The standard deviation estimate of a mean, sometimes called standard error, is derived from this as:

$$s(\bar{x}) = s(x)/\sqrt{n} \qquad \dots (2.4)$$

It is evident that as the number n of measurements is increased, so the standard error is decreased, leading to greater confidence in the estimate $\bar{x}$ of the true quantity.

2.1.5 Estimate of the uncertainty

The reliability of an estimate can be expressed as an *uncertainty interval* in which the true value should be expected to fall with a specified level of confidence or probability. In statistical terminology this is called a confidence interval. The uncertainty interval which contains an estimate x is $x \pm \mathrm{u}(x)$, where $\mathrm{u}(x)$ is called the *uncertainty*, $x - \mathrm{u}(x)$ and $x + \mathrm{u}(x)$ are called the *uncertainty limits*, and the difference $2\mathrm{u}(x)$ between these limits is called the *range of uncertainty*. Normally the probability levels are 95 % and 99 %.

A true quantity estimate of $\bar{x}$, the mean of n measurements, could then be stated as:

True quantity $= \bar{x} \pm \mathrm{u}(\bar{x})$, n measurements, 95 % probability;

when $n = 1$, $\bar{x}$ becomes the single measurement x.

If the standard deviation σ is known from long experience, then the uncertainty is also known. That referring to 95 % probability is given by:

$$\mathrm{u}(\bar{x}) = 1{,}96\sigma(\bar{x}) = 1{,}96\sigma(x)/\sqrt{n} \qquad \dots (2.5)$$

As before, $\bar{x}$ becomes the single measurement x when $n = 1$. The value 1,96 is the value of the standard normal deviate for a two-sided probability of 95 % (see annex C).

If, however, the standard deviation of individual measurements has been estimated as $s(x)$, based on Φ degrees of freedom, then the uncertainty should be estimated as:

$$u(\bar{x}) = t_{95,\Phi} s(\bar{x}) = t_{95,\Phi} s(x)/\sqrt{n} \qquad \ldots (2.6)$$

Once again, when $n = 1$, $\bar{x}$ becomes the single measurement x.

Here $t_{95,\Phi}$ is the value of the t-distribution for a two-sided probability of 95 %, corresponding to a standard deviation estimate based on Φ degrees of freedom (see annex B). In this context, degrees of freedom should be regarded as the number of independent measurements from which the standard deviation was estimated. Given n measurements, therefore, s would be based on $\Phi = (n - 1)$ degrees of freedom, since one degree of freedom was already accounted for in estimating the mean.

The t-distribution is a function of the degrees of freedom, and the t-value for a given probability will decrease in magnitude as Φ increases. As Φ tends towards infinity, so the t-distribution tends towards a Gaussian distribution. Values of 2 and 3 are sometimes used as approximations of the t-values corresponding to 95 % and 99 % probability respectively. These values are appropriate for estimates based on 10 to 20 measurements.

2.1.6 Estimate of repeatability

Repeatability is the term used for uncertainty which relates not to individual measurements or measurement means as in 2.1.5, but to the *difference* between two individual measurements. Since the standard deviation of the difference between two measurements x_1 and x_2 (see 2.1.8) is:

$$\sigma(x_1 - x_2) = \sqrt{2}\ \sigma(x_1) = \sqrt{2}\ \sigma(x_2) \qquad \ldots (2.7)$$

then the repeatability estimate r is given by:

$$r = \sqrt{2}\ u(x) \qquad \ldots (2.8)$$

In this case $u(x)$ refers to individual measurements x_i rather than the mean $\bar{x}$, and equations (2.5) and (2.6) would become:

$$U(x) = 1{,}96\sigma(x) \qquad \ldots (2.9)$$

and

$$u(x) = t_{95,\Phi} s(x) \qquad \ldots (2.10)$$

Note that a repeatability value, to be used in practice, should be derived from an independent set of measurements which excludes the pair of values in question. The standard deviation estimate should be based on at least 20 and preferably 30 or more degrees of freedom.

2.1.7 Estimate of maximum range

It is possible to extend the concept of repeatability (the uncertainty for the difference between two measurements) by considering the distribution of a range of three or more measurements. For this it is necessary to refer to the limiting values $E_1(n)$ or $E_2(n, \Phi)$ of a range of measurements with unit standard deviation corresponding to a chosen probability level (see annex A).

The upper limit of the range of n measurements, knowing the standard deviation $\sigma(x)$, is given by:

$$W = \sigma(x)\ E_1(n) \qquad \ldots (2.11)$$

Where the standard deviation is estimated as $s(x)$ (see 2.1.4) based on Φ degrees of freedom, from an independent exercise excluding the measurements in question, the limit is estimated to be:

$$w = s(x)\ E_2(n, \Phi) \qquad \ldots (2.12)$$

In either case, the limit calculated corresponds to the maximum range (n measurements) to be expected in practice with the given probability. The limit corresponding to 95 % probability may be used as a test to establish statistical control (see 2.2.2). A rogue value can also be identified in this way (see 2.2.3), but should be confirmed by the use of one of the outlier tests given in annex D. As with repeatability, an estimate of maximum range to be used in practice should be based on at least 20 and preferably 30 or more degrees of freedom, and should exclude the measurements in question.

2.1.8 Combination of errors

Consider an indirect measurement y which is calculated from, say, m intermediate measurements x_1, x_2 ... x_m according to the function:

$$y = F(x_1,\ x_2 \dots x_m) \qquad \dots (2.13)$$

If the m intermediate measurements are algebraically independent, that is, no one can be calculated from the others, then the statistics of the indirect measurement may be derived as shown below.

2.1.8.1 The estimate $\bar{y}$ of the true value (see 2.1.3) can be calculated by substitution of the appropriate means into equation (2.13), that is:

$$\bar{y} \simeq F(\bar{x}_1,\ \bar{x}_2 \dots \bar{x}_m) \qquad \dots (2.14)$$

This approximation applies to functions F which are approximately linear.

2.1.8.2 The estimate $s(y)$ of the standard deviation of y (see 2.1.4) is given by:

$$s^2(y) = \left[\frac{\partial F}{\partial x_1} s(x_1)\right]^2 + \left[\frac{\partial F}{\partial x_2} s(x_2)\right]^2 + \dots + \left[\frac{\partial F}{\partial x_m} s(x_m)\right]^2 \qquad \dots (2.15)$$

where the sensitivity coefficients $\partial F/\partial x_i$ are evaluated at the known or mean values of x_i.

Note that the standard deviation estimates used in this expression could be in terms of either individual measurements [equation (2.2)] or mean values [equation (2.4)]. Furthermore, the expression is valid if one or more of the standard deviation values is known as $\sigma(x_i)$, rather than estimated as $s(x_i)$.

2.1.8.3 The estimate $\mathrm{u}(y)$ of the uncertainty of y (see 2.1.5) is similar in form to equation (2.13), that is:

$$\mathrm{u}^2(y) = \left[\frac{\partial F}{\partial x_1} \mathrm{u}(x_1)\right]^2 + \left[\frac{\partial F}{\partial x_2} \mathrm{u}(x_2)\right]^2 + \dots + \left[\frac{\partial F}{\partial x_m} \mathrm{u}(x_m)\right]^2 \qquad \dots (2.16)$$

Once again, the uncertainty estimates used in this expression can be in terms of individual measurements or mean values, and could include known values of uncertainty $\mathrm{u}(x_i)$.

2.2 Measurement procedure

2.2.1 Introduction

In order to monitor meter performance through a statistically based control chart, in general terms the procedure should be carried out as follows:

a) establish statistical control;

b) take measurements in the proving run conducted under the operating conditions required;

c) test the measurements for reliability and use them to create new performance charts, or add to performance charts previously created;

d) add the measurements to control charts in progress, or use the measurements to create new control charts if sufficient measurements have been accumulated in the "learning period".

2.2.2 Statistical control

A measurement taken under undefined or variable operating conditions will not yield meaningful statistics. In order to establish statistical control, great care should be taken that factors such as temperature and flowrate are correctly measured, and that all external influences have been identified.

It is very often difficult to establish statistical control quantitatively. It may be possible, however, to examine performance charts and calculate the maximum allowable range for a set of measurements obtained under the given operating conditions (see 2.1.7). At the very least, it is essential that the measurement procedure is clearly understood and that equipment is operating correctly.

2.2.3 Measurement reliability

A set of n repeated measurements having been obtained, they should be examined for outliers (rogue values). It should be stressed, however, that measurements should not freely be discarded. An attempt should always be made to find a reason for the extreme values, after which corrective action can be taken. Given no further information on the scatter of the measurements, Dixon's or Grubbs' outlier test may be used (see annex D). In the event that an outlier is detected by this means, then it should be disregarded and further measurements obtained. It should also be confirmed that the extreme value was not due to a change in an uncontrolled variable such as temperature or flowrate (see 2.2.2).

The scatter of the K-factor may have already been determined for the operating conditions under which the set of measurements was obtained (see 2.2.4). In that case the uncertainty limits are known, and if a measurement were to fall outside the limit corresponding to 95 % probability, it should be regarded as a rogue value. When only two measurements are available, and their difference exceeds the repeatability (see 2.1.6), then both measurements are suspect. Similarly, the extreme values of a range of n measurements would be suspect if an observed range exceeded the maximum (see 2.1.7).

2.2.4 Performance charts

The performance of a meter can be represented diagrammatically on a performance chart. Figure 1 is an example, in which mean meter factor is given as a function of only one operating condition, namely flowrate. Variability is expressed in figure 1 as the range of n repeated measurements (typically, $n = 5$ or 10), but could also have been expressed as the uncertainty interval.

A separate performance chart should be drawn for each meter and product, and should refer to a stated set of operating conditions (for example range of temperature). In the case of central proving, however, in which it is possible to take measurements covering a wide range of operating conditions on the same class of meter, the "performance charts" may take the form of a matrix or surface in which meter factor is a function of two or more operating variables. (See 3.3.)

2.2.5 Control charts

2.2.5.1 Chart preparation

Following a sufficient learning period (for example 15 sets of proving runs), the true value estimate of K-factor can be represented on a control chart. Figure 2 is an example, in which each entry is a mean of 5 K-factors from four proving runs. The warning and action limits are the uncertainty limits, estimated at the end of the learning period, corresponding to 95 % and 99 % probability, respectively. It would be reasonable to expect that 5 % of the results would lie outside the warning limits and 1 % outside the action limits if the process were in statistical control.

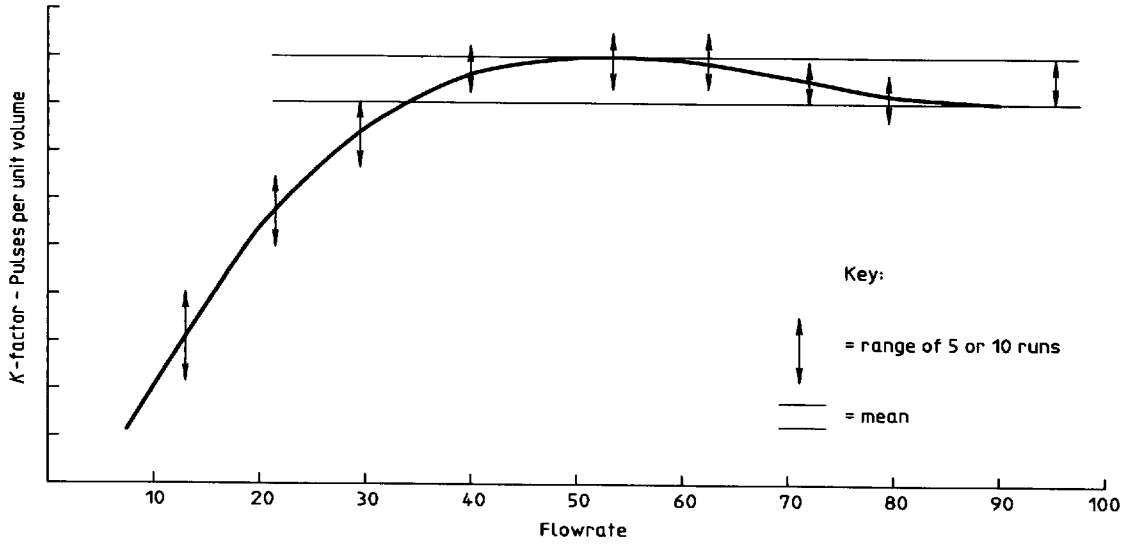


Figure 1 — Performance chart — *K*-factor versus flowrate [showing scatter (range) of 5 to 10 consecutive runs]

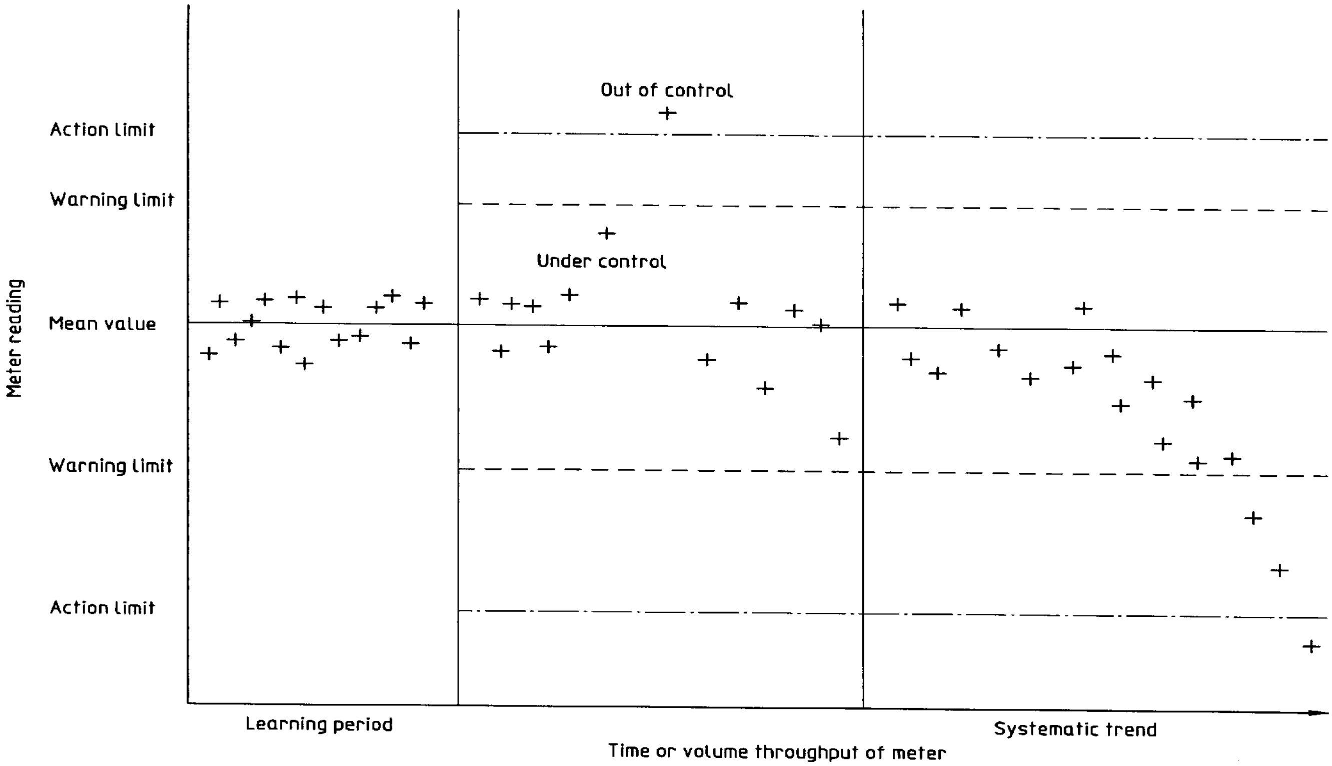


Figure 2 — Control chart (general)

Figure 2 shows entries which indicate whether the measurements are "in control" or not, and also gives an example of a systematic trend with time or volume throughput. It should be noted that if a systematic trend is evident, the control chart can be recreated with an appropriate shift in mean value and control limits. This would, for example, take account of meter component wear over a long period of time.

2.2.5.2 Parallel control charts

Control charts referring to different measurements, for example K-factor and flowrate, can be constructed with a common scale of time or volume throughput. This will allow the correlation of one measurement with another, and may indicate a reason for major deviations in K-factor.

2.2.5.3 Moving-average control charts

Long-term meter performance can be monitored by using moving averages. After the learning period, each entry on the control chart (see figure 2) will be the average of, say, the last ten sets of proving runs. This will provide a moving "window" which will be less affected by short-term deviations, and from which it is possible to identify a slope change (long-term drift) or a step change (constant shift in K-factor).

2.2.5.4 Non-linear control charts

The performance chart in figure 1 becomes a control chart if warning and action limits are placed on each side of the line. In this case the learning period includes the measurements on which the performance chart was based. From these measurements, and previous experience when available, it is possible to estimate the standard deviation of K-factor for a given range of flowrates. Control limits can then be estimated and drawn on the chart.

2.2.5.5 Reconciliation charts

Long-term meter performance can be monitored in meter/tank or meter/meter comparisons of volume throughput. When a meter's readings are regularly compared with another measuring technique (for example tank gauging) on the same volume of liquid, then the volume of throughput is accumulated for each measuring device and the difference between them expressed as a percentage of the cumulative sum. The true value on a reconciliation chart will be zero, and as the cumulative sums of volume increase, so the percentage difference will be expected to tend towards zero. If not, meter failure or bias would be indicated.

Section 3: Central proving

3.1 Collection of data

3.1.1 Proving conditions

3.1.1.1 General

Initial and periodic provings may be carried out on different oils which are stored in the centralized proving station in order to cover, when required, a wide range of viscosities.

During the proving operation, the meter factor of the meter versus flowrate and of the meter versus viscosity should be established. As the meter factor will depend upon pressure p, temperature t and flowrate Q, it is necessary to maintain the pressure and temperature as constant as possible.

During the proving operation, the meter factor of a positive displacement meter versus flowrate shall be established. If the meter will be used on products covering a wide range of viscosities < 20 cSt, a curve of meter factors for each product type shall be established.

Turbine- and other inferential meters are in essence Reynolds-number related in their behaviour. The meter factor should therefore preferentially be established versus Reynolds number or Q/D.

If there is any variation in the pressure and temperature it should be sufficiently small to ensure that it does not contribute more than ± 0,01 % to the overall uncertainty in meter factor. The variation should be less than the uncertainty of the pipe prover reference volume, which is usually of the order of 0,02 % to 0,05 %.

3.1.1.2 Pressure

The pressure should be measured at (or close to) the meter and at the prover outlet. Its value should be kept constant within the limits ± 50 kPa.

3.1.1.3 Temperature

The temperature of the liquid should be measured at (or close to) the meter and at the prover. Sufficient product should be passed or circulated through the system initially in order to ensure thermally stable conditions. Any variations in temperature of less than 0,1 °C between the meter and prover may be considered as instrument errors rather than variations in liquid temperature.

3.1.1.4 Flowrate

The mean flowrate should be computed from the volume of the prover at reference conditions of temperature and pressure using the time interval between the first and last detector signals.

3.1.1.5 Viscosity

Determine the density and viscosity (at a minimum of two different temperatures, e.g. 20 °C and 40 °C) of the oil for each proving operation.

3.1.1.6 Meter data

The number of pulses generated by the meter at the various flowrates is totalled by an electronic counter which is started and stopped by the two detector signals respectively. In order to achieve a maximum uncertainty of 0,01 %, a suitable pulse interpolation technique may be used.

3.1.1.7 Number of provings

Each calibration for the same flowrate should be repeated at least twice, further measurements being required if the scatter is unacceptably large (see 3.2.2).

3.1.1.8 Number of flowrates required

For any given fluid the meter should be tested at no less than five flowrates, which will include the maximum and minimum flowrates. The minimum number of flowrates covering the operating range is defined in relation to the actual performance of the meter; for example, if the meter factor varies considerably with flowrate then a larger number of flowrates will be required in the test procedure.

3.1.1.9 Number of viscosities

Where possible, tests should be carried out on the actual fluids and over the flow range and under the same conditions of temperature and pressure that would be experienced in operation. The choice of the viscosities and their spacings depends on the type of meter being proved, i.e. the sensitivity of meter factor to viscosity variation, and the range of viscosities encountered in operation.

Although the viscosities normally chosen should be equally spaced over the operating range of the meter, it may be necessary to select additional viscosities at points where the meter factor is particularly sensitive to viscosity variation.

3.1.2 Test report

The results and conditions concerning the initial and periodic proving tests should be set down in a report that includes the following information:

a) identification of the centralized proving station;

b) reference volume of the pipe prover used;

c) meter characteristics (manufacturer, model and serial number);

d) exact details of the conditions under which the various tests were performed.

3.2 Reliability of data collected and resulting values

3.2.1 Central proving operational conditions

Although it is implied in 3.1.1.1 that conditions are held constant within a test in a central proving station, products are often recycled with slight variations in temperature. However, it is possible to compute the viscosity from the temperature.

If the uncertainty in meter factor (see 3.2.3) is known, during a test any variation in the parameters should not produce a variation in meter factor greater than a given percentage of this uncertainty. A typical value of this percentage is 30 %.

3.2.2 Reliability of data collected

The reliability of the data collected should be checked by means of the following tests before estimating the meter factor and the scatter on this estimate.

3.2.2.1 Outlier test

At each flow and viscosity (Q, ν) n $(n \geqslant 3)$ provings are made.

Dixon's test (see D.1) or Grubbs' test (see D.2) can be used to discard outliers (rogue values).

3.2.2.2 Tests for short-term variability or scatter

There are two methods for testing the scatter or variation in the meter factor: a repeatability test and a range test.

3.2.2.2.1 Test for repeatability

The absolute difference between two measurements at each (Q, ν) point can be tested against the repeatability r. If r is unknown, then a given percentage of the mean MF_m of the measurements should be used in its place. A typical value is 0,05 %.

If the absolute value of the difference is less than or equal to r, then the two measurements shall be retained. If the absolute difference exceeds r, then at least three additional measurements should be obtained, giving a total of n measurements. The test is repeated, but now using the absolute difference between the most divergent value and the average of the remaining $n - 1$ measurements.

If the absolute difference is less than or equal to

$$r \cdot \sqrt{\frac{n}{2(n-1)}}$$

then all the measurements shall be retained. If the absolute difference exceeds

$$r \cdot \sqrt{\frac{n}{2(n-1)}}$$

the most divergent measurement shall be rejected and the procedure repeated until an acceptable set of measurements is obtained. However, if two or more measurements out of 20 have been rejected, then the proving run should be stopped for investigation.

3.2.2.2.2 Test of the range

The range of $n > 2$ measurements at each (Q, ν) point can be tested against a maximum value of w, where MF_{max} and MF_{min} are respectively the largest and smallest measurements. If the standard deviation of the meter factor measurements is known or can be estimated from previous measurements, then w can be calculated from equation (2.11) or equation (2.12) (see 2.1.7), using tabulated values of $E_1(n)$ or $E_2(n, \Phi)$ corresponding to 95 % probability. If the standard deviation is unknown, then a given percentage of the mean value MF_m of the measurements should be used in place of w. A typical value is 0,05 %.

If $(MF_{max} - MF_{min})$ is less than or equal to w, then all the measurements shall be retained. If $(MF_{max} - MF_{min})$ exceeds w, then the most divergent measurement shall be rejected and the procedure repeated until an acceptable set of measurements is obtained. In that event w will require recalculation to take account of the new value of n. However, if two or more measurements out of not more than 20 have been rejected, then the proving run should be stopped for investigation.

If σ or s is not known, then test the measurements by applying the ratio:

$$\frac{MF_{max} - MF_{min}}{MF_{max} + MF_{min}}$$

If the ratio is less than 0,000 25 then the results are acceptable.

3.2.3 Resulting values

At each proving point (Q, ν) the meter factor value is the arithmetic mean of the retained measurements after the tests defined in 3.2.2.

The uncertainty on meter factor determination (arithmetic mean), estimated as described above, is calculated with equations (2.5) and (2.6) of 2.1.5 (Section 2).

3.2.4 Variation in meter factor with (Q, ν)

When meter factors have been established over the entire range of the flowrate and viscosity, tests should be performed in order to check the variation of the meter factor. Different ranges of flowrates are considered as corresponding to different ranges of viscosities.

The ratio

$$\frac{MF_{max} - MF_{min}}{MF_{max} + MF_{min}}$$

is computed.

The ratio should be less than a given limit, which is typically 0,025 %, where MF_{max} and MF_{min} are the maximum and minimum values of meter factor, respectively, over the entire range of proving.

3.3 Performance charts

3.3.1 General

A centralized proving station is a fixed installation comprising a number of pipe provers for proving and testing turbine and displacement meters.

Measurements are carried out in order to establish the performance characteristic for the meter under test. The appropriate meter factors, which are determined for different oil flowrates and ratios Q/ν, can be represented by means of polynomials, tables or a matrix.

3.3.2 Preparation of data

3.3.2.1 Flowrate Q

The flowrate Q, expressed in cubic metres per hour, is calculated from the time T_1 (in seconds) required to displace the prover spheroid between the two detectors which define the prover loop volume, V_p (in litres) at the standard conditions of temperature t_{0p} and pressure p_{0p}. The flowrate at temperature t and pressure p is given by

$$Q = 3{,}6 \frac{V_p}{T_1} [1 + C_{tp}(t - t_{0p}) + C_{pp}(p - p_{0p})] \qquad \ldots (3.17)$$

where C_{tp} and C_{pp} are the temperature and pressure corrections, respectively, of the prover.

3.3.2.2 Viscosity

The oil viscosity ν, expressed in centistokes, at proving temperature t is either directly measured at the proving temperature or computed from a formula such as

$$\log_{10} \cdot \log_{10}[\nu + f(\nu)] = A - B \cdot \log_{10}(t + 273{,}15) \qquad \ldots (3.18)$$

where A and B are constants calculated from the oil viscosity at two different temperatures (see ANSI/ASTM D341).

3.3.2.3 Meter characteristics

3.3.2.3.1 *K*-factor

The *K*-factor, which is expressed in pulses per unit volume (here pulses per cubic metre), is calculated from equation (3.19) on the assumption that the temperature and pressure at the meter and prover are the same.

$$K = \frac{N}{V_p} \cdot \frac{1 + C_{tm}(t - t_{0m}) + C_{pm}(p - p_{0m})}{1 + C_{tp}(t - t_{0p}) + C_{pp}(p - p_{0p})} \qquad \ldots (3.19)$$

where

- N is the number of pulses counted during a proving run;
- V_p is the base volume of the prover at standard conditions;
- C_{tm} is the correction for the expansion of the meter due to temperature;
- C_{pm} is the correction for the expansion of the meter due to pressure;
- C_{tp} is the correction for the expansion of the prover due to temperature;
- C_{pp} is the correction for the expansion of the prover due to pressure;
- t is the temperature of the liquid at the meter and prover;
- p is the pressure of the liquid at the meter and prover;
- t_0 is the reference temperature (at standard conditions);
- p_0 is the reference pressure (at standard conditions).

3.3.2.3.2 Meter correction factor

The meter correction factor, which is dimensionless, is normally the ratio between a nominal *K*-factor permanently stored in the meter processor and the new *K*-factor.

$$\text{Meter correction factor} = \frac{K_{nominal}}{K} \qquad \ldots (3.20)$$

3.3.2.3.3 Relative error

The relative error (E) of the turbine meter is normally defined as:

$$E = \frac{\text{indicated volume} - \text{true volume}}{\text{true volume}} \qquad \ldots (3.21)$$

It can be expressed as:

$$E = \frac{1 - \text{MF}}{\text{MF}} \qquad \ldots (3.22)$$

3.3.3 Meter calibration and performance charts

3.3.3.1 Calibration curve for turbine and displacement meters

3.3.3.1.1 PD meters

As positive displacement (PD) meters are little influenced by the viscosity of the product above 20 cSt, the meter factors can be considered to be mainly a function of flowrate Q.

These meters are proved with an oil of the same category as that for which they are used. The performance charts can represented MF, K or E as a function of flowrate. See performance chart No. 1 (figure 3) showing relative error versus flowrate.

3.3.3.1.2 Turbine meters

When using oil of the same category with a kinematic viscosity less than 1 cSt or when viscosity variations are small, i.e. oils that do not produce meter factor variations greater than 0,1 %, then the meter factor is only a function of flowrate. The performance charts can represent MF, K or E as a function of flowrate. See performance chart No. 1 (figure 3) showing relative error versus flowrate.

On multiproduct oils or when large viscosity variations occur, for turbine meters, for instance helicoidal two-blade turbine meters, a universal calibration curve (UCC) may be plotted (see 3.3.3.2). The performance charts can represent MF as a function of Q/ν (see figure 4).

With meters for which it is not possible to plot a UCC, tables can be established as a function of flowrate and viscosity. The performance charts can represent MF, K or E as a function of flowrate at a given viscosity (see figure 5).

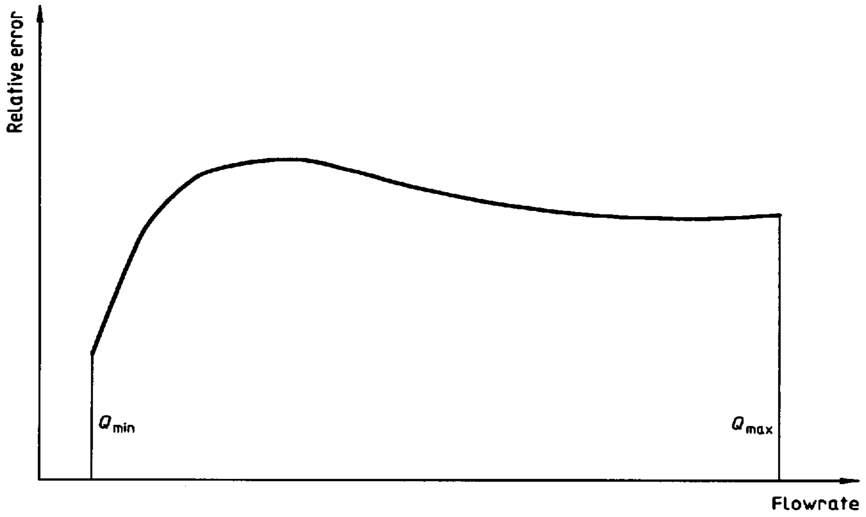


Figure 3 — Performance chart No. 1

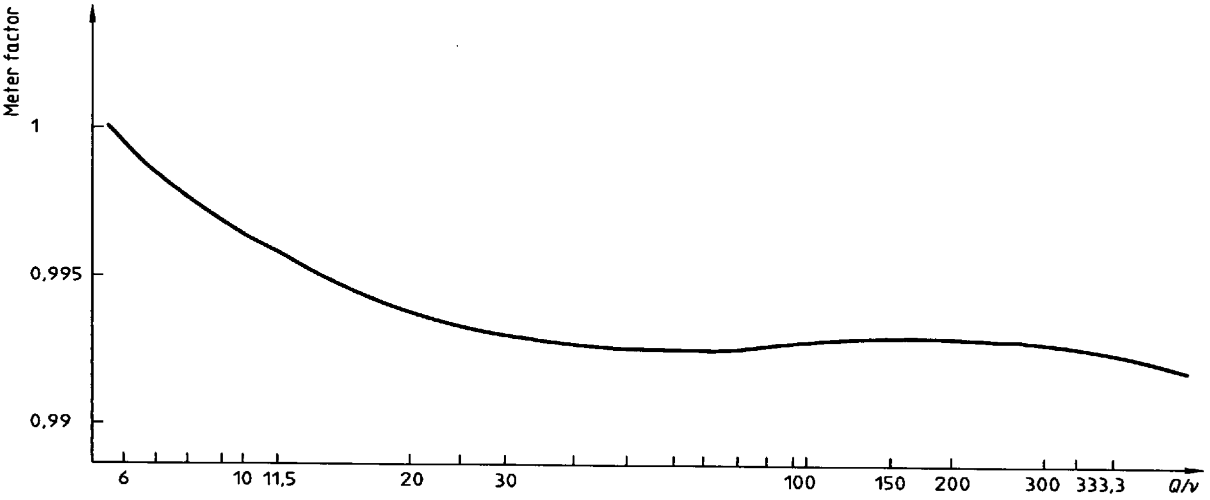


Figure 4 — Performance chart No. 2

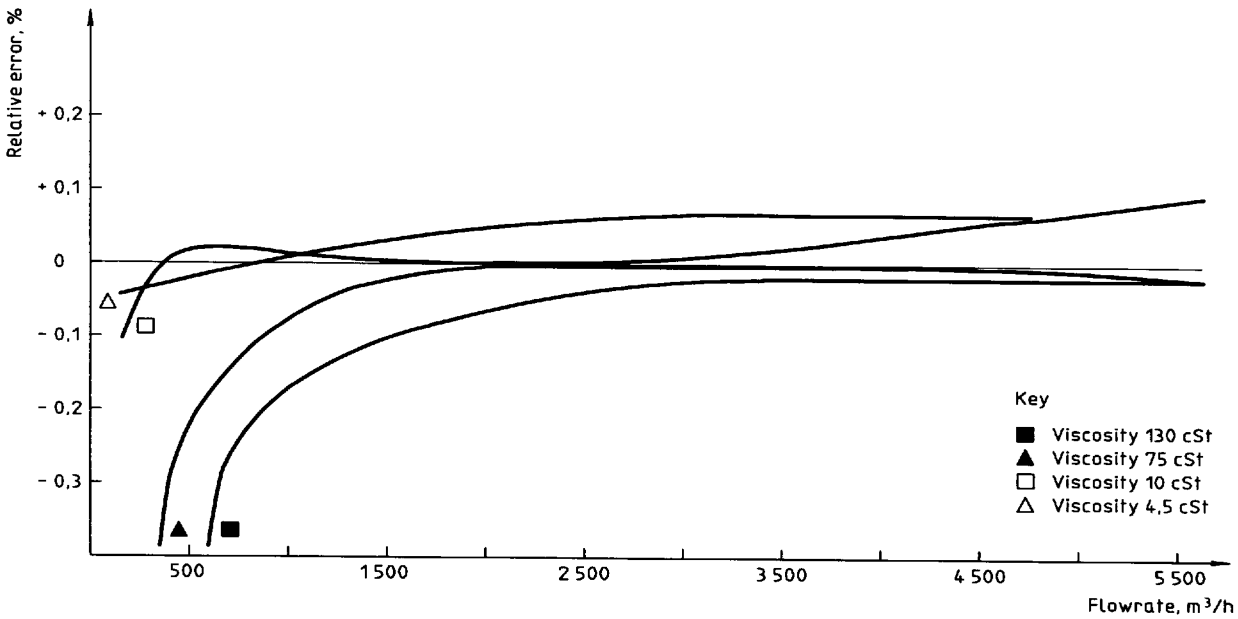
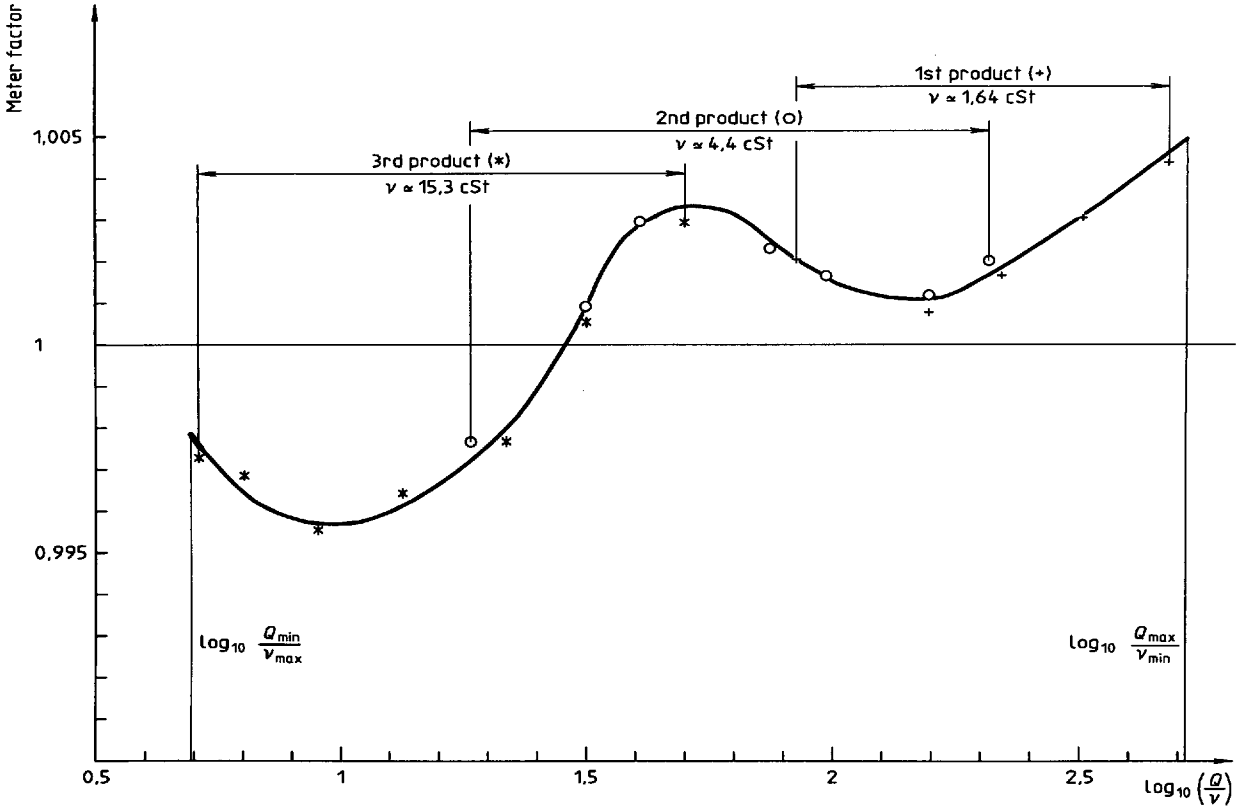


Figure 5 — Performance chart No. 3, $E = f(Q)$

3.3.3.2 Universal calibration curve for turbine meters

The results obtained for different products, flowrates and viscosities are used to establish a curve (UCC) giving MF versus the ratio Q/ν. This curve can be represented by a polynomial depending on the variable Q/ν (see figure 6).

Results can also be presented as a direct matrix (see table 1 or 2).



Polynomial equation:

$$MF = a_0 + a_1 \cdot \log(Q/\nu) + a_2 \cdot \log^2(Q/\nu) + ... + a_6 \cdot \log^6(Q/\nu)$$

$a_0 = 0,847\ 995\ 41$ $a_1 = 0,753\ 462\ 37$ $a_2 = -1,474\ 550\ 3$
$a_3 = 1,428\ 418\ 5$ $a_4 = 1,474\ 550\ 3$ $a_5 = -0,018\ 590\ 46$
$a_6 = 1,428\ 418\ 5$

Figure 6 — Performance chart No. 4 — UCC polynomial

3.3.3.2.1 Polynomial method

For a given pair [$\log_{10}(Q/\nu)$, MF], a polynomial of a given degree can be established to produce the best fit for these points. The method generally used is that of least squares (see annex E).

The polynomial degree should be chosen in accordance with the curve shape; generally polynomials of degree 4 to degree 6 are sufficient. The random uncertainty of the polynomial should be less than ± 0,1 % (see annex E).

It is necessary to check that the number of points (n) in function of the retained degree (d) is sufficient. A good choice is to take at least twice the polynomial degree plus one [$n = 2(d + 1)$]. The point spacing should be as regular as possible.

3.3.3.2.2 Direct matrix

From the UCC, a double-entry table (Q, ν) can be established giving meter factor as a function of flowrate and viscosity.

To obtain a meter factor for a flowrate or viscosity value which does not appear in the matrix, an interpolation will be necessary between the nearest appropriate values.

3.3.3.2.3 Table

A table may be prepared from the data for calibration of a turbine meter, which is similar to a direct matrix but has smaller viscosity increments.

Table 1 — Performance chart No. 5 — Direct matrix

Viscosity ν (cSt)	**Meter factor** Flowrate (m^3/h)							
	100	200	300	400	500	600	700	800
1,00	1,001 7	1,001 3	1,002 8	1,004 2	1,004 4	1,003 2	1,000 5	0,996 3
2,00	1,002 5	1,001 7	1,001 1	1,001 3	1,002 0	1,002 8	1,003 6	1,004 2
4,00	1,000 1	1,002 5	1,002 3	1,001 7	1,001 3	1,001 1	1,001 1	1,001 3
6,00	0,997 9	1,001 5	1,002 5	1,002 5	1,002 1	1,001 7	1,001 4	1,001 2
8,00	0,996 8	1,000 1	1,001 9	1,002 5	1,002 5	1,002 3	1,002 0	1,001 7
10,00	0,996 4	0,998 9	1,001 0	1,002 1	1,002 5	1,002 6	1,002 4	1,002 2
12,00	0,996 5	0,997 9	1,000 1	1,001 5	1,002 2	1,002 5	1,002 6	1,002 5
14,00	0,996 8	0,997 2	0,999 2	1,000 8	1,001 7	1,002 3	1,002 5	1,002 6
16,00	0,997 1	0,996 8	0,998 5	1,000 1	1,001 2	1,001 9	1,002 3	1,002 5

NOTE — The polynomial chart is the same as for performance chart No. 4.

Table 2 — Performance chart No. 6

Viscosity ν (cSt)	Meter factor								
	Flowrate (m^3/h)								
	50	100	150	200	250	300	350	400	450
1	0,997 3	0,997 9	0,998 9						
2	0,996 6	0,997 3	0,997 2	0,997 9	0,998 7				
3	0,994 4	0,997 4	0,997 3	0,997 1	0,997 4	0,997 9	0,998 5	0,998 9	0,998 9
4	0,992 9	0,996 6	0,997 5	0,997 3	0,997 1	0,997 2	0,997 5	0,997 9	0,998 3
5	0,992 4	0,995 5	0,997 2	0,997 5	0,997 3	0,997 2	0,997 2	0,997 3	0,997 6
6	0,992 6	0,994 4	0,996 6	0,997 4	0,997 5	0,997 3	0,997 2	0,997 1	0,997 2
7	0,993 2	0,993 5	0,995 9	0,997 1	0,997 4	0,997 4	0,997 3	0,997 2	0,997 1
8	0,994 0	0,992 9	0,995 1	0,996 6	0,997 3	0,997 5	0,997 4	0,997 3	0,997 2
9	0,994 7	0,992 5	0,994 4	0,996 1	0,997 0	0,997 4	0,997 5	0,997 4	0,997 3
10	0,995 3	0,992 4	0,993 8	0,995 5	0,996 6	0,997 2	0,997 4	0,997 5	0,997 4
11	0,995 5	0,992 4	0,993 3	0,994 9	0,996 2	0,996 9	0,997 3	0,997 5	0,997 5
12	0,995 3	0,992 6	0,992 9	0,994 4	0,995 7	0,996 6	0,997 1	0,997 4	0,997 5
13		0,992 8	0,992 6	0,994 0	0,995 3	0,996 3	0,996 9	0,997 2	0,997 4
14		0,993 2	0,992 5	0,993 5	0,994 8	0,995 9	0,996 6	0,997 1	0,997 3
15		0,993 6	0,992 4	0,993 2	0,994 4	0,995 5	0,996 3	0,996 9	0,997 2
16		0,994 0	0,992 4	0,992 9	0,994 0	0,995 1	0,996 0	0,996 6	0,997 0
17		0,994 4	0,992 4	0,992 7	0,993 7	0,994 8	0,995 7	0,996 3	0,996 8
18		0,994 7	0,992 6	0,992 5	0,993 4	0,994 4	0,995 3	0,996 1	0,996 6
19		0,995 0	0,992 7	0,992 4	0,993 1	0,994 1	0,995 0	0,995 8	0,996 4
20		0,995 3	0,992 9	0,992 4	0,992 9	0,993 8	0,994 7	0,995 5	0,996 1
21		0,995 4	0,993 2	0,992 4	0,992 7	0,993 5	0,994 4	0,995 2	0,995 9
22		0,995 5	0,993 4	0,992 4	0,992 6	0,993 3	0,994 2	0,994 9	0,995 6
23		0,995 5	0,993 7	0,992 5	0,992 5	0,993 1	0,993 9	0,994 7	0,995 4
24		0,995 3	0,994 0	0,992 6	0,992 4	0,992 9	0,993 7	0,994 4	0,995 1
25			0,994 2	0,992 7	0,992 4	0,992 8	0,993 4	0,994 2	0,994 9
26			0,994 5	0,992 8	0,992 4	0,992 6	0,993 2	0,994 0	0,994 7
27			0,994 7	0,993 0	0,992 4	0,992 5	0,993 1	0,993 7	0,994 4
28			0,994 9	0,993 2	0,992 4	0,992 5	0,992 9	0,993 5	0,994 2
29			0,995 1	0,993 4	0,992 5	0,992 4	0,992 8	0,993 4	0,994 0
30			0,995 3	0,993 6	0,992 6	0,992 4	0,992 7	0,993 2	0,993 8
31			0,995 4	0,993 8	0,992 7	0,992 4	0,992 6	0,993 0	0,993 6
32			0,995 5	0,994 0	0,992 8	0,992 4	0,992 5	0,992 9	0,993 5
33			0,995 5	0,994 2	0,992 9	0,992 4	0,992 4	0,992 8	0,993 3
34			0,995 5	0,994 4	0,993 0	0,992 4	0,992 4	0,992 7	0,993 2
35			0,995 5	0,994 6	0,993 2	0,992 5	0,992 4	0,992 6	0,993 0
36			0,995 3	0,994 7	0,993 3	0,992 6	0,992 4	0,992 5	0,992 9
37			0,995 2	0,994 9	0,993 5	0,992 6	0,992 4	0,992 5	0,992 8
38				0,995 0	0,993 7	0,992 7	0,992 4	0,992 4	0,992 7
39				0,995 2	0,993 8	0,992 8	0,992 4	0,992 4	0,992 6
40				0,995 3	0,994 0	0,992 9	0,992 4	0,992 4	0,992 6

3.4 Control charts and tests

Control charts and tests can be made for meter factor, K-factor or relative error, depending on the type of application (see 2.2.5).

3.4.1 Quality of meters

Meters which are proved in a centralized proving station should have performance characteristics which are sufficiently stable in terms of time and throughput to achieve a good repeatability.

3.4.2 Frequency of proving

The frequency of proving may be a function of time or a function of total volume metered. The meter should be proved as soon as one of these two limits is reached.

The meter should be re-proved if there is any defect in metering, even if no limit has been reached.

3.4.2.1 Re-proving as a function of time

The meter-factor drift is usually on the order of 0,1 % per year. The time interval between two consecutive provings should be judiciously chosen in order that the proving will be significant.

The time interval is mainly based on the operational experience of the meter, usually six months to one year even if the meter has not been used.

3.4.2.2 Re-proving as a function of volume

A re-proving of the meter is also required in terms of the volume measured by the meter. The limit to this volume before re-proving is based on the experience of the meter's behaviour as indicated by the control charts.

3.4.3 Control charts and tests corresponding to any meter

The meter-factor variations of a meter from one proving to another can be used to monitor the meter.

If the following criteria are met, the meter can be used; if not, an investigation of the meter is necessary.

a) The difference between E_{max} and E_{min} of the new calibration curve should be less than 0,5 % (see figure 7).

b) The difference between the old and new curves should be less than 0,1 % (see figure 8).

3.4.4 Control charts and tests via polynomials

The meter-factor variation analysis for an individual meter is evaluated by the use of the polynomials $\mathrm{MF} = f(Q/\nu)$ or with control charts (see figures 9 and 10). The following criteria should be met.

a) The difference between MF_{max} and MF_{min} of the new calibration curve should be less than 0,5 (see figure 9), that is:

$$200 \frac{(\mathrm{MF}_{max} - \mathrm{MF}_{min})}{(\mathrm{MF}_{max} + \mathrm{MF}_{min})} \leqslant 0{,}5$$

b) The random uncertainty of the polynomial (see annex E) should be less than $\pm$ 0,1 %.

c) The maximum difference calculated from the old and new polynomials should be less than 0,1 %.

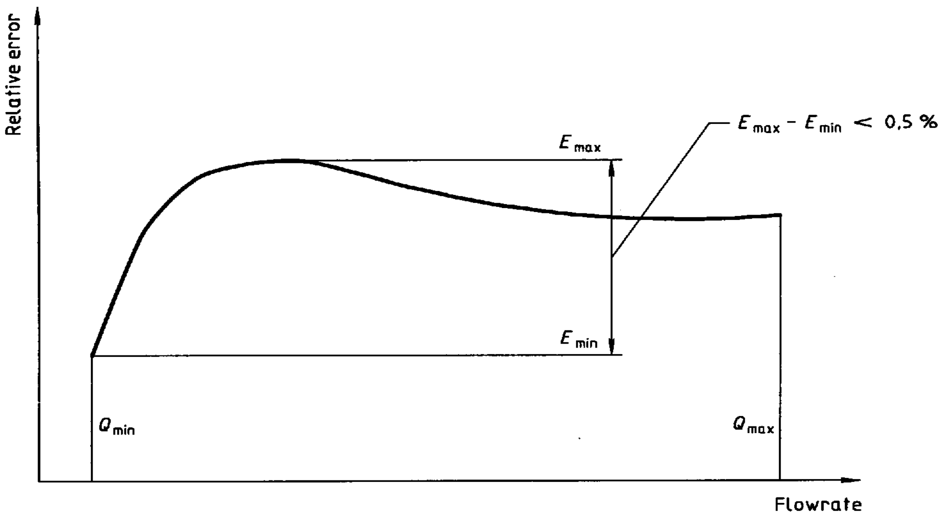


Figure 7 — Control chart No. 1

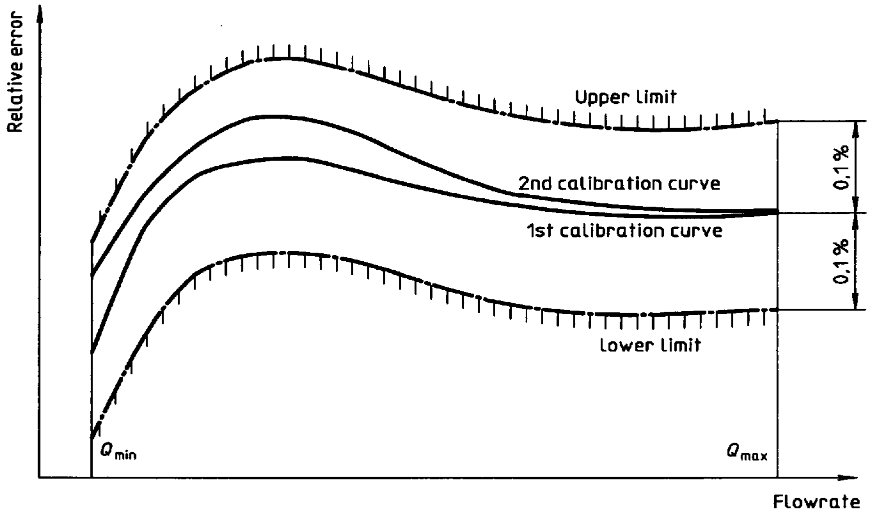


Figure 8 — Control chart No. 2

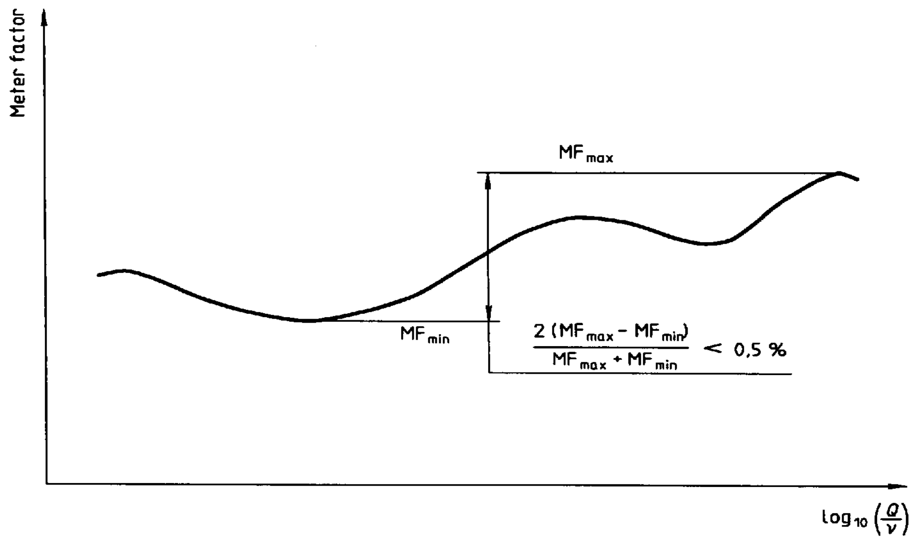


Figure 9 — Control chart No. 3

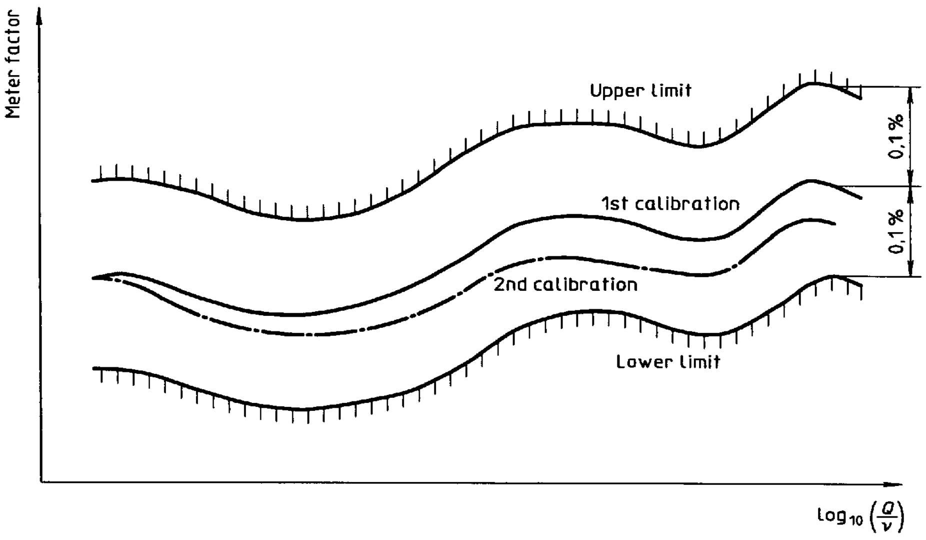


Figure 10 — Control chart No. 4

3.4.5 Control charts via direct matrix

The meter-factor variation analysis for an individual meter is made directly with the matrix. The following criteria should be met.

a) See 3.4.4 a).

b) The maximum difference between the old and new meter-factor values corresponding to the same inputs (Q, ν) should be calculated. This difference should be less than 0,1 %.

3.5 Worked examples

3.5.1 Scope of examples

Examples of central proving calculations and the use of control charts are given in this section as follows:

Example 1: Test for outliers
Example 2: Test for repeatability
Example 3: Test of the range
Example 4: Resulting values
Example 5: Preparation of data
Example 6: Universal calibration curve

3.5.2 Example 1: Test for outliers

3.5.2.1 Given

A 10-inch turbine meter proved with a unidirectional pipe prover with a base volume at 20 °C and 100 kPa (1 bar) between detectors of 9 955,4 litres gave the following results:

No.	Meter factor
1	1,001 5
2	1,001 4
3	1,002 2
4	1,001 3

3.5.2.2 Find

Any outliers?

3.5.2.3 Solution

a) **Step 1**

Rearrange measurements in order of ascending magnitude and apply Dixon's Test for outliers (see annex D) at the 95 % probability level.

x_1	No. 4	1,001 3
x_2	No. 2	1,001 4
x_3	No. 1	1,001 5
x_4	No. 3	1,002 2

b) **Step 2**

$$R_{10} = \frac{x_4 - x_3}{x_4 - x_1} = \frac{1,002\ 2 - 1,001\ 5}{1,002\ 2 - 1,001\ 3} = \frac{7}{9} = 0,777$$

where R is the Dixon ratio.

As 0,777 is larger than the critical value of 0,765, measurement x_4 is rejected as an outlier.

3.5.3 Example 2: Test for repeatability

3.5.3.1 Given

The same turbine meter as in example 1 gave the following results.

No.	Meter factor
1	$x_1 = 0,995\ 8$
2	$x_2 = 0,996\ 3$

The estimated repeatability for this type of meter is

$r = 0,000\ 4$

3.5.3.2 Find

Are these two measurements acceptable?

3.5.3.3 Solution

a) **Step 1**

Calculate

$$x_2 - x_1 = 0,996\ 3 - 0,995\ 8 = 0,000\ 5$$

As 0,000 5 is larger than $r = 0,000\ 4$, three additional measurements are made:

$x_3 = 0,995\ 6$
$x_4 = 0,995\ 7$
$x_5 = 0,995\ 7$

b) **Step 2**

Rearrange the five measurements in order of ascending magnitude to find the most divergent value.

0,995 6 ...... 0,000 1
0,995 7
0,995 7
0,995 8
0,996 3 ...... 0,000 5

The most divergent value is 0,996 3.

c) **Step 3**

Calculate

$$\frac{0{,}995\,6 + 0{,}995\,7 + 0{,}995\,7 + 0{,}995\,8}{4} = 0{,}995\,7$$

$$0{,}996\,3 - 0{,}995\,7 = 0{,}000\,6$$

$$r \cdot \sqrt{\frac{n}{2(n-1)}} = 0{,}000\,4 \cdot \sqrt{\frac{5}{2(5-1)}} = 0{,}000\,3$$

As 0,000 6 is larger than r, then x_2 is rejected.

3.5.4 Example 3: Test of the range

3.5.4.1 Given

The same turbine meter as in example 1 gave the following measurements.

No.	Meter factor
1	0,995 8
2	0,995 9
3	0,997 2

3.5.4.2 Find

Are these measurements acceptable?

3.5.4.3 Solutions

a) **Case 1**

Standard deviation is known ($\sigma = 0{,}000\,4$).

$$MF_{max} - MF_{min} = 0{,}997\,2 - 0{,}995\,8 = 0{,}001\,4$$

The limiting value for 95 % probability $E_1(n) = 3{,}31$ for $n = 3$ (see annex A, table A.1). Thus $\sigma \cdot E_1(n) = 0{,}000\,4 \times 3{,}31 = 0{,}001\,324$.

As 0,001 4 is larger than 0,001 324, the third (most divergent) measurement is not retained.

b) **Case 2**

Standard deviation has been estimated as $s = 0{,}000\,4$ with 20 degrees of freedom, from an independent exercise excluding the measurements in question.

$$MF_{max} - MF_{min} = 0{,}997\,2 - 0{,}995\,8 = 0{,}001\,4$$

The limiting value for 95 % probability $E_2(3,\ 20) = 3{,}58$ (see annex A, table A.2). Thus $s \cdot E_2(n,\ \Phi) = 0{,}000\,4 \times 3{,}58 = 0{,}001\,43$.

As 0,001 4 is smaller than 0,001 43, the measurements are retained.

c) **Case 3**

σ or s is not known, but the critical value for a range of three values is given as 0,05 % of the mean.

$$MF_{max} - MF_{min} = 0{,}997\,2 - 0{,}995\,8 = 0{,}001\,4$$

$$0{,}05\ \%\ \text{of the mean} = 0{,}000\,5\,\frac{(0{,}995\,8 + 0{,}995\,4 + 0{,}997\,2)}{3} = 0{,}000\,5$$

As 0,001 4 is larger than 0,000 5 (see 3.2.2.2.2), the third (most divergent) measurement is not retained.

3.5.5 Example 4: Resulting values

3.5.5.1 Given

The same turbine meter as in example 1 gave the following results for one point (Q, ν):

No.	Meter factor
1	0,995 7
2	0,995 9
3	0,996 2

3.5.5.2 Find

What is the mean value of the meter factor? What is the estimated standard deviation?

3.5.5.3 Solution

a) **Step 1**

Find the mean.

$$\bar{x} = \frac{1}{n} \sum x_i = 0{,}995\,93$$

b) **Step 2**

Find the standard deviation:

$$s(x) = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

n	x_i	$(x_i - \bar{x})$	$(\bar{x} - x_i)^2$
1	0,995 7	− 0,000 23	0,000 000 052 9
2	0,995 9	− 0,000 03	0,000 000 000 9
3	0,996 2	− 0,000 27	0,000 000 072 9
			0,000 000 126 7

$$s(x) = 0{,}000\,25$$

c) **Step 3**

Find the uncertainty of a single measurement $u(x) = t_{95,\ n-1}s(x)$:

$$u(x) = 4{,}303 \times 0{,}000\ 25 = 0{,}001\ 08$$

Find the uncertainty of the mean of three measurements $u(\bar{x}) = t_{95,\ n-1}s(x)\sqrt{n}$:

$$u(\bar{x}) = \frac{4{,}303 \times 0{,}000\ 25}{\sqrt{3}} = 0{,}000\ 6$$

3.5.6 Example 5: Preparation of data

3.5.6.1 Given

The unidirectional prover used in a centralized station has the following characteristics:

$V_p = 2\ 502{,}5$ litres

$C_{tp} = 35 \times 10^{-6}$ °C

$C_{pp} = 25 \times 10^{-6}$ bar

$t_{0p} = 20$ °C

p_{0p} = atmospheric pressure

The pulse interpolation technique used is the double chronometer method (see figure 11).

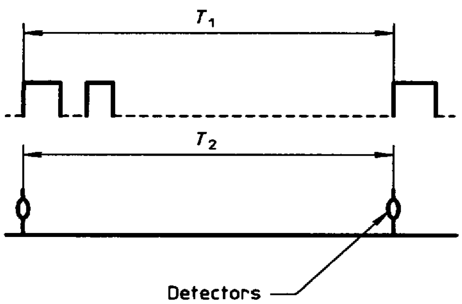


Figure 11 — Double chronometer method of pulse interpolation

The product used to prove the turbine meter is a distillate having a viscosity of 4,10 cSt at 20 °C.

The laboratory has provided the following equation to calculate the viscosity of the product at a given temperature:

$$\log_{10}\log_{10}(\nu t + 0{,}7) = A - B \cdot \log_{10}(273{,}15 + t)$$

where

$A = 10{,}252$

$B = 4{,}223$

The turbine meter has the following characteristics:

pulse value = 0,5 litre/pulse

$K\text{-factor}_{nominal} = 1/0,5 = 2$ pulses/litre

$C_{tm} = 69 \times 10^{-6}$ °C

$C_{pm} = 0$

The test results are given in table 3.

Table 3 — Example 5: Test results for turbine meter

No.	Temperature °C	Pressure bar	T_1 10^{-4} s	T_2 10^{-4} s	Number of pulses N
1	9,4	3,0	201 576	201 594	5 016
2	9,6	3,0	200 126	200 120	5 016
3	10,0	3,0	335 234	335 266	5 023
4	10,6	3,0	335 352	335 368	5 024
5	10,7	3,0	496 172	496 183	5 024

3.5.6.2 Find

For test point No. 1 calculate:

— flowrate;

— viscosity;

— K-factor;

— meter factor.

3.5.6.3 Solution

a) **Flowrate,** Q, in cubic metres per hour

$$Q = \frac{V_p \times 3,6}{T_2} \left[1 + \left(35 \times 10^{-6}\right)(t - 20) + \left(25 \times 10^{-6}\right)p\right]$$

where

$t = 9,4$ °C

$p = 3,0$ bar

$T_2 = 201\ 594 \times 10^{-4}$ s

$$Q = \frac{2502,5 \times 3,6}{20,159\ 4} \left[1 + \left(35 \times 10^{-6}\right)(9,4 - 20) + \left(25 \times 10^{-6}\right) \times 3\right]$$

$$= 446,888 \left[1 - \left(3,71 \times 10^{-4}\right) + \left(7,5 \times 10^{-5}\right)\right]$$

$$= 446,888 \times 0,999\ 704 = 446,756 \text{ m}^3/\text{h}$$

b) **Viscosity**

$$\log_{10}\log_{10}(\nu_t + 0{,}7) = 10{,}252 - 4{,}223 \times \log_{10}(273{,}15 + t)$$

where

$t = 9{,}4$ °C

$\nu_t = 5{,}55$ cSt

c) ***K*-factor**

$$K = \frac{N}{V_p} \cdot \frac{T_2}{T_1} \cdot \frac{1 + C_{tm}(t - t_{0m}) + C_{pm}(p - p_{0m})}{1 + C_{tp}(t - t_{0p}) + C_{pp}(p - p_{0p})}$$

$$= \frac{5\ 016}{2\ 502{,}5} \times \frac{201\ 594}{201\ 576} \times \frac{1 + \left(69 \times 10^{-6}\right)(9{,}4 - 20)}{1 + \left(35 \times 10^{-6}\right)(9{,}4 - 20) + \left(25 \times 10^{-6}\right) \times 3}$$

$$= 2{,}004\ 395\ 6 \times 1{,}000\ 089\ 3 \times \frac{0{,}999\ 268\ 6}{0{,}999\ 704}$$

$$= 2{,}003\ 7 \text{ pulses/litre}$$

d) **Meter factor**

$$\text{MF} = \frac{2}{K} = \frac{2}{2{,}003\ 7} = 0{,}998\ 2$$

3.5.7 Example 6: Universal calibration curve (UCC)

3.5.7.1 Given

A 6-inch turbine meter was proved each year for three years (1978, 1979, 1980) with three products in a centralized station.

The characteristics of the prover are:

— unidirectional

— $V_p = 2\ 503{,}2$ litres (for 1978 and 1979) and 2 503,5 litres after recalibration in 1980

— $C_{pt} = 49 \times 10^{-6}$ °C

— $C_{pp} = 31 \times 10^{-6}$ bar

The products used to prove the turbine meter were:

— jet fuel

— distillate

— light fuel oil

The centralized station has given a test report for each year, in which are indicated

— product and viscosity at 20 °C;

— calculated flowrate, in cubic metres per hour;

— calculated kinematic viscosity at temperature of the test;

— $\log_{10}(Q/\nu)$;

— meter factor;

— coefficients of the polynomial equation (UCC).

Attached to this test report are the

— performance chart (see figures 12 to 17);

— tables of meter factors (see tables 4 to 9);

— UCC.

3.5.7.2 Find

MF_{max} and MF_{min}; the following criteria should be met.

a) **Rule No. 1:**

$$\frac{2(MF_{max} - MF_{min})}{MF_{max} + MF_{min}} < 0{,}005\ 0$$

for an oil with a viscosity range 3 cSt to 30 cSt.

b) **Rule No. 2:**

The random uncertainty of the polynomial is $< \pm$ 0,1 %.

c) **Rule No. 3:**

The maximum difference between the polynomials is < 0,1 %.

3.5.7.3 Solutions

3.5.7.3.1 Analysis of 1978 Test Report

a) **Rule No. 1:**

From table of meter factors, find the maximum and minimum values of the meter factor:

$MF_{max} = 0{,}998\ 7$

$MF_{min} = 0{,}994\ 3$

Calculate the ratio:

$$\frac{2(0{,}998\ 7 - 0{,}994\ 3)}{0{,}998\ 7 + 0{,}994\ 3} = 0{,}004\ 4$$

As this ratio is smaller than 0,5 %, the turbine meter can be used.

b) **Rule No. 2:**

From table 6 the random uncertainty of the polynomial is ± 0,04 %. As this value is < ± 0,1 %, the polynomial can be used.

MF_{max} and MF_{min} can also be found on the performance chart.

3.5.7.3.2 Analysis of 1979 Test Report

a) **Rule No. 1:**

On the table of meter factors, find the maximum and minimum values of the meter factors:

$MF_{max} = 0{,}998\ 5$

$MF_{min} = 0{,}993\ 8$

Calculate the ratio:

$$\frac{2(0{,}998\ 5 - 0{,}993\ 8)}{0{,}998\ 5 + 0{,}993\ 8} = 0{,}004\ 7$$

As this ratio is < 0,5 %, the turbine meter satisfies Rule No. 1.

b) **Rule No. 2:**

From table 9 the random uncertainty of the polynomial is ± 0,02 %. As this value is < ± 0,1 %, the polynomial can be used.

c) **Rule No. 3:**

— Using the polynomial method, the difference between the two polynomials (1978 and 1979) (see figure 18) is directly calculated for all the values of $\log_{10}(Q/\nu)$. The results appear on the control chart.

The maximum observed difference between $\log_{10}(Q_{min}/\nu_{max})$ and $\log_{10}(Q_{max}/\nu_{min})$ is 0,08 %. As it is < 0,1 %, the turbine meter satisfies Rule No. 3.

— Using the direct matrix method

ν (cSt)	Flowrate (m^3/h) 100			250			400		
	1978	1979	Diff.	1978	1979	Diff.	1978	1979	Diff.
5	0,994 8	0,995 2	0,000 4	0,996 3	0,996 5	0,000 2	0,996 7	0,997 4	0,000 7
10	0,994 3	0,993 7	0,000 6	0,995 2	0,995 7	0,000 5	0,996 0	0,996 5	0,000 5
15	0,994 8	0,994 2	0,000 6	0,994 6	0,994 7	0,000 1	0,995 3	0,995 9	0,000 6
20	0,995 8	0,995 6	0,000 2	0,994 3	0,994 0	0,000 3	0,994 8	0,995 2	0,000 4
25				0,994 3	0,993 7	0,000 6	0,994 5	0,994 6	0,000 1
30				0,994 4	0,993 7	0,000 7	0,994 3	0,994 1	0,000 2
35				0,994 6	0,994 0	0,000 6	0,994 3	0,995 8	0,000 5
40				0,995 0	0,994 5	0,000 5	0,994 3	0,993 7	0,000 6

The maximum observed difference is 0,07 %; the turbine meter thus satisfies Rule No. 3.

d) **Conclusions:**

As rules No. 1, 2 and 3 are satisfied, the turbine meter can be used.

3.5.7.3.3 Analysis of 1980 Test Report

a) **Rule No. 1:**

On the table of meter factors, find the maximum and minimum values of the meter factor.

$MF_{max} = 0{,}998\ 9$

$MF_{min} = 0{,}992\ 4$

Calculate the ratio:

$$\frac{2(0{,}998\ 9 - 0{,}992\ 4)}{0{,}998\ 9 + 0{,}992\ 4} = 0{,}006\ 5$$

As this ratio is larger than 0,5 %, the turbine meter does not satisfy the rule.

b) **Rule No. 2:**

From table 12 the random uncertainty of the polynomial is ± 0,16 %. As this value is > ± 0,1 %, the polynomial cannot be used.

c) **Rule No. 3:**

The difference between the two polynomials (1979 and 1980) (see figure 19) is directly calculated for all the values of $\log_{10}(Q/\nu)$. The maximum observed difference is 0,13 % within the range $\log_{10}(Q_{min}/\nu_{max})$ and $\log_{10}(Q_{max}/\nu_{min})$. As this value is > 0,1 %, the turbine meter does not satisfy the rule.

d) **Conclusions:**

The turbine meter cannot be used; it should be repaired by the manufacturer and re-submitted for calibration.

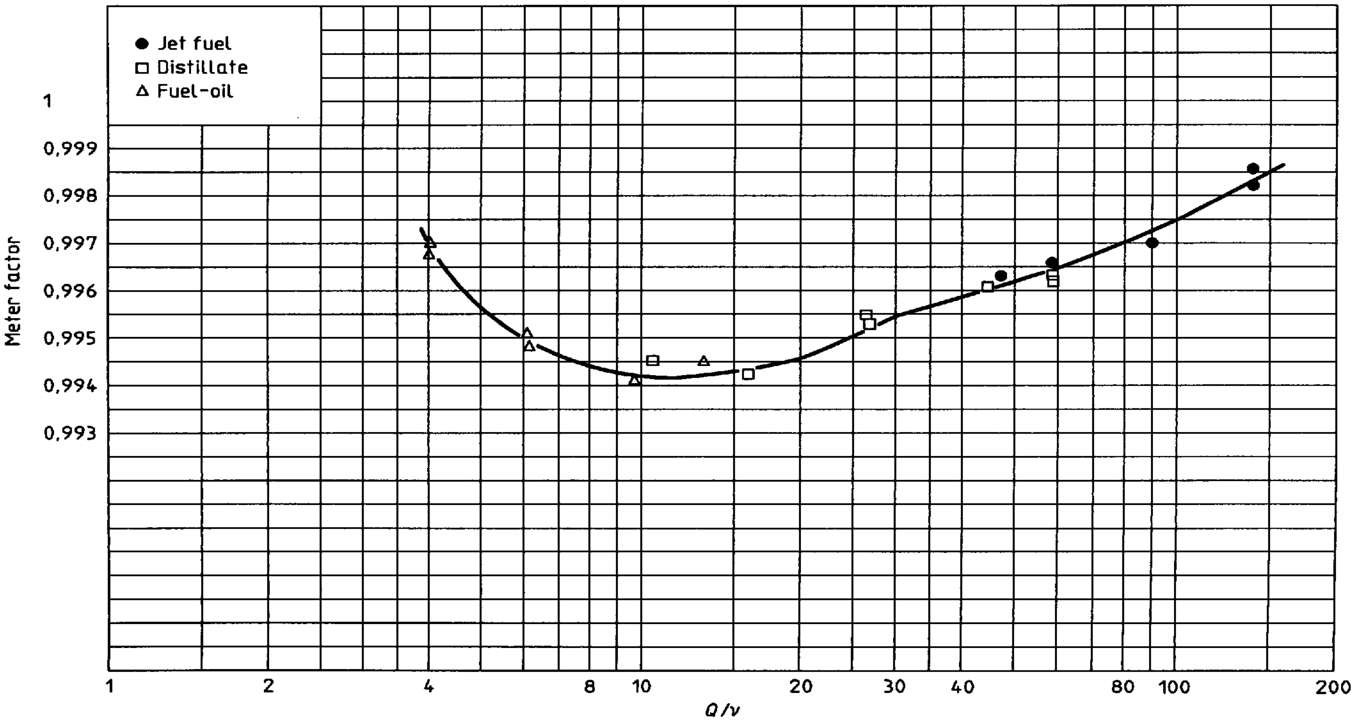


Figure 12 — Performance chart, turbine No. 310, 1978

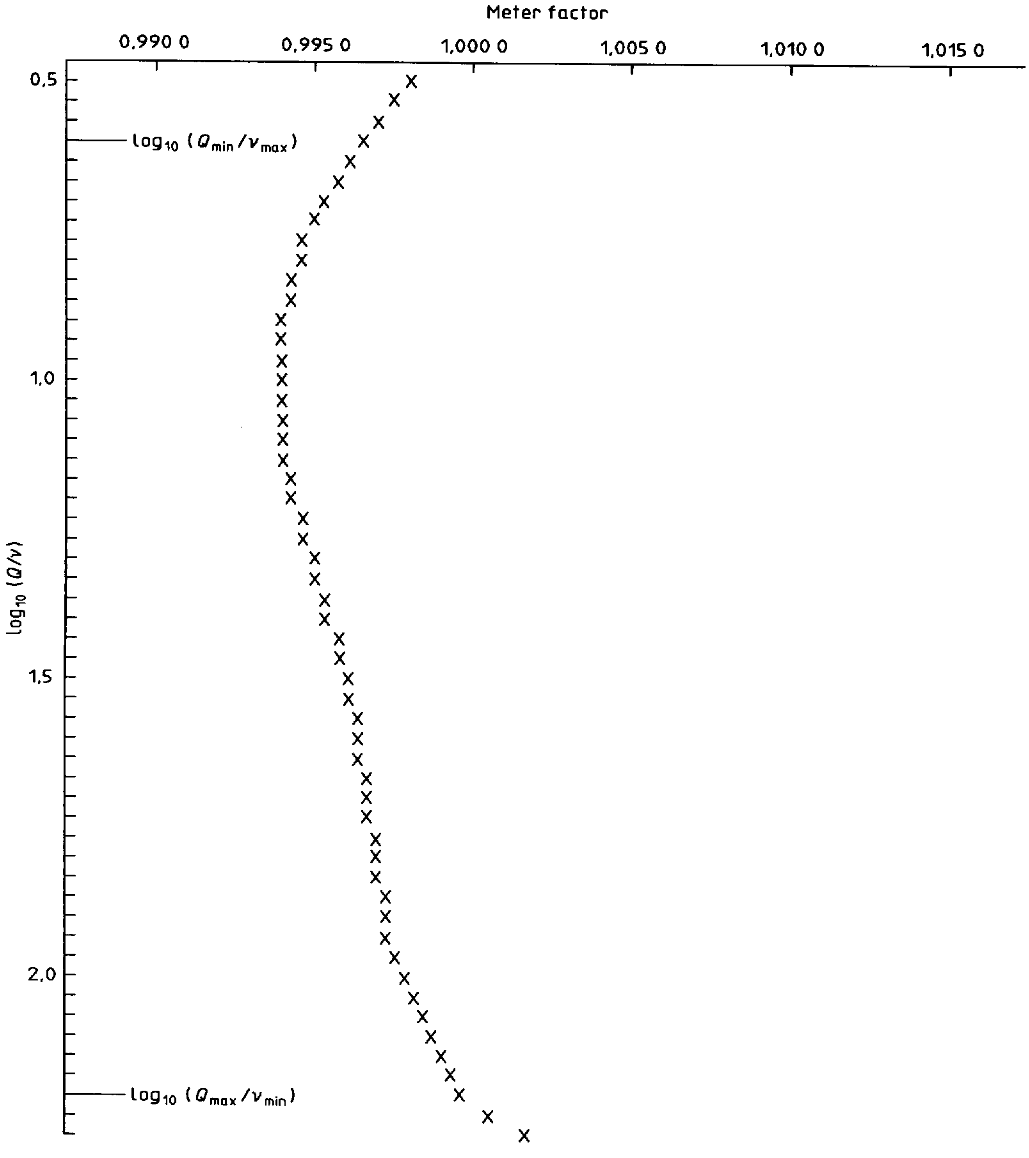


Figure 13 — Log(Q/ν) vs. meter factor, turbine No. 310, 1978

Table 4 — Performance chart, meter No. 310 test report, 1978

Prover loop	**Meter**
$V_p = 2\ 503{,}2$ litres $C_{tp} = 49 \times 10^{-6}$ °C $C_{pp} = 31 \times 10^{-6}$ bar	turbine meter No. 310 type: 6-inch $p = 0{,}5$ litre/pulse

Product	**Calculated flowrate** m³/h	**Calculated kinematic viscosity at t** cSt	$\text{Log}_{10}(Q/\nu)$	**Meter factor**
Jet fuel $\nu_{20} = 1{,}70$ cSt	273,50	1,98	2,140	0,998 2
	282,58	1,97	2,157	0,998 4
	178,14	1,97	1,956	0,997 0
	178,17	1,97	1,956	0,997 0
	116,10	1,97	1,770	0,996 5
	116,55	1,95	1,776	0,996 5
	79,13	1,94	1,611	0,996 3
	79,44	1,93	1,614	0,996 3
Distillate $\nu_{20} = 4{,}40$ cSt	371,42	5,37	1,840	0,996 5
	371,65	5,34	1,843	0,996 3
	244,50	5,33	1,662	0,995 9
	244,79	5,31	1,664	0,995 9
	141,36	5,29	1,427	0,995 3
	141,46	5,28	1,428	0,995 5
	87,27	5,25	1,221	0,994 3
	87,18	5,22	1,223	0,994 3
	53,39	5,19	1,012	0,994 5
	52,92	5,17	1,010	0,994 5
Fuel oil $\nu_{20} = 45{,}50$ cSt	413,86	30,90	1,127	0,994 5
	413,91	30,60	1,131	0,994 5
	294,10	30,10	0,990	0,994 1
	294,99	30,10	0,991	0,994 1
	183,58	30,00	0,787	0,994 9
	183,83	30,00	0,787	0,995 1
	120,19	29,80	0,606	0,996 8
	120,20	29,70	0,607	0,997 0

Universal Calibration Curve: polynomial equation

$$\text{MF} = a_0 + a_1 \cdot \log_{10}(Q/\nu) + a_2 \cdot [\log_{10}(Q/\nu)]^2 + ... + a_6 \cdot [\log_{10}(Q/\nu)]^6$$

where

$a_0 = 1{,}017\ 619$ $a_1 = -6{,}510\ 977$ (E-2) $a_2 = 7{,}846\ 935$ (E-2)
$a_3 = -6{,}678\ 370$ (E-2) $a_4 = 0{,}045\ 565\ 26$ $a_5 = -1{,}851\ 974$
$a_6 = 3{,}025\ 942$ (E-3)

Table 5 — Meter factor for turbine meter No. 310, 1978

ν	Flowrate (m^3/h)								
(cSt)	50	100	150	200	250	300	350	400	450
1	0,996 3	0,997 1	0,998 7						
2	0,995 2	0,996 3	0,996 7	0,997 1	0,997 7				
3	0,994 6	0,995 7	0,996 3	0,996 5	0,996 8	0,997 1	0,997 5	0,998 0	0,998 7
4	0,994 3	0,995 2	0,995 9	0,996 3	0,996 5	0,996 7	0,996 8	0,997 1	0,997 4
5	0,994 3	0,994 8	0,995 5	0,996 0	0,996 3	0,996 4	0,996 6	0,996 7	0,996 9
6	0,994 4	0,994 6	0,995 2	0,995 7	0,997 6	0,996 3	0,996 4	0,996 5	0,996 7
7	0,994 6	0,994 4	0,995 0	0,995 5	0,995 8	0,996 1	0,996 3	0,996 4	0,996 5
8	0,995 0	0,994 3	0,994 7	0,995 2	0,995 6	0,995 9	0,996 1	0,996 3	0,996 4
9	0,995 4	0,994 3	0,994 6	0,995 0	0,995 4	0,995 7	0,995 9	0,996 1	0,996 3
10	0,995 8	0,994 3	0,994 5	0,994 8	0,995 2	0,995 5	0,995 8	0,996 0	0,996 1
11	0,996 2	0,994 3	0,994 4	0,994 7	0,995 1	0,995 4	0,995 6	0,995 8	0,996 0
12	0,996 7	0,994 4	0,994 3	0,994 6	0,994 9	0,995 2	0,995 5	0,995 7	0,995 9
13		0,994 5	0,994 3	0,994 5	0,994 8	0,995 1	0,995 4	0,995 6	0,995 8
14		0,994 6	0,994 3	0,994 4	0,994 7	0,995 0	0,995 2	0,995 5	0,995 6
15		0,994 8	0,994 3	0,994 3	0,994 6	0,994 8	0,995 1	0,995 3	0,995 5
16		0,995 0	0,994 3	0,994 3	0,994 5	0,994 7	0,995 0	0,995 2	0,995 4
17		0,995 2	0,994 3	0,994 3	0,994 4	0,994 7	0,994 9	0,995 1	0,995 3
18		0,995 4	0,994 4	0,994 3	0,994 4	0,994 6	0,994 8	0,995 0	0,995 2
19		0,995 6	0,994 5	0,994 3	0,994 3	0,994 5	0,994 7	0,994 9	0,995 1
20		0,995 8	0,994 6	0,994 3	0,994 3	0,994 5	0,994 6	0,994 8	0,995 0
21		0,996 0	0,994 6	0,994 3	0,994 3	0,994 4	0,994 6	0,994 8	0,995 0
22		0,996 2	0,994 7	0,994 3	0,994 3	0,994 4	0,994 5	0,994 7	0,994 9
23		0,996 5	0,994 9	0,994 4	0,994 3	0,994 3	0,994 5	0,994 6	0,994 8
24		0,996 7	0,995 0	0,994 4	0,994 3	0,994 3	0,994 4	0,994 6	0,994 7
25			0,995 1	0,994 5	0,994 3	0,994 3	0,994 4	0,994 5	0,994 7
26			0,995 2	0,994 5	0,994 3	0,994 3	0,994 4	0,994 5	0,994 6
27			0,995 4	0,994 6	0,994 3	0,994 3	0,994 3	0,994 4	0,994 6
28			0,995 5	0,994 6	0,994 3	0,994 3	0,994 3	0,994 4	0,994 5
29			0,995 6	0,994 7	0,994 4	0,994 3	0,994 3	0,994 4	0,994 5
30			0,995 8	0,994 8	0,994 4	0,994 3	0,994 3	0,994 3	0,994 5
31			0,995 9	0,994 9	0,994 4	0,994 3	0,994 3	0,994 3	0,994 4
32			0,996 1	0,995 0	0,994 5	0,994 3	0,994 3	0,994 3	0,994 4
33			0,996 2	0,995 1	0,994 5	0,994 3	0,994 3	0,994 3	0,994 4
34			0,996 4	0,995 2	0,994 6	0,994 3	0,994 3	0,994 3	0,994 3
35			0,996 5	0,995 3	0,994 6	0,994 4	0,994 3	0,994 3	0,994 3
36			0,996 7	0,995 4	0,994 7	0,994 4	0,994 3	0,994 3	0,994 3
37			0,996 9	0,995 5	0,994 8	0,994 4	0,994 3	0,994 3	0,994 3
38				0,995 6	0,994 8	0,994 5	0,994 3	0,994 3	0,994 3
39				0,995 7	0,994 9	0,994 5	0,994 3	0,994 3	0,994 3
40				0,995 8	0,995 0	0,994 6	0,994 4	0,994 3	0,994 3

Table 6 — Random uncertainty of polynomial for turbine No. 310, 1978

$\log_{10}(Q/\nu)$	Meter factor		Deviation
	y	$\hat{y}$ (polynomial)	$y - \hat{y}$ (polynomial)
2,140	0,998 2	0,998 203	– 0,000 003
2,157	0,998 4	0,998 422	– 0,000 022
1,956	0,997 0	0,996 894	0,000 106
1,956	0,997 0	0,996 894	0,000 106
1,770	0,996 5	0,996 416	0,000 084
1,776	0,996 5	0,996 429	0,000 071
1,611	0,996 3	0,996 003	0,000 297
1,614	0,996 3	0,996 012	0,000 288
1,840	0,996 5	0,996 570	– 0,000 070
1,843	0,996 3	0,996 577	– 0,000 277
1,622	0,995 9	0,996 152	– 0,000 252
1,664	0,995 9	0,996 157	– 0,000 257
1,427	0,995 3	0,995 342	– 0,000 042
1,428	0,995 5	0,995 346	0,000 155
1,221	0,994 3	0,994 579	– 0,000 279
1,223	0,994 3	0,994 586	– 0,000 286
1,012	0,994 5	0,994 260	0,000 240
1,010	0,994 5	0,994 261	0,000 239
1,127	0,994 5	0,994 354	0,000 153
1,131	0,994 5	0,994 354	0,000 146
0,990	0,994 1	0,994 275	– 0,000 175
0,991	0,994 1	0,994 274	– 0,000 174
0,787	0,994 9	0,995 033	– 0,000 133
0,787	0,995 1	0,995 033	0,000 067
0,606	0,996 8	0,996 898	– 0,000 098
0,607	9,997 0	0,996 884	0,000 116

Standard deviation s

$$s = \sqrt{\frac{1}{\Phi} \sum_{i=1}^{n} (y_i - \hat{y})^2}$$

where

$$\Sigma (y - \hat{y})^2 = 0{,}000\,000\,9$$

Degrees of freedom $\Phi = n - j = 26 - 6 = 20$ where

number of measurements $n = 26$;

degrees of polynomial $j = 6$.

$$s = \sqrt{\frac{1}{20} \times 0{,}000\,000\,9}$$

$$s = 0{,}000\,21$$

$t_{95,\,\Phi} = 2{,}086$

Random uncertainty $t_{95,\,\Phi} s = 2{,}086 \times 0{,}000\,21 = 0{,}000\,44$ (± 0,04 %)

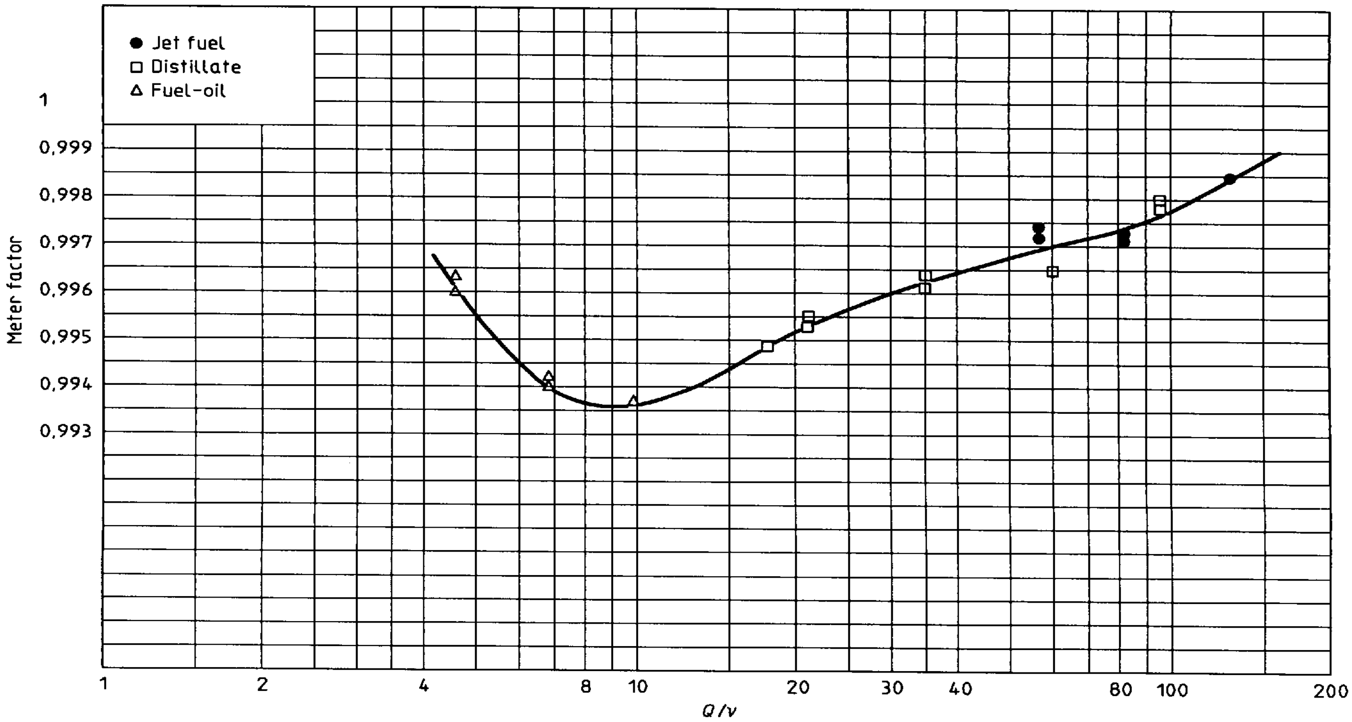


Figure 14 — Performance chart, turbine No. 310, 1979

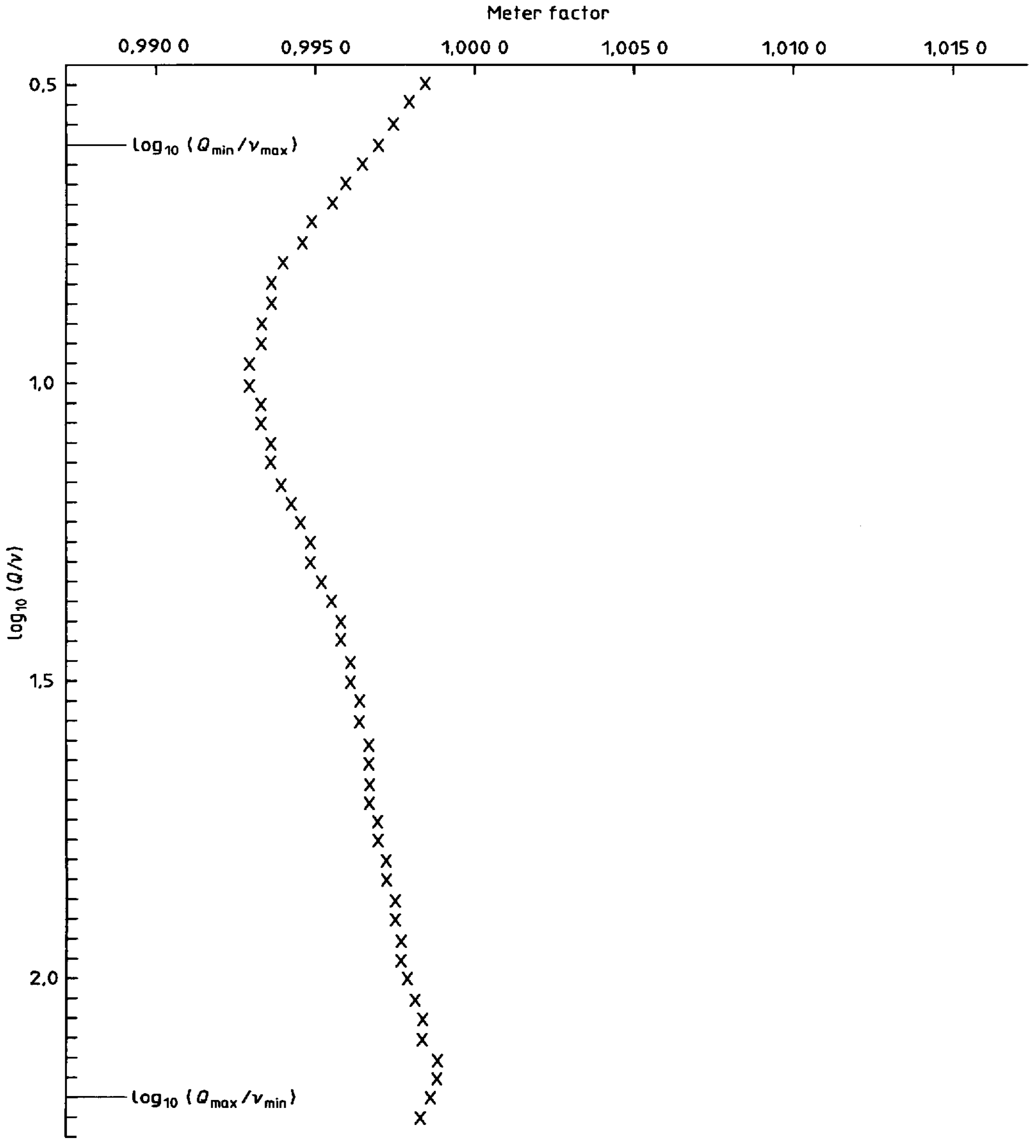


Figure 15 — Log(Q/v) vs. meter factor, turbine No. 310, 1979

Table 7 — Performance chart, meter No. 310 test report, 1979

Prover loop	**Meter**
$V_p = 2\ 503{,}2$ litres $C_{tp} = 49 \times 10^{-6}$ °C $C_{pp} = 31 \times 10^{-6}$ bar	turbine meter No. 310 type: 6-inch $p = 0{,}5$ litre/pulse

Product	**Calculated flowrate** m³/h	**Calculated kinematic viscosity at t** cSt	$\text{Log}_{10}(Q/\nu)$	**Meter factor**
Jet fuel $\nu_{20} = 1{,}81$ cSt	275,03 274,75 176,33 179,23 119,27 113,20	2,13 2,12 2,12 2,11 2,10 2,10	2,111 2,113 1,920 1,929 1,754 1,732	0,998 4 0,998 4 0,997 2 0,997 4 0,997 2 0,997 4
Distillate $\nu_{20} = 3{,}89$ cSt	391,71 392,14 242,06 242,66 142,64 142,60 86,71 86,78 72,18 72,08	4,10 4,10 4,09 4,09 4,07 4,07 4,06 4,05 4,03 4,01	1,980 1,981 1,772 1,773 1,545 1,545 1,330 1,331 1,253 1,255	0,998 0 0,997 8 0,996 5 0,996 5 0,996 1 0,996 3 0,995 5 0,995 3 0,994 9 0,994 9
Fuel oil $\nu_{20} = 40{,}00$ cSt	405,88 406,16 273,13 272,84 183,25 183,67	41,10 40,90 40,70 40,50 40,30 40,00	0,995 0,997 0,827 0,828 0,658 0,662	0,993 7 0,993 7 0,994 1 0,994 3 0,996 3 0,996 1

Universal Calibration Curve: polynomial equation

$$MF = a_0 + a_1 \cdot \log_{10}(Q/\nu) + a_2 \cdot [\log_{10}(Q/\nu)]^2 + \dots + a_6 \cdot [\log_{10}(Q/\nu)]^6$$

where

$a_0 = 0{,}952\ 782\ 6$ $a_1 = 0{,}319\ 288\ 7$ $a_2 = -0{,}808\ 241\ 4$
$a_3 = 0{,}954\ 389\ 4$ $a_4 = -0{,}579\ 113\ 8$ $a_5 = 0{,}175\ 774\ 8$
$a_6 = 0{,}021\ 165\ 95$

Table 8 — Meter factor for turbine meter No. 310, 1979

ν (cSt)	Flowrate (m^3/h) 50	100	150	200	250	300	350	400	450
1	0,996 7	0,997 9	0,998 5						
2	0,995 7	0,996 7	0,997 2	0,997 9	0,998 4				
3	0,994 7	0,996 2	0,996 7	0,997 0	0,997 4	0,997 9	0,998 2	0,998 5	0,998 5
4	0,994 0	0,995 7	0,996 4	0,996 7	0,996 9	0,997 2	0,997 5	0,997 9	0,998 1
5	0,993 7	0,995 2	0,996 1	0,996 5	0,996 7	0,996 9	0,997 1	0,997 4	0,997 6
6	0,993 7	0,994 7	0,995 7	0,996 2	0,996 5	0,996 7	0,996 9	0,997 0	0,997 2
7	0,994 0	0,994 3	0,995 4	0,996 0	0,996 3	0,996 5	0,996 7	0,996 8	0,997 0
8	0,994 5	0,994 0	0,995 0	0,995 7	0,996 2	0,996 4	0,996 6	0,996 7	0,996 8
9	0,995 0	0,993 8	0,994 7	0,995 5	0,995 9	0,996 2	0,996 4	0,996 6	0,996 7
10	0,995 6	0,993 7	0,994 4	0,995 2	0,995 7	0,996 1	0,996 3	0,996 5	0,996 6
11	0,996 2	0,993 7	0,994 2	0,994 9	0,995 5	0,995 9	0,996 2	0,996 4	0,996 5
12	0,996 8	0,993 7	0,994 0	0,994 7	0,995 3	0,995 7	0,996 0	0,996 2	0,996 4
13		0,993 9	0,993 9	0,994 5	0,995 1	0,995 6	0,995 9	0,996 1	0,996 3
14		0,994 0	0,993 8	0,994 3	0,994 9	0,995 4	0,995 7	0,996 0	0,996 2
15		0,994 2	0,993 7	0,994 1	0,994 7	0,995 2	0,995 6	0,995 9	0,996 1
16		0,994 5	0,993 7	0,994 0	0,994 5	0,995 0	0,995 4	0,995 7	0,996 0
17		0,994 7	0,993 7	0,993 9	0,994 4	0,994 9	0,995 3	0,995 6	0,995 9
18		0,995 0	0,993 7	0,993 8	0,994 2	0,994 7	0,995 1	0,995 5	0,995 7
19		0,995 3	0,993 8	0,993 7	0,994 1	0,994 6	0,995 0	0,995 3	0,995 6
20		0,995 6	0,993 9	0,993 7	0,994 0	0,994 4	0,994 8	0,995 2	0,995 5
21		0,995 9	0,994 0	0,993 7	0,993 9	0,994 3	0,994 7	0,995 1	0,995 4
22		0,996 2	0,994 2	0,993 7	0,993 8	0,994 2	0,994 6	0,994 9	0,995 3
23		0,996 5	0,994 3	0,993 7	0,993 8	0,994 1	0,994 5	0,994 8	0,995 1
24		0,996 8	0,994 5	0,993 7	0,993 7	0,994 0	0,994 4	0,994 7	0,995 0
25			0,994 7	0,993 8	0,993 7	0,993 9	0,994 3	0,994 6	0,994 9
26			0,994 8	0,993 9	0,993 7	0,993 9	0,994 2	0,994 5	0,994 8
27			0,995 0	0,993 9	0,993 7	0,993 8	0,994 1	0,994 4	0,994 7
28			0,995 2	0,994 0	0,993 7	0,993 8	0,994 0	0,994 3	0,994 6
29			0,995 4	0,994 1	0,993 7	0,993 7	0,993 9	0,994 2	0,994 5
30			0,995 6	0,994 2	0,993 7	0,993 7	0,993 9	0,994 1	0,994 4
31			0,995 8	0,994 4	0,993 8	0,993 7	0,993 8	0,994 1	0,994 3
32			0,996 0	0,994 5	0,993 8	0,993 7	0,993 8	0,994 0	0,994 3
33			0,996 2	0,994 6	0,993 9	0,993 7	0,993 7	0,993 9	0,994 2
34			0,996 4	0,994 7	0,994 0	0,993 7	0,993 7	0,993 9	0,994 1
35			0,996 6	0,994 9	0,994 0	0,993 7	0,993 7	0,993 8	0,994 1
36			0,996 8	0,995 0	0,994 1	0,993 7	0,993 7	0,993 8	0,994 0
37			0,997 0	0,995 2	0,994 2	0,993 8	0,993 7	0,993 8	0,993 9
38				0,995 3	0,994 3	0,993 8	0,993 7	0,993 7	0,993 9
39				0,995 5	0,994 4	0,993 9	0,993 7	0,993 7	0,993 9
40				0,995 6	0,994 5	0,993 9	0,993 7	0,993 7	0,993 8

Table 9 — Random uncertainty of polynomial for turbine No. 310, 1979

$\log_{10}(Q/\nu)$	Meter factor		Deviation
	y	$\hat{y}$ (polynomial)	$y - \hat{y}$ (polynomial)
2,111	0,998 4	0,998 415	– 0,000 015
2,113	0,998 4	0,998 423	– 0,000 023
1,920	0,997 2	0,997 437	– 0,000 237
1,929	0,997 4	0,997 480	– 0,000 080
1,754	0,997 2	0,996 836	0,000 364
1,732	0,997 4	0,996 780	0,000 621
1,980	0,998 0	0,997 773	0,000 258
1,981	0,997 8	0,997 748	0,000 052
1,772	0,996 5	0,996 885	– 0,000 385
1,773	0,996 5	0,996 888	– 0,000 388
1,545	0,996 1	0,996 311	– 0,000 211
1,545	0,996 3	0,996 311	– 0,000 011
1,330	0,995 5	0,995 362	0,000 138
1,331	0,995 3	0,995 367	– 0,000 067
1,253	0,994 9	0,994 899	0,000 001
1,255	0,994 9	0,994 911	– 0,000 011
0,995	0,993 7	0,993 703	– 0,000 003
0,997	0,993 7	0,993 705	– 0,000 005
0,827	0,994 1	0,994 201	– 0,000 101
0,828	0,994 3	0,994 194	0,000 106
0,658	0,996 3	0,996 232	0,000 068
0,662	0,996 1	0,996 170	– 0,000 070

Standard deviation s

$$s = \sqrt{\frac{1}{\Phi} \sum_{i=1}^{1} \left(y_i - \hat{y}\right)^2}$$

where

$$\Sigma\left(y - \hat{y}\right)^2 = 0{,}000\ 000\ 2$$

Degrees of freedom $\Phi = n - j = 22 - 6 = 16$ where

number of points $n = 22$;

degrees of polynomial $j = 6$.

$$s = \sqrt{\frac{1}{16} \times 0{,}000\ 000\ 2}$$

$$s = 0{,}000\ 11$$

$t_{95,\Phi} = 2{,}120$

Random uncertainty $t_{95,\Phi}s = 2{,}120 \times 0{,}000\ 11 = 0{,}000\ 23$ ($\pm$ 0,02 %)

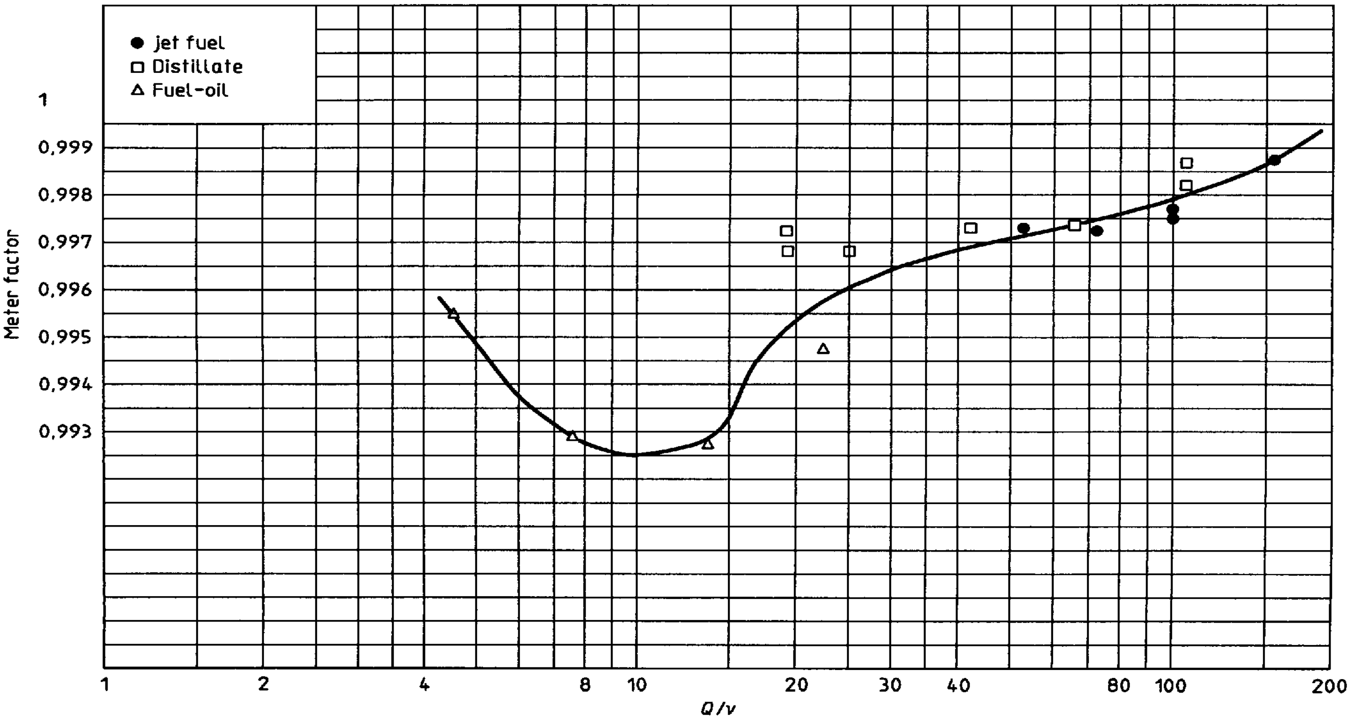


Figure 16 — Performance chart, turbine No. 310, 1980

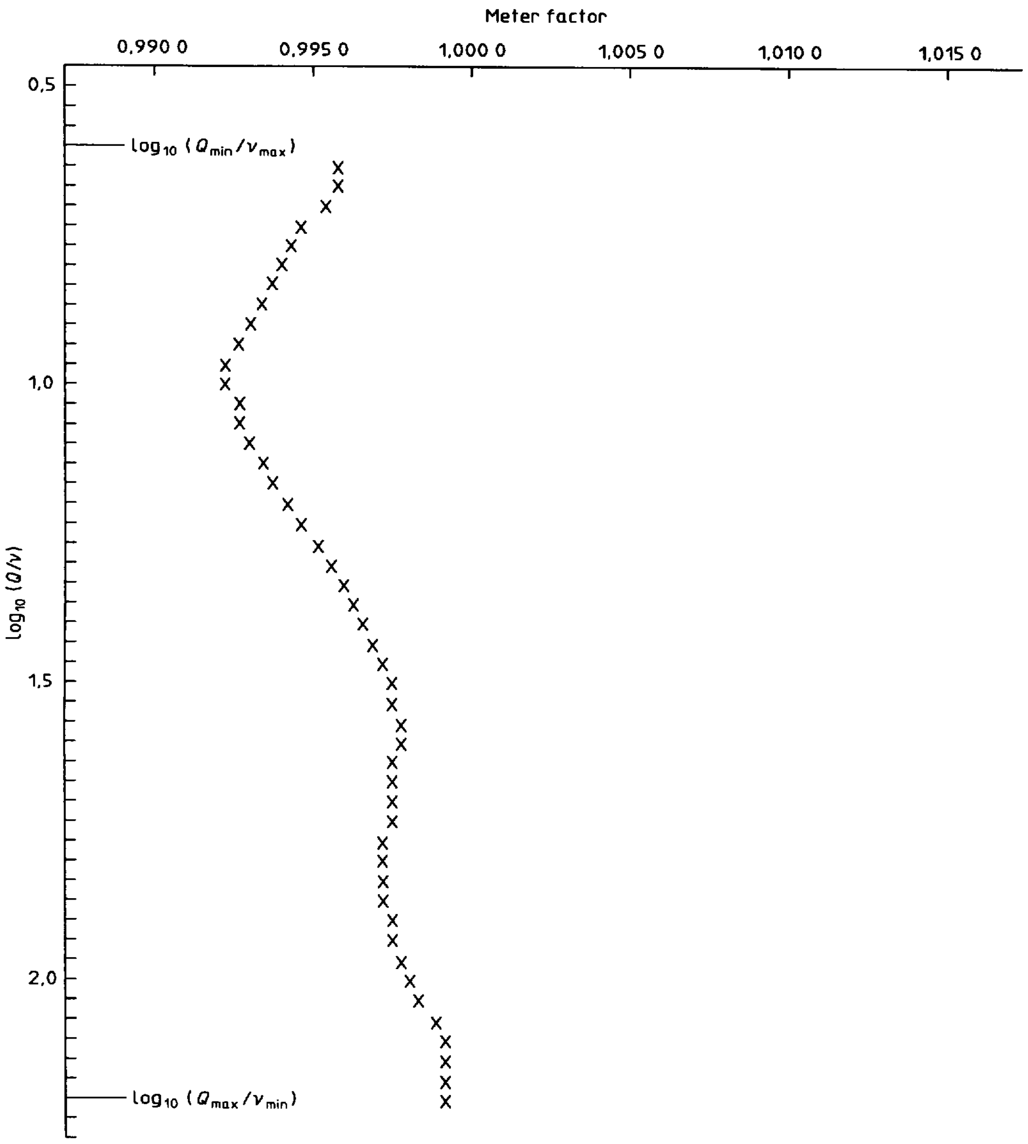


Figure 17 — Log(Q/ν) vs. meter factor, turbine No. 310, 1980

Table 10 — Performance chart, meter No. 310 test report, 1980

Prover loop	**Meter**
V_p = 2 503,5 litres $C_{tp} = 49 \times 10^{-6}$ °C $C_{pp} = 31 \times 10^{-6}$ bar	turbine meter No. 310 type: 6-inch p = 0,5 litre/pulse

Product	**Calculated flowrate** m³/h	**Calculated kinematic viscosity at t** cSt	$\text{Log}_{10}(Q/\nu)$	**Meter factor**
Jet fuel	234,57	1,49	2,197	0,998 8
ν_{20} = 1,70 cSt	234,46	1,49	2,197	0,998 8
	148,68	1,48	2,002	0,997 5
	148,60	1,48	2,002	0,997 7
	107,10	1,47	1,862	0,997 2
	106,42	1,46	1,863	0,997 2
	77,05	1,45	1,725	0,997 3
	76,81	1,44	1,727	0,997 3
Distillate	447,57	4,11	2,037	0,998 2
ν_{20} = 4,50 cSt	447,33	4,06	2,042	0,998 6
	267,70	4,02	1,823	0,997 4
	268,55	4,11	1,815	0,997 3
	99,91	3,96	1,402	0,996 8
	99,92	3,93	1,405	0,996 9
	165,03	3,90	1,626	0,997 3
	165,01	3,87	1,630	0,997 4
	72,99	3,78	1,286	0,996 9
	73,11	3,75	1,290	0,997 2
Fuel oil	368,38	16,20	1,357	0,994 7
ν_{20} = 17,00 cSt	368,37	16,10	1,359	0,994 8
	211,20	15,50	1,134	0,992 8
	211,11	15,10	1,146	0,992 8
	116,06	15,10	0,886	0,992 9
	116,12	15,00	0,889	0,992 9
	66,91	14,70	0,658	0,995 5
	65,95	14,30	0,664	0,995 5

Universal Calibration Curve: polynomial equation

$$MF = a_0 + a_1 \cdot \log_{10}(Q/\nu) + a_2 \cdot [\log_{10}(Q/\nu)]^2 + \ldots + a_6 \cdot [\log_{10}(Q/\nu)]^6$$

where

$a_0 = 0{,}648\ 226\ 9$ $a_1 = 1{,}858\ 232$ $a_2 = -3{,}909\ 047$
$a_3 = 4{,}136\ 455$ $a_4 = -2{,}337\ 616$ $a_5 = 0{,}674\ 177\ 4$
$a_6 = -7{,}803\ 245$ (E-2)

Table 11 — Meter factor for turbine meter No. 310, 1980

ν	Flowrate (m^3/h)								
(cSt)	50	100	150	200	250	300	350	400	450
1	0,997 3	0,997 9	0,998 9						
2	0,996 6	0,997 3	0,997 2	0,997 9	0,998 7				
3	0,994 4	0,997 4	0,997 3	0,997 1	0,997 4	0,997 9	0,998 5	0,998 9	0,998 9
4	0,992 9	0,996 6	0,997 5	0,997 3	0,997 1	0,997 2	0,997 5	0,997 9	0,998 3
5	0,992 4	0,995 5	0,997 2	0,997 5	0,997 3	0,997 2	0,997 2	0,997 3	0,997 6
6	0,992 6	0,994 4	0,996 6	0,997 4	0,997 5	0,997 3	0,997 2	0,997 1	0,997 2
7	0,993 2	0,993 5	0,995 9	0,997 1	0,997 4	0,997 4	0,997 3	0,997 2	0,997 1
8	0,994 0	0,992 9	0,995 1	0,996 6	0,997 3	0,997 5	0,997 4	0,997 3	0,997 2
9	0,994 7	0,992 5	0,994 4	0,996 1	0,997 0	0,997 4	0,997 5	0,997 4	0,997 3
10	0,995 3	0,992 4	0,993 8	0,995 5	0,996 6	0,997 2	0,997 4	0,997 5	0,997 4
11	0,995 5	0,992 4	0,993 3	0,994 9	0,996 2	0,996 9	0,997 3	0,997 5	0,997 5
12	0,995 3	0,992 6	0,992 9	0,994 4	0,995 7	0,996 6	0,997 1	0,997 4	0,997 5
13		0,992 8	0,992 6	0,994 0	0,995 3	0,996 3	0,996 9	0,997 2	0,997 4
14		0,993 2	0,992 5	0,993 5	0,994 8	0,995 9	0,996 6	0,997 1	0,997 3
15		0,993 6	0,992 4	0,993 2	0,994 4	0,995 5	0,996 3	0,996 9	0,997 2
16		0,994 0	0,992 4	0,992 9	0,994 0	0,995 1	0,996 0	0,996 6	0,997 0
17		0,994 4	0,992 4	0,992 7	0,993 7	0,994 8	0,995 7	0,996 3	0,996 8
18		0,994 7	0,992 6	0,992 5	0,993 4	0,994 4	0,995 3	0,996 1	0,996 6
19		0,995 0	0,992 7	0,992 4	0,993 1	0,994 1	0,995 0	0,995 8	0,996 4
20		0,995 3	0,992 9	0,992 4	0,992 9	0,993 8	0,994 7	0,995 5	0,996 1
21		0,995 4	0,993 2	0,992 4	0,992 7	0,993 5	0,994 4	0,995 2	0,995 9
22		0,995 5	0,993 4	0,992 4	0,992 6	0,993 3	0,994 2	0,994 9	0,995 6
23		0,995 5	0,993 7	0,992 5	0,992 5	0,993 1	0,993 9	0,994 7	0,995 4
24		0,995 3	0,994 0	0,992 6	0,992 4	0,992 9	0,993 7	0,994 4	0,995 1
25			0,994 2	0,992 7	0,992 4	0,992 8	0,993 4	0,994 2	0,994 9
26			0,994 5	0,992 8	0,992 4	0,992 6	0,993 2	0,994 0	0,994 7
27			0,994 7	0,993 0	0,992 4	0,992 5	0,993 1	0,993 7	0,994 4
28			0,994 9	0,993 2	0,992 4	0,992 5	0,992 9	0,993 5	0,994 2
29			0,995 1	0,993 4	0,992 5	0,992 4	0,992 8	0,993 4	0,994 0
30			0,995 3	0,993 6	0,992 6	0,992 4	0,992 7	0,993 2	0,993 8
31			0,995 4	0,993 8	0,992 7	0,992 4	0,992 6	0,993 0	0,993 6
32			0,995 5	0,994 0	0,992 8	0,992 4	0,992 5	0,992 9	0,993 5
33			0,995 5	0,994 2	0,992 9	0,992 4	0,992 4	0,992 8	0,993 3
34			0,995 5	0,994 4	0,993 0	0,992 4	0,992 4	0,992 7	0,993 2
35			0,995 5	0,994 6	0,993 2	0,992 5	0,992 4	0,992 6	0,993 0
36			0,995 3	0,994 7	0,993 3	0,992 6	0,992 4	0,992 5	0,992 9
37			0,995 2	0,994 9	0,993 5	0,992 6	0,992 4	0,992 5	0,992 8
38				0,995 0	0,993 7	0,992 7	0,992 4	0,992 4	0,992 7
39				0,995 2	0,993 8	0,992 8	0,992 4	0,992 4	0,992 6
40				0,995 3	0,994 0	0,992 9	0,992 4	0,992 4	0,992 6

Table 12 — Random uncertainty of polynomial for turbine No. 310, 1980

$\log_{10}(Q/\nu)$	Meter factor		Deviation
	y	$\hat{y}$ (polynomial)	$y - \hat{y}$ (polynomial)
2,197	0,998 8	0,998 799	0,000 001
2,197	0,998 8	0,998 799	0,000 001
2,002	0,997 5	0,997 903	– 0,000 403
2,002	0,997 7	0,997 903	– 0,000 203
1,862	0,997 2	0,997 199	0,000 001
1,863	0,997 2	0,997 201	– 0,000 001
1,725	0,997 3	0,997 267	0,000 033
1,727	0,997 3	0,997 264	0,000 036
2,037	0,998 2	0,998 187	0,000 013
2,042	0,998 6	0,998 229	0,000 371
1,823	0,997 4	0,997 160	0,000 240
1,815	0,997 3	0,997 159	0,000 141
1,402	0,996 8	0,996 624	0,000 176
1,405	0,996 9	0,996 651	0,000 249
1,626	0,997 3	0,997 438	– 0,000 138
1,630	0,997 4	0,997 434	– 0,000 034
1,286	0,996 9	0,995 310	0,001 590
1,290	0,997 2	0,995 363	0,001 838
1,357	0,994 7	0,996 173	0,001 473
1,359	0,994 8	0,996 195	– 0,001 395
1,134	0,992 8	0,993 337	– 0,000 537
1,146	0,992 8	0,993 476	– 0,000 676
0,886	0,992 9	0,992 817	0,000 084
0,889	9,992 9	0,992 790	0,000 110
0,658	0,995 5	0,995 523	– 0,000 023
0,664	0,995 5	0,995 501	– 0,000 012

Standard deviation s

$$s = \sqrt{\frac{1}{\Phi} \sum_{i=1}^{1} (y_i - \hat{y})^2}$$

where

$$\Sigma(y - \hat{y})^2 = 0{,}000\ 011\ 32$$

Degrees of freedom $\Phi = n - j = 26 - 6 = 20$ where

number of points $n = 26$;

degrees of polynomial $j = 6$.

$$s = \sqrt{\frac{1}{20} \times 0{,}000\ 011\ 32}$$

$$s = 0{,}000\ 75$$

$t_{95,\Phi} = 2{,}086$

Random uncertainty $t_{95,\Phi}s = 2{,}086 \times 0{,}000\ 75 = 0{,}001\ 57$ (± 0,16 %)

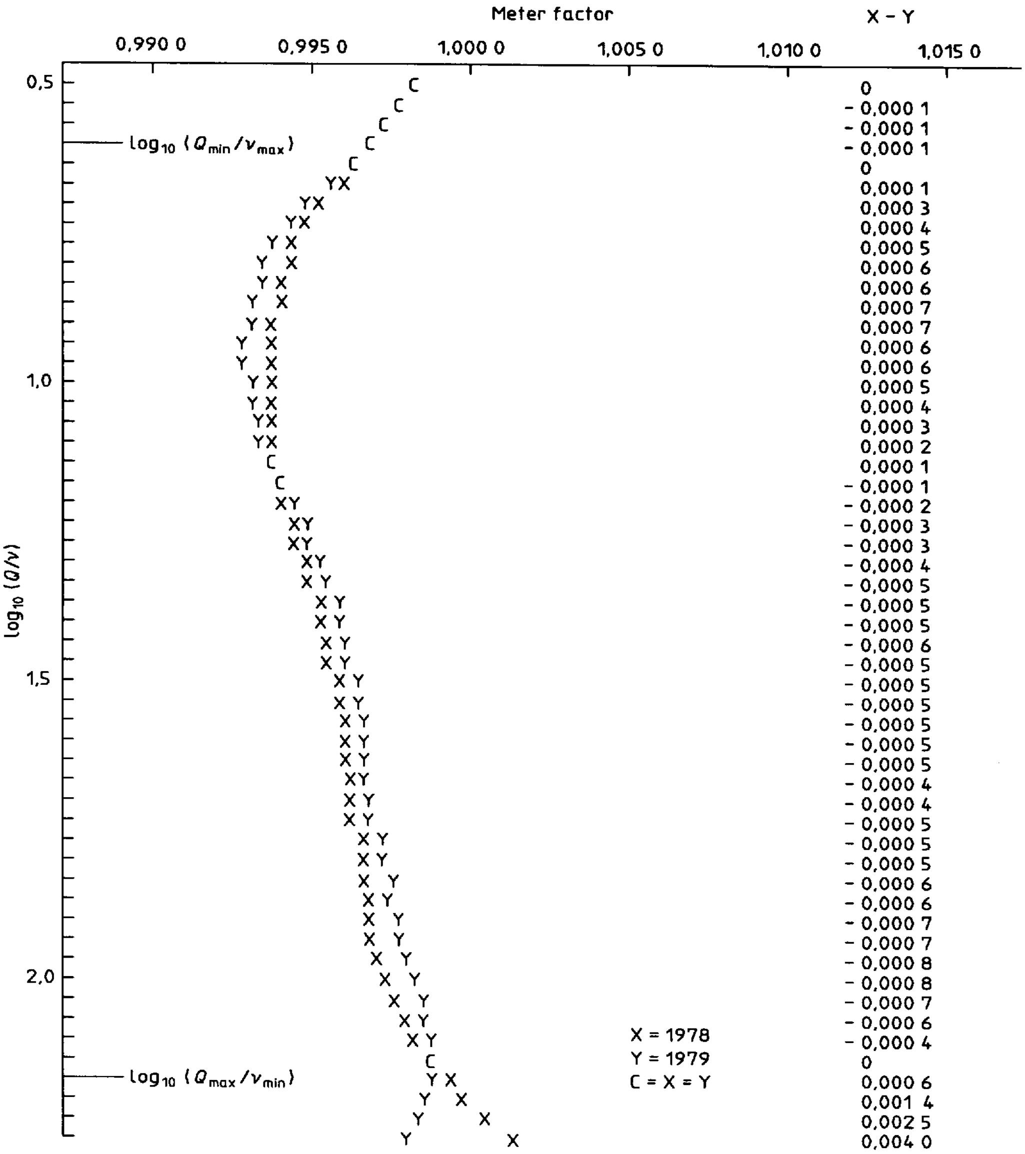


Figure 18 — Comparison of 1978 data with 1979 data, log(Q/ν) vs. meter factor, turbine No. 310

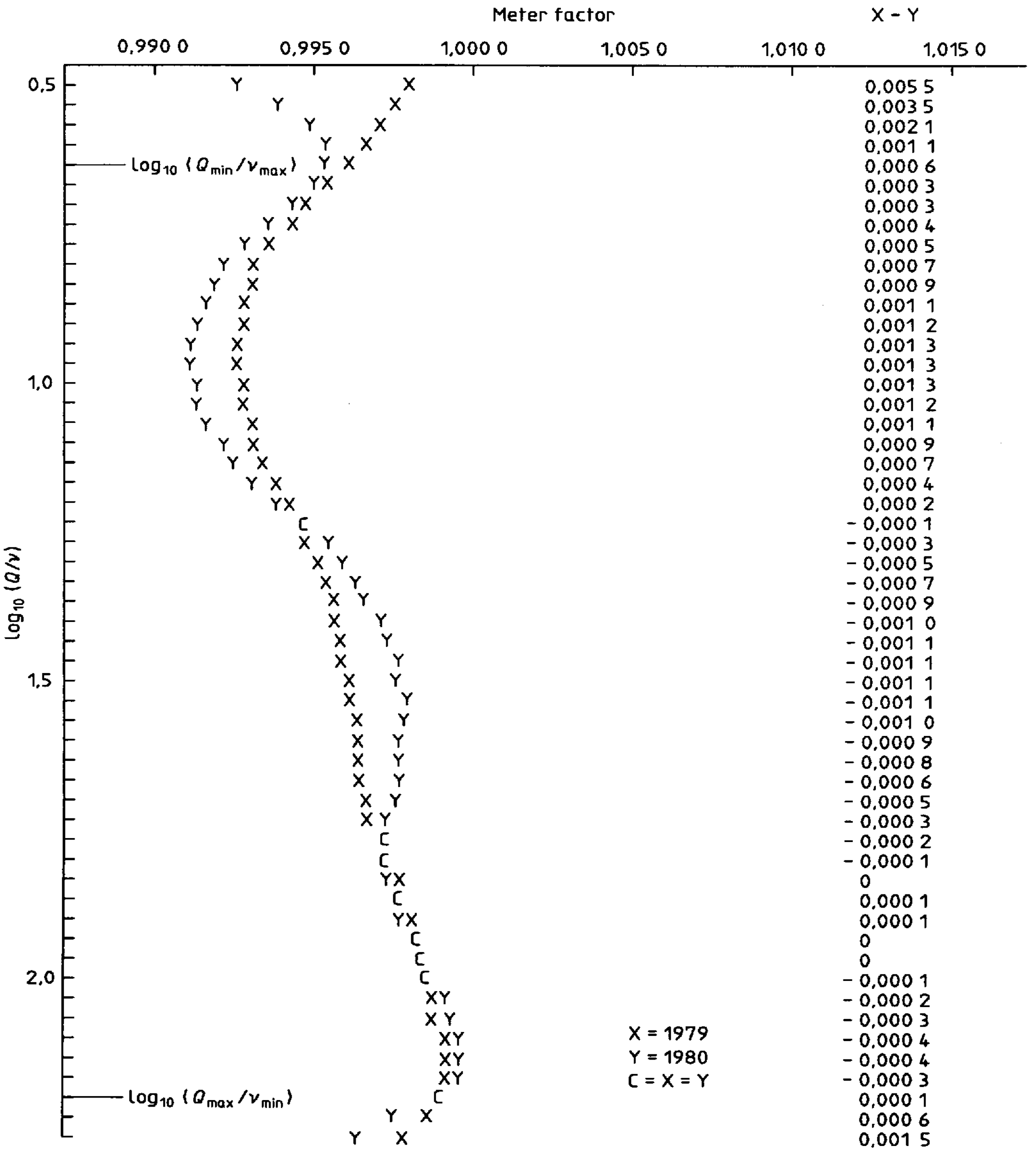


Figure 19 — Comparison of 1979 data with 1980 data, log(Q/ν) vs. meter factor, turbine No. 310

Section 4: On-line proving

4.1 Collection of data

4.1.1 Proving conditions

4.1.1.1 General

All quantities which may affect the K-factor of the meter should be measured and recorded. These quantities include the nature of the fluid, the temperature, pressure and flowrate at the meter under test. The fluid should be free from air bubbles and solid contaminants at all times.

Some knowledge of the probable measurement uncertainty is involved in statistical control. This knowledge may be obtained from initial proving, or from experience with similar meters. Failing this, it should be estimated from a preliminary proving exercise.

4.1.1.2 Sensitivity of meter

The installation should be free from any flow disturbances which might significantly affect the proving. In particular, none of the quantities affecting the K-factor should change during the proving by an amount corresponding to more than a fraction of approximately 0,3 of the probable uncertainty. The sensitivity of the meter to these disturbances may be assessed from either the initial proving or preliminary on-line proving. For example, if the measurement uncertainty is $\pm$ 0,5 %, the permissible effect of a change in flowrate during a proving test is $\pm$ 0,015 %. Examination of the initial proving may indicate that the K-factor will change by 0,1 % for a corresponding flowrate change of 50 %. The permissible change in flowrate during a proving test is then $\pm$ 0,015/0,1 = $\pm$ 7,5 % (see figure 20).

It is evident that a meter whose K-factor varies more rapidly than this with flowrate will require even closer control. A similar calculation may be made for the effects of temperature and pressure, if the corresponding performance charts are available.

4.1.1.3 Meter data

A single proving consists of determining the number of meter pulses during one operation of the prover. If this number is large enough (for example > 10 000) so that an error of one pulse does not represent a significant proportion of the allowable error, simple counting will suffice; if not, a recommended pulse interpolation technique should be used.

4.1.1.4 Prover volume corrections

The predetermined volume of the prover should be corrected for changes due to differences in pressure and temperature from the standard conditions for which this volume was measured. The number of pulses is then divided by the corrected volume to determine the K-factor of the meter. A set of provings (five or more) should be conducted over a short period of time during which all quantities are held as constant as possible. In particular, flowrate should not vary by an amount which corresponds to a change in K-factor of more than 0,3 of the allowable uncertainty.

4.1.1.5 Number of provings

The five or more provings constitute a set appropriate to the conditions of testing: specified fluid, measured pressure, measured temperature, measured flowrate. If operating conditions permit, the testing conditions should be changed to other constant values and the set of provings repeated. If operating conditions change for externally imposed reasons, the meter should be re-proved for each significant change.

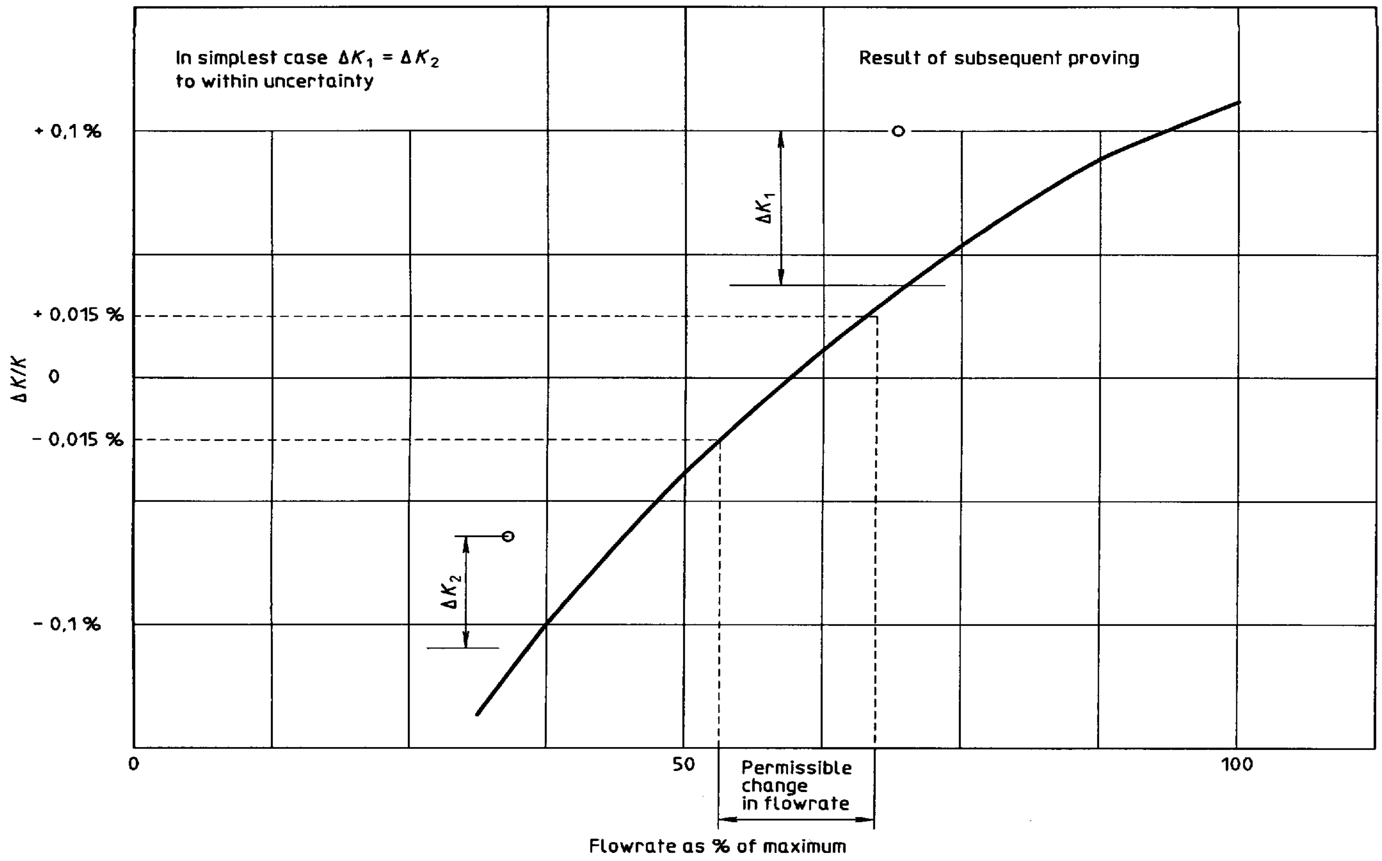


Figure 20 — Permissible change in flowrate during proving

4.1.2 Effect of flowrate

4.1.2.1 Controlling conditions

In practice, only one condition can usually be changed at will, namely the flowrate; changes in flowrate may cause changes in pressure. In the case of meters mounted in pipelines, it may be possible to change the product also.

4.1.2.2 Number of flowrates

Provings should be made over the same, or greater, range of flowrate that the meter is required to measure. The number of different flowrates at which the meter is proved will depend on a number of factors, e.g. the required accuracy, the sensitivity of K-factor to flowrate, the extent of the flowrate range and the amount of information in the form of the flowrate/K-factor graph.

4.1.2.3 Re-proving

Where the meter characteristics have not been previously determined in a central station, initial provings on-line should be carried out at a suitable number and over a wide range of flowrates.

Subsequent provings however may be carried out at a reduced number of different flowrates, provided the maximum and minimum are included.

4.2 Reliability of data collected

4.2.1 Operating conditions

With no prior information, the reliability of the data can only be based on user experience. Measurements of operating conditions should be verified for consistency — in particular, temperature measurements at the prover and at the meter under test should agree to within limits imposed by the accuracy of their calibration (e.g. 0,2 °C) and should vary only slowly and steadily with changing conditions. Similarly, pressure measurements should be consistent and any pressure differences along the flow loop should change smoothly with changing flowrate.

Pressure and temperature readings should either be constant or vary only slowly with time before a proving is attempted. The flow loop should be checked for absence of air. Pressure and temperature should be essentially constant during each proving run. Any small changes should not correspond to more than 0,3 of the allowable uncertainty in the K-factor.

4.2.2 Data analysis

4.2.2.1 Effects of random errors

When these operational criteria have been satisfactorily met, it is possible to examine the statistical properties of the K-factor and particularly the variability of the K-factor during a single series of proving runs. Small variability indicates no excessive random error, and data can be accepted for further processing. A large scatter and particularly a set of data containing outliers (see 2.2.3) indicates a defective system. Defects can be localized if additional information is available; thus if a second meter proved on the same system has a low scatter, it is likely that the first meter is defective and the prover satisfactory.

4.2.2.2 K-factor — flowrate curves

Another check for error is in the relationship between the K-factor and flowrate (see figure 21). Initially the shape of the graph is unpredictable, but it should form a smooth curve which changes shape only slowly throughout the life of the meter.

4.2.2.3 Monitoring proving conditions

Probably the best check on the reliability of data is to repeat at least one set of flowrate conditions at the beginning and end of the proving cycle. If the mean K-factors are within the limits calculated from the scatter of the individual sets, it is then clear that conditions are well controlled and the meter has no significant calibration drift over the period of testing. A somewhat larger change may be attributed to either poorly controlled conditions or meter drift. Experience, particularly from proving tests on other meters, should distinguish the probable cause.

4.3 Performance charts

4.3.1 General

Because a meter is unlikely to be proved under identical conditions on different occasions, direct comparison of K-factors obtained on different occasions will not necessarily give an accurate indication of the extent to which the K-factor may have changed with time. It is therefore necessary to establish performance charts as soon as possible in the proving cycle, so as to correct for changes in proving conditions on different occasions.

4.3.2 Initial proving data

An elementary example of a performance chart is a graph of the change in K-factor versus flowrate for a single product at a fixed temperature and pressure (figure 21). Such a graph may be constructed from initial proving data. A subsequent proving at a particular flowrate will provide a K-factor which may be compared with the K-factor from the graph. The flowrate in the subsequent proving need not be identical with any flowrate in the initial proving, provided that it is within the range of initial flowrates.

4.3.3 Prior data

In the absence of initial proving, it may be possible to construct a performance chart from a series of on-line provings, provided that each is carried out over a range of conditions and that these conditions overlap. Thus in the elementary example of figure 21 three sets of provings have been made on different occasions. Each set provides a section of the K-factor vs. flowrate graph. The sections are displaced, but can be made to overlap by multiplying the K-factors in each series by a different arbitrary factor. The factor may be determined in several ways depending on the form of the data. If for example the overlapping flowrates in the first and second sets of figure 21 are common, then determine the ratio in K-factor for each set, take the mean and use this ratio as the correction factor. If however the flowrates are not common, the readings of one set should be interpolated for comparison with those of the other prior to determining the ratio. After multiplication, the sections combine to form a graph of K-factor vs. flowrate which can be used for comparison with any subsequent series of provings. If in the different series of provings other factors (for example temperature, pressure, nature of product) vary significantly, the elementary graph of K-factor vs. flowrate will need to be expanded into a full performance chart covering all the variables.

4.4 Control charts

4.4.1 General

Control charts are intended to indicate the change in K-factor with either time or total volume of liquid passed by the meter. In the simplest case, the K-factor will change by the same amount under all conditions. In this case, comparison of the K-factor at any flowrate with the original performance graph will provide a unique indication of the change in K-factor, ΔK (see figure 20).

Values of ΔK may be plotted on a control chart of ΔK versus either time or total volume (see figure 22). Experimental values may show several features.

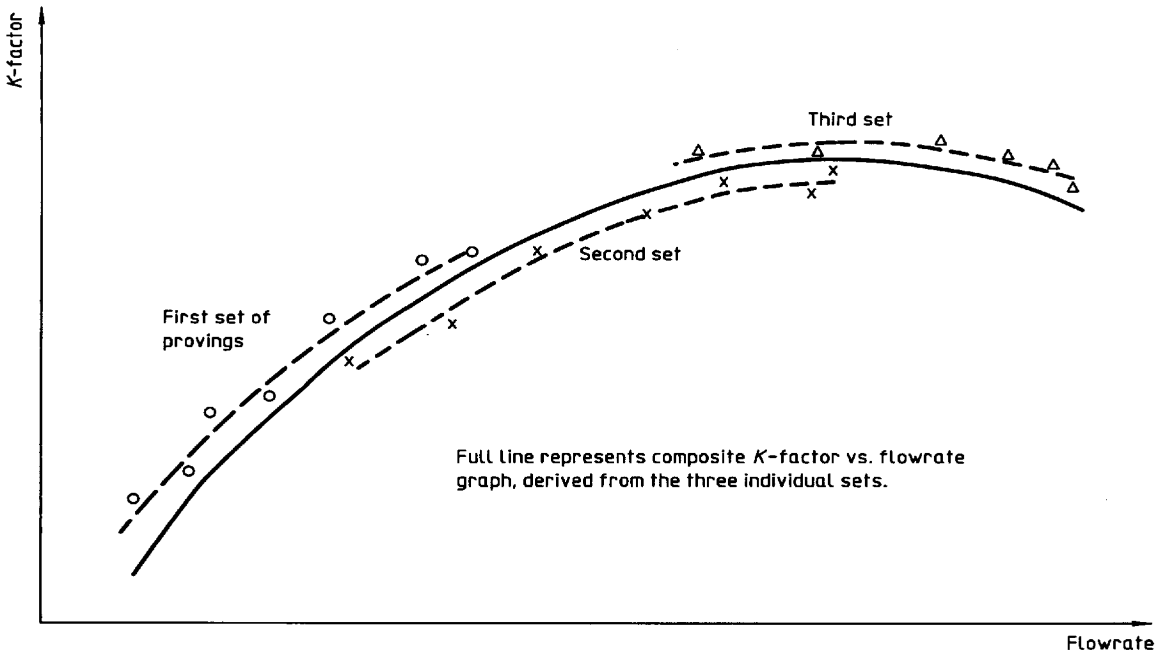


Figure 21 — Performance chart from series of on-line provings

4.4.1.1 Even in the absence of any real change in K-factor, the results are expected to show some random variation due to the uncertainties of (a) the performance chart and (b) the on-line proving. The expected level of variation is indicated by the warning limits shown in figure 22.

Variations within these limits are of no significance and no action is indicated.

4.4.1.2 Results may show large random fluctuations far outside the limit lines. If these variations are purely random it will not in general be possible to correct for them, and flow measurements made between provings will be subject to a corresponding uncertainty, larger than the (short-term) measurement uncertainty in a single series. There is some evidence that long-term random variations may exceed 0,25 % when the short-term uncertainty is 0,05 %.

4.4.1.3 Results may show a systematic trend with changes in time or volume. The K-factor on any one occasion may be repeatable to within the corresponding uncertainty limits, but the mean value may change by more than these limits between one occasion and another. The example in figure 22 shows no such trend.

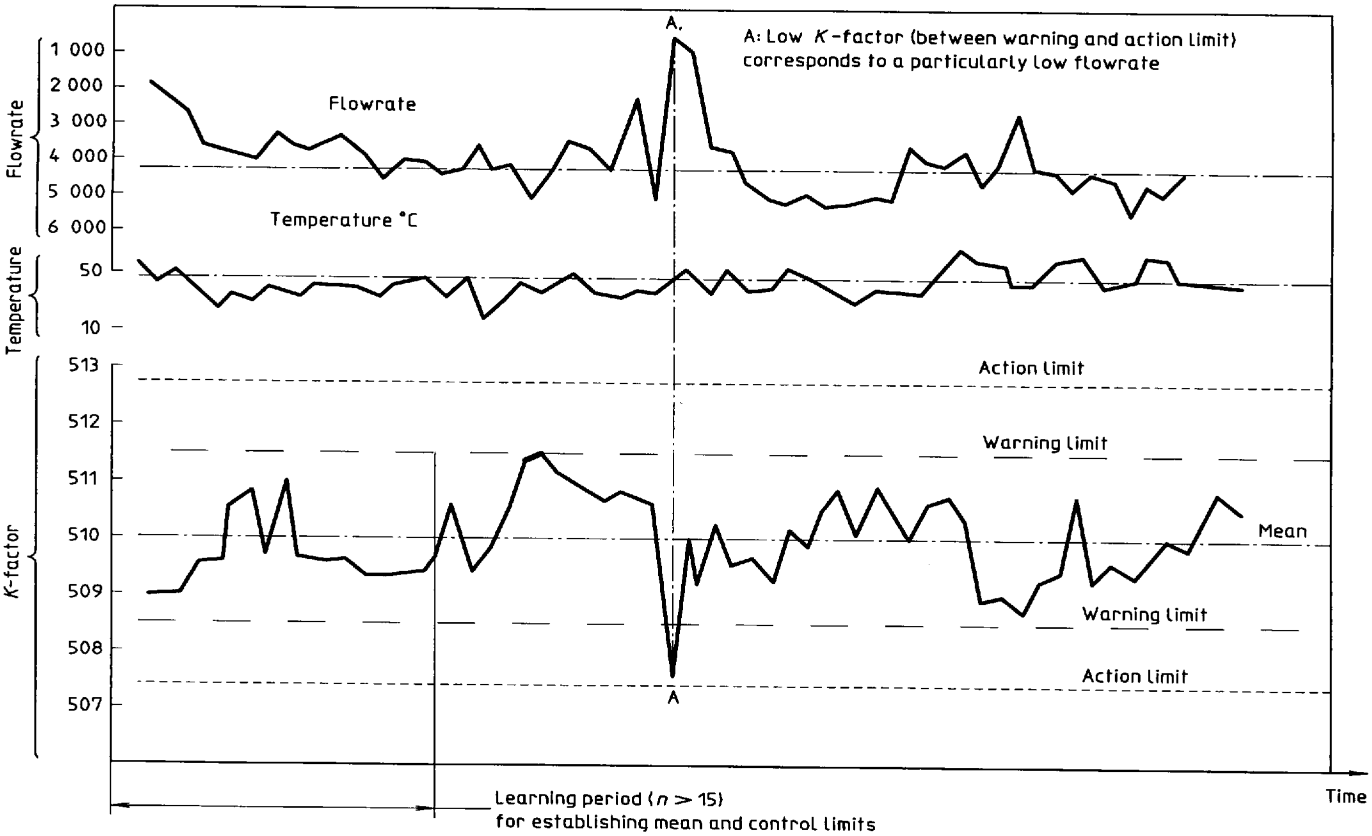


Figure 22 — K-factor (mean of 5 or 10 consecutive runs) versus time

4.4.2 Use of control charts

4.4.2.1 General

The control chart may be used (a) to update the K-factor of the meter at appropriate intervals and (b) to indicate when it is appropriate to maintain or replace a meter. The action is required in the three situations of 4.4.1 as follows.

4.4.2.1.1 For the situation described in 4.4.1.1, no action of any kind is indicated. Neither a change in K-factor nor replacement of the meter by another of the same type would be expected to improve accuracy.

4.4.2.1.2 For the situation described in 4.4.1.2, there is again no advantage in changing the K-factor or in replacing the meter by another of the same type. However, if the variation in K-factor is excessive, it may be advantageous either to control measurement variables more closely (e.g. temperature) or to replace the meter with one less sensitive to any change in conditions.

4.4.2.1.3 For the situation described in 4.4.1.3, several actions may be taken to improve accuracy. These are:

a) adjust the K-factor used in flow calculation at suitable intervals to correspond with the recent proving;

b) as a refinement, calculate a trend line for K-factor versus time and interpolate values of K from this;

c) remove the meter from service, and either service it or replace it as indicated by inspection.

4.4.2.2 Modified procedure when performance charts are not available

When performance data is not available prior to on-line proving, some quality control may still be undertaken. The effect of changes in conditions (e.g. flowrate) is not quantified, but a correlation is made between large changes in conditions and corresponding changes in K-factor.

A value of K-factor outside the action limits may be tolerated if it coincides with an extreme condition and is not supported by later tests at more nearly average conditions. An example is given in figure 22. The mean K-factor derived by averaging each set of consecutive proving runs is plotted against time (date) or volume throughput. Straight lines are drawn between each point.

The flowrate and temperature obtained during the proving can be monitored and plotted on the same axis. The mean of the K-factor for not less than 15 consecutive dates is then determined, using equation (2.6) in 2.1.5.

The inner and outer control limits estimated from the 15 meter factors (learning period) are then established as horizontal lines running parallel to the time axis, using equation (2.6).

If the subsequent meter factors are inside the inner control limits then the system is considered to be "in control". If any K-factor is outside the outer control limits, then the K-factor is considered to be in error due to an assignable cause other than random scatter.

The flowrate and temperature values can be used to establish whether any variation in K-factor outside the outer control limits is due to a significant variation in these two parameters. Temperature variation usually results in changes in viscosity of the liquid and the expansion or contraction of the meter casing.

Thus at point A in figure 22 the K-factor falls outside the warning limits though within the action limits. The flowrate graph in the upper part shows an unusually low flowrate; subsequent tests at high and more uniform values of flowrate show only small random fluctuations in K-factor, well within the warning limits. Hence no action is called for.

It should be noted however, that in the absence of correlations between K-factor and flowrate, the warning limits will be set wider to accommodate the effect of the flowrate changes. The control chart is therefore somewhat less sensitive to any real changes in meter performance than a control chart derived from a performance chart (figures 21 and 23).

4.4.2.3 Correction for effect of changing flowrates without using a separate performance chart

When it is possible to vary the flowrate during on-line proving, a graph such as figure 23 may be constructed. This may be regarded as a combined performance/control chart.

The *K*-factor (mean of *n* consecutive runs) versus flowrate curves are plotted at intervals of time and the maximum upper and lower limits are determined by examination. The "best fit" mean curve is then drawn and the upper and lower limits quantified. These correspond to the action limits.

This control chart can be used either for monitoring meter or prover malfunction or to provide a database for generating an equation for continuous linearization of the meter factor by a microprocessor.

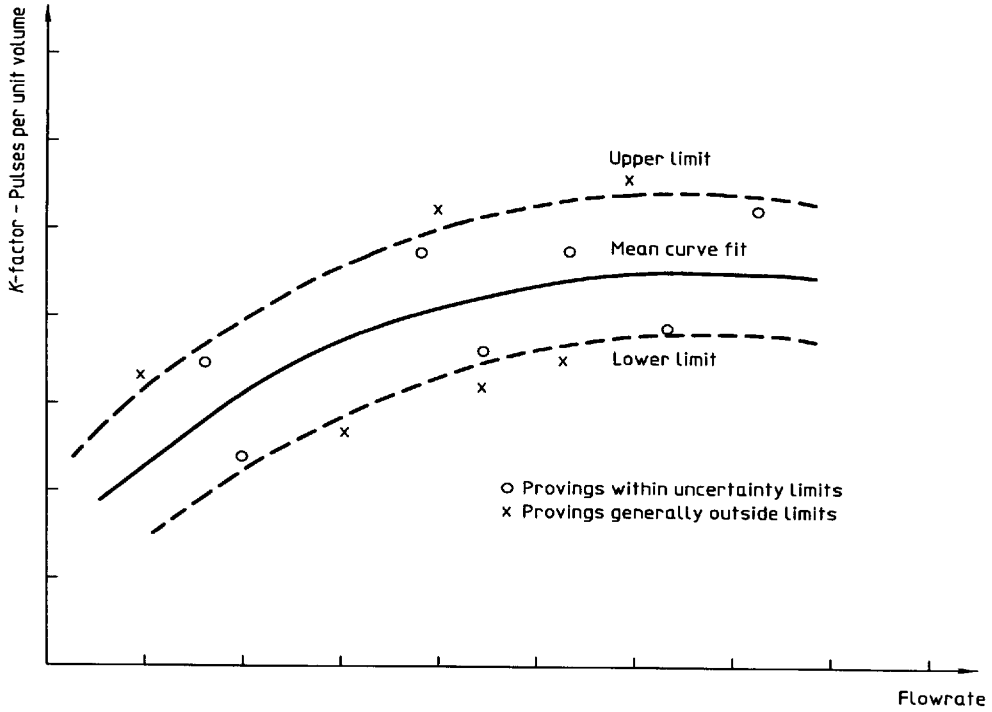


Figure 23 — Combined performance/control chart

4.4.2.4 Control charts showing systematic trends — Moving average

When a K-factor shows a significant trend on the control chart, it is useful to compute a moving average. The uncertainty limits for the average K-factor are determined for a moving average of ten consecutive proving sets carried out over a period of time. Any significant drift can then be monitored.

Control limits for the moving average K-factors are estimated from equation (2.6) as:

$$\pm\, t_{95,n-1} s(x)/\sqrt{10}$$

where $s(x)/\sqrt{10}$ is the standard deviation of the moving average [see equation (2.4)].

These limits are shown in figure 24. Since the moving averages are not independent of each other, the control limits will be wider than required and will correspond to a probability higher than the 95 % associated with warning limits. For this reason warning limits should be regarded as action limits in "moving average" control charts.

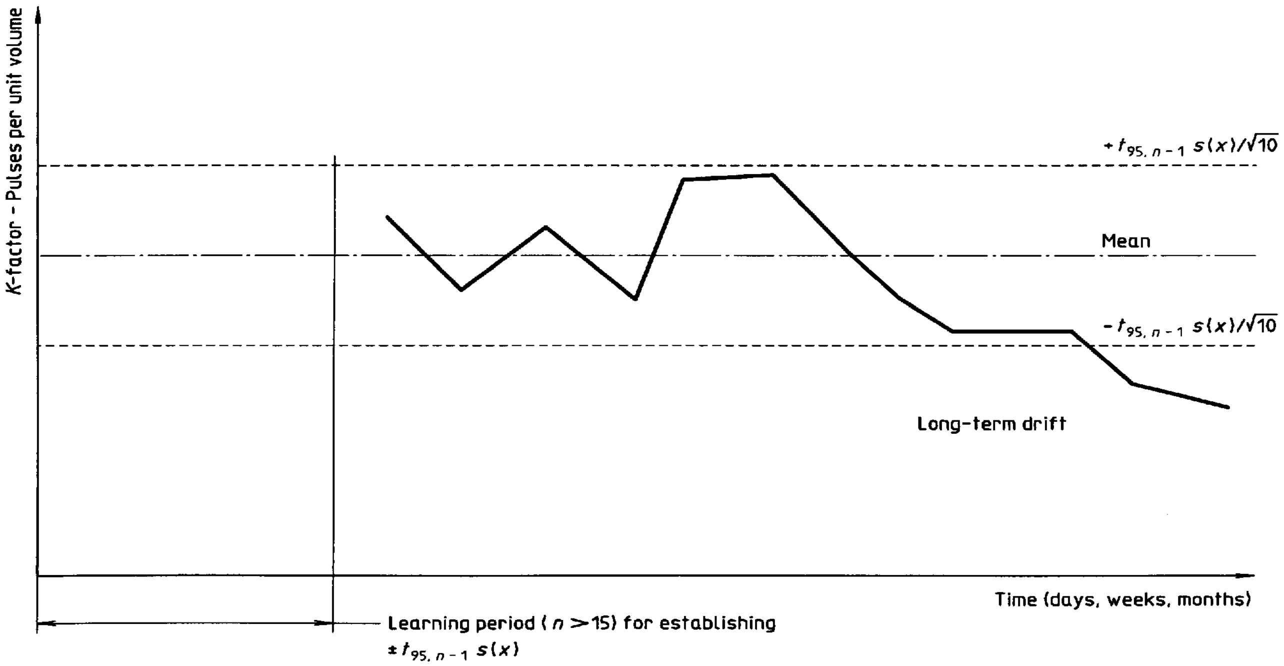


Figure 24 — Moving-average meter factor versus time

Each time a new K-factor is determined, the moving average is re-estimated from the last ten consecutive K-factors and plotted on the moving average chart. Any plotting which drifts outside the control limits may be due to:

— meter bearing wear;

— meter prover malfunction (valves, detectors, etc.).

The mean K-factor can be set in the meter scaler and only changed when there is evidence of a long-term variation or drift.

4.5 Worked examples

4.5.1 Scope of examples

Examples of on-line proving calculations and the use of control charts are given in this section as follows:

1 Test for outliers

2 Estimation of random uncertainty of K-factor

3 Establishing a control chart for a meter within its linear range

4 Establishing a control chart for a meter outside its linear range, including the use of "normalizing" techniques

5 Estimation of combined random and systematic uncertainties of metered quantities

4.5.2 Example 1 — Test for outliers

4.5.2.1 Given

An 8-inch turbine meter proved with a bidirectional pipe prover with 12-inch internal bore, 0,375-inch wall thickness and a base volume at 15 °C and 1 bar between detectors of 3 985,31 litres gave the following results:

No.	K-factor (pulses/litre)
1	6,147 0
2	6,142 2
3	6,143 5
4	6,142 5
5	6,143 2
6	6,143 2
7	6,143 2
8	6,142 7
9	6,142 0
10	6,142 2
11	6,142 2

4.5.2.2 Find

Any outliers?

4.5.2.3 Solution

a) **Step 1:** Rearrange measurements in order of ascending magnitude and apply Dixon's Test for outliers (see D.1); 95 % probability level.

X_1	No. 9	6,142 0
X_2	No. 2	6,142 2
X_3	No. 10	6,142 2
X_4	No. 11	6,142 2
X_5	No. 4	6,142 5
X_6	No. 8	6,142 7
X_7	No. 5	6,143 2
X_8	No. 6	6,143 2
X_9	No. 7	6,143 2
X_{10}	No. 3	6,143 5
X_{11}	No. 1	6,147 0

b) **Step 2**

1) $$R_{21} = \frac{X_{11} - X_9}{X_{11} - X_2} = \frac{6{,}147\,0 - 6{,}143\,2}{6{,}147\,0 - 6{,}142\,2} = 0{,}792$$

where R is the Dixon ratio (see annex D for explanation of subscripts).

As 0,792 is larger than the critical value of 0,576, measurement X_{11} is rejected as an outlier.

2) $$R_{11} = \frac{X_{10} - X_9}{X_{10} - X_2} = \frac{6{,}143\,5 - 6{,}143\,2}{6{,}143\,5 - 6{,}142\,2} = 0{,}231$$

As 0,231 is less than 0,477, then measurement X_{10} is acceptable.

4.5.3 Example 2 — Estimation of random uncertainty of K-factor

4.5.3.1 Given

Using results given in example 1, excluding measurement number 1 (as it was rejected as an outlier).

4.5.3.2 Find

What is

— the mean value of the K-factor?

— the estimated standard deviation?

— the range of uncertainty due to random error about a single measurement of K-factor?

— the range of uncertainty for the mean value of K-factor?

4.5.3.3 Solution

a) Calculate standard deviation using:

No.	y_i	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$
1		[rejected as outlier]	
2	6,142 2	– 0,000 5	0,000 000 25
3	6,143 5	0,000 8	0,000 000 64
4	6,142 5	– 0,000 2	0,000 000 04
5	6,143 2	0,000 5	0,000 000 25
6	6,143 2	0,000 5	0,000 000 25
7	6,143 2	0,000 5	0,000 000 25
8	6,142 7	0,000 0	0,000 000 00
9	6,142 0	– 0,000 7	0,000 000 49
10	6,142 2	– 0,000 5	0,000 000 25
11	6,142 2	– 0,000 5	0,000 000 25
		$\Sigma(y_i - \bar{y})^2 =$	0,000 002 67

b) Mean

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = 6{,}142\ 7 \text{ pulses/litre}$$

c) Standard deviation

$$s(y) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - y)^2}$$

$$s(y) = \sqrt{\frac{0{,}000\ 002\ 67}{9}}$$

$$s(y) = 0{,}000\ 54$$

d) Uncertainty of single measurement

$$\text{u}(y) = t_{95,n-1}\, s(y)$$

$$\text{u}(y) = 2{,}262 \times 0{,}00054 = \pm\ 0{,}001\ 22$$

e) Uncertainty of mean

$$\text{u}(\bar{y}) = t_{95,n-1}\, s(y)/\sqrt{n}$$

$$\text{u}(\bar{y}) = 2{,}262 \times 0{,}000\ 54\ /\ \sqrt{10} = \pm\ 0{,}000\ 39$$

4.5.4 Example 3 — Establishing a control chart for a meter operating within its linear range

4.5.4.1 Given

A number of sets ($m = 10$) of consecutive meter provings were carried out at regular intervals. The meter was proved between 80 % to 90 % of the maximum flowrate in the linear range where the K-factor variation is less than ± 0,1 %. The temperature varied by less than 5 °C throughout the proving period. (See figure 25.)

	Mean K-factor (pulse/litre) of each set
1st week	6,144 6
2nd week	6,139 6
3rd week	6,142 0
4th week	6,143 3
5th week	6,137 0
6th week	6,140 9
7th week	6,145 9
8th week	6,147 0
9th week	6,168 5
10th week	6,142 0
11th week	6,138 3

4.5.4.2 Find

Are there any outlying mean K-factors?

4.5.4.3 Solution

a) **Step 1:** Rearrange the measurements in ascending order of magnitude and apply Dixon's Test for outliers (see D.1) at 95 % probability level.

X_1	6,137 0
X_2	6,138 3
X_3	6,139 6
X_4	6,140 9
X_5	6,142 0
X_6	6,142 0
X_7	6,143 3
X_8	6,144 6
X_9	6,145 9
X_{10}	6,147 0
X_{11}	6,168 5

b) **Step 2**

$$R = \frac{X_{11} - X_9}{X_{11} - X_2} = \frac{6,168\ 5 - 6,145\ 9}{6,168\ 5 - 6,138\ 3} = 0,748$$

where R is the Dixon ratio.

As 0,748 is larger than the critical value of 0,576, then reject $X_{11} = 6,168\ 5$ (9th week).

There were no further outliers when the test was repeated.

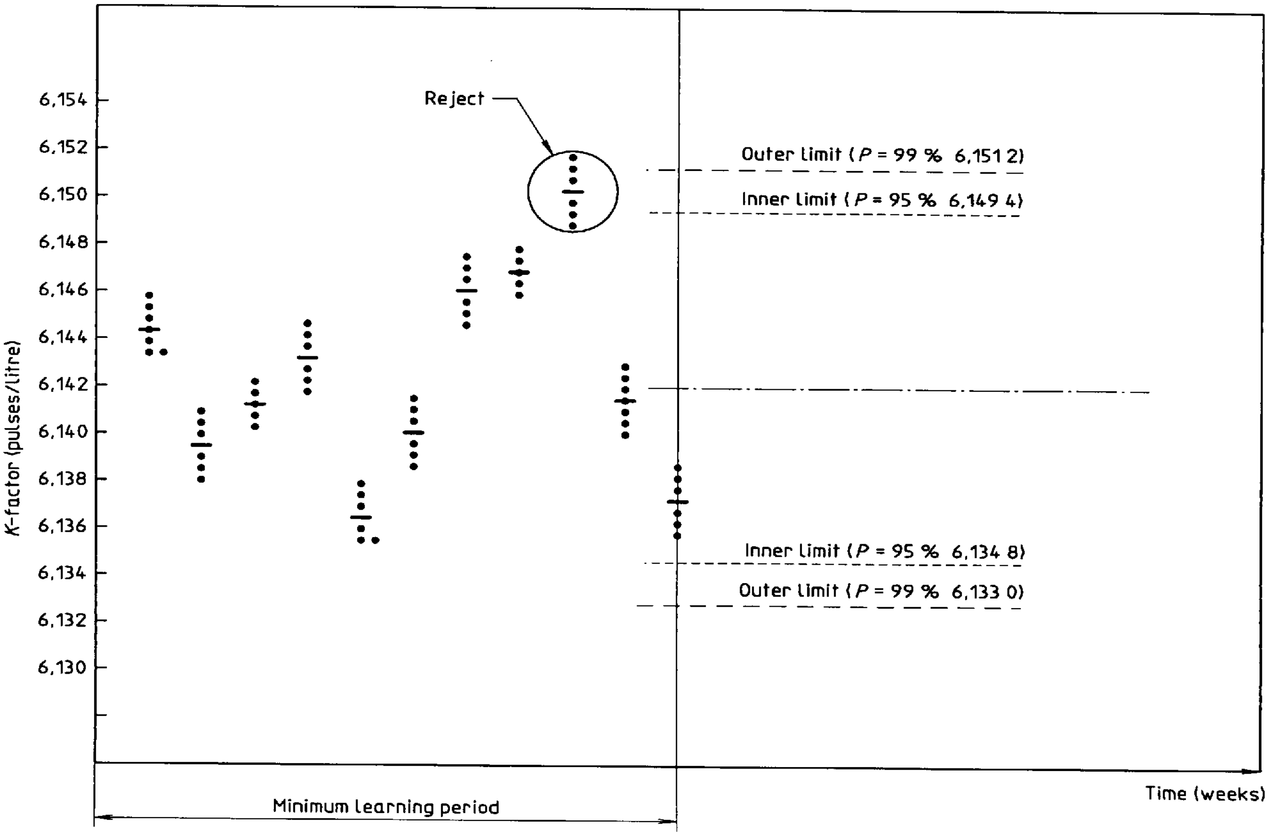


Figure 25 — Control chart for meter operating within linear range

4.5.4.4 Given

Use previous results, excluding measurement for 9th week, as follows:

	Mean K-factor (pulse/litre)
1st week	6,144 6
2nd week	6,139 6
3rd week	6,142 0
4th week	6,143 3
5th week	6,137 0
6th week	6,140 9
7th week	6,145 9
8th week	6,147 0
10th week	6,142 0
11th week	6,138 3

4.5.4.5 Find

What is

— the mean value of K-factor for the control chart;

— the upper and lower (P = 95 %) control limits (inner);

— the upper and lower (P = 99 %) control limits (outer).

4.5.4.6 Solution

a) Mean $\bar{z} = \frac{1}{m} \sum_{i=1}^{m} z_i = 6{,}142\ 1$ pulses/litre

$$s(z) = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (z_i - \bar{z})^2} = 0{,}003\ 24$$

b) $u(z) = t_{95,m-1} s(z) = 2{,}262 \times 0{,}003\ 24 = \pm\ 0{,}007\ 3$

Lower limit = 6,142 1 − 0,007 3 = 6,134 8

Upper limit = 6,142 1 + 0,007 3 = 6,149 4

c) $u(z) = t_{99,m-1} s(z) = 3{,}250 \times 0{,}003\ 24 = \pm\ 0{,}010\ 5$

Lower limit = 6,142 1 − 0,010 5 = 6,131 6

Upper limit = 6,142 1 + 0,010 5 = 6,152 6

4.5.5 Example 4 — Establishing a control chart for a meter operating outside its linear range and with varying temperatures

4.5.5.1 Given

A number of sets (m = 1) of consecutive meter provings carried out at regular intervals. The meter was proved under varying conditions of flowrate and temperature (see table 13, columns 1–4).

4.5.5.2 Find

What are the mean K-factors corresponding to flowrate and temperature or viscosity?

4.5.5.3 Solution

a) **Performance chart (based on previous experience)**

The turbine meter has been proved over a range of flowrates at a number of fixed (arbitrary) temperatures. A best-fit curve is drawn through the K-factors for each temperature (see figure 26). Interpolated values of K-factor are estimated at other temperatures so as to provide a performance chart for the meter.

A matrix (see figure 26) is then prepared from the performance chart data, from which the K-factors can be calculated for each combination of flowrate and temperature or viscosity.

b) **Equation generated from proving data**

In the absence of performance curves based on previous experience, the mean K-factor can be estimated by generating the best curve fit for values of K-factor in terms of flowrate divided by viscosity, using the following relationships:

— Linear regression: $y = a + bx$

— Exponential: $y = a\,e^{bx}$

— Logarithmic: $y = a + b \ln x$

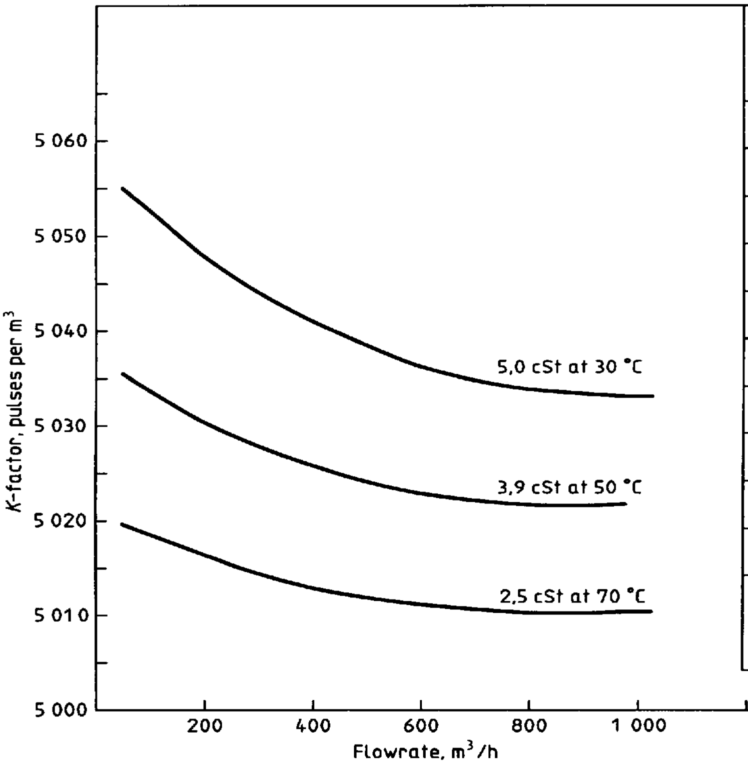
— Power: $y = ax^b$

The curve fit selected is that which shows the maximum correlation coefficient.

Table 13 — Example 4 — Normalizing meter factor

(1)	(2)	(3)	(4)	(5)	(6)	(6a)	(7)	(7a)	(8)	(8a)
Date	**Flowrate** Q m³/h	**Temp.** °C	**Meas. K-factor** (mean) K	ν cSt	K-factor **Interpolated** (matrix) K_1	**Calculated** (equation) K_1 1)	$K_1 - \overline{K}_1$	$K_1 - \overline{K}_1$ 1)	$K - (K_1 - \overline{K}_1)$	$K - (K_1 - \overline{K}_1)$ 1)
3.4.67	900	50	5 025,0	3,9	5 020,5	5 020,0	− 0,2	− 0,9	5 025,2	5 025,9
4.4.67	1 000	50	5 021,0	3,9	5 020,0	5 019,3	− 0,7	− 1,6	5 021,7	5 022,6
5.4.67	800	52	5 014,9	3,7	5 020,0	5 020,4	− 0,7	− 0,5	5 015,6	5 015,4
6.4.67	950	60	5 017,7	3,0	5 015,2	5 018,0	− 5,5	− 2,9	5 023,2	5 020,6
7.4.67	900	55	5 021,4	3,4	5 017,9	5 019,1	− 2,8	− 1,8	5 024,2	5 023,2
8.4.67	1 000	50	5 015,0	3,9	5 020,0	5 019,3	− 0,7	− 1,6	5 015,7	5 016,6
9.4.67	900	55	5 016,2	3,4	5 017,9	5 019,1	− 2,8	− 1,8	5 019,0	5 018,0
10.4.67	200	55	5 028,0	3,4	5 026,7	5 028,7	+ 6,0	+ 7,8	5 022,0	5 020,2
11.4.67	950	60	5 018,0	3,0	5 015,2	5 018,0	− 5,5	− 2,9	5 023,5	5 020,9
12.4.67	800	50	5 014,2	3,9	5 021,0	5 020,7	+ 0,3	− 0,2	5 013,9	5 014,4
13.4.67	900	45	5 025,6	4,4	5 023,3	5 020,8	+ 2,6	− 0,1	5 023,0	5 025,7
14.4.67	100	40	5 039,0	5,1	5 043,0	5 035,7	+ 22,3	+ 14,8	5 016,7	5 024,2
15.4.67	1 000	50	5 021,0	3,9	5 020,0	5 019,3	− 0,7	− 1,6	5 021,7	5 022,6
16.4.67	950	50	5 020,3	3,9	5 020,3	5 019,6	− 0,4	− 1,3	5 020,7	5 021,6
17.4.67	1 000	55	5 026,0	3,4	5 017,5	5 018,4	− 3,2	− 2,5	5 029,2	5 028,5
18.4.67	300	65	5 017,5	2,7	5 018,2	5 024,7	− 2,5	+ 3,8	5 020,0	5 013,7
19.4.67	950	50	5 026,5	3,9	5 020,3	5 019,6	− 0,4	− 1,3	5 026,9	5 027,8
20.4.67	800	52	5 018,0	3,7	5 020,0	5 020,4	− 0,7	− 0,5	5 018,7	5 018,5
21.4.67	1 000	50	5 015,3	3,9	5 020,0	5 019,3	− 0,7	− 1,6	5 016,0	5 016,9
22.4.67	1 000	55	5 018,0	3,4	5 017,5	5 018,4	− 3,2	− 2,5	5 021,2	5 020,5
Mean			5 020,9		5 020,7	5 020,9			5 020,9	5 020,9
Standard deviation (s)			6,031						4,029	4,290

1) Calculated from the equation $K_1 = 5\,054{,}8(Q/\nu)^{-0{,}001\,27}$



Flowrate m^3/h	K-factor, pulses per m^3 40 °C 1)	50 °C (3,9 cSt)	60 °C 1)	70 °C (2,5 cSt)
100	5 043,0	5 033,5	5 026,0	5 018,5
200	5 038,8	5 030,0	5 023,3	5 016,5
300	5 035,3	5 027,5	5 021,3	5 015,0
400	5 032,8	5 025,5	5 019,5	5 013,5
500	5 030,8	5 024,0	5 018,3	5 012,5
600	5 029,0	5 022,5	5 017,0	5 011,5
700	5 027,8	5 021,5	5 016,3	5 011,0
800	5 026,8	5 021,0	5 015,8	5 010,5
900	5 026,0	5 020,5	5 015,3	5 010,0
1 000	5 025,5	5 020,0	5 015,0	5 010,0

1) Interpolated values.
NOTE – Meter factors are the mean of ten consecutive proving runs.

Figure 26 — Chart A — Meter performance curves — Meter turbine proved with pipe prover

4.5.5.4 Find

What are the K-factors corrected for the effect of changes in flow conditions?

4.5.5.4.1 Solution

a) **Step 1:** Estimate the viscosity corresponding to the temperature at each proving, by reference to a suitable log/viscosity chart where no performance chart is available.

b) **Step 2:** Calculate values of K-factor by either interpolating from the performance chart or using the K-factor and flowrate (Q)/viscosity (ν) best-fit equation (table 13, column 6 or 6a).

$$K_1 = 5\ 054{,}8(Q/\nu)^{-0{,}001\ 27}$$

c) **Step 3:** Calculate the mean K-factor = 5 020,9 and K_1 = 5 020.

d) **Step 4:** Calculate $(K_1 - \overline{K}_1)$ (see table 13, column 7 or 7a).

e) **Step 5:** Calculate $K - (K_1 - \overline{K}_1)$ (see table 13, column 8 or 8a).

4.5.5.5 Find

What is

— the mean value of the normalized K-factor (corrected for influences due to variations in flow conditions);

— the upper and lower (P = 95 %) control limits (inner);

— the upper and lower (P = 99 %) control limits (outer).

4.5.5.5.1 Solution

a) Mean $\bar{z}_i = \frac{1}{m} \sum_{i=1}^{m} z_i = 5\,020{,}9$ pulses/m^3

b) $s(z) = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (z_i - \bar{z})^2} = 4{,}029$ or $4{,}290$

c) $u(z) = t_{95,m-1} s(z) = 2{,}093 \times 4{,}029 = \pm\, 8{,}4$ (by interpolation)

or = 2,093 × 4,290 = ± 9,0 (by calculation)

Lower limit = 5 020,9 − 8,4 = 5 012,5 pulses/m^3 (interpolation)

or = 5 020,9 − 9,0 = 5 011,9 (calculation)

Upper limit = 5 020,9 + 8,4 = 5 029,3 pulses/m^3 (interpolation)

or = 5 020,9 + 9,0 = 5 029,9 (calculation)

d) $u(z) = t_{99,m-1} s(z) = 3{,}250 \times 4{,}029 = \pm\, 13{,}1$ (interpolation)

or = 3,250 × 4,290 = ± 13,9 (calculation)

Lower limit = 5 020,9 − 13,1 = 5 007,8 pulses/m^3 (interpolation)

or = 5 020,9 − 13,9 = 5 007,0 (calculation)

Upper limit = 5 020,9 + 13,1 = 5 034,0 pulses/m^3 (interpolation)

or = 5 020,9 + 13,9 = 5 034,8 (calculation)

4.5.5.6 Find

What is

— the mean value of the "normalized" K-factor (corrected for influences due to non-linearity) for the control chart;

— the upper and lower (P = 95 %) control limits (inner);

— the upper and lower (P = 99 %) control limits (outer).

4.5.5.6.1 Solution

From table 13, column 7 (see figure 27).

NOTE 1 Figure 27 shows actual and normalized values plotted on a control chart with upper and lower limits.

a) Mean $\bar{z} = \frac{1}{m} \sum_{i=1}^{m} z_i = 5\,020{,}9$ pulses/m^3

b) $s(z) = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (z_i - \bar{z})^2} = 4{,}029$

c) $u(z) = t_{95,m-1} s(z) = 2{,}093 \times 4{,}029 = \pm\ 8{,}4$ pulses/m^3

Lower limit = 5 020,9 − 8,4 = 5 012,5

Upper limit = 5 020,9 + 8,4 = 5 029,3

d) $u(z) = t_{99,m-1} s(z) = 2{,}861 \times 4{,}029 = \pm\ 11{,}5$ pulses/m^3

Lower limit = 5 020,9 − 11,5 = 5 009,4

Upper limit = 5 020,9 + 11,5 = 5 032,4

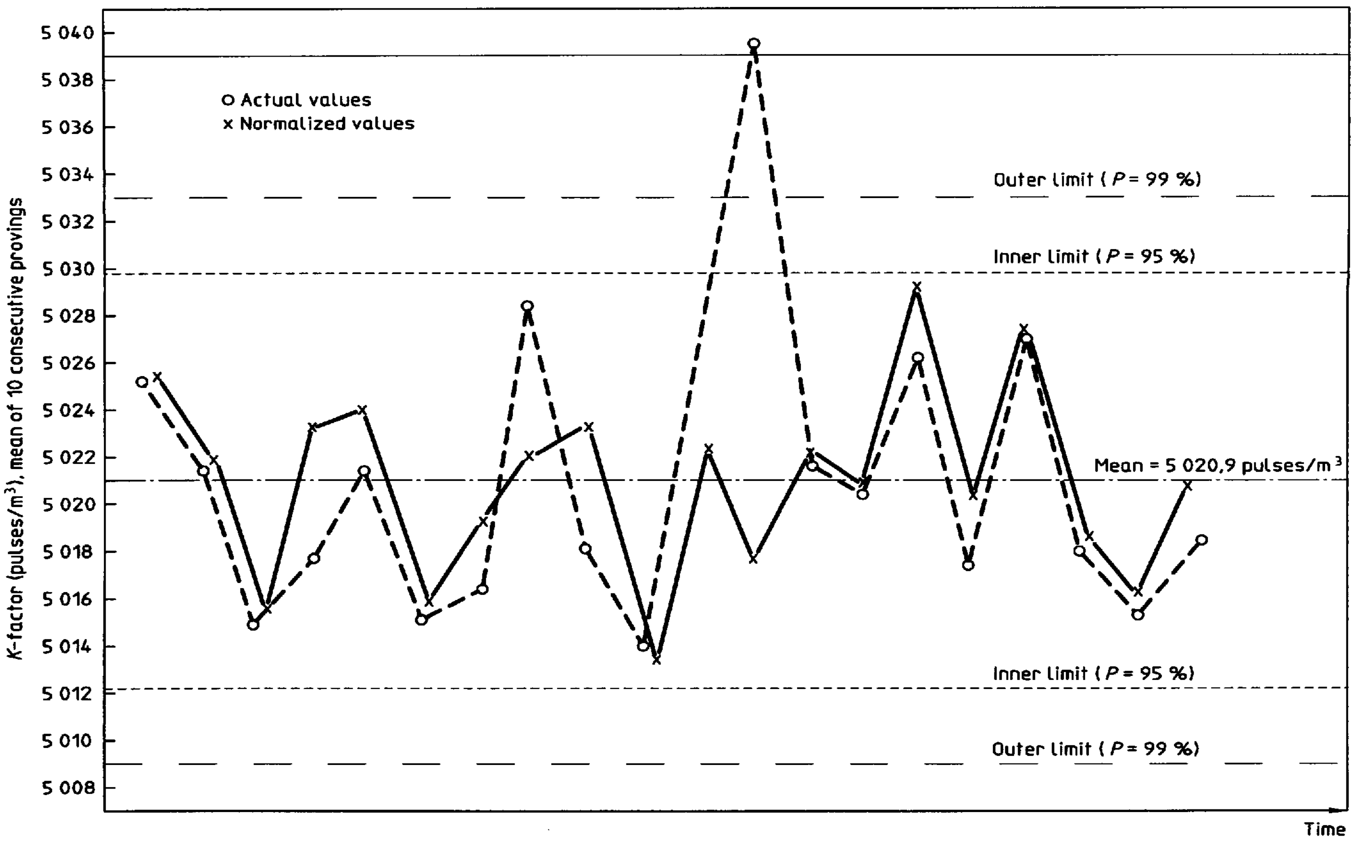


Figure 27 — Control chart — Meter operating outside linear range

4.5.6 Example 5 — Uncertainty of metered quantity (combined random and systematic uncertainty)

4.5.6.1 Given

An 8-inch turbine meter was proved with the same pipe prover on the same grade of crude oil over a period of several weeks.

4.5.6.2 Find

What is the uncertainty of a single K-factor over the proving period?

4.5.6.3 Solution

4.5.6.3.1 Estimate the short-term uncertainty $u(x_1)$ expressed as a percentage. Assuming a typical value (example 2) which holds for all sets of 10 consecutive provings:

$$\mathrm{u}(x_1) = \frac{\mathrm{u}(y)}{\bar{y}} \times 100$$

$$= \frac{0{,}001\ 22}{6{,}142\ 7} \times 100$$

$$= \pm\ 0{,}02\ \%$$

4.5.6.3.2 Estimate the long-term uncertainty $\mathrm{u}(x_2)$ based on 10 sets ($m = 10$) (Example 3), expressed as a percentage:

$$\mathrm{u}(x_2) = \frac{\mathrm{u}(z)}{\bar{z}} \times 100$$

$$= \frac{0{,}007\ 3}{6{,}142\ 7} \times 100$$

$$= \pm\ 0{,}12\ \%$$

4.5.6.3.3 Uncertainty of the pipe prover $\mathrm{u}(x_3)$ is typically ± 0,05 % and normally considered as a fixed systematic error.

As the same prover was used in deriving all the K-factors, the uncertainty should be treated as a bias (and added algebraically).

NOTE 2 The magnitude of the long-term uncertainty $\mathrm{u}(x_2)$ is largely dependent on the linearity or variation in K-factor with changes in flowrate and viscosity (temperature). The estimated value of ± 0,12 % was derived from a meter which was proved under similar flow conditions. In Example 4 it can be seen that the long-term uncertainty is of the order of ± 0,25 % when proved under varying flow conditions.

4.5.6.3.4 Uncertainty of a single K-factor, treating $\mathrm{u}(x_3)$ as a bias:

$$\mathrm{u}(K) = \sqrt{\mathrm{u}(x_1)^2 + \mathrm{u}(x_2)^2} + \mathrm{u}(x_3)$$

$$= \sqrt{(0{,}02)^2 + (0{,}12)^2} + 0{,}05$$

$$= \pm\ 0{,}17\ \%$$

Section 5: Secondary control

5.1 Comparison between meter and tank

5.1.1 General

By comparing the quantities of oil transferred into or out of a vertical cylindrical storage tank with the quantities measured through a meter, it is possible to monitor any significant change in the performance of the meter.

This method is often referred to as "secondary control", as it is designed to detect only gross errors usually associated with the mechanical failure of the meter's moving parts.

5.1.2 Principle of monitoring system

The volumes at standard conditions (15 °C or 20 °C) of both the tank transfer quantity and meter throughput are compared, and the difference expressed as a percentage of the total volume of the transfer.

This percentage difference is then compared with a value of the combined uncertainty of the tank and meter quantities computed from equations (5.23) and (5.24).

The component uncertainties for the level gauging, temperature measurement, tank calibration and meter derived from previous experience are used in the equation.

If the difference between the meter and tank quantities exceeds the computed value of the combined uncertainty, the meter should be investigated for damage.

5.1.3 Tank gauging uncertainty

The uncertainty of the quantity contained in a vertical cylindrical tank may be computed using a simplified equation based on the following assumptions:

a) Tank calibration has been made in accordance with national or international standards.

b) Tables include or indicate necessary corrections, such as

— tank wall expansion under liquid head;

— shell temperature correction for heated products;

— floating roof immersion correction;

— bottom deflection.

c) Tank floor does not move significantly during transfer.

When these conditions are met, the error due to the calibration table should not exceed ± 0,05 %.

The tank gauging uncertainty may then be calculated using equation (5.23):

$$(E_T)_{95} = 0{,}05 + \frac{1}{10(h_2 - h_1)} \sqrt{2E_h^2 + \left(E_t^2 + 0{,}5^2\right)\left(h_1^2 + h_2^2\right)} \qquad \ldots (5.23)$$

where

$(E_T)_{95}$	is the % uncertainty of the transfer tank quantity;
0,05 %	is the tank calibration error;
h_1	is the high-liquid level, in metres;
h_2	is the low-liquid level, in metres;
E_h	is the gauging error, in millimetres;
E_t	is the temperature error, in °C;
0,5 %	is the uncertainty on the volume correction factor.

5.1.4 Meter uncertainty

The uncertainty of the quantity measured by a meter may be computed using a simplified equation:

$$(E_M)_{95} = \sqrt{E_M^2 + 0{,}01\left(E_t^2 + 0{,}5^2\right)} \qquad \ldots (5.24)$$

where

$(E_M)_{95}$	is the % uncertainty of the metered quantity;
E_M	is the meter volumetric error, in percent;
E_t	is the temperature error, in °C;
0,5 %	is the uncertainty on the volume correction factor.

5.1.5 Calculation of uncertainty of transfer

The combined uncertainty of transfer (shore tank and meter uncertainty) may be computed from equation (5.25).

$$(E)_{95} = \sqrt{(E_T)_{95}^2 + (E_M)_{95}^2} \qquad \ldots (5.25)$$

5.1.6 Tables of uncertainty

For convenience, tables showing the computed values of uncertainties on tanks and meters can be prepared from equations (5.23) and (5.24) for various values of gauging, temperature and meter error.

Examples are shown in tables 14 and 15.

Table 16 gives combined values of transfer uncertainty.

EXAMPLE — Tank gauging uncertainty, using equation (5.23).

$$(E_T)_{95} = 0{,}05 + \frac{1}{10(h_2 - h_1)} \sqrt{2 \times 4^2 + \left(0{,}7^2 + 0{,}5^2\right)\left(h_1^2 + h_2^2\right)}$$

where

± 0,05 % = tank gauging calibration error;

± 4 mm = gauging error;

± 0,7 °C = temperature error;

± 0,5 % = volume correction factor error.

Table 14 — Computed uncertainty for vertical cylindrical tanks at 15 °C

		Tank gauging uncertainty, %													
		h_1 (m) after or before transfer													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
h_2 (m) before or after transfer	2	0,65													
	3	0,36	0,70												
	4	0,27	0,39	0,76											
	5	0,23	0,29	0,43	0,84										
	6	0,20	0,25	0,32	0,47	0,93									
	7	0,19	0,22	0,27	0,35	0,52	1,02								
	8	0,18	0,20	0,24	0,29	0,38	0,58	1,12							
	9	0,17	0,19	0,22	0,25	0,31	0,41	0,62	1,23						
	10	0,17	0,18	0,20	0,23	0,27	0,34	0,45	0,67	1,34					
	11	0,16	0,17	0,19	0,22	0,25	0,29	0,36	0,48	0,72	1,45				
	12	0,16	0,17	0,18	0,20	0,23	0,26	0,31	0,39	0,52	0,78	1,56			
	13	0,16	0,17	0,18	0,19	0,22	0,24	0,28	0,34	0,42	0,56	0,84	1,67		
	14	0,15	0,16	0,17	0,19	0,20	0,23	0,26	0,30	0,36	0,45	0,59	0,89	1,79	
	15	0,15	0,16	0,17	0,18	0,20	0,22	0,24	0,27	0,32	0,38	0,47	0,63	0,95	1,90
	16	0,15	0,16	0,17	0,18	0,19	0,21	0,23	0,26	0,29	0,34	0,40	0,50	0,67	1,01
	17	0,15	0,16	0,16	0,17	0,19	0,20	0,22	0,24	0,27	0,31	0,36	0,43	0,53	0,71
	18	0,15	0,15	0,16	0,17	0,18	0,19	0,21	0,23	0,25	0,28	0,32	0,37	0,45	0,56

EXAMPLE — Meter uncertainty, using equation (5.24).

$$(E_M)_{95} = \sqrt{E_M^2 + 0{,}01\left(E_t^2 + 0{,}5^2\right)}$$

where

$(E_M)_{95}$	is the % uncertainty of the measured quantity;
E_M	is the meter volumetric error, in %
E_t	is the temperature error, in °C;
0,5 %	is the uncertainty in the volume correction factor.

Table 15 — Computed meter uncertainty at 15 °C

Temperature error, E_t, °C	Meter volumetric error, E_M, %													
	0	0,02	0,04	0,06	0,08	0,10	0,12	0,14	0,16	0,18	0,20	0,22	0,24	0,26
0	0,05	0,05	0,06	0,08	0,09	0,11	0,13	0,15	0,17	0,19	0,21	0,23	0,25	0,26
0,2	0,05	0,06	0,07	0,08	0,10	0,11	0,13	0,15	0,17	0,19	0,21	0,23	0,25	0,26
0,4	0,06	0,07	0,08	0,09	0,10	0,12	0,14	0,15	0,17	0,19	0,21	0,23	0,25	0,27
0,6	0,08	0,08	0,09	0,10	0,11	0,13	0,14	0,16	0,18	0,20	0,21	0,23	0,25	0,27
0,8	0,09	0,10	0,10	0,11	0,12	0,14	0,15	0,17	0,19	0,20	0,22	0,24	0,26	0,28
1,0	0,11	0,11	0,12	0,13	0,14	0,15	0,16	0,18	0,20	0,21	0,23	0,25	0,26	0,28
1,2	0,13	0,13	0,14	0,14	0,15	0,16	0,18	0,19	0,21	0,22	0,24	0,26	0,27	0,29
1,4	0,15	0,15	0,15	0,16	0,17	0,18	0,19	0,20	0,22	0,23	0,25	0,27	0,28	0,30
1,6	0,17	0,17	0,17	0,18	0,19	0,20	0,21	0,22	0,23	0,25	0,26	0,28	0,29	0,31
1,8	0,19	0,19	0,19	0,20	0,20	0,21	0,22	0,23	0,25	0,26	0,27	0,29	0,30	0,32
2,0	0,21	0,21	0,21	0,21	0,22	0,23	0,24	0,25	0,26	0,27	0,29	0,30	0,32	0,33

EXAMPLE — Uncertainty of transfer for tank and meter, using equation (5.25).

$$(E)_{95} = \sqrt{(E_T)_{95}^2 + (E_M)_{95}^2}$$

where

$(E)_{95}$ is the % uncertainty of transfer;

$(E_T)_{95}$ is the % uncertainty of the transfer tank quantity;

$(E_M)_{95}$ is the % uncertainty of the metered quantity.

Table 16 — Combined values of transfer uncertainty for tank and meter

Tank quantity uncertainty, $(E_T)_{95}$, %	Metered quantity uncertainty, $(E_M)_{95}$, %													
	0,06	0,08	0,10	0,12	0,14	0,16	0,18	0,20	0,22	0,24	0,26	0,28	0,30	0,32
0,14	0,15	0,16	0,17	0,18	0,20	0,21	0,23	0,24	0,26	0,28	0,30	0,31	0,33	0,35
0,16	0,17	0,18	0,19	0,20	0,21	0,23	0,24	0,26	0,27	0,29	0,31	0,32	0,34	0,36
0,18	0,19	0,20	0,21	0,22	0,23	0,24	0,25	0,27	0,28	0,30	0,32	0,33	0,35	0,37
0,20	0,21	0,22	0,22	0,23	0,24	0,26	0,27	0,28	0,30	0,31	0,33	0,35	0,36	0,38
0,22	0,23	0,23	0,24	0,25	0,26	0,27	0,28	0,30	0,31	0,33	0,34	0,36	0,37	0,39
0,24	0,25	0,25	0,26	0,27	0,28	0,29	0,30	0,31	0,33	0,34	0,35	0,37	0,38	0,40
0,26	0,27	0,27	0,28	0,29	0,30	0,31	0,32	0,33	0,34	0,35	0,37	0,38	0,40	0,41
0,28	0,29	0,29	0,30	0,30	0,30	0,32	0,33	0,34	0,36	0,37	0,38	0,40	0,41	0,43
0,30	0,31	0,31	0,32	0,32	0,33	0,34	0,35	0,36	0,37	0,38	0,40	0,41	0,42	0,44
0,32	0,33	0,33	0,34	0,34	0,35	0,36	0,37	0,38	0,39	0,40	0,41	0,43	0,44	0,45
0,34	0,35	0,35	0,35	0,36	0,37	0,38	0,38	0,39	0,40	0,42	0,43	0,44	0,45	0,47
0,36	0,36	0,37	0,37	0,38	0,39	0,39	0,40	0,41	0,42	0,43	0,44	0,46	0,47	0,48
0,38	0,38	0,39	0,39	0,40	0,40	0,41	0,42	0,43	0,44	0,45	0,46	0,47	0,48	0,50
0,40	0,40	0,41	0,41	0,42	0,42	0,43	0,44	0,45	0,46	0,47	0,48	0,49	0,50	0,51
0,42	0,42	0,43	0,43	0,44	0,44	0,45	0,46	0,47	0,47	0,48	0,49	0,50	0,52	0,53
0,44	0,44	0,45	0,45	0,46	0,46	0,47	0,48	0,48	0,49	0,50	0,51	0,52	0,53	0,54
0,46	0,46	0,47	0,47	0,48	0,48	0,49	0,49	0,50	0,51	0,52	0,53	0,54	0,55	0,56
0,48	0,48	0,49	0,49	0,49	0,50	0,51	0,51	0,52	0,53	0,54	0,55	0,56	0,57	0,58

Annex A
(informative)

Statistical tables

Annex A presents table A.1, table A.2 and table A.3 for use in statistical calculations.

Table A.1 — Distribution of the range

Number of measurements n	Conversion factor $D(n)$	Limiting values $E_1(n)$	
		$P = 95$ %	$P = 99$ %
2	1,128	2,77	3,64
3	1,693	3,31	4,12
4	2,059	3,63	4,40
5	2,326	3,86	4,60
6	2,534	4,03	4,76
7	2,704	4,17	4,88
8	2,847	4,29	4,99
9	2,970	4,39	5,08
10	3,078	4,47	5,16
11	3,173	4,55	5,23
12	3,258	4,62	5,29
13	3,336	4,68	5,35
14	3,407	4,74	5,40
15	3,472	4,80	5,45
16	3,532	4,85	5,49
17	3,588	4,89	5,54
18	3,640	4,93	5,57
19	3,689	4,97	5,61
20	3,735	5,01	5,65

NOTE — From Tippett L.H.C., *Biometrica,* **17,** 1925, p. 364 and Pearson E.S., *Biometrica,* **32,** 1942, p. 301.

Table A.2 — Limiting values $E_2(n, \Phi)$, $P = 0{,}95$

$d\Phi$	n: 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	17,97	26,98	32,82	37,08	40,41	43,12	45,40	47,36	49,07	50,59	51,96	53,20	54,33	55,36	56,32	57,22	58,04	58,83	59,56
2	6,08	8,33	9,80	10,88	11,74	12,44	13,03	13,54	13,99	14,39	14,75	15,08	15,38	15,65	15,91	16,14	16,37	16,57	16,77
3	4,50	5,91	6,82	7,50	8,04	8,48	8,85	9,18	9,46	9,72	9,95	10,15	10,35	10,52	10,69	10,84	10,98	11,11	11,24
4	3,93	5,04	5,76	6,29	6,71	7,05	7,35	7,60	7,83	8,03	8,21	8,37	8,52	8,66	8,79	8,91	9,03	9,13	9,23
5	3,64	4,60	5,22	5,67	6,03	6,33	6,58	6,80	6,99	7,17	7,32	7,47	7,60	7,72	7,83	7,93	8,03	8,12	8,21
6	3,46	4,34	4,90	5,30	5,63	5,90	6,12	6,32	6,49	6,65	6,79	6,92	7,03	7,14	7,24	7,34	7,43	7,51	7,59
7	3,34	4,16	4,68	5,06	5,36	5,61	5,82	6,00	6,16	6,30	6,43	6,55	6,66	6,76	6,85	6,94	7,02	7,10	7,17
8	3,26	4,04	4,53	4,89	5,17	5,40	5,60	5,77	5,92	6,05	6,18	6,29	6,39	6,48	6,57	6,65	6,73	6,80	6,87
9	3,20	3,95	4,41	4,76	5,02	5,24	5,43	5,59	5,74	5,87	5,98	6,09	6,19	6,28	6,36	6,44	6,51	6,58	6,64
10	3,15	3,88	4,33	4,65	4,91	5,12	5,30	5,46	5,60	5,72	5,83	5,93	6,03	6,11	6,19	6,27	6,34	6,40	6,47
11	3,11	3,82	4,26	4,57	4,82	5,03	5,20	5,35	5,49	5,61	5,71	5,81	5,90	5,98	6,06	6,13	6,20	6,27	6,33
12	3,08	3,77	4,20	4,51	4,75	4,95	5,12	5,27	5,39	5,51	5,61	5,71	5,80	5,88	5,95	6,02	6,09	6,15	6,21
13	3,06	3,73	4,15	4,45	4,69	4,88	5,05	5,19	5,32	5,43	5,53	5,63	5,71	5,79	5,86	5,93	5,99	6,05	6,11
14	3,03	3,70	4,11	4,41	4,64	4,83	4,99	5,13	5,25	5,36	5,46	5,55	5,64	5,71	5,79	5,85	5,91	5,97	6,03
15	3,01	3,67	4,08	4,37	4,59	4,78	4,94	5,08	5,20	5,31	5,40	5,49	5,57	5,65	5,72	5,78	5,85	5,90	5,96
16	3,00	3,65	4,05	4,33	4,56	4,74	4,90	5,03	5,15	5,26	5,35	5,44	5,52	5,59	5,66	5,73	5,79	5,84	5,90
17	2,98	3,63	4,02	4,30	4,52	4,70	4,86	4,99	5,11	5,21	5,31	5,39	5,47	5,54	5,61	5,67	5,73	5,79	5,84
18	2,97	3,61	4,00	4,28	4,49	4,67	4,82	4,96	5,07	5,17	5,27	5,35	5,43	5,50	5,57	5,63	5,69	5,74	5,79
19	2,96	3,59	3,98	4,25	4,47	4,65	4,79	4,92	5,04	5,14	5,23	5,31	5,39	5,46	5,53	5,59	5,65	5,70	5,75
20	2,95	3,58	3,96	4,23	4,45	4,62	4,77	4,90	5,01	5,11	5,20	5,28	5,36	5,43	5,49	5,55	5,61	5,66	5,71
24	2,92	3,35	3,90	4,17	4,37	4,54	4,68	4,81	4,92	5,01	5,10	5,18	5,25	5,32	5,38	5,44	5,49	5,55	5,59
30	2,89	3,49	3,85	4,10	4,30	4,46	4,60	4,72	4,82	4,92	5,00	5,08	5,15	5,21	5,27	5,33	5,38	5,43	5,47
40	2,86	3,44	3,79	4,04	4,23	4,39	4,52	4,63	4,73	4,82	4,90	4,98	5,04	5,11	5,16	5,22	5,27	5,31	5,36
60	2,83	3,40	3,74	3,98	4,16	4,31	4,44	4,55	4,65	4,73	4,81	4,88	4,94	5,00	5,06	5,11	5,15	5,20	5,24
120	2,80	3,36	3,68	3,92	4,10	4,24	4,36	4,47	4,56	4,64	4,71	4,78	4,84	4,90	4,95	5,00	5,04	5,09	5,13
∞	2,77	3,31	3,63	3,86	4,03	4,17	4,29	4,39	4,47	4,55	4,62	4,68	4,74	4,80	4,85	4,89	4,93	4,97	5,01

NOTE — From Pearson E.S. and Hartley H.O. (eds.), *Biometrika Tables for Statisticians*, Cambridge University Press, Vol. 1, 1966, Table 29, pp. 192-193.

Table A.3 — Limiting values $E_2(n, \Phi)$, $P = 0{,}99$

$d\Phi$	n = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	90,03	135,0	164,3	185,6	202,2	215,8	227,2	237,0	245,6	253,2	260,0	266,2	271,8	277,0	281,8	286,3	290,4	294,3	298,0
2	14,04	19,02	22,29	24,72	26,63	28,20	29,53	30,68	31,69	32,59	33,40	34,13	34,81	35,43	36,00	36,53	37,03	37,50	37,95
3	8,26	10,62	12,17	13,33	14,24	15,00	15,64	16,20	16,69	17,13	17,53	17,89	18,22	18,52	18,81	19,07	19,32	19,55	19,77
4	6,51	8,12	9,17	9,96	10,58	11,10	11,55	11,93	12,27	12,57	12,84	13,09	13,32	13,53	13,73	13,91	14,08	14,24	14,40
5	5,70	6,98	7,80	8,42	8,91	9,32	9,67	9,97	10,24	10,48	10,70	10,89	11,08	11,24	11,40	11,55	11,68	11,81	11,93
6	5,24	6,33	7,03	7,56	7,97	8,32	8,61	8,87	9,10	9,30	9,48	9,65	9,81	9,95	10,08	10,21	10,32	10,43	10,54
7	4,95	5,92	6,54	7,01	7,37	7,68	7,94	8,17	8,37	8,55	8,71	8,86	9,00	9,12	9,24	9,35	9,46	9,55	9,65
8	4,75	5,64	6,20	6,62	6,96	7,24	7,47	7,68	7,86	8,03	8,18	8,31	8,44	8,55	8,66	8,76	8,85	8,94	9,03
9	4,60	5,43	5,96	6,35	6,66	6,91	7,13	7,33	7,49	7,65	7,78	7,91	8,03	8,13	8,23	8,33	8,41	8,49	8,57
10	4,48	5,27	5,77	6,14	6,43	6,67	6,87	7,05	7,21	7,36	7,49	7,60	7,71	7,81	7,91	7,99	8,08	8,15	8,23
11	4,39	5,15	5,62	5,97	6,25	6,48	6,67	6,84	6,99	7,13	7,25	7,36	7,46	7,56	7,65	7,73	7,81	7,88	7,95
12	4,32	5,05	5,50	5,84	6,10	6,32	6,51	6,67	6,81	6,94	7,06	7,17	7,26	7,36	7,44	7,52	7,59	7,66	7,73
13	4,26	4,96	5,40	5,73	5,98	6,19	6,37	6,53	6,67	6,79	6,90	7,01	7,10	7,19	7,27	7,35	7,42	7,48	7,55
14	4,21	4,89	5,32	5,63	5,88	6,08	6,26	6,41	6,54	6,66	6,77	6,87	6,96	7,05	7,13	7,20	7,27	7,33	7,39
15	4,17	4,84	5,25	5,56	5,80	5,99	6,16	6,31	6,44	6,55	6,66	6,76	6,84	6,93	7,00	7,07	7,14	7,20	7,26
16	4,13	4,79	5,19	5,49	5,72	5,92	6,08	6,22	6,35	6,46	6,56	6,66	6,74	6,82	6,90	6,97	7,03	7,09	7,15
17	4,10	4,74	5,14	5,43	5,66	5,85	6,01	6,15	6,27	6,38	6,48	6,57	6,66	6,73	6,81	6,87	6,94	7,00	7,05
18	4,07	4,70	5,09	5,38	5,60	5,79	5,94	6,08	6,20	6,31	6,41	6,50	6,58	6,65	6,73	6,79	6,85	6,91	6,97
19	4,05	4,67	5,05	5,33	5,55	5,73	5,89	6,02	6,14	6,25	6,34	6,43	6,51	6,58	6,65	6,72	6,78	6,84	6,89
20	4,02	4,64	5,02	5,29	5,51	5,69	5,84	5,97	6,09	6,19	6,28	6,37	6,45	6,52	6,59	6,65	6,71	6,77	6,82
24	3,96	4,55	4,91	5,17	5,37	5,54	5,69	5,81	5,92	6,02	6,11	6,19	6,26	6,33	6,39	6,45	6,51	6,56	6,61
30	3,89	4,45	4,80	5,05	5,24	5,40	5,54	5,65	5,76	5,85	5,93	6,01	6,08	6,14	6,20	6,26	6,31	6,36	6,41
40	3,82	4,37	4,70	4,93	5,11	5,26	5,39	5,50	5,60	5,69	5,76	5,83	5,90	5,96	6,02	6,07	6,12	6,16	6,21
60	3,76	4,28	4,59	4,82	4,99	5,13	5,25	5,36	5,45	5,53	5,60	5,67	5,73	5,78	5,84	5,89	5,93	5,97	6,01
120	3,70	4,20	4,50	4,71	4,87	5,01	5,12	5,21	5,30	5,37	5,44	5,50	5,56	5,61	5,66	5,71	5,75	5,79	5,83
∞	3,64	4,12	4,40	4,60	4,76	4,88	4,99	5,08	5,16	5,23	5,29	5,35	5,40	5,45	5,49	5,54	5,57	5,61	5,65

NOTE — From Pearson E.S. and Hartley H.O. (eds.) *Biometrika Tables for Statisticians*, Cambridge University Press, Vol. 1, 1966, Table 29, pp.192-193.

Annex B
(informative)

t-distribution values for 95 % and 99 % probability (two-sided)

Annex B presents *t*-distribution values in table B.1.

Table B.1 — *t*-distribution values

Degrees of freedom Φ	$t_{95,\Phi}$	$t_{99,\Phi}$
1	12,706	63,657
2	4,303	9,925
3	3,182	5,841
4	2,776	4,604
5	2,571	4,032
6	2,447	3,707
7	2,365	3,499
8	2,306	3,355
9	2,262	3,250
10	2,228	3,169
11	2,201	3,106
12	2,179	3,055
13	2,160	3,012
14	2,145	2,977
15	2,131	2,947
16	2,120	2,921
17	2,110	2,898
18	2,101	2,878
19	2,093	2,861
20	2,086	2,845
21	2,080	2,831
22	2,074	2,819
23	2,069	2,807
24	2,064	2,797
25	2,060	2,787
26	2,056	2,779
27	2,052	2,771
28	2,048	2,763
29	2,045	2,756
30	2,042	2,750
40	2,021	2,704
60	2,000	2,660
120	1,980	2,617
∞	1,960	2,576

NOTE — From Fisher and Yates, *Statistical Tables for Biological, Agricultural and Medical Research.*

Annex C
(informative)

Normal (Gaussian) distribution

Consider a set of n repeated measurements x_i lying in the range a to b so that $a < x_i < b$. If the total range were to be split into p equal sub-ranges of length $dx = (b - a)/p$, a frequency histogram can be drawn. This consists of a series of p contiguous rectangles, with base equal to the sub-range dx and height proportional to the number of measurements falling in that range (see figure C.1).

The height of each rectangle could just as easily represent the proportion of the total number falling in the sub-range, or the relative frequency. The total area of the histogram would then be 1, and the area in each rectangle would become the probability of a measurement falling in the sub-range.

Now consider the number of measurements n becoming very large, and the length dx of each sub-range becoming very small. A continuous line drawn through the mid-point of the tops of each rectangle, which represent the relative frequency of measurements, would give a bell-shaped curve similar to that shown in figure C.2.

For the normal distribution the curve is symmetrical about the true mean μ, and has the formula:

$$f(x) = \frac{1}{\sigma\sqrt{2\Pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

where σ is the standard deviation. The area under the curve once again represents probability. Each of the hatched regions shown, for example, has an area:

$$P = \int_{-\infty}^{\mu - c} f(x)\ \mathrm{d}(x) = \int_{\mu + c}^{+\infty} f(x)\ \mathrm{d}(x)$$

When $c = 1{,}96\sigma$, the probability P (one hatched area) will be 0,025, or 2,5 % of the total area under the curve.

Now if measurements x_i follow the normal distribution with mean μ and standard deviation σ, then values u_i will follow a normal distribution with zero mean and unit standard deviation, where:

$$u_i = \frac{x_i - \mu}{\sigma}$$

The value u_i is termed the "standard normal deviate", and has been tabulated for different probabilities P. For a double-sided probability $P = 0{,}05$, however, the standard normal deviate has a value 1,96. This probability is represented by both hatched areas in the distribution shown, and includes all values of x which differ from the mean μ by more than $1{,}96\sigma$. The corresponding value for a probability $P = 0{,}01$ is $u_i = 2{,}576$.

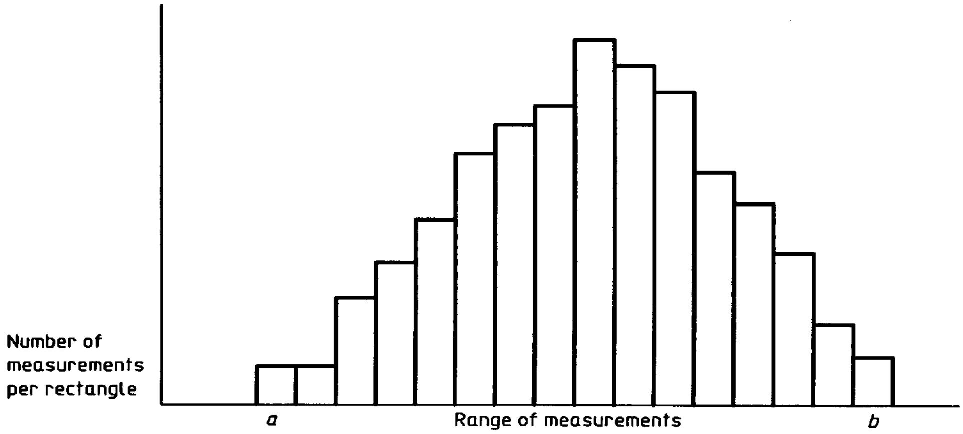


Figure C.1 — Frequency histogram

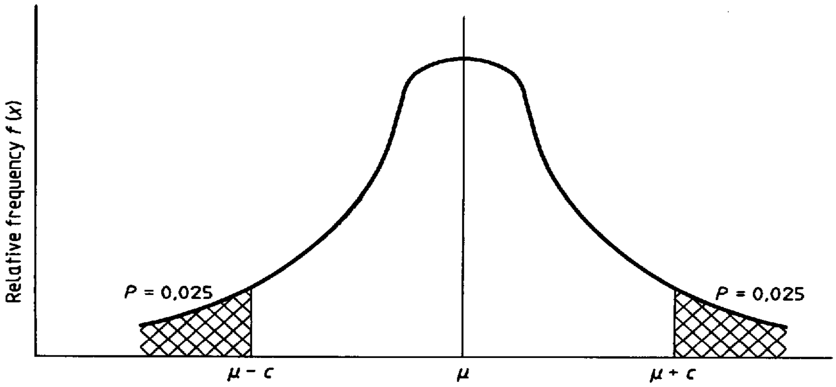


Figure C.2 — Bell-shaped distribution curve

Annex D
(informative)

Outlier tests

D.1 Procedure for Dixon's Test for outliers

Table D.1 — For use with Dixon's Test for outliers

Number of measurements n	Critical values $P = 0,95$	Critical values $P = 0,99$	Test criterion Low values	Test criterion High values
3	0,941	0,988		
4	0,765	0,889		
5	0,642	0,780	$R_{10} = \frac{x_2 - x_1}{x_n - x_1}$ or	$\frac{x_n - x_{n-1}}{x_n - x_1}$
6	0,560	0,698		
7	0,507	0,637		
8	0,554	0,683		
9	0,512	0,635	$R_{11} = \frac{x_2 - x_1}{x_{n-1} - x_1}$ or	$\frac{x_n - x_{n-1}}{x_n - x_2}$
10	0,477	0,597		
11	0,576	0,679		
12	0,546	0,642	$R_{21} = \frac{x_3 - x_1}{x_{n-1} - x_1}$ or	$\frac{x_n - x_{n-2}}{x_n - x_2}$
13	0,521	0,615		
14	0,546	0,641		
15	0,525	0,616		
16	0,507	0,595		
17	0,490	0,577		
18	0,475	0,561		
19	0,462	0,547		
20	0,450	0,535	$R_{22} = \frac{x_3 - x_1}{x_{n-2} - x_1}$ or	$\frac{x_n - x_{n-2}}{x_n - x_3}$
21	0,440	0,524		
22	0,430	0,514		
23	0,421	0,505		
24	0,413	0,497		
25	0,406	0,489		

NOTE — From *Biometrics*, **9**, 1953, p. 89.

D.1.1 Arrange the set of measurements x_i in ascending order of magnitude $x_1, x_2, ..., x_n$.

D.1.2 Using table D.1, choose the appropriate test criterion, depending on the value of n and whether the measurement in question is low or high.

D.1.3 Calculate the Dixon ratio R. If this exceeds the critical ratio at the 5 % probability ($P = 0,95$), then the measurement in question is highly suspect and could possibly be rejected.

D.1.4 If the critical ratio at the 1 % probability level ($P = 0{,}99$) is exceeded, then the measurement in question should be discarded. When a measurement is rejected, the outlier test should be repeated.

NOTE 3 The two subscripts in the Dixon ratio R refer to the differences in the numerator and denominator respectively.

D.2 Procedure for Grubbs Test for outliers

Table D.2 — For use with Grubb's Test for outliers

Number of measurements	Critical values		Test criterion
n	$P = 0{,}95$	$P = 0{,}99$	
3	1,15	1,15	
4	1,46	1,49	
5	1,67	1,75	
6	1,82	1,94	
7	1,94	2,10	
8	2,03	2,22	
9	2,11	2,32	
10	2,18	2,41	
11	2,23	2,48	
12	2,29	2,55	
13	2,33	2,61	$G_1 = (\bar{x} - x_1)/s$
14	2,37	2,66	or
15	2,41	2,71	$G_n = (x_n - \bar{x})/s$
16	2,44	2,75	
17	2,47	2,79	
18	2,50	2,82	
19	2,53	2,85	
20	2,56	2,88	
21	2,58	2,91	
22	2,60	2,94	
23	2,62	2,96	
24	2,64	2,99	
25	2,66	3,01	

NOTE — From *Annals of Mathematical Statistics*, **21**, 1950, p. 27.

D.2.1 Calculate x and s (equations (2.1) (see 2.1.3) and (2.2) (see 2.1.4) respectively), and arrange the set of measurements x_i in ascending order of magnitude $x_1, x_2, ..., x_n$.

D.2.2 Using table D.2, calculate the appropriate test criterion, depending on whether the measurement in question is low or high.

D.2.3 If the test criterion exceeds the critical value at the 5 % probability level ($P = 0{,}95$), then the measurement in question is highly suspect and could possibly be rejected. If the critical value at the 1 % probability level ($P = 0{,}99$) is exceeded, then the measurement in question should be discarded. When a measurement is rejected, the outlier test should be repeated.

NOTE 4 The critical values can be approximated by formulae, allowing the test to be more easily applied when using a programmable calculator. The formulae take the form:

$$G = a_0 + a_1 n^{-1/2} + a_2 n^{-1} + a_3 n^{-3/2} + a_4 n^{-2}$$

Corresponding coefficients are as follows:

	$P = 0{,}95$	$P = 0{,}99$
a_0	3,945 2	4,275 5
a_1	− 9,165 7	− 8,112 4
a_2	18,839	13,348
a_3	− 28,907	− 26,218
a_4	16,023	19,389

Annex E
(informative)

Random uncertainty of polynomial approximation

Given

(x_i, y_i) of a sample of n points;

$\hat{y} = p(x)$ a polynomial which fits these points;

J the degree of the polynomial $p(x)$

The random uncertainty of the polynomial is given by:

$t_{95,\Phi} s$

where

$t_{95,\Phi}$ is the Student's t-value for a probability level of 95 %;

$\Phi = N - J$ is the number of degrees of freedom;

$s^2 = \frac{1}{N-J} \sum_{i=1}^{n} [y_i - p(x_i)]^2$

Annex F
(informative)

Bibliography

[1] ISO 7278-3:1986, *Liquid hydrocarbons — Dynamic measurement — Proving systems for volumetric meters — Part 3: Pulse interpolation techniques.*

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