International Standard ISO 3443/2

INTERNATIONAL ORGANIZATION FOR STANDARDIZATION•МЕЖДУНАРОДНАЯ ОРГАНИЗАЦИЯ ПО СТАНДАРТИЗАЦИИ•ORGANISATION INTERNATIONALE DE NORMALISATION

Tolerances for building — Part 2 : Statistical basis for predicting fit between components having a normal distribution of sizes

Tolérances pour le bâtiment — Partie 2 : Base statistique pour la prévision de possibilités d'assemblage entre composants, relevant d'une distribution normale des dimensions

First edition — 1979-07-15

UDC 69 : 721.01 **Ref. No. ISO 3443/2-1979 (E)**

Descriptors : buildings, construction, joints, compatibility, fits, dimensional tolerances, statistical analysis.

3443/2-1979 (E)

Price based on 5 pages

FOREWORD

ISO (the International Organization for Standardization) is a worldwide federation of national standards institutes (ISO member bodies). The work of developing International Standards is carried out through ISO technical committees. Every member body interested in a subject for which a technical committee has been set up has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work.

Draft International Standards adopted by the technical committees are circulated to the member bodies for approval before their acceptance as International Standards by the ISO Council.

International Standard ISO 3443/2 was developed by Technical Committee ISO/TC 59, *Building construction*, and was circulated to the member bodies in October 1977.

It has been approved by the member bodies of the following countries :

Australia	Hungary	Portugal
Austria	Israel	Romania
Belgium	Italy	South Africa, Rep. of
Canada	Japan	Spain
Czechoslovakia	Korea, Rep. of	Sweden
Denmark	Mexico	United Kingdom
Egypt, Arab Rep. of	New Zealand	USSR
Finland	Norway	
Germany, F.R.	Poland	

The member bodies of the following countries expressed disapproval of the document on technical grounds :

France
Netherlands

This International Standard forms one of a series concerning tolerances for components and construction of predictable variability. This series includes the following :

ISO 1803, *Tolerances for building — Vocabulary.*[1)]

ISO 3443/1, *Tolerances for building — Part 1 : Basic principles for evaluation and specification.*[2)]

ISO 4464, *Tolerances for building — Identification of tolerances and their relationship.*[3)]

It is envisaged that a group of standards in the series will deal with calculation methods for relating tolerances, work sizes and joint width.[4)] This International Standard describes the statistical basis of such calculation methods, and subsequent standards within the group will describe modifications and additions to this statistical basis to take account of various factors that arise in practice.

The annex is for information and does not form an integral part of this International Standard.

1) Under revision.

2) At present at the stage of draft.

3) In preparation.

4) The expression "joint width" is used in this International Standard as it is the term currently used. In this case, it should be made clear that it indicates the notion which is expressed in ISO 2444 by the less commonly used term "joint clearance" as follows :

joint clearance : The distance between the joint faces of two components set side by side, i.e. the distance considered in order to achieve fit.

Tolerances for building — Part 2 : Statistical basis for predicting fit between components having a normal distribution of sizes

1 SCOPE

This International Standard describes the fundamental characteristics of dimensional variability in building and of the particular case of combination of random unrelated variables; it sets out the need to relate dimensional variability to the limits imposed on joint widths by the need for satisfactory functioning.

2 FIELD OF APPLICATION

This International Standard applies to all forms of building construction that have predictable variability which follows a Gaussian distribution.

3 REFERENCE

ISO 3207, *Statistical interpretation of data — Determination of a statistical tolerance interval.*

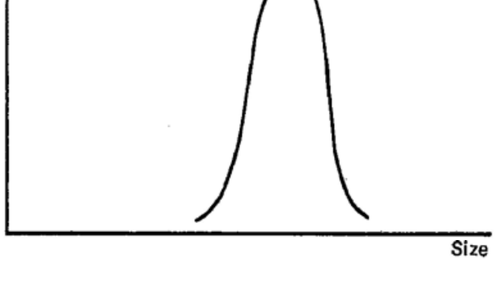
4 GENERAL

Although this International Standard does not deal in detail with the design of joints between components, it recognises that a given joint design will have certain associated limits within which the required joint width must lie if it is to function satisfactorily. The joint width achieved in a given assembly of components will be determined by the dimensional variability (deviations, errors, inaccuracies) in that assembly. The calculation of "fit" is essentially a process of reconciling the required joint width range with the joint width that is predicted to result from dimensional variability. Thus, the dimensional flexibility of a jointing technique is expressed in terms of its maximum and minimum clearance capabilities, i.e. the limits of clearance within which performance can be maintained.

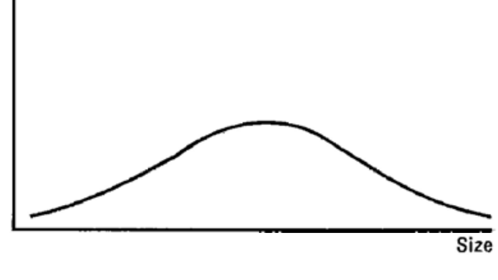
Exceeding either limit results in a "misfit". The design or selection of a jointing technique should therefore include the aim of matching its clearance capability with the clearance predicted to occur. The calculation of "fit" is relevant both to the derivation of a suitable work size for a component and to proposed uses of an existing component, of known work size, in a known situation.

5 PROBABILITY AND INDUCED DEVIATIONS

In many production and erection processes, the achieved sizes in a sufficient number of attempts follow the so-called normal distribution, the density function of which is depicted by the Gaussian curve (see figure 1).



Small standard deviation



Large standard deviation

FIGURE 1 — **Gaussian curves (Normal distribution for different standard deviations)**

A normal distribution has two parameters, the mean and the standard deviation. The probability density curve is symmetrical about the mean, at which point the peak occurs. The standard deviation is a quantity that represents the spread of the curve (see figure 2).

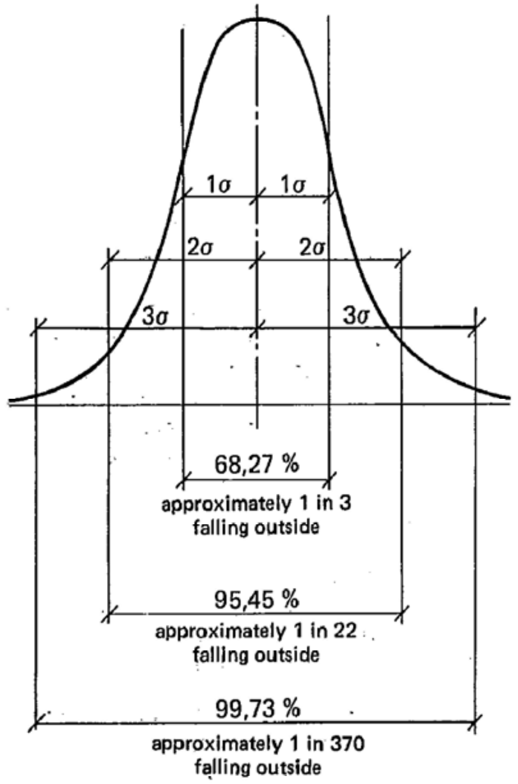


FIGURE 2 — **Limits corresponding to 1, 2 and 3 times the standard deviation**

If the mean value is displaced in relation to the specified value *B*, there is said to be a systematic deviation [see figure 3b)]. If the values apply to sizes which are distributed normally with the parameters mean μ_s and standard deviation σ_s, then the deviations are distributed normally with parameters $\mu_d = \mu_s - B$ and $\sigma_d = \sigma_s$. A systematic deviation implies that μ_d is different from zero.

If the parameters are known, the probability of failure (defects) corresponding to given limits is the sum of the two probabilities of either limit being infringed [see figure 3c)].

These two parameters for populations of types of construction or components cannot be precisely known and have to be estimated from samples, since by definition population data relate to infinite populations. The parameters can be estimated with sufficient precision from samples of adequate size (see ISO 3207) of such construction or components. The data so obtained relate to "populations", and the question of representativeness of small samples of construction such as occur on site does not arise.

6 COMBINATION OF RANDOM VARIABLES

In any assembly of components in building, a number of dimensional variabilities combine to produce the total variability operating (for example, variability in size and variability in position). In most cases these are the result of quite separate operations and can therefore be considered as occurring independently and at random. The occurrence of an extreme deviation value in any operation is infrequent. The simultaneous occurrence of two or more extreme values is many times more infrequent.

This aspect of probability, together with the chance that different deviations may compensate for each other, is taken into account in the statistical theory of random accumulated errors. The effect of so combining independent variables is that the probability of exceeding a given multiple of standard deviation remains the same for the combined variability as it had been for each constituent.

This theory relies upon the measurement of all variability in terms of the standard deviation, as described above. It states that the standard deviation of the total variability (combined effect of several variables) is equal to the square root of the sum of the squares of the individual standard deviations of the separate variables :

$$\sigma_t = \sqrt{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2}$$

The standard deviation has been shown to correspond to the limit that is exceeded by approximately one item in three. If the standard deviation of the variability in joint width due to component deviations is calculated by the above formula, it can be multiplied by a suitable factor to give the limits on joint width corresponding to any appropriate risk of misfit.

This assumes that component deviations follow a normal distribution, without finite limits being imposed. If limits are applied, for example, in manufacture, and the few units whose sizes exceed them are rejected and do not reach the site, the risk of misfit is marginally better than calculated.

Thus the effect of the total of all the variabilities in any assembly upon the joints in that assembly can be assessed in terms of the probability of either joint limit (required minimum width or required maximum width) being exceeded. By this means a basis is provided for the selection of target dimensions (for example work sizes), for the selection of jointing techniques (i.e. joint width ranges) and for the control of variability.

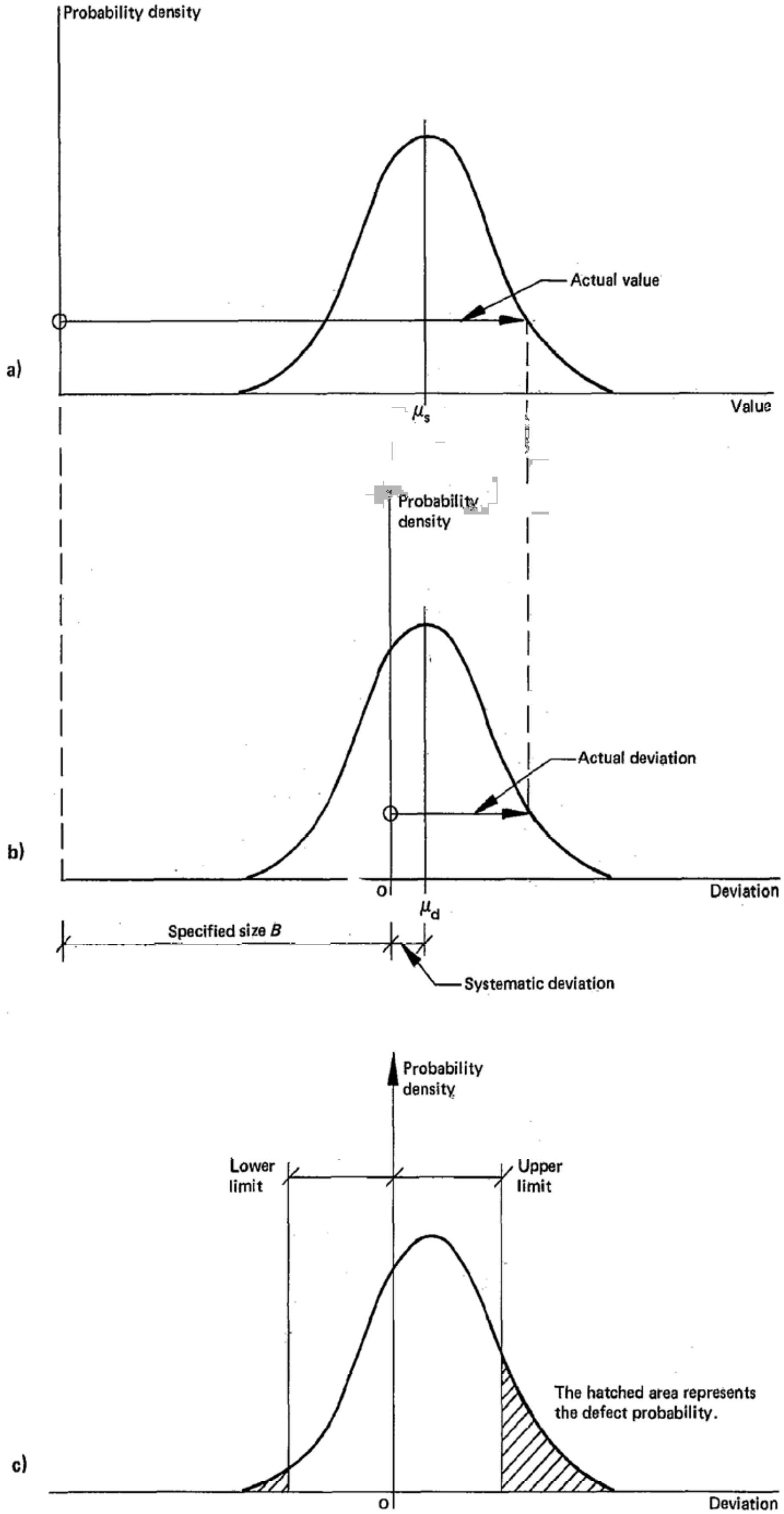


FIGURE 3 – **Distribution of values (a) and distribution of deviations (b), with illustration of limits (c).**

ANNEX

SAMPLING AND THE CALCULATION OF THE STANDARD DEVIATION

A.1 GENERAL

The measurement of samples is generally a routine process in which no thought is given to the individual significance of the values found. The primary need is for the sample to be gathered at random, so that it can be regarded as representative of the body of items from which it was drawn[1]. The patterns associated with probability emerge only when statistical processes are applied to the resultant data, and the properties of the distribution are calculated. The two characteristics most commonly required are the arithmetic mean of the values, and the standard deviation as a measure of their variability or dispersion. The techniques described in this annex apply only to variable processes having a normal distribution of deviation.

The normal distribution of values described in clause 5 is the pattern that appears when a sufficient number of random attempts to achieve a target are measured assuming the values to be unbiased. The more observations that are made, the closer the pattern resembles that predicted by theory. A similar relationship applies when a sample is used to estimate the properties of the population from which it was drawn. The information from a sample that contains only a few items is unreliable. This means that in such cases the population parameters can be surmised only as broad limiting values, i.e. the population mean and standard deviation can be predicted as being likely to lie within a certain range around the calculated sample mean and standard deviation. As the sample size increases, so the range of likely positions for the true value of the population attribute diminishes.

These ranges of values are defined by confidence limits applied to the mean and standard deviation calculated from a sample. They are, for example, derived for 95 % confidence, meaning that there is a 5 % chance that the true values for the population lie beyond the limits. Lower confidence levels may prove to be more economic for the building industry. Confidence limits for the mean and standard deviation are tabulated for various sample sizes in ISO 3207.

A.2 ESTIMATION OF THE MEAN AND STANDARD DEVIATION OF A POPULATION FROM A SERIES OF OBSERVED VALUES

The following symbols apply :

x_i is an observed value;

$\bar{x}$ is the arithmetic mean of the series of observed values considered (this is also the estimated mean of the population);

n is the number of observations in the series;

s is the estimated standard deviation of the population.

The basic expressions are as follows :

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

However, the calculation of the standard deviation from this expression is laborious if a large number of observations is involved. The arithmetic can be simplified by using the following short cuts.

a) In calculating the mean and standard deviation of a series of observations, a constant may be subtracted from every observation for the purpose of computation, provided that this constant is added to the computed mean. This procedure is known as changing the origin, and after the computations have been completed the constant must be added to the computed mean to refer it again to the former origin; the standard deviation is unaffected by the change of origin.

b) The observations may all be multiplied (or divided) by the same factor, provided that the computed mean and the standard deviation are divided (or multiplied) by the same factor. The units in which the computations are carried out are usually known as working units.

c) The expression $\sum_{i=1}^{n} (x_i - \bar{x})^2$ is the sum of the squares of the deviations of the observed values from their mean value.

The computation of this expression may be performed more simply as follows.

1) Sum the squares of the observations [in the new units if adjustments have been made as described in a) and b)].

1) Under certain circumstances, precautions may be needed to ensure randomness. It may be necessary to select items according to tables of random numbers, and techniques exist for examining abnormal results and rejecting them if genuinely unrepresentative. Further guidance is available from textbooks on statistics and quality control.

2) Subtract from this : $\frac{1}{n}$ x the square of the sum of the observations (in new units, if adjusted).

The complete expression for the standard deviation in this form is written thus :

$$s = \sqrt{\frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n-1}}$$

or, in a more suitable form for a desk calculator,

$$s = \sqrt{\frac{n\,\Sigma x_i^2 - (\Sigma x_i)^2}{n\,(n-1)}}$$

A.3 EXAMPLE (limited to ten observations for simplicity)

It is required to calculate the mean and standard deviation from the following data :

A series of actual sizes of a component, measured in millimetres :

1 491,5	1 489,0	1 487,0	1 490,0	1 489,0
1 492,0	1 489,0	1 490,5	1 487,0	1 489,5

Taking the origin as 1 490 and multiplying each value by 2, the data become, in working units,*

+ 3	− 2	− 6	0	− 2
+ 4	− 2	+ 1	− 6	− 1

Sum of deviations from new origin (paying regard to sign)

$= -11$ working units

Sum of squares of deviations from new origin

$= 9 + 4 + 36 + 0 + 4 + 16 + 4 + 1 + 36 + 1$
$= 111$ (working units)^2

Sum of squares of deviations from mean

$= 111 - \frac{11^2}{10} = 111 - 12{,}1$
$= 98{,}9$ (working units)^2

Standard deviation, s

$= \sqrt{\frac{98{,}9}{9}}$
$= \sqrt{11}$
$= 3{,}32$ working units

Mean, $\bar{x}$, in millimetres,

$= 1\,490 - \frac{11}{2 \times 10}$
$= 1\,489{,}45$

Standard deviation, s, in millimetres,

$= \frac{3{,}32}{2}$
$= 1{,}66$

* In this example, the chosen working unit is 2 X 1 mm.