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Electric cables — Calculation of current rating — Cable current rating calculations using finite element method

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British Standards

National foreword

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The British Standards which implement international publications referred to in this document may be found in the *BSI Catalogue* under the section entitled "International Standards Correspondence Index", or by using the "Search" facility of the *BSI Electronic Catalogue* or of British Standards Online.

Summary of pages

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ELECTRIC CABLES – CALCULATIONS FOR CURRENT RATINGS – FINITE ELEMENT METHOD

FOREWORD

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IEC 62095, which is a technical report, has been prepared by IEC technical committee 20: Electric cables.

The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
20/600/DTR	20/634/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this publication will remain unchanged until 2014. At this date, the publication will be

- reconfirmed;
- withdrawn;
- replaced by a revised edition, or
- amended.

ELECTRIC CABLES – CALCULATIONS FOR CURRENT RATINGS – FINITE ELEMENT METHOD

1 Introduction

1.1 General

The most important tasks in cable current rating calculations are the determination of the conductor temperature for a given current loading or, conversely, the determination of the tolerable load current for a given conductor temperature. In order to perform these tasks the heat generated within the cable and the rate of its dissipation away from the conductor, for a given conductor material and given load, must be calculated. The ability of the surrounding medium to dissipate heat plays a very important role in these determinations and varies widely because of factors such as soil composition, moisture content, ambient temperature and wind conditions. The heat is transferred through the cable and its surroundings in several ways. For underground installations the heat is transferred by conduction from the conductor, insulation, screens and other metallic parts. It is possible to quantify the heat transfer processes in terms of the appropriate heat transfer equation as shown in Annex A (equation A.1).

Current rating calculations for power cables require a solution of the heat transfer equations which define a functional relationship between the conductor current and the temperature within the cable and its surroundings. The challenge in solving these equations analytically often stems from the difficulty of computing the temperature distribution in the soil surrounding the cable. An analytical solution can be obtained when a cable is represented as a line source placed in an infinite homogenous surrounding medium. Since this is not a practical assumption for cable installations, another assumption is often used; namely, that the earth surface is an isotherm. In practical cases, the depth of burial of the cables is in the order of ten times their external diameter, and for the usual temperature range reached by such cables, the assumption of an isothermal earth surface is a reasonable one. In cases where this hypothesis does not hold; namely, for large cable diameters and cables located close to the ground surface, a correction to the solution has to be used or numerical methods should be applied.

With the isothermal surface boundary, the steady-state heat conduction equations can be solved assuming that the cable is located in a uniform semi-infinite medium.

Methods of solving the heat conduction equations are described in IEC 60287 (steady-state conditions) [1] and IEC 60853 (cyclic conditions), for most practical applications. When these methods cannot be applied, the heat conduction equations can be solved using numerical approaches. One such approach, particularly suitable for the analysis of underground cables, is the finite element method presented in this document. The cases when the use of the finite element method is recommended are discussed next.

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[1] IEC 60287 has been withdrawn and replaced by a series of publications (see item 2 of the Bibliography).

1.2 Field of application

In classical cable rating calculations, the heat conduction equation is solved under several simplifying assumptions [1] [2]. This limits the field of the applicability of the analytical methods. The limitations of the classical methods will be apparent from a few examples. In the analytical methods described in IEC 60287 [2], IEC 60853-1 [3] and IEC 60853-2 [4], the case of a group of cables is dealt with on the basis of the restricted application of superposition. To apply this principle, it must be assumed that the presence of another cable, even if it is not loaded, does not disturb the heat flux path from the first cable, nor the generation of heat within it. This allows separate computations to be performed for each cable with the final temperature rise being an algebraic sum of the temperature rises due to cable itself and the rise caused by the other cables. Such a procedure is reasonably correct when the cables are separated from each other. When this is not the case, for example for cables in touching formation, the temperature rise caused by simultaneous operation of all cables should be considered. A direct solution of the heat conduction equation employing the finite element method offers such a possibility.

Numerical methods also permit more accurate modelling of the region's boundaries for example, a convective boundary at the earth surface, constant heat flux circular boundaries for heat or water pipes in the vicinity of the cables, or an isothermal boundary at the water level at the bottom of a trench. Thus, when an isothermal boundary cannot be assumed, for example, for cables installed in shallow troughs or directly buried not far from the ground surface, the finite element method provides a suitable tool for the thermal analysis.

Perhaps the most obvious case when the analytical approximations fail is when the medium surrounding the cable is composed of several materials having different thermal resistivities. Figure 2 shows an example of such situation. This is an actual cable installation where not only were several soil characteristics present, but also, a vertical convective boundary had to be dealt with. The non-uniform soil conditions and non-isothermal boundaries are handled easily by the finite element method. The computational efficiency of this approach is also quite satisfying. With presently available personal computers, calculations involving networks with several thousand nodes can be performed in a matter of minutes.

There are also advantages in using the finite element method in the transient analysis. The analytical approach for transient calculations is described in IEC 60853-1 and IEC 60853-2. In this document, separate computations are performed for the internal and the external parts of the cable. Coupling between internal and external circuits was achieved by assuming that the heat flow into the soil is proportional to the attainment factor of the transient between the conductor and the outer surface of the cable. The validity of the methods did not rest on an analytical proof, but on an empirical agreement of the responses given by the recommended circuits and the temperature transients calculated by more lengthy but more accurate computer-based methods. Here, again, the finite element method offers a solution with minimal simplifying assumptions.

It should be noted that the value selected for the thermal resistivity of the soil, and its temperature, will have a significant influence on any calculated current rating or cable temperature. In many cases there is little to be gained by using a ‘more accurate' method of calculation if soil conditions are not known with a degree of certainty.

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2 Figures between brackets refer to the bibliography.

1.3 Information obtained from the finite element method

The usual cable rating problem is to compute the permissible conductor current so that the maximum conductor temperature does not exceed a specified value. Numerical methods, on the other hand, are used to compute the temperature distribution within the cable and its surroundings given heat generated within the cable (this is particularly useful when we need to determine the temperature field and specific isotherms around the cable). However, when numerical methods are used to determine cable rating, an iterative approach has to be used for the purpose. This is accomplished by specifying a certain conductor current and calculating the corresponding conductor temperature. Then, the current is adjusted and the calculation repeated until the specified temperature is found convergent within a specified tolerance.

An explanation of the finite element method is given in Clause 2 followed by the discussion of input requirements in Clause 3. In Clause 4, several examples where the application of the finite element approach is advisable are presented.

1.4 Alternative methods

Although this report concentrates on the use of finite element methods for the calculation of heat transfer through the materials surrounding buried cables, other numerical methods are available. These include finite difference methods, boundary element methods, the superposition method described in Electra 87 [5] and the approaches combining conformal transformation and the finite difference method.

Finite difference methods (FDM) are frequently used in the study of electric stress distribution in cable joints and terminations. It has been shown that FDM is more suitable than FEM for three dimensional cable problems. This is because difficulties can arise when using FEM to model long thin objects, such as cables, in three dimensions. However, FDM is intended for use with rectangular elements and hence is not well suited for modelling curved surfaces.

Boundary element methods need less effort in defining the input data and use less computer time than FEM. However, transient analysis cannot be performed using boundary element methods.

The superposition method described in Electra 87 for the calculation of the response of single core cables to a step function thermal transient has a number of advantages over FEM. These include the following:

a) it requires relatively little modelling data, typically less than 100 nodes compared with 1000 nodes for FEM. The method is therefore more suitable for real time rating systems. The one-dimensional temperature field can be derived using numerically stable methods. Hence, relatively large time steps can be used without introducing significant errors;

b) approximate methods can be developed to use this approach when two different cable backfills exist;

c) the method can be used as a basis for calculating transient temperatures for three dimensional problems such as occur in cable joint bays and systems with separate water cooling;

d) it can be used to calculate mutual heating between crossing cables;

e) it is suitable for studying the effect of temperature dependant material properties such as conductor resistance, dielectric losses and soil thermal resistivity.

Although this superposition method is suitable for many cable rating problems, it is not well suited to problems involving a large number of cables and complex geometry.

The approach applying conformal transformation was proposed by CIGRE WG 21.02 and is described in Electra 98 [6]. Germay and Mushamalirwa [7] compared the finite element method with four approaches based on a conformal transformation of the z-plane perpendicular to the cable axes into a w-plane, in order to transform the circular boundaries of the cables into horizontal straight segments to facilitate the solution. However, the conformal transformation method has several drawbacks. The major one is that the equations describing the transformed network are equivalent to finite difference equations obtained by discretising the heat equation in the w-plane and, hence, the complexity of a numerical solution of the heat conduction problem is not avoided. Another drawback is that both the earth and cable surfaces are assumed to be isothermal. In addition, transformation of the boundaries between regions with different resistivities point by point is very laborious and the resulting computer software cannot efficiently handle more than four cables in one installation.

2 Overview of the finite element method

The finite element method is a numerical technique for solving partial differential equations. Among many physical phenomena described by such equations, the heat conduction problem and heat and mass transfer in the vicinity of power cables have been addressed in the literature ([8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21]). The fundamental concept of the finite element method is that temperature can be approximated by a discrete model composed of a set of continuous functions defined over a finite number of sub-domains. The piecewise continuous functions are defined using the values of temperature at a finite number of points in the region of interest.

The discrete solution is constructed as follows.

a) A finite number of points in the solution region is identified. These points are called nodal points or nodes.
b) The value of the temperature at each node is denoted as variable which is to be determined.
c) The region of interest is divided into a finite number of sub-regions called elements. These elements are connected at common nodal points and collectively approximate the shape of the region.
d) Temperature is approximated over each element by a polynomial that is defined using nodal values of the temperature. A different polynomial is defined for each element, but the element polynomials are selected in such a way that continuity is maintained along the element boundaries. The nodal values are computed so that they provide the "best" approximation possible to the true temperature distribution. This approach results in a matrix equation whose solution vector contains coefficients of the approximating polynomials. The solution vector of the algebraic equations gives the required nodal temperatures. The answer is then known throughout the solution region.

The construction of a discrete solution can be illustrated by using a one-dimensional example of the temperature distribution in a fin, shown in Figure 1 [22].

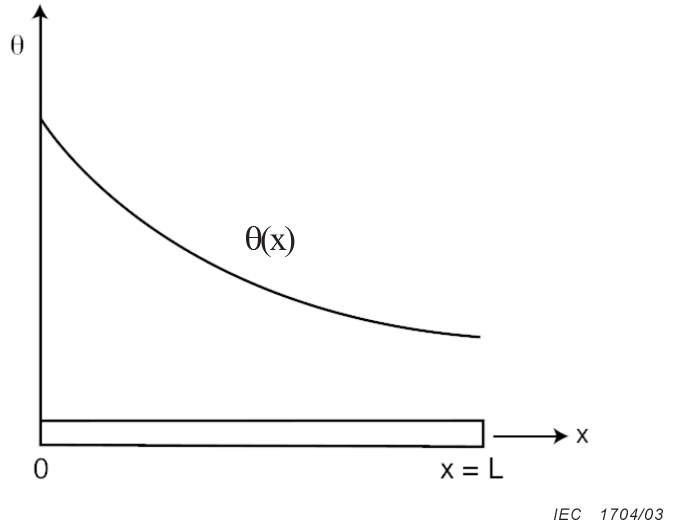


Figure 1 – Temperature distribution in a one dimensional fin

The continuous function is the temperature distribution $\theta(x)$ and the domain is the interval [0, *L]* along the *x*-axis. The nodes are shown in Figure 2a (they do not have to be equally spaced). The values of $\theta(x)$ are then specified at each node point. These values are shown graphically in Figure 2b and are labelled to match the node numbers, $\theta_1, \theta_2, ..., \theta_5$.

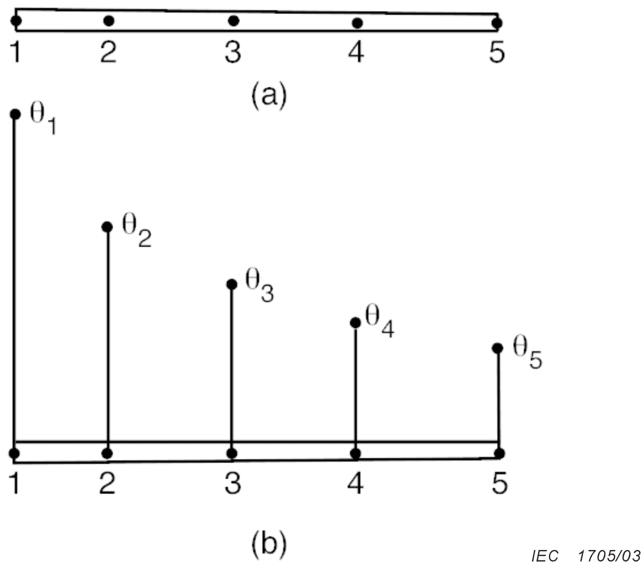


Figure 2 – The nodal points and the assumed values of $\theta(x)$

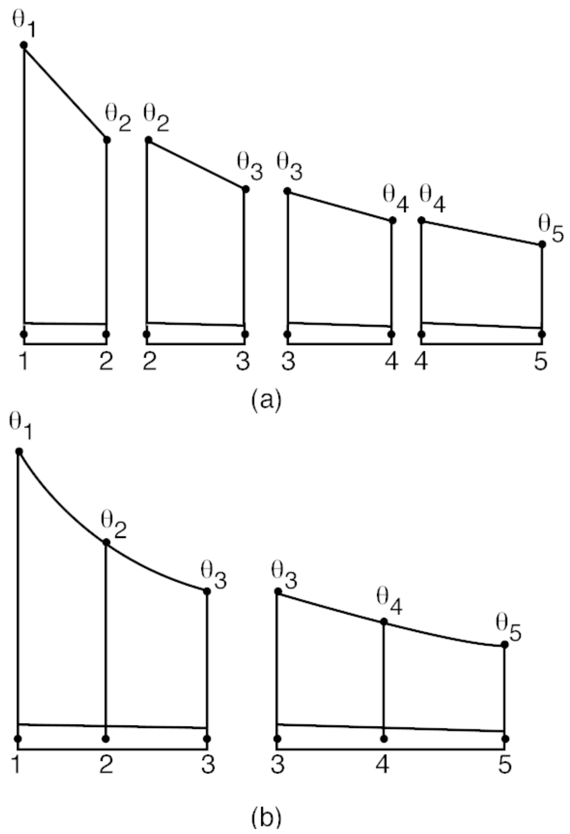


Figure 3 – Division of the domain into elements

The division of the domain into elements can proceed in two ways. We can limit each element to two nodes yielding four elements (Figure 3a), or we can divide the domain into two elements each with three nodes (Figure 3b). The element polynomial is defined using the values of $\theta(x)$ at the element nodal points. If we subdivide the region into four elements, there will be two nodes per element, and the element function will be linear in x. The final approximation to $\theta(x)$ would consist of four piecewise continuous linear functions, each defined over a single element (Figure 4a).

The division of the domain into two elements allows the element function to be a quadratic equation. The final approximation to $\theta(x)$ in this case would be two piecewise continuous quadratic functions (Figure 4b). The element functions constitute a piecewise continuous approximation because the slopes of these two quadratic functions are not necessarily the same at node three.

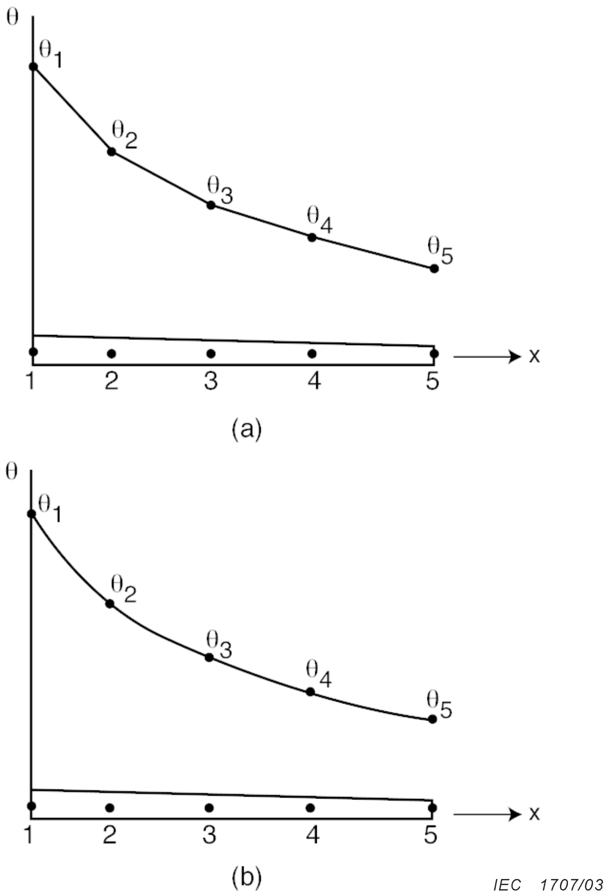


Figure 4 – Discrete models for one-dimensional temperature distribution

Generally, the temperature distribution is unknown, and we wish to determine the values of this quantity at certain points. The temperatures $\theta_1, \theta_2, ..., \theta_n$ at the nodal points are unknown and they have to be determined first. The procedure for the determination of these temperatures is described in Annex A.

In cable rating applications, two-dimensional elements are most commonly used. The elements in the two-dimensional domain are functions of *x* and *y* and are generally either triangular or quadrilateral in shape. The element function becomes a plane (Figure 5) or a curved surface (Figure 6). The plane is associated with the minimum number of element nodes, which is three for the triangle and four for the quadrilateral.

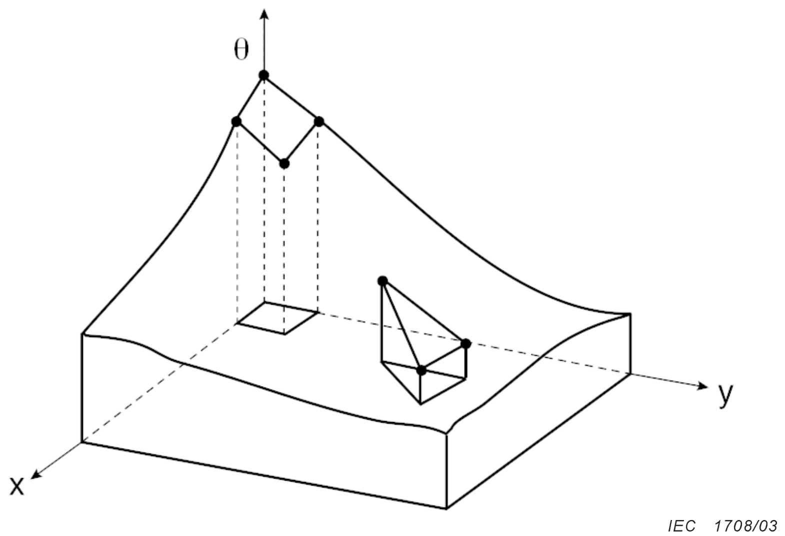


Figure 5 – Modelling of a two-dimensional scalar function using triangular or quadrilateral elements

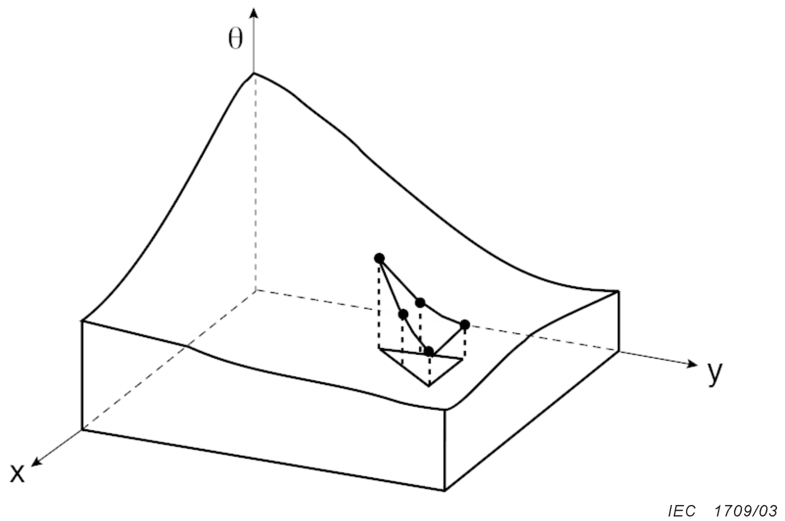


Figure 6 – Modelling of a two-dimensional scalar function using a quadratic triangular element

The element function can be a curved surface when more than the minimum number of nodes is used. An excess number of nodes also allows the elements to have curved boundaries.

3 Practical considerations when applying the finite element method for cable rating calculations

When the finite element program is used for the analysis of an underground cable installation, the user retains the control over several parameters which influence the accuracy of the calculations. These are: (1) size of the region to be discretised, (2) size of the elements constructed by mesh generator, (3) type and location of region boundaries, (4) representation of cable losses, and (5) selection of the time step in transient analysis. These topics are reviewed below.

3.1 Selection of the region to be discretised

Location of boundaries is an important consideration in numerical studies. The earth's surface is an obvious boundary, but the region underneath is infinite. The objective is to select a large enough region so that the calculated values along the boundaries agree with those that exist in the physical problem. For the cable rating problem, this means that the side and bottom boundaries must be selected in such a way that the nodal temperatures at those boundaries all have the same value and the temperature gradient across the boundary is equal zero.

Experience plus a study of how others modelled similar infinite regions is probably the best guide. In our experience, a rectangular field 10 m wide and 5 m deep, with the cables located in the centre, gives satisfactory results in the majority of practical cases.

For transient analysis, the radius of the soil, out to which heat disperses, will increase with time and for practical purposes it is sufficient to consider only that radius within which a sensible temperature rise occurs. This radius can be estimated from the following equation assuming that each cable is a line source of heat:

$$\theta_{r,t} = \frac{W_I \rho_s}{4\pi}\left[-Ei\left(\frac{-r^2}{4\delta t}\right)\right]$$

where $\theta_{r,t}$ is the threshold temperature value at the distance r from the cable axis and the remaining notation is that used in IEC 60853. The value of θ_{r2} can be taken as 0,1 K when the number of cables is not greater than 3 and suitably smaller for a large number of cables. The above equation is applied for each cable. The region to be discretised will be an envelope around all the circles.

3.2 Element sizes

In most of the commercial mesh generators, the user retains some sort of control of the element sizes. This is achieved by specifying the spacing between boundary nodes for various parts of the network (cables, backfill, soil, etc.). This spacing can be varied to obtain desired element sizes (the elements should be smallest closer to the cables). The smaller the element size, the more accurate the results. However, the computing time rises exponentially with the number of elements in the model. Hence, a compromise is required in the selection of the suitable element size. Since the smallest elements are usually associated with the cable itself, their maximum size cannot be greater than the dimensions of the part of the cable which is fitted with the elements. Figure 7a shows an example of 6 cables located in a concrete duct bank and Figures 7b and 7c show two finite element representations of the two middle cables in this installation. In Figure 7b, the largest permissible element size is used, and in Figure 7c much smaller elements are applied. The 80°C isotherm is shown for both cases to show the effect of the approximations.

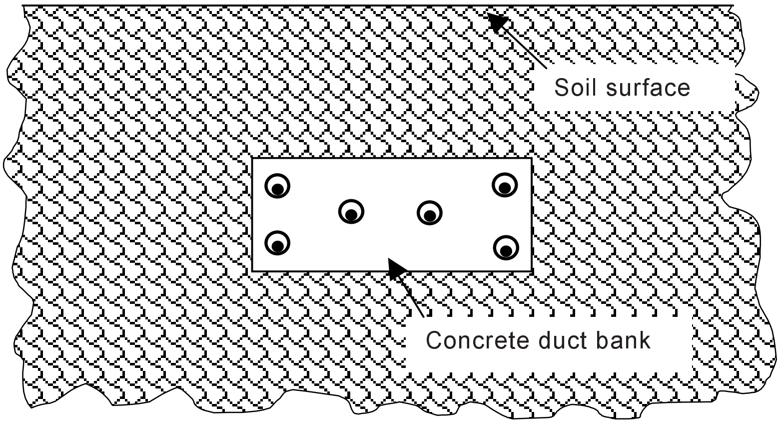


Figure 7(a) – Example of 6 cables in a concrete duct bank, installation

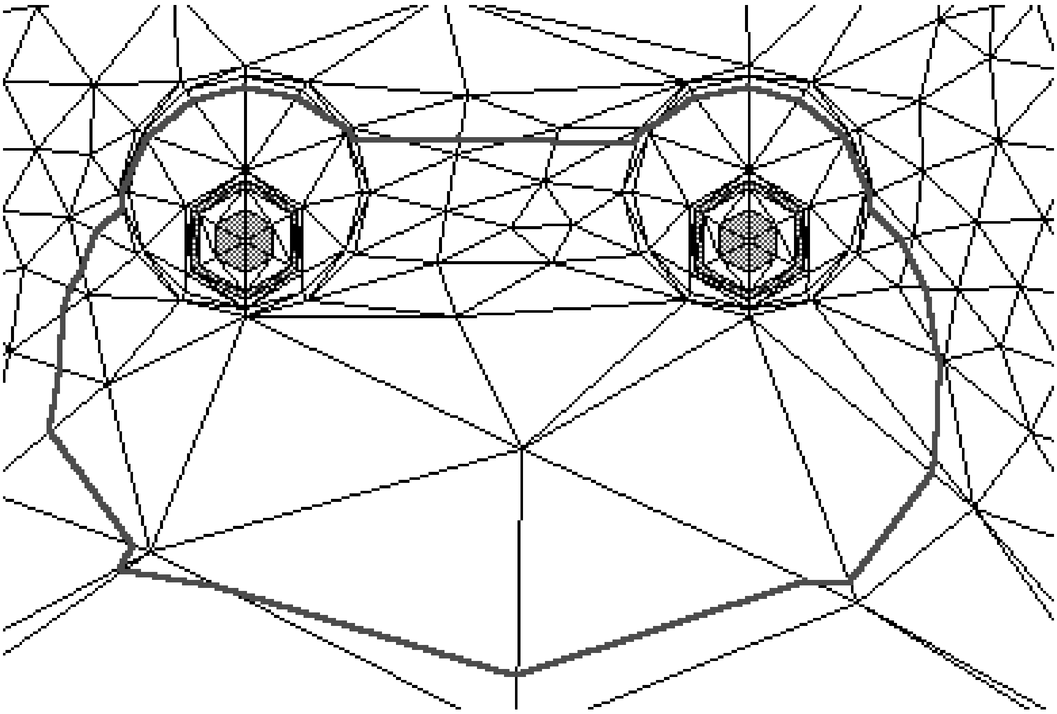


Figure 7(b) – Example of 6 cables in a concrete duct bank, coarse mesh

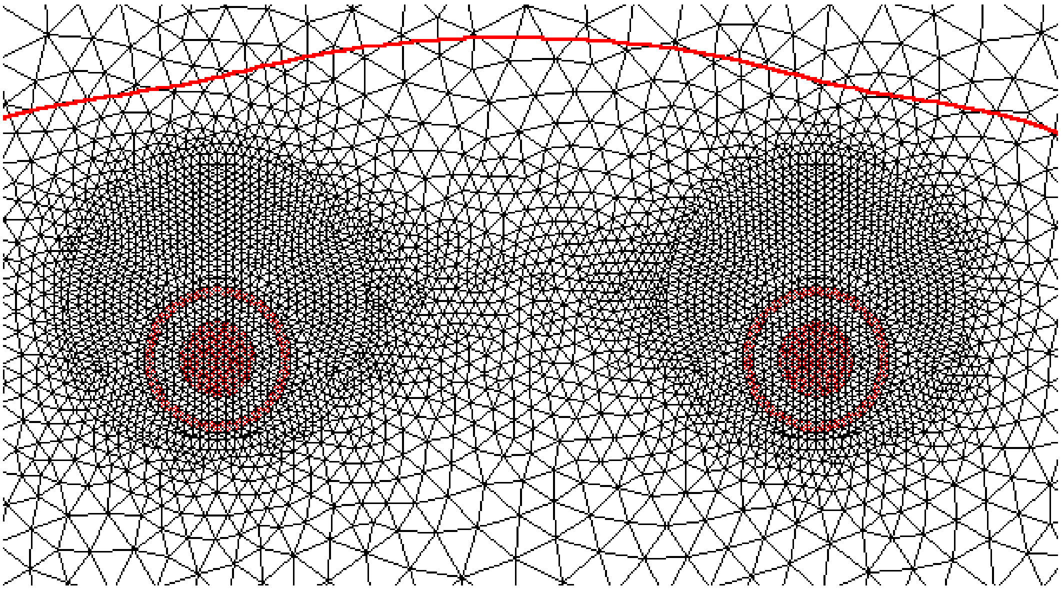


Figure 7(c) – Example of 6 cables in a concrete duct bank, fine mesh

Figure 7 – Example of meshing a finite element model

As can be seen, the location and shape of the isotherm is somewhat different in both cases. The conductor temperature in case (b) is 93,3°C and in case (c) 96,5°C. The calculations took about 25 times longer in case (c) than in case (b). This particular example confirms a general observation that a fairly coarse network can give satisfactory results if a precision of a few degrees is acceptable.

3.3 Boundary conditions

Unlike in the case of the classical cable rating computations, where the isothermal earth surface boundary is assumed, the finite element method allows representation of different boundary conditions and arbitrary boundary locations. Both straight line and curved boundaries can be represented. In particular, circular boundaries representing either cable, water or steam pipe surfaces can easily be handled. In some applications, only the external thermal resistance of the cable may be of interest and the circular boundary representation of the cable can be used in this case.

Three different boundary conditions are applicable for cable current rating calculations using the finite element method. If the temperature is known along a portion of the boundary, then the isothermal condition exists. This temperature may be a function of the surface length. The information required by the finite element software is the value of the boundary temperature. It should be noted that if the conditions represented in the IEC 60287 approach are to be modelled using the finite element method, this temperature is the ambient temperature at the depth of the cable burial.

If heat is gained or lost at the boundary due to convection, the convective boundary exists. Such a boundary should be used when large diameter cables are installed close to the ground surface. In this case, the user will be required to specify the value of the heat convection coefficient and air ambient temperature. The convection heat transfer at the earth surface includes natural and forced convection. Normally, forced convection is much stronger than natural convection. Determination of the convection coefficient is a very important task in computation of ratings of cables using the finite element method. The value of this coefficient varies between 2 $W/m^2 \cdot K$ and 25 $W/m^2 \cdot K$ for free convection and between 25 $W/m^2 \cdot K$ and 250 $W/m^2 \cdot K$ for forced convection. The lower the value of this coefficient, the more severe heating occurs in the ground.

The third type of the boundary which can occur in cable rating calculations is the constant heat flux boundary. Such a boundary will most likely be required when there are other heat sources in the vicinity of the cables being examined and their heat generation is known.

3.4 Representation of cable losses

Conductor, sheath and dielectric losses are represented in the finite element studies as heat sources and provision should be made to vary these as required with time and/or temperature. Values of these losses are recalculated at each time step, using methods given in IEC 60287.

Conductor, sheath and armour losses are temperature dependent. Therefore, an iterative procedure is required. Usually, 3 to 4 iterations are sufficient to obtain the required accuracy.

For transient calculations in the finite element method, each node contains an initial temperature rise at $t = 0$. In the general case, where the transient due to the dielectric loss may not have reached its steady-state, these initial temperature rises (relative to the ambient temperature outside of the mesh) must be obtained from a prior calculation. From the beginning of the transient the computation must in all cases take account of both the Joule and the dielectric losses. The two usual situations are as follows:

a) the transient due to dielectric losses has reached a steady-state (the voltage has been applied for a very long time). The initial temperature rise at each node is put equal to the steady-state temperature rise at that point caused by the dielectric losses only;
b) the voltage is applied to the cable at the same time as the load current. In this case, the initial temperature rises are zero and the dielectric loss generators should be allocated their proper values from time zero.

3.5 Selection of a time step

Since, in general, the computations involve evaluation of temperatures in increments of time, care must be taken in the selection of the time step. In principle, one should select as large a time interval as possible to reduce the amount of computation. Unfortunately, the size of the time step can affect the accuracy of the computations if too large a step is used. Electra 87 and Libondi [23] provide some guidance on the selection of suitable values. The duration of the time step, $\Delta\tau$, will depend on

a) the time constant, $\sum T \cdot \sum Q$ of the network (defined as the product of its total thermal resistance (between conductor and outer surface) and its total thermal capacitance (whole cable)),

b) time elapsed from the beginning of the transient, τ, and

c) the location of the time τ with relation to the shape of the load curve being applied.

NOTE Requirement c) can be illustrated as shown in Figure 8.

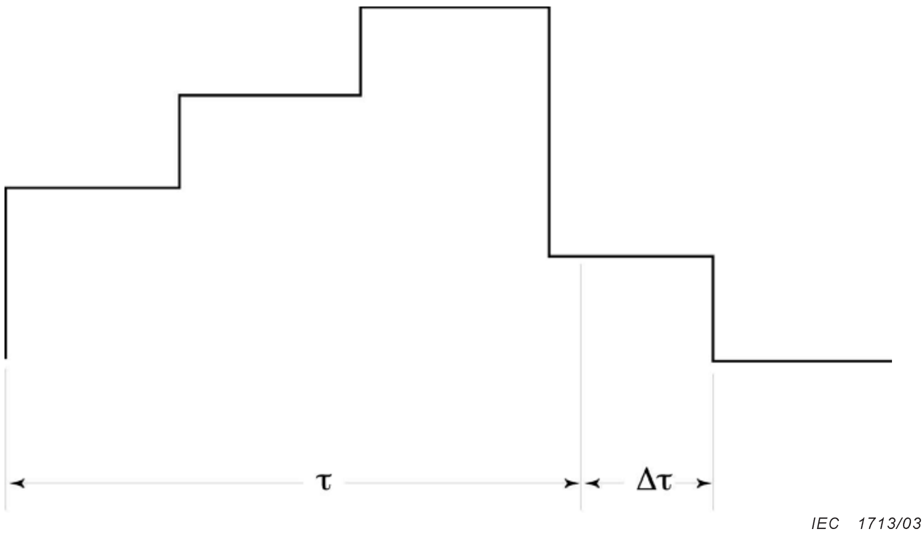


Figure 8 – Relationship between the time step, the load curve and the time elapsed from the beginning of the transient

The following conditions are suggested for the selection of the time step $\Delta\tau$ (Electra 87):

$$\log_{10}\frac{\Delta\tau}{\Sigma T\cdot\Sigma Q}=\frac{1}{3}\log_{10}\frac{\tau}{\Sigma T\cdot\Sigma Q}-1{,}58 \qquad \text{for} \qquad \tau<\frac{1}{3}\Sigma T\cdot\Sigma Q$$

$$\log_{10}\frac{\Delta\tau}{\Sigma T\cdot\Sigma Q}=\frac{1}{3}\log_{10}\frac{\tau}{\Sigma T\cdot\Sigma Q}-1{,}25 \qquad \text{for} \qquad \tau>\frac{1}{3}\Sigma T\cdot\Sigma Q$$

Adjusting the time step automatically during the computations is the preferable approach.

4 Examples of application of the finite element method for cable rating calculations

The following examples show applications of the finite element method for current rating calculations of underground cables. The first example examines a standard cable installation which can easily be solved using the analytical approach described in IEC 60287. The remaining two examples show the situations when analytical methods are not advisable.

4.1 Example 1

Consider a 30 kV three-core, XLPE-insulated cable with 300 mm^2 copper conductor and a lead sheath. All thermal and electrical parameters are as specified in IEC 60287 (1982).

The cable is directly buried as shown in Figure 9.

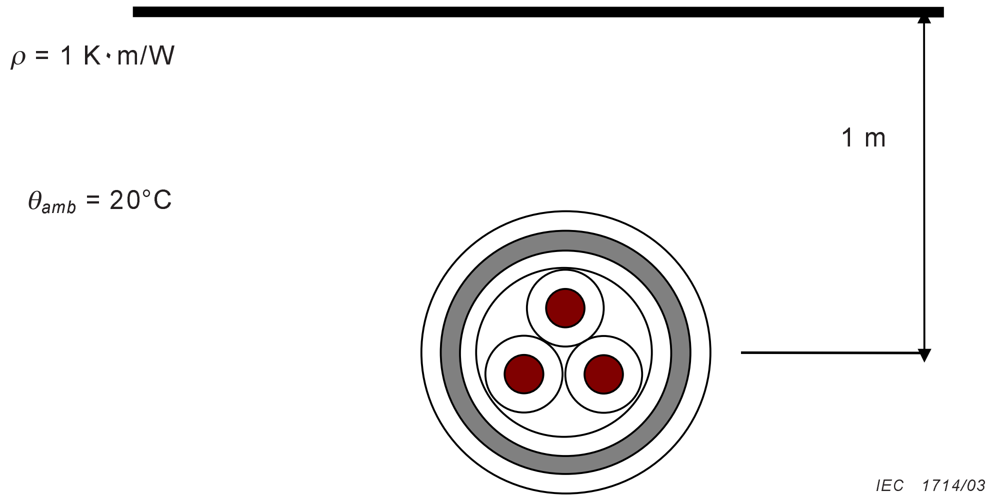


Figure 9 – Laying conditions for the finite element study in example 1

Soil ambient temperature is 20°C. Thermal resistivity of the soil is shown in Figure 9.

The results of the analysis performed using the IEC 60287 method and the finite element approach are summarised in Table 1.

Table 1 – Comparison of the IEC 60287 and the finite element results for cables in example 1

Calculation method	Temperature °C		
	Conductor	Sheath	External
IEC 60287	90	71,1	66,4
Finite element	89,3	70,5	66,5

The results are remarkably close for this standard cable installation.

4.2 Example 2

This example shows an installation where with different cable types, several soil layers and a convective vertical boundary, see Figure 10. This system cannot be examined with the use of the methods described in IEC 60287, but can be easily handled by the finite element method. The isotherms computed with the finite element program are shown in Figure 11.

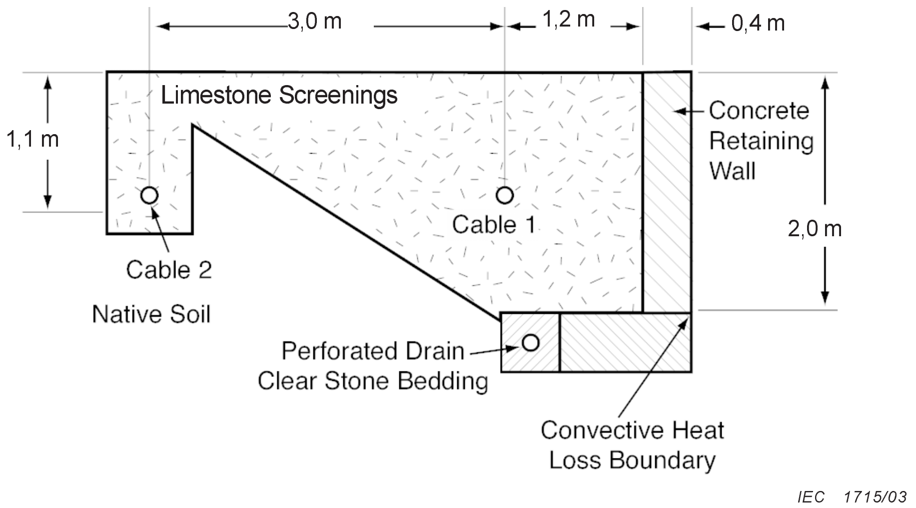


Figure 10 – Installation for example 2

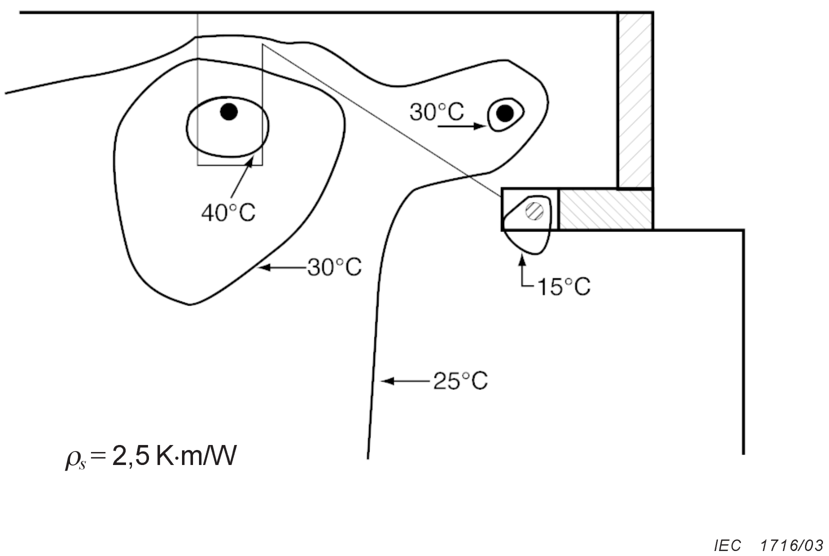


Figure 11 – Isotherms for the system in Figure 10

4.3 Example 3

When cables are installed in sand-filled troughs, either completely buried or with the cover flush with the ground surface, there is a danger that the sand will dry out and remain dry for long periods. The cable external thermal resistance may then be very high and the cable may reach undesirably high temperatures. IEC 60287 advises to calculate the cable rating using a value of 2,5 K·m/W for the thermal resistivity of the sand filling unless a specially selected filling has been used for which the dry resistivity is known. When cables are located in shallow troughs, a convective boundary has to be modelled. To illustrate the difference in the temperature computed using the IEC 60287 approach and the finite element method, consider an installation shown in Figure 12.

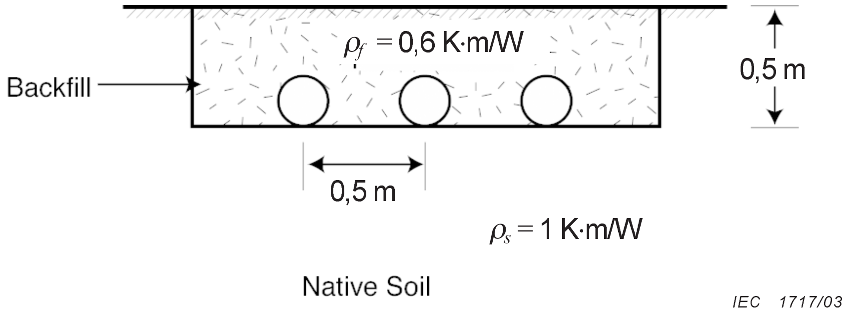


Figure 12 – Large cables located in a shallow trough

Table 2 shows the results obtained applying the IEC 60287 approach and the finite element method. Ambient temperature of 20°C is assumed.

Table 2 – Conductor temperature of the middle cable obtained with the IEC 60287 and the finite element methods

Calculation method	Convection coefficient W/m^2·K			
	2	**5**	**20**	**80**
IEC 60287	70	70	70	70
Finite element	95	81	72	69

This demonstrates that the temperature differences can be quite substantial. In this example, the IEC result corresponds to a very high convection coefficient. A brief discussion on the selection of the convection coefficient can be found in King and Halfter [24].

Annex A

Development of equations

A.1 Heat transfer equations

If the thermal resistance is constant, the heat conduction equation in a solid can be written as

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+W_{\text{int}}\,\rho=\frac{1}{\delta}\frac{\partial\theta}{\partial t} \quad \text{(A.1)}$$

where

θ	is the unknown temperature (°C);
$\delta=1/\rho c$	is the thermal diffusivity of the medium (m^2/s);
c	is the volumetric specific heat of the material (J/m^3);
ρ	is the thermal resistivity of the material (K·m/W);
W_{int}	is the heat generation rate in the cable (W/m).

The boundary conditions associated with (A.1) can be expressed in two different forms. If the temperature is known along a portion of the boundary, then

$$\theta=\theta_B(s) \quad \text{(A.2)}$$

where θ_B is the boundary temperature that may be a function of the surface length *s*. If heat is gained or lost at the boundary due to convection $h(\theta-\theta_{amb})$ or a heat flux *q*, then

$$\frac{1}{\rho}\frac{\partial\theta}{\partial n}+q+h(\theta-\theta_{amb})=0 \quad \text{(A.3)}$$

where *n* is the direction of the normal to the boundary surface, *h* is a convection coefficient, and θ is an unknown boundary temperature.

In cable rating computation, the temperature of the conductor is usually given and the maximum current flowing in the conductor is sought. Thus, when the conductor heat loss is the only energy source in the cable, we have $W_{\text{int}}=I^2R$ and equation (A.1) is used to solve for *I* with the specified boundary conditions.

As mentioned earlier, the challenge in solving equation (A.1) analytically stems mostly from the difficulty of computing the temperature distribution in the soil surrounding the cable. In the analytical methods used in IEC 60287, the case of a group of cables is dealt with on the basis of the restricted application of superposition. This assumes that the presence of another cable, even if it is not loaded, does not disturb the heat flux path from the first cable, nor the generation of heat within it. This allows separate computations to be performed for each cable with the final temperature rise being an algebraic sum of the temperature rises due to cable itself and the rise caused by the other cables. Such a procedure is not theoretically correct and, for better precision, the temperature rise caused by simultaneous operation of all cables should be considered. Direct solution of the heat conduction equation employing numerical methods offers such a possibility.

Numerical methods allow not only better representation of the mutual heating effects, but also permit more accurate modeling of the region's boundaries (e.g., a convective boundary at the earth surface, constant heat flux circular boundaries for heat or water pipes in the vicinity of the cables, or an isothermal boundary at the water level at the bottom of the trench).

In the reminder of this Annex, the solution to equations (A.1 – A.3) using the finite element method is developed.

A.2 Approximating polynomials

For the purpose of introducing the method and explaining how it is used in cable rating computations, we will use the simplest and the most common shape for two-dimensional elements, the triangle. In this document, the words "triangle", "element" and "finite element" will be used interchangeably.

Consider a simple triangular element shown in Figure A.1.

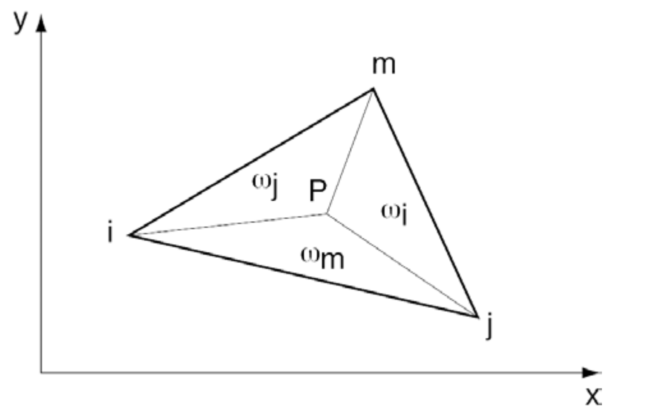


Figure A.1 – Area co-ordinates

For this element, the temperature θ at any point inside can be uniquely specified as [15]

$$\theta = A\omega_i + B\omega_j + C\omega_m \qquad \text{(A.4)}$$

where ω_i, ω_j and ω_m are the area co-ordinates defined as in Figure A.1. These area co-ordinates define uniquely the position of any point *P* inside the triangle *ijm*. To determine the constant *A*, the temperature at node *i* is written as (Eq. A.4)

$$\theta_i = 1 \times A + 0 \times B + 0 \times C$$

This gives $A = \theta_i$. Similarly, for nodes *j* and *m*, we obtain: $B = \theta_j$ and $C = \theta_m$. Therefore,

$$\theta = \omega_i\theta_i + \omega_j\theta_j + \omega_m\theta_m = [\omega_i, \omega_j, \omega_m]\cdot\begin{bmatrix}\theta_i \\ \theta_j \\ \theta_m\end{bmatrix} = N^e \cdot \Theta^e \qquad \text{(A.5)}$$

Assuming that the time derivatives are prescribed functions of the space co-ordinates at any particular instant of time, we can write the time derivative for the temperature within each element as

$$\frac{\partial\theta}{\partial t} = \omega_i\frac{\partial\theta_i}{\partial t} + \omega_j\frac{\partial\theta_j}{\partial t} + \omega_m\frac{\partial\theta_m}{\partial t} = [\omega_i, \omega_j, \omega_m]\times\begin{bmatrix}\frac{\partial\theta_i}{\partial t} \\ \frac{\partial\theta_j}{\partial t} \\ \frac{\partial\theta_m}{\partial t}\end{bmatrix} = N^e \times \frac{\partial\Theta^e}{\partial t} \qquad \text{(A.6)}$$

since N^e is a function of the co-ordinate system and not the time.

The relationship between area co-ordinates and Cartesian co-ordinates is

$$\begin{bmatrix}x \\ y \\ 1\end{bmatrix} = \begin{bmatrix}x_i & x_j & x_m \\ y_i & y_j & y_m \\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}\omega_i \\ \omega_j \\ \omega_m\end{bmatrix}$$

The inverse relationship yields the coefficients of vector N^e:

$$\begin{bmatrix}\omega_i \\ \omega_j \\ \omega_m\end{bmatrix} = \frac{1}{2A}\begin{bmatrix}(y_j - y_m) & (x_m - x_j) & (x_j y_m - x_m y_j) \\ (y_m - y_i) & (x_i - x_m) & (x_m y_i - x_i y_m) \\ (y_i - y_j) & (x_j - x_i) & (x_i y_j - x_j y_i)\end{bmatrix}\begin{bmatrix}x \\ y \\ 1\end{bmatrix} \qquad \text{(A.7)}$$

where *A* is the area of the triangle.

We can observe from equations (A.5) and (A.7) that the temperature is a linear function in *x* and *y*. This means that the gradients in either *x* or *y* directions are constant. A constant gradient within any element means that many small elements have to be used to approximate a rapid change in the value of θ.

A.3 Finite element equations

In the previous section we have learnt how to compute the temperature at any point inside an element if the temperature values at the nodes are known. To obtain node temperatures, we use a property, known in the variational calculus, that states that the minimisation of the functional [25]

$$\chi = \int_s \frac{1}{2\rho}\left[(\nabla\theta)^t\nabla\theta + \left(W_{\text{int}} - c\frac{d\theta}{dt}\right)\theta\right]dS + \int_c\left[q\theta + \frac{1}{2}h(\theta - \theta_{amb})^2\right]dC \qquad \text{(A.8)}$$

over the area *S* bounded by the closed curve *C*, where the superscript *t* denotes transposition and

$$\nabla\theta = \begin{bmatrix} \frac{\partial\theta}{\partial x} \\ \frac{\partial\theta}{\partial y} \end{bmatrix}$$

requires that the differential equation (A.1) with the boundary conditions (A.2) and (A.3) be satisfied. Therefore, any temperature distribution that makes χ a minimum also satisfies the governing differential equations and therefore is a solution to the problem being studied.

Equation (A.8) is a starting point for determining the temperature at each node. We minimise (A.8) by using our set of element functions, each defined over a single element and written in terms of the nodal values. The nodal values θ_n are the unknown values in our formulation. These values are obtained by taking derivatives of χ with respect to each θ_n and equating them to zero.

Recalling that functions θ are defined over each individual element, the integrals in (A.8) must be separated into integrals over the individual elements and the derivatives computed for each element; that is

$$\chi = \sum_{e=1}^{E} \chi^e \tag{A.9}$$

where χ^e is the functional defined for element *e*, and *E* is the total number of elements.

Let us consider a single element first. As any element contributes to only three of the differentials associated with its nodes, these contributions can be listed as

$$\left(\frac{\partial\chi}{\partial\theta_n}\right)^e = \begin{bmatrix} \frac{\partial\chi^e}{\partial\theta_i} \\ \frac{\partial\chi^e}{\partial\theta_j} \\ \frac{\partial\chi^e}{\partial\theta_m} \end{bmatrix} \tag{A.10}$$

The derivatives in equation (A.10) cannot be evaluated until the integrals in (A.8) have been written in terms of the nodal values, Θ^e. This is done by first computing the derivatives of θ with respect to *x* and *y*. Only two of the area co-ordinates are independent. Assuming that these are ω_i and ω_j, we have

$$\nabla\theta = \begin{bmatrix} \frac{\partial\theta}{\partial x} \\ \frac{\partial\theta}{\partial y} \end{bmatrix} = J \begin{bmatrix} \frac{\partial\theta}{\partial\omega_i} \\ \frac{\partial\theta}{\partial\omega_j} \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} (y_j - y_m) & (y_m - y_i) \\ (x_m - x_j) & (x_i - x_m) \end{bmatrix} \begin{bmatrix} \frac{\partial\theta}{\partial\omega_i} \\ \frac{\partial\theta}{\partial\omega_j} \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} b_i & b_j \\ a_i & a_j \end{bmatrix} \begin{bmatrix} \frac{\partial\theta}{\partial\omega_i} \\ \frac{\partial\theta}{\partial\omega_j} \end{bmatrix} \tag{A.11}$$

where the Jacobian J is obtained by differentiating equation (A.7). Further, from equation (A.5) and the fact that $\omega_i + \omega_j + \omega_m = 1$, we obtain

$$\begin{bmatrix} \dfrac{\partial\theta}{\partial\omega_i} \\ \dfrac{\partial\theta}{\partial\omega_j} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \theta_i \\ \theta_j \\ \theta_m \end{bmatrix} = V\Theta^{e} \tag{A.12}$$

Thus, for a single element, we have

$$\nabla\theta = J \times V \times \Theta^{e} \tag{A.13}$$

Substituting (A.13) into (A.8), with S and C corresponding to a single element, and differentiating with respect to Θ^{e}, after some routine but tedious computations, equation (A.10) can be written as

$$\left(\frac{\partial\chi}{\partial\theta_n}\right)^{e} = h^{e}\Theta^{e} + q^{e}\frac{\partial\Theta^{e}}{\partial t} - k^{e} \tag{A.14}$$

Denoting by d_{ij}, d_{jm} and d_{mi} the distance between nodes ij, jm and mi, respectively, the element conductivity matrix is equal to

$$h^{e} = \frac{1}{4A\rho}\left\{\begin{bmatrix} a_i^2 & a_i a_j & a_i a_m \\ a_i a_j & a_j^2 & a_j a_m \\ a_i a_m & a_j a_m & a_m^2 \end{bmatrix} + \begin{bmatrix} b_i^2 & b_i b_j & b_i b_m \\ b_i b_j & b_j^2 & b_j b_m \\ b_i b_m & b_j b_m & b_m^2 \end{bmatrix}\right\}$$

$$+\frac{hd_{ij}}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{hd_{jm}}{6}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} + \frac{hd_{mi}}{6}\begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix} \tag{A.15}$$

$$a_i = x_m - x_j \; a_j = x_i - x_m \; a_m = x_j - x_i$$

$$b_i = y_j - y_m \; b_j = y_m - y_i \; b_m = y_i - y_j$$

If there is no convective boundary along any segment of the element, the relevant term in equation (A.15) is omitted (see example 3 above).

The element capacity matrix is given by

$$q^{e} = \frac{cA}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \tag{A.16}$$

and the element heat generation vector is equal to

$$k^e = \frac{W_{\text{int}} A}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} + \frac{(h\theta_{amb}+q)d_{ij}}{2}\begin{bmatrix}1\\1\\0\end{bmatrix} + \frac{(h\theta_{amb}+q)d_{jm}}{2}\begin{bmatrix}0\\1\\1\end{bmatrix} + \frac{(h\theta_{amb}+q)d_{mi}}{2}\begin{bmatrix}1\\0\\1\end{bmatrix} \tag{A.17}$$

Here again, the last three terms apply only if the appropriate boundary exists along the element edge. Factor $W_{\text{int}} A$ represents the total heat in W/m generated in the element.

Performing computations given by equations (A.14-A.17) for each element, we finally obtain the following set of linear algebraic equations for the whole region:

$$\frac{\partial \chi}{\partial \Theta} = \sum_{e=1}^{E}\left(\frac{\partial \chi}{\partial \theta_n}\right)^e = H\Theta + Q\frac{\partial \Theta}{\partial t} - K = 0 \tag{A.18}$$

In this equation, *H* is the heat conductivity matrix, *Q* the heat capacity matrix, Θ and $\frac{\partial \Theta}{\partial t}$ are vectors containing the nodal temperatures and their derivatives, *K* is a vector which expresses the distribution of heat sources and heat sinks over the region under consideration.

In the steady state analysis, equation (A.18) simplifies to:

$$H\Theta - K = 0 \tag{A.19}$$

The set of ordinary differential equations (A.18) which define the discretised problem can be solved using one of the many recursion schemes. There are two popular procedures for solving these equations to obtain the values of Θ at each point in time. The first is to approximate the time derivative using a finite difference scheme. The alternate procedure is to use finite elements defined in the time domain. Flatabo [15] used the mid-interval Crank-Nicolson finite difference algorithm for the solution of this equation. This method requires an iteration within each time step. Here, we propose to use Lees' [26] three-level, time-stepping scheme in which the discretised equation is replaced by the recurrence relationship

$$\Theta^{n+1} = -\left[\frac{H^n}{3} + \frac{Q^n}{2\Delta\tau}\right]^{-1}\left[\frac{H^n\Theta^n}{3} + \frac{H^n\Theta^{n-1}}{3} - \frac{Q^n\Theta^{n-1}}{2\Delta\tau} - K^n\right] \tag{A.20}$$

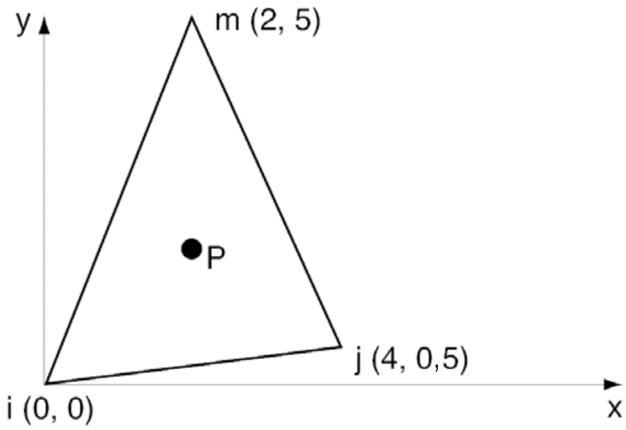
where the superscript *n* refers to the time level and $\Delta\tau$ is the time step. The procedure is unconditionally stable and has the advantage of producing the solution at time level $n+1$ without the need for any iteration as the coefficient matrices are evaluated at level *n*. The initial conditions have to be specified and the first time step iteration is performed by a modified version of equation (A.20) requiring only one previous time step solution.

A.4 Examples

The following examples illustrate how the finite element method is implemented for the solution of a heat conduction problem.

A.4.1 Example A1

Consider a triangular element shown in Figure A.2. We will evaluate the element equation and calculate the value of the temperature at point *P* for the following nodal temperature values: $\theta_i = 40$ °C, $\theta_j = 34$ °C and $\theta_m = 46$ °C. *P* is located at (2,0, 1,5).



IEC 1719/03

Figure A.2 – Illustration for example A1

The temperature θ is given by equation (A.5) in this Annex with the shape function described by equation (A.7). First, we need to compute the area of the triangle. This is obtained from

$$2A = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_m & y_m \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 4 & 0{,}5 \\ 1 & 2 & 5 \end{vmatrix} = 19$$

From equation (A.7),

$$\begin{bmatrix} \omega_i \\ \omega_j \\ \omega_m \end{bmatrix} = \frac{1}{19}\begin{bmatrix} (0{,}5-5) & (2-4) & (4\times5-2\times0{,}5) \\ (5-0) & (0-2) & (2\times0-0\times5) \\ (0-0{,}5) & (4-0) & (0\times0{,}5-4\times0) \end{bmatrix}\begin{bmatrix} 2 \\ 1{,}5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0{,}368 \\ 0{,}368 \\ 0{,}264 \end{bmatrix}$$

The temperature at point *P* is obtained from equation (A.5):

$$\theta = 0{,}368\times40 + 0{,}358\times34 + 0{,}264\times46 = 39{,}4\ °\text{C}$$

In triangular elements, temperature varies linearly between any two nodes. Any line of constant temperature is a straight line and intersects two sides of the element. The only exception is when all nodes have the same value. These two properties make it easy to locate isothermal contour lines.

A.4.2 Example A2

We will determine the 41°C contour line for the triangular element used in example A1.

The temperature isotherm for 41°C intersects sides *im* and *mj*. The coordinates at which this isotherm intersects the sides of the triangle are obtained from the following simple ratios:

$$\frac{46-41}{46-34}=\frac{2-x}{2-4} \quad \text{or} \quad x=2{,}83$$

and

$$\frac{46-41}{46-34}=\frac{5-y}{5-0{,}5} \quad \text{or} \quad y=3{,}12$$

The contour is shown in Figure A.3.

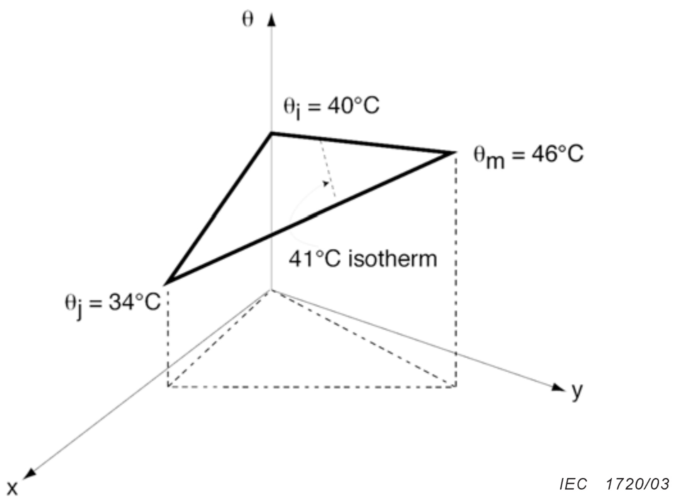


Figure A.3 – 41°C isothermal contour

A.4.3 Example A3

Consider the element examined in example A1. Assume that this element experiences convection on surface *ij* and a constant heat flux on surface *mi*. We will calculate element matrices given the numerical dimensions and properties shown in Figure A.4.

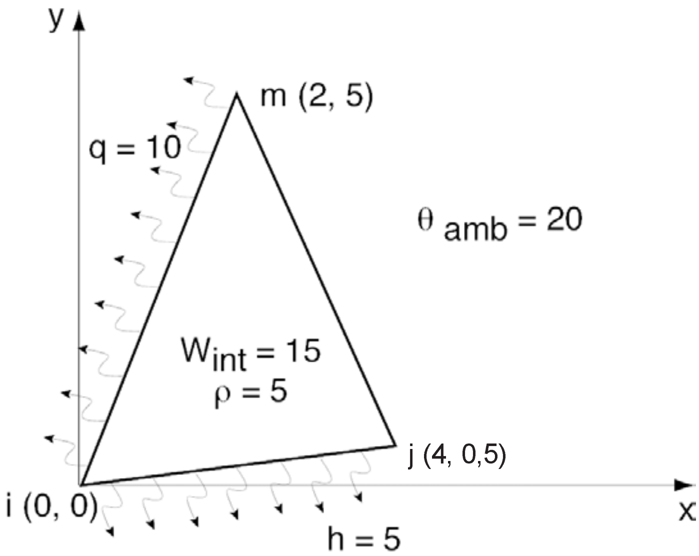


Figure A.4 – Illustration for example 3

The lengths of the boundary sides are

$$d_{ij} = \sqrt{(0-4)^2 + (0-0{,}5)^2} = 4{,}03 \qquad d_{mi} = \sqrt{(2-0)^2 + (5-0)^2} = 5{,}39$$

Substituting the numerical values in equation (A.15), we obtain the following conductivity matrix:

$$a_i = 2-4 = -2 \qquad a_j = 0-2 = -2 \qquad a_m = 4-0 = 4$$

$$b_i = 0{,}5-5 = -4{,}5 \qquad b_j = 5-0 = 5 \qquad b_m = 0-0{,}5 = -0{,}5$$

$$\mathrm{h}^e = \frac{1}{4\times 9{,}5\times 5}\left\{\begin{bmatrix} (-2)^2 & (-2)(-2) & (-2)(4) \\ (-2)(-2) & (-2)^2 & (-2)(4) \\ (-2)(4) & (-2)(4) & (4)^2 \end{bmatrix} + \begin{bmatrix} (-4{,}5)^2 & (-4{,}5)(5) & (-4{,}5)(-0{,}5) \\ (-4{,}5)(5) & (5)^2 & (5)(-0{,}5) \\ (-4{,}5)(-0{,}5) & (5)(-0{,}5) & (-0{,}5)^2 \end{bmatrix}\right\}$$

$$+\begin{bmatrix} \frac{5\times 4{,}03}{3} & \frac{5\times 4{,}03}{6} & 0 \\ \frac{5\times 4{,}03}{6} & \frac{5\times 4{,}03}{3} & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6{,}85 & 3{,}26 & -0{,}03 \\ 3{,}26 & 6{,}87 & -0{,}06 \\ -0{,}03 & -0{,}06 & 0{,}09 \end{bmatrix}$$

The heat generation vector is obtained from equation (A.17):

$$\mathrm{k}^e = \frac{15\times 9{,}5}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} + \frac{5\times 20\times 4{,}03}{2}\begin{bmatrix}1\\1\\0\end{bmatrix} + \frac{10\times 5{,}39}{2}\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}276\\249\\74{,}5\end{bmatrix}$$

A.4.4 Example A4

We will consider now a domain composed of three elements, one being the same as examined in example 3 and two adjacent elements as shown in Figure A.5.

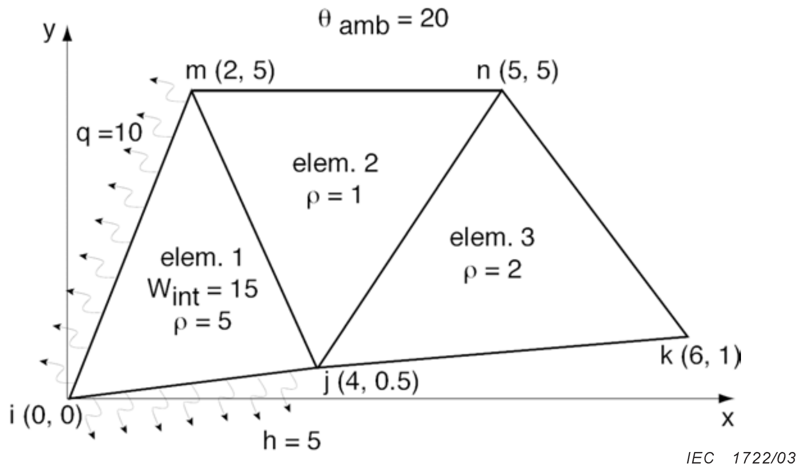


Figure A.5 – Illustration to example 4

We will determine nodal temperatures in the steady-state for this system assuming that the other boundary surfaces have zero temperature gradient.

The matrix for element 1 was obtained in example A3 and is equal to

$$h_1^e = \begin{bmatrix} 6{,}85 & 3{,}26 & -0{,}03 \\ 3{,}26 & 6{,}87 & -0{,}06 \\ -0{,}03 & -0{,}06 & 0{,}09 \end{bmatrix}$$

The element matrices for elements 2 (nodes *j, n, m*) and 3 (nodes *j, k, n*) are obtained from equation (A.15):

$$h_2^e = \begin{bmatrix} 0{,}33 & -0{,}22 & -0{,}11 \\ -0{,}22 & 0{,}90 & -0{,}68 \\ -0{,}11 & -0{,}68 & 0{,}79 \end{bmatrix} \quad \text{and} \quad h_3^e = \begin{bmatrix} 0{,}5 & -0{,}5 & 0 \\ -0{,}5 & 0{,}63 & -0{,}13 \\ 0 & -0{,}13 & 0{,}13 \end{bmatrix}$$

Since there are 5 nodes in this system, the matrix **H** will have 5 rows and 5 columns and is equal to

i j m n k

$$\mathbf{H}=\begin{bmatrix}6{,}85 & 3{,}26 & -0{,}03 & 0 & 0\\ 3{,}26 & 6{,}87 & -0{,}06 & 0 & 0\\ -0{,}03 & -0{,}06 & 0{,}09 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}+\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0 & 0{,}33 & -0{,}11 & -0{,}22 & 0\\ 0 & -0{,}11 & 0{,}79 & -0{,}68 & 0\\ 0 & -0{,}22 & -0{,}68 & 0{,}90 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$+\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0 & 0{,}5 & 0 & 0 & -0{,}5\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0{,}13 & -0{,}13\\ 0 & -0{,}5 & 0 & -0{,}13 & 0{,}63\end{bmatrix}=\begin{bmatrix}6{,}85 & 3{,}26 & -0{,}03 & 0 & 0\\ 3{,}26 & 7{,}7 & -0{,}17 & -0{,}22 & -0{,}5\\ -0{,}03 & -0{,}17 & 0{,}88 & -0{,}68 & 0\\ 0 & -0{,}22 & -0{,}68 & 1{,}03 & -0{,}13\\ 0 & -0{,}5 & 0 & -0{,}13 & 0{,}63\end{bmatrix}$$

Since elements 2 and 3 do not generate any heat and have zero temperature gradient, vector K is the same as obtained in example A4 with the components corresponding to nodes *n* and *k* equal to zero; that is

$$\mathrm{K}=\begin{bmatrix}276\\ 249\\ 74{,}5\\ 0\\ 0\end{bmatrix}$$

With the conductance matrix and heat generation vector given above, the following nodal temperatures are obtained by solving equations (A.19):

$$\theta_i = 24{,}6\ °\text{C},\quad \theta_j = 34{,}9\ °\text{C},\quad \theta_m = 211{,}9\ °\text{C},\quad \theta_n = 154{,}9\ °\text{C},\quad \theta_k = 59{,}7\ °\text{C}$$

The following example illustrates an application of the finite element method for modelling of cables in extended backfills and duct banks.

A.4.5 Example A5

The following procedure to obtain the geometric factor for cables located in duct banks and backfills was proposed by El-Kady and Horrocks [14].

Consider the thermal circuit configuration given in Figure A.6 where the cable bank is represented by a rectangular cross-sectional surface *C* of height *h* and with *w*. For this configuration, the total thermal resistance between the duct bank surface and the ground ambient is given by

$$T=-\frac{\rho_s\left(\theta_c-\theta_{amb}\right)}{\int_C \frac{\partial\theta}{\partial n}\,ds} \qquad \text{(A.21)}$$

where ρ_s is the thermal resistivity of the soil, *C* represents the duct bank surface and $\partial\theta/\partial n$ denotes differentiation along the normal to *C*.

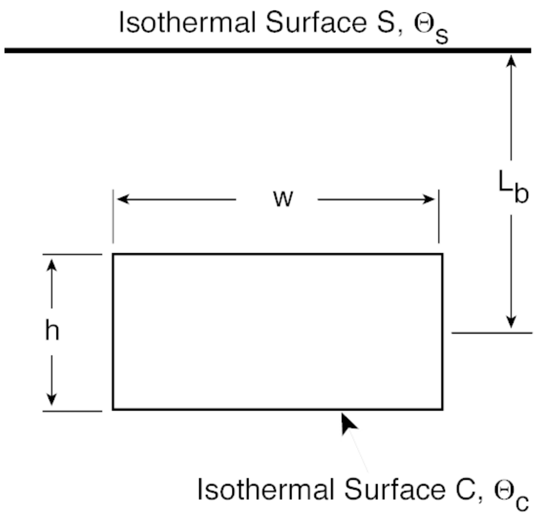


Figure A.6 – Thermal circuit configuration in example 5

In the finite element solution, the medium surrounding the surface *C* is partitioned into small triangles constituting a finite element grid such that the first grid layer, enclosing the bank surface *C*, is carefully structured, as shown in Figure A.7, to attain an efficient subsequent evaluation of equation (A.21).

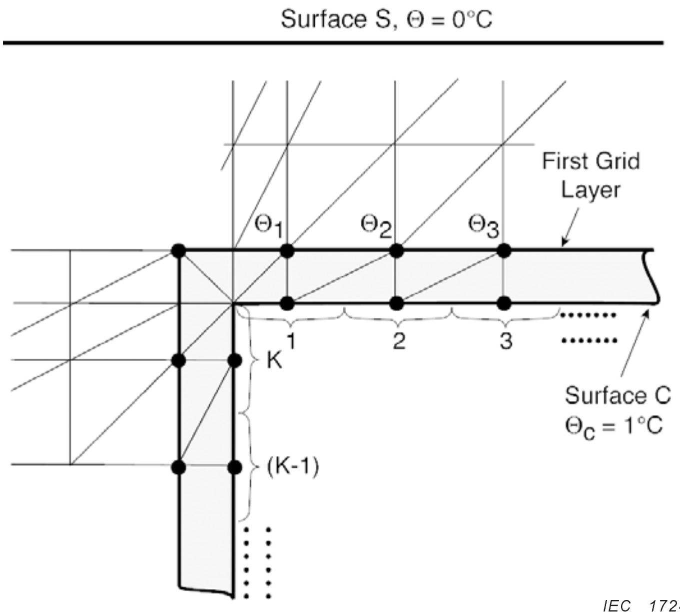


Figure A.7 – Finite element grid structure for a outer layer of a duct bank

The surface *C* is partitioned into *K* small segments, as shown in Figure A.7, where the temperatures $\theta_1, \theta_2, ...$ of the middle points of the first grid layer (which constitute nodes of the finite element grid) are evaluated. The accuracy of the solution can be controlled by adjusting the size of the elements of the grid. Equation (A.21) can now be written in the discretised form

$$T = -\frac{\rho_s}{\sum_{i \in I_c} \frac{\Delta\theta_i}{\Delta n_i} \frac{\Delta S_i}{\theta_{ci} - \theta_{amb}}} \qquad (A.22)$$

where, as shown in Figure A.7, θ_i is the temperature of segment *i* along the first finite element grid layer surrounding the duct bank surface, θ_{ci} is the temperature at the duct bank surface *C* of segment *i*, and I_c is the index set of segments along *C*. By choosing $\Delta S_i / \Delta n_i = 1$ for all *i*, equation (A.22) reduces to

$$T = \frac{\rho_s}{2\pi} G = -\frac{\rho_s}{\sum_{i \in I_c} \frac{\theta_i - \theta_{ci}}{\theta_{ci} - \theta_{amb}}} \qquad (A.23)$$

Hence,

$$G = \frac{2\pi}{\sum_{i \in I_c} \frac{\theta_{ci} - \theta_i}{\theta_{ci} - \theta_{amb}}} \qquad (A.24)$$

Equation (A.24) provides the value of the geometric factor in terms of the temperature results from the finite element analysis. We note that no assumption that the surface *C* is isothermal was required. If, in fact, the duct bank surface is an isotherm, then $\theta_{ci} = \theta_c$ for all *i* in equation (A.24) leading to

$$G = \frac{2\pi(\theta_c - \theta_{amb})}{\sum_{i \in I_c} (\theta_c - \theta_i)} \qquad (A.25)$$

If we set $\theta_c = 1$ and $\theta_{amb} = 0$, equation (A.25) further simplifies to

$$G = \frac{2\pi}{K - \sum_{i \in I_c} \theta_i} \qquad (A.26)$$

Equation (A.26) was used by El-Kady and Horrocks [14] to obtain the extended values of the geometric factor for duct banks and backfills.

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