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Guidance on the application of statistical methods for determining the properties of masonry products

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National foreword

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Guidance on the application of statistical methods for determining the properties of masonry products

Guide pour l'application de méthodes statistiques pour la détermination des propriétés des éléments de maçonnerie

Leitfaden für die Anwendung statistischer Methoden zur Bestimmung der Eigenschaften von Mauerwerk Produkten

This Technical Report was approved by CEN on 24 August 2015. It has been drawn up by the Technical Committee CEN/TC 125.

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European Foreword

This document (CEN/TR 16886:2016) has been prepared by Technical Committee CEN/TC 125 “Masonry”, the secretariat of which is held by BSI.

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Introduction

This document is informative for the guidance of manufacturers and Notified Bodies (NBs), who want to use statistical methods for the evaluation of conformity and Factory Production Control of masonry products. Its use is optional. Other statistical methods and non-statistical methods may be used.

Quality control of building materials and components is an indispensable part of an overall concept of structural reliability. As quality control is generally a time-consuming and expensive task, various operational techniques and activities have been developed to fulfil safety requirements in buildings. Properly employed statistical methods are one way to provide efficient, economic and effective means of quality control.

Background: "The terms and definitions in EN 1990 (*Eurocode: Basis of structural design*) are derived from ISO 2394 (*General principles on reliability for structures*). For the design of structures, EN 1996-1-1 (*Eurocode 6: Design of masonry structures — Part 1-1: General rules for reinforced and unreinforced masonry structures*) is intended to be used together with EN 1990. ISO 12491 (*Statistical methods for quality control of building materials and components*) gives general principles for the application of statistical methods for the quality control of building materials and components, in compliance with the safety and serviceability requirements of ISO 2394. ISO 12491 is applicable to all buildings and other civil engineering works, existing or under construction, whatever nature or combination of materials used, e.g. concrete, steel, wood, bricks. The EN 771 series specifies that one method of satisfying the conformity criterion laid down in these product standards is to use the approach given in ISO 12491."

This Technical Report gives guidance on how a statistical evaluation can be put into practice based on the background of ISO 12491.

A simplified method is also given based on information obtained from practice about the possible distribution in production for specific product characteristics.

The method may also be used for the evaluation of different properties at the different stages of the factory production control (FPC) with the aim to minimize testing costs for the manufacturer and to ensure that the requirements are fulfilled.

Detailed examples are given in Annex C. For other more sophisticated techniques and specific problems, other international standards can be applied.

The initial draft of this document was prepared by the joint working group CEN/TC 125/TG 5 and the Sector Group 10 of Notified Bodies under the Construction Products Directive. The CEN/TR is a tool available for manufacturers and Notified Bodies.

It is laid down in the hEN's of masonry products that the manufacturer should demonstrate compliance for his product with the requirements of the harmonized product standards.

The purpose of this Technical Report is to put statistical evaluation into practice. Detailed examples are given in the annexes.

1 Scope

In the masonry unit standards and in national legislation, some properties need to be declared based on a certain fractile and confidence level. To demonstrate compliance with that a statistical tool can be used.

The purpose of this Technical Report is to exemplify how a statistical tool can be used in practice. This document should not contradict nor extend the scope of the work and role of a Notified Body, nor impose additional burdens on the manufacturer, beyond those laid down in the Construction Products Regulation and the product standards.

Mechanical and other properties of building materials and components are in the report described by random variables with a certain type of probability distribution. The popular normal distribution (Laplace-Gauss distribution) is given in Annex A. Normal distribution may be used to approximate many actual symmetrical distributions. When a remarkable asymmetry is observed, then another type of distribution reflecting this asymmetry should be considered, leading to a more complex method to demonstrate compliance with the product standard. More information on the normality test of Shapiro-Wilk is given in Annex D.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

EN 1990, *Eurocode - Basis of structural design*

EN 1996 (all parts), *Eurocode 6 — Design of masonry structures*

3 Terms, definitions and symbols

For the purposes of this document, the following terms, definitions and symbols apply.

3.1 Terms and definitions

3.1.1
unit
defined quantity of building material, component or element that can be individually considered and separately tested

3.1.2
population
totality of units under consideration

3.1.3
variable
X
variable which can take any of the values of a specified set of values and with which is associated a probability distribution

3.1.4
probability distribution
function which gives the probability that a variable *X* takes any given value (in the case of a discrete variable) or belongs to a given set of values (in the case of a continuous variable)

3.1.5
distribution function
Π(*x*)
function giving, for every value of *x*, the probability that the variable *X* is less than or equal to *x*:

$$\Pi(x) = P,(X \leq x)$$

3.1.6
(probability) density function
f(*x*)
derivative (when it exists) of the distribution function

3.1.7
parameter (population)
quantity used in describing the distribution of a random variable in a population

3.1.8
fractile
x
if *X* is a continuous variable and *p* is a real number between 0 and 1, the *p*-fractile is the value of a variable *X* for which the distribution function equals *p*

Note 1 to entry: Thus x_p is a fractile if $P,(X \leq x_p) = p$

3.1.9
population mean
μ
for a continuous variable *X* having the probability density *f(x)*, the mean, if it exists, is given by:

$$\mu = \int \times f(x)dx$$

the integral being extended over the interval(s) of variation of the variable *X*

3.1.10
population variance
σ²
for a continuous variable *X* having the probability density function *f(x)*, the variance, if it exists, is given by

$$\sigma^2 = \int (x - \mu)^2 f(x)dx$$

the integral being extended over the interval(s) of variation of the variable *X*

3.1.11
population standard deviation
σ
positive square root of the population variance σ^2

3.1.12
normal distribution
probability distribution of a continuous variable *X*, the probability density function of which is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

3.1.13
random sample
one or more sampling units taken from a population in such a way that each unit of the population has the same probability of being taken

3.1.14
sample size
n
number of sampling units in the sample

3.1.15
sample mean
$\mathbf{x_m}$
sum of *n* values x_i of sampling units divided by the sample size *n*

$$x_m = \frac{\sum x_i}{n}$$

3.1.16
sample variance
$\mathbf{s^2}$
sum of *n* squared deviations from the sample mean x_m divided by the sample size *n* minus 1

$$s^2 = \frac{\Sigma(x_i - x_m)^2}{n-1}$$

3.1.17
sample standard deviation
s
positive square root of the sample variance s^2

3.1.18
estimation
operation of assigning, from observations on a sample, numerical values to the parameter of a distribution chosen as the statistical model of the population from which this sample was taken

3.1.19
estimator
function of a set of the sample random variables used to estimate a population parameter

3.1.20
estimate
value of an estimator obtained as a result of an estimation

3.1.21
confidence level
γ
given value of the probability associated with a confidence interval

3.1.22
lot
definite quantity of units, manufactured or produced under the same conditions which are presumed uniform

3.1.23
isolated lot
lot separated from the sequence of lots in which it was produced or collected, and not forming part of a current sequence of inspection lots

3.1.24
conforming unit
unit which satisfies all the specified requirements

3.1.25
non-conforming unit
unit containing at least one non-conformity which causes the unit not to satisfy specified requirements

3.1.26
sampling inspection
inspection in which decisions are made to accept or not accept a lot, based on results of a sample selected from that lot

3.1.27
sampling plan
plan in accordance with which one or more samples are taken in order to obtain information and the possibility of reaching a decision concerning the acceptance of the lot

3.2 Symbols

k_n is the acceptance coefficient

k_1 is the acceptance coefficient one-sided tolerance interval

k_2 is the acceptance coefficient two-sided tolerance interval

k_c is the corrected acceptance coefficient

k_k is the acceptance coefficient for known standard deviation

k_u is the acceptance coefficient for unknown standard deviation

n is the number of test samples within the spot sample

x_m is the mean test result

x_i is the test result for test sample i

i is the number of the individual test sample

x_{est} is the test result of the estimated normal distribution of the spot sample

s is the standard deviation of the test results

s_s	is the standard deviation of the test results of a spot sample
σ	is the known standard deviation
l	is the number of inspection lots
$\lambda_{10,dry,unit}$	is the thermal conductivity of the unit
p	is the fractile
γ	is the confidence level

4 General

It is specified in the product standards that the manufacturer should demonstrate compliance for his product with the requirements of the relevant European standard and with the declared values for the product properties by carrying out both:

a) product type determination, which can be type testing, type calculation, reference to tabulated values or descriptive documentation of the product;

b) factory production control (FPC).

If a manufacturer of masonry elements intends to declare that the units are Category I units, then the units shall fulfil the definition of Category I units which is 'Units with a declared compressive strength with a probability of failure to reach it not exceeding 5 %', which means that the manufacturer is declaring that the customer can be 95 % confident that the delivered units fulfilled the declared compressive strength. To be able to demonstrate this, the manufacturer can operate a FPC that includes a statistical evaluation.

The confidence level for a property shall be fixed depending on how important the property is in a building. The higher the confidence level is the lower is the risk that the product does not fulfil the declared values. When dealing with the safety of a building it is necessary to presuppose a minimum confidence level fulfilled by the used products, otherwise the partial safety factors cannot be fixed.

Confidence levels other than 95 % can be used, e.g. the safety system specified in EN 1990 to which the Eurocode for masonry (EN 1996 series) refers to for safety aspects, is based on the assumption that declared values for the used product properties fulfil a confidence level of 75 %.

For characteristics, where a certain minimum confidence level is not fixed in a technical specification or in a contract to be fulfilled, the manufacturer is free to fix the confidence level he will operate with, and the higher the chosen level is the lower the risk that the manufacturer is running that the delivered products do not fulfil the declared values. The risk the manufacturer is running is fixed by a combination of the actual variation in test results over time, the frequencies of checking and testing, the way the FPC system is developed and how close the declared value is to the tested values.

In the product standard the conformity criteria are related to a 'consignment', that is a delivery to a building site. The product standard defines a declared value as a value that the manufacturer is confident in achieving, bearing in mind the precision of the tests and the variability of the production process, and when the declared values are accompanying the product to the building site, they are valid for the delivered consignment. Since it is impractical to test each consignment, the manufacturer should plan the FPC system in such a way that the effect of the variations of product characteristics during the production is taken into account when declaring the characteristics for the consignment. In some production processes products are naturally separated into batches and a consignment is quite often only a part of a batch. If a production is based on a continuous flow a consignment is only a part of the continuous production.

5 Statistical evaluation

5.1 Factory production control

The FPC system can be developed in such a way that the checking procedures are:

— mainly related to the process only (full process control and consequently only a small amount of finished product testing); or

— mainly related to the finished products only (and consequently limited process control); or

— a combination of both.

It can even be so that the amount of process control and finished product testing varies depending on the property to be assessed. If the test for the property is low cost, e.g. a test of dimensions, and if the property is less important in relation to the end use then it might be the right solution to use finished product testing. But if the testing of the property is expensive, e.g. frost resistance tests, then the solution might be to base the assessment on process control using proxy tests.

The manufacturer defines the product groups. A product group consists of products from one manufacturer having common values for one or more characteristics. That means that the products belonging to a product group might differ according to the characteristics in question. If a product group is defined, then the FPC system should ensure that all types of units within a group are controlled and over time also in the finished product testing, if that is part of the FPC.

Depending on the way the FPC system is developed (mainly related to process control only, mainly related to finished product testing only or a combination of both) a selection of these should be considered.

Samples, taken during the process and finished product samples need to be representative for the inspection lot. For that reason the sampling procedure is important and so should be specified. When the frequency of testing is fixing the size of the inspection lot and thereby the manufacturer's risk the frequency should be carefully considered, decided and recorded. If test results and FPC system give evidence of problems then the frequencies can be reconsidered and reduced compared to the ones used.

5.2 Finished product testing

5.2.1 General

When testing the finished product in FPC, it is possible to use alternative test methods if a correlation can be established between the alternative test method and the reference test method or if a safe relationship can be demonstrated when using the alternative method compared to the reference method.

It is also important to notice that a test result of a spot sample (see 5.2.3) is representing an inspection lot (see 5.2.2). If an evaluated test result is not conforming, the whole production since the last test should be looked upon as non-conforming. For that reason it can be recommended that for properties where the reference test is time consuming and might be costly, alternative tests or proxy tests that are less time consuming and costly are used. By doing so the time span between the tests can be shortened and the amount of products covered by a non-conforming test result will be less and thereby reduce the manufacturer's risk.

The amount of products produced between two tests is an inspection lot. The frequency of testing can vary from one property to another and thereby the inspection lot can vary from one property to another.

5.2.2 Inspection lot

The production is divided into inspection lots.

An inspection lot shall consist of units produced under uniform conditions:

- same raw materials;
- same dimensions;
- same production process.

If a certain characteristic is the same for multiple units, where the dimension has no influence, these units can belong to the same product family.

This means that an inspection lot for the characteristic in question can only consist of products belonging to the same product group.

The manufacturer decides on the size of the inspection lot from:

- raw material mixing lots; or
- number/volume of units; or
- number of production days.

Independent of the way the size of the inspection lot is decided, it shall be possible to draw a representative spot sample.

5.2.3 Spot sampling and sample sizes

When the inspection lot has been decided, the sampling procedure for a spot sample shall be fixed in such a way that the spot sample is representative for the inspection lot as shown in the example of Figure 1.

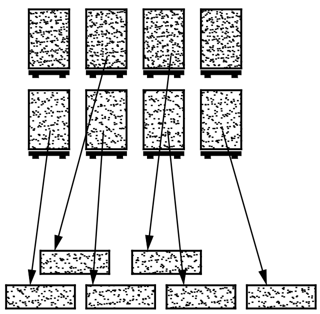

Figure 1 — An example of representative sampling

Sampling procedures for stacks and banded packs are given in the European product standard. It is also possible to sample from the conveyer belt or, in the case of fired units, after the kiln.

The number of units in the spot sample is decided by the manufacturer. If a minimum number of units has been fixed then this should be accepted.

By deciding on the size of the inspection lot the manufacturer is fixing the frequencies of tests to be done. The size of the inspection lot should be decided based on:

- how close the declared value is to the test value;
- the deviation of the test values;
- how much process control is going on.

These decisions allow the manufacturer to manage their own risks.

5.2.4 Production types

A production, which is naturally separated into batches, is named a batch production. In the case of the batch production the properties of the units may change batch by batch. A batch is normally looked upon as a separate inspection lot. If the process control minimizes the changes from one batch to another, an inspection lot can cover more than one batch. An example of a batch production is shown in Figure 2.

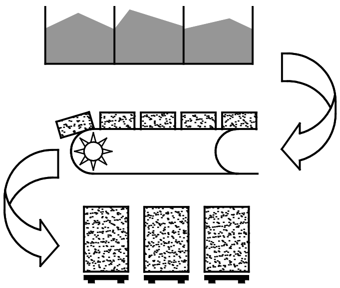

Figure 2 — Example of batch production

A production, which is based on a continuous flow, is named a series production. An example of a series production is given in Figure 3. In the case of series production the properties of the units are the same within a series. A series production usually contains more than one inspection lot.

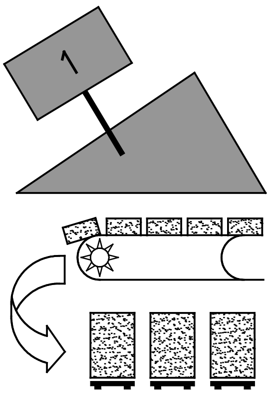

Figure 3 — Example of series production

5.2.5 Control method A: Batch control

When a batch production is in operation, then the FPC system needs to be based on a batch control, which means, that each batch is controlled separately as shown in Figure 4.

When dealing with the evaluation of test results, the acceptance coefficient k_n is given in Tables 1 and 2 (5.2.7). These tables show that there is a great difference in using k_n for three or for six test results and for that reason it is recommended to operate with spot sample sizes of at least six units.

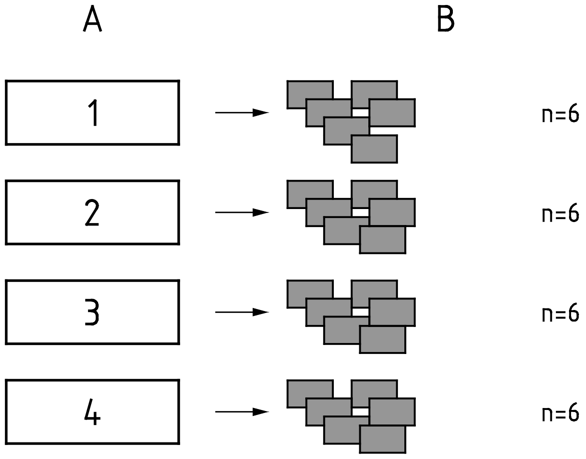

Figure 4 — Example of Method A: Each inspection lot is evaluated individually

5.2.6 Control method B:'Rolling' inspection

In a series production there are a series of inspection lots, which should not exceed a total number of five. In the example in Figure 5 four are used.

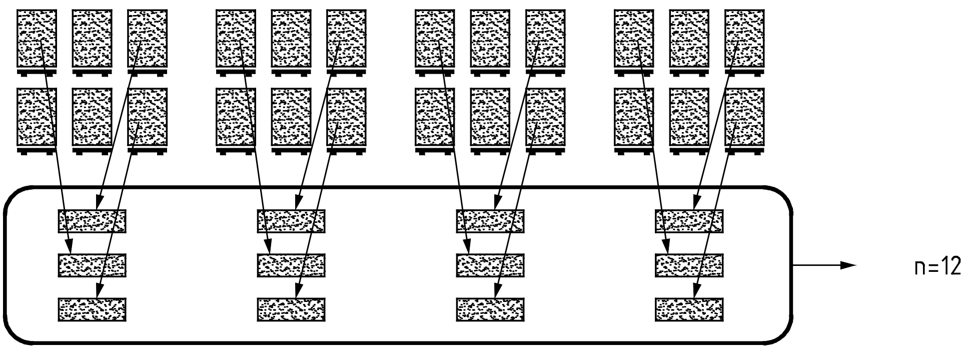

Figure 5 — Example with 4 inspection lots in a series

For the first inspection lot a spot sample size of three is taken and tested. For the second inspection lot three new samples are taken, tested and evaluated together with the ones from the first inspection lot and therefore the spot sample size will be six. For the third inspection lot three new samples are taken, tested and evaluated together with the ones from the first and second inspection lot and therefore the spot sample size will be nine. For the fourth inspection lot three new samples are taken and tested and evaluated together with the ones from the first three inspection lots and therefore the spot sample size will be 12. For the fifth inspection lot three new samples are taken, tested and evaluated together with the ones from the second, third and fourth inspection lots and therefore the spot sample size will be 12. The described rolling system will continue for the following inspection lots. The rolling system is illustrated in Figure 6. When dealing with the evaluation of test results the acceptance coefficient k_n is given in Tables 1 and 2 (5.2.7). These tables show that there is a great difference for 6 and 12 test results, and the number of tests to be done is half compared to the batch control when the size of the inspection lot is the same. Another possibility is to half the size of the inspection lot and therefore to reduce the number of units covered by non-conformity, if that occurs.

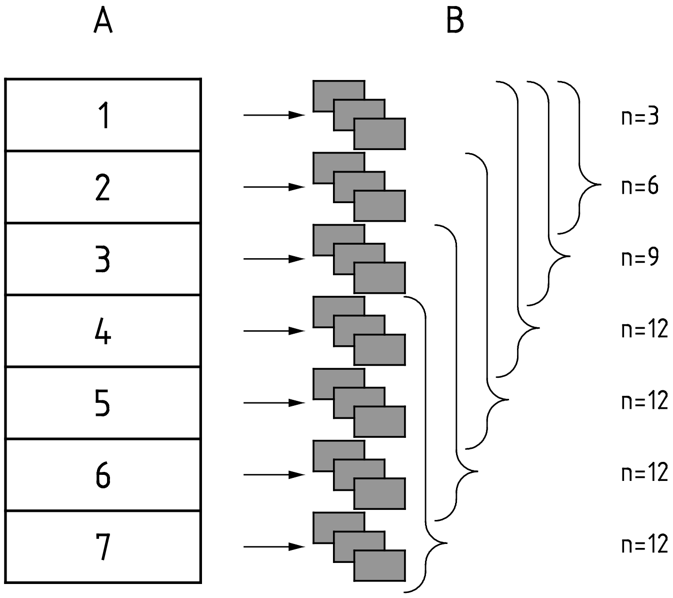

Figure 6 — Example of method B,'Rolling' inspection: series of four inspection lots

Another possibility is the so-called 'progressive' sampling procedure (see Figure 7). For each of the first to fifth inspection lots a spot size of one sample is taken and tested. These lots are evaluated together. For the sixth and following inspection lots one additional sample is taken, tested and evaluated together with the ones from the previous inspection lots. The spot size is gradually increased from 5 to 15 samples.

From then on, one additional sample is taken from each next inspection lot but the spot sample is limited to the last 15 samples. The spot sample size continues to be 15.

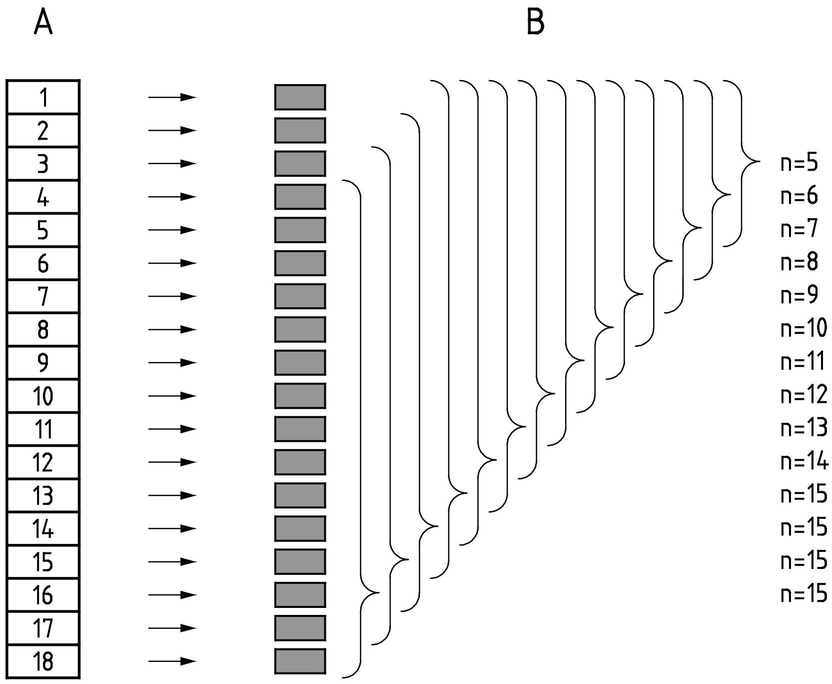

Figure 7 — Example of method B,'Rolling' inspection 'Progressive' sampling: series of 15 inspection lots

5.2.7 Evaluation of test results

Where and when possible and applicable, the results of the checks and testing should be interpreted by means of statistical techniques, by attributes or by variables to verify the product characteristics and to determine if the production conforms to the compliance criteria and the products conform to the declared values. One method of satisfying this conformity criterion is to use the approach given in ISO 12491. This approach is shown in detail in this subclause.

When using the test results of a spot sample with a limited number of samples to estimate the characteristics of the production there are some uncertainties. The deviation within the test results is one uncertainty and how representative the spot sample is for the production is another uncertainty. The first uncertainty is dealt with in the evaluation by taking into account the standard deviation *s* of the test results of the spot sample. The second uncertainty is dealt with by using an acceptance coefficient k_n. The acceptance coefficient k_n can be regarded as a factor minimizing the statistical uncertainties from spot sampling. k_n is dependent on several factors:

— the number of samples in the inspection lot *n*;

— the confidence level γ;

— the fractile *p* [(a)];

— the standard deviation is unknown. The symbol used is k_u;

— the standard deviation is known. The symbol used is k_k;

— one-sided limit evaluation. The symbol used is k_1;

— two-sided limit evaluation. The symbol used is k_2.

When evaluating the test results from a spot sample, the following procedure should be used:

Calculate the mean value of the test results using Formula (1):

$$x_m = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (1)$$

where

x_m is the mean test result

x_i is the test result for test sample i

n is the number of test samples within the spot sample

i is the number of the individual test sample

Calculate the standard deviation s_s for the test results of the spot sample using Formula (2):

$$s = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - x_m\right)^2}{n-1}} \quad (2)$$

where

s	is the standard deviation for the test results
n	is the number of test samples within the spot sample
i	is the number of the individual test sample
x_i	is the test result for test sample i
x_m	is the mean test result

The mean test result x_m and the calculated standard deviation of the test results s are the specific values of the corresponding estimator of the population mean μ and standard deviation σ.

Be aware that a 5 % characteristic value corresponds with a fractile $P = 95$ and a 95 % characteristic value also corresponds with a fractile $P = 95$. A 50 % characteristic value corresponds with a fractile $P = 50$.

If the standard deviation is unknown and if the test results shall be compared with a lower limit value then calculate the test result of the estimated normal distribution x_{est} using Formula (3):

$$X_{est} = X_m - K_{1,u} \times S_s \quad (3)$$

If the standard deviation is unknown and if the test results shall be compared with an upper limit value then calculate the test result of the estimated normal distribution x_{est} using Formula (4):

$$X_{est} = X_m + K_{1,u} \times S_s \quad (4)$$

If the standard deviation is unknown and if the test results shall be compared with a two-sided limit value then calculate the test result of the estimated normal distribution x_{est} using Formula (5):

$$X_{est} = X_m \pm K_{2,u} \times S_s \quad (5)$$

If the standard deviation σ is known and if the test results shall be compared with a lower limit value then calculate the test result of the estimated normal distribution x_{est} using Formula (6):

$$X_{est} = X_m - K_{1,k} \times \sigma \quad (6)$$

If the standard deviation σ is known and if the test results shall be compared with an upper limit value then calculate the test result of the estimated normal distribution x_{est} using Formula (7):

$$X_{est} = X_m + K_{1,k} \times \sigma \quad (7)$$

If the standard deviation σ is known and if the test results shall be compared with a two-sided upper limit value then calculate the test result of the estimated normal distribution x_{est} using Formula (8):

$$X_{est} = X_m \pm K_{2,k} \times \sigma \quad (8)$$

where

x_{est}	is the test result of the estimated normal distribution of the spot sample
x_m	is the mean test result
$k_{1,u}$	is the acceptance coefficient for unknown standard deviation and one-sided limit evaluation to be taken from Table 1 or 2 or the relevant tables in Annex B
$k_{2,u}$	is the acceptance coefficient for unknown standard deviation and two-sided limit evaluation to be taken from the relevant tables in Annex B
s_s	is the standard deviation for the test results of the spot sample
$k_{1,k}$	is the acceptance coefficient for known standard deviation and one-sided limit evaluation to be taken from Table 1 or 2 or the relevant tables in Annex B
$k_{2,k}$	is the acceptance coefficient for known standard deviation and two-sided limit evaluation to be taken from the relevant tables in Annex B
σ	is the known standard deviation

Table 1 — k_1 for 50 % characteristic value (50 % fractile) and 95 % confidence level

Standard deviation	***n*=3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**14**	**15**
Unknown	1,69	1,18	0,95	0,82	0,74	0,67	0,62	0,58	0,55	0,52	0,47	0,46
Known	0,95	0,82	0,74	0,67	0,62	0,58	0,55	0,52	0,50	0,48	0,44	0,43

Table 2 — k_1 for 5 % characteristic value (95 % fractile) and 95 % confidence level

Standard deviation	***n*=3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**14**	**15**
Unknown	7,66	5,14	4,20	3,71	3,40	3,19	3,03	2,91	2,82	2,74	2,62	2,57
Known	2,60	2,47	2,38	2,32	2,27	2,23	2,19	2,17	2,14	2,12	2,09	2,07

More tables for the acceptance coefficients k_1 and k_2, depending on the used fractile p (50 %, 75 %, 90 %, 95 %) and the used confidence level γ (50 %, 75 %, 90 %, 95 %) for known and unknown standard deviation are given in Annex B.

The method of using the acceptance coefficient for known standard deviation k_k is only valid when the standard deviation s_s of the spot sample corresponds to Formula (9):

$$0,63\ \sigma \leq s_s \leq 1,37\ \sigma \tag{9}$$

If, as part of the evaluation, it turns out that $s_s > 1,37\ \sigma$, the manufacturer shall either restart or continue working with the unknown acceptance coefficient k_u. This means that the inspection lots shall be treated separately.

If, as part of the evaluation, it turns out that $s_s < 0,63\ \sigma$, the producer may decide:

- to restart, or
- to continue working with the unknown acceptance coefficient k_u, or
- to continue working with the known acceptance coefficients, knowing he is evaluating on the safe side

5.2.8 How to come from unknown to known standard deviation?

In control method A (5.2.5) the standard deviation of the population is considered to be unknown at least for the first 40 test samples and the acceptance coefficient k_u is taken from tables for unknown standard deviation. For the next 80 test samples the standard deviation can be considered to be known, but the used acceptance coefficient is corrected (k_c). The acceptance coefficient for the known standard deviation k_k is taken from tables for known standard deviation. The corrected acceptance coefficient k_c is calculated by a linear interpolation between the acceptance coefficient k_u and k_k. The known standard deviation σ is calculated based on at least the first 40 test results.

In control method B (5.2.6) the standard deviation of the population is considered to be unknown at least for the first 20 test samples and the acceptance coefficient k_u is taken from tables for unknown standard deviation. For the next 40 test samples the standard deviation can be considered to be known, but the used acceptance coefficient is corrected (k_c) as mentioned in the previous section. The acceptance coefficient for the known standard deviation k_k is taken from tables for known standard deviation. The known standard deviation σ is calculated based on at least the first 20 test results.

If 'progressive sampling' is used the standard deviation of the population is considered to be unknown for at least the first 30 test samples and the acceptance coefficient k_u is taken from tables for unknown standard deviation. For the next 30 test samples the standard deviation can be considered to be known, but the used acceptance coefficient is corrected (k_c) as mentioned in the first section of this clause. The acceptance coefficient for the known standard deviation k_k is taken from tables for known standard deviation. The known standard deviation σ is calculated based on at least the first 30 test results.

5.2.9 Conformity

After calculating x_{est} by testing the inspection lots the result shall be compared with either the declared value (DV) or a lower (LL) or upper limit (UL) depending on the property.

For instance:

- for compressive strength it is the declared value or the lower limit. The declared value needs to be equal to or lower than the lower limit value.

- for process control properties it can be the upper and lower declared value or the upper and lower limit.

- for thermal values it is the declared value or the upper limit. The declared value needs to be equal to or higher than the lower limit value.

5.2.10 Simple and conservative approach

A simple and conservative approach can be to evaluate single test results for at least one year for a given property and calculate the mean value and the standard deviation, then fix a band in which new test results shall fit in. The upper band limit and lower band limit can then be two times the standard deviation away from the mean value. Then the declared value is recommended to be 0,4 times the standard deviation away from the respective band limits. If non-conformity occurs the evaluation of at least the last year of single test results, including the non-conforming values, should be repeated and the band limit values adjusted accordingly. The same should happen for the declared value. The non-conforming inspection lot can be treated as described in 5.2.11 using control method A.

5.2.11 Non-conforming products

When an evaluation of the test results of the last spot sample is leading to non-conformity, it is important to avoid the whole inspection lot being mixed up with the other inspection lots. The non-conforming inspection lot shall be treated separately. It may be reclassified by the manufacturer and given different declared values. If it is not segregated the whole stock shall be treated as non-conforming. For that reason a procedure for dealing with non-conforming products should be developed.

It should be in the interest of the manufacturer to avoid that the same non-conformity occurs again. When non-conformity occurs, it is important to try to identify the reason why, otherwise it is difficult to find out what to do to avoid it happening again. Testing can be part of the identification.

To ensure that the personnel managing the production know what to do when check and measuring values are passing the limit values, it is important to have the necessary instructions documented.

Non-conformities will normally result in higher frequencies than the ones used. The background for that is to reduce the size of the next batch that might also not comply.

5.2.12 Guidance

5.2.12.1 How to use the different possibilities?

A manufacturer is producing units in two different ways:

a) Product 1 is a special unit produced very rarely and only in small quantities. The characteristics of the product can vary from production to production.

b) Product 2 is one of the core units of the production site. It is produced in a series of variable length – sometimes only two days of production – but it is produced within short-time intervals.

   For product 1 it is obvious to use control method A (batch control). For product 2 both control methods A and B can be used. For product 2 it is even possible to use control method A for some properties and control method B for other properties. If using method B a re-declaration in connection with a non-conformity is possible based on test results obtained by testing a new spot sample taken at random from the inspection lot following control method A. However it is necessary to keep the test results leading to the non-conformity in the method B control system when evaluating the next spot sample.

The following details can be used when planning the setup of the FPC system:

Control method A:

1) verification of separate inspection lots.

2) inspection lots are defined to be the full production series.

3) the minimum sample size of the spot sample is 6 units ($n \geq 6$).

4) level of confidence for compressive strength for Category I units shall be 95 %. For gross dry density or net dry density used as a proxy property to thermal conductivity a confidence level of 50 % or 90 % may be chosen.

5) if the spot sample size is 6 units, the acceptance constant k_n for mean compressive strength at a 95 % confidence level is $k_{1,u}$ = 0,82 for unknown standard deviation and $k_{1,k}$ = 0,67 for known standard deviation.

6) if the spot sample size is 6 units, the acceptance constant k_n for 5 % characteristic compressive strength at a 95 % confidence level is $k_{1,u}$ = 3,71 for unknown standard deviation and $k_{1,k}$ = 2,32 for known standard deviation.

7) if the spot sample size is 6 units, the acceptance constant k_n for mean compressive strength at a 75 % confidence level (Category II units) is $k_{1,u}$ = 0,30 for unknown standard deviation and $k_{1,k}$ = 0,28 for known standard deviation.

Control method B:

1) verification of series of inspection lots.

2) inspection lots can be defined as the units produced within 1 production week/five days.

3) the minimum sample size of the spot sample is 3 units ($n \geq 3$).

4) size of series are 4 inspection lots (l = 4).

5) in case of n = 3, the sample size used for evaluation of each inspection lot is 12.

6) level of confidence for compressive strength for Category I units shall be 95 %. For gross dry density used as a proxy property to thermal conductivity a confidence level of 50 % or 90 % may be chosen.

7) if the spot sample size is 3 units, the acceptance constant k_n for mean compressive strength at a 95 % confidence level is $k_{1,u}$ = 0,52 for unknown standard deviation and $k_{1,k}$ = 0,47 for known standard deviation. If a sample size of a spot sample is raised to 6 units instead of 3 then the acceptance constant k_n for mean compressive strength is $k_{1,u}$ = 0,35 for unknown standard deviation and $k_{1,k}$ = 0,34 for known standard deviation.

8) if the spot sample size is 3 units, the acceptance constant k_n for 5 % characteristic compressive strength at a 95 % confidence level is $k_{1,u}$ = 2,74 for unknown standard deviation and $k_{1,k}$ = 2,12 for known standard deviation. If a sample size of a spot sample is raised to 6 units instead of 3 then the acceptance constant k_n for mean compressive strength is $k_{1,u}$ = 2,31 for unknown standard deviation and $k_{1,k}$ = 1,98 for known standard deviation.

9) if the spot sample size is 3 units, the acceptance constant k_n for mean compressive strength at a 75 % confidence level (Category II units) is $k_{1,u}$ = 0,20 for unknown standard deviation and $k_{1,k}$ = 0,19 for known standard deviation. If a sample size of a spot sample is raised to 6 units instead of 3 then the acceptance constant k_n for mean compressive strength is $k_{1,u}$ = 0,14 for unknown standard deviation and $k_{1,k}$ = 0,14 for known standard deviation.

5.2.12.2 What to do with an inspection lot where the evaluated test results for one or more properties are leading to non-conformity?

Control method A:

1) discard the inspection lot; or

2) sample a new and larger spot sample (e.g. 12 instead of 6), test the sample for the properties leading to a non-conformity and evaluate the test results using a reduced acceptance constant (e.g. 0,52 instead of 0,82) in accordance with the higher number of units in the test sample; or

3) change the declaration of the units based on product type determination by type testing.

Control method B:

1) discard the inspection lot; or

2) sample a new larger spot sample (e.g. ≥ 6 instead of 3 units) using control method A and evaluate the test results using a reduced acceptance constant, in accordance with the number of the units in the test sample and change the declaration accordingly. (*)

   (*) Always keep the results of the inspection lot within the system when evaluating the next inspection lot or start from the very beginning.

When a non-conformity is identified in the finished product testing it is not possible to take any corrective actions for the tested inspection lot. It can only be discarded or re-declared. The longer the production process of the units lasts, the larger the number of units produced before it is possible to correct the process, leading again to a larger number of units to be discarded or re-declared.

Consideration should be given to identifying the most economical way to arrange the control through the right mix of process control and finished product testing, and to consider also the possibility of using internal proxy tests in the process control.

The manufacturer may define product groups. A product group consists of products from one manufacturer having common values for one or more characteristics. That means that the products belonging to a product group may differ in accordance with the characteristics in question. If a product group is defined, then the FPC system should ensure that all types of units within a group are controlled and over time also by the finished product testing, if this is part of the FPC.

For process control the evaluation procedure described in 5.2.6 may be used, when appropriate.

5.2.12.3 Example of proxy test

A manufacturer of units would like to declare the thermal conductivity, $\lambda_{10,dry,unit}$, of the unit. By carrying out tests for masonry made of specific units it is possible for these units to establish a relationship between the thermal conductivity, $\lambda_{10,dry,unit}$, and the gross dry density of the units. By testing and controlling the gross dry density it is possible to declare the thermal conductivity,

$\lambda_{10,dry,unit}$, of the unit. The gross dry density is used as a proxy property for the thermal conductivity.

If the declared thermal conductivity value shall be a 50 % fractile with a confidence level of 50 % the test results of the spot samples shall be evaluated, e.g. by the calculation procedures described in 5.2.6 using Table B.1 or B.5.

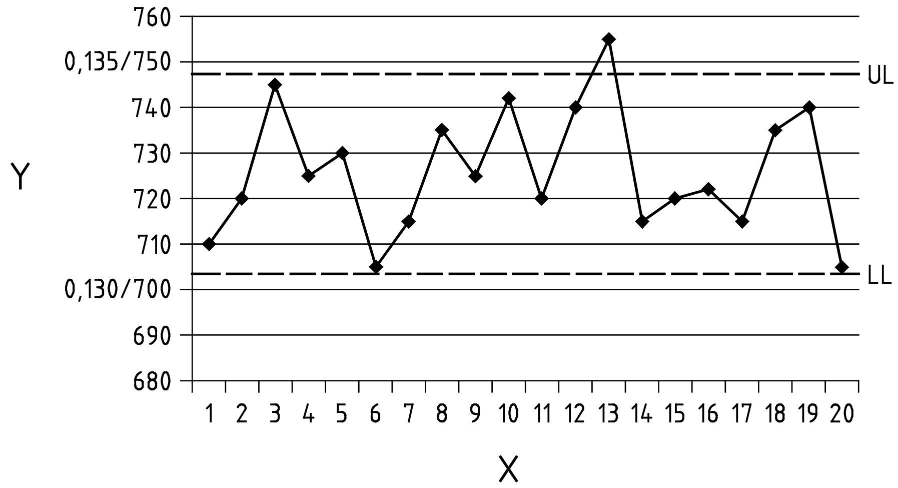

Figure 8 — Example of variation in the gross dry density of the units over time

6 Product type determination

If non-conformity occurs in control method A, the inspection lot may be re-declared. If the reference test methods and the sampling procedure for type testing are used, then the test can be regarded as a type test for the product type determination.

If non-conformity occurs in control method B and the inspection lot is re-declared following control method A using the reference test methods and the sampling procedure for type testing, then the test can be regarded as an type test for the product type determination.

Annex A
(informative)

Normal distribution (Laplace-Gauss distribution)

The normal distribution is given by the formula:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-u}{\sigma} \right)^2 \right]$$

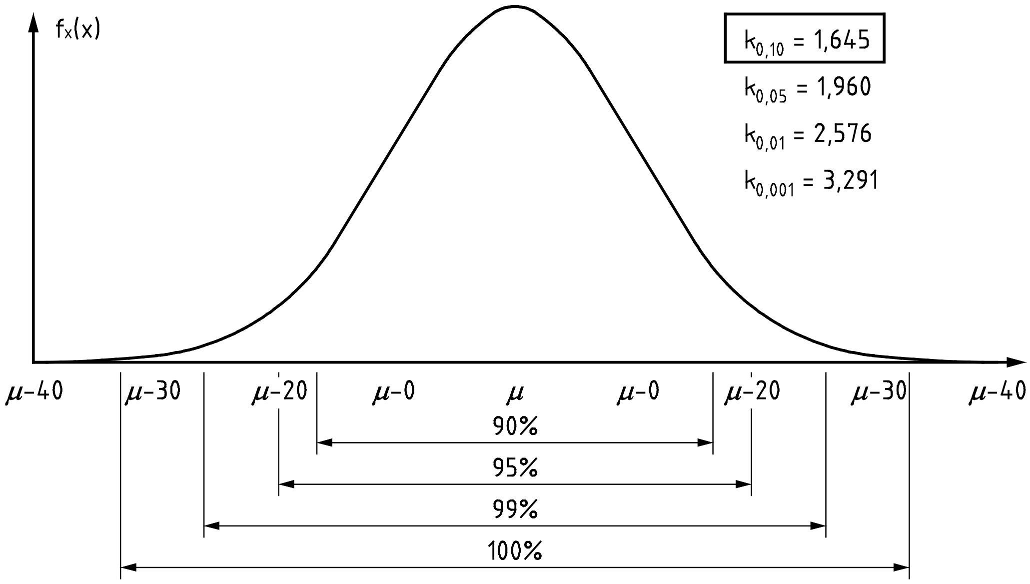

Figure A.1 —Normal distribution (Laplace-Gauss distribution)

This normal distribution is a fundamental type of symmetrical distribution defined on an unlimited interval, which is fully described by two parameters:

- the mean μ;
- the variance σ^2.

All the information derived from a given random sample x_1, x_2, …,x_n of the size n, taken from a normal population, is completely described by two sample characteristics only:

- the sample mean x_m;
- the sample variance s^2.

These characteristics are specific values of the corresponding estimators of the population mean and variance.

Annex B
(informative)

Tables for acceptance coefficient k_n depending on the used fractile p and confidence level γ (taken from ISO 16269-6:2005)

Table B.1 — k_1 for one-sided statistical tolerance, standard deviation: known and confidence level γ = 50 %

n	fractile: *p*			
	0,50	***0,75***	***0,90***	***0,95***
2	0,000	0,675	1,282	1,645
3	0,000	0,675	1,282	1,645
4	0,000	0,675	1,282	1,645
5	0,000	0,675	1,282	1,645
6	0,000	0,675	1,282	1,645
7	0,000	0,675	1,282	1,645
8	0,000	0,675	1,282	1,645
9	0,000	0,675	1,282	1,645
10	0,000	0,675	1,282	1,645
11	0,000	0,675	1,282	1,645
12	0,000	0,675	1,282	1,645
13	0,000	0,675	1,282	1,645
14	0,000	0,675	1,282	1,645
15	0,000	0,675	1,282	1,645
16	0,000	0,675	1,282	1,645
17	0,000	0,675	1,282	1,645
18	0,000	0,675	1,282	1,645
19	0,000	0,675	1,282	1,645
20	0,000	0,675	1,282	1,645

n	fractile: *p*			
	0,50	***0,75***	***0,90***	***0,95***
21	0,000	0,675	1,282	1,645
22	0,000	0,675	1,282	1,645
23	0,000	0,675	1,282	1,645
24	0,000	0,675	1,282	1,645
25	0,000	0,675	1,282	1,645
26	0,000	0,675	1,282	1,645
27	0,000	0,675	1,282	1,645
28	0,000	0,675	1,282	1,645
29	0,000	0,675	1,282	1,645
30	0,000	0,675	1,282	1,645
35	0,000	0,675	1,282	1,645
40	0,000	0,675	1,282	1,645
45	0,000	0,675	1,282	1,645
50	0,000	0,675	1,282	1,645
60	0,000	0,675	1,282	1,645
70	0,000	0,675	1,282	1,645
80	0,000	0,675	1,282	1,645
90	0,000	0,675	1,282	1,645
100	0,000	0,675	1,282	1,645

Table B.2 — k_1 for one-sided statistical tolerance, standard deviation: known and confidence level γ = 75 %

n	fractile: *p*				n	fractile: *p*			
	0,50	*0,75*	*0,90*	*0,95*		*0,50*	*0,75*	*0,90*	*0,95*
2	0,477	1,152	1,759	2,122	21	0,148	0,823	1,430	1,793
3	0,390	1,064	1,671	2,035	22	0,144	0,819	1,426	1,789
4	0,388	1,012	1,619	1,983	23	0,141	0,816	1,423	1,786
5	0,302	0,977	1,584	1,947	24	0,138	0,813	1,420	1,783
6	0,276	0,950	1,557	1,921	25	0,136	0,810	1,417	1,781
7	0,255	0,930	1,537	1,900	26	0,133	0,807	1,414	1,778
8	0,239	0,913	1,521	1,884	27	0,131	0,805	1,412	1,776
9	0,225	0,900	1,507	1,870	28	0,128	0,802	1,410	1,773
10	0,214	0,888	1,495	1,859	29	0,126	0,800	1,408	1,771
11	0,204	0,878	1,485	1,849	30	0,124	0,798	1,405	1,768
12	0,195	0,870	1,477	1,840	35	0,115	0,789	1,396	1,759
13	0,188	0,862	1,469	1,832	40	0,107	0,782	1,389	1,752
14	0,181	0,855	1,462	1,826	45	0,101	0,776	1,383	1,746
15	0,175	0,849	1,456	1,820	50	0,096	0,770	1,377	1,741
16	0,169	0,844	1,451	1,814	60	0,088	0,762	1,369	1,732
17	0,164	0,839	1,446	1,809	70	0,081	0,756	1,363	1,726
18	0,159	0,834	1,441	1,804	80	0,076	0,75	1,357	1,721
19	0,155	0,830	1,437	1,800	90	0,072	0,746	1,353	1,716
20	0,151	0,826	1,433	1,796	100	0,068	0,742	1,35	1,713

Table B.3 — k_1 for one-sided statistical tolerance, standard deviation: known and confidence level γ = 90 %

n	fractile: *p*				n	fractile: *p*			
	0,50	*0,75*	*0,90*	*0,95*		*0,50*	*0,75*	*0,90*	*0,95*
2	0,907	1,581	2,188	2,552	21	0,281	0,955	1,562	1,926
3	0,740	1,415	2,022	2,385	22	0,274	0,948	1,555	1,919
4	0,641	1,316	1,923	2,286	23	0,268	0,943	1,550	1,913
5	0,574	1,248	1,855	2,218	24	0,262	0,937	1,544	1,907
6	0,524	1,198	1,805	2,169	25	0,257	0,932	1,539	1,902
7	0,485	1,159	1,766	2,130	26	0,252	0,926	1,533	1,897
8	0,454	1,128	1,735	2,098	27	0,248	0,922	1,529	1,893
9	0,428	1,102	1,709	2,073	28	0,243	0,917	1,524	1,888
10	0,406	1,080	1,687	2,051	29	0,239	0,913	1,520	1,884
11	0,387	1,061	1,668	2,032	30	0,234	0,909	1,516	1,879
12	0,370	1,045	1,652	2,015	35	0,217	0,892	1,499	1,862
13	0,356	1,030	1,637	2,001	40	0,203	0,878	1,485	1,848
14	0,343	1,017	1,625	1,998	45	0,192	0,866	1,473	1,836
15	0,331	1,006	1,613	1,976	50	0,182	0,856	1,463	1,827
16	0,321	0,995	1,602	1,966	60	0,166	0,840	1,447	1,811
17	0,311	0,986	1,593	1,956	70	0,154	0,828	1,435	1,799
18	0,303	0,977	1,584	1,947	80	0,144	0,818	1,425	1,789
19	0,295	0,969	1,576	1,939	90	0,136	0,810	1,417	1,780
20	0,287	0,962	1,569	1,932	100	0,129	0,803	1,410	1,774

Table B.4 — k_1 for one-sided statistical tolerance, standard deviation: known and confidence level γ = 95 %

n	fractile: *p*				n	fractile: *p*			
	0,50	***0,75***	***0,90***	***0,95***		***0,50***	***0,75***	***0,90***	***0,95***
2	1,164	1,838	2,445	2,828	21	0,360	1,035	1,642	2,005
3	0,950	1,625	2,232	2,595	22	0,351	1,026	1,633	1,996
4	0,823	1,497	2,104	2,468	23	0,344	1,019	1,626	1,989
5	0,736	1,411	2,018	2,381	24	0,336	1,011	1,618	1,981
6	0,672	1,346	1,954	2,317	25	0,330	1,005	1,612	1,975
7	0,622	1,297	1,904	2,267	26	0,323	0,998	1,605	1,968
8	0,582	1,257	1,864	2,227	27	0,317	0,992	1,599	1,962
9	0,549	1,223	1,830	2,194	28	0,311	0,986	1,593	1,956
10	0,521	1,195	1,802	2,166	29	0,306	0,981	1,588	1,951
11	0,496	1,171	1,778	2,141	30	0,301	0,975	1,582	1,946
12	0,475	1,150	1,757	2,120	35	0,279	0,953	1,560	1,923
13	0,457	1,131	1,738	2,102	40	0,261	0,935	1,542	1,905
14	0,440	1,115	1,722	2,085	45	0,246	0,920	1,527	1,891
15	0,425	1,100	1,707	2,070	50	0,233	0,908	1,515	1,878
16	0,412	1,086	1,693	2,057	60	0,213	0,887	1,494	1,858
17	0,399	1,074	1,691	2,044	70	0,197	0,872	1,479	1,842
18	0,388	1,063	1,670	2,033	80	0,184	0,859	1,466	1,829
19	0,378	1,052	1,659	2,023	90	0,174	0,848	1,455	1,819
20	0,368	1,043	1,650	2,013	100	0,165	0,839	1,447	1,810

Table B.5 — k_1 for one-sided statistical tolerance, standard deviation: unknown and confidence level γ = 50 %

n	fractile: *p*					n	fractile: *p*			
	0,50	***0,75***	***0,90***	***0,95***			***0,50***	***0,75***	***0,90***	***0,95***
2	0,000	0,888	1,785	2,339		21	0,000	0,685	1,301	1,671
3	0,000	0,774	1,499	1,939		22	0,000	0,684	1,300	1,669
4	0,000	0,739	1,419	1,830		23	0,000	0,684	1,299	1,668
5	0,000	0,722	1,382	1,780		24	0,000	0,683	1,298	1,667
6	0,000	0,712	1,361	1,751		25	0,000	0,683	1,298	1,666
7	0,000	0,706	1,347	1,732		26	0,000	0,682	1,297	1,665
8	0,000	0,701	1,337	1,719		27	0,000	0,682	1,297	1,665
9	0,000	0,698	1,330	1,710		28	0,000	0,682	1,296	1,664
10	0,000	0,695	1,325	1,702		29	0,000	0,682	1,296	1,663
11	0,000	0,693	1,320	1,696		30	0,000	0,681	1,295	1,662
12	0,000	0,692	1,317	1,691		35	0,000	0,680	1,293	1,660
13	0,000	0,690	1,314	1,687		40	0,000	0,680	1,292	1,658
14	0,000	0,689	1,311	1,684		45	0,000	0,679	1,290	1,657
15	0,000	0,688	1,309	1,681		50	0,000	0,679	1,290	1,655
16	0,000	0,678	1,307	1,679		60	0,000	0,678	1,288	1,654
17	0,000	0,686	1,306	1,677		70	0,000	0,678	1,287	1,652
18	0,000	0,686	1,304	1,675		80	0,000	0,677	1,287	1,652
19	0,000	0,685	1,303	1,673		90	0,000	0,677	1,286	1,651
20	0,000	0,685	1,302	1,672		100	0,000	0,677	1,286	1,650

Table B.6 — k_1 for one-sided statistical tolerance, standard deviation: unknown and confidence level γ = 75 %

n	fractile: *p*				n	fractile: *p*			
	0,50	*0,75*	*0,90*	*0,95*		*0,50*	*0,75*	*0,90*	*0,95*
2	0,708	2,225	3,993	5,122	21	0,151	0,860	1,522	1,924
3	0,472	1,465	2,502	3,152	22	0,147	0,854	1,514	1,916
4	0,383	1,256	2,134	2,681	23	0,144	0,850	1,509	1,909
5	0,332	1,152	1,962	2,464	24	0,140	0,846	1,503	1,902
6	0,297	1,088	1,860	2,336	25	0,138	0,842	1,498	1,896
7	0,272	1,044	1,791	2,251	26	0,135	0,838	1,492	1,889
8	0,252	1,011	1,740	2,189	27	0,133	0,835	1,488	1,884
9	0,236	0,985	1,702	2,142	28	0,130	0,831	1,483	1,879
10	0,223	0,964	1,671	2,104	29	0,128	0,828	1,479	1,874
11	0,212	0,947	1,646	2,074	30	0,125	0,825	1,475	1,869
12	0,202	0,933	1,625	2,048	35	0,116	0,813	1,458	1,850
13	0,193	0,920	1,607	2,026	40	0,108	0,803	1,445	1,834
14	0,186	0,909	1,591	2,008	45	0,102	0,795	1,435	1,822
15	0,179	0,900	1,578	1,991	50	0,097	0,789	1,426	1,811
16	0,173	0,891	1,566	1,977	60	0,088	0,778	1,412	1,795
17	0,168	0,884	1,555	1,964	70	0,082	0,770	1,401	1,783
18	0,163	0,877	1,545	1,952	80	0,076	0,763	1,393	1,773
19	0,158	0,870	1,536	1,942	90	0,072	0,758	1,386	1,765
20	0,154	0,865	1,529	1,932	100	0,068	0,753	1,380	1,758

Table B.7 — k_1 for one-sided statistical tolerance, standard deviation: unknown and confidence level γ = 90 %

n	fractile: *p*				n	fractile: *p*			
	0,50	*0,75*	*0,90*	*0,95*		*0,50*	*0,75*	*0,90*	*0,95*
2	2,177	5,843	10,253	13,090	21	0,290	1,036	1,752	2,191
3	1,089	2,603	4,259	5,312	22	0,283	1,026	1,737	2,174
4	0,819	1,973	3,188	3,957	23	0,277	1,017	1,725	2,160
5	0,686	1,698	2,743	3,400	24	0,270	1,008	1,713	2,146
6	0,603	1,540	2,494	3,092	25	0,265	1,001	1,703	2,134
7	0,545	1,436	2,333	2,894	26	0,259	0,993	1,692	2,121
8	0,501	1,360	2,219	2,755	27	0,254	0,986	1,683	2,110
9	0,466	1,303	2,133	2,650	28	0,249	0,979	1,674	2,099
10	0,438	1,257	2,066	2,569	29	0,245	0,973	1,666	2,090
11	0,414	1,220	2,012	2,503	30	0,240	0,967	1,658	2,080
12	0,394	1,189	1,967	2,449	35	0,221	0,943	1,624	2,041
13	0,377	1,162	1,929	2,403	40	0,207	0,923	1,598	2,011
14	0,361	1,139	1,896	2,364	45	0,194	0,907	1,577	1,986
15	0,348	1,119	1,867	2,329	50	0,184	0,894	1,560	1,966
16	0,336	1,101	1,842	2,299	60	0,168	0,873	1,533	1,934
17	0,325	1,085	1,820	2,273	70	0,155	0,857	1,512	1,910
18	0,315	1,071	1,800	2,249	80	0,145	0,845	1,495	1,890
19	0,306	1,058	1,782	2,228	90	0,137	0,834	1,482	1,875
20	0,297	1,046	1,766	2,208	100	0,130	0,825	1,471	1,862

Table B.8 — k_1 for one-sided statistical tolerance, standard deviation: unknown and confidence level γ = 95 %

n	fractile: *p*				n	fractile: *p*			
	0,50	***0,75***	***0,90***	***0,95***		***0,50***	***0,75***	***0,90***	***0,95***
2	4,465	11,763	20,582	26,260	21	0,377	1,153	1,907	2,373
3	1,686	3,807	6,156	7,656	22	0,367	1,138	1,887	2,349
4	1,177	2,618	4,162	5,144	23	0,359	1,126	1,870	2,330
5	0,954	2,150	3,407	4,203	24	0,350	1,114	1,853	2,310
6	0,823	1,896	3,007	3,708	25	0,343	1,104	1,839	2,293
7	0,735	1,733	2,756	3,400	26	0,335	1,093	1,825	2,276
8	0,670	1,618	2,582	3,188	27	0,329	1,084	1,813	2,261
9	0,620	1,533	2,454	3,032	28	0,322	1,075	1,800	2,246
10	0,580	1,466	2,355	2,911	29	0,317	1,067	1,789	2,233
11	0,547	1,412	2,276	2,815	30	0,311	1,059	1,778	2,220
12	0,519	1,367	2,211	2,737	35	0,286	1,026	1,733	2,167
13	0,495	1,329	2,156	2,671	40	0,267	1,000	1,698	2,126
14	0,474	1,296	2,109	2,615	45	0,251	0,978	1,669	2,093
15	0,455	1,268	2,069	2,567	50	0,238	0,961	1,646	2,065
16	0,439	1,243	2,033	2,524	60	0,216	0,933	1,609	2,023
17	0,424	1,221	2,002	2,487	70	0,200	0,912	1,582	1,990
18	0,411	1,201	1,974	2,453	80	0,187	0,895	1,560	1,965
19	0,398	1,183	1,949	2,424	90	0,176	0,882	1,542	1,944
20	0,387	1,167	1,926	2,397	100	0,167	0,870	1,527	1,927

Table B.9 — k_2 for two-sided statistical tolerance, standard deviation: known and confidence level γ = 50 %

n	fractile: *p*				n	fractile: *p*			
	0,50	*0,75*	*0,90*	*0,95*		*0,50*	*0,75*	*0,90*	*0,95*
2	0,755	1,282	1,823	2,164	21	0,683	1,164	1,663	1,982
3	0,727	1,238	1,766	2,100	22	0,682	1,163	1,662	1,981
4	0,714	1,216	1,737	2,067	23	0,682	1,163	1,662	1,980
5	0,706	1,203	1,719	2,046	24	0,681	1,162	1,661	1,979
6	0,701	1,195	1,707	2,033	25	0,681	1,162	1,661	1,978
7	0,697	1,188	1,698	2,023	26	0,681	1,161	1,660	1,977
8	0,694	1,184	1,692	2,015	27	0,681	1,161	1,660	1,977
9	0,692	1,180	1,686	2,009	28	0,680	1,160	1,659	1,976
10	0,690	1,177	1,682	2,004	29	0,680	1,160	1,659	1,976
11	0,689	1,175	1,679	2,000	30	0,680	1,160	1,658	1,975
12	0,688	1,173	1,676	1,997	35	0,679	1,158	1,656	1,973
13	0,687	1,171	1,674	1,994	40	0,679	1,157	1,655	1,972
14	0,686	1,170	1,672	1,992	45	0,678	1,157	1,654	1,970
15	0,685	1,168	1,670	1,990	50	0,678	1,156	1,653	1,969
16	0,685	1,167	1,669	1,988	60	0,678	1,155	1,652	1,968
17	0,684	1,166	1,667	1,986	70	0,677	1,155	1,651	1,967
18	0,684	1,165	1,666	1,985	80	0,677	1,154	1,650	1,966
19	0,683	1,165	1,665	1,984	90	0,677	1,154	1,650	1,965
20	0,683	1,164	1,664	1,983	100	0,677	1,153	1,649	1,965

Table B.10 — k_2 for two-sided statistical tolerance, standard deviation: known and confidence level γ = 75 %

n	fractile: *p*				n	fractile: *p*			
	0,50	*0,75*	*0,90*	*0,95*		*0,50*	*0,75*	*0,90*	*0,95*
2	0,919	1,520	2,106	2,464	21	0,697	1,187	1,697	2,021
3	0,834	1,402	1,971	2,323	22	0,695	1,185	1,694	2,018
4	0,792	1,340	1,897	2,244	23	0,695	1,184	1,692	2,016
5	0,768	1,303	1,850	2,194	24	0,694	1,183	1,690	2,013
6	0,752	1,278	1,818	2,158	25	0,693	1,182	1,684	2,011
7	0,741	1,260	1,794	2,132	26	0,692	1,180	1,678	2,009
8	0,732	1,246	1,776	2,112	27	0,692	1,179	1,681	2,008
9	0,726	1,236	1,762	2,096	28	0,691	1,178	1,684	2,006
10	0,721	1,227	1,751	2,083	29	0,691	1,177	1,683	2,005
11	0,716	1,220	1,742	2,073	30	0,690	1,176	1,681	2,003
12	0,713	1,214	1,734	2,064	35	0,688	1,173	1,676	1,997
13	0,710	1,209	1,727	2,056	40	0,686	1,170	1,672	1,992
14	0,707	1,205	1,722	2,050	45	0,685	1,168	1,669	1,989
15	0,705	1,202	1,717	2,044	50	0,684	1,166	1,667	1,986
16	0,703	1,198	1,712	2,039	60	0,682	1,164	1,663	1,982
17	0,702	1,196	1,708	2,034	70	0,681	1,162	1,661	1,979
18	0,700	1,193	1,705	2,030	80	0,681	1,16	1,659	1,977
19	0,699	1,191	1,702	2,027	90	0,68	1,159	1,657	1,975
20	0,698	1,189	1,699	2,024	100	0,679	1,158	1,656	1,973

Table B.11 — k_2 for two-sided statistical tolerance, standard deviation: known and confidence level γ = 90 %

n	fractile: *p*				n	fractile: *p*			
	0,50	*0,75*	*0,90*	*0,95*		*0,50*	*0,75*	*0,90*	*0,95*
2	1,187	1,842	2,446	2,809	21	0,720	1,226	1,749	2,081
3	1,013	1,640	2,236	2,597	22	0,717	1,222	1,744	2,075
4	0,924	1,527	2,114	2,473	23	0,716	1,219	1,740	2,071
5	0,872	1,456	2,034	2,390	24	0,714	1,216	1,736	2,066
6	0,837	1,407	1,977	2,330	25	0,713	1,214	1,733	2,062
7	0,813	1,371	1,935	2,285	26	0,711	1,211	1,729	2,058
8	0,795	1,344	1,902	2,250	27	0,710	1,209	1,726	2,055
9	0,781	1,323	1,875	2,222	28	0,708	1,207	1,723	2,052
10	0,770	1,306	1,854	2,198	29	0,707	1,205	1,721	2,049
11	0,761	1,292	1,836	2,179	30	0,706	1,203	1,718	2,046
12	0,754	1,281	1,821	2,162	35	0,701	1,195	1,708	2,034
13	0,758	1,271	1,809	2,148	40	0,698	1,190	1,700	2,025
14	0,742	1,262	1,797	2,136	45	0,695	1,185	1,694	2,018
15	0,738	1,255	1,788	2,125	50	0,693	1,182	1,689	2,012
16	0,734	1,248	1,779	2,115	60	0,690	1,177	1,682	2,004
17	0,730	1,243	1,772	2,107	70	0,688	1,173	1,677	1,998
18	0,727	1,237	1,765	2,099	80	0,686	1,170	1,673	1,993
19	0,724	1,233	1,759	2,092	90	0,685	1,168	1,670	1,990
20	0,722	1,229	1,753	2,086	100	0,684	1,166	1,667	1,987

Table B.12— k_2 for two-sided statistical tolerance, standard deviation: known and confidence level γ = 95 %

n	fractile: *p*				n	fractile: *p*			
	0,50	***0,75***	***0,90***	***0,95***		***0,50***	***0,75***	***0,90***	***0,95***
2	1,393	2,062	2,668	3,031	21	0,739	1,256	1,790	2,128
3	1,160	1,812	2,415	2,777	22	0,736	1,251	1,783	2,120
4	1,036	1,668	2,265	2,627	23	0,733	1,247	1,778	2,114
5	0,960	1,574	2,165	2,525	24	0,730	1,243	1,772	2,108
6	0,910	1,509	2,093	2,451	25	0,728	1,240	1,768	2,103
7	0,875	1,460	2,039	2,395	26	0,726	1,236	1,763	2,097
8	0,894	0,423	1,996	2,350	27	0,724	1,233	1,759	2,093
9	0,828	1,394	1,961	2,313	28	0,722	1,230	1,755	2,088
10	0,812	1,370	1,933	2,283	29	0,721	1,228	1,752	2,084
11	0,799	1,351	1,909	2,258	30	0,719	1,225	1,748	2,080
12	0,788	1,334	1,889	2,236	35	0,713	1,214	1,733	2,063
13	0,779	1,320	1,872	2,218	40	0,708	1,206	1,723	2,051
14	0,772	1,308	1,857	2,201	45	0,704	1,200	1,714	2,041
15	0,765	1,298	1,844	2,187	50	0,701	1,195	1,708	2,033
16	0,759	1,289	1,832	2,174	60	0,697	1,188	1,697	2,022
17	0,754	1,281	1,822	2,163	70	0,694	1,182	1,690	2,013
18	0,749	1,274	1,812	2,152	80	0,691	1,178	1,684	2,007
19	0,745	1,267	1,804	2,143	90	0,689	1,175	1,680	2,002
20	0,742	1,261	1,797	2,135	100	0,688	1,173	1,677	1,998

Table B.13 — k_2 for two-sided statistical tolerance, standard deviation: unknown and confidence level γ = 50 %

n	fractile: *p*				n	fractile: *p*			
	0,50	*0,75*	*0,90*	*0,95*		*0,50*	*0,75*	*0,90*	*0,95*
2	1,243	2,057	2,870	3,376	21	0,703	1,198	1,711	2,038
3	0,943	1,582	2,229	2,635	22	0,701	1,195	1,708	2,034
4	0,853	1,441	2,040	2,416	23	0,700	1,193	1,706	2,031
5	0,809	1,370	1,946	2,308	24	0,699	1,191	1,703	2,028
6	0,782	1,328	1,889	2,243	25	0,698	1,190	1,701	2,026
7	0,765	1,300	1,851	2,199	26	0,697	1,188	1,698	2,023
8	0,752	1,279	1,823	2,168	27	0,697	1,187	1,696	2,021
9	0,743	1,264	1,802	2,143	28	0,696	1,186	1,694	2,018
10	0,735	1,252	1,786	2,124	29	0,695	1,185	1,693	2,016
11	0,730	1,242	1,772	2,109	30	0,694	1,183	1,691	2,014
12	0,725	1,234	1,761	2,096	35	0,691	1,179	1,685	2,007
13	0,721	1,227	1,752	2,086	40	0,689	1,175	1,680	2,001
14	0,717	1,222	1,744	2,077	45	0,688	1,172	1,676	1,997
15	0,714	1,217	1,738	2,069	50	0,686	1,170	1,673	1,993
16	0,712	1,212	1,732	2,062	60	0,684	1,167	1,668	1,988
17	0,709	1,209	1,727	2,056	70	0,683	1,165	1,665	1,984
18	0,707	1,205	1,722	2,051	80	0,682	1,163	1,662	1,981
19	0,706	1,202	1,718	2,046	90	0,681	1,162	1,661	1,979
20	0,704	1,200	1,714	2,042	100	0,681	1,160	1,659	1,977

Table B.14 — k_2 for two-sided statistical tolerance, standard deviation: unknown and confidence level γ = 75 %

n	fractile: *p*				n	fractile: *p*			
	0,50	*0,75*	*0,90*	*0,95*		*0,50*	*0,75*	*0,90*	*0,95*
2	2,674	4,394	6,109	7,178	21	0,788	1,343	1,918	2,284
3	1,492	2,487	3,489	4,117	22	0,784	1,336	1,908	2,273
4	1,211	2,036	2,872	3,397	23	0,781	1,331	1,900	2,264
5	1,083	1,829	2,590	3,069	24	0,777	1,325	1,892	2,254
6	1,009	1,709	2,425	2,877	25	0,774	1,320	1,886	2,246
7	0,961	1,630	2,316	2,750	26	0,771	1,315	1,879	2,238
8	0,926	1,573	2,238	2,659	27	0,769	1,311	1,873	2,231
9	0,900	1,530	2,179	2,590	28	0,766	1,306	1,867	2,224
10	0,880	1,497	2,133	2,536	29	0,764	1,303	1,862	2,218
11	0,864	1,469	2,095	2,492	30	0,762	1,299	1,857	2,211
12	0,850	1,447	2,064	2,456	35	0,753	1,284	1,835	2,186
13	0,839	1,428	2,038	2,425	40	0,747	1,273	1,819	2,167
14	0,829	1,412	2,015	2,399	45	0,741	1,263	1,806	2,152
15	0,821	1,398	1,996	2,376	50	0,737	1,256	1,795	2,139
16	0,814	1,386	1,979	2,356	60	0,730	1,244	1,779	2,119
17	0,807	1,375	1,964	2,338	70	0,725	1,236	1,766	2,105
18	0,802	1,366	1,950	2,322	80	0,721	1,229	1,757	2,093
19	0,797	1,357	1,938	2,308	90	0,718	1,223	1,749	2,084
20	0,792	1,349	1,927	2,295	100	0,715	1,219	1,742	2,076

Table B.15 — k_2 for two-sided statistical tolerance, standard deviation: unknown and confidence level γ = 90 %

n	fractile: *p*				n	fractile: *p*			
	0,50	***0,75***	***0,90***	***0,95***		***0,50***	***0,75***	***0,90***	***0,95***
2	6,809	11,166	15,513	18,221	21	0,880	1,500	2,142	2,550
3	2,492	4,135	5,789	6,824	22	0,873	1,487	2,124	2,529
4	1,766	2,954	4,158	4,913	23	0,867	1,477	2,110	2,512
5	1,473	2,478	3,500	4,143	24	0,861	1,466	2,095	2,494
6	1,314	2,218	3,141	3,723	25	0,856	1,458	2,083	2,480
7	1,213	2,053	2,913	3,456	26	0,850	1,449	2,070	2,465
8	1,144	1,939	2,755	3,270	27	0,846	1,442	2,059	2,452
9	1,093	1,854	2,637	3,133	28	0,841	1,434	2,048	2,439
10	1,053	1,789	2,546	3,026	29	0,837	1,427	2,039	2,428
11	1,022	1,737	2,474	2,941	30	0,833	1,420	2,029	2,417
12	0,996	1,694	2,414	2,871	35	0,817	1,393	1,991	2,372
13	0,975	1,659	2,365	2,813	40	0,805	1,372	1,962	2,337
14	0,957	1,628	2,322	2,763	45	0,795	1,356	1,938	2,309
15	0,941	1,602	2,286	2,720	50	0,787	1,342	1,919	2,286
16	0,928	1,580	2,254	2,683	60	0,775	1,321	1,889	2,250
17	0,916	1,560	2,226	2,650	70	0,766	1,306	1,867	2,224
18	0,905	1,542	2,201	2,620	80	0,759	1,294	1,849	2,203
19	0,896	1,526	2,179	2,594	90	0,753	1,284	1,835	2,187
20	0,887	1,512	2,159	2,570	100	0,748	1,276	1,824	2,173

Table B.16 — k_2 for two-sided statistical tolerance, standard deviation: unknown and confidence level γ = 95 %

n	fractile: *p*				n	fractile: *p*			
	0,50	***0,75***	***0,90***	***0,95***		***0,50***	***0,75***	***0,90***	***0,95***
2	13,652	22,383	31,093	36,520	21	0,944	1,608	2,296	2,733
3	3,585	5,938	8,306	9,789	22	0,934	1,591	2,272	2,705
4	2,288	3,819	5,369	6,342	23	0,926	1,577	2,253	2,682
5	1,812	3,041	4,291	5,077	24	0,918	1,563	2,233	2,659
6	1,566	2,639	3,733	4,423	25	0,911	1,552	2,217	2,639
7	1,416	2,392	3,390	4,020	26	0,904	1,540	2,200	2,619
8	1,314	2,224	3,157	3,746	27	0,898	1,530	2,186	2,602
9	1,240	2,101	2,987	3,546	28	0,892	1,519	2,171	2,585
10	1,183	2,008	2,857	3,394	29	0,887	1,511	2,159	2,570
11	1,139	1,935	2,754	3,273	30	0,881	1,502	2,146	2,555
12	1,103	1,875	2,671	3,175	35	0,860	1,466	2,095	2,495
13	1,074	1,825	2,602	3,094	40	0,844	1,438	2,056	2,449
14	1,049	1,784	2,543	3,025	45	0,831	1,417	2,025	2,412
15	1,027	1,748	2,493	2,965	50	0,821	1,399	2,000	2,382
16	1,009	1,717	2,449	2,914	60	0,804	1,371	1,960	2,336
17	0,992	1,689	2,411	2,869	70	0,792	1,351	1,931	2,301
18	0,978	1,665	2,377	2,829	80	0,783	1,335	1,909	2,274
19	0,965	1,644	2,347	2,793	90	0,776	1,322	1,890	2,252
20	0,954	1,625	2,319	2,761	100	0,769	1,312	1,875	2,234

Annex C
(informative)

Examples of statistical evaluation

C.1 Example 1

Example of statistical analysis of compressive strength using batch control:

The fractile P = 50 %

The confidence level γ = 95 %

The number of series of inspection lots is l = 1

One-sided tolerance interval, lower limit

The declared mean compressive strength is 15 N/mm²

For all inspection lots a sample size of 6 are taken, tested and evaluated, inspection lot by inspection lot (x_m (Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (3)) in accordance with 5.2.7 using $k_{1,u}$ taken from Table B.8 (p: 50 % and γ: 95 %)).

For the first 42 samples (1 – 7 inspection lots), the standard deviation of the population is considered to be unknown and the $k_{1,u}$ factor taken from Table B.8 (p: 50 % and γ: 95 %) is 0,823. For the inspection lots 8 – 20 the standard deviation can be considered as known, but the used acceptance coefficient is corrected (k_c). The acceptance coefficient for the known standard deviation $k_{1,k}$ is taken from Table B.4 (p: 50 % and γ: 95 %) and is 0,672. The corrected acceptance coefficient k_c is calculated by a linear interpolation between the acceptance coefficient $k_{1,u}$ and $k_{1,k}$ taking into account the considered inspection lot. The known standard deviation σ is calculated based on the first 42 test results.

From inspection lot 21 onwards 6 samples are taken from each next inspection lot and the test results are evaluated inspection lot by inspection lot (x_m (Formula (1)), known standard deviation σ and x_{est} (Formula (6))in accordance with 5.2.7 using $k_{1,k}$ taken from Table B.4 (p: 50 % and γ: 95 %)).

After each evaluation the result shall be compared with the lower limit value (e.g. the declared value) decided by the manufacturer.

If there is non-conformity due to great differences between the test results, the estimated value is highlighted by a red signal at the right side (batch 21). A non-conforming inspection lot shall be treated separately as described in 5.2.11.

Table C.1 — Example 1: One sided tolerance interval – lower limit – method A

EXAMPLE 1

ONE SIDED TOLERANCE INTERVAL-lower limit

METHOD A : use at least 6 testresults per inspection lot

fractile p	50	Start correction	7	Series of inspection lots	1	Declared Value	15
confidence level	95	End correction	20			1,409	1,858

Inspection lot	test 1	test 2	test 3	test 4	test 5	test 6	n	Xm	Ss	$k_{1,u}$	k_c	$k_{1,k}$	σ	Xest	Xest	Xest	Equation OK?
1	18,1	17,9	18,3	19,4	17,7	19,2	6	18,43	0,70	0,823				17,85			
2	16,1	18,4	18,8	17,6	15,7	17	6	17,27	1,24	0,823				16,25			
3	17,7	19,1	17,9	18,1	15,7	18,1	6	17,77	1,12	0,823				16,84			
4	21,4	20,8	19,6	18,5	18,6	18,1	6	19,50	1,35	0,823				18,39			
5	19,5	20,8	19,7	21,1	19,4	18,6	6	19,85	0,94	0,823				19,08			
6	19,9	19,3	20,1	18,8	21,1	20,6	6	19,97	0,84	0,823				19,28			
7	17,2	19,2	17,0	18,2	19,4	17,1	6	18,02	1,09	0,823	0,823	0,672	1,409	17,12	16,86		
8	20,2	17,8	16,3	19,9	21,9	22,6	6	19,78	2,39		0,811				18,64		
9	21,0	14,7	19,0	18,9	18,8	19,3	6	18,62	2,09		0,800				17,49		
10	20,2	17,3	22,3	21,3	22,4	22,6	6	21,02	2,03		0,788				19,91		
11	20,0	22,8	21,2	19,9	20,1	22,6	6	21,10	1,33		0,777				20,01		
12	23,8	20,6	20,8	17,1	21	15,5	6	19,80	3,00		0,765				18,72		
13	21,1	21,0	20,1	19,1	23,4	21,5	6	21,03	1,44		0,753				19,97		
14	21,0	16,5	17,2	15,8	21,7	19,5	6	18,62	2,47		0,742				17,57		
15	18,8	20,5	20,3	20,1	21,7	17,1	6	19,75	1,59		0,730				18,72		
16	19,7	19,5	20,0	18,1	20,3	17,2	6	19,13	1,21		0,718				18,12		
17	20,4	19,6	20,1	20,8	14,5	18,2	6	18,93	2,35		0,707				17,94		
18	20,5	21,4	20,4	22,4	19,1	18,6	6	20,40	1,41		0,695				19,42		
19	19,8	20,5	18,1	19,2	19,1	18,8	6	19,25	0,83		0,684				18,29		
20	19,7	17,6	16,9	20,9	18,4	18	6	18,58	1,47		0,672	0,672	1,858		17,33	17,33	OK
21	16,5	14,2	15,3	16,4	14,3	16,4	6	15,52	1,08			0,672				14,27	OK
22	21,3	21,4	21,3	18,9	21,2	20,8	6	20,82	0,96			0,672				19,57	OK
23	19,3	17,5	18,3	17,2	19,1	16,7	6	18,02	1,06			0,672				16,77	OK
24	18,5	21,8	19,4	17,2	21,3	17,4	6	19,27	1,94			0,672				18,02	OK

C.2 Example 2

Example of statistical analysis of compressive strength using 'Rolling' inspection

The fractile P = 50 %

The confidence level γ *= 95 %*

The number of series of inspection lots is l = 4

One-sided tolerance interval, lower limit

The declared mean compressive strength is 15 N/mm^2

For the first inspection lot a sample size of three samples are taken, tested and evaluated (x_m (Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (3)) in accordance with 5.2.7 using $k_{1,u}$ taken from Table B.8 (p: 50 % and γ: 95 %)). For the next and the following two inspection lots three additional samples are taken and tested and evaluated together with the ones from the previous inspection lots (x_m (Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (3)) in accordance with 5.2.7 using $k_{1,u}$ taken from Table B.8 (p: 50 % and γ: 95 %)). By doing so the spot sample size evaluated together is gradually increased from 3 to 12 samples.

From then on, three additional samples are taken from each next inspection lot and evaluated together with the ones from the previous inspection lots (x_m(Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (3)) in accordance with 5.2.7 using $k_{1,u}$ taken from Table B.8 (p: 50 % and γ: 95 %)) but the spot sample size is limited to the last 12 samples. The spot sample size continues to be 12.

For the first 21 samples (1 – 7 inspection lot), the standard deviation of the population is considered to be unknown and the $k_{1,u}$ factor taken from Table B.8 (p: 50 % and γ: 95 %) is 0,519. For the inspection lots 8 – 20 the standard deviation can be considered as known, but the used acceptance coefficient is corrected (k_c). The acceptance coefficient for the known standard deviation $k_{1,k}$ is taken from Table B.4 (p: 50 % and γ: 95 %) and is 0,475. The corrected acceptance coefficient k_c is calculated by a linear interpolation between the acceptance coefficient $k_{1,u}$ and $k_{1,k}$ taking into account the considered inspection lot. The known standard deviation σ is calculated based on the first 21 test results.

From inspection lot 21 and so on 3 additional samples are taken from each next inspection lot and the test results are evaluated together with the ones from the previous inspection lots (x_m (Formula (1)), known standard deviation σ and x_{est} (Formula (6)) in accordance with 5.2.7 using $k_{1,k}$ taken from Table B.4 (p: 50 % and γ: 95 %)) and the spot sample size is still limited to the last 12 samples.

After each evaluation the result shall be compared with the lower limit value (e.g. the declared value) decided by the manufacturer.

Part of the evaluation is also to check that the standard deviation s_s of the spot sample corresponds to the following formula:

$$0,63\ \sigma \le s_s \le 1,37\ \sigma$$

In the last column it is indicated whether the mentioned formula fits or does not fit (indicated by 'NOK') (see 5.2.7)

If there is a non-conformity due to great differences between the test results, the estimated value is highlighted by a red signal at the right side (batch 1). A non-conforming inspection lot shall be treated separately as described in 5.2.11.

Table C.2 — Example 2: One sided tolerance interval – lower limit

EXAMPLE 2

ONE SIDED TOLERANCE INTERVAL-lower limit

fractile p	50	Start correction	7	Series of inspection lots	4	Declared Value	15	
confidence level	95	End correction	20			2,325		

Inspection lot	test 1	test 2	test 3	n	Xm	Ss	$k_{1,u}$	k_c	$k_{1,k}$	σ	Xest	Xest	Xest
1	20,2	17,8	16,3	3	18,10	1,97	1,69				14,78		
2	21,0	14,7	19,0	6	18,17	2,39	0,82				16,20		
3	20,2	17,3	22,3	9	18,76	2,43	0,62				17,25		
4	20,0	22,8	21,2	12	19,40	2,45	0,52				18,13		
5	23,8	20,6	20,8	12	20,31	2,45	0,52				19,03		
6	21,1	21,0	20,1	12	20,93	1,63	0,52				20,09		
7	21,0	16,5	17,2	12	20,51	2,02	0,52	0,52	0,48	2,32	19,46	19,30	
8	18,8	20,5	20,3	12	20,14	1,91		0,52	0,48			18,94	
9	19,7	19,5	20,0	12	19,64	1,47		0,51	0,48			18,45	
10	20,4	19,6	20,1	12	19,47	1,35		0,51	0,48			18,28	
11	20,5	21,4	20,4	12	20,10	0,65		0,51	0,48			18,93	
12	19,8	20,5	18,1	12	20,00	0,79		0,50	0,48			18,83	
13	19,7	17,6	16,9	12	19,58	1,35		0,50	0,48			18,42	
14	18,7	20,0	19,7	12	19,44	1,34		0,50	0,48			18,29	
15	21,3	21,4	21,3	12	19,58	1,49		0,49	0,48			18,44	
16	19,3	17,5	18,3	12	19,31	1,55		0,49	0,48			18,17	
17	18,5	21,8	19,4	12	19,77	1,41		0,49	0,48			18,64	
18	19,9	21,9	22,6	12	20,27	1,66		0,48	0,48			19,15	
19	18,9	18,8	19,3	12	19,68	1,59		0,48	0,48			18,57	
20	21,3	22,4	22,6	12	20,62	1,62		0,48	0,48	1,76		19,78	19,78
21	19,9	20,1	22,6	12	20,86	1,53			0,48				20,02
22	17,1	21,0	15,5	12	19,96	2,21			0,48				19,12
23	19,1	23,4	21,5	12	20,54	2,37			0,48				19,70
24	15,8	21,7	19,5	12	19,77	2,54			0,48				18,93
25	20,1	21,7	17,1	12	19,46	2,57			0,475				18,62
26	18,1	20,3	17,2	12	19,63	2,27			0,475				18,79
27	20,8	14,5	18,2	12	18,75	2,32			0,475				17,91
28	22,4	19,1	18,6	12	19,01	2,21			0,475				18,17
29	19,2	19,1	18,8	12	18,86	1,94			0,475				18,02
30	20,9	18,4	18,0	12	19,00	1,93			0,475				18,16

C.3 Example 3

Example of statistical analysis of compressive strength using a special type of 'Rolling' inspection: 'Progressive Sampling'

The fractile P = 95 %

The confidence level γ = 95 %

The number of series of inspection lots is l = 15

One-sided tolerance interval, lower limit

The declared 5 % characteristic compressive strength is 4 N/mm²

For each of the first to fifth inspection lots a spot size of one sample is taken and tested. These inspection lots are evaluated together (x_m (Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (3)) in accordance with 5.2.7 using $k_{1,u}$ taken from Table B.8 (p: 95 % and γ: 95 %)). For the sixth and following inspection lots one additional sample is taken, tested and evaluated together with the ones from the previous inspection lots (x_m (Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (3)) in accordance with 5.2.7 using $k_{1,u}$ taken from Table B.8 (p: 95 % and γ: 95 %)). The spot size is gradually increased from 5 to 15 samples.

From then on, one additional sample is taken from each next inspection lot and evaluated together with the ones from the previous inspection lots (x_m (Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (3)) in accordance with 5.2.7 using $k_{1,u}$ taken from Table B.8 (p: 95 % and γ: 95 %)) but the spot sample size is limited to the last 15 samples. The spot sample size continues to be 15.

For the first 30 samples, the standard deviation of the population is considered to be unknown and the $k_{1,u}$ factor taken from Table B.8 (p: 95 % and γ: 95 %) is 2,567. For the inspection lots 30 – 60 the standard deviation can be considered as known, but the used acceptance coefficient is corrected (k_c). The acceptance coefficient for the known standard deviation $k_{1,k}$ is taken from Table B.4 (p: 95 % and γ: 95 %) and is 2,070. The corrected acceptance coefficient k_c is calculated by a linear interpolation between the acceptance coefficient $k_{1,u}$ and $k_{1,k}$ taking into account the considered inspection lot. The known standard deviation σ is calculated based on the first 21 test results.

From inspection lot 61 and so on one additional sample is taken from each next inspection lot and the test results are evaluated together with the ones from the previous inspection lots (x_m (Formula (1)), known standard deviation σ and x_{est} (Formula (6)) in accordance with 5.2.7 using $k_{1,k}$ taken from Table B.4 (p: 95 % and γ: 95 %)) and the spot sample size is still limited to the last 15 samples.

After each evaluation the result shall be compared with the lower limit value (e.g. the declared value) decided by the manufacturer.

Part of the evaluation is also to check that the standard deviation s_s of the spot sample corresponds to the following formula:

$$0{,}63\ \sigma \le s_s \le 1{,}37\ \sigma$$

In the last column it is indicated whether the mentioned formula fits (indicated by 'OK') or does not fit (indicated by 'NOK'). At batch 64 there is a non-conformity ($s_s > 1{,}37\ \sigma$). The manufacturer shall restart or to continue working with the acceptance coefficient $k_{1,u}$. This means that the inspection lots shall be treated separately.

If there is a non-conformity due to great differences between the test results, the estimated value is highlighted by a red signal at the right side (batch 2, 3, 4, 34 and 64). A non-conforming inspection lot shall be treated separately as described in 5.2.11.

Table C.3 — Example 3: One sided tolerance interval – lower limit – method B

EXAMPLE 3

ONE SIDED TOLERANCE INTERVAL-lower limit METHOD B: progressive sampling : use only 1 testresult per inspection lot

fractile p	95	Start correction	30	Series of inspection lots	15	Declared Value	4	
confidence level	95	End correction	60			1,009		0,974

Inspection lot	test 1	test 2	test 3	n	Xm	Ss	$k_{1,u}$	k_c	$k_{1,k}$	σ	Xest	Xest	Xest	Equation OK?
1	6,78			1	6,78	#DEEL/0!								
2	8,36			2	7,57	1,11	26,260				-21,69			
3	8,64			3	7,92	1,00	7,656				0,26			
4	8,41			4	8,05	0,85	5,144				3,66			
5	8,25			5	8,09	0,74	4,203				4,96			
6	7,44			6	7,98	0,72	3,708				5,32			
7	8,57			7	8,06	0,69	3,400				5,71			
8	6,25			8	7,84	0,91	3,188				4,95			
9	7,83			9	7,84	0,85	3,032				5,27			
10	7,40			10	7,79	0,81	2,911				5,44			
11	8,57			11	7,86	0,80	2,815				5,60			
12	8,27			12	7,90	0,77	2,737				5,78			
13	7,67			13	7,88	0,74	2,671				5,89			
14	8,13			14	7,90	0,72	2,615				6,02			
15	6,53			15	7,81	0,78	2,567				5,81			
16	7,85			15	7,88	0,72	2,567				6,02			
17	7,20			15	7,80	0,73	2,567				5,93			
18	7,95			15	7,75	0,69	2,567				5,97			
19	7,39			15	7,69	0,67	2,567				5,95			
20	5,96			15	7,53	0,79	2,567				5,51			
21	6,26			15	7,46	0,85	2,567				5,26			
22	5,60			15	7,26	0,92	2,567				4,90			
23	7,15			15	7,32	0,88	2,567				5,07			
24	6,01			15	7,20	0,93	2,567				4,82			
25	7,88			15	7,23	0,94	2,567				4,81			
26	6,01			15	7,06	0,91	2,567				4,72			
27	5,91			15	6,90	0,89	2,567				4,61			
28	7,05			15	6,86	0,87	2,567				4,63			
29	5,51			15	6,69	0,86	2,567				4,48			
30	5,87			15	6,64	0,88	2,567	2,567	2,070	1,009	4,37	4,05		
31	7,59			15	6,62	0,86		2,550				4,05		
32	8,64			15	6,72	1,00		2,534				4,16		
33	6,72			15	6,64	0,94		2,517				4,10		
34	4,91			15	6,47	1,01		2,501				3,95		
35	8,21			15	6,62	1,09		2,484				4,12		
36	8,48			15	6,77	1,19		2,468				4,28		
37	7,87			15	6,92	1,17		2,451				4,45		
38	7,36			15	6,94	1,18		2,434				4,48		
39	8,11			15	7,08	1,18		2,418				4,64		
40	6,50			15	6,98	1,17		2,401				4,56		
41	7,38			15	7,07	1,14		2,385				4,67		
42	7,78			15	7,20	1,11		2,368				4,81		
43	8,11			15	7,27	1,13		2,352				4,90		
44	8,70			15	7,48	1,08		2,335				5,13		
45	7,46			15	7,59	0,98		2,319				5,25		
46	7,76			15	7,60	0,98		2,302				5,28		
47	6,46			15	7,45	0,98		2,285				5,15		
48	7,78			15	7,52	0,96		2,269				5,24		
49	6,73			15	7,65	0,68		2,252				5,37		
50	7,47			15	7,60	0,66		2,236				5,34		
51	6,98			15	7,50	0,63		2,219				5,26		
52	6,35			15	7,40	0,68		2,203				5,17		
53	6,35			15	7,33	0,74		2,186				5,12		
54	5,53			15	7,16	0,83		2,169				4,97		
55	6,68			15	7,17	0,83		2,153				5,00		
56	5,86			15	7,07	0,89		2,136				4,91		
57	7,40			15	7,04	0,87		2,120				4,90		
58	6,42			15	6,93	0,83		2,103				4,81		
59	5,80			15	6,74	0,72		2,087				4,63		
60	6,57			15	6,68	0,69		2,070	2,070	0,974		4,66	4,66	OK
61	5,71			15	6,54	0,67			2,070				4,52	OK
62	5,78			15	6,49	0,69			2,070				4,48	OK
63	10,46			15	6,67	1,21			2,070				4,66	OK
64	11,26			15	6,97	1,69			2,070				4,96	NOK

C.4 Example 4

Example of statistical analysis of net dry density using 'Rolling' inspection.

The fractile P = 50 %

The confidence level γ = 50 %

The number of series of inspection lots is l = 5 One-sided tolerance interval, upper limit

The declared mean net dry density is 1 400 kg/m³

For the first inspection lot a sample size of three samples are taken, tested and evaluated (x_m (Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (4)) in accordance with 5.2.7 using $k_{1,u}$ taken from Table B.5 (p: 50 % and γ: 50 %)). For the following three inspection lots three additional samples are taken, tested and evaluated together with the ones from the previous inspection lots (x_m (Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (4)) in accordance with 5.2.7 using $k_{1,u}$ taken from Table B.5 (p: 50 % and γ: 50 %)). By doing so the spot sample size evaluated together is gradually increased from 3 to 15 samples.

From then on, three additional samples are taken from each next inspection lot and evaluated together with the ones from the previous inspection lots (x_m (Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (4)) in accordance with 5.2.7 using $k_{1,u}$ taken from Table B.5 (p: 50 % and γ: 50 %)) but the spot sample size is limited to the last 15 samples. The spot sample size continues to be 15.

For the first 21 samples (1 – 7 inspection lot), the standard deviation of the population is considered to be unknown and the $k_{1,u}$ factor taken from Table B.5 (p: 50 % and γ: 50 %) is 0,000, which means that $x_{est} = x_m$. For the inspection lots 8 – 20 the standard deviation can be considered as known. The acceptance coefficient for the known standard deviation $k_{1,k}$ is taken from Table B.1 (p: 50 % and γ: 50 %) and is 0,000, which means that $x_{est} = x_m$. The known standard deviation σ is calculated based on the first 21 test results.

From inspection lot 21 and so on 3 additional samples are taken from each next inspection lot and the test results are evaluated together with the ones from the previous inspection lots (x_m (Formula (1)), known standard deviation σ and x_{est}(Formula (7)) in accordance with 5.2.7 using $k_{1,k}$ taken from Table B.1 (p: 50 % and γ: 50 %)) and the spot sample size is still limited to the last 15 samples.

After each evaluation the result shall be compared with the lower limit value (e.g. the declared value) decided by the manufacturer.

Part of the evaluation is also to check that the standard deviation s_s of the spot sample corresponds to the following formula:

$$0{,}63\ \sigma \le s_s \le 1{,}37\ \sigma$$

In the last column it is indicated whether the mentioned formula fits (indicated by 'OK') or does not fit (indicated by 'NOK').

If there is a non-conformity due to great differences between the test results, the estimated value is highlighted by a red signal at the right side. A non-conforming inspection lot shall be treated separately as described in 5.2.11.

The last column makes the link to EN 1745, Table A.1 to calculate the $\lambda_{10,dry}$ (50/50) value of the material.

Table C.4 — Example 4: One sided tolerance interval – upper limit – method B

EXAMPLE 4									
ONE SIDED TOLERANCE INTERVAL-upper limit	METHOD B: use at least 3 testresults per inspection lot								
fractile p	50	Start correction	7	Series of inspection lots	5	Declared Value	1400		0,35
confidence level	50	End correction	20			41,8569		40,433	

Inspection lot	test 1	test 2	test 3	n	Xm	Ss	$k_{1,u}$	k_c	$k_{1,k}$	σ	Xest	Xest	Xest	Equation OK?	$\lambda_{(10,dry)}$ (50/50)
1	1206,00	1264,0	1362,0	3	1277,33	78,85	0,000				1277,33				0,2908829
2	1326,00	1344,0	1314,0	6	1302,67	57,86	0,000				1302,67				0,301179
3	1344,00	1360,0	1386,0	9	1322,89	55,90	0,000				1322,89				0,3093979
4	1318,00	1324,0	1276,0	12	1318,67	49,55	0,000				1318,67				0,3076819
5	1344,00	1338,0	1364,0	15	1324,67	45,94	0,000				1324,67				0,3101205
6	1252,00	1310,0	1324,0	15	1328,27	33,47	0,000				1328,27				0,3115836
7	1306,00	1312,0	1304,0	15	1324,13	34,15	0,000	0,000	0,000	41,857	1324,13	1324,13			0,310
8	1336,00	1316,0	1324,0	15	1316,53	26,93		0,000				1316,53			0,307
9	1264,00	1384,0	1272,0	15	1316,67	35,61		0,000				1316,67			0,307
10	1236,00	1270,0	1336,0	15	1303,07	38,35		0,000				1303,07			0,301
11	1274,00	1324,0	1338,0	15	1306,40	37,64		0,000				1306,40			0,303
12	1352,00	1392,0	1370,0	15	1319,20	46,90		0,000				1319,20			0,308
13	1336,00	1342,0	1278,0	15	1317,87	48,51		0,000				1317,87			0,307
14	1256,00	1276,0	1304,0	15	1312,27	45,41		0,000				1312,27			0,305
15	1326,00	1344,0	1344,0	15	1323,73	38,67		0,000				1323,73			0,310
16	1256,00	1360,0	1314,0	15	1323,33	41,62		0,000				1323,33			0,310
17	1344,00	1324,0	1332,0	15	1315,73	33,99		0,000				1315,73			0,306
18	1336,00	1338,0	1384,0	15	1322,53	36,37		0,000				1322,53			0,309
19	1366,00	1310,0	1382,0	15	1337,33	31,49		0,000				1337,33			0,315
20	1282,00	1312,0	1364,0	15	1333,60	35,73		0,000	0,000	40,433		1333,60	1333,60	OK	0,314
21	1396,00	1316,0	1364,0	15	1343,33	32,16			0,000				1343,33	OK	0,318
22	1330,00	1384,0	1376,0	15	1349,33	34,16			0,000				1349,33	OK	0,320
23	1342,00	1270,0	1236,0	15	1335,33	47,11			0,000				1335,33	OK	0,314
24	1324,00	1324,0	1238,0	15	1323,87	50,05			0,000				1323,87	OK	0,310
25	1276,00	1392,0	1336,0	15	1326,93	52,45			0,000				1326,93	OK	0,311
26	1346,00	1342,0	1334,0	15	1323,33	48,32			0,000				1323,33	OK	0,310
27	1310,00	1364,0	1324,0	15	1317,20	44,29			0,000				1317,20	OK	0,307
28	1344,00	1324,0	1344,0	15	1328,13	35,57			0,000				1328,13	OK	0,312
29	1344,00	1304,0	1344,0	15	1335,20	26,58			0,000				1335,20	OK	0,314

C.5 Example 5

Example of two-sided statistical analysis of a characteristic in process control using 'Rolling' inspection

The fractile P = 50 %

The confidence level γ = 75 %

The number of series of inspection lots is l = 4

Two-sided tolerance interval

The manufacturer wants to have the mean value of an imaginary property in process control between 242 and 247

For the first inspection lot a sample size of three samples are taken and tested and evaluated (x_m (Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (5)) in accordance with 5.2.7 using $k_{2,u}$ taken from Table B.14 (*p*: 50 % and *γ*: 75 %)). For the following four inspection lots three additional samples are taken, tested and evaluated together with the ones from the previous inspection lots (x_m (Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (5)) in accordance with 5.2.7 using $k_{2,u}$ taken from Table B.14 (*p*: 50 % and *γ*: 75 %)). By doing so the spot sample size evaluated together is gradually increased from 3 to 12 samples.

From then on, three additional samples are taken from each next inspection lot and evaluated together with the ones from the previous inspection lots (x_m (Formula (1)), standard deviation s_s (Formula (2)) and x_{est} (Formula (5)) in accordance with 5.2.7 using $k_{2,u}$ taken from Table B.10 (*p*:

50 % and γ: 75 %)) but the spot sample size is limited to the last 12 samples. The spot sample size continues to be 12.

For the first 21 samples (1 – 7 inspection lot), the standard deviation of the population is considered to be unknown and the $k_{2,u}$ factor taken from Table B.10 (p: 50 % and γ: 75 %) is 0,850. For the inspection lots 8 – 20 the standard deviation can be considered as known, but the used acceptance coefficient is corrected (k_c). The acceptance coefficient for the known standard deviation $k_{2,k}$ is taken from Table B.10 (p: 50 % and γ: 75 %) and is 0,713. The corrected acceptance coefficient k_c is calculated by a linear interpolation between the acceptance coefficient $k_{2,u}$ and $k_{2,k}$ taking into account the considered inspection lot. The known standard deviation σ is calculated based on the first 21 test results.

From inspection lot 21 onwards 3 additional samples are taken from each next inspection lot and the test results are evaluated together with the ones from the previous inspection lots (x_m (Formula (1)), known standard deviation σ and x_{est} (Formula (8)) in accordance with 5.2.7 using $k_{2,k}$ taken from Table B.10 (p: 50 % and γ: 75 %)) and the spot sample size is still limited to the last 12 samples.

After each evaluation the result shall be compared with the lower limit value (e.g. the declared value) decided by the manufacturer.

Part of the evaluation is also to check that the standard deviation s_s of the spot sample corresponds to the following formula:

$$0{,}63\ \sigma \le s_s \le 1{,}37\ \sigma$$

In the last column it is indicated whether the mentioned formula fits (indicated by ‘OK’) or does not fit (indicated by ‘NOK’).

If there is a non-conformity due to great differences between the test results, the estimated value is highlighted by a red signal at the right side (batch 5, 6, 7, 14, 18, 19, 20, 23, 24 and 25). A non-conforming inspection lot gives a warning to the manufacturer to take corrective actions.

Table C.5 — Example 5: Two sided tolerance interval – lower limit

EXAMPLE5

2,166

TWO SIDED TOLERANCE INTERVAL-lower limit

fractile p	50	Start correction	7	Series of inspection lots	4	Declared Value lower limit	242
confidence level	75	End correction	20			Declared Value upper limit	247

1,677

Inspection lot	test 1	test 2	test 3	n	Xm	Ss	$k_{1,u}$	k_c	$k_{1,k}$	σ	Xest lower limit	Xest upper limit	Xest lower limit	Xest upper limit	Xest lower limit	Xest upper limit	Equation OK?
1	244	245	246	3	245,00	1,00	1,492				243,508	246,492					
2	245	247	246	6	245,50	1,05	1,009				244,442	246,5582					
3	246	247	247	9	245,89	1,05	0,9				244,940	246,8376					
4	247	246	246	12	246,00	0,95	0,85				245,190	246,8104					
5	242	242	241	12	245,17	2,21	0,85				243,289	247,0441					
6	241	242	242	12	244,08	2,57	0,85				241,895	246,2718					
7	245	246	247	12	243,92	2,43	0,85	0,850	0,713	2,166	241,852	245,9816	242,076	245,7576			
8	247	246	246	12	243,92	2,43		0,839					242,099	245,7347			
9	243	244	243	12	244,33	2,10		0,829					242,538	246,1286			
10	244	244	243	12	244,83	1,53		0,818					243,061	246,6058			
11	246	246	246	12	244,83	1,47		0,808					243,084	246,5829			
12	245	245	246	12	244,58	1,24		0,797					242,857	246,3101			
13	245	245	244	12	244,92	1,00		0,787					243,213	246,6206			
14	246	245	245	12	245,33	0,65		0,776					243,652	247,0145			
15	245	244	243	12	244,83	0,83		0,766					243,175	246,4916			
16	244	245	244	12	244,58	0,79		0,755					242,948	246,2188			
17	248	248	247	12	245,33	1,61		0,745					243,721	246,946			
18	247	246	248	12	245,75	1,82		0,734					244,160	247,3398			
19	245	245	245	12	246,00	1,54		0,724					244,433	247,567			
20	244	245	245	12	246,08	1,44		0,713	0,713	1,677			244,888	247,2791	244,888	247,279	OK
21	246	245	248	12	245,75	1,29			0,713						244,554	246,946	OK
22	245	246	248	12	245,58	1,24			0,713						244,388	246,779	OK
23	245	246	247	12	245,83	1,27			0,713						244,638	247,029	OK
24	247	246	247	12	246,33	1,07			0,713						245,138	247,529	OK
25	245	244	244	12	245,83	1,27			0,713						244,638	247,029	OK
26	243	244	245	12	245,25	1,36			0,713						244,054	246,446	OK

Annex D
(informative)

Normality test of SHAPIRO – WILK

D.1 General

A certain number of techniques exist which permit the verification of the hypothesis of normality. The assumption of a normal distribution of the variable *X* can be tested using various normality tests: a random sample is compared with the theoretical model of the normal distribution and observed deviations are tested to determine whether they are significant or not. If the deviations are insignificant, then the assumption of normal distribution is accepted, otherwise it is rejected. The recommended significance level α to be used in building is 0,05 (then the risk of acceptance of a wrong hypotheses has a suitable value).

It shall be noted that for high ratios σ/μ there is a non-negligible probability of the occurrence of negative values of the variable *X*. If *X* shall be positive (e.g. compressive strength, density), then other theoretical models for the probability distribution may be more suitable.

In the case where the hypotheses of normality should be rejected, the obvious method to follow is to resort to non-parametric tests or to use suitable transformations for obtaining normally distributed populations, for example: 1/x, log (x+ a), $\sqrt{(\mathrm{x}+a)}$.

D.2 Normality test of SHAPIRO-WILK

The test is effective for $5 \le n \le 50$

Calculate the standard deviation *s* for the test results

$$s = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - x_m\right)^2}{n-1}}$$

Put the test results in order of size (from small to large)

Calculate the differences $d_i = x_{(n+1-i)} - x_i$

If the number of test results *n* is pair, there are k = *n*/2 differences

If the number of test results *n* is impair, there are k = (*n*-1)/2 differences and the median test result is not used

Calculate the value *b*

$$b = \sum_{i=1}^{k} ai\, di$$ (a_i is defined in Table D.1)

Calculate the value *W*:

$$W = \frac{b^2}{n.s^2}$$ (to calculate with a minimum of three decimals)

The reference values $W_{0,95}$ (risk α = 0,05, confidence level: 1-α = 0,95) are given in Table D.2.

If $W \ge W_{0,95}$ the hypotheses of normality is accepted.

Table D.1 — Values for the coefficient a_i

		n																			
		5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
i	1	0.6646	0.6431	0.6233	0.6052	0.5888	0.5739	0.5601	0.5475	0.5359	0.5251	0.5150	0.5056	0.4968	0.4886	0.4808	0.4734	0.4643	0.4590	0.4542	0.4493
	2	0.2413	0.2806	0.3031	0.3164	0.3244	0.3291	0.3315	0.3325	0.3325	0.3318	0.3306	0.3290	0.3273	0.3253	0.3232	0.3211	0.3185	0.3156	0.3126	0.3098
	3	0.0000	0.0875	0.1401	0.1743	0.1976	0.2141	0.2260	0.2347	0.2412	0.2460	0.2495	0.2521	0.2540	0.2553	0.2561	0.2565	0.2578	0.2571	0.2563	0.2554
	4			0.0000	0.0561	0.0947	0.1224	0.1429	0.1586	0.1707	0.1802	0.1878	0.1939	0.1988	0.2027	0.2059	0.2085	0.2119	0.2131	0.2139	0.2145
	5					0.0000	0.0399	0.0695	0.0922	0.1099	0.1240	0.1353	0.1447	0.1524	0.1587	0.1641	0.1686	0.1736	0.1764	0.1787	0.1807
	6							0.0000	0.0303	0.0539	0.0727	0.0880	0.1005	0.1109	0.1197	0.1271	0.1334	0.1399	0.1443	0.1480	0.1512
	7									0.0000	0.0240	0.0433	0.0593	0.0725	0.0837	0.0932	0.1013	0.1092	0.1150	0.1201	0.1245
	8											0.0000	0.0196	0.0359	0.0496	0.0612	0.0711	0.0804	0.0878	0.0941	0.0997
	9													0.0000	0.0163	0.0303	0.0422	0.0530	0.0618	0.0696	0.0764
	10															0.0000	0.0140	0.0263	0.0368	0.0459	0.0539
	11																	0.0000	0.0122	0.0228	0.0321
	12																			0.0000	0.0107

		n																									
		25	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
	1	0.4450	0.4407	0.4366	0.4328	0.4291	0.4254	0.4220	0.4188	0.4156	0.4127	0.4096	04068	0.4040	0.4015	0.3989	0.3964	0.3940	0.3917	0.3894	0.3872	0.3850	0.3830	0.3808	0.3789	0.3770	0.3751
	2	0.3069	0.3043	0.3018	0.2992	0.2968	0.2944	0.2921	0.2898	0.2876	0.2854	0.2834	0.2813	0.2794	0.2274	0.2755	0.2737	0.2719	0.2701	0.2684	0.2667	0.2651	0.2635	0.2620	0.2604	0.2589	0.2574
	3	02543	0.2533	0.2522	0.2510	0.2499	0.2487	0.2475	0.2463	0.2451	0.2439	0.2427	0.2415	0.2403	0.2391	0.2380	0.2368	0.2357	0.2345	0.2334	0.2323	0.2313	0.2302	0.2291	0.2281	0.2271	0.2260
	4	0.2148	0.2151	0.2152	0.2151	0.2150	0.2148	0.2145	0.2141	0.2137	0.2132	0.2127	0.2121	0.2116	0.2110	0.2104	0.2098	0.2091	0.2085	0.2078	0.2072	0.2065	0.2058	0.2052	0.2045	0.2038	0.2032
	5	0.1822	0.1836	0.1848	0.1857	0.1864	0.1870	0.1874	0.1878	0.1880	0.1882	0.1883	0.1883	0.1883	0.1881	0.1880	0.1878	0.1876	0.1874	0.1871	0.1868	0.1865	0.1862	0.1859	0.1855	0.1851	0.1847
i	**6**	0.1539	0.1563	0.1584	0.1601	0.1616	0.1630	0.1641	0.1651	0.1660	0.1667	0.1673	0.1678	0.1683	0.1686	0.1689	0.1691	0.1693	0.1694	0.1695	0.1695	0.1695	0.1695	0.1695	0.1693	0.1692	0.1691
	7	0.1283	0.1316	0.1346	0.1372	0.1395	0.1415	0.1433	0.1449	0.1463	0.1475	0.1487	0.1496	0.1505	0.1513	0.1520	0.1526	0.1531	0.1535	0.1539	0.1542	0.1545	0.1548	0.1550	0.1551	0.1553	0.1554
	8	0.1045	0.1089	0.1128	0.1162	0.1192	0.1219	0.1243	0.1265	0.1284	0.1301	0.1317	0.1331	0.1344	0.1356	0.1366	0.1376	0.1384	0.1392	0.1398	0.1405	0.1410	0.1415	0.1420	0.1423	0.1427	0.1430
	9	0.0823	0.0876	0.0923	0.0965	0.1002	0.1036	0.1066	0.1093	0.1118	0.1140	0.1160	0.1179	0.1196	0.1211	0.1225	0.1237	0.1249	0.1259	0.1269	0.1278	0.1286	0.1293	0.1300	0.1306	0.1312	0.1317
	10	0.0610	0.0672	0.0728	0.0778	0.0822	0.0862	0.0899	0.0931	0.0961	0.0988	0.1013	0.1036	0.1056	0.1075	0.1092	0.1108	0.1123	0.1136	0.1149	0.1160	0.1170	0.1180	0.1189	0.1197	0.1205	0.1212
	11	0.0403	0.0476	0.0540	0.0598	0.0650	0.0697	0.0739	0.0777	0.0812	0.0844	0.0873	0.0900	0.0924	0.0947	0.0967	0.0986	0.1004	0.1020	0.1035	0.1049	0.1062	0.1073	0.1085	0.1095	0.1105	0.1113
	12	0.0200	0.0284	0.0358	0.0424	0.0483	0.0537	0.0585	0.0629	0.0669	0.0706	0.0739	0.0770	0.0798	0.0824	0.0848	0.0870	0.0891	0.0909	0.0927	0.0943	0.0959	0.0972	0.0986	0.0998	0.1010	0.1020
	13	0.0000	0.0094	0.0178	0.0253	0.0320	0.0381	0.0435	0.0485	0.0530	0.0572	0.0610	0.0645	0.0677	0.0706	0.0733	0.0759	0.0782	0.8040	0.0824	0.0842	0.0860	0.0876	0.0892	0.0906	0.0919	0.0932
	14			0.0000	0.0084	0.0159	0.0227	0.0289	0.0344	0.0395	0.0441	0.0484	0.0523	0.0559	0.0592	0.0622	0.0651	0.0677	0.0701	0.0724	0.0745	0.0765	0.0783	0.0801	0.0817	0.0832	0.0846
	15					0.0000	0.0076	0.0144	0.0206	0.0262	0.0314	0.0361	0.0404	0.0444	0.0481	0.0515	0.0546	0.0575	0.0602	0.0628	0.0651	0.0673	0.0694	0.0713	0.0731	0.0748	0.0764
	16							0.0000	0.0068	0.0131	0.0187	0.0239	0.0287	0.0331	0.0372	0.0409	0.0444	0.0476	0.0506	0.0534	0.0560	0.0584	0.0607	0.0628	0.0648	0.0667	0.0685
	17									0.0000	0.0062	0.0119	0.0172	0.0220	0.0264	0.0305	0.0343	0.0379	0.0411	0.0442	0.0471	0.0497	0.0522	0.0546	0.0568	0.0588	0.0608
	18											0.0000	0.0057	0.0110	0.0158	0.0203	0.0244	0.0283	0.0318	0.0352	0.0383	0.0412	0.0439	0.0465	0.0489	0.0511	0.0532
	19													0.0000	0.0053	0.0101	0.0146	0.0188	0.0227	0.0263	0.0296	0.0328	0.0357	0.0385	0.0411	0.0436	0.0459
	20															0.0000	0.0049	0.0094	0.0136	0.0175	0.0211	0.0245	0.0277	0.0307	0.0335	0.0361	0.0386
	21																	0.0000	0.0045	0.0087	0.0126	0.0163	0.0197	0.0229	0.0259	0.0288	0.0314
	22																			0.0000	0.0042	0.0081	0.0118	0.0153	0.0185	0.0215	0.0244
	23																					0.0000	0.0039	0.0076	0.0111	0.0143	0.0174
	24																							0.0000	0.0037	0.0071	0.0104
	25																									0.0000	0.0035

Table D.2 — Reference values for $W_{0,95}$ (risk α = 0,05, confidence level: 1-α = 0,95)

n	$W_{0,95}$		n	$W_{0,95}$
1			26	0,920
2			27	0,923
3			28	0,924
4			29	0,926
5	0,767		30	0,927
6	0,788		31	0,929
7	0,803		32	0,930
8	0,818		33	0,931
9	0,829		34	0,933
10	0,842		35	0,934
11	0,850		36	0,935
12	0,859		37	0,936
13	0,866		38	0,938
14	0,874		39	0,939
15	0,881		40	0,940
16	0,887		41	0,941
17	0,892		42	0,942
18	0,897		43	0,943
19	0,901		44	0,945
20	0,905		45	0,945
21	0,908		46	0,945
22	0,911		47	0,946
23	0,914		48	0,947
24	0,916		49	0,947
25	0,918		50	0,950

Bibliography

[1] EN 771-1:2011, *Specification for masonry units — Part 1: Clay masonry units*

[2] EN 771-2:2011, *Specification for masonry units — Part 2: Calcium silicate masonry units*

[3] EN 771-3:2011, *Specification for masonry units — Part 3: Aggregate concrete masonry units (Dense and lightweight aggregates)*

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[5] EN 771-5:2011, *Specification for masonry units — Part 5: Manufactured stone masonry units*

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