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Recommendations for the co-ordination of dimensions in building —

Combinations of sizes

Confirmed
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Co-operating organizations

The following Government departments, professional, scientific and industrial organizations co-operated in the preparation of this document.

Agrément Board*
Asbestos Cement Manufacturers' Association
Association of Building Technicians
Association of Industrialized Building Component Manufacturers Ltd.*
Brick Development Association
British Ceramic Research Association
British Constructional Steelwork Association
British Plastics Federation
British Railways Board
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Concrete Society Ltd.
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Scottish Development Department
The Society of Railing and Balustrade Manufacturers
Timber Research and Development Association

The Government departments and scientific and industrial organizations marked with an asterisk in the above list were directly represented on the committee entrusted with the preparation of this PD document.

Summary of pages

This document comprises a front cover, an inside front cover, pages i to iv, pages 1 to 21 and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

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Amendments issued since publication

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Synopsis

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Example: combinations of 3 and 4 showing how all numbers from 6 onwards (the critical number) are filled and different arrangements occur.

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How computers can identify combinations. What computer programs exist.

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Summary of contents

Combinations can help everybody in the construction industry.

They promote economy in building.

Different arrangements are possible. Table of possible arrangements gives up to 8 + 8 of two different sizes.

Critical number (CN) is the sparking point.

CN is a good guide to combinability and flexibility of a component range.

The table of CN gives CN of pairs and trios of component sizes 2 to 30. The table narrows the search.

The “combigraph” gives in relation to spaces:

composition of all combinations using up to three component sizes;

total number of all combinations;

critical number

Problem-analysis check lists for manufacturers, designers and builders define and solve their problems.

Computers work out combinations effectively but preliminary sifting is desirable for economy and understanding.

Combinations can help in other industries.

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1 Introduction

The Introduction to PD 6444-1[1)] states that in choosing a range of sizes for a modular building component, the objective is "the development of a limited range of sizes giving maximum flexibility of assembly for the component". Flexibility is given as "the measure of the number of different basic spaces that a component range can fill, both singly and in various assembly combinations".

The range of sizes for a modular component is expressed in terms of co-ordinating spaces or, for single dimensions, co-ordinating sizes. Thus, in the terminology of dimensional co-ordination, basic spaces are filled by the co-ordinating spaces of individual components. A full explanation of basic space, and co-ordinating space concepts is given in PD 6444.

Components that can be assembled or coupled in combinations, both vertically and horizontally, (so that a small range of such components can fill a wide range of basic spaces), are termed "additive" components. ***The object of this document is to provide further guidance on the selection of limited ranges of co-ordinating sizes for additive components, by an introduction to the use of combinations of numbers, or sizes.*** Other than simple arithmetic, no mathematics are required.

Only one linear dimension, of the basic space or component co-ordinating space, has been considered in the worked examples contained in the document. However, the same procedures can be applied in the selection of sizes for other dimensions.

The guidance contained in this document can assist manufacturers in choosing the right co-ordinating sizes to make, and designers in selecting the most appropriate sizes from those available.

In substance, this document is a simplified and abbreviated version of a book[2)] and, of necessity, the scope of the guidance given cannot be as extensive as that of the book itself. It does, however, contain some new information which is not in the book.

[1)] PD 6444, "*Recommendation for the co-ordination of dimensions in building. Basic spaces for structure, external envelope and internal sub-division*", Part 1, "*(Functional Groups 1, 2 & 3)*".

[2)] "Combinations of numbers in building". Philip Dunstone, The Estates Gazette Ltd., London.

2 Definitions

For the purposes of this document, the following definitions, which are taken from BS 2900[3)], apply:

Term	Definition
	Co-ordination
dimensional co-ordination	The application of a range of related dimensions to the sizing of building components and assemblies and the buildings incorporating them.
	The Building Reference System
reference system	A system of points, lines and planes to which sizes and positions of a building component or assembly may be related.
reference plane	A plane of a reference system.
basic space	A space bounded by reference planes, assigned to receive a building component or assembly including, where appropriate, allowance for joints and tolerances.
	The Component Reference System
co-ordinating plane	A plane by reference to which a building component or assembly is co-ordinated with another.
co-ordinating space	A space bounded by co-ordinating planes, allocated to a building component or assembly, including allowance for joints and tolerances.
co-ordinating dimension	1) A dimension of a co-ordinating space. 2) A dimension which is common to two or more building components to permit their assembly.
co-ordinating size	The size of a co-ordinating dimension.
	The Building Component and its Sizes
building component	Building material formed as a distinct unit.
modular building component	A building component whose co-ordinating sizes are in accordance with BS 4011[a].

[a] BS 4011, "*Recommendations for the co-ordination of dimensions in building. Basic sizes for building components and assemblies*".

[3)] BS 2900, "*Recommendations for the co-ordination of dimensions in building. Glossary of terms*".

3 Putting components together

When components and assemblies are put together on site to form a building, they fill spaces. Each space and each component is of a definite size and the components are used additively to fill spaces. This is equally true of traditional buildings as of dimensionally co-ordinated ones designed to the controlling reference system given in BS 4330[a].

The following are examples of components being used additively to fill spaces.

In the examples whole numbers only are used, but these may be regarded as units of any size required. In order to find the ***physical*** sizes of components or spaces, it is therefore necessary to multiply by the size of the unit selected. For example, in working in units of 100 mm, a space of 29 units long would be 29 × 100 mm = 2 900 mm long; with 300 mm units, a component 5 units long would be 5 × 300 mm = 1 500 mm long.

For simplicity, examples are worked for sizes of one horizontal dimension only, with the height dimension remaining constant; but the same procedures can be applied for selecting the sizes of other dimensions.

3.1 Single components

NOTE Although this part of the discussion may seem obvious, it is necessary for an understanding of what follows.

A single component, say 3 units long, is represented thus:

3 

When added together, components 3 units long will fill a space 15 units long. In practical terms, panels 300 mm long will make a wall 1 500 mm long. (See Figure 1.)

15 

Figure 1— Components 3 units long will fill a space 15 units long

[a] BS 4330, "*Recommendations for the co-ordination of dimensions in building. Controlling dimensions.*

They will not completely fill spaces 16 or 17 units long. (See Figure 2.)

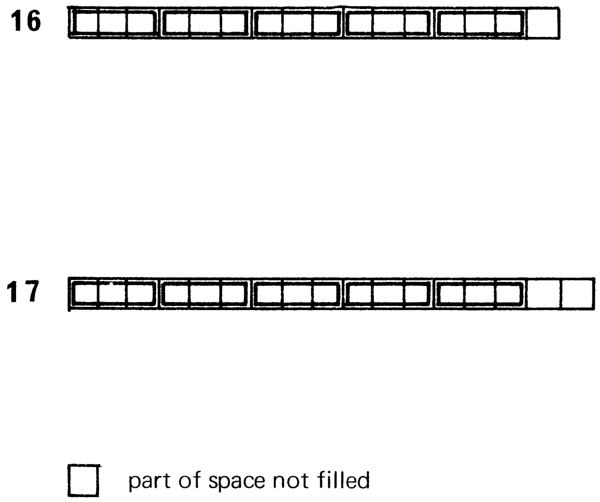


Figure 2 — Components 3 units long will not completely fill spaces 16 or 17 units long

The obvious rule is that the length of the space is divided by the length of the component:

15 ÷ 3 = 5 (no remainder — space filled)

16 ÷ 3 = 5 (remainder 1 — space not completely filled)

17 ÷ 3 = 5 (remainder 2 — space not completely filled)

3.2 Two components

For the purposes of this document, component relationships fall into two categories:

Components which have common factors

Components which have no common factors.

The first of two components, say 2 units long, is represented, as before by a rectangle thus:

2 

The second component, say 6 units long, is represented by a hatched one thus:

6 

3.2.1 ***Two components having common factors***

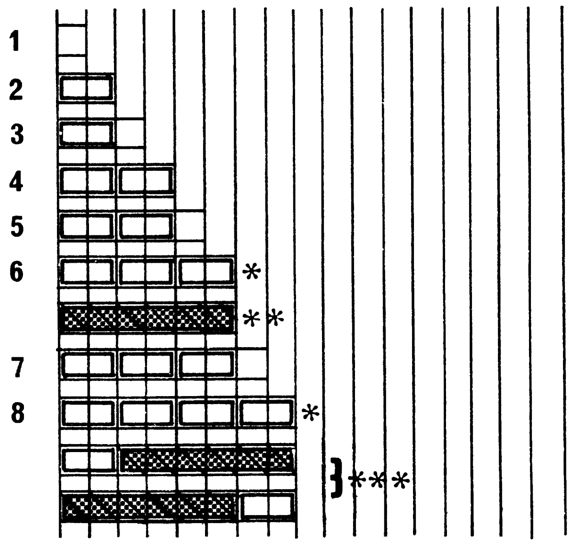
In combining units of

2 

and units of

6 

it will be found that they fill all spaces divisible by 2, but do not fill odd numbered spaces. (See Figure 3.)



and so on — spaces of a length divisible by 2 only are filled.

part of space not filled

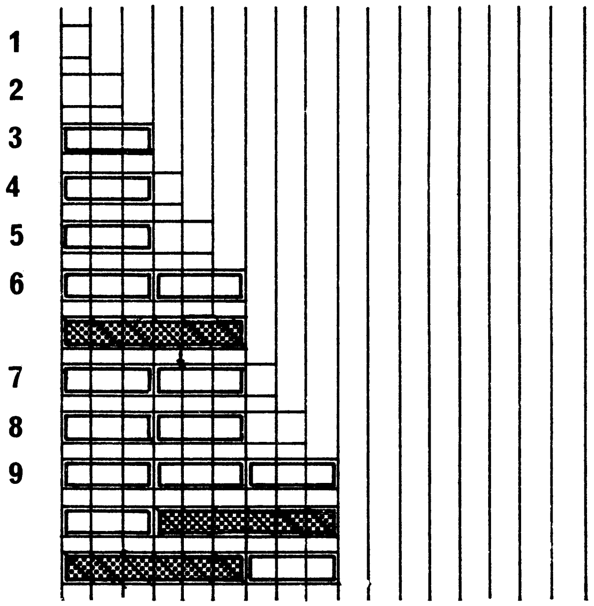
* first combination (single component)

** second combination (single component)

*** second combination (both components) — two different arrangements

Figure 3 — Combinations of 2 and 6

Units of 3 and units of 6 will similarly fill all spaces divisible by 3 but no others. (See Figure 4.)



and soon — spaces of length divisible by 3 only are filled.

part of space not filled

Figure 4 — Combinations of 3 and 6

3.2.2 ***Two components having no common factors***

Components with no common factors are termed "relatively prime". Relatively prime components combine at a particular point to fill ***every*** space.

This particular point we call the ***critical number.***

Being ***relatively*** prime has nothing to do with prime numbers directly.

If 3 and 4 are combined, they are said to be relatively prime, because they have no common factors, although 3 is a prime number and 4 is not. 2 and 5 are at the same time relatively prime ***and*** prime numbers. Similarly, 9 and 10 are relatively prime though ***neither*** is a prime number.

The results of combining the relatively prime numbers 3 and 4 are very different from those produced by components which have common factors like 2 and 6 or 3 and 6. 3 and 4 combine to fill every space from 6 onwards, but combinations of 2 and 6 and of 3 and 6 never fill every space and have no critical numbers.

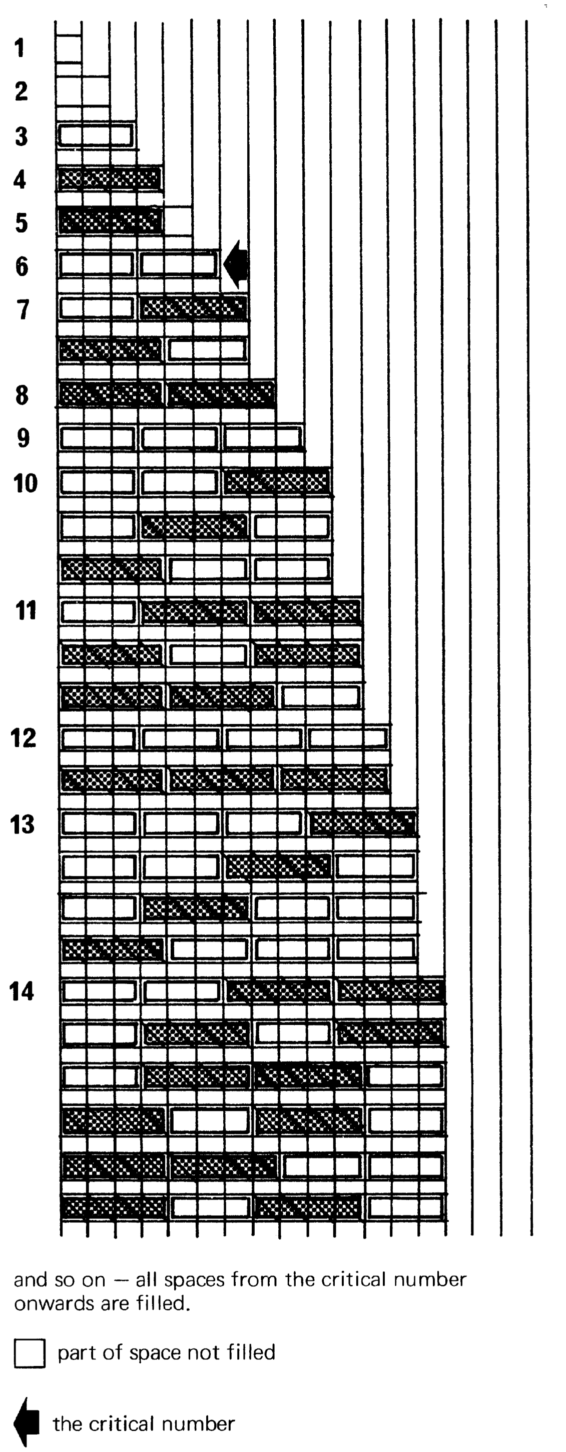


Figure 5 — Combinations of 3 and 4

3.3 Arrangements

With two-component combinations, as distinct from multiples of a single component, different ***arrangements*** will be possible. Spaces 7 and 10-14 in the last example illustrate this.

In filling space 10, three "pieces" are used: ***two*** of size 3 and ***one*** of size 4, resulting in three different possible arrangements.

Similarly, in filling space 11, ***three*** pieces are used. In space 12 ***three*** or ***four*** pieces of the same size resulting in only one possible arrangement of each are used — and so on.

The number of possible arrangements produced by any two-component combination (up to a maximum number of eight of each) is shown in the table of possible arrangements. (See Figure 6.)

Examples of these in Figure 5 are:

Space 14 which has ***two*** pieces of one component (Size A) and ***two*** of the other (Size B) for which the table shows that there are 6 arrangements. The 6 arrangements are illustrated.

Space 13 which has ***one*** piece of one component (Size A) and ***three*** pieces of the other (Size B) for which the table gives 4 arrangements. These arrangements are illustrated.

As noted above, whenever one or both components is represented by one piece, the total number of arrangements is the same as that of the number of pieces. An example of this is space 11 which is filled:

		Pieces
Size 3	(Size A)	1
Size 4	(Size B)	2
Total (= total number of arrangements — see table)		3

pieces of component size A \ pieces of component size B	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	2	3	4	5	6	7	8	9
2		6	10	15	21	28	36	45
3			20	35	56	84	120	165
4				70	126	210	330	495
5					252	462	792	1287
6						924	1716	3003
7							3432	6435
8								12870

Figure 6 — Table of possible arrangements

4 More about critical numbers

It has been shown that component sizes which are relatively prime will combine to fill every space after a point called the critical number. Conversely, component sizes which have common factors never fill every space.

This is why the common practice of using sizes which are in a doubling series never produce combinations, which fill every space without the use of a secondary "in-fill" component or of "make-up" jointing techniques.

In order to select sizes which, in combination, will fill the maximum number of spaces, it is first necessary to know more about the critical number.

The following examples are confined to combinations of two components; critical number is abbreviated to CN.

With two components which are relatively prime (have no common factors) the critical number (CN) is found by deducting 1 from each component and then multiplying them together:

CN of 3 and 4	=	$(3 - 1) \times (4 - 1)$
	=	2×3
	=	6 (as in **3.2.2**)

Or similarly, using the formula:

CN	=	$(a - 1)\ (b - 1)$
CN of 3 and 4	=	$(3 - 1) \times (4 - 1)$
	=	2×3
	=	6

The relatively prime components not only fill every space from their critical number onwards, but also approximately half the number of spaces below CN. Thus 3 and 4, having a CN of 6, fill 2 spaces below their CN.

5 Table of critical numbers

Reference has already been made to CN, the critical number, or that number from which all spaces are filled.

The critical number is always a good guide to the combinability of the component sizes selected and therefore the flexibility of the component range.

A table of critical numbers is published in the book referred to in the Introduction. This gives CN for any 2 or any 3 sizes between 2 and 30. An extract from the table is given in Appendix A of this document.

The only rule in using the table is that the sizes must be put into ascending order.

A live problem illustrates how the table is used:

A manufacturer already makes, say, an enamelled metal panel in two widths, one of 900 mm (9 units) and one of 1 600 mm (16 units). These have a CN of 120, i.e. (9 – 1) (16 – 1) = 8 × 15 = 120. He wants to choose a third panel, near in size to the other two, which reduces the critical number, i.e. begins to fill spaces as early as possible.

The table gives the following:

Sizes (units of 100 mm)		
7, 9, 16	CN = 48	(same as 7 and 9 only, because 7 + 9 = 16)
8, 9, 16	CN = 56	(same as 8 and 9 only, because 8 × 2 = 16)
[a]9, $\boxed{10,}$ 16	CN = 34	Lowest CN
9, 11, 16	CN = 47	As shown in Appendix A
9, 12, 16	CN = 48	
9, 13, 16	CN = 47	
9, 14, 16	CN = 50	
9, 15, 16	CN = 54	
9, 16, 17	CN = 57	
9, 16, 18	CN = 120	(same as 9 and 16 only, because 9 × 2 = 18)
9, 16, 19	CN = 59	

[a] 10 (100 mm) is therefore the best third co-ordinating size to make and, from 34 (3 400 mm) upwards, the three sizes 9, 10 and 16 fill every size.

6 Problem analysis check lists

It is possible to compile check lists to analyse typical problems.

One of these for manufacturers is reproduced in simplified form from the book referred to in the Introduction.

1)

Q. Minimum component size?

a. possible

b. desirable

A. *a.* 500 mm (5 units)

b. 800 mm (8 units)

2)

Q. Maximum component size?

a. possible

b. desirable

A. *a.* 3 000 mm (30 units)

b. 2 400 mm (24 units)

3)

Q. Can components be fixed on either edge (turned through 90°)?

A. No.

4)

Q. Are there certain component sizes which ***must*** be made?

A. 1 200 mm (12 units)

5)

Q. Must any component sizes be complementary (e.g. 300 mm and 500 mm out of 800 mm board)?

A. No.

6)

Q. Minimum number of component sizes (range)?

A. Two.

7)

Q. Maximum number of component sizes (range)?

A. Three

8)

Q. Minimum CN desirable

A. 3 000 mm (30 units)

9)

Q. Are there certain dimensions which ***must*** be covered?

A. 3 000 mm (30 units)

Solution:

Taking 5 (the lowest size) and 12 (the size which ***must*** be made), the table shows that suitable third sizes are:

Size	CN
13	22
14	24
16	24
21	29

As 21 is nearest to the maximum size, the suggested range is 5, 12 and 21 (500 mm, 1 200 mm and 2 100 mm) and this satisfies all the stated requirements.

Factors which the manufacturer must take into account, amongst other things, are:

1) What are the manufacturing limits (size, weight and design) imposed by plant, handling, storing, transporting, and the physical properties and sizes of the “raw” materials?

2) Can edge-sizes be used in turning corners, for example, are they or can they be made to co-ordinating sizes that conform to BS 4011[4)]

3) What are the limitations (size, weight and design) imposed by site plant, handling, storing, transporting, supporting, fixing and protecting and by Codes of Practice?

4) What are the design limitations imposed by compliance with the Building Regulations, fire and other regulations and with British Standards?

5) What are the demands at home and overseas?

6) What are the economics of plant, handling, storing, transporting and of selling the products?

[4)] BS 4011, “*Recommendations for the co-ordination of dimensions in building. Basic sizes for building components and assemblies*”.

7 Identifying combinations

By using the information given previously about two relatively prime component sizes the critical number (the point at which they combine to fill every space) may be found. The fact that they fill approximately half the number of spaces below the critical number is also known.

The next steps are to identify ***which*** spaces are filled, the actual combinations that fill them, and the possible arrangements of those combinations.

This is done by means of a simple graph, for which the name "combigraph" has been adopted.

8 The "combigraph"

8.1 Take a piece of squared paper. (See Figure 7.)

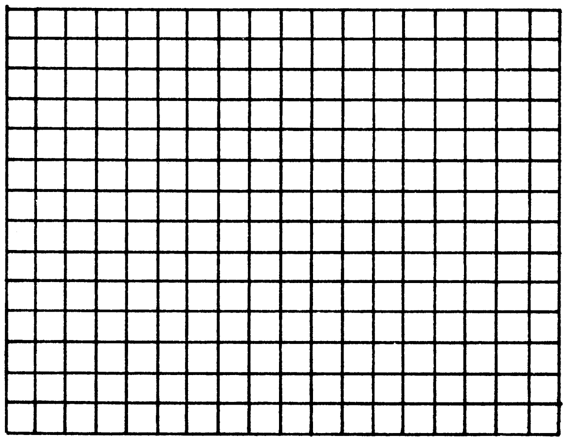


Figure 7 — See 8.1

8.2 Mark the boundaries of the graph (the x and y axes) and number the lines and verticals. (See Figure 8.)

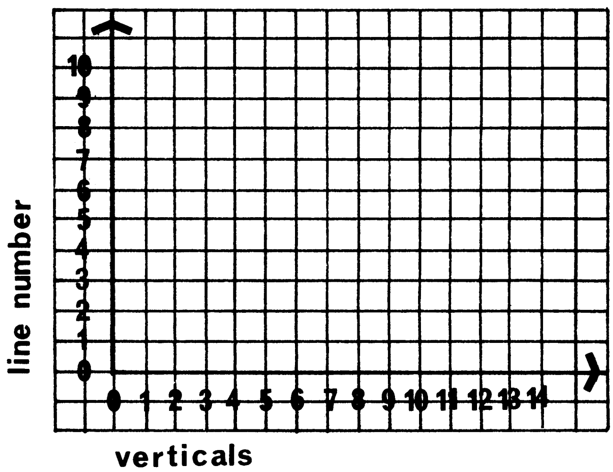


Figure 8 — See 8.2

8.3 Choose, as an example, component sizes of 3 and 5. Show these on the graph.

8.4 Firstly, show component size 3. This will be represented by and plotted as follows:

The first goes on line 1 above 3 on the bottom scale representing 1 × 3 = 3. This is on vertical 3.

The second on line 2 above 6 (2 × 3 = 6). On vertical 6.

The third on line 3 above 9 (3 × 3 = 9). On vertical 9.

The fourth on line 4 above 12 (4 × 3 = 12). On vertical 12.

And so on.

Figure 9 illustrates these plottings.

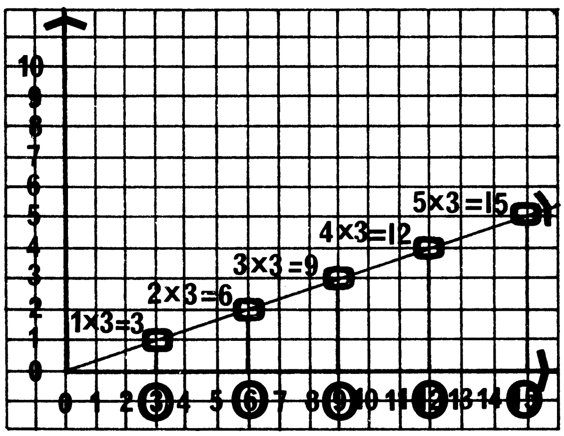


Figure 9 — Plotting of component size 3

As a check, the symbols should lie on a straight sloping line.

8.5 Similarly, size 5 will be represented by ♦ and plotted as follows:

The first ♦ goes on line 1 above 5 on bottom scale representing 1 × 5 = 5.

The second ♦ on line 2 above 10 (2 × 5 = 10).

The third ♦ on line 3 above 15 (3 × 5 = 15).

The fourth ♦ on line 4 above 20 (4 × 5 = 20).

And so on.

Figure 10 illustrates these plottings.

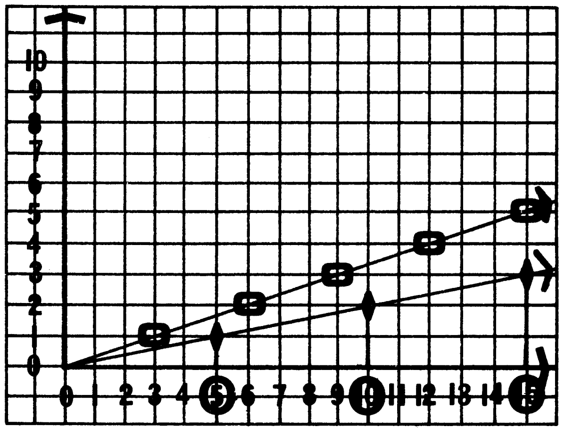


Figure 10 — Plotting of components sizes 3 and 5

Again, the ♦ symbols should lie on a straight sloping line.

8.6 The ***combinations*** which arise from using both 3 and 5 can now be plotted. These are represented by **O** and are plotted between the rays formed by putting 3 and 5 on the graph.

The **O** symbols go:

None on line 1.

One on line 2 dividing the distance between and into 2 equal parts. (See Figure 11.)

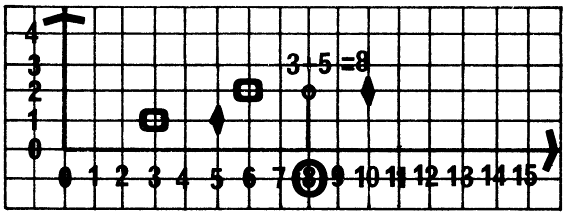


Figure 11 — See 8.6

Two on line 3 dividing the distance between and into 3 equal parts. (See Figure 12.)

Three on line 4 dividing to into 4 equal parts. And so on.

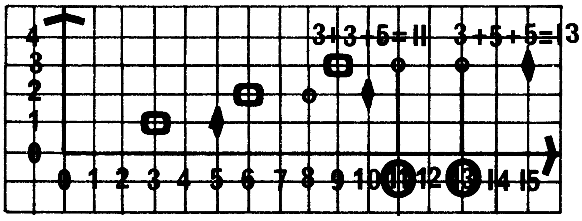


Figure12 — See 8.6

The regular patterns which the **O** symbols make will be apparent. In fact a typical "combigraph" pattern is illustrated in Figure 13.

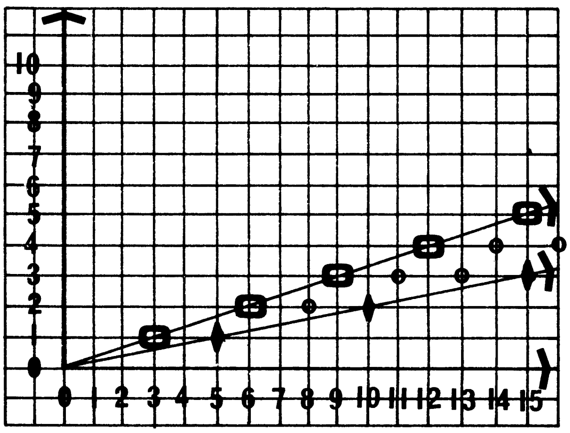


Figure 13 — Completed plotting of all combinations of sizes 3 and 5

8.7 Deductions to be drawn from the graph

Several things may now be deduced from the graph:

Every represents a combination using component size 3 only.

Every represents a combination using component size 5 only.

Every **O** represents a combination of sizes 3 3 and 5.

The total number of different combinations is given. For example, two symbols on vertical 15 indicate that two different combinations fill space 15. Similarly and **O** on vertical 18 show the two combinations that fill space 18.

The critical number (8) is illustrated, 7 being the last blank vertical.

There are 3 combinations below CN. (Approximately half the number of spaces below the CN.)

As an introduction to the subject, the above is sufficient but much has been left out and, for a wider understanding, recourse must be had to the book referred to in the Introduction.

9 "Combigraph" and block diagrams combined

The graph may now be linked with the block method of showing combinations used at first. (See Figure 14.)

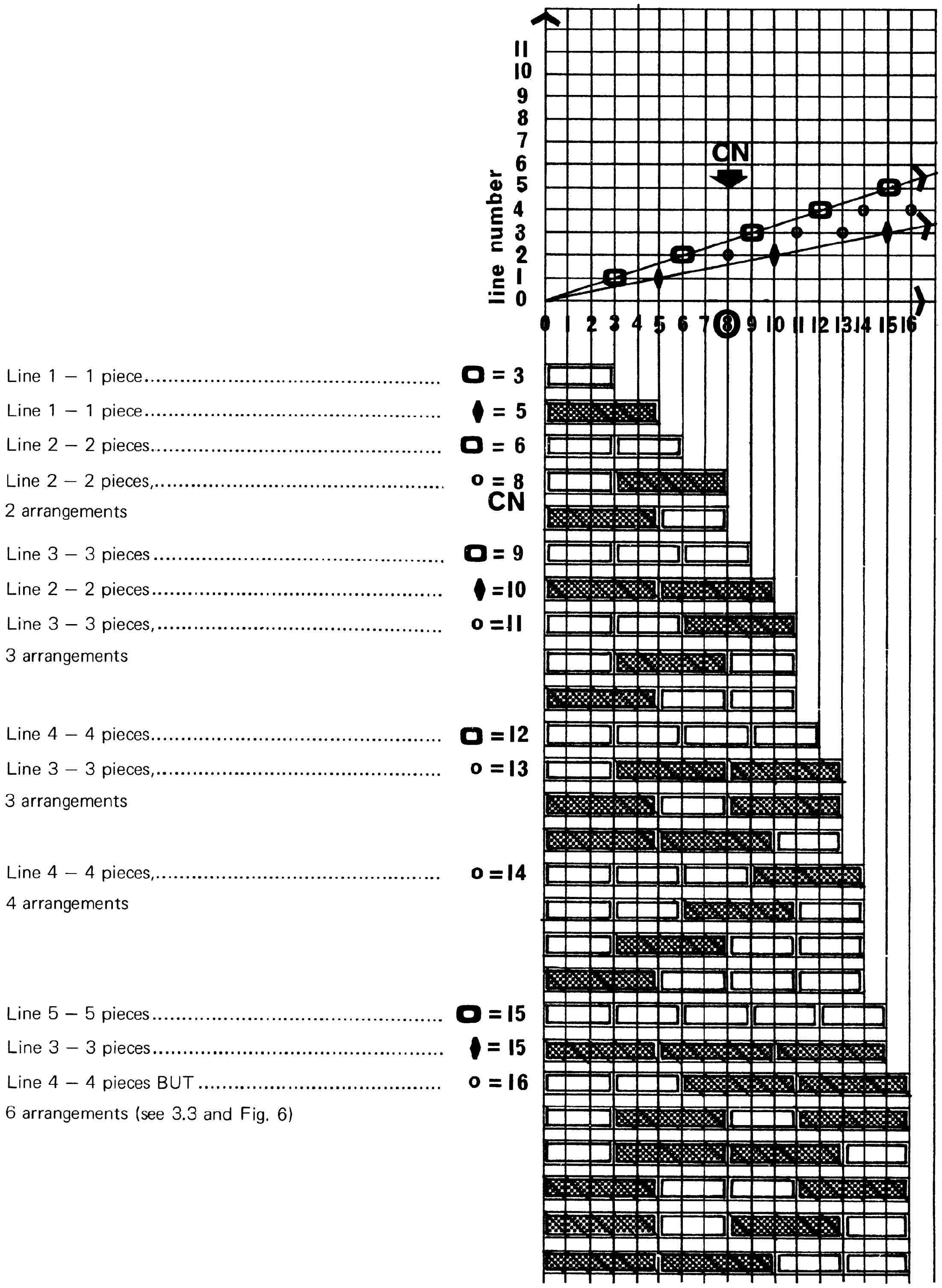


Figure 14 — "Combigraph" and block diagrams combined. Component sizes 3 and 5

10 Combinations and arrangements using two components — summary

Summarizing what has been deduced from the diagrams:

1) The total number of different combinations is given.

2) The actual combinations can be read off.

3) The CN is illustrated.

4) The total number of pieces is the same as the line number on which a combination symbol lies.

5) The number of arrangements is equal to the number of the line on which the symbol lies ***so long as one of the components is represented by only one piece,*** in which case the number of arrangements is therefore the same as the number of pieces.

6) When both components are each represented by more than one piece, the table (Figure 6) gives the number of arrangements.

11 Three and more component sizes

The method can deal with three and more component sizes — the diagrams just become a little more crowded and less easy to read.

12 Use of computers

The computer can deal most efficiently with combinations. Computer programs exist which will give:

Details of all combinations for any given dimension using up to six component sizes and recording in addition the total number of combinations and symmetry.

The ***number*** of possible combinations if a complete list is not required.

Critical number using up to six component sizes.

The total number of possible arrangements.

Before using a computer, however, it is desirable to understand the principles and to do some of the preliminary sifting using the table of critical numbers.

13 Use by all industries

The methods described are applicable to all manufacturers whose products are used additively; the guidance given is not confined to the construction industry.

Appendix A Table of critical numbers

Rules:

1) Put component sizes into ascending order.
2) Apply as in key to find critical number.

KEY

This position gives critical number for TWO component sizes CN = (a − 1) (b − 1)

Remaining positions give the critical number using a THIRD component size according to this layout.

	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

EXAMPLE

9 x 11 only (= 8 x 10) 'X' is shown where these have common factors

9 x 11 x 16 (see clause 4)

9 & 11 & 17

9 & 11 & 22

80				
	38	44	36	44
47	42		44	
47		49	62	58
51		53		47

11

9

(Blank position indicates that critical number given by THIRD component size is the same as that given by the first two.)

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