International Standard 2602

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Statistical interpretation of test results — Estimation of the mean — Confidence interval

Interprétation statistique de résultats d'essais — Estimation de la moyenne — Intervalle de confiance

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards institutes (ISO member bodies). The work of developing International Standards is carried out through ISO technical committees. Every member body interested in a subject for which a technical committee has been set up has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work.

Draft International Standards adopted by the technical committees are circulated to the member bodies for approval before their acceptance as International Standards by the ISO Council.

International Standard ISO 2602 was developed by Technical Committee ISO/TC 69, *Applications of statistical methods*.

This second edition was submitted directly to the ISO Council, in accordance with clause 5.10.1 of part 1 of the Directives for the technical work of ISO. It cancels and replaces the first edition (i.e. ISO 2602-1973), which has been approved by the member bodies of the following countries :

Australia	India	Portugal
Austria	Ireland	Romania
Belgium	Israel	South Africa, Rep. of
Czechoslovakia	Italy	Sweden
Egypt, Arab Rep. of	Japan	Switzerland
France	Netherlands	Thailand
Germany, F.R.	New Zealand	United Kingdom
Hungary	Poland	USSR

No member body had expressed disapproval of the document.

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Statistical interpretation of test results — Estimation of the mean — Confidence interval

Second edition

0 Introduction

The scope of this International Standard is limited to a special question. It concerns only the estimation of the mean of a normal population on the basis of a series of tests applied to a random sample of individuals drawn from this population, and deals only with the case where the variance of the population is unknown. It is not concerned with the calculation of an interval containing, with a fixed probability, at least a given fraction of the population (statistical tolerance limits).

It is recalled that ISO 2854 relates to the following collection of problems (including the problem treated in this International Standard) :

- estimation of a mean and of the difference between two means (the variances being either known or unknown);
- comparison of a mean with a given value and of two means with one another (the variances being either known or unknown, but equal);
- estimation of a variance and of the ratio of two variances;
- comparison of a variance with a given value and of two variances with one another.

The test methods generally provide for several determinations which are carried out :

- on the same item (where the test is not destructive);
- on distinct portions of a very homogeneous product (a liquid, for example);
- on distinct items sampled from an aggregate with a certain amount of variability.

In the first two cases, the deviations between the results obtained depend only upon the repeatability of the method. In the third case, they depend also on the variability of the product itself.

The statistical treatment of the results allows the calculation of an interval which contains, with a given probability, the mean of the population of results that would be obtained from a very large number of determinations, carried out under the same conditions. In the case of items with a variability, this International Standard assumes that the individuals on which the determinations are carried out constitute a random sample from the original population and may be considered as independent.

The interval so calculated is called the confidence interval for the mean. Associated with it is a confidence level (sometimes termed a confidence coefficient), which is the probability, usually expressed as a percentage, that the interval does contain the mean of the population. Only the 95 % and 99 % levels are provided for in this International Standard.

1 Scope

This International Standard specifies the statistical treatment of test results needed to calculate a confidence interval for the mean of a population.

2 Field of application

The test results are expressed by measurements of a continuous character. This International Standard does not cover tests of a qualitative character (for example presence or absence of a property, number of defectives, etc.).

The probability distribution taken as a mathematical model for the total population is a normal distribution for which parameters, mean m and standard deviation σ, are unknown.

The normality assumption is very widely satisfied : the distribution of the results obtained under test conditions is generally a normal or nearly normal distribution.

It may, however, be useful to check the validity of the assumption of normality by means of appropriate methods[1)].

The calculations may be simplified by a change of the origin or the unit of the test results but it is dangerous to round off these results.

1) This subject is in preparation.

It is not permissible to discard any observations or to apply any corrections to apparently doubtful observations without a justification based on experimental, technical or other evident grounds which should be clearly stated.

The test method may be subject to systematic errors, the determination of which is not taken into consideration here. It should be noted, however, that the existence of such errors may invalidate the methods which follow. In particular, if there is an unsuspected bias the increase of the sample size n has no influence on bias. The methods that are treated in ISO 2854 may be useful in certain cases for identifying systematic errors.

3 References

ISO 2854, *Statistical treatment of data — Problems of estimation and tests of means and variances*.

ISO 3534, *Statistics — Vocabulary and symbols.*

4 Definitions and symbols

The vocabulary and symbols used in this International Standard are in conformity with ISO 3534.

5 Estimation of the mean

5.1 Case of ungrouped results

After the discarding of any doubtful results, the series comprises n measurements x_i (where $i = 1, 2, 3, \ldots, n$), some of which may have the same value.

The mean m of the underlying normal distribution is estimated by the arithmetic mean $\bar{x}$ of the n results :

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

5.2 Case of results grouped in classes

When the number of results is sufficiently high (above 50 for example), it may be advantageous to group them into classes of the same width. In certain cases, the results may also have been directly obtained grouped into classes.

The frequency of the ith class, i.e. the number of results in class i, is denoted by n_i.

The number of classes being denoted by k, we have :

$$n = \sum_{i=1}^{k} n_i$$

The midpoint of class i is designated by y_i. The mean m is then estimated by the weighted mean of all midpoints of classes :

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{k} n_i y_i$$

6 Confidence interval for the mean

The confidence interval for the population mean is calculated from the estimates of the mean and of the standard deviation.

The alternative method of calculating the confidence interval by use of the range is given in the annex.

6.1 Estimation of the standard deviation

6.1.1 Case of ungrouped results

The estimate of the standard deviation σ, calculated from the squares of the deviations from the arithmetic mean, is given by the formula :

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

where

x_i is the value of the ith measurement ($i = 1, 2, 3, \ldots, n$);

n is the total number of measurements;

$\bar{x}$ is the arithmetic mean of the n measurements, calculated as in clause 5.1.

For ease of calculation, the use of the following formula is recommended :

$$s = \sqrt{\frac{1}{n-1} \left[\sum_{i=1}^{n} x_i^2 - \frac{1}{n} \left(\sum_{i=1}^{n} x_i \right)^2 \right]}$$

6.1.2 Case of grouped results

In the case of grouping by classes, the formula for the estimate of the standard deviation is written :

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{k} n_i (y_i - \bar{y})^2}$$

For ease of calculation, the use of the following formula is recommended :

$$s = \sqrt{\frac{1}{n-1}\left[\sum_{i=1}^{k} n_i y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{k} n_i y_i\right)^2\right]}$$

where

y_i is the mid-point of the ith class ($i = 1, 2, 3, \ldots, k$);

k is the number of classes;

n is the total number of measurements;

$\bar{y}$ is the weighted mean of all mid-points of classes calculated as in sub-clause 5.2.

In the case of grouped results, the calculated value of s should be corrected ("Sheppard's correction"). As this correction is of secondary importance, it has not been mentioned here.

6.2 Confidence interval for the mean

For a chosen confidence level (95 % or 99 %), according to the specific case, a two-sided or a one-sided confidence interval has to be determined.

6.2.1 Two-sided confidence interval

The two-sided confidence interval for the population mean is defined by the following double inequality :

a) at the confidence level 95 % :

$$\bar{x} - \frac{t_{0,975}}{\sqrt{n}} s < m < \bar{x} + \frac{t_{0,975}}{\sqrt{n}} s$$

b) at the confidence level 99 %

$$\bar{x} - \frac{t_{0,995}}{\sqrt{n}} s < m < \bar{x} + \frac{t_{0,995}}{\sqrt{n}} s$$

6.2.2 One-sided confidence interval

The one-sided confidence interval for the population mean is defined by one or other of the following inequalities :

a) at the confidence level 95 % :

$$m < \bar{x} + \frac{t_{0,95}}{\sqrt{n}} s$$

or

$$m > \bar{x} - \frac{t_{0,95}}{\sqrt{n}} s$$

b) at the confidence level 99 % :

$$m < \bar{x} + \frac{t_{0,99}}{\sqrt{n}} s$$

or

$$m > \bar{x} - \frac{t_{0,99}}{\sqrt{n}} s$$

with $\bar{x}$, if necessary, replaced by $\bar{y}$, in the case of results grouped in classes.

The values $t_{0,975}$, $t_{0,995}$, $t_{0,95}$, $t_{0,99}$ are those of Student's t distribution with $\nu = n + 1$ degrees of freedom.

These values are given in table 1.

This table gives also the values of ratios

$$\frac{t_{0,975}}{\sqrt{n}}, \frac{t_{0,995}}{\sqrt{n}}, \frac{t_{0,95}}{\sqrt{n}}, \frac{t_{0,99}}{\sqrt{n}}$$

When values of n are greater than 60, it is preferable to calculate the value of t by linear interpolation from $\frac{120}{n}$ using table 2.

Example :

$n = 250$

$$\frac{120}{n} = 0,48$$

$$t_{0,995} = 2,576 + 0,48 + (2,617 - 2,576)$$
$$= 2,596$$

7 Presentation of the results

7.1 Give the expression of the mean according to 5.1 or 5.2.

7.2 Express the confidence interval in the form of the double inequality of 6.2.1 or one of the inequalities of 6.2.2, stating the confidence level (95 % or 99 %). Indicate the number of results discarded as being doubtful and the reasons for discarding.

Table 1 — Values of $t_{1-\alpha}$ and of the ratio $t_{1-\alpha}/\sqrt{n}$

	Confidence level Two-sided case		Confidence level One-sided case		Confidence level Two-sided case		Confidence level One-sided case	
	95 %	99 %	95 %	99 %	95 %	99 %	95 %	99 %
n	$t_{0,975}$	$t_{0,995}$	$t_{0,95}$	$t_{0,99}$	$\frac{t_{0,975}}{\sqrt{n}}$	$\frac{t_{0,995}}{\sqrt{n}}$	$\frac{t_{0,95}}{\sqrt{n}}$	$\frac{t_{0,99}}{\sqrt{n}}$
2	12,71	63,66	6,314	31,82	8,985	45,013	4,465	22,501
3	4,303	9,925	2,920	6,965	2,484	5,730	1,686	4,021
4	3,182	5,841	2,353	4,541	1,591	2,920	1,177	2,270
5	2,776	4,604	2,132	3,747	1,242	2,059	0,953	1,676
6	2,571	4,032	2,015	3,365	1,049	1,646	0,823	1,374
7	2,447	3,707	1,943	3,143	0,925	1,401	0,734	1,188
8	2,365	3,499	1,895	2,998	0,836	1,237	0,670	1,060
9	2,306	3,355	1,860	2,896	0,769	1,118	0,620	0,966
10	2,262	3,250	1,833	2,821	0,715	1,028	0,580	0,892
11	2,228	3,169	1,812	2,764	0,672	0,956	0,546	0,833
12	2,201	3,106	1,796	2,718	0,635	0,897	0,518	0,785
13	2,179	3,055	1,782	2,681	0,604	0,847	0,494	0,744
14	2,160	3,012	1,771	2,650	0,577	0,805	0,473	0,708
15	2,145	2,977	1,761	2,624	0,554	0,769	0,455	0,668
16	2,131	2,947	1,753	2,602	0,533	0,737	0,438	0,651
17	2,120	2,921	1,746	2,583	0,514	0,708	0,423	0,627
18	2,110	2,898	1,740	2,567	0,497	0,683	0,410	0,605
19	2,101	2,878	1,734	2,552	0,482	0,660	0,398	0,586
20	2,093	2,861	1,729	2,539	0,468	0,640	0,387	0,568
21	2,086	2,845	1,725	2,528	0,455	0,621	0,376	0,552
22	2,080	2,831	1,721	2,518	0,443	0,604	0,367	0,537
23	2,074	2,819	1,717	2,508	0,432	0,588	0,358	0,523
24	2,069	2,807	1,714	2,500	0,422	0,573	0,350	0,510
25	2,064	2,797	1,711	2,492	0,413	0,559	0,342	0,498
26	2,060	2,787	1,708	2,485	0,404	0,547	0,335	0,487
27	2,056	2,779	1,706	2,479	0,396	0,535	0,328	0,477
28	2,052	2,771	1,703	2,473	0,388	0,524	0,322	0,467
29	2,048	2,763	1,701	2,467	0,380	0,513	0,316	0,658
30	2,045	2,756	1,699	2,462	0,373	0,503	0,310	0,449
40	2,024	2,707	1,682	2,430	0,320	0,428	0,266	0,384
50	2,008	2,680	1,676	2,404	0,284	0,379	0,237	0,340
60	2,000	2,664	1,673	2,393	0,258	0,344	0,216	0,309

Table 2

n	$\frac{120}{n}$	$t_{0,975}$	$t_{0,995}$	$t_{0,95}$	$t_{0,99}$
60	2	2,000	2,664	1,673	2,393
120	1	1,980	2,617	1,658	2,358
∞	0	1,960	2,576	1,645	2,326

Annex

Confidence interval for the mean from the range

If the measurements are arranged in ascending order of magnitude, so that $x_1 \leqslant x_2 \leqslant \ldots \leqslant x_n$, then $w = x_n - x_1$ is defined as the sample range. Still assuming that the population is normally distributed, the confidence interval for the population mean can be determined from the sample range when the number of measurements is small, say 12 or less. The practical convenience of this calculation is that it is faster; its disadvantage is that it leads to a confidence interval which is generally wider and which is more sensitive to departures from the assumed normal form of the observations.

Two-sided confidence interval

The two-sided confidence interval for the population mean is defined by the following double inequality :

a) at the confidence level 95 % :

$$\bar{x} - q_{0,975}\, w < m < \bar{x} + q_{0,975}\, w$$

b) at the confidence level 99 % :

$$\bar{x} - q_{0,995}\, w < m < \bar{x} + q_{0,995}\, w$$

One-sided confidence interval

The one-sided confidence interval for the population mean is defined by one or other of the following inequalities :

a) at the confidence level 95 % :

$$m < \bar{x} + q_{0,95}\, w$$

or

$$m > \bar{x} - q_{0,95}\, w$$

b) at the confidence level 99 % :

$$m < \bar{x} + q_{0,99}\, w$$

or

$$m > \bar{x} - q_{0,99}\, w$$

The coefficients $q_{0,975}$, $q_{0,995}$, $q_{0,95}$, $q_{0,99}$, are given in table **3**.

Table 3

	Confidence level Two-sided case		Confidence level One-sided case	
	95 %	99 %	95 %	99 %
n	$q_{0,975}$	$q_{0,995}$	$q_{0,95}$	$q_{0,99}$
2	6,353	31,828	3,157	15,910
3	1,304	3,008	0,885	2,111
4	0,717	1,316	0,529	1,023
5	0,507	0,843	0,388	0,685
6	0,399	0,628	0,312	0,523
7	0,333	0,507	0,263	0,429
8	0,288	0,429	0,230	0,366
9	0,255	0,374	0,205	0,322
10	0,230	0,333	0,186	0,288
11	0,210	0,302	0,170	0,262
12	0,194	0,277	0,158	0,241

From : E. Lord. The use of range in place of the standard deviation in t-test (*Biometrika*, Vol. 34, 1947, pp.41-67), with entry $q_{0,95}$ ($n = 2$) corrected.

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