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ISO 1100-2

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Hydrometry — Measurement of liquid flow in open channels —

Part 2: Determination of the stage-discharge relationship

Hydrométrie — Mesurage du débit des liquides dans les canaux découverts —

Partie 2: Détermination de la relation hauteur-débit

Reference number
ISO 1100-2:2010(E)

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 1100-2 was prepared by Technical Committee ISO/TC 113, *Hydrometry*, Subcommittee SC 1, *Velocity area methods*.

This third edition cancels and replaces the second edition (ISO 1100-2:1998). Most of the clauses have been updated and technically revised. Major revisions have been made to Clause 5, including a new figure of a stage-discharge relationship and shift curves. Clause 7 has been revised to be consistent with new standards on uncertainty.

It also incorporates the Technical Corrigendum ISO 1100-2:1998/Cor.1:2000.

ISO 1100 consists of the following parts, under the general title *Hydrometry — Measurement of liquid flow in open channels*:

— *Part 1: Establishment and operation of a gauging station*

— *Part 2: Determination of the stage-discharge relationship*

Hydrometry — Measurement of liquid flow in open channels —

Part 2:
Determination of the stage-discharge relationship

1 Scope

This part of ISO 1100 specifies methods of determining the stage-discharge relationship for a gauging station. A sufficient number of discharge measurements, complete with corresponding stage measurements, are required to define a stage-discharge relationship to the accuracy required by this part of ISO 1100.

Stable and unstable channels are considered, including brief descriptions of the effects on the stage-discharge relationship of shifting controls, variable backwater and hysteresis. Methods of determining discharge for twin-gauge stations, ultrasonic velocity-measurement stations, electromagnetic velocity-measurement stations and other complex rating curves are not described in detail. These types of rating curve are described separately in other International Standards, Technical Specifications and Technical Reports, which are listed in Clause 2 and the Bibliography.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 748, *Hydrometry — Measurement of liquid flow in open channels using current-meters or floats*

ISO 772, *Hydrometry — Vocabulary and symbols*

ISO 5168, *Measurement of fluid flow — Procedures for the evaluation of uncertainties*

ISO 9123, *Measurement of liquid flow in open channels — Stage-fall-discharge relationships*

ISO 15769, *Hydrometry — Guidelines for the application of acoustic velocity meters using the Doppler and echo correlation methods*

ISO/TS 24154, *Hydrometry — Measuring river velocity and discharge with acoustic Doppler profilers*

3 Symbols

For the purposes of this document, the symbols given in ISO 772 and the following apply:

A cross-sectional area

B cross-sectional width

β power-law exponent (slope on logarithmic plot) of the rating curve

C_D coefficient of discharge

C Chezy's channel roughness coefficient

e effective gauge height of zero flow

h gauge height of the water surface

$(h - e)$ effective depth

H total head (hydraulic head)

n Manning's channel roughness coefficient

N number of stage-discharge measurements (gaugings) used to define the rating curve

p number of rating-curve parameters (Q_1, β, e) estimated from the N gaugings

P_w wetted perimeter

Q total discharge

Q_o steady-state discharge

Q_1 power-law scale factor of rating curve, equal to discharge when effective depth of flow $(h - e)$ is equal to 1

r_h hydraulic radius, equal to the effective cross-sectional area divided by the wetted perimeter, A/P_w

S standard error of estimate

S_f friction slope

S_o water surface slope corresponding to steady discharge

t time

u standard uncertainty

U expanded uncertainty

V_w velocity of a flood wave

4 Principle of the stage-discharge relationship

4.1 General

The stage-discharge relationship is the relationship at a gauging station between stage and discharge and is sometimes referred to as a rating curve or rating. The principles of the establishment and operation of a gauging station are described in ISO 1100-1.

4.2 Controls

4.2.1 General

The stage-discharge relationship for open-channel flow at a gauging station is governed by channel conditions at and downstream from the gauge, referred to as a control. Two types of control can exist, depending on channel and flow conditions. Low flows are usually controlled by a section control, whereas high flows are usually controlled by a channel control. Medium flows can be controlled by either type of control. At some stages, a combination of section and channel control might occur. These are general rules, and exceptions can and do occur. Knowledge of the channel features that control the stage-discharge relationship is important. The development of stage-discharge curves where more than one control is effective, where control features change and where the number of measurements is limited requires judgement in interpolating between measurements and in extrapolating beyond the highest or lowest measurements. This is particularly true where the controls are not permanent and tend to shift from time to time, resulting in changes in the positioning of segments of the stage-discharge relationship.

4.2.2 Section control

A section control is a specific cross-section of a stream channel, located downstream from a water-level gauge that controls the relationship between gauge height and discharge at the gauge. A section control can be a natural feature, such as a rock ledge, a gravel bar, a severe constriction in the channel or an accumulation of debris. A section control can also be a man-made feature, such as a small dam, a weir, a flume or an overflow spillway. Section controls can often be visually identified in the field by observing a riffle, or pronounced drop in the water surface, as the flow passes over the control. Frequently, as gauge height increases because of higher flows, the section control will become submerged to the extent that it no longer controls the relationship between gauge height and discharge. At this point, the riffle is no longer observable, and flow is then regulated either by another section control further downstream or by the hydraulic geometry and roughness of the channel downstream (i.e. channel control).

4.2.3 Channel control

A channel control consists of a combination of features throughout a reach at and downstream from a gauge. These features include channel size, shape, curvature, slope and roughness. The length of channel reach that controls a stage-discharge relationship varies. The stage-discharge relationship for a relatively steep channel could be controlled by a short channel reach, whereas the relationship for a flat channel could be controlled by a much longer channel reach. Additionally, the length of a channel control will vary depending on the magnitude of flow. Precise definition of the length of a channel-control reach is usually neither possible nor necessary.

4.2.4 Combination controls

At some stages, the stage-discharge relationship can be governed by a combination of section and channel controls. This usually occurs for a short range in stage between section-controlled and channel-controlled segments of the rating curve. This part of the rating curve is commonly referred to as a transition zone of the rating curve and represents the change from section control to channel control. In other instances, a combination control can consist of two section controls, where each has a partial controlling effect. More than two controls acting simultaneously are rare. In any case, combination controls and/or transition zones occur for very limited parts of a stage-discharge relationship and can usually be defined by plotting procedures. Transition zones, in particular, represent changes in the slope or shape of a stage-discharge relationship.

4.3 Governing hydraulic equations

Stage-discharge relationships are hydraulic relationships that can be defined according to the type of control that exists. Section controls, either natural or man-made, are governed by some form of the weir or flume equations. In a very general and basic form, these equations are expressed as:

$$Q = C_{\mathrm{D}} B H^{\beta} \quad (1)$$

where

Q is the discharge, in cubic metres per second;

C_D is a coefficient of discharge and includes several factors;

B is the cross-sectional width, in metres;

H is the hydraulic head, in metres;

β is a power-law exponent, dependent on the cross-sectional shape of the control section.

Stage-discharge relationships for channel controls with uniform flow are governed by the Manning or Chezy equation as it applies to the reach of the controlling channel downstream from a gauge. The Manning equation is:

$$Q = \frac{A r_h^{0,67} S_f^{0,5}}{n} \quad (2)$$

where

A is the cross-sectional area, in square metres;

r_h is the hydraulic radius, in metres;

S_f is the friction slope;

n is the channel roughness.

The Chezy equation is:

$$Q = C A r_h^{0,5} S_f^{0,5} \quad (3)$$

where C is the Chezy form of roughness.

The above equations are generally applicable for steady or quasi-steady flow. For highly unsteady flow, such as tidal or dam-break flow, equations such as the Saint-Venant unsteady-flow equations would be necessary. However, these are seldom used in the development of stage-discharge relationships and are not described in this part of ISO 1100.

4.4 Complexities of stage-discharge relationships

Stage-discharge relationships for stable controls (such as rock outcrops and man-made structures such as weirs, flumes and small dams) present few problems in their calibration provided a suitable maintenance regime can be achieved. However, complexities can arise when controls are not stable and/or when variable backwater occurs. For unstable controls, segments of a stage-discharge relationship can change position occasionally, or even frequently. This is usually a temporary condition which can be accounted for through the use of the shifting-control method.

Variable backwater can affect a stage-discharge relationship both for stable and unstable channels. Sources of backwater can be downstream reservoirs, tributaries, tides, vegetation, ice, dams and other obstructions that influence the flow at the gauging-station control.

A complexity that exists for some streams is hysteresis, which results when the water surface slope changes due to either rapidly rising or rapidly falling water levels in a channel-control reach. Hysteresis is also referred to as loop rating curves and is most pronounced in relatively flat-sloped streams. On rising stages, the water surface slope is significantly steeper than for steady-flow conditions, resulting in greater discharge than indicated by the steady-flow rating curve. The reverse is true for falling stages. See 5.8.3 for details of hysteresis rating curves.

Another complexity exists when rivers are in flood because it is often difficult to define flood-plain storage and to represent such flows in the flood-plain rating-curve section. Complex flow interactions between the main channel and flood plain often result in flow patterns that are difficult to define at the measuring section.

5 Stage-discharge calibration of a gauging station

5.1 General

The primary object of a stage-discharge gauging station is to provide a record of the discharge of the open channel or river at which the water level gauge is sited. This is achieved by measuring the stage and converting this stage to discharge by means of a stage-discharge relationship which correlates discharge and water level. In some instances, other parameters, such as index velocity, water surface fall between two gauges or rate-of-change in stage, can also be used in rating-curve calibrations (see ISO 9123 and ISO 15769). Stage-discharge relationships are usually calibrated by measuring discharge and the corresponding gauge height. Theoretical computations can also be used to aid in the shaping and positioning of the rating curve. Stage-discharge relationships from previous time periods should also be considered as an aid in the shaping of the rating curve.

5.2 Preparation of a stage-discharge relationship

5.2.1 General

The relationship between stage and discharge is defined by plotting measurements of discharge with corresponding observations of stage, taking into account whether the discharge is steady, increasing or decreasing, and also noting the rate of change in stage. This can be done either manually by plotting on paper or automatically using computerized plotting techniques. The plotting scale used could be an arithmetic scale or a logarithmic scale. Each has certain advantages and disadvantages, as explained in 5.2.3 and 5.2.4. It is customary to plot the stage as ordinate and the discharge as abscissa. However, when using the stage-discharge relationship to derive discharge from a measured value of stage, the stage is treated as the independent variable.

5.2.2 List of discharge measurements

The first step prior to plotting stage versus discharge is the preparation of a list of discharge measurements that will be used for the plot. The measurements should be checked to ensure that the recorded stages are related to a common datum and that the discharge calculations are accurate. As a minimum, this list should include 15 or more measurements, all taken during the period of analysis. More measurements would be required if the rating curve is complex because of multiple section and channel controls or if the site experiences an extreme range in stage. These measurements should be well distributed over the range of gauge heights experienced. The list should also include low and high measurements from other times that might be useful in defining the correct shape of the rating curve and/or in extrapolating the rating curve. Extreme low and high measurements should be included wherever possible.

For each discharge measurement in the list, the following items should be included:

a) a unique identification number;

b) the date of measurement;

c) the gauge height for the measurement;

d) the total discharge;

e) the accuracy of measurement, as determined by the hydrographer;

f) the rate of change in stage during the measurement, a plus sign indicating a rising stage and a minus sign indicating a falling stage.

The list of measurements could include other information; however, this is not mandatory. Table 1 shows a typical list of discharge measurements, including a number of items in addition to the mandatory items.

Table 1 — Typical list of discharge measurements made by a hydrographer using current meters and depth soundings

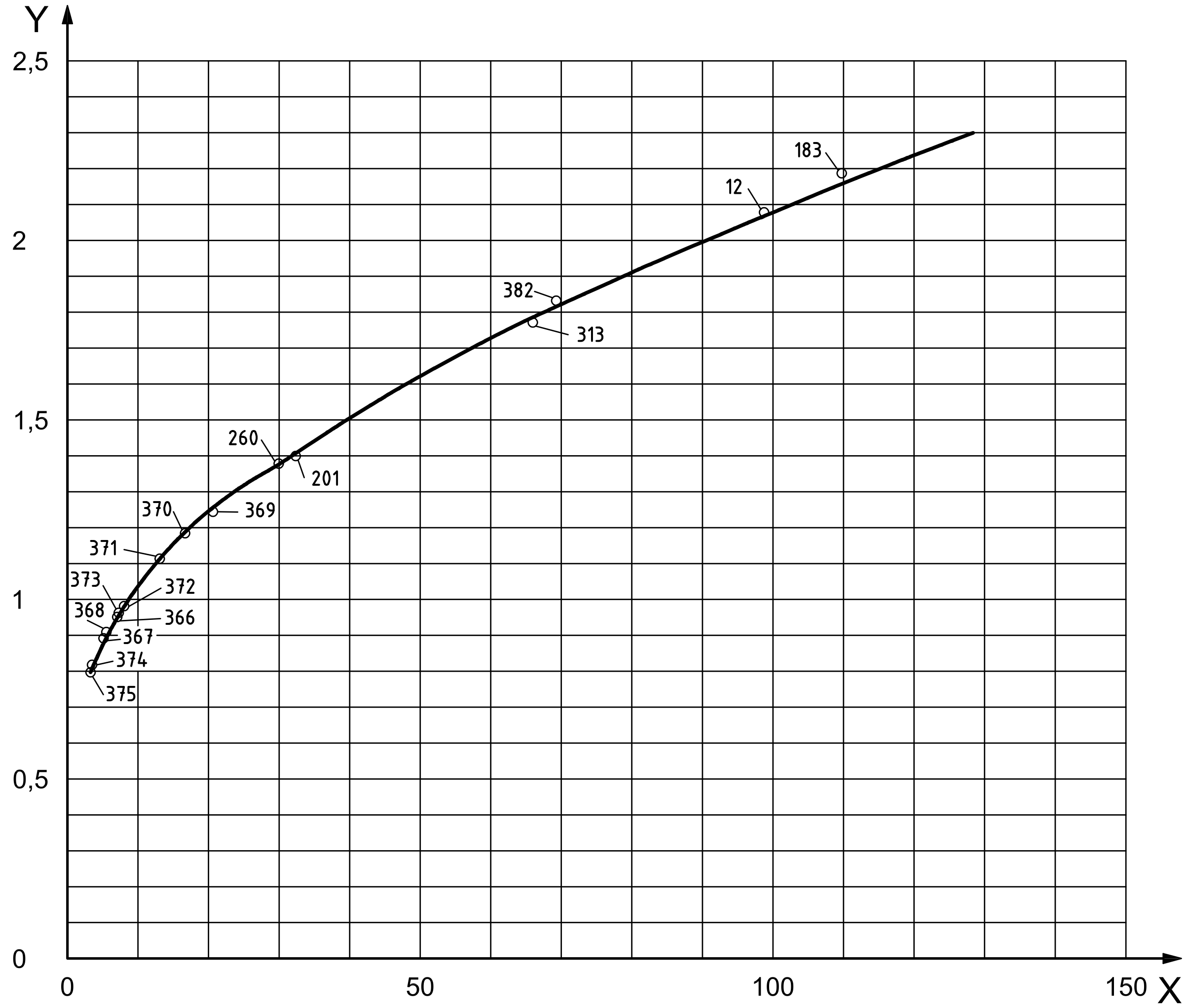
ID number	Date (yy/mm/dd)	Made by	Width m	Area m²	Mean velocity m/s	Gauge height m	Effective depth m	Discharge m³/s	Method	Number of verticals	Gauge height change m/h	Rated
12	38/04/08	MEF	36,27	77,94	1,272	2,682	2,080	99,12	0,2/0,8	22	−0,082	GOOD
183	55/02/06	GTC	33,53	78,41	1,405	2,786	2,186	110,2	0,6/0,2/0,8	22	−0,047	GOOD
201	57/02/04	AJB	28,96	21,92	1,511	2,002	1,402	33,13	0,6/0,2/0,8	21	−0,013	POOR
260	63/03/13	GMP	26,52	21,46	1,400	1,981	1,381	30,02	0,6	22	−0,020	GOOD
313	66/08/24	HFR	30,18	42,08	1,602	2,374	1,774	67,40	0,6/0,2/0,8	22	+0,006	GOOD
366	73/08/21	MAF	28,96	14,86	0,476	1,557	0,957	7,080	0,6	21	0	GOOD
367	73/10/10	MAF	28,96	13,66	0,361	1,490	0,890	4,928	0,6	21	0	GOOD
368	73/11/26	MAF	29,26	14,21	0,373	1,509	0,909	5,296	0,6	18	0	GOOD
369	74/02/19	MAF	29,87	16,26	1,291	1,838	1,238	20,99	0,6	21	0	GOOD
370	74/04/09	MAF	29,26	21,27	0,805	1,780	1,180	17,13	0,6/0,2/0,8	21	0	GOOD
371	74/05/29	MAF	29,57	19,69	0,688	1,710	1,110	13,54	0,6	21	0	GOOD
372	74/07/10	MAF	28,96	16,81	0,458	1,573	0,973	7,703	0,6	21	0	GOOD
373	74/08/22	MAF	29,26	15,79	0,481	1,570	0,970	7,590	0,6	21	0	GOOD
374	74/10/01	MAF	29,26	13,19	0,264	1,414	0,814	3,483	0,6	21	0	GOOD
375	74/11/11	MAJ	28,96	11,71	0,283	1,396	0,796	3,313	0,6	21	0	GOOD
382	75/10/01	MAF	30,48	43,76	1,598	2,432	1,832	69,95	0,2/0,8	21	+0,017	GOOD

NOTE Discharge measurements made with acoustic Doppler current profilers require additional parameters, including the number of transects and the range of discharges measured during the transects (see ISO/TS 24154).

5.2.3 Arithmetic plotting scales

The simplest type of plot uses an arithmetically divided plotting scale, as shown in Figure 1. Scale subdivisions should be chosen to cover the complete range of gauge height and discharge expected to occur at the gauging site. Scales should be subdivided in uniform increments that are easy to read and interpolate. The choice of scale should also produce a rating curve that is not unduly steep or flat. If the range in gauge height or discharge is large, it may be necessary to plot the rating curve in two or more segments to provide scales that are easily read with the necessary precision. This procedure can result in separate curves for low water, medium water and high water.

Graph paper with arithmetic scales is convenient to use and easy to read. Such scales are ideal for displaying a rating curve and have an advantage over logarithmic scales in that zero values of gauge height and/or discharge can be plotted. However, for analytical purposes, arithmetic scales have practically no advantage. A stage-discharge relationship on arithmetic scales is usually a curved line, concave downward, which is difficult to shape correctly if only a few discharge measurements are available. Logarithmic scales, on the other hand, have a number of analytical advantages as described in 5.2.4. Generally, a stage-discharge relationship is first drawn on logarithmic plotting paper for shaping and analytical purposes and then later transferred to arithmetic plotting paper if a display plot is needed.



Key

X discharge, Q, in cubic metres per second

Y effective depth, $(h - e)$, in metres

NOTE The numbers indicated against the plotted observations are the ID numbers given in Table 1.

Figure 1 — Arithmetic plot of stage-discharge relationship

5.2.4 Logarithmic plotting scales

Most stage-discharge relationships, or segments thereof, are best analysed graphically through the use of logarithmic plotting. To utilize this procedure fully, gauge height should be transformed to effective depth of flow on the control by subtracting from it the effective gauge height of zero discharge. A rating-curve segment for a given control will then tend to plot as a straight line with an equation form as described in 5.2.5.3. The slope of the straight line will conform to the type of control (section or channel), thereby providing valuable information for correctly shaping the rating-curve segment. Additionally, this feature allows the analyst to calibrate the stage-discharge relationship with fewer discharge measurements. The slope of a rating curve is the ratio of the horizontal distance to the vertical distance. This method of measuring the slope is used since the dependent variable (discharge) is always plotted as the abscissa.

Rating curves for section controls such as weirs or flumes conform to Equation (1) in 4.3 and, when plotted logarithmically, will have a slope of 1,5 or greater, depending on control shape, velocity of approach and minor variations of the coefficient of discharge. Logarithmic rating curves for most weir shapes will plot with a slope of 2 or greater. An exception is the sharp-crested rectangular weir, which plots with a slope slightly greater than 1,5. Logarithmic rating curves for section controls in natural channels will almost always have a slope of 2 or greater.

Rating curves for channel controls are governed by Equation (2) or (3) and, when plotted as effective depth versus discharge, the slope is usually between 1,5 and 2. Variations in the slope of the rating curve when channel control exists are the result of changes in roughness and friction slope as depth changes.

5.2.5 Rating-curve shape

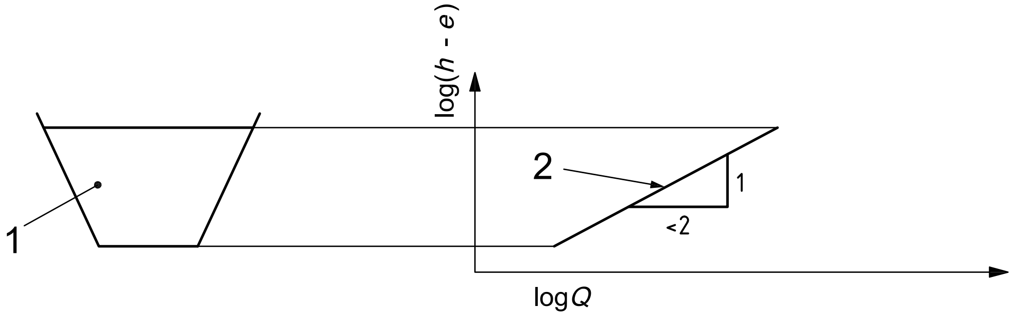
5.2.5.1 General

The details provided in 5.2.2 to 5.2.4 apply to control sections of regular shape (triangular, trapezoidal, parabolic, etc.). When a significant change in shape occurs, such as a trapezoidal section control with a small V-notch for extremely low water, there will be a change in the rating-curve slope at the point where the control shape changes. Likewise, when the control changes from section control to channel control, the logarithmic plot will show a change in slope. These changes are usually defined by short curved segments of the rating curve, referred to as transitions. This information about the plotting characteristics of a rating curve is extremely useful in the calibration and maintenance of the rating curve and in later analysis of shifting control conditions. By knowing the kind of control (section or channel), and the shape of the control, the analyst can define the correct hydraulic shape of the rating curve with greater precision. Additionally, this information allows the analyst to extrapolate accurately a rating curve or, conversely, to know when extrapolation is likely to lead to a large uncertainty.

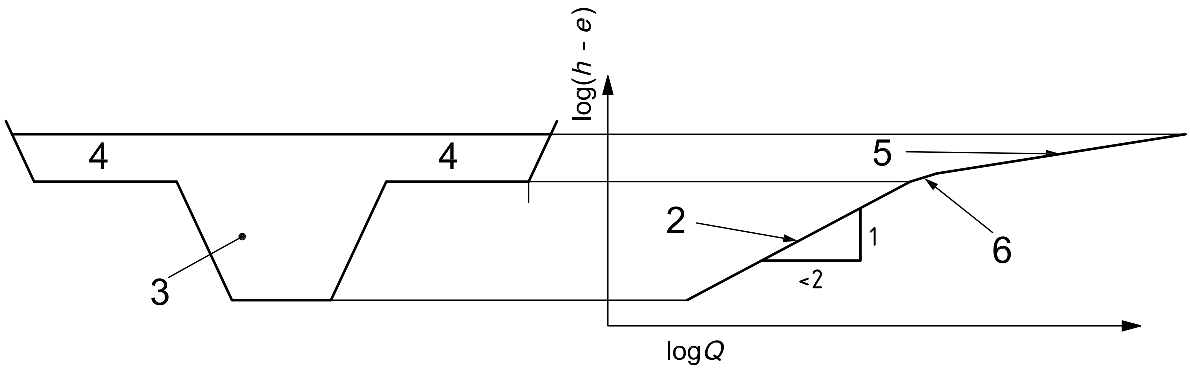
Figure 2 provides examples of a hypothetical rating curve showing the logarithmic plotting characteristics for channel and section controls and for cross-section shape changes. Figure 2 a) shows a trapezoidal channel with no flood plain and with channel-control conditions. The corresponding logarithmic plot of the rating curve, when plotted with an effective gauge height of zero flow, e, that results in a straight-line rating curve, has a slope less than 2. In Figure 2 b), flood plain has been added, which is also a channel control. There is a change in the shape of the control cross-section which results in a change in the shape of the rating curve above the bankfull stage. If the upper segment (above the transition curve) is re-plotted to the correct value of effective gauge height of zero flow, it would also have a slope less than 2. In Figure 2 c), a section control for low flow has been added. This results in a change in rating-curve shape because of the change in control. For the low-water part of the rating curve, the slope will usually be greater than 2.

Figure 3 is a logarithmic plot of an actual rating curve, using the measurements shown in Table 1. This rating curve is for a stream where section control exists throughout the range of flow, including the high-flow measurements. The effective gauge height of zero flow, e, for this stream is 0,6 m, which is subtracted from the gauge height of the measurements to define the effective depth of flow at the control. The slope of the rating curve below 1,4 m is about 4,3, which is greater than 2 and conforms to a section control. Above 1,5 m, the slope is 2,8, which also conforms to a section control. The change in slope of the rating curve above about 1,5 m is caused by a change in the shape of the control cross-section. Below about 1,4 m, the control section is essentially triangular in shape. In the range 1,4 m to 1,5 m, the control shape changes to trapezoidal, resulting in the transition curve in the rating curve. And above about 1,5 m, the control cross-section is basically trapezoidal.

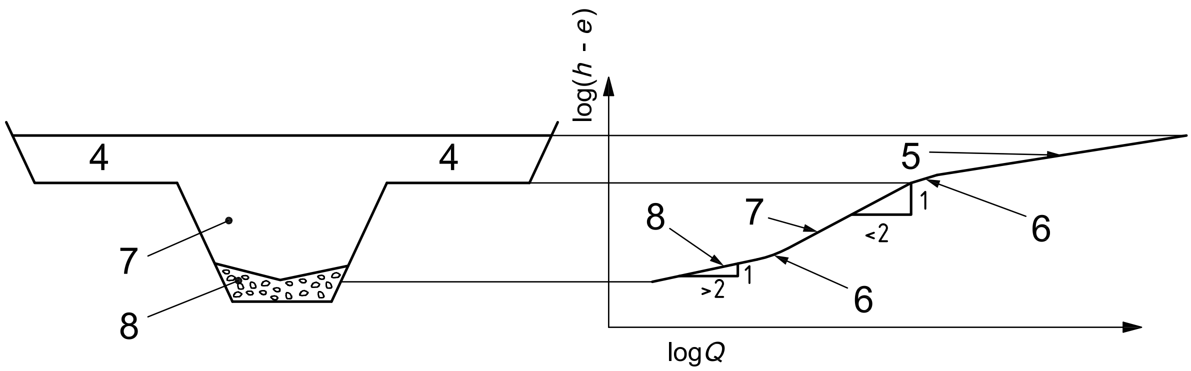
The examples shown in Figures 2 and 3 are intended to illustrate some of the principles of logarithmic plotting. The analyst should use these principles to the greatest extent possible, but should always be aware that there are probably exceptions and differences that occur at some sites.



a) Trapezoidal channel with no flood plain and with channel-control conditions



b) Flood plain added

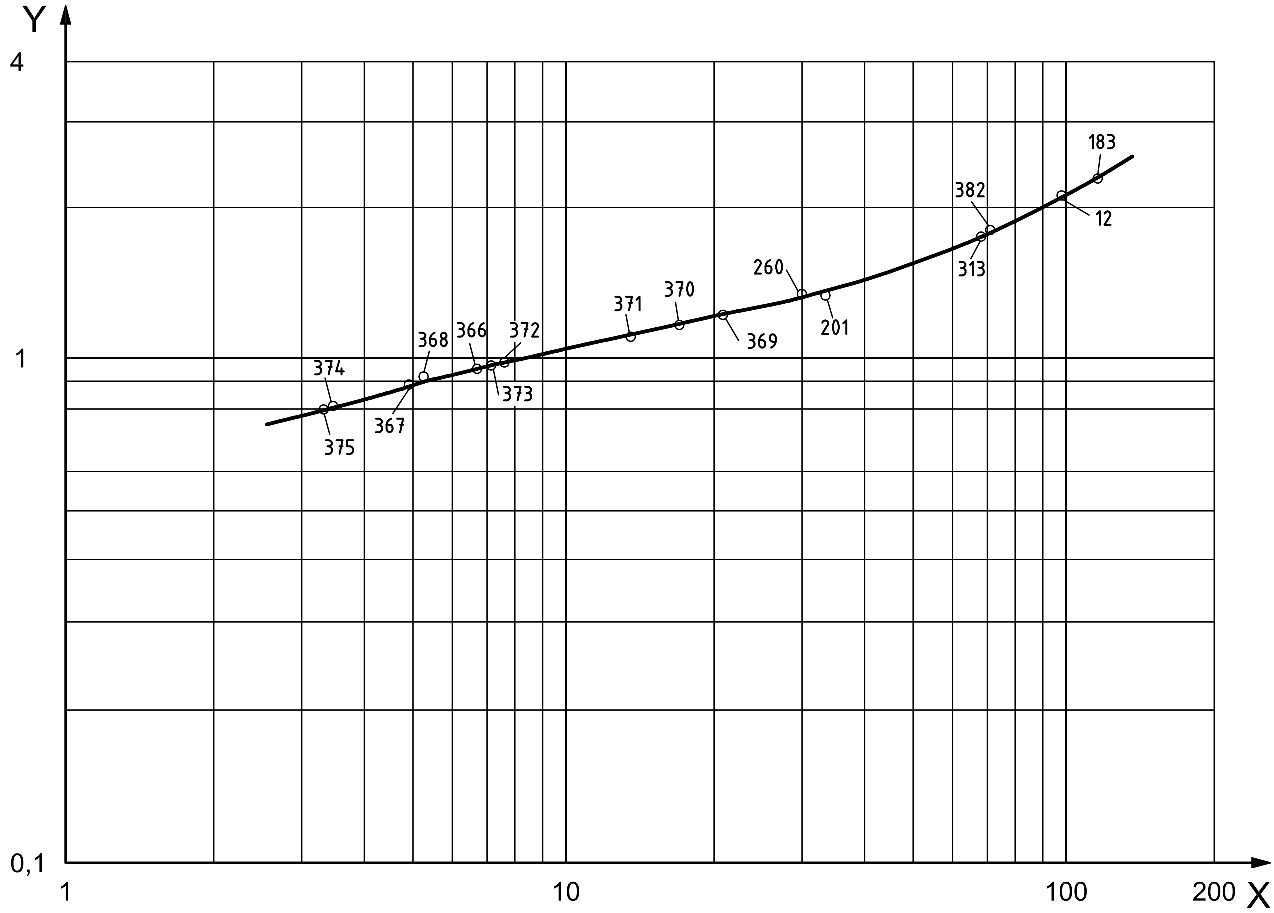


c) Section control for low flow added

Key

1 channel control (no flood plain, no section control)
2 channel-control rating curve
3 channel control (no section control)
4 flood plain
5 flood-plain rating curve
6 transition curve
7 channel control
8 section control

Figure 2 — Relationship of the channel and control properties to the rating-curve shape
(the left-hand drawing in each pair shows the channel shape, the right-hand drawing the rating-curve shape)



Key

X discharge, Q, in cubic metres per second

Y effective depth, $(h - e)$, in metres

NOTE The numbers indicated against the plotted observations are the ID numbers given in Table 1.

Figure 3 — Logarithmic plot of stage-discharge relationship

5.2.5.2 Gauge height of zero flow

The actual gauge height of zero flow is the gauge height of the lowest point in the control cross-section for a section control [sometimes referred to as the cease-to-flow (CTF) value]. For natural channels, this value can sometimes be measured in the field by measuring the depth of flow at the deepest place in the control section and subtracting this depth and the velocity head from the gauge height at the time of measurement.

The effective gauge height of zero flow is a value that, when subtracted from the mean gauge heights of the discharge measurements, will cause the logarithmic rating curve to plot as a straight line. It should be determined for each rating-curve segment. For regularly shaped controls, this value will be close to the actual gauge height of zero flow. For most controls, however, a more exact determination can be made by a trial-and-error method of plotting. A value is assumed and measurements are plotted based on this assumed value. If the resulting curve shape is concave upward, then a somewhat larger value for the effective gauge height of zero flow should be used. A somewhat smaller value should be used if the curve plots concave downward. Usually only a few trials are needed to find a value that results in a straight line for the rating-curve segment. The effective gauge height of zero flow is sometimes referred to as the logarithmic-scale offset.

5.2.5.3 Logarithmic (power-law) equation

The equation for a rating curve that plots as a straight line on logarithmic plotting paper is:

$$Q = Q_1(h - e)^{\beta} \quad (4)$$

where

$(h - e)$ is the effective depth of water on the control;

h is the gauge height of the water surface;

e is the effective gauge height of zero flow;

β is the slope of the rating curve when plotted on logarithmic paper;

Q_1 is a scale factor that is numerically equal to the discharge when the effective depth of flow $(h - e)$ is equal to 1.

5.3 Curve fitting

5.3.1 General

The curve-fitting process for stage-discharge relationships includes the actual drawing, positioning and shaping of the rating curve. Hydraulic-analysis and line-fitting applications can be used to aid in the curve-fitting process but, in the final analysis, the stage-discharge relationship shall conform to the calibration measurements. On the other hand, the rating curve should be hydraulically correct, and every calibration measurement does not need to fit on the same rating curve because of shifting control conditions that sometimes occur. The curve-fitting process should result in curve shapes that conform to control changes, as described in previous subclauses.

5.3.2 Hydraulic-equation curves

The shape of stage-discharge relationships can be defined through the use of hydraulic equations, namely Equations (1), (2) and (3) in 4.3. Where section control exists, the weir equation, Equation (1), can be used to compute rating-curve points. Coefficients of discharge, C_D, have been defined in other International Standards for certain types of weir and flume, so a reasonably accurate rating curve can be computed that will conform to correct hydraulics. For natural section controls, such as a rock outcrop or gravel bar, the coefficient of discharge can be estimated on the basis of calibration measurements. Widths and depths can be determined from a surveyed cross-section of the control section.

For segments of the rating curve that are influenced by channel control, the shape of the rating curve can be defined through the use of Equation (2) or Equation (3). An average or typical cross-section in the control reach is surveyed to define the channel characteristics of cross-sectional area and hydraulic radius. The Manning roughness, n, or the Chezy channel roughness coefficient, C, is estimated from field observations. The friction slope is estimated from channel surveys, maps or calibration measurements. Equation (2) or (3) can then be used to compute discharge for a few selected gauge heights to define the shape of the rating curve. This is a simplified procedure which assumes steady, uniform flow. More complex situations involving non-uniform flow can be analysed with various techniques of backwater curve computation. Computer programmes are available for such analyses.

For both section and channel control, the rating curve computed by the hydraulic equations is used only for defining the hydraulic shape of the rating curve. The correct position of the rating curve is defined by the calibration measurements. This procedure can also be used to aid in determining when measurements define a new rating-curve position.

5.3.3 Mathematical rating curves

The stage-discharge relationship can be defined by mathematical computations, such as regression or maximum-likelihood techniques. Several equations might be required to define the rating-curve equation, particularly if the channel geometry is complex. Care should be taken when deriving the rating curves for segment transitions. It is important that the rating-curve shape is hydraulically correct.

5.4 Combination-control stage-discharge relationships

Combination-control rating curves are also referred to as compound-control rating curves. A combination control can consist of two section controls, each of which controls a segment of the rating curve. For instance, a rock riffle section can control extremely low flows but, at higher flows, a different cross-section located downstream from the rock riffle can cause submergence of the rock riffle and become the controlling section for medium flows. The plot of such a rating curve will usually exhibit a change in slope that reflects the change in effective gauge height of zero flow for the two section controls. Additionally, there will usually be a transition curve between the two rating-curve segments which represents partial control from each of the controls. Graphical analysis of combination, or compound, controls of this type requires separate logarithmic plots for each segment of the rating curve in order to define the segments as straight lines and properly compute the rating-curve slope. When analysed in this manner, the rating-curve slope for each segment will be greater than 2.

A compound rating curve can also be a combination of section control for low flows and channel control for medium or high flows. Graphical analysis usually requires that separate plots be made for the section-control segment and the channel-control segment. A transition curve between the two segments will represent the range of flows over which there is partial control from both the section and channel controls. The slope of the section-control segment should be greater than 2 and that of the channel-control segment less than 2, when analysed in this manner.

A compound rating curve can also be a combination of more than one channel control. Figure 2 b) shows a compound rating curve with a channel control for medium and high flows and a second channel control when water overflows the banks. The transition curve between the two segments represents the situation when the flow is partially controlled by the channel geometry and partially controlled by the flood-plain geometry and vegetation. As the stage increases, the flood plain will dominate the stage-discharge relationship. This segment of the rating curve, if re-plotted with the effective gauge height of zero flow, will also have a slope of less than 2.

5.5 Stable stage-discharge relationships

A stable stage-discharge relationship is one that does not vary, or change position, over a period of time. Such a relationship results from stable channel and control conditions, which for natural channels is a relative term. Virtually all natural channels are subject to at least occasional change as a result of scour, deposition or growth of vegetation.

For stable channels and controls, the stage-discharge relationship can usually be defined easily by fitting a curve to the calibration measurements as described in previous subclauses. The example shown in Figure 3 represents a stable stage-discharge relationship as the control is a natural section of rock outcrop that is not subject to change. Shifts of this rating curve can occur, however, because of debris that might accumulate on the control.

5.6 Unstable stage-discharge relationships

Unstable stage-discharge relationships are defined as those that shift and change positions frequently. Channel geometry and friction properties, and hence the control characteristics, vary continuously as a function of time, and so also does the stage-discharge relationship. These conditions are most evident during floods and during periods when ice or vegetative growth occur. Channel scour and deposition can be a frequent occurrence in some channels due to the nature of the bed and bank materials, thus causing shifts of the rating curve. Likewise, weeds, trees and other vegetation may affect the relationship between stage and discharge during certain times of the year.

It is usually not possible to define all changes of the rating curve with discharge measurements for unstable channels and controls. Shifting-control techniques should be used to estimate the position of the rating curve during periods of time between measurements. These techniques are described in 5.7.

For some gauging stations where unstable channel conditions exist, it is sometimes advisable to install a weir or flume, if practicable, to stabilize the rating curve. In other cases, if the rating curve is unstable because of variable backwater, a velocity index gauge in conjunction with a stage gauge can be used to define the rating curve. This method, which uses acoustic velocity meters mounted on the streambed or on the side of the channel to measure an index velocity, is the most common method in use today. The method is described in detail in ISO 15769. Other, less frequently used, methods of defining an unstable rating curve include the stage-fall method which uses two stage gauges, the electromagnetic method using a full-channel-width coil, and the ultrasonic method. These methods are described in ISO 9123, ISO 9213 and ISO 6416.

5.7 Shifting controls

Shifting controls occur when channel conditions are unstable. When this condition exists, discharge measurements made at different times represent different positions of the rating curve. Frequent discharge measurements should be made during a period of shifting control to define the stage-discharge relationship, or magnitude of shifts, during that period. However, even with infrequent discharge measurements, the stage-discharge relationship can be estimated with reasonable accuracy if the available discharge measurements are supplemented with a knowledge of shifting-control behaviour.

When discharge measurements indicate a shift of the rating curve (see Clause 6), the analyst should decide if the shift is a temporary or permanent condition. If the shift is expected to last for several months or longer, it is advisable to plot a new rating curve. If the shift is a temporary condition that may change soon, the shifting-control condition should be handled by drawing a temporary shift curve to define discharge during the time of shift and until new information indicates another shift of the rating curve. Experience and knowledge of each control is the best way of knowing whether rating-curve shifts are temporary or permanent.

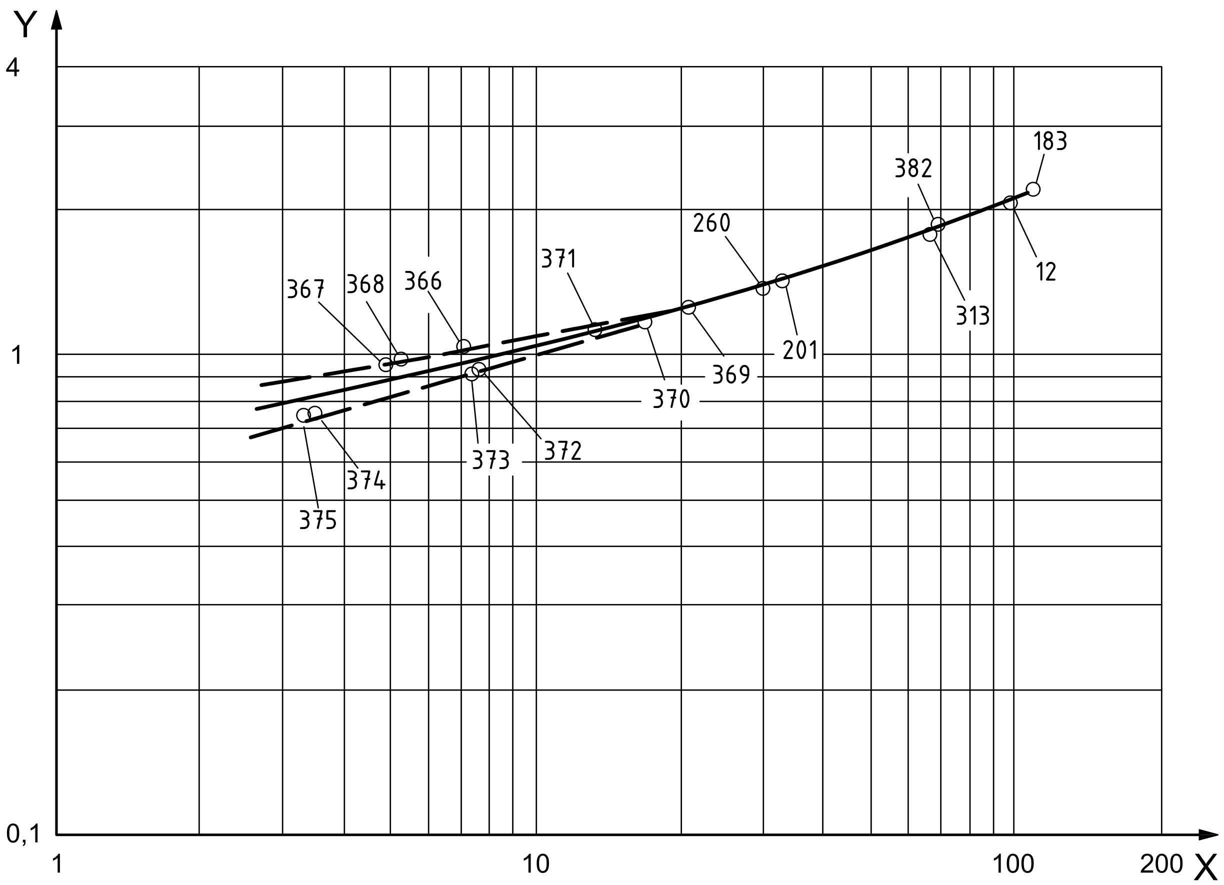
Shift curves usually have a shape which is similar to that of the original rating curve. That is why it is important to have the original, or base, rating curve shaped correctly as defined by the hydraulics of the stream channel. Scour or deposition of a natural section control results in a change in the actual and effective gauge height of zero flow. This frequently results in a shift curve that is parallel to the original rating curve when plotted on arithmetic plotting paper. That is, the difference in gauge height between the original rating curve and the shift curve is equal through the stage range controlled by the section control. This same shift curve, if plotted on logarithmic plotting paper, will be concave upward and above the original rating curve for a deposition condition, and concave downward and below the original rating curve for a scour condition.

Table 2 and Figure 4 demonstrate the use of shift curves. Measurements 366 to 368 and 372 to 375 from Table 1 have been modified to represent a period of deposition or weed growth that occurred at least between August 21 and November 26, 1973, and a period of channel scour that occurred at least between July 10 and November 11, 1974. Measurements 366 to 368 reflect a +0,05 shift from the lower end of the base rating curve; measurements 372 to 375 reflect a −0,06 shift from the base rating curve. Measurements 369 to 371, which were made between the other two sets of measurements, show that the shift did not occur in the transition zone between the two section-control segments of the rating curve. The measurements define the shift curves. The actual periods of the two shifts would be defined during a hydrographic analysis of the stage and discharge data to determine the most likely time when the deposition or weed growth and scour occurred.

Table 2 — List of discharge measurements modified from Table 1

ID number	Date (yyyy/mm/dd)	Effective depth m	Discharge m^3/s
366	1973/08/21	0,957	7,080
367	1973/10/10	0,890	4,928
368	1973/11/26	0,909	5,296
372	1974/07/10	0,973	7,703
373	1974/08/22	0,970	7,590
374	1974/10/01	0,814	3,483
375	1974/11/11	1,832	3,313

NOTE The effective depths of measurements 366 to 368 were modified to represent deposition in the stream channel and the effective depths of measurements 372 to 375 were modified to represent channel scour.



Key

X discharge, Q, in cubic metres per second

Y effective depth, $(h - e)$, in metres

NOTE 1 The upper shift curve (measurements 366 to 368) represents deposition in the channel section that controls the lower segment of the relationship (measurements 367 to 368).

NOTE 2 The lower shift curve (measurements 372 to 375) represents scour of the channel section that controls the lower segment of the relationship.

Figure 4 — Logarithmic plot of a stage-discharge relationship and shift curves representing changes in the stage-discharge relationship

For streams that shift continuously, it is usually necessary to define shift curves on the basis of discharge measurements, determinations of the gauge height of zero flow, and hydraulic characteristics of the rating curve, and then continuously adjust the shift curve between itself and another shift curve (or the base rating curve) on the basis of time. The shift curve adjustment can be uniform or proportional with respect to time or, if specific changes can be defined, the shift curve can be changed abruptly to correspond to the control change. For example, a deposit of debris on a section control can quickly wash away during a small rise in water level, thus causing a shift back to the original rating curve or to another position. This can sometimes be detected by examination of the gauge height record, where abrupt changes can signify abrupt changes to the control. Where no obvious reason can be determined for a shift curve change, it is usually assumed that the change is gradual over time, and the shift curve is adjusted with time. This is typically the case for shifts caused seasonally by the growth of aquatic vegetation.

Shifting-control procedures are complex and frequently difficult to interpret. Quite often, there is more than one logical explanation or interpretation. Experience with a given stream is important in defining the shift characteristics and in making a logical analysis.

5.8 Variable-backwater effects

5.8.1 General

Several conditions can occur in the downstream reaches of a stream to cause apparent changes to the stage-discharge relationship. Previous subclauses have detailed shifts of the control. This subclause addresses conditions of variable backwater which can cause submergence, or partial submergence, of a control and result in stage-discharge relationships that require more complex analysis.

Variable backwater can result from downstream influences such as reservoirs or tributary streams, from ice, from vegetation growth or from dynamic conditions known as hysteresis.

5.8.2 Downstream backwater influences

Downstream conditions can occasionally exist such that water levels downstream from a channel-control reach or a section control can rise sufficiently to partially submerge the control. When this happens, the control will no longer be fully effective in defining the stage-discharge relationship. A downstream reservoir can cause this if it fills enough to submerge the control. Likewise, a tributary stream that enters below or within the control reach might cause variable-backwater effects when the tributary is flowing sufficiently to submerge the control. Beaver dams or other obstructions in the channel downstream from the control can cause some degree of submergence and thereby invalidate the stage-discharge relationship.

For some conditions of downstream backwater, particularly if it is of short duration and occurs very infrequently, it is practical to analyse the discharge record using shifting-control methods. Sometimes, the extent and magnitude of the backwater can be determined by examining a graphical plot of the stage record and estimating the non-backwater stage during the period of backwater.

For variable-backwater conditions that are significant and persist for long periods of time, other measures are required to analyse the discharge record. The most common approach is to use a stage gauge and an auxiliary gauge of index velocity. The index velocity is usually determined by acoustic velocity meters that employ Doppler principles to measure velocity in one vertical or horizontal segment of the channel. A combination of index velocity and stage is then used to define the discharge relationship. This method is described in detail in ISO 15769.

Other approaches include the stage-fall method, the ultrasonic method, the electromagnetic full-channel-width coil method, and one-dimensional unsteady-flow models. These methods are described in ISO 9123, ISO 6416, ISO 9213 and ISO/TR 11627.

5.8.3 Hysteresis effects, or loop rating curves

The stage-discharge relationship for a gauging station gives the value of the normal discharge, i.e. the steady-flow discharge, for a given stage. The discharge for a particular stage can, for some rivers and streams, be greater than the normal discharge during rising stages and less than normal during falling stages because of differences in the water surface slope. This effect is known as hysteresis, or a loop rating curve. It is most pronounced for mildly sloped rivers where dynamic flow conditions are imposed by a passing flood wave.

For gauging sites where the hysteresis effect is severe, instantaneous values of the discharge determined from the steady-state rating curve can be significantly different from the true discharge. For these sites, it might be necessary to use auxiliary equipment to supplement the gauge height record in order to determine discharges accurately. A twin-gauge approach utilizing the stage-fall-discharge relationship can be used (see ISO 9123). Alternatively, a twin-gauge approach using an unsteady-flow model could be used (see ISO/TR 11627). In other situations, it might be feasible to use a velocity index relationship (see ISO 15769).

If the hysteresis effect is not severe, but of sufficient magnitude to need correction, it might be possible to use a single-gauge record of the stage in conjunction with the rate of change in the stage to compute the discharge. For certain conditions, it is possible to compute the true discharge, Q, of an unsteady flow from the steady-state discharge, Q_o, by using the following equation:

$$Q = Q_o \left(1 + \frac{1}{S_o V_w} \cdot \frac{dh}{dt}\right)^{0,5} \quad (5)$$

where

S_o is the water surface slope corresponding to steady, non-uniform flow;

V_w is the velocity of the flood wave;

$\frac{dh}{dt}$ is the rate of change of the stage with time.

The slope, S_o, can be determined from observation of gauges during conditions of steady flow. Alternatively, it can be computed approximately from Manning's or Chezy's equation.

The rate of change of the stage, dh/dt, can be obtained from the recorded observations of the stage at the gauge.

The wave velocity, V_w, is given by the equation:

$$V_w = \frac{dQ}{dA} = \frac{1}{B} \cdot \frac{dQ}{dh} \quad (6)$$

where

A is the cross-sectional area;

B is the surface width at the cross-section;

$\frac{dQ}{dh}$ can be approximated from the stage-discharge relationship.

The above conditions are valid when the rise and fall of the stream is gradual, i.e. when the rate of change in velocity (the acceleration head) can be neglected. Likewise, the velocity should not be high, so that the velocity head can safely be neglected. When a sufficient number of discharge measurements are available, it is possible to calibrate a gauging site with a family of curves by evaluating the term $1/(S_o V_w)$ as a single parameter.

5.9 Extrapolation of the stage-discharge relationship

A stage-discharge relationship should not be applied outside the range of discharge measurements upon which it is based. If, however, estimates of flow are required outside the range, it might be necessary to make an extrapolation of the rating curve. Such extrapolations, either above the highest measurement or below the lowest measurement, should be made with care and through the use of methods that help to define the shape and position of the extrapolated part of the curve. Before making any kind of rating-curve extrapolation, the channel and control should be carefully examined for some distance downstream and upstream of the gauge. Flow obstructions, contractions, expansions, debris, channel shape changes and other conditions should be noted. If abnormal channel conditions exist that cannot be accounted for in the rating-curve analysis, then a rating-curve extrapolation should not be made.

The simplest method of extrapolating a rating curve is by logarithmic plotting. To use this method correctly, the analyst should have a good understanding of the control conditions and of logarithmic plotting methods. Otherwise, this method could lead to large errors. The part of the rating curve requiring extrapolation should be plotted with the effective gauge height scale chosen so that the rating curve plots as a straight line. In addition, the analyst shall have knowledge of the type of control (section or channel) and of the shape of the control through the range of extrapolation. If the control shape does not change significantly and the channel roughness remains fairly constant, then a straight-line extrapolation of the logarithmic plot is reasonable. A logarithmic extrapolation of this type is particularly suited to channel-control conditions for medium and high flows, but should probably never be used to extrapolate more than about 1,5 times the highest measured discharge. Special care is needed when extrapolating below the lowest discharge measurement. For very low flows, when section control exists, it is very important to know the shape of the control and the gauge height of zero flow. Sometimes, it is best to plot the rating curve on arithmetic plotting paper so that the gauge height of zero flow can be plotted, especially if the rating curve is extrapolated to zero discharge.

The shape of the stage-discharge relationship can sometimes be defined through the use of the weir equation [Equation (1)], the Manning equation [Equation (2)] or the Chezy equation [Equation (3)]. These equations can be applied above the highest or below the lowest discharge measurements by using cross-section surveys, high-water-mark surveys and estimates of roughness, discharge coefficients or friction slope. Care should be taken if the shape of the cross-section changes appreciably, because friction slope can also change significantly. Where overbank flow occurs, the friction slope for flows above bankfull can differ greatly from that when the flow is within banks.

Estimates of high discharges can sometimes be made that will aid in the extrapolation of the high end of a stage-discharge relationship. The slope-area method is one such technique (see ISO 1070). Another method can be used when another gauging site exists on the same stream, either upstream or downstream. By careful accounting of additions, withdrawals and channel storage, the peak discharge can be estimated for the site where an extrapolation is required.

The velocity-area method is also sometimes used to extrapolate a rating curve. This method requires the definition of a stage-area relationship from a survey of the cross-section at the gauge as well as the definition of a stage-velocity relationship estimated in the range of the stage where the extrapolation is needed. The stage-velocity relationship can be defined accurately in the range where discharge measurements are available, but is usually difficult to estimate accurately in the range above the highest measurements. For this reason, the velocity-area method is not considered as good as the method described above using the Manning or Chezy equation.

Another approach is to use one-dimensional flow models to estimate the discharge at stages higher than existing measurements (see ISO/TR 11627). Many of the parameters for the models can be estimated from measurements made at lower stages; however, the models have the same limitations as hydraulic equations when the cross-sectional shape changes appreciably.

It is recommended that, whenever possible, extrapolations be made using two or more methods. Results can then be compared and the extrapolated part of the rating curve can be defined with added confidence.

6 Methods of testing stage-discharge relationships

A stage-discharge relationship should be checked six or more times per year by making check discharge measurements. The number of measurements and the period of time between measurements will vary depending on several factors, including the relative stability of the rating curve, hydrological events such as floods that might affect the rating curve, and other indications that the rating curve might have changed. During certain periods, such as floods or extreme drought, it is desirable to obtain additional check measurements to reduce the need for rating-curve extrapolations and to define the effects of backwater or hysteresis if they are present. Also, when a discharge measurement deviates significantly from the rating curve or from previous discharge measurements, another check measurement should be made immediately to confirm or refute the accuracy of the first discharge measurement.

Generally, when a check discharge measurement plots within a small percentage of the rating curve, it is assumed that the rating curve still applies, and no correction is made in the form of either a shift or a new rating curve. The percentage by which a measurement may deviate from the rating curve without applying a correction is usually based on the uncertainty of the discharge measurement. See ISO 748 for a description of computing discharge measurement uncertainty. If, for instance, most discharge measurements are made to 5 % uncertainty, then shifting-control techniques will not be employed unless a check measurement plots further than 5 % from the rating curve.

Another approach is to undertake a statistical analysis of the rating curve to define the dispersion (standard deviation) of the measurements around the rating curve. When two or more measurements indicate a deviation of more than two standard deviations from the rating curve, then a shift curve or a new rating curve is defined. Standard deviations are usually defined separately for each segment of a rating curve.

A bias check is also performed in some cases to define periods when the rating curve might have shifted, even though check measurements are within the specified uncertainty of discharge measurement or within two standard deviations for the rating curve. For instance, two or more measurements might plot within 5 % of the rating curve, but are all on the same side of the rating curve. Various statistical tests can be used to test for bias.

When testing and checking stage-discharge relationships, it is very important that the analyst understands why the measurements plot as they do. Without this understanding, the analyst might incorrectly apply and interpret certain statistical tests. The analyst should always consider what has been happening to the controlling stream characteristics and make decisions based on hydraulics rather than arbitrarily using statistical results.

7 Uncertainty in the stage-discharge relationship

7.1 General

This clause gives the theory and statistical equations for estimating the uncertainties in the stage-discharge relationship and in the daily mean discharge. Numerical examples of estimating the uncertainty of the stage-discharge relationship and the daily mean discharge, using the procedures in this clause, are given in Annex A.

The uncertainty in a single measurement of discharge shall be evaluated in accordance with ISO 748, the uncertainty calculations in which are based on ISO 5168. The uncertainty in the stage-discharge relationship and in continuous measurements of discharge using a stage-discharge relationship shall be evaluated in accordance with this clause, which is based on the general principles described in ISO/TR 7066-1.

7.2 Definition of uncertainty

ISO/TS 25377 sets forth the concepts, terminology and methods to be used in discussing and computing the uncertainty of hydrometric measurements. The terminology specified by ISO/TS 25377 differs slightly from that used in previous editions of this part of ISO 1100. Uncertainty is defined as a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand. The uncertainty parameter can be a standard deviation or a specified multiple of the standard deviation. The standard uncertainty is defined as "uncertainty expressed as a standard deviation". Expanded uncertainty is defined as a quantity defining an interval about the result of a measurement that can be expected to encompass a large fraction of the distribution of the values that could reasonably be attributed to the measurand. The expanded uncertainty is computed by multiplying the standard uncertainty by a coverage factor, k, typically in the range 2 to 3. The fraction of the distribution expected to be encompassed by the expanded-uncertainty interval is called the level of confidence. It should be noted that, if the distribution is assumed to be approximately normal (Gaussian), then coverage factors, k, of 1, 2, and 3 correspond to levels of confidence of about 68 %, 95 % and 99,8 %, respectively.

The expanded uncertainty and level of confidence are not to be confused with the statistical quantities "confidence interval" and "confidence level". These quantities are computed using statistical procedures and assumptions that do not always apply to the uncertainty of flow measurement. In particular, the term "confidence level" should not be used, but rather the term "level of confidence".

Previous editions of this part of ISO 1100 used the term "uncertainty" to refer to expanded relative uncertainty with a coverage factor of 2 and used the terms "standard error" and "standard deviation" to refer to standard uncertainty.

7.3 Statistical analysis of the stage-discharge relationship

7.3.1 General

The uncertainty analysis consists of comparing concurrent measurements of discharge and gauge height ("gaugings") with discharge values read from the stage-discharge relationship at the corresponding gauge heights. The uncertainty will be determined by statistical analysis of the scatter of the measurements around the rating curve. The stage-discharge relationship, being a line of best fit, should be more accurate than any of the individual gaugings. The equation of the relationship can be computed as detailed in 5.2.5.3 which assumes that the relationship plots as a straight line on logarithmic paper. If necessary, the rating curve should be divided into segments that are adequately approximated by straight lines on logarithmic paper.

It is recommended that several current meters be used to establish the stage-discharge relationship to avoid systematic bias in the relationship.

NOTE The uncertainties defined below are very similar in concept to statistical performance measures (standard errors) derived in the statistical theory of regression analysis. Statistical regression theory, however, is based on curve-fitting by the mathematical least-squares method, whereas stage-discharge rating-curve relationships are commonly developed using hydraulic reasoning in addition to mathematical fitting. Thus the term "uncertainty" is used in preference to the more restrictively defined statistical standard error.

7.3.2 Standard error of estimate

The uncertainty of the rating-curve relationship as a whole is characterized by the standard error of estimate, S (also called standard error or standard deviation of residuals), which is calculated from the dispersion of the stage-discharge data around the rating curve. The uncertainty of the discharge value computed from the rating curve for any particular value of the stage, $u(Q_\text{c}(h))$, is then calculated. If there are multiple straight-line segments in the stage-discharge relationship, this procedure is repeated for each segment.

The standard error of estimate, S, is calculated from the following equation:

$$S = \left[\frac{\sum \left(\ln Q - \ln Q_\text{c} \right)^2}{\left(N - p \right)} \right]^{0,5} \tag{7}$$

where

Q is the measured discharge;

Q_c is the corresponding discharge calculated from the rating-curve equation or rating-curve table;

N is the number of gaugings in the rating-curve segment;

p is the number of rating-curve parameters estimated from the N gaugings.

NOTE 1 The value of p depends on how many parameter values are adjusted to make the rating curve fit the gauging data. If all three parameters (Q_1, β, e) are adjusted to fit the N gaugings, then $p = 3$. If the effective gauge height of zero flow, e, is given *a priori* and another two parameters are adjusted to fit the data, then $p = 2$. If, in addition, the rating-curve slope, β, is determined *a priori*, from hydraulic considerations for example, and only the intercept Q_1 is estimated, then $p = 1$.

NOTE 2 The quantity $N - p$ is called the number of degrees of freedom. It represents the number of observations that are effective in defining the scatter of observations around the rating-curve relationship, p of the observations having been "used up", in effect, to establish the position of the rating curve.

7.3.3 Standard uncertainty

The standard uncertainty in the calculated value of $\ln Q_c$ at gauge height h, $u(\ln Q_c(h))$, is found from the following equation (see ISO/TR 7066-1):

$$u(\ln Q_c(h)) = S\left\{\frac{1}{N} + \frac{\left[\ln(h-e) - \overline{\ln(h-e)}\right]^2}{\sum\left[\ln(h-e) - \overline{\ln(h-e)}\right]^2}\right\}^{0,5} \tag{8}$$

An expanded uncertainty, $U(\ln Q_c(h))$, can be calculated from:

$$U(\ln Q_c(h)) = k \cdot u(\ln Q_c(h)) \tag{9}$$

where k is a coverage factor that provides a specified level of confidence (see Note 1). The expanded uncertainty defines an uncertainty interval around the computed value $\ln Q_c(h)$ which is expected to encompass the specified fraction of the distribution of values that could reasonably be attributed to the discharge; this interval can be expressed as:

$$\ln Q_c(h) \pm U(\ln Q_c(h)) \tag{10}$$

The uncertainties and the limits of the uncertainty interval are expressed in natural-logarithmic units. The corresponding uncertainty interval for discharges is found by taking anti-logarithms:

$$Q_c(h)\mathrm{e}^{\pm U(\ln Q_c(h))} \approx Q_c(h)[1 \pm U(\ln Q_c(h))] \tag{11}$$

where the approximate equality holds when $U(\ln Q_c(h))$ is small enough so that the linear approximation to the exponential holds.

NOTE 1 For the expanded uncertainty, the coverage factor, k, can be taken as Student's t-correction at the desired level of confidence for $N - p$ gaugings and can be taken as 2 for a level of confidence of 95 % and 20 or more gaugings. A coverage factor, k, of 1 corresponds to a level of confidence of about 68 %.

NOTE 2 The uncertainty interval limits are symmetrical in logarithmic units but not, in general, in discharge units. If the natural-logarithmic uncertainties, $U(\ln Q_c(h))$, are small, they are approximately equal to relative uncertainties in discharge units.

The expanded uncertainty, $U(\ln Q_c(h))$, with coverage factor, k, equal to 2, and the corresponding uncertainty limits on $\ln Q_c(h)$ should be calculated for each observation of $(h - e)$ related to the corresponding gauging. The limits will therefore take the form of curved lines on each side of the stage-discharge relationship and will exhibit a minimum at the mean value of $\ln(h - e)$.

NOTE 3 The uncertainty of the calculated value of $\ln Q_c$ refers to the dispersion of the set of reasonably possible positions of the rating curve based on the number of gaugings available and their accuracy. Because the gaugings are dispersed around the rating curve, with standard error of estimate S, it is expected that future discharges will be similarly dispersed around the rating curve. This dispersion would therefore have to be considered in evaluating the accuracy of predictions of future discharges using the rating curve (see 7.4).

If the stage-discharge relationship comprises two or more straight-line segments, S and $U(\ln Q_c(h))$ should be individually calculated for each segment, and the appropriate number of degrees of freedom, $(N - p)$, should be used for each segment.

It is desirable that at least 20 observations be available in each segment in order to achieve a statistically reliable estimate of S and $u(\ln Q_c(h))$.

7.4 Uncertainty of predicted discharge

The rating curve is used to compute values of discharge corresponding to values of gauge height recorded at a gauging station. The resulting computed discharge is normally considered to be a prediction of the discharge that would be observed if a gauging were made. The magnitude of the difference between the predicted value and the value that would be observed if a gauging were made is important in the practical use and interpretation of the predicted values. The dispersion of the values that could reasonably be attributed to this difference is called the uncertainty of prediction, denoted by $u(Q_p)$ (standard uncertainty) or by $U(Q_p)$ (expanded uncertainty) or by corresponding terms involving logarithms of discharge. The actual values of the predicted discharges, Q_p, are the same as the values, Q_c, computed from the rating curve at the recorded gauge height, but their interpretation is different because they are being compared with the distribution of reasonably possible individual observed discharges rather than with the distribution of reasonably possible positions of the rating curve.

There are three reasons why the predicted discharge might differ from the discharge that would be observed if a gauging were made: change in control conditions (shift in rating curve), measurement error in gauged discharge and error in recorded gauge height. The first two error sources affect the gaugings used to define the rating-curve relationship; their combined magnitude is represented by the standard error of estimate of the rating curve, S. The third error source is not reflected in the gaugings used to define the rating curve because the gauging personnel read auxiliary reference gauges in addition to the recorder to ensure that the correct gauge height is observed. Assuming that the rating curve is virtually linear over the range of reasonably possible values associated with the recorded gauge height, and using the logarithmic form of the rating-curve equation, the standard uncertainty in predicted discharge due to uncertainty in recorded gauge height is given by the following equation:

$$u(\ln Q_p(h)) = \beta \cdot u(\ln(h-e)) \quad (12)$$

where $u(\ln(h - e))$ is the uncertainty in the effective depth.

The standard uncertainty of prediction is computed by combining this uncertainty with the standard error of estimate, S, and the uncertainty in the calculated discharge, $U(\ln Q_c(h))$, in root-sum-squares (RSSs), as follows:

$$u(\ln Q_p(h)) = \sqrt{\beta^2 u(\ln(h-e))^2 + S^2 + u(\ln Q_c(h))^2} \quad (13)$$

Expanded prediction uncertainties, prediction uncertainty limits and percentage (relative) prediction uncertainties are computed in the same way as in 7.3.3. In particular, if the standard uncertainties of the natural-logarithmic quantities are small, they are equivalent to relative (percentage) uncertainties of the corresponding stages and discharges.

NOTE At some gauging stations, most likely ones with rock-ledge controls or well-maintained artificial controls, it is possible to show that the standard error of estimate represents only the measurement uncertainty of the gaugings used to establish the rating curve. In such cases, the predictions would not be affected by measurement error, and the standard error of estimate, S^2, could be omitted from the calculation of the prediction uncertainty.

7.5 Uncertainty in the daily mean discharge

The value of discharge most commonly required for design and planning purposes is the daily mean discharge. The daily mean discharge may be calculated by taking the average of the number of observations of the discharge during the 24-h period. For example, the 24 instantaneous discharges observed at 30 min after the hour throughout the day could be averaged; this corresponds to numerical integration of the discharge hydrograph by a rectangular rule.

The relative (percentage) standard uncertainty in the daily mean discharge is calculated as the discharge-weighted mean of the relative prediction uncertainties of the individual calculated discharges, using the following equation:

$$u_{\mathrm{r}}(Q_{\mathrm{dm}}) = \frac{\sum u_{\mathrm{r}}(Q_{\mathrm{p}}(h))Q_{\mathrm{c}}}{\sum Q_{\mathrm{c}}} \qquad (14)$$

where

$u_{\mathrm{r}}(Q_{\mathrm{dm}})$ is the relative (percentage) standard uncertainty in the daily mean discharge;

$u_{\mathrm{r}}(Q_{\mathrm{p}}(h))$ is the percentage standard prediction uncertainty of the discharge predicted from the rating curve, and is equal to $u(\ln Q_{\mathrm{p}}(h))$ as calculated in 7.4;

Q_{c} is the calculated value of the discharge from the rating-curve table or rating-curve equation used to calculate the daily mean discharge.

The corresponding equation for a measuring structure is the same as Equation (14)

where

$u_{\mathrm{r}}(Q_{\mathrm{p}}(h))$ is the percentage standard prediction uncertainty of the discharge predicted from the structure rating curve and is equal to:

$$\sqrt{\beta^2 u_{\mathrm{r}}(h-e)^2 + u_{\mathrm{r}}(C_{\mathrm{D}})^2}$$

$u_{\mathrm{r}}(C_{\mathrm{D}})$ being the percentage uncertainty in the coefficient of discharge for the structure.

NOTE 1 The percentage uncertainty, $u_{\mathrm{r}}(b)$, in the length of the crest (the width of the throat) has been neglected.

NOTE 2 The value of e in this case is usually zero.

Annex A
(informative)

Uncertainty in the stage-discharge relationship and in a continuous measurement of discharge

A.1 General

The uncertainty in a single determination of the discharge should be evaluated in accordance with ISO 5168. This annex deals with the uncertainty in the stage-discharge relationship as defined in ISO/TR 7066-1 and in a continuous measurement of discharge.

A.2 Example of uncertainty calculations for individual gaugings

The standard error of estimate, S, of the rating curve can be computed from Equation (7) in 7.3.2. Substituting into this equation from Table A.1, and assuming that the gauge height of zero flow, e, is given *a priori*, so that $p = 2$, the standard error of estimate, S, is calculated as follows:

$$S = [(0{,}029\ 99/(32 - 2)]^{0,5} = 0{,}031\ 62$$

This is a measure of the overall uncertainty of the rating-curve segment. An expanded uncertainty can be found by multiplying S by a coverage factor, k, to define approximate uncertainty limits enclosing a specified percentage of the points dispersed around the fitted line. When the points follow a Gaussian distribution, a coverage factor, k, equal to 2 encloses about 95 % of the points (level of confidence about 95 %); $k = 1$ gives a level of confidence of about 68 %. In this example, the expanded uncertainty for $k = 2$ is given by $U = 2S = 0{,}063\ 2$ and is measured in natural-log (ln) units. The corresponding uncertainty limits are $\ln Q_c \pm U$. Taking anti-logs yields $Q_c \exp(1 \pm U)$, which is approximately equal to $Q_c \pm UQ_c$. Thus U in natural-log units can be interpreted as a relative uncertainty, relative to the rating-curve value Q_c. U and S can thus be expressed as percentages of the rating-curve value Q_c:

$S = 3{,}16\ \%$ (68 % level of confidence) $U = 6{,}32\ \%$ (95 % level of confidence)

In logarithmic coordinates, $\ln Q_c \pm U$ defines two parallel straight lines on either side of the rating-curve segment and at a distance U ($= 2S$) from it. In other words, 95 % of the observations, on average, will be contained within these limits (6,3 %).

For those instances where the dispersion about the fitted line is not considered small, the standard uncertainty for the calculated value of $\ln Q_c$, $u(\ln Q_c(h))$, for individual values of $\ln Q_c$ at any stage, $\ln(h - e)$, can be calculated from Equation (8) in 7.3.3

Substituting the values for observation No. 1 in Table A.1 and using a coverage factor of 2 gives the following:

$$U(\ln Q_c(h)) = 2 \times 0{,}031\,62 \left(\frac{1}{32} + \frac{1{,}862\ 25}{27{,}924\ 22} \right)^{0,5}$$

$= 0{,}0198$ ln-units $= 1{,}98\ \%$ of Q_c (rounded to 2,0 % in Table A.1).

Table A.1 — Tabulated values required to calculate S

Obs No.	$(h-e)$ (e = 0,115)	Q (measured)	Q_c (from rating curve)	ln$(h-e)$ (= x)	$(x-\bar{x})^2$	ln$Q$ (= y)	lnQ_c (= y_c)	$(y-y_c)^2$	$U_r(Q_c)$ (k = 2) %
1	0,157	2,463	2,323	−1,851 51	1,862 25	0,901 38	0,842 86	0,003 42	2,0
2	0,158	2,325	2,345	−1,845 16	1,844 96	0,843 72	0,852 29	0,000 07	2,0
3	0,188	2,923	3,060	−1,671 31	1,402 92	1,072 61	1,118 41	0,002 10	1,8
4	0,192	3,242	3,160	−1,650 26	1,353 49	1,176 19	1,150 57	0,000 66	1,8
5	0,219	3,841	3,865	−1,518 68	1,064 65	1,345 73	1,351 96	0,000 04	1,7
6	0,259	4,995	4,996	−1,350 93	0,746 60	1,608 44	1,608 64	0,000 00	1,5
7	0,278	5,410	5,568	−1,280 13	0,629 28	1,688 25	1,717 04	0,000 83	1,5
8	0,279	5,422	5,598	−1,276 54	0,623 59	1,690 46	1,722 41	0,001 02	1,5
9	0,287	5,883	5,846	−1,248 27	0,579 74	1,772 07	1,765 76	0,000 04	1,4
10	0,295	6,154	6,097	−1,220 78	0,538 63	1,817 10	1,807 80	0,000 09	1,4
11	0,348	7,376	7,851	−1,055 55	0,323 41	1,998 23	2,060 64	0,003 89	1,3
12	0,405	9,832	9,902	−0,903 87	0,173 89	2,285 64	2,292 74	0,000 05	1,2
13	0,433	11,321	10,968	−0,837 02	0,122 61	2,426 66	2,394 98	0,001 00	1,2
14	0,461	12,372	12,072	−0,774 36	0,082 65	2,515 44	2,490 89	0,000 60	1,2
15	0,465	11,825	12,233	−0,765 72	0,077 76	2,470 22	2,504 14	0,001 15	1,2
16	0,501	13,826	13,711	−0,691 15	0,041 73	2,626 55	2,618 20	0,000 07	1,1
17	0,511	14,102	14,132	−0,671 39	0,034 05	2,646 32	2,648 44	0,000 00	1,1
18	0,606	19,020	18,345	−0,500 88	0,000 20	2,945 49	2,909 36	0,001 31	1,1
19	0,624	19,970	19,185	−0,471 60	0,000 23	2,994 23	2,954 13	0,001 61	1,1
20	0,632	20,280	19,563	−0,458 87	0,000 78	3,009 64	2,973 64	0,001 30	1,1
21	0,681	21,204	21,931	−0,384 19	0,010 54	3,054 19	3,087 90	0,001 14	1,1
22	0,731	23,996	24,442	−0,313 34	0,030 11	3,177 89	3,196 30	0,000 34	1,1
23	0,926	36,242	35,098	−0,076 88	0,168 09	3,590 22	3,558 14	0,001 03	1,2
24	1,225	54,591	53,855	0,202 94	0,475 83	3,999 87	3,986 30	0,000 18	1,4
25	1,411	67,327	66,859	0,344 30	0,690 83	4,209 56	4,202 59	0,000 05	1,5
26	1,646	79,050	84,631	0,498 35	0,970 65	4,370 08	4,438 30	0,004 65	1,6
27	1,895	110,783	104,989	0,639 22	1,268 07	4,707 57	4,653 86	0,002 89	1,8
28	2,517	162,814	162,095	0,923 07	1,987 91	5,092 61	5,088 18	0,000 02	2,0
29	3,150	227,600	228,478	1,147 40	2,670 83	5,427 59	5,431 44	0,000 01	2,3
30	3,165	228,800	230,145	1,152 15	2,686 38	5,432 85	5,438 71	0,000 03	2,3
31	3,191	228,500	233,044	1,160 33	2,713 27	5,431 54	5,451 23	0,000 39	2,3
32	3,225	236,600	236,854	1,170 93	2,748 30	5,466 37	5,467 44	0,000 00	2,3
			Sum =	−15,579 7	27,924 22			0,029 99	
			Mean =	−0,486 87			S =	0,031 62	

Similar computations have been made for each observation in Table A.1, and the percent uncertainty is shown in the last column. These uncertainties can be plotted on either side of the stage-discharge curve for each observed value of $(h-e)$. Such a plot on logarithmic paper will show the symmetrical limits of the 95 % uncertainty. The minimum width of these limits will occur at the mean value of ln$(h-e)$.

A.3 Example of calculating the uncertainty of the daily mean discharge

The percentage standard uncertainty for the daily mean discharge, $u_r(Q_{dm})$, can be calculated from Equation (14) in 7.5. An expanded uncertainty, $U_r(Q_{dm})$, at a level of confidence of about 95 % can be found by multiplying the standard uncertainty by a coverage factor, k, equal to 2, as follows:

$$U_r(Q_{dm}) = 2 \cdot \frac{\sum u_r(Q_p(h))Q_c}{\sum Q_c} = \frac{\sum \sqrt{\left[\beta \cdot 2u_r(h-e)\right]^2 + \left[2u_r(Q_c(h))\right]^2} \cdot Q_c}{\sum Q_c}$$

$$= 8\,049{,}57/3\,888{,}55$$

$$= 0{,}020\,7$$

Therefore, the daily mean discharge = 161,815 m^3/s ± 2,1 % (expanded uncertainty, $k = 2$, level of confidence approximately 95 %).

Table A.2 illustrates this calculation, using hourly values of discharge.

NOTE The rating-curve exponent, β, is 1,530 1. It is assumed that the control is very stable so that the standard error of estimate consists only of measurement uncertainty; thus the standard error of estimate does not need to be included in the prediction uncertainty.

Table A.2 — Typical computation of the uncertainty in the daily mean discharge, using hourly values of the discharge

Time	$(h-e)$	Q_c	$2u_r(h-e)$ %	$2u_r(Q_c)$ %	$\sqrt{[\beta \cdot 2u_r(h-e)]^2 + [2u_r(Q_c(h))]^2} \cdot Q_c$
01:00	1,110	46,314	0,4	1,3	66,99
02:00	1,450	69,707	0,3	1,5	110,34
03:00	1,856	101,699	0,2	1,7	179,69
04:00	2,178	129,906	0,2	1,9	247,55
05:00	2,405	151,186	0,2	2,0	301,95
06:00	2,565	165,850	0,2	2,0	341,41
07:00	2,674	177,814	0,2	2,1	373,22
08:00	2,767	186,328	0,2	2,1	397,33
09:00	2,814	192,255	0,2	2,1	413,16
10:00	2,876	197,717	0,1	2,2	429,18
11:00	2,929	203,339	0,1	2,2	445,08
12:00	2,952	206,867	0,1	2,2	454,42
13:00	2,967	208,478	0,1	2,2	459,02
14:00	2,950	206,653	0,1	2,2	453,81
15:00	2,911	202,487	0,1	2,2	441,97
16:00	2,860	197,084	0,1	2,2	426,71
17:00	2,800	190,793	0,2	2,1	409,08
18:00	2,730	183,543	0,2	2,1	388,96
19:00	2,632	173,558	0,2	2,1	361,61
20:00	2,513	161,697	0,2	2,0	329,69
21:00	2,380	148,788	0,2	2,0	295,70
22:00	2,250	136,534	0,2	1,9	264,23
23:00	2,142	126,635	0,2	1,9	239,42
24:00	2,049	118,320	0,2	1,8	219,04
	Sum =	3 883,552			
	Mean =	161,815			

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