INTERNATIONAL STANDARD

ISO 31-11

Second edition
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Quantities and units —

Part 11:
Mathematical signs and symbols for use in the physical sciences and technology

Grandeurs et unités —

Partie 11: Signes et symboles mathématiques à employer dans les sciences physiques et dans la technique

Reference number
ISO 31-11:1992(E)

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 31-11 was prepared by Technical Committee ISO/TC 12, *Quantities, units, symbols, conversion factors*.

This second edition cancels and replaces the first edition (ISO 31-11:1978). The major technical changes from the first edition are the following:

— a new clause on coordinate systems has been added;

— some new items have been added in the old clauses.

The scope of Technical Committee ISO/TC 12 is standardization of units and symbols for quantities and units (and mathematical symbols) used within the different fields of science and technology, giving, where necessary, definitions of the quantities and units. Standard conversion factors for converting between the various units also come under the scope of the TC. In fulfilment of this responsibility, ISO/TC 12 has prepared ISO 31.

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ISO 31 consists of the following parts, under the general title *Quantities and units*:

- *Part 0: General principles*
- *Part 1: Space and time*
- *Part 2: Periodic and related phenomena*
- *Part 3: Mechanics*
- *Part 4: Heat*
- *Part 5: Electricity and magnetism*
- *Part 6: Light and related electromagnetic radiations*
- *Part 7: Acoustics*
- *Part 8: Physical chemistry and molecular physics*
- *Part 9: Atomic and nuclear physics*
- *Part 10: Nuclear reactions and ionizing radiations*
- *Part 11: Mathematical signs and symbols for use in the physical sciences and technology*
- *Part 12: Characteristic numbers*
- *Part 13: Solid state physics*

Introduction

0.1 General

If more than one sign, symbol or expression is given for the same item, they are on an equal footing. Signs, symbols and expressions in the "Remarks" column are given for information.

Where the numbering of an item has been changed in the revision of a part of ISO 31, the number in the preceding edition is shown in parentheses below the new number for the item; a dash is used to indicate that the item in question did not appear in the preceding edition.

0.2 Variables, functions and operators

Variables, such as x, y, etc., and running numbers, such as i in $\Sigma_i x_i$, are printed in italic (sloping) type. Also parameters, such as a, b, etc., which may be considered as constant in a particular context, are printed in italic (sloping) type. The same applies to functions in general, e.g. f, g.

An explicitly defined function is, however, printed in Roman (upright) type, e.g. sin, exp, ln, Γ. Mathematical constants, the values of which never change, are printed in Roman (upright) type, e.g. $\mathrm{e} = 2{,}718\,281\,8...$; $\pi = 3{,}141\,592\,6...$; $\mathrm{i}^2 = -1$. Well defined operators are also printed in upright style, e.g. div, δ in δx and each d in $\mathrm{d}f/\mathrm{d}x$.

Numbers expressed in the form of digits are always printed upright, e.g. 351 204; 1,32; 7/8.

The argument of a function is written in parentheses after the symbol for the function, without a space between the symbol for the function and the first parenthesis, e.g. $f(x)$, $\cos(\omega t + \varphi)$. If the symbol for the function consists of two or more letters and the argument contains no operation sign, such as +; −; ×; ·; or /, the parentheses around the argument may be omitted. In these cases, there should be a thin space between the symbol for the function and the argument, e.g. ent 2,4; $\sin n\pi$; arcosh $2A$; Ei x.

If there is any risk of confusion, parentheses should always be inserted. For example, write $\cos(x) + y$ or $(\cos x) + y$; do not write $\cos x + y$, which could be mistaken for $\cos(x + y)$.

If an expression or equation must be split into two or more lines, the line-breaks should preferably be immediately after one of the signs =; +; −; ±; or ∓; or, if necessary, immediately after one of the signs ×; ·; or /. In this case, the sign works like a hyphen at the end of the first line, informing the reader that the rest will follow on the next line or even on the next page. The sign should not be repeated at the beginning of the following line; two minus signs could for example give rise to sign errors.

0.3 Scalars, vectors and tensors

Scalars, vectors and tensors are used to denote certain physical quantities. They are as such independent of the particular choice of coordinate system, whereas each component of a vector or a tensor depends on that choice.

It is important to distinguish between the "components of a vector" $\boldsymbol{a}$, i.e. a_x, a_y and a_z, and the "component vectors", i.e. $a_x\boldsymbol{e}_x$, $a_y\boldsymbol{e}_y$ and $a_z\boldsymbol{e}_z$.

The cartesian components of the position vector are equal to the cartesian coordinates of the point given by the position vector.

Instead of treating each component as a physical quantity (i.e. numerical value × unit), the vector could be written as a numerical-value vector multiplied by the unit. All units are scalars.

EXAMPLE

component F_x — numerical-value vector

$$\boldsymbol{F} = (3\ \mathrm{N}, -2\ \mathrm{N}, 5\ \mathrm{N}) = (3, -2, 5)\ \mathrm{N}$$

numerical value — unit — unit

The same considerations apply to tensors of second and higher orders.

Quantities and units —

Part 11:
Mathematical signs and symbols for use in the physical sciences and technology

1 Scope

This part of ISO 31 gives general information about mathematical signs and symbols, their meanings, verbal equivalents and applications.

The recommendations in this part of ISO 31 are intended mainly for use in the physical sciences and technology.

2 Normative reference

The following standard contains provisions which, through reference in this text, constitute provisions of this part of ISO 31. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this part of ISO 31 are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 31-0:1992, *Quantities and units — Part 0: General principles.*

3 MATHEMATICAL LOGIC

Item No.	Symbol, sign	Application	Name of symbol	Meaning, verbal equivalent and remarks
11-3.1 (*11-2.1*)	$\wedge$	$p \wedge q$	conjunction sign	p and q
11-3.2 (*11-2.2*)	$\vee$	$p \vee q$	disjunction sign	p or q (or both)
11-3.3 (*11-2.3*)	$\neg$	$\neg p$	negation sign	negation of p; not p; non p
11-3.4 (*11-2.4*)	$\Rightarrow$	$p \Rightarrow q$	implication sign	if p then q; p implies q Can also be written $q \Leftarrow p$. Sometimes $\rightarrow$ is used.
11-3.5 (*11-2.5*)	$\Leftrightarrow$	$p \Leftrightarrow q$	equivalence sign	$p \Rightarrow q$ and $q \Rightarrow p$; p is equivalent to q Sometimes $\leftrightarrow$ is used.
11-3.6 (*11-2-6*)	$\forall$	$\forall x \in A \quad p(x)$ $(\forall x \in A) \quad p(x)$	universal quantifier	for every x belonging to A, the proposition $p(x)$ is true If it is clear from the context which set A is being considered, the notation $\forall x\, p(x)$ can be used. For $x \in A$, see 11-4.1.
11-3.7 (*11-2.7*)	$\exists$	$\exists x \in A \quad p(x)$ $(\exists x \in A) \quad p(x)$	existential quantifier	there exists an x belonging to A for which $p(x)$ is true If it is clear from the context which set A is being considered, the notation $\exists x\, p(x)$ can be used. For $x \in A$, see 11-4.1. $\exists!$ or $\overset{1}{\exists}$ is used to indicate the existence of one and only one element for which $p(x)$ is true.

4 SETS

Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-4.1 *(11-1.1)*	$\in$	$x \in A$	x belongs to A; x is an element of the set A	
11-4.2 *(11-1.2)*	$\notin$	$y \notin A$	y does not belong to A; y is not an element of the set A	The symbol $\not\in$ is also used.
11-4.3 *(11-1.3)*	$\ni$	$A \ni x$	the set A contains x (as element)	$A \ni x$ has the same meaning as $x \in A$.
11-4.4 *(11-1.4)*	$\not\ni$	$A \not\ni y$	the set A does not contain y (as element)	$A \not\ni y$ has the same meaning as $y \notin A$. The symbol $\not\ni$ is also used.
11-4.5 *(11-1.5)*	{ }	$\{x_1, x_2, ..., x_n\}$	set with elements $x_1, x_2, ..., x_n$	Also $\{x_i : i \in I\}$, where I denotes a set of indices.
11-4.6 *(11-1.6)*	{ | }	$\{x \in A \mid p(x)\}$	set of those elements of A for which the proposition $p(x)$ is true	EXAMPLE $\{x \in \mathbb{R} \mid x \leqslant 5\}$ If it is clear from the context which set A is being considered, the notation $\{x \mid p(x)\}$ can be used. EXAMPLE $\{x \mid x \leqslant 5\}$
11-4.7 (—)	card	card (A)	number of elements in A; cardinal of A	
11-4.8 *(11-1.7)*	$\varnothing$		the empty set	
11-4.9 *(11-1.8)*	$\mathbb{N}$ **N**		the set of natural numbers; the set of positive integers and zero	$\mathbb{N} = \{0, 1, 2, 3, ...\}$ Exclusion of zero from the sets 11-4.9 to 11-4.13 is denoted by an asterisk, e.g. $\mathbb{N}^*$. $\mathbb{N}_k = \{0, 1, ..., k-1\}$
11-4.10 *(11-1.9)*	$\mathbb{Z}$ **Z**		the set of integers	$\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$ See remark to 11-4.9.
11-4.11 *(11-1.10)*	$\mathbb{Q}$ **Q**		the set of rational numbers	See remark to 11-4.9.
11-4.12 *(11-1.11)*	$\mathbb{R}$ **R**		the set of real numbers	See remark to 11-4.9.
11-4.13 *(11-1.12)*	$\mathbb{C}$ **C**		the set of complex numbers	See remark to 11-4.9.

4 SETS (*continued*)				
Item No.	**Symbol, sign**	**Application**	**Meaning, verbal equivalent**	**Remarks and examples**
11-4.14 (—)	[,]	$[a, b]$	closed interval in ℝ from a (included) to b (included)	$[a, b] = \{x \in \mathbb{R} \mid a \leqslant x \leqslant b\}$
11-4.15 (—)	] ,] (,]	$]a, b]$ $(a, b]$	left half-open interval in ℝ from a (excluded) to b (included)	$]a, b] = \{x \in \mathbb{R} \mid a < x \leqslant b\}$
11-4.16 (—)	[, [[,)	$[a, b[$ $[a, b)$	right half-open interval in ℝ from a (included) to b (excluded)	$[a, b[= \{x \in \mathbb{R} \mid a \leqslant x < b\}$
11-4.17 (—)	] , [(,)	$]a, b[$ (a, b)	open interval in ℝ from a (excluded) to b (excluded)	$]a, b[= \{x \in \mathbb{R} \mid a < x < b\}$
11-4.18 (*11-1.13*)	⊆	$B \subseteq A$	B is included in A; B is a subset of A	Every element of B belongs to A. ⊂ is also used, but see remark to 11-4.19.
11-4.19 (*11-1.14*)	⊂	$B \subset A$	B is properly included in A; B is a proper subset of A	Every element of B belongs to A, but B is not equal to A. If ⊂ is used for 11-4.18, then ⊊ shall be used for 11-4.19.
11-4.20 (*11-1.15*)	⊈	$C \nsubseteq A$	C is not included in A; C is not a subset of A	⊄ is also used. The symbols ⊈ and ⊄ are also used.
11-4.21 (*11-1.16*)	⊇	$A \supseteq B$	A includes B (as subset)	A contains every element of B. ⊃ is also used, but see remark to 11-4.22. $A \supseteq B$ has the same meaning as $B \subseteq A$.
11-4.22 (*11-1.17*)	⊃	$A \supset B$	A includes B properly	A contains every element of B, but A is not equal to B. If ⊃ is used for 11-4.21, then ⊋ shall be used for 11-4.22. $A \supset B$ has the same meaning as $B \subset A$.
11-4.23 (*11-1.18*)	⊉	$A \nsupseteq C$	A does not include C (as subset)	⊅ is also used. The symbols ⊉ and ⊅ are also used. $A \nsupseteq C$ has the same meaning as $C \nsubseteq A$.
11-4.24 (*11-1.19*)	∪	$A \cup B$	union of A and B	The set of elements which belong to A or to B or to both A and B. $A \cup B = \{x \mid x \in A \vee x \in B\}$

4 SETS (*continued*)

Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-4.25 (*11-1.20*)	$\cup$	$\bigcup_{i=1}^{n} A_i$	union of a collection of sets $A_1, \ldots, A_n$	$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n$, the set of elements belonging to at least one of the sets $A_1, \ldots, A_n$. $\bigcup_{i=1}^{n}$ and $\bigcup\limits_{i \in I}$, $\bigcup_{i \in I}$ are also used, where I denotes a set of indices.
11-4.26 (*11-1.21*)	$\cap$	$A \cap B$	intersection of A and B, read as A inter B	The set of elements which belong to both A and B. $A \cap B = \{x \mid x \in A \wedge x \in B\}$
11-4.27 (*11-1.22*)	$\bigcap$	$\bigcap_{i=1}^{n} A_i$	intersection of a collection of sets $A_1, \ldots, A_n$	$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$, the set of elements belonging to all sets $A_1, A_2, \ldots$ and A_n. $\bigcap_{i=1}^{n}$ and $\bigcap\limits_{i \in I}$, $\bigcap_{i \in I}$ are also used, where I denotes a set of indices.
11-4.28 (*11-1.23*)	$\setminus$	$A \setminus B$	difference between A and B; A minus B	The set of elements which belong to A, but not to B. $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ $A - B$ should not be used.
11-4.29 (*11-1.24*)	$\complement$	$\complement_A B$	complement of subset B of A	The set of those elements of A which do not belong to the subset B. If it is clear from the context which set A is being considered, the symbol A is often omitted. Also $\complement_A B = A \setminus B$
11-4.30 (*11-1.25*)	(,)	(a, b)	ordered pair a, b; couple a, b	$(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. $\langle a, b\rangle$ is also used.
11-4.31 (*11-1.26*)	(, ...,)	$(a_1, a_2, \ldots, a_n)$	ordered n-tuplet	$\langle a_1, a_2, \ldots, a_n\rangle$ is also used.

4 SETS (*concluded*)

Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-4.32 (*11-1.27*)	$\times$	$A \times B$	cartesian product of A and B	The set of ordered pairs (a, b) such that $a \in A$ and $b \in B$. $A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$ $A \times A \times \ldots \times A$ is denoted by A^n, where n is the number of factors in the product.
11-4.33 (—)	Δ	Δ_A	set of pairs (x, x) of $A \times A$, where $x \in A$; diagonal of the set $A \times A$	$\Delta_A = \{(x, x) \mid x \in A\}$ id_A is also used.

5 MISCELLANEOUS SIGNS AND SYMBOLS				
Item No.	**Symbol, sign**	**Application**	**Meaning, verbal equivalent**	**Remarks and examples**
11-5.1 (*11-3.1*)	$=$	$a = b$	a is equal to b	$\equiv$ may be used to emphasize that a particular equality is an identity.
11-5.2 (*11-3.2*)	$\neq$	$a \neq b$	a is not equal to b	The symbol $\nparallel$ is also used.
11-5.3 (*11-3.3*)	$\stackrel{\text{def}}{=}$	$a \stackrel{\text{def}}{=} b$	a is by definition equal to b	EXAMPLE $p \stackrel{\text{def}}{=} mv$, where p is momentum, m is mass and v is velocity. $\stackrel{\text{d}}{=}$ and $:=$ are also used.
11-5.4 (*11-3.4*)	$\triangleq$	$a \triangleq b$	a corresponds to b	EXAMPLES When $E = kT$, 1 eV $\triangleq$ 11 604,5 K. When 1 cm on a map corresponds to a length of 10 km, one may write 1 cm $\triangleq$ 10 km.
11-5.5 (*11-3.5*)	$\approx$	$a \approx b$	a is approximately equal to b	The symbol $\simeq$ is reserved for "is asymptotically equal to". See 11-7.7.
11-5.6 (*11-3.6*)	$\sim$ $\propto$	$a \sim b$ $a \propto b$	a is proportional to b	
11-5.7 (*11-3.7*)	$<$	$a < b$	a is less than b	
11-5.8 (*11-3.8*)	$>$	$b > a$	b is greater than a	
11-5.9 (*11-3.9*)	$\leqslant$	$a \leqslant b$	a is less than or equal to b	The symbols $\leq$ and $\leqq$ are also used.
11-5.10 (*11-3.10*)	$\geqslant$	$b \geqslant a$	b is greater than or equal to a	The symbols $\geq$ and $\geqq$ are also used.
11-5.11 (*11-3.11*)	$\ll$	$a \ll b$	a is much less than b	
11-5.12 (*11-3.12*)	$\gg$	$b \gg a$	b is much greater than a	
11-5.13 (*11-3.13*)	∞		infinity	

5 MISCELLANEOUS SIGNS AND SYMBOLS (*concluded*)

Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-5.14 (—)	( ) [ ] { } ⟨ ⟩	$(a+b)c$ $[a+b]c$ $\{a+b\}c$ $\langle a+b \rangle c$	$ac+bc$, parentheses $ac+bc$, square brackets $ac+bc$, braces $ac+bc$, angle brackets	In ordinary algebra the sequence of ( ), [ ], { } and ⟨ ⟩ in order of nesting is not standardized. Special uses are made of ( ), [ ], { } and ⟨ ⟩ in particular fields.
11-5.15 (—)	//	AB // CD	the line AB is parallel to the line CD	
11-5.16 (—)	⊥	AB ⊥ CD	the line AB is perpendicular to the line CD	

6 OPERATIONS			
Item No.	**Symbol, application**	**Meaning, verbal equivalent**	**Remarks and examples**
11-6.1 (*11-4.1*)	$a + b$	a plus b	
11-6.2 (*11-4.2*)	$a - b$	a minus b	
11-6.3 (—)	$a \pm b$	a plus or minus b	
11-6.4 (—)	$a \mp b$	a minus or plus b	$-(a \pm b) = -a \mp b$
11-6.5 (*11-4.3*)	$a \cdot b$ $a \times b$ ab	a multiplied by b	See also 11-4.32, 11-13.6 and 11-13.7. The sign for multiplication of numbers is a cross (×) or a dot half high (·). If a dot is used as the decimal sign, only the cross shall be used for multiplication of numbers. For decimal sign see ISO 31-0:1992, subclause 3.3.2.
11-6.6 (*11-4.4*)	$\frac{a}{b}$ a/b ab^{-1}	a divided by b	See also ISO 31-0:1992, subclause 3.1.3.
11-6.7 (*11-4.5*)	$\sum\limits_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$	Also $\sum_{i=1}^{n} a_i$, $\sum\limits_{i} a_i$, $\sum_i a_i$, $\sum a_i$
11-6.8 (*11-4.6*)	$\prod\limits_{i=1}^{n} a_i$	$a_1 \cdot a_2 \cdot \ldots \cdot a_n$	Also $\prod_{i=1}^{n} a_i$, $\prod\limits_{i} a_i$, $\prod_i a_i$, $\prod a_i$
11-6.9 (*11-4.7*)	a^p	a to the power p	
11-6.10 (*11-4.8*)	$a^{1/2}$ $a^{\frac{1}{2}}$ $\surd a$ $\sqrt{a}$	a to the power 1/2; square root of a	If $a \geqslant 0$, then $\sqrt{a} \geqslant 0$. See remark to 11-6.11.
11-6.11 (*11-4.9*)	$a^{1/n}$ $a^{\frac{1}{n}}$ $\sqrt[n]{}a$ $\sqrt[n]{a}$	a to the power $1/n$; nth root of a	If $a \geqslant 0$, then $\sqrt[n]{a} \geqslant 0$. If the symbol $\surd$ or $\sqrt[n]{}$ acts on a composite expression, parentheses shall be used to avoid ambiguity.
11-6.12 (*11-4.10*)	$\|a\|$	absolute value of a; magnitude of a; modulus of a	abs a is also used.

6 OPERATIONS (*concluded*)			
Item No.	**Symbol, application**	**Meaning, verbal equivalent**	**Remarks and examples**
11-6.13 (*11-4.11*)	sgn a	signum a	For real a: $$\operatorname{sgn} a = \begin{cases} 1 \text{ if } a > 0 \\ 0 \text{ if } a = 0 \\ -1 \text{ if } a < 0 \end{cases}$$ For complex a, see 11-10.7.
11-6.14 (*11-4.12*)	$\bar{a}$ $\langle a \rangle$	mean value of a	The method of forming the mean shall be stated if not clear from the context.
11-6.15 (*11-4.13*)	$n!$	factorial n	For $n \geqslant 1$: $n! = \prod_{k=1}^{n} k = 1 \times 2 \times 3 \times \ldots \times n$ For $n = 0$: $n! = 1$
11-6.16 (*11-4.14*)	$\binom{n}{p}$ C_n^p	binomial coefficient n, p	$\binom{n}{p} = \frac{n!}{p!(n-p)!}$
11-6.17 (*11-4.15*)	ent a E(a)	the greatest integer less than or equal to a; characteristic of a	ent 2,4 = 2 ent(−2,4) = −3 [a] or int a is sometimes used for ent a, but is now often used with the meaning "integer part of a", e.g. [2,4] = int 2,4 = 2 [−2,4] = int(−2,4) = −2

7 FUNCTIONS

Item No.	Symbol, application	Meaning, verbal equivalent	Remarks and examples
11-7.1 (*11-5.1*)	f	function f	A function may also be denoted by $x \mapsto f(x)$. Letters other than f are also used.
11-7.2 (*11-5.2*)	$f(x)$ $f(x, y, \ldots)$	value of the function f at x or at $(x, y, \ldots)$ respectively	
11-7.3 (*11-5.3*)	$f(x)\vert_a^b$ $[f(x)]_a^b$	$f(b) - f(a)$	This notation is used mainly when evaluating definite integrals.
11-7.4 (*11-5.4*)	$g \circ f$	the composite function of f and g, read as g circle f	$(g \circ f)(x) = g(f(x))$
11-7.5 (*11-5.5*)	$x \to a$	x tends to a	
11-7.6 (*11-5.6*)	$\lim\limits_{x \to a} f(x)$ $\lim_{x \to a} f(x)$	limit of $f(x)$ as x tends to a	$\lim_{x \to a} f(x) = b$ may be written $f(x) \to b$ as $x \to a$. Limits "from the right" $(x > a)$ and "from the left" $(x < a)$ may be denoted by $\lim_{x \to a+} f(x)$ and $\lim_{x \to a-} f(x)$ respectively.
11-7.7 (*11-5.7*)	$\simeq$	is asymptotically equal to	EXAMPLE $\frac{1}{\sin(x-a)} \simeq \frac{1}{x-a}$ as $x \to a$.
11-7.8 (*11-5.8*)	$\mathrm{O}(g(x))$ $f(x) = \mathrm{O}(g(x))$	$\vert f(x)/g(x)\vert$ is bounded above in the limit implied by the context; f is of the order of g	
11-7.9 (*11-5.9*)	$\mathrm{o}(g(x))$ $f(x) = \mathrm{o}(g(x))$	$f(x)/g(x) \to 0$ in the limit implied by the context; f is of lower order than g	
11-7.10 (*11-5.10*)	Δx	(finite) increment of x	
11-7.11 (*11-5.11*)	$\frac{\mathrm{d}f}{\mathrm{d}x}$ $\mathrm{d}f/\mathrm{d}x$ f'	derivative of the function f of one variable	$\mathrm{D}f$ is also used. $\frac{\mathrm{d}f(x)}{\mathrm{d}x}$, $\mathrm{d}f(x)/\mathrm{d}x$, $f'(x)$, $\mathrm{D}f(x)$ If the independent variable is time t, $\dot{f}$ is also used for $\frac{\mathrm{d}f}{\mathrm{d}t}$.

7 FUNCTIONS *(continued)*			
Item No.	**Symbol, application**	**Meaning, verbal equivalent**	**Remarks and examples**
11-7.12 (*11-5.12*)	$\left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_{x=a}$ $(\mathrm{d}f/\mathrm{d}x)_{x=a}$ $f'(a)$	value at a of the derivative of the function f	$\mathrm{D}f(a)$ is also used.
11-7.13 (*11-5.13*)	$\frac{\mathrm{d}^n f}{\mathrm{d}x^n}$ $\mathrm{d}^n f/\mathrm{d}x^n$ $f^{(n)}$	nth derivative of the function f of one variable	$\mathrm{D}^n f$ is also used. For $n = 2, 3$; f'', f''' are also used for $f^{(n)}$. If the independent variable is time t, $\ddot{f}$ is also used for $\frac{\mathrm{d}^2 f}{\mathrm{d}t^2}$.
11-7.14 (*11-5.14*)	$\frac{\partial f}{\partial x}$ $\partial f/\partial x$ $\partial_x f$	partial derivative of the function f of several variables $x, y, \ldots$ with respect to x	$\mathrm{D}_x f$ is also used. $\frac{\partial f(x, y, \ldots)}{\partial x}$, $\partial f(x, y, \ldots)/\partial x$, $\partial_x f(x, y, \ldots)$, $\mathrm{D}_x f(x, y, \ldots)$ The other independent variables may be shown as subscripts, e.g. $\left(\frac{\partial f}{\partial x}\right)_{y \ldots}$ This partial-derivative notation is extended to derivatives of higher order, e.g. $\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$ $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$
11-7.15 (*11-5.15*)	$\mathrm{d}f$	total differential of the function f	$\mathrm{d}f(x, y, \ldots) = \frac{\partial f}{\partial x}\mathrm{d}x + \frac{\partial f}{\partial y}\mathrm{d}y + \ldots$
11-7.16 (*11-5.16*)	δf	(infinitesimal) variation of the function f	
11-7.17 (*11-5.17*)	$\int f(x)\,\mathrm{d}x$	an indefinite integral of the function f	

7 FUNCTIONS (*concluded*)

Item No.	Symbol, application	Meaning, verbal equivalent	Remarks and examples
11-7.18 (*11-5.18*)	$\int_a^b f(x)\, \mathrm{d}x$ $\int_a^b f(x)\, \mathrm{d}x$	definite integral of the function f from a to b	Multiple integrals are denoted by, for example: $\int_{y=c}^{d} \int_{x=a}^{b} f(x,y)\, \mathrm{d}x\, \mathrm{d}y$ Special notations $\int_{\mathrm{C}} \int_{\mathrm{S}} \int_{\mathrm{V}} \oint$ are used for integration over a curve C, a surface S and a three-dimensional domain V, and over a closed curve or surface, respectively.
11-7.19 (*11-5.19*)	δ_{ik}	Kronecker delta symbol	$\delta_{ik} = \begin{cases} 1 \text{ for } i = k \\ 0 \text{ for } i \neq k \end{cases}$ where i and k are integers.
11-7.20 (*11-5.20*)	ε_{ijk}	Levi-Civita symbol	$\varepsilon_{ijk} = 1$ for $(i, j, k) = (1, 2, 3)$ and is completely antisymmetric in the indices, i.e. $\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = 1$ $\varepsilon_{132} = \varepsilon_{321} = \varepsilon_{213} = -1$ all others being equal to 0.
11-7.21 (*11-5.21*)	$\delta(x)$	Dirac delta distribution (function)	$\int_{-\infty}^{+\infty} f(x)\, \delta(x)\, \mathrm{d}x = f(0)$
11-7.22 (*11-5.22*)	$\varepsilon(x)$	unit step function; Heaviside function	$\varepsilon(x) = \begin{cases} 1 \text{ for } x > 0 \\ 0 \text{ for } x < 0 \end{cases}$ $\mathrm{H}(x)$ is also used. $\vartheta(t)$ is used for the unit step function of time.
11-7.23 (*11-5.23*)	$f * g$	convolution of f and g	$(f * g)(x) = \int_{-\infty}^{+\infty} f(y)\, g(x-y)\, \mathrm{d}y$

8 EXPONENTIAL AND LOGARITHMIC FUNCTIONS			
Item No.	**Sign, symbol, expression**	**Meaning**	**Remarks and examples**
11-8.1 (—)	a^x	exponential function to the base a of x	Compare 11-6.9.
11-8.2 (*11-6.1*)	e	base of natural logarithms	$\mathrm{e} = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = 2{,}718\ 281\ 8...$
11-8.3 (*11-6.2*)	e^x $\exp x$	exponential function (to the base e) of x	
11-8.4 (*11-6.3*)	$\log_a x$	logarithm to the base a of x	$\log x$ is used when the base need not be specified.
11-8.5 (*11-6.4*)	$\ln x$	$\ln x = \log_\mathrm{e} x$; natural logarithm of x	$\log x$ shall not be used in place of $\ln x$, $\lg x$, $\mathrm{lb}\, x$ or $\log_\mathrm{e} x$, $\log_{10} x$, $\log_2 x$.
11-8.6 (*11-6.5*)	$\lg x$	$\lg x = \log_{10} x$; common (decimal) logarithm of x	See remark to 11-8.5.
11-8.7 (*11-6.6*)	$\mathrm{lb}\, x$	$\mathrm{lb}\, x = \log_2 x$; binary logarithm of x	See remark to 11-8.5.

9 CIRCULAR AND HYPERBOLIC FUNCTIONS			
Item No.	**Sign, symbol, expression**	**Meaning**	**Remarks and examples**
11-9.1 *(11-7.1)*	π	ratio of the circumference of a circle to its diameter	$\pi = 3{,}141\ 592\ 6...$
11-9.2 *(11-7.2)*	$\sin x$	sine of x	$(\sin x)^n$, $(\cos x)^n$, etc., are often written $\sin^n x$, $\cos^n x$, etc.
11-9.3 *(11-7.3)*	$\cos x$	cosine of x	
11-9.4 *(11-7.4)*	$\tan x$	tangent of x	$\mathrm{tg}\ x$ is still used.
11-9.5 *(11-7.5)*	$\cot x$	cotangent of x	$\cot x = 1/\tan x$
11-9.6 *(11-7.6)*	$\sec x$	secant of x	$\sec x = 1/\cos x$
11-9.7 *(11-7.7)*	$\csc x$	cosecant of x	$\mathrm{cosec}\ x$ is also used. $\csc x = 1/\sin x$
11-9.8 *(11-7.8)*	$\arcsin x$	arc sine of x	$y = \arcsin x \Leftrightarrow x = \sin y,\ -\pi/2 \leqslant y \leqslant \pi/2$ The function arcsin is the inverse of the function sin with the restriction mentioned above. See remark following 11-9.13.
11-9.9 *(11-7.9)*	$\arccos x$	arc cosine of x	$y = \arccos x \Leftrightarrow x = \cos y,\ 0 \leqslant y \leqslant \pi$ The function arccos is the inverse of the function cos with the restriction mentioned above. See remark following 11-9.13.
11-9.10 *(11-7.10)*	$\arctan x$	arc tangent of x	$\mathrm{arctg}\ x$ is still used. $y = \arctan x \Leftrightarrow x = \tan y,\ -\pi/2 < y < \pi/2$ The function arctan is the inverse of the function tan with the restriction mentioned above. See remark following 11-9.13.
11-9.11 *(11-7.11)*	$\mathrm{arccot}\ x$	arc cotangent of x	$y = \mathrm{arccot}\ x \Leftrightarrow x = \cot y,\ 0 < y < \pi$ The function arccot is the inverse of the function cot with the restriction mentioned above. See remark following 11-9.13.

9 CIRCULAR AND HYPERBOLIC FUNCTIONS (*continued*)			
Item No.	**Sign, symbol, expression**	**Meaning**	**Remarks and examples**
11-9.12 (*11-7.12*)	arcsec x	arc secant of x	$y = \text{arcsec}\, x \Leftrightarrow x = \sec y$, $0 \leqslant y \leqslant \pi$, $y \neq \pi/2$ The function arcsec is the inverse of the function sec with the restriction mentioned above. See remark following 11-9.13.
11-9.13 (*11-7.13*)	arccsc x	arc cosecant of x	arccosec x is also used. $y = \text{arccsc}\, x \Leftrightarrow x = \csc y$, $-\pi/2 \leqslant y \leqslant \pi/2$, $y \neq 0$ The function arccsc is the inverse of the function csc with the restriction mentioned above. **Remark on 11-9.8 to 11-9.13.** The notations $\sin^{-1}x$, $\cos^{-1}x$, etc., for the inverse circular functions shall not be used because they may be mistaken for $(\sin x)^{-1}$, $(\cos x)^{-1}$, etc.
11-9.14 (*11-7.14*)	sinh x	hyperbolic sine of x	sh x is also used.
11-9.15 (*11-7.15*)	cosh x	hyperbolic cosine of x	ch x is also used.
11-9.16 (*11-7.16*)	tanh x	hyperbolic tangent of x	th x is also used.
11-9.17 (*11-7.17*)	coth x	hyperbolic cotangent of x	$\coth x = 1/\tanh x$
11-9.18 (*11-7.18*)	sech x	hyperbolic secant of x	$\text{sech}\, x = 1/\cosh x$
11-9.19 (*11-7.19*)	csch x	hyperbolic cosecant of x	cosech x is also used. $\text{csch}\, x = 1/\sinh x$
11-9.20 (*11-7.20*)	arsinh x	inverse hyperbolic sine of x	arsh x and argsh x are also used. $y = \text{arsinh}\, x \Leftrightarrow x = \sinh y$ The function arsinh is the inverse of the function sinh. See remarks following 11-9.25.
11-9.21 (*11-7.21*)	arcosh x	inverse hyperbolic cosine of x	arch x and argch x are also used. $y = \text{arcosh}\, x \Leftrightarrow x = \cosh y$, $y \geqslant 0$ The function arcosh is the inverse of the function cosh with the restriction mentioned above. See remarks following 11-9.25.

9 CIRCULAR AND HYPERBOLIC FUNCTIONS (*concluded*)			
Item No.	**Sign, symbol, expression**	**Meaning**	**Remarks and examples**
11-9.22 (*11-7.22*)	artanh x	inverse hyperbolic tangent of x	arth x and argth x are also used. $y = \text{artanh}\, x \Leftrightarrow x = \tanh y$ The function artanh is the inverse of the function tanh. See remarks following 11-9.25.
11-9.23 (*11-7.23*)	arcoth x	inverse hyperbolic cotangent of x	argcoth x is also used. $y = \text{arcoth}\, x \Leftrightarrow x = \coth y,\ y \neq 0$ The function arcoth is the inverse of the function coth with the restriction mentioned above. See remarks following 11-9.25.
11-9.24 (*11-7.24*)	arsech x	inverse hyperbolic secant of x	$y = \text{arsech}\, x \Leftrightarrow x = \text{sech}\, y,\ y \geqslant 0$ The function arsech is the inverse of the function sech with the restriction mentioned above. See remarks following 11-9.25.
11-9.25 (*11-7.25*)	arcsch x	inverse hyperbolic cosecant of x	arcosech x is also used. $y = \text{arcsch}\, x \Leftrightarrow x = \text{csch}\, y,\ y \neq 0$ The function arcsch is the inverse of the function csch with the restriction mentioned above. **Remarks on 11-9.20 to 11-9.25.** arsinh, arcosh, etc., are also called area hyperbolic sine, area hyperbolic cosine and so on, because the argument is an area. The notations $\sinh^{-1}x$, $\cosh^{-1}x$, etc., for the inverse hyperbolic functions shall not be used because they may be mistaken for $(\sinh x)^{-1}$, $(\cosh x)^{-1}$, etc.

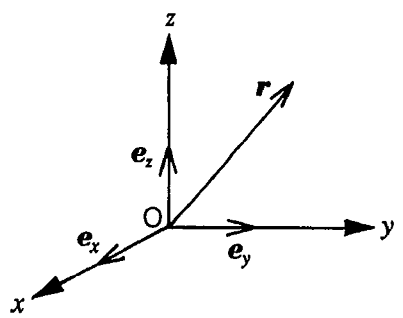
10 COMPLEX NUMBERS			
Item No.	**Sign, symbol, expression**	**Meaning**	**Remarks and examples**
11-10.1 (*11-8.1*)	i j	imaginary unit, $i^2 = -1$	In electrotechnology, j is generally used.
11-10.2 (*11-8.2*)	Re z	real part of z	
11-10.3 (*11-8.3*)	Im z	imaginary part of z	$z = x + iy$, where $x = \operatorname{Re} z$ and $y = \operatorname{Im} z$.
11-10.4 (*11-8.4*)	$\lvert z \rvert$	absolute value of z; modulus of z	mod z is also used.
11-10.5 (*11-8.5*)	arg z	argument of z; phase of z	$z = re^{i\varphi}$, where $r = \lvert z \rvert$ and $\varphi = \arg z$, i.e. $\operatorname{Re} z = r \cos \varphi$ and $\operatorname{Im} z = r \sin \varphi$.
11-10.6 (*11-8.6*)	z^*	(complex) conjugate of z	Sometimes $\bar{z}$ is used instead of z^*.
11-10.7 (*11-8.7*)	sgn z	signum z	$\operatorname{sgn} z = z/\lvert z \rvert = \exp(i \arg z)$ for $z \neq 0$, $\operatorname{sgn} z = 0$ for $z = 0$.

11 MATRICES			
Item No.	**Sign, symbol, expression**	**Meaning**	**Remarks and examples**
			The use of capital letters in this section is not intended to imply that matrices or matrix elements cannot be written with lower-case letters.
11-11.1 (*11-9.1*)	$\boldsymbol{A}$ $\begin{pmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{m1} & \cdots & A_{mn} \end{pmatrix}$	matrix $\boldsymbol{A}$ of type m by n	$\boldsymbol{A}$ is the matrix with the elements A_{ij}; m is the number of rows and n is the number of columns. $\boldsymbol{A} = (A_{ij})$ is also used. Square brackets are also used instead of parentheses.
11-11.2 (*11-9.2*)	$\boldsymbol{AB}$	product of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$	$(\boldsymbol{AB})_{ik} = \sum_j A_{ij}B_{jk}$ where the number of columns of $\boldsymbol{A}$ must be equal to the number of rows of $\boldsymbol{B}$.
11-11.3 (*11-9.3*)	$\boldsymbol{E}$ $\boldsymbol{I}$	unit matrix	Any square matrix for which $E_{ik} = \delta_{ik}$. See 11-7.19.
11-11.4 (*11-9.4*)	$\boldsymbol{A}^{-1}$	inverse of a square matrix $\boldsymbol{A}$	$\boldsymbol{AA}^{-1} = \boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{E}$
11-11.5 (*11-9.5*)	$\boldsymbol{A}^{\mathrm{T}}$ $\tilde{\boldsymbol{A}}$	transpose matrix of $\boldsymbol{A}$	$\left(\boldsymbol{A}^{\mathrm{T}}\right)_{ik} = A_{ki}$
11-11.6 (*11-9.6*)	$\boldsymbol{A}^*$	complex conjugate matrix of $\boldsymbol{A}$	$(\boldsymbol{A}^*)_{ik} = (A_{ik})^* = A_{ik}^*$ In mathematics, $\bar{\boldsymbol{A}}$ is often used.
11-11.7 (*11-9.7*)	$\boldsymbol{A}^{\mathrm{H}}$ $\boldsymbol{A}^{\dagger}$	Hermitian conjugate matrix of $\boldsymbol{A}$	$\left(\boldsymbol{A}^{\mathrm{H}}\right)_{ik} = (A_{ki})^* = A_{ki}^*$ In mathematics, $\boldsymbol{A}^*$ is often used.
11-11.8 (*11-9.8*)	$\det \boldsymbol{A}$ $\begin{vmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{n1} & \cdots & A_{nn} \end{vmatrix}$	determinant of a square matrix $\boldsymbol{A}$	
11-11.9 (*11-9.9*)	$\mathrm{tr}\,\boldsymbol{A}$	trace of a square matrix $\boldsymbol{A}$	$\mathrm{tr}\,\boldsymbol{A} = \sum_i A_{ii}$
11-11.10 (*11-9.10*)	$\lVert\boldsymbol{A}\rVert$	norm of the matrix $\boldsymbol{A}$	Several matrix norms can be defined, e.g. the Euclidean norm $\lVert\boldsymbol{A}\rVert = \left(\mathrm{tr}\left(\boldsymbol{AA}^{\mathrm{H}}\right)\right)^{1/2}$

12 COORDINATE SYSTEMS

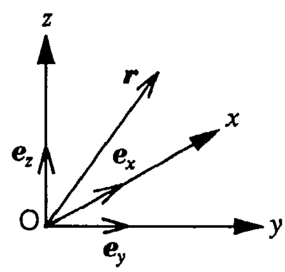
Item No.	Coordinates	Position vector and its differential	Name of coordinate system	Remarks
11-12.1 (—)	x, y, z	$\boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$; $\mathrm{d}\boldsymbol{r} = \mathrm{d}x\,\boldsymbol{e}_x + \mathrm{d}y\,\boldsymbol{e}_y + \mathrm{d}z\,\boldsymbol{e}_z$	cartesian coordinates	$\boldsymbol{e}_x$, $\boldsymbol{e}_y$ and $\boldsymbol{e}_z$ form an orthonormal right-handed system. See figure 1.
11-12.2 (—)	ϱ, φ, z	$\boldsymbol{r} = \varrho\boldsymbol{e}_\varrho + z\boldsymbol{e}_z$; $\mathrm{d}\boldsymbol{r} = \mathrm{d}\varrho\,\boldsymbol{e}_\varrho + \varrho\,\mathrm{d}\varphi\,\boldsymbol{e}_\varphi + \mathrm{d}z\,\boldsymbol{e}_z$	cylindrical coordinates	$\boldsymbol{e}_\varrho(\varphi)$, $\boldsymbol{e}_\varphi(\varphi)$ and $\boldsymbol{e}_z$ form an orthonormal right-handed system. See figures 3 and 4. If $z = 0$, then ϱ and φ are the polar coordinates.
11-12.3 (—)	r, ϑ, φ	$\boldsymbol{r} = r\boldsymbol{e}_r$; $\mathrm{d}\boldsymbol{r} = \mathrm{d}r\,\boldsymbol{e}_r + r\,\mathrm{d}\vartheta\,\boldsymbol{e}_\vartheta + r\sin\vartheta\,\mathrm{d}\varphi\,\boldsymbol{e}_\varphi$	spherical coordinates	$\boldsymbol{e}_r(\vartheta, \varphi)$, $\boldsymbol{e}_\vartheta(\vartheta, \varphi)$ and $\boldsymbol{e}_\varphi(\varphi)$ form an orthonormal right-handed system. See figures 3 and 5.

NOTE 1 If, exceptionally, a left-handed system (see figure 2) is used for certain purposes, this shall be clearly stated to avoid the risk of sign errors.



The x-axis is pointing towards the viewer.

Figure 1 — Right-handed cartesian coordinate system



The x-axis is pointing away from the viewer.

Figure 2 — Left-handed cartesian coordinate system

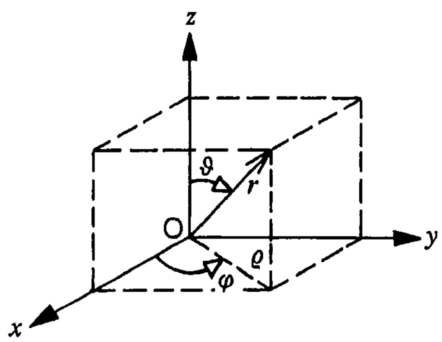


Figure 3 — Oxyz is a right-handed coordinate system

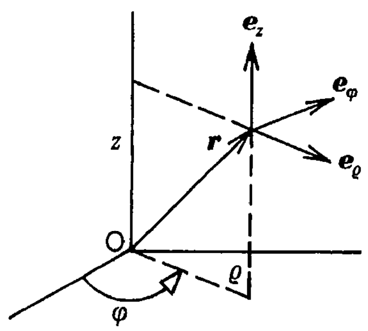


Figure 4 — Right-handed cylindrical coordinates

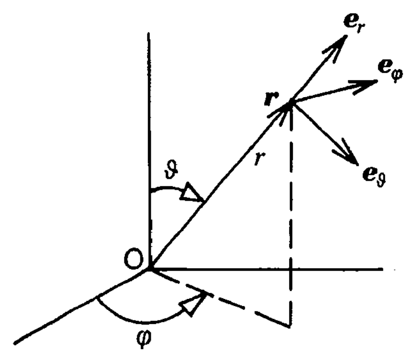


Figure 5 — Right-handed spherical coordinates

13 VECTORS AND TENSORS

Item No.	Sign, symbol, expression	Meaning	Remarks and examples
			In this section, only cartesian (orthonormal) coordinates in ordinary space are considered. The more general case requiring covariant and contravariant representations is not treated here. The cartesian coordinates are denoted x, y, z or x_1, x_2, x_3. In the latter case, indices i, j, k, l ranging from 1 to 3 are used, and the following summation convention is adopted: if such an index appears twice in a term, summation over the range of this index is understood. **Remarks to 11-13.1 to 11-13.20** A scalar is a tensor of zero order and a vector is a tensor of the first order. Vectors and tensors are often represented by general symbols for their components, e.g. a_i for a vector, T_{ij} for a tensor of the second order, $a_i b_j$ for a dyadic product.
11-13.1 *(11-10.1)*	$\boldsymbol{a}$ $\vec{a}$	vector $\boldsymbol{a}$	An arrow above the letter symbol can be used instead of bold face type to indicate a vector. Any vector $\boldsymbol{a}$ can be multiplied by a scalar or a number k, i.e. $k\boldsymbol{a}$.
11-13.2 *(11-10.2)*	a $\lvert\boldsymbol{a}\rvert$	magnitude of the vector $\boldsymbol{a}$	$\lVert\boldsymbol{a}\rVert$ is also used.
11-13.3 *(11-10.3)*	$\boldsymbol{e}_a$	unit vector in the direction of $\boldsymbol{a}$	$\boldsymbol{e}_a = \boldsymbol{a}/\lvert\boldsymbol{a}\rvert$ $\boldsymbol{a} = a\boldsymbol{e}_a$
11-13.4 *(11-10.4)*	$\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$ $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ $\boldsymbol{e}_i$	unit vectors in the directions of the cartesian coordinate axes	
11-13.5 *(11-10.5)*	a_x, a_y, a_z a_i	cartesian components of vector $\boldsymbol{a}$	$\boldsymbol{a} = a_x\boldsymbol{e}_x + a_y\boldsymbol{e}_y + a_z\boldsymbol{e}_z$ $a_x\boldsymbol{e}_x$, etc., are the component vectors. If it is clear from the context which are the base vectors, the vector can be written $\boldsymbol{a} = (a_x, a_y, a_z)$. $\boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$ is the radius vector (position vector).

13 VECTORS AND TENSORS *(continued)*			
Item No.	**Sign, symbol, expression**	**Meaning**	**Remarks and examples**
11-13.6 *(11-10.6)*	$\boldsymbol{a} \cdot \boldsymbol{b}$	scalar product of $\boldsymbol{a}$ and $\boldsymbol{b}$	$\boldsymbol{a} \cdot \boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z$ $\boldsymbol{a} \cdot \boldsymbol{b} = \sum_i a_i b_i$ often written $a_i b_i$ by convention (see remark preceding 11-13.1). $\boldsymbol{a} \cdot \boldsymbol{a} = \boldsymbol{a}^2 = \lvert \boldsymbol{a} \rvert^2 = a^2$ In special fields, $(\boldsymbol{a}, \boldsymbol{b})$ is also used.
11-13.7 *(11-10.7)*	$\boldsymbol{a} \times \boldsymbol{b}$	vector product of $\boldsymbol{a}$ and $\boldsymbol{b}$	The components are $(\boldsymbol{a} \times \boldsymbol{b})_x = a_y b_z - a_z b_y$, etc., in a right-handed cartesian coordinate system. $(\boldsymbol{a} \times \boldsymbol{b})_i = \sum_j \sum_k \varepsilon_{ijk} a_j b_k$ For ε_{ijk}, see 11-7.20.
11-13.8 *(11-10.8)*	$\boldsymbol{\nabla}$ $\vec{\nabla}$	nabla operator	$\boldsymbol{\nabla} = \boldsymbol{e}_x \frac{\partial}{\partial x} + \boldsymbol{e}_y \frac{\partial}{\partial y} + \boldsymbol{e}_z \frac{\partial}{\partial z} = \sum_i \boldsymbol{e}_i \frac{\partial}{\partial x_i}$ $\frac{\partial}{\partial \boldsymbol{r}}$ is also used.
11-13.9 *(11-10.9)*	$\boldsymbol{\nabla}\varphi$ **grad** φ	gradient of φ	grad φ is also used. $\boldsymbol{\nabla}\varphi = \sum_i \boldsymbol{e}_i \frac{\partial \varphi}{\partial x_i}$
11-13.10 *(11-10.10)*	$\boldsymbol{\nabla} \cdot \boldsymbol{a}$ div $\boldsymbol{a}$	divergence of $\boldsymbol{a}$	$\boldsymbol{\nabla} \cdot \boldsymbol{a} = \sum_i \frac{\partial a_i}{\partial x_i}$
11-13.11 *(11-10.11)*	$\boldsymbol{\nabla} \times \boldsymbol{a}$ **rot** $\boldsymbol{a}$ **curl** $\boldsymbol{a}$	rotation of $\boldsymbol{a}$; curl of $\boldsymbol{a}$	The components are $(\boldsymbol{\nabla} \times \boldsymbol{a})_x = \frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}$, etc. rot $\boldsymbol{a}$, curl $\boldsymbol{a}$ are also used. $(\boldsymbol{\nabla} \times \boldsymbol{a})_i = \sum_j \sum_k \varepsilon_{ijk} \frac{\partial a_k}{\partial x_j}$ For ε_{ijk}, see 11-7.20.
11-13.12 *(11-10.12)*	$\boldsymbol{\nabla}^2$ Δ	Laplacian	$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

13 VECTORS AND TENSORS (*concluded*)			
Item No.	**Sign, symbol, expression**	**Meaning**	**Remarks and examples**
11-13.13 (*11-10.13*)	$\square$	Dalembertian	$\square = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$
11-13.14 (*11-10.14*)	$\boldsymbol{T}$ $\overset{\rightrightarrows}{T}$	tensor $\boldsymbol{T}$ of the second order	Two arrows above the letter symbol can be used instead of bold face sans serif type to indicate a tensor of the second order.
11-13.15 (*11-10.15*)	$T_{xx}, T_{xy}, \ldots, T_{zz}$ T_{ij}	cartesian components of tensor $\boldsymbol{T}$	$\boldsymbol{T} = T_{xx}\boldsymbol{e}_x\boldsymbol{e}_x + T_{xy}\boldsymbol{e}_x\boldsymbol{e}_y + \ldots$ $T_{xx}\boldsymbol{e}_x\boldsymbol{e}_x$, etc., are the component tensors. If it is clear from the context which are the base vectors, the tensor can be written $\boldsymbol{T} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}$
11-13.16 (*11-10.16*)	$\boldsymbol{ab}$ $\boldsymbol{a} \otimes \boldsymbol{b}$	dyadic product; tensor product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$	tensor of the second order with components $(\boldsymbol{ab})_{ij} = a_i b_j$
11-13.17 (*11-10.17*)	$\boldsymbol{T} \otimes \boldsymbol{S}$	tensor product of two tensors $\boldsymbol{T}$ and $\boldsymbol{S}$ of the second order	tensor of the fourth order with components $(\boldsymbol{T} \otimes \boldsymbol{S})_{ijkl} = T_{ij}S_{kl}$
11-13.18 (*11-10.18*)	$\boldsymbol{T} \cdot \boldsymbol{S}$	inner product of two tensors $\boldsymbol{T}$ and $\boldsymbol{S}$ of the second order	tensor of the second order with components $(\boldsymbol{T} \cdot \boldsymbol{S})_{ik} = \sum_j T_{ij}S_{jk}$
11-13.19 (*11-10.19*)	$\boldsymbol{T} \cdot \boldsymbol{a}$	inner product of a tensor $\boldsymbol{T}$ of the second order and a vector $\boldsymbol{a}$	vector with components $(\boldsymbol{T} \cdot \boldsymbol{a})_i = \sum_j T_{ij}a_j$
11-13.20 (*11-10.20*)	$\boldsymbol{T} : \boldsymbol{S}$	scalar product of two tensors $\boldsymbol{T}$ and $\boldsymbol{S}$ of the second order	scalar quantity $\boldsymbol{T} : \boldsymbol{S} = \sum_i \sum_j T_{ij}S_{ji}$

14 SPECIAL FUNCTIONS

Item No.	Sign, symbol, expression	Meaning	Remarks and examples
11-14.1 (*11-11.1*)	$\mathrm{J}_l(x)$	cylindrical Bessel functions (of the first kind)	solutions of $x^2y'' + xy' + (x^2 - l^2)y = 0$ $\mathrm{J}_l(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{l+2k}}{k!\Gamma(l+k+1)}$ $(l \geqslant 0)$ For Γ, see 11-14.19.
11-14.2 (*11-11.2*)	$\mathrm{N}_l(x)$	cylindrical Neumann functions; cylindrical Bessel functions of the second kind	$\mathrm{N}_l(x) = \lim_{k \to l} \frac{\mathrm{J}_k(x) \cos k\pi - \mathrm{J}_{-k}(x)}{\sin k\pi}$ $\mathrm{Y}_l(x)$ is also used.
11-14.3 (*11-11.3*)	$\mathrm{H}_l^{(1)}(x)$ $\mathrm{H}_l^{(2)}(x)$	cylindrical Hankel functions; cylindrical Bessel functions of the third kind	$\mathrm{H}_l^{(1)}(x) = \mathrm{J}_l(x) + \mathrm{i}\mathrm{N}_l(x)$ $\mathrm{H}_l^{(2)}(x) = \mathrm{J}_l(x) - \mathrm{i}\mathrm{N}_l(x)$
11-14.4 (*11-11.4*)	$\mathrm{I}_l(x)$ $\mathrm{K}_l(x)$	modified cylindrical Bessel functions	solutions of $x^2y'' + xy' - (x^2 + l^2)y = 0$ $\mathrm{I}_l(x) = \mathrm{i}^{-l}\mathrm{J}_l(\mathrm{i}x)$ $\mathrm{K}_l(x) = (\pi/2)\mathrm{i}^{l+1}(\mathrm{J}_l(\mathrm{i}x) + \mathrm{i}\mathrm{N}_l(\mathrm{i}x))$
11-14.5 (*11-11.5*)	$\mathrm{j}_l(x)$	spherical Bessel functions (of the first kind)	solutions of $x^2y'' + 2xy' + [x^2 - l(l+1)]y = 0$ $(l \geqslant 0)$ $\mathrm{j}_l(x) = (\pi/2x)^{1/2}\mathrm{J}_{l+1/2}(x)$
11-14.6 (*11-11.6*)	$\mathrm{n}_l(x)$	spherical Neumann functions; spherical Bessel functions of the second kind	$\mathrm{n}_l(x) = (\pi/2x)^{1/2}\mathrm{N}_{l+1/2}(x)$ $\mathrm{y}_l(x)$ is also used.
11-14.7 (*11-11.7*)	$\mathrm{h}_l^{(1)}(x)$ $\mathrm{h}_l^{(2)}(x)$	spherical Hankel functions; spherical Bessel functions of the third kind	$\mathrm{h}_l^{(1)}(x) = \mathrm{j}_l(x) + \mathrm{i}\mathrm{n}_l(x) = (\pi/2x)^{1/2}\mathrm{H}_{l+1/2}^{(1)}(x)$ $\mathrm{h}_l^{(2)}(x) = \mathrm{j}_l(x) - \mathrm{i}\mathrm{n}_l(x) = (\pi/2x)^{1/2}\mathrm{H}_{l+1/2}^{(2)}(x)$ Modified spherical Bessel functions are denoted $\mathrm{i}_l(x)$ and $\mathrm{k}_l(x)$ respectively; compare 11-14.4.
11-14.8 (*11-11.8*)	$\mathrm{P}_l(x)$	Legendre polynomials	solutions of $(1 - x^2)y'' - 2xy' + l(l+1)y = 0$ $\mathrm{P}_l(x) = \frac{1}{2^l l!} \frac{\mathrm{d}^l}{\mathrm{d}x^l}(x^2 - 1)^l$ $(l \in \mathbb{N})$

14 SPECIAL FUNCTIONS *(continued)*			
Item No.	**Sign, symbol, expression**	**Meaning**	**Remarks and examples**
11-14.9 (*11-11.9*)	$\mathrm{P}_l^m(x)$	associated Legendre functions	solutions of $(1-x^2)y'' - 2xy' + \left[l(l+1) - \frac{m^2}{1-x^2}\right]y = 0$ $\mathrm{P}_l^m(x) = (1-x^2)^{m/2} \frac{\mathrm{d}^m}{\mathrm{d}x^m} \mathrm{P}_l(x)$ $(l, m \in \mathbb{N};\ m \leqslant l)$
11-14.10 (*11-11.10*)	$\mathrm{Y}_l^m(\vartheta, \varphi)$	spherical harmonics	solutions of $\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial y}{\partial\vartheta}\right) + \frac{1}{\sin^2\vartheta}\frac{\partial^2 y}{\partial\varphi^2} + l(l+1)y = 0$ $\mathrm{Y}_l^m(\vartheta, \varphi) = (-1)^m \left[\frac{(2l+1)}{4\pi}\frac{(l-\lvert m\rvert)!}{(l+\lvert m\rvert)!}\right]^{1/2} \times \mathrm{P}_l^{\lvert m\rvert}(\cos\vartheta)\mathrm{e}^{\mathrm{i}m\varphi}$ $(l, \lvert m\rvert \in \mathbb{N};\ \lvert m\rvert \leqslant l)$
11-14.11 (*11-11.11*)	$\mathrm{H}_n(x)$	Hermite polynomials	solutions of $y'' - 2xy' + 2ny = 0$ $\mathrm{H}_n(x) = (-1)^n \mathrm{e}^{x^2} \frac{\mathrm{d}^n}{\mathrm{d}x^n} \mathrm{e}^{-x^2}$ $(n \in \mathbb{N})$
11-14.12 (*11-11.12*)	$\mathrm{L}_n(x)$	Laguerre polynomials	solutions of $xy'' + (1-x)y' + ny = 0$ $\mathrm{L}_n(x) = \mathrm{e}^x \frac{\mathrm{d}^n}{\mathrm{d}x^n}(x^n\mathrm{e}^{-x})$ $(n \in \mathbb{N})$
11-14.13 (*11-11.13*)	$\mathrm{L}_n^m(x)$	associated Laguerre polynomials	solutions of $xy'' + (m+1-x)y' + (n-m)y = 0$ $\mathrm{L}_n^m(x) = \frac{\mathrm{d}^m}{\mathrm{d}x^m}\mathrm{L}_n(x)$ $(m, n \in \mathbb{N};\ m \leqslant n)$
11-14.14 (*11-11.14*)	$\mathrm{F}(a, b; c; x)$	hypergeometric functions	solutions of $x(1-x)y'' + [c - (a+b+1)x]y' - aby = 0$ $\mathrm{F}(a, b; c; x) = 1 + \frac{ab}{c}x + \frac{a(a+1)b(b+1)}{2!c(c+1)}x^2 + \ldots$

14 SPECIAL FUNCTIONS *(continued)*			
Item No.	**Sign, symbol, expression**	**Meaning**	**Remarks and examples**
11-14.15 *(11-11.15)*	$\mathrm{F}(a; c; x)$	confluent hypergeometric functions	solutions of $xy'' + (c - x)y' - ay = 0$ $\mathrm{F}(a; c; x) = 1 + \frac{a}{c}x + \frac{a(a+1)}{2!c(c+1)}x^2 + \ldots$
11-14.16 *(11-11.16)*	$\mathrm{F}(k, \varphi)$	(incomplete) elliptic integral of the first kind	$\mathrm{F}(k, \varphi) = \int_0^{\varphi} \frac{\mathrm{d}\vartheta}{\sqrt{1 - k^2 \sin^2\vartheta}}$ $\mathrm{K}(k) = \mathrm{F}(k, \pi/2) \quad (0 < k < 1)$ is the complete elliptic integral of the first kind.
11-14.17 *(11-11.17)*	$\mathrm{E}(k, \varphi)$	(incomplete) elliptic integral of the second kind	$\mathrm{E}(k, \varphi) = \int_0^{\varphi} \sqrt{1 - k^2 \sin^2\vartheta}\, \mathrm{d}\vartheta$ $\mathrm{E}(k) = \mathrm{E}(k, \pi/2) \quad (0 < k < 1)$ is the complete elliptic integral of the second kind.
11-14.18 *(11-11.18)*	$\Pi(k, n, \varphi)$	(incomplete) elliptic integral of the third kind	$\Pi(k, n, \varphi) = \int_0^{\varphi} \frac{\mathrm{d}\vartheta}{\left(1 + n \sin^2\vartheta\right)\sqrt{1 - k^2 \sin^2\vartheta}}$ $\Pi(k, n, \pi/2) \quad (0 < k < 1)$ is the complete elliptic integral of the third kind.
11-14.19 *(11-11.19)*	$\Gamma(x)$	gamma function	$\Gamma(x) = \int_0^{\infty} t^{x-1} \mathrm{e}^{-t}\, \mathrm{d}t \quad (x > 0)$ $\Gamma(n + 1) = n! \quad (n \in \mathbb{N})$
11-14.20 (—)	$\mathrm{B}(x, y)$	beta function	$\mathrm{B}(x, y) = \int_0^1 t^{x-1}(1 - t)^{y-1}\, \mathrm{d}t$ $(x, y \in \mathbb{R}; x > 0, y > 0)$ $\mathrm{B}(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x + y)$
11-14.21 *(11-11.20)*	$\mathrm{Ei}\, x$	exponential integral	$\mathrm{Ei}\, x = \int_x^{\infty} \frac{\mathrm{e}^{-t}}{t}\, \mathrm{d}t$

14 SPECIAL FUNCTIONS (*concluded*)			
Item No.	**Sign, symbol, expression**	**Meaning**	**Remarks and examples**
11-14.22 (*11-11.21*)	erf x	error function	$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt$ $\operatorname{erf}(\infty) = 1$ The function erfc $x = 1 - \operatorname{erf} x$ is called the complementary error function. In statistics the distribution function $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} \, dt$ is used.
11-14.23 (*11-11.22*)	$\zeta(x)$	Riemann zeta function	$\zeta(x) = \frac{1}{1^x} + \frac{1}{2^x} + \frac{1}{3^x} + \ldots \quad (x > 1)$

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