



BSI Standards Publication

# Space systems — Orbit determination and estimation — Process for describing techniques

**National foreword**

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**Space systems — Orbit determination  
and estimation — Process for  
describing techniques**

*Systèmes spatiaux — Détermination et estimation de l'orbite —  
Processus pour la description des techniques*





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## Foreword

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The committee responsible for this document is ISO/TC 20, *Aircraft and space vehicles*, Subcommittee SC 14, *Space systems and operations*.

## Introduction

This Technical Report prescribes the manner in which satellite owners/operators describe techniques used to determine orbits from active and passive observations and the manner in which they estimate satellite orbit evolution.

The same data inputs lead to different predictions when they are used in different models. Satellite owners/operators shall often accept orbit descriptions developed with physical models that others employ. The differences in orbit propagation as a result of using different physical models and numerical techniques can be significant. Safe and cooperative operations among those who operate satellites demand that each satellite owner/operator understand the differences among their approaches to orbit determination and propagation.





# Space systems — Orbit determination and estimation — Process for describing techniques

## 1 Scope

This Technical Report prescribes the manner in which orbit determination and estimation techniques are to be described so that parties can plan operations with sufficient margin to accommodate different individual approaches to orbit determination and estimation. This Technical Report does not require the exchange of orbit data nor does it prescribe a method of performing orbit determination. It only prescribes the information that shall accompany such data so that collaborating satellite owners/operators understand the similarities and differences between their independent orbit determination processes.

All satellite owners/operators are entitled to a preferred approach to physical approximations, numerical implementation, and computational execution of orbit determination and estimation of future states of their satellites. Mission demands should determine the architecture (speed of execution, required precision, etc.). This Technical Report will enable stakeholders to describe their techniques in a manner that is uniformly understood. Implementation details that can have proprietary or competitive advantage need not be revealed.

## 2 Symbols and abbreviated terms

— BDRF	Bidirectional Reflectance Function
— FPA	Flight Path Angle
— GPS	Global Positioning System
— HEO	High Earth Orbit
— IOD	Initial Orbital Determination
— LEO	Low Earth Orbit
— LS	Least Squares
— OD	Orbital Determination
— RAAN	Right Ascension of the Ascending Node
— RMS	Root Mean Square
— SP	Sequential Processing
— TLE	Two-line Elements
— UTC	Coordinated Universal Time

## 3 Background

### 3.1 General

Satellite orbit determination (OD) estimates the position and velocity of an orbiting object from discrete observations. The set of observations includes external measurements from terrestrial or space-based sensors and measurements from instruments on the satellite itself. Satellite orbit propagation estimates the future state of motion of a satellite whose orbit has been determined from past observations. Though

a satellite's motion is described by a set of ideal equations of motion representing physical hypotheses, the observations used in OD are subject to systematic and random uncertainties. Therefore, OD and propagation are probabilistic and can only approximately describe the satellite's motion. The degree of approximation that can be tolerated depends on the intended use of the orbital information.

A spacecraft is influenced by a variety of external forces, including terrestrial gravity, atmospheric drag, multibody gravitation, solar radiation pressure, tides, and spacecraft thrusters. Selection of forces for modelling depends on the accuracy and precision required from the OD process and the amount of available data. The complex modelling of these forces results in a highly nonlinear set of dynamical equations. Many physical and computational uncertainties limit the accuracy and precision of the spacecraft state that can be determined. Similarly, the observational data are inherently nonlinear with respect to the state of motion of the spacecraft and some influences might not have been included in models of the observation of the state of motion.

Satellite OD and propagation are stochastic estimation problems because observations are inherently noisy and uncertain and because not all of the phenomena that influence satellite motion are clearly discernable. Estimation is the process of extracting a desired time-varying signal from statistically noisy observations accumulated over time. Estimation encompasses data smoothing, which is statistical inference from past observations; filtering, which infers the signal from past observations and current observations; and prediction or propagation, which employs past and current observations to infer the future of the signal.

This Technical Report and related ISO documents employ the term "orbit data." Orbit data encompasses all forms of data that contribute to determining the orbits of satellites and that report the outcomes of orbit determination in order to estimate the future trajectory of a satellite. This includes observations of satellite states of motion either through active illumination, as with radars, or through passive observation of electromagnetic energy emitted or reflected from satellites, as with telescopes.

It is desirable to keep each space orbit standard as simple as possible, treating the form and content of orbit data exchange, description of the modelling approach, and other relevant but independent aspects individually. It is hoped that this will develop a sufficient body of standards incrementally, not complicating matters for which there is consensus with matters that might be contentious.

Most in the space community employ a variation of only a few major architectures. These architectures are cited in many texts and references that need not be enumerated in this document.

OD begins with observations from specified locations and produces spacecraft position and velocity, all quantities subject to quantifiable uncertainty.

## **3.2 Initial orbit determination**

Initial OD (IOD) methods input tracking measurements with tracking platform locations, and output spacecraft position and velocity estimates. No a priori orbit estimate is required. Associated solution error magnitudes can be very large. IOD methods are sometimes nonlinear methods and are often trivial to implement. Measurement editing is typically not performed during IOD calculations because there are insufficient observations. Operationally, the OD process is frequently begun, or restarted, with IOD. IOD methods were derived by various authors: LaPlace, Poincaré, Gauss, Lagrange, Lambert, Gibbs, Herrick, Williams, Stumpp, Lancaster, Blanchard, Gooding, and Smith. Restarting techniques are most easily accomplished by using a solution from another technique.

## **3.3 Subsequent orbit determination**

### **3.3.1 Least squares differential corrections**

Least squares (LS) methods input tracking measurements with tracking platform locations and an a priori orbit estimate, and output a refined orbit estimate. Associated solution error magnitudes are by definition small when compared to IOD outputs. LS methods consist of an iterative sequence of corrections where sequence convergence is defined as a function of tracking measurement residual root mean square

(RMS). Each correction is characterized by a minimization of the sum of squares of tracking measurement residuals. The LS method was derived first by Gauss in 1795 and then independently by Legendre.

### 3.3.2 Sequential processing

Sequential processing (SP) methods are distinguished from LS processing methods in that batches of data are considered sequentially, collecting a set of observations over a specified time interval and batch-processing one interval after the next. SP can be thought of as a moving time window whose contents are captured and processed at intervals, independent of previously processed batches of data. The analysis does not include process noise inputs and calculations. It is in no way equivalent to filter processing, in which each new observation is added to past observations, improving estimates in a rigorous, traceable manner.

### 3.3.3 Filter processing

Filter methods output refined state estimates sequentially at each observation time. Filter methods are forward-time recursive sequential methods consisting of a repeating pattern of time updates of the state of motion estimate and measurement updates of the state of motion estimate. The filter time update propagates the state estimate forward, and the filter measurement update incorporates the next measurement. The recursive pattern includes an important interval of filter initialization. Filter-smoother methods are backward-time recursive sequential methods consisting of a repeating pattern of state estimate refinement using filter outputs and backwards transition. Time transitions for both filter and smoother are dominated most significantly by numerical orbit propagators. The search for sequential processing was begun by Wiener, Kalman, Bucy, and others.

## 3.4 Required information for orbit determination

### 3.4.1 Observations

When observation data are communicated for collaborative or independent determination of satellite orbits, the observation types upon which that information is based shall be included. Several types of ground-based, airborne, and space-based sensor observations are routinely used in orbit determination. [Table 1](#) describes the various observation types and sources.

**Table 1 — Space surveillance observation product description**

Content	Source
two angles and slant range	Radars
two angles	Baker-Nunn cameras, telescopes, binoculars, visual sightings
Azimuth	Direction finders
Time of closest approach	Radars, radio receivers [for transmitting (Doppler) satellites]
Range, angles, and rates	Radars
Pseudorange and carrier phase, as well as single, double, and triple differences of these basic measurement types	GPS or onboard inertial sensors
Direction cosines	Interferometric radars

#### 3.4.1.1 Observation location information

When data are communicated for collaborative or independent determination of satellite orbits, the following information about the observation location and measuring devices shall be communicated:

- facility location latitude, longitude, altitude, and the reference from which such are measured, (e.g. WGS-84);
- tracking station identification (ID);
- elevation cutoff;
- measurement biases;
- transponder delay for downlinked information.

#### 3.4.1.2 Satellite information

When data are communicated for collaborative or independent determination of satellite orbits, the following information about the satellite subject shall be included:

- a priori state estimate;
- tracking data ID;
- force model parameters;
- covariance matrix;
- general accelerations;
- transponder delay.

#### 3.4.1.3 Estimation parameters and control

When data are communicated for collaborative or independent determination of satellite orbits, the following information about estimation parameters and control shall be included:

- estimation parameters;
- global force model controls;
- integration controls;
- database controls;
- observation uncertainties.

#### 3.4.2 Tracking data selection and editing

When data are communicated for collaborative or independent determination of satellite orbits, the provider shall state whether data were edited and what the criteria were for tracking data selection.

#### 3.4.3 Widely used OD schemes

When a widely used, consensus-validated, and authoritatively documented OD scheme is employed, the requirements of this Technical Report can be satisfied by citing that documentation and the specific parameter sets that the data provider employed within that scheme, which vary with scheme and version. Some widely used OD schemes that are acceptable are cited in [Annex A](#). The list is not exhaustive.

### **3.4.4 Required information for orbit propagation or prediction**

The following subclauses enumerate and describe standard alternatives for information acceptable under this Technical Report.

#### **3.4.4.1 Force models**

Spacecraft are affected by several different conservative and non-conservative forces. Non-conservative phenomena dissipate spacecraft energy, for example by doing work on and heating the atmosphere.

##### **3.4.4.1.1 Gravitation**

Descriptions of an orbit propagation or prediction scheme shall include complete information about gravitational field characteristics employed. That description shall be based on the following formalism.

##### **3.4.4.1.2 Earth gravity**

The Earth's gravitational field shall be described in terms of a Jacobi polynomial expansion of finite order and degree. Jacobi polynomials are a complete, orthonormal set over the unit sphere. There are two angular degrees of freedom, equivalent to latitude and longitude. Any analytic function within that space can be represented by a weighted doubly infinite series of Jacobi polynomials.

##### **3.4.4.1.2.1 Two-body motion**

Two-body motion or Keplerian motion considers only the force of gravity from the Earth. Both the spacecraft and the Earth are considered point masses, with all mass concentrated at their centres of mass. This is the lowest-order zonal harmonic approximation.

##### **3.4.4.1.3 Zonal harmonics**

###### **3.4.4.1.3.1 J<sub>2</sub>**

The J<sub>2</sub> perturbation (first-order) accounts for secular (constant rate over time) variations in the orbit elements due to Earth oblateness, mainly nodal precession and rotation of the semi-major axis of orbit elements that are otherwise those of unperturbed, Newtonian orbits. J<sub>2</sub> is a zonal harmonic coefficient in an infinite Jacobi polynomial series representation of the Earth's gravity field. It represents the dominant effects of Earth oblateness. The even zonal harmonic coefficients of the gravity field are the only coefficients that result in secular changes in satellite orbital elements. The J<sub>2</sub> propagator includes only the dominant first-order secular effects.

###### **3.4.4.1.3.2 J<sub>4</sub>**

The J<sub>4</sub> perturbation (second-order) accounts for secular variations in the orbit elements due to Earth oblateness. The effects of J<sub>4</sub> are approximately 1 000 times smaller than J<sub>2</sub> and are a result of Earth oblateness.

###### **3.4.4.1.3.3 Generalized zonal harmonics**

It is impractical to determine the weights (coefficients) for a mathematically complete Jacobi polynomial series representation; therefore the series is truncated at meaningful (in terms of precision of the representation of the gravity field) order (latitudinal) and degree (longitudinal). If the order and degree are equal, the truncation is "square." Since gravitational and other perturbations are not necessarily symmetrical in latitude and longitude, the best approximation for a given application is not necessarily square. Static elements of the gravity field are the gravitation of the fixed portions of the distribution of the Earth's mass. The static gravity field is not uniform. Dynamic elements of the gravity are caused by the fluid elements of the Earth's core and by variations in the distribution of water. There are solid and ocean tides.

#### **3.4.4.1.4 Multibody gravitation**

Certain phenomena, such as libration points, only exist with more than two gravitationally interacting bodies. Descriptions of spacecraft orbit propagation or prediction schemes shall include information about third-body or multiple-body gravitational interactions if such are considered.

##### **3.4.4.1.4.1 Lunar gravitation**

Descriptions of spacecraft orbit propagation or prediction shall state whether Lunar influences were considered and how they were described.

##### **3.4.4.1.4.2 Restricted three-body problem**

The restricted three-body problem considers one of the participating bodies to be a point mass. The data set shall state whether such approximations were employed.

##### **3.4.4.1.4.3 Other gravitational influences**

The data set shall state whether other massive bodies were considered beyond the Earth, the Moon, and the satellite of interest and how those influences were approximated.

#### **3.4.4.1.5 Atmospheric resistance**

Gas-dynamic resistance can be a significant dissipative force in low Earth orbits (LEOs). It is usually sufficient to represent it as aerodynamic drag, the product of dynamic pressure, aggregated drag coefficient, and cross-sectional area. Since dynamic pressure is proportional to gas density, the minimum description of atmospheric drag shall include the information described in [3.4.4.1.4.1](#) through [3.4.4.1.4.2](#).

##### **3.4.4.1.5.1 Drag coefficient**

Drag coefficient depends upon satellite geometry, orientation, and gas-dynamic regime described by Knudsen number (ratio of object characteristic dimension to gas mean free path) and Mach number (ratio of object speed to acoustic propagation speed). When describing how atmospheric resistance is represented, data providers shall provide the value of drag coefficient employed or, if drag is allowed to vary, the manner in which drag coefficient varies. If gas-dynamic drag is approximated differently, the scheme shall be described. If gas-dynamic drag is not considered, that shall be stated explicitly.

##### **3.4.4.1.5.2 Atmospheric density model**

Density within the Earth's atmosphere varies temporally and spatially. Those variations are important in LEO. Some acceptable and most-often used atmospheric density models are as follows:

- 1976 Standard Harris-Priester;
- Jacchia 1970 and 1971;
- Jaccia-Roberts;
- MSIS (Several Versions and extensions).

These models can also include measurable parameters that are “proxies” for the variation of atmospheric parameters. These include solar flux/geomagnetic particle flux which can be inferred from the meteorological observables

- daily F10.7,
- average F10.7, and
- geomagnetic index.



### 3.4.4.2 Radiation pressure

Momentum transfer from photons to satellites can be an important force for HEOs. Radiation pressure depends on the area and surface characteristics of the satellite and the nature of the incident radiative fluxes. The Sun is the predominant direct source of electromagnetic radiation, but the Earth and the Moon also emit and reflect electromagnetic radiation. The minimum description of radiation pressure is as follows:

- solar radiation pressure coefficient;
- area-to-mass ratio;
- satellite bidirectional reflectance function (BDRF) or equivalent;
- shadow and shape factor models;
- eclipse models (cylindrical, dual-cone);
- Earth, lunar, and other-body albedo and intensity at the satellite.

### 3.4.5 Numerical or analytical approach

Orbit propagation or prediction has evolved synchronously with advances in computational capability. Initially, force models were greatly simplified, and most important non-gravitational forces were approximated analytically. These generally linearized approaches were valid only over short intervals or for small variations from two-body Keplerian motion. Even though more precise numerical integration became feasible, execution times were too long and computation was too expensive to employ numerics regularly. A number of semi-analytical techniques emerged. These reduced numerical complexity (with some compromise to precision) by providing formulae from which significant elements of the propagation work flow could be extracted. Purely numerical techniques are not used frequently. These suffer only the physical approximations made in describing important phenomena and numerical phenomena common to all discrete computations. Analytical, numerical, and semi-analytical orbit propagation techniques are distinguished. Semi-analytical and analytical approaches are considered to be specific “propagators”. This subclause applies to numerically derived orbit predictions.

Orbital products depend co-equally on the quality and distribution of inputs, the manner in which conservative and dissipative forces are described, and the manner in which computations are performed. [Annex C](#) cites representative numerical integration schemes.

## 3.5 Orbit elements

### 3.5.1 General

Six independent quantities and orbit elements describe the orbit of a satellite. A seventh variable designates the satellite location at a specific time of interest (epoch). There are many different sets of orbit elements. Each is best suited for a particular application, such as aiming antennas, ease of manipulation in various coordinate systems, or estimating orbits from different types of measurements. This subclause applies to mean orbits, the sets of parameters that emerge from the smoothing, filtering, or predictive estimation schemes.

The traditionally used set of orbital elements is called the set of Keplerian elements; Keplerian elements parameters can be encoded as text in a number of formats. The most common of them is the NASA/NORAD “two-line elements” (TLE) format, originally designed for use with 80-column punched cards, but still in use because it is the most common format.

Data providers shall specify completely the set of orbit elements employed. The following subclauses illustrate some choices. Some of these orbital elements are paired, and only certain combinations are valid.

### 3.5.2 Orbit size and shape

Table 2 outlines equivalent pairs of orbit geometry and satellite position parameters.

**Table 2 — Orbit pairs and satellite position parameters**

Element pair	Description
Semi-major axis/eccentricity	Semi-major axis is half the length of the major (longest) axis of the orbital ellipse. Eccentricity describes the shape of the ellipse (a real number $> = 0$ and $< 1$ , where $0 =$ a circular orbit).
Apogee radius/perigee radius	Measured from the centre of the Earth to the points of maximum and minimum radius in the orbit.
Apogee altitude/perigee altitude	Measured from the “surface” of the Earth to the points of maximum and minimum radius in the orbit. For these values, the surface of the Earth is modelled as a sphere whose radius equals the equatorial radius of the Earth.
Period/eccentricity	Period is the duration of one orbit, based on assumed two-body motion. Eccentricity is defined above.
Mean motion (revs/day)/eccentricity	Mean motion (revs/day) identifies the number of orbits per day (86 400 s per period), based on assumed two-body motion. Eccentricity is defined above.

### 3.5.3 Orbit orientation

Table 3 defines the three elements of orbit orientation.

**Table 3 — Orbit orientation elements**

Element	Description
Inclination	The angle between the angular momentum vector (perpendicular to the plane of the orbit) and the inertial $z$ axis.
Argument of perigee	The angle from the ascending node to the eccentricity vector (lowest point of orbit) measured in the direction of the satellite’s motion and in the orbit plane. The eccentricity vector points from the centre of the Earth to perigee with a magnitude equal to the eccentricity of the orbit. For a circular orbit, the argument of perigee is defined to be zero (perigee at the ascending node).
Right ascension of the ascending node (RAAN)/longitude of the ascending node	RAAN ( $\Omega$ ) is the angle from the inertial $x$ axis to the ascending node measured in a right-handed sense about the inertial $z$ axis in the equatorial plane. In the case of an equatorial orbit, the ascending node is defined to be directed along the reference frame’s positive $x$ axis, thus $\Omega = 0$ .  Longitude of the ascending node is the Earth-fixed longitude where the satellite crosses the inertial equator (the intersection of the ground track and the inertial equator) from south to north. The specified ascending node crossing is assumed to be at, or prior to, the initial condition of the orbit in the same nodal revolution.



### 3.5.4 Satellite location

The elements described in [Table 4](#) can be used to specify satellite location .

**Table 4 — Satellite location elements and descriptions**

Element	Description
True anomaly	The angle from the eccentricity vector (points toward perigee) to the satellite position vector, measured in the direction of satellite motion and in the orbit plane.
Mean anomaly	The angle from the eccentricity vector to a position vector where the satellite would be if it were always moving at its average angular rate.
Eccentric anomaly	An angle measured with an origin at the centre of the ellipse from the direction of perigee to a point on a circumscribing circle from which a line perpendicular to the semi-major axis intersects the position of the satellite on the ellipse.
Argument of latitude	The sum of the true anomaly and the argument of perigee.
Time past ascending node	The elapsed time since the last ascending node crossing based on assumed two-body motion.
Time past perigee	The elapsed time since the last perigee passage based on assumed two-body motion.

## 3.6 Coordinate systems

This Technical Report distinguishes between coordinate systems and reference frames. A system is “a set of prescriptions and conventions together with the modelling requirements to define, at any time, a triad of axes.” <sup>1)</sup> A reference frame is the realization of a certain coordinate set within the overall description of a system.

There are many different coordinate systems. Each has some particular advantage for a user community. Transforming essential quantities, such as orbit elements or satellite attitude, from one coordinate system to another is one of the major sources of error in space operations. Generally, a specific set of orbit elements accompanies a coordinate system. The coordinate system in which data elements reside shall be described completely.

### 3.6.1 Cartesian

Cartesian coordinates are often used, for example, to specify the initial position and velocity of the satellite:

- position: specify the  $x$ ,  $y$ , and  $z$  components of the satellite’s position;
- velocity: specify the  $x$ ,  $y$ , and  $z$  components of the satellite’s velocity.

### 3.6.2 Equinoctial

The equinoctial coordinate type uses the centre of the Earth as the origin and the plane of the satellite’s orbit as the reference plane.

The advantage of this element set is that singularities are limited to retrograde equatorial orbits, parabolic/hyperbolic orbits, and collision orbits. The Keplerian element RAAN is undefined when the inclination is zero and is numerically unstable for an inclination near zero. As the inclination approaches zero, the line of nodes becomes indeterminate. The Keplerian element argument of perigee becomes singular when the eccentricity is zero. As eccentricity approaches zero, the line of apsides becomes

1) Vallado, *Fundamentals of Astrodynamics and Applications*, 2005, p. 151.

indeterminate. The Air Force Satellite Control Network typically solves for the equinoctial elements during the orbit estimation process. [Table 5](#) defines an orbit in this system.

**Table 5 — Equinoctial orbit elements**

Element(s)	Description
Semi-major axis/mean motion	Semi-major axis is half the length of the major axis of the orbital ellipse. Mean motion is the average angular rate of the satellite based on two-body motion.
$h/k/p/q$	$h/k$ collectively describe the shape of the satellite's orbit and the position of perigee. $p/q$ collectively describe the orientation of the satellite's orbit plane.
Mean longitude	Specifies a satellite's position within its orbit at epoch and equals the sum of the classical RAAN, argument of perigee, and mean anomaly.
Formulation	Retrograde, which has its singularity at an inclination of $0^\circ$ , or Posigrade, which has its singularity at an inclination of $180^\circ$ .

### 3.6.3 Delaunay variables

The Delaunay variables coordinate type uses a set of canonical action-angle variables that are commonly used in general perturbation theories. The element set consists of three conjugate action-angle pairs. Lowercase letters represent the angles and uppercase letters represent the conjugate actions. There are two options for the representation of each action variable. The default representation gives the canonical actions used in Hamilton's equations of motion. The other representation, which divides the actions by the square root of the central-body gravitational constant, yields a geometric version of the Delaunay set that is independent of the central body.

- $L$  is related to the two-body orbital energy;
- $G$  is the magnitude of the orbital angular momentum;
- $H$  is the  $z$  component of the orbital angular momentum.

The above components are expressed in terms of distance squared, divided by time, where distance is measured in standard units and time is measured in seconds. The angles are as follows:

- $l$  is the mean anomaly;
- $g$  is the argument of perigee;
- $h$  is the right ascension of the ascending node.

### 3.6.4 Mixed spherical coordinate system

The mixed spherical coordinate type uses a variation of the spherical elements that combines Earth-fixed position parameters with inertial velocity parameters. These are also known as DODS elements.

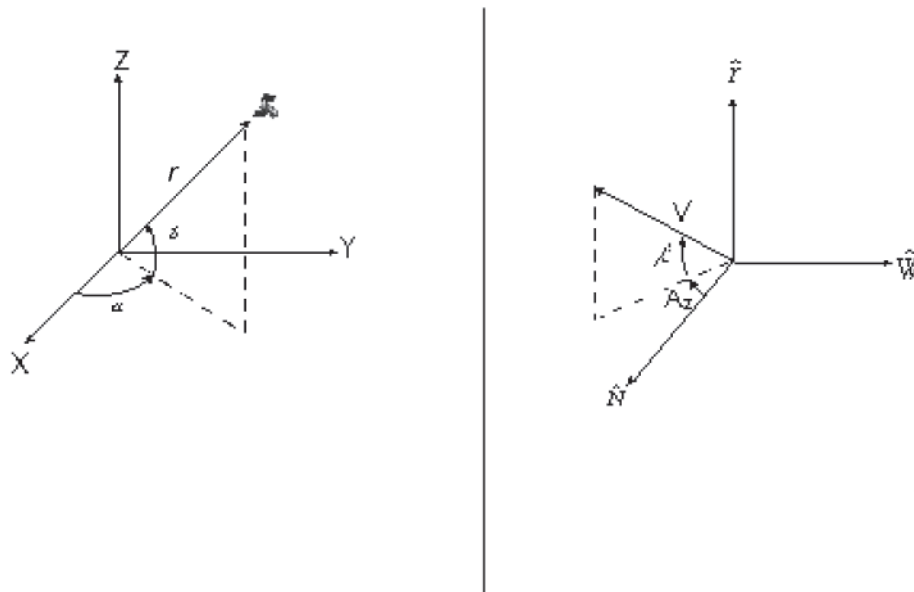
The mixed spherical orbital elements are shown in [Table 6](#).

**Table 6 — Mixed spherical orbit elements**

<b>Element</b>	<b>Description</b>
Longitude	Measured from $-180^\circ$ to $+360^\circ$
Geodetic latitude	Measured from $-90^\circ$ to $+90^\circ$ . The geodetic latitude of a point is the angle between (1) the normal to the reference ellipsoid that passes through the satellite position and (2) the equatorial plane.
Altitude	The object's position above or below the reference ellipsoid. Altitude is measured along a normal to the surface of the reference ellipsoid.
Flight path angle	Horizontal (Hor FPA) or vertical (Ver FPA) flight path angle. The angle between the inertial velocity vector and the radius vector (vertical) or the complement of this angle (horizontal).
Azimuth	The angle in the satellite local horizontal plane between the projection of the inertial velocity vector onto this plane and the local north direction measured as positive in the clockwise direction.
Velocity	The magnitude of the inertial velocity vector.

### 3.6.5 Spherical coordinate system

A spherical coordinate system defines the path of an orbit using polar rather than rectangular coordinates. As shown in [Figure 1](#), the first two elements depend on whether the coordinate system is fixed or inertial.



**Key**

- |          |                                    |           |                       |
|----------|------------------------------------|-----------|-----------------------|
| $r$      | radius                             | $\hat{r}$ | radial direction      |
| $\alpha$ | right ascension, longitude (fixed) | $\hat{N}$ | local North direction |
| $\delta$ | declination, latitude (fixed)      | $\hat{W}$ | local West direction  |
|          |                                    | $V$       | velocity magnitude    |
|          |                                    | $\beta$   | horizontal FPA        |
|          |                                    | $A_z$     | flight path azimuth   |

**Figure 1 — Position and velocity for spherical coordinate elements**

[Table 7](#) lists the spherical orbital elements.

**Table 7 — Spherical orbital elements**

Element	Description
Right ascension (inertial) longitude (fixed)	Right ascension is defined as the angle from the $x$ axis to the projection of the satellite position vector in the equatorial plane measured as positive in the direction of the $y$ axis.
Declination (inertial)/latitude (fixed)	Declination is defined as the angle between the satellite position vector and the inertial equatorial plane measured as positive toward the positive inertial $z$ axis.
Radius	The magnitude of the satellite position vector.
Flight path angle	Horizontal (Hor FPA) or vertical (Ver FPA) flight path angle. The angle between the velocity vector and the radius vector (vertical) or the complement of this angle (horizontal).
Azimuth	The angle in the satellite local horizontal plane between the projection of the velocity vector onto this plane and the local north direction measured as positive in the clockwise direction.
Velocity	The magnitude of the velocity vector.

### 3.6.6 Geodetic

The geodetic orbital elements described in [Table 8](#) are referenced to geodetic coordinates (latitude, longitude, altitude above some reference geoid.)

**Table 8 — Geodetic orbital elements**

Element	Description
Radius or altitude	Radius is measured from the centre of the Earth and specified as distance above or below the reference ellipsoid. Altitude is measured along an outward normal to the surface of the ellipsoid.
Latitude	Measured in degrees from $-90^\circ$ to $+90^\circ$ . The geodetic latitude of a point is the angle between the normal to the reference ellipsoid and the equatorial plane.
Longitude	Measured in degrees from $-360^\circ$ to $+360^\circ$ . The longitude of a point is the angle between the projection of the position vector in the equatorial plane and the prime meridian.
Radius rate or altitude rate	The rate of change of the radius or altitude.
Latitude rate	The rate of change of the satellite's latitude.
Longitude rate	The rate of change of the satellite's longitude.

### 3.7 Reference frames

All orbital parameters shall be anchored to an appropriate frame of reference. A reference frame is a set of three orthogonal axes from which distances and angles are measured. Reference frame issues dominate the exchange of orbital data. Reference frames can be fixed, either in inertial space or to a reference object, such as the Earth. Reference frames can also be associated with moving and accelerating objects or points. Certain astrodynamics problems are more amenable to analytical or numerical solution in some reference frames rather than in others. It is very important to note that although the solution can be easier to obtain when solved in a solution-convenient reference frame, difficulties reemerge when transforming the answer to an operationally meaningful reference frame.

Both celestial and inertial reference frames are employed so as to describe or estimate satellite orbits. Terrestrial frames are bound to the Earth, which itself is dynamic in inertial space. Celestial frames are bound generally to extremely distant objects which for most purposes, are stationary on the celestial sphere. The vernal equinox is one such reference (first point of Aries) that lies within the ecliptic plane, defined by the Earth's mean orbit about the Sun. Since the Earth's axis is inclined relative to the ecliptic (and the inclination is not constant), the right ascension, celestial longitude, and celestial latitude of an object are different from any latitude and longitude on the Earth, and the elevation angle from any point on the Earth is different from right ascension. This Technical Report prescribes use of a terrestrial or mixed terrestrial/orbital reference frame, within which satellite observations are accomplished. Orbit information providers shall also provide characteristics of the reference frame at epoch of the orbital information. [Annex B](#) cites examples of standard reference frames acceptable within this standard. These are not the only acceptable reference frames.

### 3.8 State variables, mean orbits, and covariance

Every orbit estimation process begins with the selection and definition of state variables. State variables are the products of orbit determination. They form a one-dimensional column vector. Classically, the state of an object is just its state of motion, described completely in Newtonian mechanics by its position and velocity. The existence of non-conservative forces and perturbations that cannot be described simply by point mass inverse square Newtonian gravitation expands the number of state variables necessary to estimate an object's motion. Since all sources of uncertainty cannot be explained or even recognized, a fictitious "consider variable" is sometimes augmented to the state vector to capture uncertainties otherwise unaccountable within a tractable set of physically meaningful state variables.

Mean orbits are the sets of parameters that emerge from the smoothing, filtering, or predictive estimation schemes. There are as many different possible mean orbits as there are permutations of the quantities and functions discussed in [3.5](#). and [3.6](#).

Covariances are measures of the interdependence of uncertainties in orbit state variables relative to their mean values, the degree to which changes in one are related to changes in another. Covariances are, therefore, symmetric matrices. The correlation coefficient is the binary covariance of two random variables divided by the product of their individual variances, so that it varies from  $-1$  to  $+1$ . If a correlation coefficient is zero, the two variables change independently of each other and are uncorrelated. The sign of a covariance element indicates whether the changes in the two variables are in the same direction or not.

Orbital data provided for independent or collaborative orbit propagation under auspices of this Technical Report shall include both mean orbits and covariances. The information package shall also describe broadly the formalism employed to develop mean elements and covariances: least squares (batch or sequential) or filtering.

### 3.9 Orbit propagators

Orbit propagators are comprehensive tools that combine physical models, all of the characteristics in [3.5](#) and data input/output utilities. There are three types of orbit propagators: analytic, semi-analytic, and numerical. Analytic propagators use a closed-form solution of the time-dependent motion of a satellite to produce ephemeris or to provide directly the position and velocity of a satellite at a particular time. Numerical propagators numerically integrate the equations of motion for the satellite. Semi-analytic schemes employ some closed-form approximations and some numerical integration. Within each category, propagators differ in choices of alternatives.

When a widely used, consensus-validated, and authoritatively documented propagator is employed, the requirements of this Technical Report can be satisfied by citing that documentation and the specific parameter sets that the data provider employed within that propagator, which vary with propagator and version. [Annex D](#) cites representative well-documented propagator schemes acceptable within this standard. These are examples only. Other approaches adequately described can also be acceptable.

## 4 Documentary requirements

The requirements of this Technical Report shall be reported in the formats attached ([Annex D](#)). This Technical Report is completely satisfied only when all elements of information have been provided. When this is not possible or is precluded by industrial, local, or national policies, partial compliance is encouraged. For partial compliance, any data elements that cannot be reported shall be so labelled with the associated reason (industrial security, national policy, etc.). Specific orbit data message requirements for metadata shall also be satisfied where required by international standard.

## Annex A (informative)

### Representative widely used orbit determination and estimation tool sets

The representative widely used orbit determination and estimation tool sets are as follows:

- **Precision Orbit Determination System (PODS)** – The propagator for a PODS uses estimation algorithms to determine spacecraft orbits based on observation data from ground-based and space-based sensors; Optimized, Bayesian, weighted LS estimation with high-fidelity atmospheric modelling.
- **Goddard Trajectory Determination System (GTDS)** – Batch, mainframe computer, initially Fortran; Bayesian, weighted, least squares estimator.
- **Draper Semianalytic Satellite Theory (DSST)** – This is a very comprehensive mean element approach.
- **Real Time Orbit Determination (RTOD)** – This is an extended, sequential Kalman filter capable of determining four satellite orbits simultaneously in real time.
- **Orbit Determination ToolKit (ODTK)** – This is a Kalman filter approach that accommodates measurement and process noise consistently. It is a suite of tools for initial orbit determination, filtering, and smoothing that accepts a variety of common measurement types.
- **Orbit Determination Error Analysis System (ODEAS)** – Batch and sequential least squares, sequential Kalman filter.
- **Orbit/Covariance Estimation Analysis (OCEAN)** – Full numerical filtering capable of parallel processing of independent phenomena and events.

## Annex B (informative)

### Representative reference frames

Representative coordinate reference frames are as follows:

- Earth-centred, Earth-fixed (ECEF): Orthogonal system from the centre of the Earth with vertical axis through the geographic North Pole and horizontal axis fixed at 90° longitude;
- Earth-centred, orbit-based, inertial (Perifocal): Centred on the centre of the Earth, but in the orbital plane of a designated satellite;
- Orbital frame: Reference frame affixed to a point on the orbit of a designated satellite with the vertical axis in the nadir direction;
- Body-fixed: Reference frame affixed to a designated satellite with vertical axis in the nadir direction;
- J2000:  $x$  toward mean vernal equinox,  $z$  along Earth's mean rotational axis on 1 Jan 2000, 12:00:00.00 UTC.;
- B1950:  $x$  toward mean vernal equinox,  $z$  along Earth's mean rotational axis on 31 Dec 1949, 22:09:07.2;
- True equator, mean equinox of date (or epoch):  $x$  points along the mean vernal equinox and  $z$  points along the true rotation axis of the Earth at the specified date (or coordinate epoch). Not recommended, since this was conceived for computational convenience during early space operations. Although some data are presented in this frame, there is no consensus or authoritative definition;
- Mean of epoch (or date):  $x$  points toward the mean vernal equinox and  $z$  points along the mean rotation axis of the Earth at coordinate epoch (or orbit epoch);
- True of epoch (or date or reference date): toward the true vernal equinox and  $z$  points along the true rotation axis of the Earth at coordinate epoch (or orbit epoch or specified reference date);
- Alignment of epoch: An inertial reference frame coincident with Earth-centred fixed-at-coordinate epoch.



## **Annex C** (informative)

### **Representative numerical integration schemes**

Representative numerical integration schemes include

- Runge-Kutta of order  $N$  (RK- $N$ ),
- Runge-Kutta-Fehlberg of order  $N$  (RKF- $N$ ),
- Bulirsch Stoer,
- Gauss-Jackson of order  $N$ ,
- variation of parameters in universal variables,
- predictor-corrector (full- or pseudo-correction),
- step-size control (fixed, adaptive),
- time regularization (time steps proportional to eccentric or true anomaly), and
- interpolation scheme (variation of parameters, Lagrange, Hermitian).

## Annex D (informative)

### Sample data sheet

Each entry category requires additional hierarchical information and narrative explanation.

ORBIT DETERMINATION AND PROPAGATION MODEL AND PROCESS DESCRIPTION WORK SHEET  
 ISO STANDARD XXX-XXX

INFORMATION ELEMENT	SUBELEMENT	DESCRIPTION						
ORBIT DETERMINATION								
INITIAL ORBIT DETERMINATION SCHEME								
SUBSEQUENT ORBIT DETERMINATION SCHEME								
OBSERVATIONS	OBSERVED QUANTITIES							
	FACILITY AND LOCATION INFORMATION							
	SATELLITE INFORMATION							
	ESTIMATION PARAMETERS AND CONTROLS							
	TRACKING DATA SELECTION AND EDITING							
ORBIT PROPAGATION								
FORCE MODEL	GRAVITATION							
	ATMOSPHERIC RESISTANCE							
	RADIATION PRESSURE							
	RELATIVISTIC AND OTHER INFLUENCES							
NUMERICAL OR ANALYTICAL APPROACH								
ORBIT ELEMENT SET								
COORDINATE SYSTEM								
REFERENCE FRAMES								
STATE VARIABLES AND COVARIANCE MATRIX INFORMATION								

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- [3] ANALYSIS Space Mission and Design, 3rd ed, James R. Wertz and Wiley J. Larson, Space Technology Library, Microcosm Press, 2009
- [4] SPACECRAFT SYSTEMS ENGINEERING. *Peter Fortescue, John Stark, and Graham Swinerd*. Wiley, London, Third Edition, 2003





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