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Sampling procedures for inspection by variables —

Part 4: Procedures for assessment of declared quality levels

Règles d'échantillonnage pour les contrôles par mesures —

Partie 4: Procédures pour l'évaluation des niveaux déclarés de qualité

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 3951-4 was prepared by Technical Committee ISO/TC 69, *Application of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

ISO 3951 consists of the following parts, under the general title *Sampling procedures for inspection by variables*:

— *Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

— *Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

— *Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

— *Part 4: Procedures for assessment of declared quality levels*

— *Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)*

Introduction

The procedures in this part of ISO 3951 differ in their scope from the procedures in ISO 3951 Parts 1, 2, 3 and 5. The acceptance sampling procedures that are specified in ISO 3951 Parts 1, 2, 3 and 5 are intended to be used in bilateral agreements between two parties. Those acceptance sampling procedures are intended to be used as simple, pragmatic rules for releasing product after inspection of only a limited sample of a consignment, and therefore the procedures do not make reference (either explicitly or implicitly) to any formally declared quality level.

Under acceptance sampling, there is no sharp borderline between quality levels that should be considered acceptable and qualities that should be rejected by the procedure. For the procedures in ISO 3951 Parts 1, 2, 3 and 5, the two parties agree upon some limiting quality level (AQL) which is the worst tolerable process average when a continuing series of lots is submitted. The switching rules and the sampling schemes in those four standards are designed to encourage the suppliers to have process averages consistently better than the AQL selected. In order to keep sample sizes moderate, the protection against accepting individual lots of inferior quality may be less than that provided by sampling plans targeted for sentencing individual lots.

Procedures in ISO 3951 Parts 1, 2, 3 and 5 are well suited for acceptance sampling purposes, but they should not be used in reviews, audits, etc. to verify a quality that has been declared for some entity. The main reason is that the procedures have been indexed in terms of quality levels that are relevant solely for the pragmatic purposes of acceptance sampling, and the various risks have been balanced accordingly.

The procedures in this part of ISO 3951 have been developed as a response to the growing need for sampling procedures suitable for formal, systematic inspections such as reviews or audits. When performing such a formal inspection, it is necessary for the authority to consider the risk of reaching an incorrect conclusion, and to take this risk into account in planning and executing the review/audit/testing, etc.

This part of ISO 3951 provides guidance and rules to assist the user in taking this risk into account in an informed manner.

The rules in this part of ISO 3951 have been devised such that there is only a small, limited risk of contradicting the declared quality level when in fact the actual level conforms to the declared level.

If it were also desired that there should be a similarly small risk of not contradicting the declared quality level when in fact the actual quality level does not conform to the declared quality level, then it would be necessary to investigate a rather large sample. Therefore, in order to obtain the benefit of a moderate sample size, the procedures in this part of ISO 3951 have been devised in such a way that they allow a somewhat higher risk of failing to contradict the declared quality level when in fact the actual quality level does not conform to the declared quality level.

The wording of the result of the assessment should reflect this unbalance between the risks of reaching incorrect conclusions.

When the sample result contradicts the declared quality level, *there is strong evidence of nonconformance to the declared quality level.*

When the sample result does not contradict the declared quality level, this should be understood as “we have not, in this limited sample, found strong evidence of nonconformance to the declared quality level”.

Sampling procedures for inspection by variables —

Part 4:
Procedures for assessment of declared quality levels

1 Scope

This part of ISO 3951 establishes sampling plans and procedures by variables that can be used to assess whether the quality level of an entity (lot, process, etc.) conforms to a declared value. The sampling plans have been devised so that their operating characteristic curves match those of the corresponding attributes plans in ISO 2859-4 as closely as possible, so that the choice between using sampling by attributes and sampling by variables is not influenced by attempts to increase the chance of accepting an incorrectly declared quality level. In this part of ISO 3951, there is a risk of between 1,4 % and 8,2 % of contradicting a correct declared quality level. The risk is 10 % of failing to contradict an incorrect declared quality level which is related to the limiting quality ratio (see Clause 4). Sampling plans are provided corresponding to three levels of discriminatory ability, and for the cases of unknown and known process standard deviation.

In contrast to the procedures in the other parts of ISO 3951, the procedures in this part of ISO 3951 are not applicable to acceptance assessment of lots. Generally, the balancing of the risks of reaching incorrect conclusions in assessment procedures will differ from the balancing in the procedures for acceptance sampling.

This part of ISO 3951 may be used for various forms of quality inspection in situations where objective evidence of conformity to some declared quality level is to be provided by means of inspection of a sample. The procedures are applicable to entities such as lots, process output, etc. that allow random samples of individual items to be taken from the entity.

The sampling plans provided in this part of ISO 3951 are applicable, but not limited, to inspection of a variety of products such as

— end items,

— components and raw materials,

— operations,

— materials in process,

— supplies in storage,

— maintenance operations,

— data or records, and

— administrative procedures.

The procedures are intended to be used when the quality characteristics are measurable variables that are independent and normally distributed, and where the quantity of interest is the fraction of items that are nonconforming.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2859-4:2002, *Sampling procedures for inspection by attributes — Part 4: Procedures for assessment of declared quality levels*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 3951-2: 2006, *Sampling procedures for inspection by variables — Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

ISO 9000, *Quality management systems — Fundamentals and vocabulary*

3 Terms, definitions, symbols and abbreviations

3.1 Terms and definitions

For the purposes of this part of ISO 3951, the terms and definitions given in ISO 3534-1, ISO 3534-2, ISO 3951-2 and ISO 9000 and the following apply.

3.1.1
quality ratio
ratio of the actual to the declared quality level of the entity under investigation

3.1.2
limiting quality ratio
LQR
value of the quality ratio that is limited to a small risk (10 % in this part of ISO 3951) of failing to contradict an incorrect declared quality level

3.2 Symbols and abbreviated terms

The symbols and abbreviated terms used in this part of ISO 3951 are as follows:

$B_v(.)$ Distribution function of the symmetric beta distribution with both parameters equal to v

$B(v,v)$ Beta function with both arguments equal to v, i.e. $B(v,v)=\Gamma(v)\Gamma(v)/\Gamma(v)$ where $\Gamma(v)$ is the gamma function (see below)

D Declared quality level (as a symbol)

DQL Declared Quality Level (as an acronym)

k_s Form k acceptability constant under the "s" method, used when the sample standard deviation is unknown

k_σ Form k acceptability constant under the "σ" method, used when the process standard deviation is presumed to be known

L Lower specification limit (as a subscript, denotes the value at L)

LQR Limiting Quality Ratio (as an acronym)

m Number of quality characteristics, all assumed to be independent and normally distributed

n_s Sample size under the "s" method

n_σ Sample size under the "σ" method

OC Operating Characteristic

p Process fraction nonconforming in the entity

$\hat{p}$ Estimate of the fraction nonconforming in the entity

$\hat{p}_{\mathrm{c}}$ Estimate of the combined fraction nonconforming at both specification limits, i.e. $\hat{p}_{\mathrm{c}} = \hat{p}_L + \hat{p}_U$

p^* Form p^* acceptability constant (for both the "s" and "σ" methods)

Q Quality statistic (see 7.2.2 and 7.3.2)

s Sample standard deviation

U Upper specification limit (as a subscript, denotes the value at U)

$\bar{x}$ Sample mean

$\Phi(.)$ Standard normal distribution function

$\Gamma(v)$ Gamma function, defined by $\Gamma(v) = \int_0^\infty t^{v-1}\exp(-t)\mathrm{d}t$ for $v > 0$

σ Process standard deviation

4 Principles

In any assessment procedure based on sampling, there will be an inherent uncertainty due to possible sampling fluctuations. The procedures in this part of ISO 3951 have been conceived so as to lead to contradiction of the declared quality level only when there is sufficient evidence to support a conclusion that the actual quality is poorer than the declared quality level.

The plans have been devised in such a way that their operating characteristic curves match those of the corresponding attributes plans ISO 2859-4 as closely as possible. Details of the matching method are given in Annex A. The attributes plans of ISO 2859-4 were selected such that when the actual quality level is equal to or better than the declared quality level, the risk is less than 5 % of contradicting the declared value. It follows that when the actual quality level is worse than the declared quality level, there is a risk that the procedures will fail to contradict an incorrect declared quality level. Owing to the fact that the match between corresponding OC curves in ISO 2859-4 and ISO 3951-4 is imperfect, the corresponding risk in this part of ISO 3951 varies around 5 %.

This risk depends on the value of the quality ratio, i.e. the ratio between the actual and the declared quality level. The limiting quality ratio, LQR, is introduced to denote the highest quality ratio considered tolerable. When the actual quality level is LQR times the declared quality level, the procedures in this part of ISO 3951 have a risk of 10 % of failing to contradict the declared quality level (corresponding to a 90 % probability of contradicting the incorrect declared quality level).

Three LQR levels I, II and III are considered; details of the three LQR levels provided in this part of ISO 3951 are given in 6.1. Sampling plans are provided for both the case where the process standard deviation is unknown (the "s" method) and the case where it is known (the "σ" method). (See ISO 3951-2 for details on the implementation of sampling by variables plans.)

The sampling plans provided in this part of ISO 3951 are indexed by the limiting quality ratio (LQR) level and the declared quality level (DQL) and are provided in Table 1.

5 Declared quality level (DQL)

The DQL together with the LQR level is used for indexing the sampling plans provided in this part of ISO 3951. The values of DQL in the tables are known as preferred DQLs. The series of preferred DQL values correspond to the series of preferred AQLs for inspection for nonconforming items given in ISO 3951-1.

There shall be a sound basis for the DQL used. The DQL shall not be deliberately overstated or understated.

When a DQL is designated for a certain type of nonconformity, it indicates that the supplier has good reason to believe that the quality is not worse than this designated value.

CAUTION — When the DQL is estimated from a sample taken from the entity of interest, the procedures in this International Standard shall not be used. Such a verification of an estimate from a sample requires that the sample size and inspection result be taken into account in order to incorporate the uncertainty associated with the estimate. This uncertainty affects the assessment of the risks of making incorrect conclusions on the actual status of the entity of interest. Such verification usually requires larger sample sizes than those used in the procedures described in this part of ISO 3951.

Table 1 — Master table of sampling plans

DQL	LQR Level I					LQR Level II					LQR Level III				
in % nonconforming items	n_s	k_s	n_σ	k_σ	100 p^*	n_s	k_s	n_σ	k_σ	100 p^*	n_s	k_s	n_σ	k_σ	100 p^*
0,010	132	3,286	23	3,277	0,040 31	←					←				
0,015	117	3,156	21	3,143	0,064 05	←					←				
0,025	101	3,016	20	3,003	0,103 0	179	3,148	33	3,140	0,071 38	←				
0,040	86	2,879	19	2,867	0,161 4	158	3,012	31	3,003	0,113 6	258	3,187	46	3,181	0,065 03
0,065	73	2,728	17	2,710	0,260 4	132	2,867	29	2,858	0,181 7	223	3,051	44	3,045	0,103 5
0,10	60	2,573	16	2,556	0,415 6	112	2,723	27	2,712	0,285 4	189	2,912	40	2,905	0,163 2
0,15	50	2,412	15	2,393	0,662 1	93	2,565	25	2,553	0,458 7	160	2,762	37	2,754	0,261 8
0,25	40	2,237	13	2,211	1,070	76	2,400	23	2,387	0,732 7	134	2,614	34	2,604	0,410 3
0,40	31	2,061	12	2,033	1,685	61	2,230	20	2,212	1,162	110	2,449	31	2,438	0,659 8
0,65	24	1,863	11	1,830	2,747	48	2,043	18	2,021	1,876	89	2,279	28	2,266	1,052
1,0	18	1,659	9	1,611	4,376	37	1,853	16	1,827	2,962	70	2,101	26	2,087	1,667
1,5	13	1,426	8	1,367	7,199	27	1,636	14	1,604	4,802	54	1,904	23	1,886	2,688
2,5	9	1,189	7	1,114	11,44	20	1,411	12	1,370	7,626	41	1,702	20	1,680	4,238
4,0	6	0,887	6	0,786	19,45	13	1,195	8	1,127	11,42	30	1,471	17	1,442	6,857
6,5	4	0,536	3	0,379	32,13	9	0,869	8	0,801	19,60	21	1,227	14	1,190	10,85
10	3	0,044	2	0,021	48,79	6	0,497	4	0,402	32,11	14	0,935	9	0,877	17,61

The plans are indexed by the declared quality level (DQL) of nonconforming product and limiting quality ratio (LQR) levels.

← Use the sampling plan to the left, which corresponds to a higher limiting quality ratio as no sampling plan exists for this level of the limiting quality ratio.

6 Sampling plans

6.1 LQR (limiting quality ratio) levels

6.1.1 Level I

Level I may be used when a smaller sample size is desirable. For Level I sampling plans, the limiting quality ratios range in value from 7,6 to 14,1. For example, if the declared quality level is 1,0 % nonconforming items, and the actual quality level is 12,2 times this declared quality level, then the risk is 10 % of failing to contradict the declared quality level (see Table 2).

Table 2 — Level I plans, limiting quality ratios (LQRs) and probabilities of falsely contradicting correctly declared quality levels (DQLs)

DQL in % nonconforming items	"s" method				"σ" method				100 p *
	n_s	k_s	LQR	Probability of falsely contradicting a correct DQL in %	n_σ	k_σ	LQR	Probability of falsely contradicting a correct DQL in %	
0,010	132	3,286	13,6	2,5	23	3,277	13,1	1,7	0,040 31
0,015	117	3,156	14,1	2,1	21	3,143	14,0	1,5	0,064 05
0,025	101	3,016	13,5	2,4	20	3,003	13,2	1,6	0,103 0
0,040	86	2,879	13,2	2,6	19	2,867	12,6	1,7	0,161 4
0,065	73	2,728	12,9	2,7	17	2,710	12,6	1,8	0,260 4
0,10	60	2,573	13,3	2,7	16	2,556	12,7	1,6	0,415 6
0,15	50	2,412	13,7	2,3	15	2,393	13,1	1,3	0,662 1
0,25	40	2,237	13,1	2,7	13	2,211	12,7	1,6	1,070
0,40	31	2,061	12,7	3,1	12	2,033	12,0	1,6	1,685
0,65	24	1,863	12,2	3,2	11	1,830	11,5	1,5	2,747
1,0	18	1,659	12,2	3,2	9	1,611	11,8	1,6	4,376
1,5	13	1,426	12,5	2,9	8	1,367	12,0	1,2	7,199
2,5	9	1,189	11,1	3,6	7	1,114	10,6	1,3	11,44
4,0	6	0,887	10,3	3,4	6	0,786	9,9	0,91	19,45
6,5	4	0,536	8,9	3,1	3	0,379	9,9	2,5	32,13
10	3	0,044	7,6	1,6	2	0,021	8,1	3,7	48,79

EXAMPLE Suppose the "s" method plan $n_s = 60$, $k_s = 2{,}573$ is used, corresponding to a declared quality level (DQL) of 0,10 % nonconforming items. For this plan, there is a risk of 10,0 % of failing to contradict this DQL when the actual quality level is 13,3 (LQR) times the declared quality level, i.e. if the actual quality level is 1,33 % nonconforming items.

If, on the contrary, the actual quality level had been the DQL, i.e. if the actual quality level is 0,10 % nonconforming items, then there is a risk of 2,7 % of falsely contradicting this correct DQL.

6.1.2 Level II

Level II is the standard level that shall be used unless specific conditions warrant the use of another level. For Level II sampling plans, the limiting quality ratios range in value from 5,34 to 7,48. For example, if the declared quality level is 0,10 % nonconforming items, and the actual quality level is 7,05 times the declared quality level, then the risk of failing to contradict the declared quality level under the "s" method is 10,0 % (see Table 3).

Table 3 — Level II plans, limiting quality ratios (LQRs) and probabilities of falsely contradicting correctly declared quality levels (DQLs)

DQL in % nonconforming items	"s" method				"σ" method				100 p *
	n_s	k_s	LQR	Probability of falsely contradicting a correct DQL in %	n_σ	k_σ	LQR	Probability of falsely contradicting a correct DQL in %	
0,025	179	3,148	7,22	3,4	33	3,140	7,07	2,5	0,0713 8
0,040	158	3,012	7,06	3,4	31	3,003	6,95	2,6	0,113 6
0,065	132	2,867	6,97	3,7	29	2,858	6,76	2,7	0,181 7
0,10	112	2,723	7,05	3,6	27	2,712	6,84	2,5	0,285 4
0,15	93	2,565	7,48	3,0	25	2,553	7,21	1,9	0,458 7
0,25	76	2,400	7,10	3,5	23	2,387	6,80	2,2	0,732 7
0,40	61	2,230	6,95	3,8	20	2,212	6,77	2,5	1,162
0,65	48	2,043	6,76	4,0	18	2,021	6,59	2,5	1,876
1,0	37	1,853	6,78	3,9	16	1,827	6,60	2,3	2,962
1,5	27	1,636	7,14	3,4	14	1,604	6,90	1,7	4,802
2,5	20	1,411	6,48	3,9	12	1,370	6,35	2,0	7,626
4,0	13	1,195	6,04	5,9	8	1,127	6,25	3,9	11,42
6,5	9	0,869	5,66	4,6	8	0,801	5,60	2,2	19,60
10	6	0,497	5,34	3,2	4	0,402	5,94	3,9	32,11

EXAMPLE Suppose the "s" method plan $n_s = 112$, $k_s = 2{,}723$ is used, corresponding to a declared quality level (DQL) of 0,10 % nonconforming items. For this plan, there is a risk of 10,0 % of failing to contradict this DQL when the actual quality level is 7,05 (LQR) times the declared quality level, i.e. if the actual quality level is 0,705 % nonconforming items.

If, on the contrary, the actual quality level is equal to the DQL, i.e. if the actual quality level is 0,10 % nonconforming items, then there is a risk of 3,6 % of falsely contradicting this correct DQL.

6.1.3 Level III

Level III is for situations where a smaller LQR is desired, at the expense of a larger sample size. For Level III sampling plans, the limiting quality ratios range in value from 4,72 to 5,97. For example, if the declared quality level is 0,10 % nonconforming items and the actual quality level is 5,30 times this declared quality level, i.e. 0,530 %, then under the "σ" method there is a risk of 10 % of failing to contradict the declared quality level (see Table 4).

Table 4 — Level III plans, limiting quality ratios (LQRs) and probabilities of falsely contradicting correctly declared quality levels (DQLs)

DQL in % nonconforming items	"s" method				"σ" method				100 p*
	n_s	k_s	LQR	Probability of falsely contradicting a correct DQL in %	n_σ	k_σ	LQR	Probability of falsely contradicting a correct DQL in %	
0,040	258	3,187	5,63	2,8	46	3,181	5,54	2,1	0,065 03
0,065	223	3,051	5,57	2,9	44	3,045	5,43	2,1	0,103 5
0,10	189	2,912	5,41	3,4	40	2,905	5,30	2,5	0,163 2
0,15	160	2,762	5,61	2,9	37	2,754	5,49	2,0	0,261 8
0,25	134	2,614	5,82	2,5	34	2,604	5,71	1,7	0,410 3
0,40	110	2,449	5,57	3,0	31	2,438	5,45	2,0	0,659 8
0,65	89	2,279	5,49	3,1	28	2,266	5,37	2,1	1,052
1,0	70	2,101	5,30	3,6	26	2,087	5,11	2,2	1,667
1,5	54	1,904	5,45	3,1	23	1,886	5,27	1,7	2,688
2,5	41	1,702	5,61	2,7	20	1,680	5,45	1,4	4,238
4,0	30	1,471	5,97	1,9	17	1,442	5,86	0,9	6,857
6,5	21	1,227	5,01	3,3	14	1,190	4,96	1,8	10,85
10	14	0,935	4,72	3,3	9	0,877	5,02	2,8	17,61

EXAMPLE Suppose the "σ" method plan $n_\sigma = 40$, $k_\sigma = 2{,}905$ is chosen, corresponding to a declared quality level (DQL) of 0,10 % nonconforming items. For this plan, there is a 10 % risk of failing to contradict this DQL when the actual quality level is 5, 30 (LQR) times the declared quality level, i.e. if the actual quality level is 0,530 % nonconforming items.

If, on the contrary, the actual quality level is equal to the DQL, i.e. if the actual quality level is 0,10 % nonconforming items, then there is a risk of 2,5 % of falsely contradicting this correct DQL.

6.2 Selection of an "s" method sampling plan

Given the chosen DQL and LQR levels, use Table 1 to select an "s" method single sampling plan.

EXAMPLE For example, if the process standard deviation is unknown and LQR Level II is chosen with a DQL of 0,65 % nonconforming items, Table 1 yields an "s" method sampling plan with a sample size of 48, and a Form k acceptability constant of 2,043 (or, equivalently, a Form p* acceptability constant of 0,018 76), which, from Table 3, provides an LQR of 6,76.

If the declared quality level is not one of the tabulated values, then the next higher tabulated value of DQL shall be used to select the plan.

NOTE This will result in a limiting quality ratio that is somewhat higher and a probability of falsely contradicting a correct declared quality level that is somewhat lower than the values given in Tables 2 to 4 (see 8.2).

6.3 Selection of a "σ" method sampling plan

Given the chosen DQL and LQR levels, use Table 1 to select a "σ" method single sampling plan.

EXAMPLE For example, if the process standard deviation is presumed to be known and LQR Level II is chosen with a DQL of 0,65 % nonconforming items, Table 1 yields a "σ" method sampling plan with sample size 18 and Form k acceptability constant 2,021, which provides an LQR of 6,59 (see Table 3).

If the declared quality level is not one of the tabulated values, then the next higher tabulated value of DQL shall be used to select the plan.

NOTE This will result in a limiting quality ratio that is somewhat higher and a probability of falsely contradicting a correct declared quality level that is somewhat lower than the values given in Tables 2 to 4 (see 8.2).

7 Operating a sampling plan

7.1 Sample selection

The sample shall be selected by simple random sampling. If the sample size exceeds the size of the entity under investigation, then all items of the entity shall be inspected.

7.2 Rules for contradicting a declared quality level: "s" method

7.2.1 General

Determine the applicable sampling plan (n_s, k_s), or equivalently (n_s, p^*), from Table 1.

If the sample size equals or exceeds the size of the entity under investigation, then the DQL shall be verified by comparing it to the actual quality level determined by inspecting all items in the entity.

Otherwise, select a random sample of size n_s. For each item in the sample, measure the value of the quality characteristic x. Calculate the sample mean $\bar{x}$ and the sample standard deviation s.

7.2.2 Single specification limit

For a single upper specification limit U, calculate the quality statistic $Q = (U - \bar{x})/s$.

For a single lower specification limit L, calculate the quality statistic $Q = (\bar{x} - L)/s$.

If $Q \geq k_s$, the declared quality level has not been contradicted. If $Q < k_s$, the declared quality level has been contradicted.

EXAMPLE A Level I DQL of 0,25 % is to be used, with an upper specification limit $U = 11{,}5$. The quality characteristic is normally distributed with unknown process standard deviation. From Table 1, it is seen that a sample size $n_s = 40$ is required and that the accompanying Form k acceptability constant $k_s = 2{,}237$. Suppose that the random sample of 40 items from the entity yields a sample mean $\bar{x} = 10{,}62$ and a sample standard deviation $s = 0{,}442$. The quality statistic $Q = (11{,}5 - 10{,}62)/0{,}442 = 1{,}991$. As $Q < k_s$, the declared quality level has been contradicted.

An example of a single specification limit for known process standard deviation is given in B.2.

7.2.3 Double specification limits under combined control

In the case of double specification limits U and L under combined control, calculate

$$\hat{p}_U = B_{(n-2)/2}\left[\frac{1}{2}\left(1-\frac{U-\bar{x}}{s}\frac{\sqrt{n}}{n-1}\right)\right] = B_{(n-2)/2}\left[\frac{1}{2}\left(1-\frac{\sqrt{n}Q_U}{n-1}\right)\right]$$

$$\hat{p}_L = B_{(n-2)/2}\left[\frac{1}{2}\left(1-\frac{\bar{x}-L}{s}\frac{\sqrt{n}}{n-1}\right)\right] = B_{(n-2)/2}\left[\frac{1}{2}\left(1-\frac{\sqrt{n}Q_L}{n-1}\right)\right]$$

and

$$\hat{p}_{\mathrm{c}} = \hat{p}_U + \hat{p}_L$$

where $B_v(a) = \int_0^a \frac{t^{v-1}(1-t)^{v-1}}{B(v,v)}\mathrm{d}t$, with $B(v,v) = \frac{\Gamma(v)\Gamma(v)}{\Gamma(2v)}$ and $\Gamma(v) = \int_0^\infty t^{v-1}\exp(-t)\ \mathrm{d}t$.

NOTE 1 See K.3 of ISO 3951-2:2006 for an accurate normal approximation to $\hat{p}_U$ and $\hat{p}_L$.

NOTE 2 $B_v(a)$ is taken to be equal to 0 when $a < 0$ or equal to 1 when $a > 1$.

If $\hat{p}_{\mathrm{c}} \leq p^*$, the declared quality level has not been contradicted; if $\hat{p}_{\mathrm{c}} > p^*$, the declared quality level has been contradicted.

EXAMPLE A Level II DQL of 1,0 % is to be used with double specification limits $L = 40{,}00$ and $U = 40{,}80$. The quality characteristic is normally distributed with unknown process standard deviation. From Table 1, it is seen that a sample size $n_s = 37$ is required and that the accompanying Form p^* acceptability constant $p^* = 0{,}029\ 62$. Suppose that the random sample of 37 items from the entity yields a sample mean $\bar{x} = 40{,}328$ and a sample standard deviation $s = 0{,}154$.

The upper and lower quality statistics are calculated as

$Q_U = (40{,}800 - 40{,}332) / 0{,}154 = 3{,}039$ and

$Q_L = (40{,}328 - 40{,}000)/0{,}154 = 2{,}130$, respectively.

The corresponding estimates of the fractions nonconforming at the two limits are

$$\hat{p}_U = B_{(n-2)/2}\left[\frac{1}{2}\left\{1-\sqrt{n}Q_U/(n-1)\right\}\right] = B_{17,5}\left[\frac{1}{2}\left\{1-\sqrt{37}\times 3{,}039/36\right\}\right] = B_{17,5}[0{,}243\ 3] = 0{,}000\ 58$$

and

$$\hat{p}_L = B_{(n-2)/2}\left[\frac{1}{2}\left\{1-\sqrt{n}Q_L/(n-1)\right\}\right] = B_{17,5}\left[\frac{1}{2}\left\{1-\sqrt{37}\times 2{,}130/36\right\}\right] = B_{17,5}[0{,}3201] = 0{,}014\ 36$$

The sum of these estimates is $\hat{p}_{\mathrm{c}} = \hat{p}_U + \hat{p}_L = 0{,}000\ 58 + 0{,}014\ 36 = 0{,}014\ 94$. As $\hat{p}_{\mathrm{c}} \leq p^*$, the declared quality level has not been contradicted.

An example of double specification limits under combined control for unknown process standard deviation is given in B.1.

7.2.4 Double specification limits under separate control

For double specification limits under separate control, there will be separate DQLs applying to each limit, say D_U for the upper limit and D_L for the lower limit. Denote the Form k plans for these DQLs (n_U, k_U) and (n_L, k_L) respectively. Denote the sample means and sample standard deviations yielded from random samples of size n_U and n_L by $\bar{x}_U$, s_U and $\bar{x}_L$, s_L respectively. Calculate $Q_U = (U - \bar{x}_U)/s_U$ and $Q_L = (\bar{x}_L - L)/s_L$. If $Q_U \geq k_U$ and $Q_L \geq k_L$, the declared quality levels have not been contradicted; otherwise, at least one of the declared quality levels has been contradicted.

EXAMPLE Double specification limits are to be controlled separately with a Level II DQL of 0,65 % at the upper limit $U = 3{,}125$ and a Level III DQL of 0,25 % at the lower limit $L = 3{,}100$. The quality characteristic is normally distributed with unknown process standard deviation. From Table 1, it is seen that the appropriate Form k plans are $n_U = 48$, $k_U = 2{,}043$ for the upper limit and $n_L = 134$, $k_L = 2{,}614$ for the lower limit. Suppose that the random sample of 48 items from the entity yields a sample mean $\bar{x}_U = 3{,}1173$ and a sample standard deviation $s_U = 0{,}002\,91$, and that a sample of size 134 from the same entity yields a sample mean $\bar{x}_L = 3{,}116\,9$ and a standard deviation $s_L = 0{,}003\,07$. The upper and lower quality statistics are calculated as $Q_U = (3{,}125 - 3{,}117\,3)/0{,}00291 = 2{,}646$ and $Q_L = (3{,}116\,9 - 3{,}100)/0{,}003\,07 = 5{,}505$ respectively. As $Q_U > k_U$ and $Q_L > k_L$, the declared quality levels are not contradicted.

An example of double specification limits under separate control for unknown process standard deviation is given in B.3.

7.2.5 Double specification limits under complex control

Double specification limits under complex control consists of combined control of both limits together with separate control of one of the limits. There will be a DQL for the combined fraction nonconforming at the two limits and a DQL for the fraction nonconforming at the limit that is under separate control. Suppose without loss of generality that the separately controlled limit is the upper limit, and denote the Form p^* plans by $n_{\mathrm{c}}, p_{\mathrm{c}}^*$ and n_U, p_U^* for the combined part of the complex requirement and for the upper limit respectively. A random sample of size n_{c} is drawn, which yields a sample mean of $\bar{x}_{\mathrm{c}}$ and a sample standard deviation of s_{c}. A second random sample of size n_U is drawn, which yields a sample mean of $\bar{x}$ and a sample standard deviation of s.

Calculate

$$\hat{p}_{\mathrm{c}} = \hat{p}_{\mathrm{c},U} + \hat{p}_{\mathrm{c},L} = B_{(n_{\mathrm{c}}-2)/2}\left[\frac{1}{2}\left(1 - \frac{U - \bar{x}_{\mathrm{c}}}{s_{\mathrm{c}}}\frac{\sqrt{n_{\mathrm{c}}}}{n_{\mathrm{c}}-1}\right)\right] + B_{(n_{\mathrm{c}}-2)/2}\left[\frac{1}{2}\left(1 - \frac{\bar{x}_{\mathrm{c}} - L}{s_{\mathrm{c}}}\frac{\sqrt{n_{\mathrm{c}}}}{n_{\mathrm{c}}-1}\right)\right]$$

$$\hat{p}_U = B_{(n_U-2)/2}\left[\frac{1}{2}\left(1 - \frac{U - \bar{x}}{s}\frac{\sqrt{n_U}}{n_U - 1}\right)\right]$$

If $\hat{p}_{\mathrm{c}} \leq p_{\mathrm{c}}^*$ and $\hat{p}_U \leq p_U^*$, the declared quality levels have not been contradicted; otherwise, the pair of declared quality levels for complex control has been contradicted.

NOTE If, instead, it were the lower limit that was under separate control then, if $\hat{p}_{\mathrm{c}} \leq p_{\mathrm{c}}^*$ and $\hat{p}_L \leq p_L^*$, the declared quality levels have not been contradicted; otherwise, at least one of the declared quality levels has been contradicted.

EXAMPLE This example is a modification of the example of 7.2.4. Complex control of double specification limits is to be used with $U = 3{,}125$ and $L = 3{,}100$. A Level 2 DQL of 0,65 % applies to both limits combined and a Level 3 DQL of 0,25 % applies to the lower limit. The quality characteristic is normally distributed with unknown process standard deviation. From Table 1, it is seen that the appropriate Form p^* plans are $n_{\mathrm{c}} = 48$, $p_{\mathrm{c}}^* = 0{,}018\,76$ for both limits combined and $n_L = 134$, $p_L^* = 0{,}004\,103$ for the lower limit. Suppose that the random sample of 48 items from the entity yields a sample mean $\bar{x}_{\mathrm{c}} = 3{,}117\,3$ and a sample standard deviation $s_{\mathrm{c}} = 0{,}002\,91$, and that a sample of size 134 from the same entity yields a sample mean $\bar{x}_L = 3{,}116\,9$ and a standard deviation $s_L = 0{,}004\,07$. The upper and lower quality statistics for the combined control part of the specification are $Q_U = (3{,}125 - 3{,}117\,3)/0{,}002\,91 = 2{,}646$ and $Q_L = (3{,}116\,9 - 3{,}100)/0{,}002\,91 = 2{,}371$ respectively.

The combined estimated process fraction nonconforming is

$$\begin{aligned}\hat{p}_c &= \hat{p}_{c,U} + \hat{p}_{c,L} \\ &= B_{(n_c-2)/2}\left[\frac{1}{2}\left(1-\frac{U-\bar{x}_c}{s_c}\frac{\sqrt{n_c}}{n_c-1}\right)\right] + B_{(n_c-2)/2}\left[\frac{1}{2}\left(1-\frac{\bar{x}_c-L}{s_c}\frac{\sqrt{n_c}}{n_c-1}\right)\right] \\ &= B_{23}\left[\frac{1}{2}\left(1-\frac{2{,}646\sqrt{48}}{47}\right)\right] + B_{23}\left[\frac{1}{2}\left(1-\frac{2{,}371\sqrt{48}}{47}\right)\right] \\ &= B_{23}(0{,}305\,0) + B_{23}(0{,}325\,2) \\ &= 0{,}003\,07 + 0{,}007\,43 \\ &= 0{,}010\,5\end{aligned}$$

The estimated fraction nonconforming at the lower limit is

$$\begin{aligned}\hat{p}_L &= B_{(n_L-2)/2}\left[\frac{1}{2}\left(1-Q_L\frac{\sqrt{n_L}}{n_L-1}\right)\right] \\ &= B_{66}\left[\frac{1}{2}\left(1-2{,}371\frac{\sqrt{134}}{133}\right)\right] \\ &= B_{66}(0{,}396\,8) \\ &= 0{,}008\,4\end{aligned}$$

Although $\hat{p}_c$ is less than p_c^*, $\hat{p}_L$ is greater than p_L^*, so the pair of declared quality levels for complex control are contradicted.

An example of double specification limits under complex control for unknown process standard deviation is given in B.4.

7.2.6 Multivariate "s" method for independent quality characteristics

For m independent and normally distributed quality characteristics of equal importance to the quality of the items, having unknown process standard deviations and being assessed against a single DQL, determine the "s" method Form p^* sampling plan (n_s, p^*) in the same way as for a single quality characteristic. Select a random sample of size n_s and measure the m quality characteristics on each member of the sample. Calculate the estimates $\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_m$ of the fractions nonconforming as in 7.2.3. Then calculate the estimate of the overall fraction nonconforming as

$$\hat{p} = 1-(1-\hat{p}_1)(1-\hat{p}_2)\ldots(1-\hat{p}_m)$$

If $\hat{p} \le p^*$, the declared quality level has not been contradicted; if $\hat{p} > p^*$, the declared quality level has been contradicted.

EXAMPLE Suppose for two independent and normally distributed quality characteristics x and y, a DQL of 4 % nonconforming at Level II is to be assessed. Table 1 shows that the appropriate plan has sample size 13 and Form p^* acceptability constant 0,114 2. A random sample of size 13 is selected, and x and y are measured on each item in the sample. The sample means $\bar{x}$ and $\bar{y}$ and the sample standard deviations s_x and s_y are calculated. Suppose that, from these, the values of $\hat{p}_x$ and $\hat{p}_y$ are calculated and found to be $\hat{p}_x = 0{,}047\,7$ and $\hat{p}_y = 0{,}047\,7$. The combined fraction nonconforming is estimated as $\hat{p} = 1-(1-\hat{p}_x)(1-\hat{p}_y) = 1 - 0{,}952\,3 \times 0{,}978\,2 = 0{,}068\,5$. As this is less than p^*, the declared quality level has not been contradicted.

7.3 Rules for contradicting a declared quality level: "σ" method

7.3.1 General

Determine the applicable sampling plan (n_σ, k_σ), or equivalently (n_σ, p^*), from Table 1.

If the sample size equals or exceeds the size of the entity under investigation, then the DQL shall be verified by comparing it to the actual quality level determined by inspecting all items in the entity.

Otherwise, select a random sample of size n_σ. For each item in the sample, measure the value of the quality characteristic x. Calculate the sample mean $\bar{x}$ and the sample standard deviation s.

NOTE The purpose of calculating the sample standard deviation when the process standard deviation is presumed to be known is to ensure that the presumption is reasonable. In case of doubt, the "s" method should be used.

7.3.2 Single specification limit

For a single upper specification limit U, calculate the quality statistic $Q = (U - \bar{x}) / \sigma$.

For a single lower specification limit L, calculate the quality statistic $Q = (\bar{x} - L) / \sigma$.

If $Q \geq k_\sigma$, the declared quality level has not been contradicted. If $Q < k_\sigma$, the declared quality level has been contradicted.

EXAMPLE A Level I DQL of 0,25 % is to be used, with an upper specification limit $U = 11{,}5$. The quality characteristic is normally distributed with a presumed known process standard deviation $\sigma = 0{,}453$. From Table 1, it is seen that a sample size $n_\sigma = 13$ is required and that the accompanying Form k acceptability constant is $k_\sigma = 2{,}211$. Suppose that the random sample of 13 items from the entity yields a sample mean $\bar{x} = 10{,}62$ and a sample standard deviation $s = 0{,}439$. The fact that $s = 0{,}439$ gives no cause to doubt the presumption that the process standard deviation is 0,453, so we continue to use the "σ" method. [This may be confirmed objectively by carrying out a two-sided test of the hypothesis that $\sigma^2 = (0{,}453)^2$ against the alternative hypothesis that $\sigma^2 \neq (0{,}453)^2$ using a procedure such as that in Table E —*Comparison of a variance or of a standard deviation with a given value* of ISO 2854:1976.] The quality statistic $Q = (11{,}5 - 10{,}62)/0{,}453 = 1{,}943$. However, as $Q < k_\sigma$, the declared quality level has been contradicted.

7.3.3 Double specification limits under combined control

In the case of double specification limits U and L under combined control, calculate

$$\hat{p}_U = \Phi\left(\frac{\bar{x} - U}{\sigma}\sqrt{\frac{n_\sigma}{n_\sigma - 1}}\right) = \Phi\left(-Q_U\sqrt{\frac{n_\sigma}{n_\sigma - 1}}\right)$$

$$\hat{p}_L = \Phi\left(\frac{L - \bar{x}}{\sigma}\sqrt{\frac{n_\sigma}{n_\sigma - 1}}\right) = \Phi\left(-Q_L\sqrt{\frac{n_\sigma}{n_\sigma - 1}}\right)$$

and

$$\hat{p}_{\mathrm{c}} = \hat{p}_U + \hat{p}_L$$

where $\Phi(.)$ denotes the distribution function of the standard normal distribution.

If $\hat{p}_{\mathrm{c}} \leq p^*$, the declared quality level has not been contradicted; if $\hat{p}_{\mathrm{c}} > p^*$, the declared quality level has been contradicted.

EXAMPLE A Level II DQL of 1,0 % is to be used with double specification limits $L = 40{,}00$ and $U = 40{,}80$. The quality characteristic is normally distributed with a standard deviation that is presumed to be stable and equal to 0,138. From Table 1, it is seen that a sample size $n_\sigma = 16$ is required and that the accompanying Form p^* acceptability constant $p^* = 0{,}029\,62$. Suppose that the random sample of 16 items from the entity yields a sample mean $\bar{x} = 40{,}328$ and a sample standard deviation $s = 0{,}150$. The value $s = 0{,}150$ gives no reason to doubt that $\sigma = 0{,}138$. The upper and lower quality statistics are calculated as $Q_U = (40{,}800 - 40{,}328)/0{,}138 = 3{,}420$ and $Q_L = (40{,}328 - 40{,}000)/0{,}138 = 2{,}377$ respectively. The corresponding estimates of the fractions nonconforming at the two limits are

$$\hat{p}_U = \Phi\left(-Q_U\sqrt{\frac{n_\sigma}{n_\sigma - 1}}\right) = \Phi\left(-3{,}420\sqrt{\frac{16}{15}}\right) = \Phi(-3{,}532) = 0{,}000\,206$$

and

$$\hat{p}_L = \Phi\left(-Q_L\sqrt{\frac{n_\sigma}{n_\sigma - 1}}\right) = \Phi\left(-2{,}337\sqrt{\frac{16}{15}}\right) = \Phi(-2{,}414) = 0{,}007\,889$$

The sum of these estimates is $\hat{p}_{\mathrm{c}} = \hat{p}_U + \hat{p}_L = 0{,}000\,206 + 0{,}007\,889 = 0{,}008\,095$. As $\hat{p}_{\mathrm{c}} \le p^*$, the declared quality level has not been contradicted.

7.3.4 Double specification limits under separate control

For double specification limits under separate control, there will be separate DQLs applying to each limit, say D_U for the upper limit and D_L for the lower limit. Denote the Form k plans for these DQLs by (n_U, k_U) and (n_L, k_L) respectively. Denote the sample means arising from random samples of size n_U and n_L by $\bar{x}_U$ and $\bar{x}_L$ respectively. Calculate $Q_U = (U - \bar{x}_U)/\sigma$ and $Q_L = (\bar{x}_L - L)/\sigma$. If $Q_U \ge k_U$ and $Q_L \ge k_L$, the declared quality levels have not been contradicted; otherwise, at least one of the declared quality levels has been contradicted.

EXAMPLE Separate control of double specification limits is to be used with a Level II DQL of 0,65 % at the upper limit $U = 3{,}125$ and a Level III DQL of 0,25 % at the lower limit $L = 3{,}100$. The quality characteristic is normally distributed with a process standard deviation that is presumed to be known and equal to 0,003 10. From Table 1, it is seen that the appropriate Form k "σ" method plans are $n_U = 18$, $k_U = 2{,}021$ for the upper limit and $n_L = 34$, $k_L = 2{,}604$ for the lower limit. Suppose that the random sample of 18 items from the entity yields a sample mean $\bar{x}_U = 3{,}117\,3$ and a sample standard deviation $s_U = 0{,}002\,91$, and that a sample of size 34 from the same entity yields a sample mean $\bar{x}_L = 3{,}116\,9$ and a standard deviation $s_L = 0{,}003\,07$. Neither of these standard deviations gives rise to doubt about the presumed value of σ, so we continue to use the "σ" method. The upper and lower quality statistics are calculated as $Q_U = (3{,}125 - 3{,}117\,3)/0{,}003\,10 = 2{,}484$ and $Q_L = (3{,}116\,9 - 3{,}100)/0{,}003\,10 = 5{,}452$ respectively. As $Q_U > k_U$ and $Q_L > k_L$, the declared quality levels are not contradicted.

7.3.5 Double specification limits under complex control

Complex control of double specification limits consists of combined control of both limits together with separate control of one of the limits. There will be a DQL for the combined fraction nonconforming at the two limits and a DQL for the fraction nonconforming at the limit that is under separate control. Suppose that the Form p^* "σ" method plan for the combined part of the complex requirement is n_{c}, p^*_{c}. Suppose without loss of generality that the separately controlled limit is the upper limit, and that the appropriate plan for this limit is n_U, p^*_U.

A random sample of size n_{c} is drawn, which yields a sample mean of $\bar{x}_{\mathrm{c}}$ and a sample standard deviation of s_{c}. A second random sample of size n_U is drawn, which yields a sample mean of $\bar{x}$ and a sample standard deviation of s. Provided that neither of the values s_{c} nor s casts doubt on the presumed value of σ, we continue to use the "σ" method as follows.

Calculate

$$\hat{p}_{\mathrm{c}} = \hat{p}_{\mathrm{c},U} + \hat{p}_{\mathrm{c},L} = \Phi\left(\frac{\bar{x}_{\mathrm{c}} - U}{\sigma}\sqrt{\frac{n_{\mathrm{c}}}{n_{\mathrm{c}} - 1}}\right) + \Phi\left(\frac{L - \bar{x}_{\mathrm{c}}}{\sigma}\sqrt{\frac{n_{\mathrm{c}}}{n_{\mathrm{c}} - 1}}\right) = \Phi\left(-Q_{\mathrm{c},U}\sqrt{\frac{n_{\mathrm{c}}}{n_{\mathrm{c}} - 1}}\right) + \Phi\left(-Q_{\mathrm{c},L}\sqrt{\frac{n_{\mathrm{c}}}{n_{\mathrm{c}} - 1}}\right)$$

and

$$\hat{p}_U = \Phi\left(\frac{\bar{x}_U - U}{\sigma}\sqrt{\frac{n_U}{n_U - 1}}\right) = \Phi\left(-Q_U\sqrt{\frac{n_U}{n_U - 1}}\right)$$

If $\hat{p}_{\mathrm{c}} \le p_{\mathrm{c}}^*$ and $\hat{p}_U \le p_U^*$, the declared quality levels have not been contradicted; otherwise, at least one of the declared quality levels has been contradicted.

NOTE If, instead, the lower limit were under separate control then, if $\hat{p}_{\mathrm{c}} \le p_{\mathrm{c}}^*$ and $\hat{p}_L \le p_L^*$, the declared quality levels have not been contradicted; otherwise, at least one of the declared quality levels has been contradicted.

EXAMPLE This example is a modification of the example of 7.3.4. Complex control of double specification limits is to be used, with $U = 3{,}125$ and $L = 3{,}100$. A Level II DQL of 0,65 % applies to both limits combined and a Level III DQL of 0,25 % applies to the lower limit. The quality characteristic is normally distributed with known process standard deviation 0,003 10. From Table 1, it is seen that the appropriate Form p^* plans are $n_{\mathrm{c}} = 18$, $p_{\mathrm{c}}^* = 0{,}018\,76$ for both limits combined and $n_L = 34$, $p_L^* = 0{,}004\,103$ for the lower limit. Suppose that the random sample of 18 items from the entity yields a sample mean $\bar{x}_U = 3{,}117\,3$ and a sample standard deviation $s_U = 0{,}002\,91$, and that a sample of size 34 from the same entity yields a sample mean $\bar{x}_L = 3{,}116\,9$ and a standard deviation $s_L = 0{,}003\,07$. Again, neither of these standard deviations gives rise to doubt about the presumed value of σ. The upper and lower quality statistics for the combined control part of the specification are respectively

$$Q_{\mathrm{c},U} = (3{,}125 - 3{,}117\,3)/\,(0{,}003\,10) = 2{,}484 \text{ and } Q_{\mathrm{c},L} = (3{,}117\,3 - 3{,}100)/\,(0{,}003\,10) = 5{,}581$$

The estimated fraction nonconforming for the combined part of the specification is

$$\begin{aligned}
\hat{p}_{\mathrm{c}} &= \hat{p}_{\mathrm{c},U} + \hat{p}_{\mathrm{c},L} \\
&= \Phi\left(-Q_{\mathrm{c},U}\sqrt{\frac{n_{\mathrm{c}}}{n_{\mathrm{c}} - 1}}\right) + \Phi\left(-Q_{\mathrm{c},L}\sqrt{\frac{n_{\mathrm{c}}}{n_{\mathrm{c}} - 1}}\right) \\
&= \Phi\left(-2{,}484\sqrt{\frac{18}{17}}\right) + \Phi\left(-5{,}581\sqrt{\frac{18}{17}}\right) \\
&= \Phi(-2{,}556) + \Phi(-5{,}743) \\
&= 0{,}005\,294 + 0{,}000\,000 \\
&= 0{,}005\,294
\end{aligned}$$

The estimated fraction nonconforming at the lower limit is

$$\begin{aligned}
\hat{p}_L &= \Phi\left(\frac{L - \bar{x}_L}{\sigma}\sqrt{\frac{n_L}{n_L - 1}}\right) \\
&= \Phi\left(\frac{3{,}100\,0 - 3{,}116\,9}{0{,}003\,10}\sqrt{\frac{34}{33}}\right) \\
&= \Phi\left(\frac{3{,}100\,0 - 3{,}116\,9}{0{,}003\,10}\sqrt{\frac{34}{33}}\right) \\
&= \Phi(-5{,}534) \\
&= 0{,}000\,000
\end{aligned}$$

As $\hat{p}_{\mathrm{c}} < p_{\mathrm{c}}^*$ and $\hat{p}_L < p_L^*$, the declared quality levels for complex control are not contradicted.

7.3.6 Multivariate "σ" method for independent quality characteristics

For m independent and normally distributed quality characteristics having known process standard deviations and being assessed against a single DQL, determine the "σ" method Form p^* sampling plan (n_σ, p^*) in the same way as for a single quality characteristic. Select a random sample of size n_σ and measure the m quality

characteristics on each member of the sample. Calculate the estimates $\hat{p}_1$, $\hat{p}_2$, ..., $\hat{p}_m$ of the fractions nonconforming as in 7.3.3. Then calculate the estimate of the overall fraction nonconforming as

$$\hat{p} = 1-(1-\hat{p}_1)(1-\hat{p}_2)...(1-\hat{p}_m)$$

If $\hat{p} \leq p^*$, the declared quality level has not been contradicted; if $\hat{p} > p^*$, the declared quality level has been contradicted.

EXAMPLE Suppose for two independent and normally distributed quality characteristics x and y, a DQL of 4 % nonconforming for known process standard deviations σ_x and σ_y at Level II is to be assessed. Table 1 shows that the appropriate plan has sample size 8 and Form p^* acceptability constant 0,114 2. A random sample of size 8 is selected, and x and y are measured on each item in the sample. The sample means $\bar{x}$ and $\bar{y}$ and the sample standard deviations s_x and s_y are calculated. Suppose it is found, after two applications of a formal testing procedure such as that in Table E of ISO 2854:1976, that both sample standard deviations are entirely consistent with the presumed values of σ. Suppose also that, from the values of the specification limits for x and y, together with the values of $\bar{x}$, $\bar{y}$, σ_x and σ_y, the values of $\hat{p}_x$ and $\hat{p}_y$ are calculated and found to be $\hat{p}_x = 0,47\,7$ and $\hat{p}_y = 0,021\,8$. The combined fraction nonconforming is estimated as $\hat{p} = 1-(1-\hat{p}_x)(1-\hat{p}_y) = 1 - 0,952\,3 \times 0,978\,2 = 0,068\,5$. As this is less than p^*, the declared quality level has not been contradicted.

7.4 Disposition of nonconforming items

Any nonconforming items found in the sample shall not be returned to the rest of the items unless the nonconforming items are brought to a conforming condition and applicable administrative rules are followed.

8 Further information

8.1 Curves showing approximate probability of contradiction

The curves shown in Figure 1 indicate the approximate probability that a sample result will lead to contradiction of the declared quality level. The curves give the approximate probability of contradiction as a function of the quality ratio.

The curves in Figure 1 refer to situations where the declared quality level is one of the preferred values. For non-preferred values of the DQL, the information in Figure 1 does not apply.

8.2 Tables indicating discriminatory ability

Tables 5 to 10 provide additional information about the probabilities of contradicting incorrect declared quality levels for different values of the quality ratio.

For each individual sampling plan, Tables 2 to 4 show the value of the limiting quality ratio (LQR) that corresponds to a risk of failing to contradict the declared quality level. This LQR together with the information presented in Tables 5 to 10 may be used to assess the discriminatory ability of each sampling plan.

Tables 2 to 4 also show the probability that the sample result will (falsely) contradict the declared quality level when the actual quality level is equal to the DQL.

NOTE The values in Tables 2 to 10 have been determined under the assumption that the sample size is only a small fraction of the entity under investigation. The values in these tables are valid when the sample size is less than or equal to 1/10 of the entity. When the sample size is a larger fraction of the entity under investigation, the actual discriminatory ability will be better than indicated by the values in Tables 2 to 10. In particular, the actual limiting quality ratio will be smaller than the value indicated in Tables 2 to 4, and moreover, the actual probability of falsely contradicting a correct DQL will also be smaller than that indicated in Tables 2 to 4.

The values in Tables 2 to 10 apply to situations where the DQL used is one of the preferred values of DQL. If the DQL used is not one of the preferred DQLs, then the next higher preferred DQL should be used to select the sampling plan. This results in a change to the balance of risks. On one hand, the risk of falsely

contradicting a correct declared DQL will be less than that given in Tables 2 to 4. On the other hand, the actual LQR will be higher than the tabulated value of LQR for the preferred DQL.

The actual LQR, $R_{LQ,a}$, is given by the formula:

$$R_{LQ,a} = R_{LQ,p} \times \frac{Q_{DL,p}}{Q_{DL,np}}$$

where

$R_{LQ,p}$ is the preferred limiting quality ratio;

$Q_{DL,p}$ is the preferred declared quality level;

$Q_{DL,np}$ is the non-preferred declared quality level.

The quality level corresponding to a risk of failing to contradict the actual (i.e. non-preferred) DQL remains as given by the sampling plan and is determined as this preferred DQL times its tabulated LQR.

Tables 5 to 10 can still be applied to non-preferred DQLs with the understanding that the actual quality level is the quality ratio, given in Tables 2 to 4, times the preferred DQL used (see the example below).

EXAMPLE Suppose a DQL of 0,125 % nonconforming items is to be assessed at LQR Level II when the process standard deviation is unknown. As this is a non-preferred DQL, and the next higher preferred DQL is 0,15 %, Table 1 indicates that the sampling plan $n = 93$, $k_s = 2{,}565$ is to be used.

From Table 3, it can be concluded that there is less than a 3,0 % risk of falsely contradicting the (non-preferred) DQL of 0,125 % nonconforming items. Furthermore, there will be a risk of failing to contradict the non-preferred DQL when the actual quality level is 7,48 times 0,15 %, i.e. 1,122 %. For the non-preferred DQL, the actual LQR is $7{,}48 \times (0{,}15/0{,}125) = 8{,}98$. In other words, there will be a risk of failing to contradict the non-preferred DQL when the actual quality level is 8,98 times higher ($8{,}98 \times 0{,}125$ % being equal to 1,122 %).

Using Table 7, for a quality ratio of 5,0 and the preferred DQL of 0,15 % (corresponding to an actual quality level of $5{,}0 \times 0{,}15\ \% = 0{,}75\ \%$), the probability of contradicting the non-preferred DQL of 0,125 % is 71,8 %. Table 7 can similarly be used to find the probability of contradicting the non-preferred DQL for seven other values of the quality ratio.

Table 5 — Probability (%) of contradicting a DQL for different values of the quality ratio for LQR Level I plans, "s" method

Quality ratio	Declared quality level (DQL) in % nonconforming items															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10
1,0	2,5	2,1	2,4	2,6	2,7	2,7	2,3	2,7	3,1	3,2	3,2	2,9	3,6	3,4	3,1	1,6
1,5	6,6	5,8	6,3	6,8	7,0	6,8	6,0	6,7	7,4	7,6	7,5	6,6	8,0	7,5	7,0	4,2
3,0	24,4	22,5	23,8	24,9	25,4	24,4	22,4	23,9	25,1	25,6	25,0	22,6	25,6	24,8	24,9	20,0
5,0	47,7	45,2	47,1	48,4	49,2	47,6	45,0	47,0	48,3	49,2	48,4	45,2	49,7	49,8	52,8	52,7
7,5	68,1	65,8	67,8	68,9	69,8	68,2	65,9	68,0	69,1	70,4	69,9	67,3	72,2	74,1	80,3	89,2
10,0	80,4	78,7	80,4	81,3	82,1	80,9	79,2	81,0	82,0	83,2	83,1	81,5	86,0	88,6	94,9	—
15,0	92,2	91,4	92,4	92,9	93,5	92,9	92,2	93,3	93,9	94,8	95,1	94,8	97,2	98,9	99,999 8	—
20,0	96,7	96,3	96,9	97,2	97,5	97,2	97,0	97,6	97,9	98,4	98,6	98,7	99,6	99,98	—	—

EXAMPLE Suppose the plan corresponding to a declared quality level of 0,04 % nonconforming items is used. For a quality ratio of 10 (the actual quality level is 10 times the declared quality level, i.e. 0,4 % nonconforming items) there is a probability of 81,3 % that this sampling plan will indicate contradiction of the declared quality level.

Table 6 — Probability (%) of contradicting a DQL for different values of the quality ratio for LQR Level I plans, "σ" method

Quality ratio	Declared quality level (DQL) in % nonconforming items															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10
1,0	1,7	1,5	1,6	1,7	1,8	1,6	1,3	1,6	1,6	1,5	1,6	1,2	1,3	0,9	2,5	3,7
1,5	5,2	4,7	5,0	5,3	5,5	5,0	4,1	4,8	4,8	4,7	4,7	3,6	3,9	3,0	5,6	7,5
3,0	22,9	20,9	22,1	23,1	23,4	22,2	19,8	21,2	21,9	21,9	20.9	17,6	19,5	17,0	20,3	23,8
5,0	47,4	44,2	46,4	48,1	48,1	46,8	43,9	45,6	47,1	48,0	46,0	41,9	46,2	44,6	44,8	51,2
7,5	68,8	65,6	68,0	69,8	69,7	69,0	66,7	68,1	70,1	71,7	69,7	66,8	72,6	73,9	72,6	83,7
10,0	81,5	78,9	81,0	82,6	82,5	82,1	80,6	81,7	83,6	85,3	83,9	82,5	87,8	90,4	90,7	—
15,0	93,1	91,7	93,0	93,9	93,9	93,9	93,4	94,0	95,1	96,2	95,8	95,8	98,2	99,5	99,997	—
20,0	97,2	96,5	97,2	97,7	97,7	97,8	97,6	97,9	98,5	99,0	99,0	99,1	99,8	99,997	—	—

EXAMPLE Suppose the plan corresponding to a declared quality level of 0,10 % nonconforming items is used. For a quality ratio of 15 (the actual quality level is 15 times the declared quality level, i.e. 1,5 % nonconforming items), there is a probability of 93,9 % that this sampling plan will indicate contradiction of the declared quality level.

Table 7 — Probability (%) of contradicting a DQL for different values of the quality ratio for LQR Level II plans, "s" method

Quality ratio	Declared quality level (DQL) in % nonconforming items													
	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10
1,0	3,4	3,4	3,7	3,6	3,0	3,5	3,8	4,0	3,9	3,4	3,9	5,9	4,6	3,2
1,5	10,8	10,9	11,6	11,2	9,6	10,8	11,3	11,8	11,4	9,9	11,3	14,8	12,6	9,8
2,0	21,0	21,3	22,3	21,6	18,9	20,8	21,6	22,2	21,6	18,9	21,3	26,0	23,6	19,9
3,0	43,0	43,7	44,7	43,7	39,8	42,6	43,6	44,6	43,7	39,6	43,8	49,1	48,0	45,2
4,0	61,2	62,2	63,1	62,1	58,1	61,1	62,1	63,3	62,5	58,4	63,3	68,1	69,0	69,2
5,0	74,3	75,3	76,0	75,2	71,8	74,5	75,5	76,7	76,1	72,7	77,5	81,4	83,7	86,1
7,5	91,1	91,8	92,1	91,8	90,1	91,7	92,3	93,1	93,1	91,6	94,5	96,2	98,1	99,6
10,0	96,8	97,2	97,4	97,3	96,6	97,3	97,6	98,1	98,1	97,7	98,9	99,5	99,9	100

EXAMPLE Suppose the plan corresponding to a declared quality level of 0,10 % nonconforming items is used. For a quality ratio of 7,5 (the actual quality level is 7,5 times the declared quality level, i.e. 0,75 % nonconforming items), there is a probability of 91,8 % that this sampling plan will indicate contradiction of the declared quality level.

Table 8 — Probability (%) of contradicting a DQL for different values of the quality ratio for LQR Level II plans, "σ" method

Quality ratio	Declared quality level (DQL) in % nonconforming items													
	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10
1,0	2,5	2,6	2,7	2,5	1,9	2,2	2,5	2,5	2,3	1,7	2,0	3,9	2,2	3,9
1,5	9,3	9,5	9,8	9,2	7,5	8,5	9,0	9,1	8,5	6,7	7,8	11,3	8,1	10,2
2,0	19,4	19,7	20,4	19,4	16,5	18,3	18,9	19,2	18,2	15,0	17,1	21,6	17,9	19,0
3,0	42,1	42,8	44,0	42,6	38,4	41,4	42,0	42,7	41,5	36,6	40,5	44,6	43,4	40,3
4,0	61,3	62,1	63,4	62,2	58,1	61,4	61,9	62,9	62,0	57,3	62,0	64,6	67,2	61,7
5,0	74,9	75,6	76,9	76,0	72,7	75,7	76,0	77,2	76,7	73,1	77,7	79,0	83,7	78,9
7,5	91,7	92,2	93,0	92,7	91,2	92,9	93,1	93,9	93,9	92,8	95,3	95,6	98,5	98,4
10,0	97,2	97,5	97,8	97,7	97,2	98,0	98,0	98,4	98,5	98,3	99,2	99,3	99,96	100

EXAMPLE Suppose the plan corresponding to a declared quality level of 0,15 % nonconforming items is used. For a quality ratio of 5 (the actual quality level is 5 times the declared quality level, i.e. 0,75 % nonconforming items) there is a probability of 72,7 % that this sampling plan will indicate contradiction of the declared quality level.

Table 9 — Probability (%) of contradicting a DQL for different values of the quality ratio for LQR Level III plans, "s" method

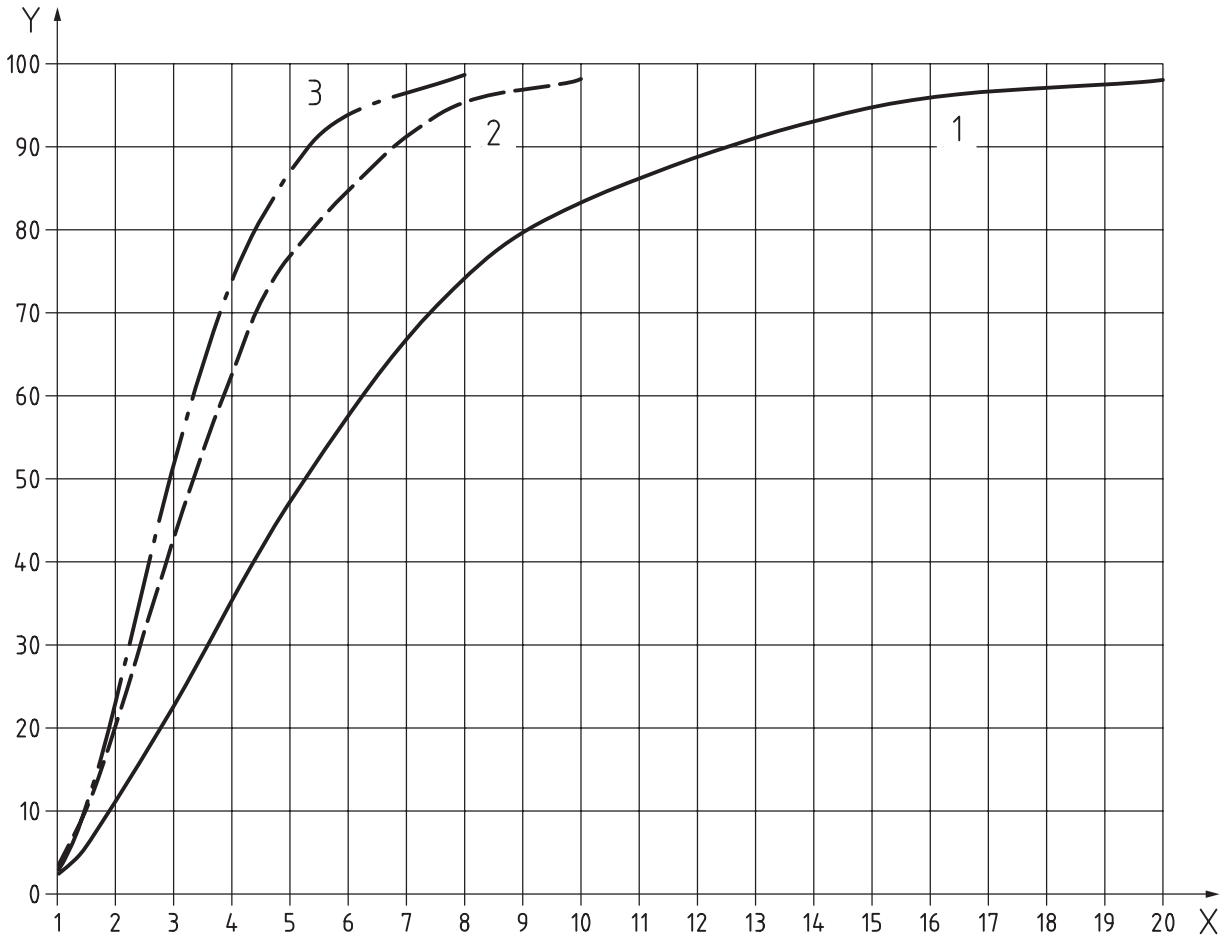
Quality ratio	Declared quality level (DQL) in % nonconforming items												
	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10
1,0	2,8	2,9	3,4	2,9	2,5	3,0	3,1	3,6	3,1	2,7	1,9	3,3	3,3
1,5	11,3	11,6	12,8	11,4	10,2	11,5	11,8	13,0	11,6	10,2	7,7	12,0	12,1
2,0	24,3	24,7	26,7	24,3	22,2	24,3	24,8	26,7	24,3	21,9	17,5	25,2	25,8
3,0	52,2	52,7	55,0	52,0	49,1	52,0	52,7	55,1	52,1	49,0	42,5	54,7	56,8
4,0	72,8	73,3	75,1	72,7	70,1	72,8	73,6	75,6	73,4	70,9	65,1	77,1	80,1
5,0	85,2	85,6	86,9	85,2	83,4	85,5	86,1	87,6	86,3	84,7	80,8	89,9	92,5
6,0	92,1	92,4	93,2	92,2	91,0	92,5	93,0	94,0	93,3	92,5	90,2	96,0	97,7
8,0	97,8	97,9	98,2	97,9	97,5	98,0	98,3	98,6	98,5	98,4	97,8	99,5	99,9

EXAMPLE Suppose the plan corresponding to a declared quality level of 0,40 % nonconforming items is used. For a quality ratio of 5 (the actual quality level is 5 times the declared quality level, i.e. 2,0 % nonconforming items) there is a probability of 85,5 % that this sampling plan will indicate contradiction of the declared quality level.

Table 10 — Probability (%) of contradicting a DQL for different values of the quality ratio for LQR Level III plans, "σ" method

Quality ratio	Declared quality level (DQL) in % nonconforming items												
	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10
1,0	2,1	2,1	2,5	2,0	1,7	2,0	2,1	2,2	1,7	1,4	0,9	1,8	2,8
1,5	9,9	9,9	11,1	9,7	8,4	9,5	9,6	10,2	8,7	7,3	5,1	8,6	10,4
2,0	22,9	23,1	25,0	22,5	20,1	22,1	22,5	23,9	21,1	18,5	13,8	21,0	22,7
3,0	51,7	52,5	54,7	51,5	48,1	51,2	51,9	54,6	51,0	47,3	39,6	52,2	52,1
4,0	73,1	74,0	75,8	73,2	70,4	73,3	74,0	76,8	74,2	71,2	64,3	76,8	75,8
5,0	85,8	86,6	87,7	86,1	84,2	86,3	86,9	89,1	87,6	85,9	81,3	90,4	89,8
6,0	92,6	93,2	93,9	92,9	91,8	93,2	93,7	95,1	94,4	93,5	91,0	96,5	96,4
8,0	98,0	98,3	98,5	98,2	97,8	98,4	98,6	99,1	98,9	98,8	98,2	99,7	99,7

EXAMPLE Suppose the plan corresponding to a declared quality level of 0,25 % nonconforming items is used. For a quality ratio of 4 (the actual quality level is 4 times the declared quality level, i.e. 1,0 % nonconforming items) there is a probability of 70,4 % that this sampling plan will indicate contradiction of the declared quality level.



Key

X quality ratio
Y probability of contradicting a DQL, in percent
1 Level I
2 Level II
3 Level III

Figure 1 — Curves showing the approximate probability of contradicting a DQL for different values of the quality ratio

Annex A
(informative)

Method of matching variables plans to attributes plans

A.1 Notation

$P_a(p;n,\mathrm{Ac})$ Probability of acceptance using sampling by attributes at process quality level p

$P_u(p;n_s,k_s)$ Probability of acceptance using sampling by variables for unknown process standard deviation at process quality level p

$P_k(p;n_\sigma,k_\sigma)$ Probability of acceptance using sampling by variables for known process standard deviation at process quality level p

A.2 Design objective

The purpose of matching the operating characteristic (OC) curve of each variables plan to the OC curve of the corresponding single sampling attributes plan is so that, when the quality characteristic is normally distributed, the only reasons for choosing one in preference to the other is for economy or administrative convenience. However, there are many design criteria that could be used to perform the matching. The method used here is to minimize the integral of the absolute difference between the OC values after weighting by the sum of the OC values. The reason for the weighting is to give greater importance to discrepancies between the OC values at low values of p, because the lower the value of p, the longer the average time before a switch to tightened inspection (and a different OC curve) takes place.

The objective function to be minimized for unknown process standard deviation is thus

$$\int_0^1 \{P_a(p;n,\mathrm{Ac}) + P_u(p;n_s,k_s)\} \,|\, P_a(p;n,\mathrm{Ac}) - P_u(p;n_s,k_s) \,|\, \mathrm{d}p = \int_0^1 |P_a^2(p;n,\mathrm{Ac}) - P_u^2(p;n_s,k_s) \,|\, \mathrm{d}p$$

Similarly, for known process standard deviation, the objective function to be minimized is

$$\int_0^1 \{P_a(p;n,\mathrm{Ac}) + P_k(p;n_\sigma,k_\sigma)\} \,|\, P_a(p;n,\mathrm{Ac}) - P_k(p;n_\sigma,k_\sigma) \,|\, \mathrm{d}p = \int_0^1 |P_a^2(p;n,\mathrm{Ac}) - P_k^2(p;n_\sigma,k_\sigma) \,|\, \mathrm{d}p$$

The method can therefore be interpreted in general as minimizing the area between the squared OC values for sampling by attributes and the squared OC values for sampling by variables.

Annex B
(informative)

Examples of use of the procedures

B.1 Example 1: Double specification limits under combined control for unknown process standard deviation

The quality control department of a manufacturing plant has expressed concern about one particular component dimension that stubbornly continues to show a combined percent nonconforming of about 1,5 % at its two specification limits. The management decided to purchase a new machining tool that is claimed to be able to reduce this percentage to 0,1 %. After the tool was installed, the management decided to assess whether the claim was justified. The process standard deviation of the measurements on the component is unknown for the new tool, and the component dimension has lower and upper specification limits of 42,7 mm and 43,0 mm respectively. Measurement error is negligible.

Management decided to use this part of ISO 3951 to evaluate the claim by selecting a declared quality level (DQL) of 0,1 %. Management also wanted the probability of a positive evaluation of the claim to be small in the case of no reduction of the percentage nonconforming. Therefore, Level III of the LQR was chosen, to assure good discrimination between quality levels. From Table 4, it was found that, for LQR Level III and the declared quality level DQL = 0,1 %, the sampling plan for unknown process standard deviation has sample size $n_s = 189$ and limiting value $p^* = 0{,}163\,2$ % for the estimated process fraction nonconforming. This plan was proposed for the internal review.

A random sample of size $n_s = 189$ components was taken and the component dimension measured for each sampled item. The sample mean and sample standard deviation were found to be $\bar{x} = 42{,}781$ and $s = 0{,}026\,9$ respectively. Following the procedures in 7.2.3,

$$Q_U = \frac{U - \bar{x}}{s} = \frac{43{,}0 - 42{,}781}{0{,}0269} = 8{,}141$$

$$Q_L = \frac{\bar{x} - L}{s} = \frac{42{,}781 - 42{,}7}{0{,}0269} = 3{,}011$$

$$\hat{p}_U = B_{(n-2)/2}\left[\tfrac{1}{2}\left(1 - \frac{\sqrt{n}Q_U}{n-1}\right)\right] = B_{93{,}5}\left[0{,}202\,3\right] = 0{,}000\,000$$

$$\hat{p}_L = B_{(n-2)/2}\left[\tfrac{1}{2}\left(1 - \frac{\sqrt{n}Q_L}{n-1}\right)\right] = B_{93{,}5}\left[0{,}389\,9\right] = 0{,}001\,165$$

and

$$\hat{p}_{\mathrm{C}} = \hat{p}_U + \hat{p}_L = 0{,}001\,165 = 0{,}116\,5\ \%$$

As $\hat{p}_{\mathrm{C}} < p^*$, the claim can be considered to have been verified, even though $\hat{p}_{\mathrm{C}} > 0{,}1$ %.

From Table 4, it is found for this plan that there is a risk of 3,4 % of contradicting a correct declared quality level (0,1 % nonconforming), and a risk of 10,0 % of failing to contradict when the actual quality level is 0,541 % nonconforming [i.e. the actual quality level is 5,41 (the LQR) times worse than the declared quality level]. For additional information on the discriminatory ability of this sampling plan, refer to Table 9 of this part of ISO 3951.

B.2 Example 2: Single specification limit for known process standard deviation

The senior management of a chain of high street banks have declared that no more than 4 % of service times should exceed 5 minutes. The manager of a branch of a bank is concerned that one of his tellers, who is currently on probation, provides a service that is too slow. Experience has shown that the natural logarithms of service times are closely approximated by a normal distribution with a standard deviation of 0,50 log-minutes. The manager wishes to obtain objective evidence of the suspect teller's incompetence, so he instigates a study of her service times to see if he would be correct to declare that more than 4 % of her service times exceed 5 minutes.

For this example, a low risk is required of incorrectly rejecting the declared quality level; on the other hand, a higher risk may be tolerated of incorrectly accepting that the teller achieves the desired level of performance. Accordingly, the manager decides to select a Level III plan. From Table 4, he finds the following plan for known process standard deviation:

$n_{\Phi} = 17$, $k_{\Phi} = 1{,}442$

The random sample of 17 service times is given below in minutes:

1,083	1,283	1,583	1,367	2,333	2,883	2,117	3,083	1,967
2,517	5,750	2,317	2,950	3,983	6,400	1,517	2,883	

The natural logarithms of these times, presumably to approximately follow a normal distribution, are:

0,079 73	0,249 20	0,459 32	0,312 62	0,847 15	1,058 83	0,750 00	1,125 90	0,676 51
0,923 07	1,749 20	0,840 27	1,081 81	1,382 04	1,856 30	0,416 73	1,058 83	

The sample mean and sample standard deviation of the logarithms are found to be

$\bar{x} = 0{,}874\ 56$

and

$s = 0{,}496\ 24$ respectively.

The value of the sample standard deviation is very close to the presumed value of the process standard deviation, so there is no evidence to doubt this presumption.

In terms of the natural logarithms of service times, the upper specification limit is $U = \ln 5 = 1{,}609\ 44$.

The upper quality statistic is

$$Q = (U - \bar{x}) / \sigma;\ = (1{,}609\ 44 - 0{,}874\ 56)/0{,}50 = 1{,}469\ 76$$

As $Q > k_{\sigma}$, the declared quality level has not been contradicted, i.e. there is no evidence that the teller has more than 4 % of service times exceeding 5 minutes.

NOTE This conclusion is reached despite 2 of the 17 service times exceeding 5 minutes. This demonstrates that strong evidence of contradiction of a DQL is required before a contradiction can be made using plans from this International Standard. In this respect, the sample size, 17, is rather small.

From Table 4, it is found for this plan that there is a risk of 0,9 % of contradicting a correct declared quality level (4,0 % nonconforming), and a risk of 10,0 % of failing to contradict when the actual quality level is 23,44 % nonconforming [i.e. the actual quality level is 5,86 (the LQR) times worse than the declared quality level]. For additional information on the discriminatory ability of this sampling plan, refer to Table 10 of this part of ISO 3951.

B.3 Example 3: Double specification limits under separate control for unknown process standard deviation

A bottle filling plant is having problems with the filling process, owing to variation in the height of the bottles. The bottles have specification limits 24,0 ± 0,2 cm. Violating these limits has different effects. If the bottles are too tall, the filling nozzle can damage the bottle and possibly itself, whereas if the bottle is too short, the nozzle does not fit tightly on the bottle and the bottle will not be properly filled. Management requires no more than 0,1 % of the bottles to be taller than 24,2 cm and no more than 0,4 % of bottles to be shorter than 23,8 cm. The bottle supplier declares that these requirements are being met. Management decides to test these declared quality levels on the next large consignment of bottles.

Management of the filling plant is anxious to maintain its good relationship with its bottle supplier, and therefore requires the probability of contradicting correctly declared quality levels to be low. Accordingly, a plan from this part of ISO 3951 can be used.

Level II is chosen. From Table 3, the plan ($n_{s,U} = 112$, $k_{s,U} = 2{,}723$) is selected for the upper limit and ($n_{s,L} = 61$, $k_{s,L} = 2{,}230$) for the lower limit. The random samples result in sample means and sample standard deviations ($\bar{x}_U = 23{,}881$, $s_U = 0{,}065\,5$) for the upper limit and ($\bar{x}_L = 23{,}947$, $s_L = 0{,}062\,6$) for the lower limit. Hence,

$$Q_U = (U - \bar{x}_U)/s_U = (24{,}2 - 23{,}881)/0{,}065\,5 = 4{,}870$$

and

$$Q_L = (\bar{x}_L - L)/s_L = (23{,}947 - 23{,}8)/0{,}0626 = 2{,}348$$

As $Q_U > k_{s,U}$ and $Q_L > k_{s,L}$, the claim by the bottle supplier is accepted.

B.4 Example 4: Double specification limits under complex control for unknown process standard deviation

This example is a modification of the previous example, with a slightly tighter constraint. Suppose that management requires no more than 0,1 % of the bottles to be taller than 24,2 cm and no more than a *total* of 0,4 % of bottles to be taller than 24,2 cm or shorter than 23,8 cm. Form p^* is used in order to be able to carry out the required tests.

Level II is chosen again. From Table 3, the plan ($n_{s,U} = 112$, $p^*_U = 0{,}002\,854$) is selected for the upper limit and ($n_{s,C} = 61$, $p^*_C = 0{,}011\,62$) for the two limits combined. The random samples result in sample means and sample standard deviations ($\bar{x}_U = 23{,}881$, $s_U = 0{,}065\,5$) for the upper limit and ($\bar{x}_C = 23{,}922$, $s_C = 0{,}063\,9$) for the two limits combined. Hence

$$\hat{p}_U = B_{(n_{s,U}-2)/2}\left[\frac{1}{2}\left(1 - \frac{U - \bar{x}_U}{s_U}\frac{\sqrt{n_{s,U}}}{n_{s,U} - 1}\right)\right]$$

$$= B_{55}\left[\frac{1}{2}\left(1 - \frac{24{,}2 - 23{,}881}{0{,}065\,5}\frac{\sqrt{112}}{111}\right)\right]$$

$$= B_{55}(0{,}267\,830)$$

$$= 0{,}000\,000$$

and

$$\begin{aligned}
\hat{p}_{\mathrm{c}} &= \hat{p}_{\mathrm{c},U} + \hat{p}_{\mathrm{c},L} \\
&= B_{(n_{\mathrm{c}}-2)/2}\left[\frac{1}{2}\left(1-\frac{U-\bar{x}_{\mathrm{c}}}{s_{\mathrm{c}}}\frac{\sqrt{n_{s,\mathrm{c}}}}{n_{s,\mathrm{c}}-1}\right)\right] + B_{(n_{\mathrm{c}}-2)/2}\left[\frac{1}{2}\left(1-\frac{\bar{x}_{\mathrm{c}}-L}{s_{\mathrm{c}}}\frac{\sqrt{n_{s,\mathrm{c}}}}{n_{s,\mathrm{c}}-1}\right)\right] \\
&= B_{29,5}\left[\frac{1}{2}\left(1-\frac{24{,}2-23{,}922}{0{,}063\,9}\frac{\sqrt{61}}{60}\right)\right] + B_{29,5}\left[\frac{1}{2}\left(1-\frac{23{,}922-23{,}8}{0{,}063\,9}\frac{\sqrt{61}}{60}\right)\right] \\
&= B_{29,5}\left(0{,}216\,843\right) + B_{29,5}\left(0{,}375\,737\right) \\
&= 0{,}000\,001 + 0{,}026\,722 \\
&= 0{,}026\,723
\end{aligned}$$

Thus, $\hat{p}_U < p_U^*$ but $\hat{p}_{\mathrm{c}} > p_{\mathrm{c}}^*$. The claim by the bottle supplier is therefore not accepted because the proportion of bottles that fail to conform to specification is excessive.

Bibliography

[1] ISO 2854:1976, *Statistical interpretation of data — Techniques of estimation and tests relating to means and variances*

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