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Guide to

Assessment of position, size and departure from nominal form of geometric features

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Committees responsible for this British Standard

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Contents

Foreword

This British Standard has been prepared under the direction of the Advanced Manufacturing Technology Standards Policy Committee.

This standard is intended for use by manufacturers of coordinate measuring machines (CMMs) and software writers within the CMM industry.

It covers the assessment of geometric form of workpieces measured with a CMM. It gives information and guidance to promote the better use of CMMs for this purpose by the adoption of reliable software that provides well presented comprehensive information.

This British Standard contains mathematical concepts and notation; therefore it is assumed that the execution of its guidance is entrusted to appropriately qualified and experienced personnel.

A British Standard does not purport to include all the necessary provisions of a contract. Users of British Standards are responsible for their correct application.

Compliance with a British Standard does not of itself confer immunity from legal obligations.

Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 17 and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

1 Scope

This British Standard provides information and guidance to manufacturers of coordinate measuring machines (CMMs), and particularly software writers within the CMM industry. It contains recommendations for determining the position, size and departure from nominal form of geometric features, given measurements of coordinates of points on a workpiece. The features covered correspond to the following geometric elements: lines, planes, circles, spheres, cylinders and cones.

This standard is concerned with software implementations of algorithms based on sound mathematical and computational principles, rather than automated versions of manual or graphical assessment procedures.

This standard does not provide detailed guidance on methods for treating data gathered in an unstable environment.

This standard does not cover secondary attributes, i.e. measures derived from departures from form of the above geometric elements, such as parallelism, concentricity or orthogonality.

This standard primarily relates to CMMs that operate with a right-handed Cartesian coordinate system. It also relates to other measurement systems that provide such coordinates. Some of the guidance provided also applies to other coordinate systems.

2 Definitions

For the purposes of this British Standard the following definitions apply.

2.1
algorithm

a step-by-step description in mathematical or other unambiguous terms of a process for solving a particular problem, e.g. the determination of the parameters describing a geometric feature

2.2
centroid

the point described by coordinates which are the arithmetic means of the corresponding coordinates of the data points

2.3
circularity

measure of departure from nominal form for a mathematical circle

2.4
cylindricity

measure of departure from nominal form for a mathematical cylinder

2.5
data point

either a raw data point or a raw data point that has been processed in some way

2.6
departure from nominal form

overall measure of the deviation of a workpiece from nominal form

NOTE The departure from nominal form is defined as the spread, or in terms of the spread.

2.7
deviation

the straight-line (Euclidean) distance of a data point from the reference, measured normal to the geometric element. The distance is regarded as positive or negative according to which side of the element the point lies. Where appropriate it is negative if the point lies in the material of the workpiece, and positive otherwise. In the case of a line in two dimensions or a plane all points on one side are taken to have a positive deviation and those on the other a negative deviation. The deviation for the *i*th data point is denoted by e_i

2.8
direction cosines (of a line)

the cosines of the angles between a line and the Cartesian axes

2.9
geometric element

line, plane, circle, cylinder, cone or sphere

2.10
geometric feature

(part of) an object nominally in the shape of one of the geometric elements

2.11
line

a straight line in two or three dimensions

2.12
measurement procedure

a strategy for obtaining a representative set of points on a workpiece

2.13
nominal form

the ideal geometric object of which the geometric feature under test is a machined or otherwise manufactured manifestation, e.g. sphere

2.14
normal

a line passing through a point on a curve or surface and perpendicular to the tangent line or plane at the point

NOTE A normal conventionally points out of the material.

2.15
outlier

a data point that is not regarded as a member of a set of data points representative of the geometric feature

NOTE An outlier may arise from malfunction of the CMM or an error on the part of its operator.

2.16
parameters

algebraic variables representing the size and position of a geometric element, e.g. the radius and centre coordinates of a circle

2.17
parameter values

numerical values of parameters

2.18
parametrization

a choice of algebraic variables to represent a geometric element

2.19
pre-processing

operations upon measured data intended to render it more suitable for purposes of assessment of form

2.20
range of the deviations

the difference between the largest and smallest signed deviations, i.e. $\max_i e_i - \min_i e_i$

2.21
raw data

measured coordinates of points on the boundary or surface of the geometric feature

2.22
reference

a computed geometric element to be used as a basis for assessment

2.23
representative set of points

a set of points that, for the purposes of the assessment, adequately represent the geometric feature

2.24
residual

a measure of the error of fit of a reference or trial reference at a point. At the ith point the residual is denoted by res_i

2.25
root mean square (r.m.s.) deviation

the square root of the quotient of the sum of the squares of the deviations and the number of degrees of freedom υ, i.e. $\sqrt{(\Sigma_i e_i^2/\upsilon)}$

2.26
software, software implementation

a computer implementation of an algorithm

2.27
spread

a measure of the scatter of the deviations

NOTE Two useful definitions are the range of the deviations and the root mean square deviation. A further measure of spread is a suitable multiple of the root mean square deviation.

2.28
uniform pseudorandom number generator

an algorithm or software for producing a sequence of numbers that, according to statistical tests, appear to be samples from a rectangular distribution

2.29
workpiece

the object or component under test, containing the geometric feature being assessed

3 Symbols and abbreviations

For the purposes of this British Standard the following symbols and abbreviations apply. Several meanings are given to some of the symbols and the specific meaning is implied in each case by the context in which the symbols are used.

a	Direction cosine for x.
b	Direction cosine for y.
c	Direction cosine for z.
c	Constant in circle equation.
C	Circle.
C	Cylinder.
C	Cone.
d	Distance of a point from a geometric element.
e_i	Deviation of the ith data point from a reference.
E	Objective function used in computing a reference.
f	Parameter in circle equation.
f	Intermediate variable in distance formulae.
F	Measure of departure from nominal form.
g	Parameter in circle equation.
g	Intermediate variable in distance formula.

G	Point on L or P closest to the centroid of the data points.
h	Distance between two parallel planes.
h	Height of frustum of cone.
i	Subscript for data point.
l	Length of generator of frustum of cone.
L	Straight line.
n	Number of parameters necessary to describe a geometric element, which normally is also the same as the mathematical minimum number of points required to define the element.
n_c	Number of approximately uniformly spaced parallel planes.
n_p	Number of measurements on or near a plane.
N	Number of measured points on a workpiece.
P	Plane.
q	Number of lobes on a nominally circular feature.
r	Radius of circle, sphere or cylinder.
r	Radial coordinate in a cylindrical or spherical coordinate system.
r_1, r_2	Radii of ends of frustum of cone.
r_i	Distance of ith data point from centre of reference circle.
res_i	Residual evaluated at the ith data point.
s	Difference between number of measurements on successive planes in a measurement procedure for a cone.
s	Distance from the surface of a cone to a point on its axis.
S	Sphere
S	Set of data points.
t	Parameter proportional to distance.
u, v, w	Intermediate variables in distance formulae.
x	First Cartesian coordinate.
$\bar{x}$	Arithmetic mean of values of x_i (= $\Sigma_i x_i/N$).
x_0, x_1, x_2	x-coordinates of locating points for a straight line or axis of a cylinder or a cone.
x_p	x-coordinate of a general point on a geometric feature when computing distance from the point to a geometric element.
y	Second Cartesian coordinate.
$\bar{y}$	Arithmetic mean of values of y_i (= $\Sigma_i y_i/N$).
y_0, y_1, y_2	y-coordinates of locating points for a straight line or axis of a cylinder or a cone.
y_p	y-coordinate of a general point on a geometric feature when computing distance from the point to a geometric element.
z	Third Cartesian coordinate.
$\bar{z}$	Arithmetic mean of values of z_i (= $\Sigma_i z_i/N$).
z_0, z_1, z_2	z-coordinate of locating points for a straight line or axis of a cylinder or a cone.
z_p	z-coordinate of a general point on a geometric feature when computing distance from the point to a geometric element.
θ	Bearing angle for cylindrical or spherical coordinates.
v	Number of degrees of freedom, given by $N - n$.
ϕ	Azimuth angle for spherical coordinates.
ψ	Apex angle of cone (equal to twice the angle between the cone's generator and axis).

4 Outline of guide

In order to obtain a reliable assessment of geometric form in any particular case, the corresponding geometric element should first be represented, i.e. parametrized, in a mathematically sound way. Recommended parametrizations are given in clause **5**. It is recommended that the assessment process itself be carried out in four stages:

a) apply an appropriate measurement procedure, i.e. a strategy for obtaining a representative set of measurements on the workpiece (see clause **6**);

b) (optionally) pre-process the data, i.e. replace the measured data by modified values in order, for example, to smooth the data, to remove inappropriate points or to compensate for environmental effects (see clause **7**);

c) compute the reference (e.g. an approximating circle in terms of its centre coordinates and radius), to give position and size (see clause **8**);

d) assess, in terms of the reference, the departure from nominal form (see clause **9**).

Once the assessment has been carried out, it is recommended that the software provides the information detailed in clause **10**. To avoid unnecessary numerical inaccuracies during the assessment, software writers should adopt the recommendations given in clause **11**.

NOTE Appendix A gives mathematical formulae for the distance of a point to a geometric element described by one or other of the parametrizations given in clause **5**. These formulae should be useful to the software writer in preparing algorithms for assessing departure from nominal form when using the recommended parametrizations.

5 Parametrization of geometric elements

5.1 General

This clause is concerned with the manner in which the position and, where relevant, orientation and size of each geometric element considered in this standard should be described in mathematical terms. For a reliable assessment to be carried out the workpiece should be adequately represented by a set of measured data points in a Cartesian coordinate system (see **6.2**). The geometric element that is to act as a reference for the data is described in terms of this system. The description consists of assigning numerical values to parameters that define the geometric element.

It is possible to parametrize each of the geometric elements in more than one way. The parametrizations given here are recommended as being generally applicable. They have the property that small changes in the geometric element usually result in correspondingly small changes in the parameter values. Certain other parametrizations may be equally sound, although it should be noted that the use of some parametrizations can yield unreliable results.

Example. It is possible to parametrize a cone in terms of position of the vertex, direction of the axis and the angle that the cone generator makes with the axis. For cones with only a gentle taper, e.g. a tapered shaft, the vertex position may be far from the data points. Slight changes in the data could result in large changes in the vertex position of the computed reference. Therefore this parametrization is not recommended.

NOTE For lines and circles nominally in a specified plane, the data points should first be projected into an (x, y) Cartesian coordinate system in that plane. The recommendations in **5.2** then apply to that coordinate system. Following computation, the description of the geometric feature should then be transformed to the original coordinate system.

5.2 Specific geometries

5.2.1 *Lines in a specified plane*

5.2.1.1 *General*. A line should be specified by either:

a) one point on the line and information about the orientation of the line; or

b) two points on the line.

In the recommendations given in **5.2.1.2** and **5.2.1.3** the data itself is used to determine the points to take.

NOTE 1 It is not recommended to use the gradient of the line and the intercept with a coordinate axis because of numerical difficulties that occur when the line is parallel or nearly parallel to the axis.

NOTE 2 It is not recommended to use distance of the line from the origin and the gradient of the line because this parametrization is numerically unstable, particularly when the span of the data points is small compared with the distance of the points from the origin.

5.2.1.2 *One point and the direction cosines*. A line L, related to a set of data points, should be specified by:

a) a point (x_0, y_0) on L; and

b) its direction cosines (a, b).

NOTE 1 The point (x_0, y_0) should be taken at or near G, the point on L closest to the centroid of the data points.

NOTE 2 Any point (x, y) on L satisfies the equation:

$(x, y) = (x_0, y_0) + t\,(a, b)$

for some value of t.

5.2.1.3 *Two points on the line*. A line L, related to a set of data points, should be specified by two points (x_1, y_1), (x_2, y_2) that:

a) lie on L; and

b) are such that all data points lie between the two lines perpendicular to L passing through (x_1, y_2) and (x_2, y_2), respectively; and

c) are as close together as (reasonably) possible.

NOTE Any point (x, y) on L satisfies the equation:

$(x, y) = (1 - t)\,(x_1, y_1) + t(x_2, y_2)$

for some value of t.

5.2.2 *Lines in three dimensions*

5.2.2.1 *General*. The parametrization of lines in three dimensions is very similar to that of lines in the plane (see **5.2.1**). The parametrization of a cylinder or cone requires the specification of an axis, so this clause will have relevance to **5.2.7** and **5.2.8**.

5.2.2.2 *One point and the direction cosines*. A line L, related to a set of data points, should be specified by:

a) a point (x_0, y_0, z_0) on L; and

b) its direction cosines (a, b, c).

NOTE 1 The point (x_0, y_0, z_0) should be taken at or near G, the point on L closest to the centroid of the data points.

NOTE 2 Any point (x, y, z) on L satisfies the equation:

$(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$

for some value of t.

5.2.2.3 *Two points on the line*. A line L, related to a set of data points, should be specified by two points (x_1, y_1, z_1), (x_2, y_2, z_2) that:

a) lie on L; and

b) are such that all data points lie between the two planes perpendicular to L passing through (x_1, y_1, z_1) and (x_2, y_2, z_2), respectively; and

c) are as close together as (reasonably) possible.

NOTE Any point (x, y, z) on L satisfies the equation:

$(x, y, z) = (1 - t)\,(x_1, y_1, z_1) + t(x_2, y_2, z_2)$

for some value of t.

5.2.3 *Planes*

5.2.3.1 *General.* A plane should be specified by a point on the plane and either:

a) the direction cosines of the normal to the plane; or

b) a point on the normal to the plane passing through the first point.

5.2.3.2 *Point on the plane and direction cosines of the normal.* A plane *P*, related to a set of data points, should be specified by:

a) a point (x_0, y_0, z_0) on *P*; and

b) the direction cosines (a, b, c) of the normal to *P*.

NOTE 1 The point (x_0, y_0, z_0) should be taken at or near *G*, the point on *P* closest to the centroid of the data points.

NOTE 2 Any point (x, y, z) on *P* satisfies the equation:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

5.2.3.3 *Point on the plane and a second point on the normal.* A plane *P*, related to a set of data points, should be specified by:

a) a point (x_0, y_0, z_0) on *P*; and

b) a point (x_1, y_1, z_1) on the normal to *P* at (x_0, y_0, z_0).

NOTE 1 The point (x_0, y_0, z_0) should be taken at or near *G*, the point on *P* closest to the centroid of the data points.

NOTE 2 The point (x_1, y_1, z_1) should be determined such that its distance from *P* is comparable with the span of the data points.

NOTE 3 Any point (x, y, z) on *P* satisfies the equation:

$$(x_1 - x_0)(x - x_0) + (y_1 - y_0)(y - y_0) + (z_1 - z_0)(z - z_0) = 0$$

5.2.4 *Circles in a specified plane*

5.2.4.1 *Centre and radius.* A circle *C* in the plane should be specified by its centre (x_0, y_0) and its radius *r*.

NOTE 1 Any point (x, y) on *C* satisfies the equation:

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

NOTE 2 Numerical inaccuracies are likely to arise in the use of this parametrization for a circle related to a set of data points lying on or near an arc whose length is much smaller than its radius.

5.2.5 *Circles in three dimensions*

5.2.5.1 *General.* A circle in three dimensions should be specified by its centre and radius, and the plane in which it lies. Since the centre of the circle lies in the plane, this point should be used in specifying the plane.

5.2.5.2 *Centre, radius and plane.* A circle *C* should be specified by:

a) its centre (x_0, y_0, z_0); and

b) its radius *r*; and either

1) the direction cosines (a, b, c) of the normal to the plane containing *C*; or

2) a point (x_1, y_1, z_1) on the normal at the centre of *C* to the plane containing *C*.

NOTE 1 The point (x_1, y_1, z_1) should be chosen such that its distance from the centre is comparable to the radius.

NOTE 2 Numerical inaccuracies are likely to arise in the use of this parametrization for a circle related to a set of data points lying on or near an arc whose length is much smaller than its radius.

5.2.6 *Spheres.* A sphere *S* should be specified by its centre (x_0, y_0, z_0) and its radius *r*.

NOTE 1 Any point (x, y, z) on *S* satisfies the equation:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

NOTE 2 Numerical inaccuracies are likely to arise in the use of this parametrization for a sphere related to a set of data points that span a region whose area is small compared with the surface area of the sphere.

5.2.7 *Cylinders.* A cylinder *C*, related to a set of data points, should be specified by:

a) the axis of *C*, specified according to either **5.2.2.2** or **5.2.2.3**; and

b) its radius *r*.

NOTE 1 If the axis is specified according to **5.2.2.2**, the point (x_0, y_0, z_0) should be taken close to the midpoint of the part of the axis that is enclosed by the data.

NOTE 2 Numerical inaccuracies are likely to arise in the use of this parametrization for a cylinder related to a set of data points that, when orthogonally projected onto a plane perpendicular to the cylinder axis, lie on or near an arc whose length is much smaller than the cylinder radius.

5.2.8 *Cones*

5.2.8.1 *General.* A cone should be specified by its axis, angle, and information about where on the axis the cone is situated. The use of the vertex is not recommended in general (see example in **5.1**).

5.2.8.2 *Axis, angle and distance from a point on the axis to the cone surface.* A cone *C*, related to a set of data points, should be specified by:

a) the axis of *C*, specified according to either **5.2.2.2** or **5.2.2.3**; and

b) the apex angle ψ of the cone; and

c) the distance *s* to the surface of *C* from a point (x_0, y_0, z_0) on the cone axis.

NOTE 1 If the axis is specified according to **5.2.2.2**, the point (x_0, y_0, z_0) should be taken close to the midpoint of the part of the axis that is enclosed by the data.

NOTE 2 Numerical inaccuracies are likely to arise in the use of this parametrization for a cone related to a set of data points that, when projected from the cone vertex onto a plane perpendicular to the cone axis, lie on or near an arc whose length is much smaller than the radius of the intersection of the cone and plane.

6 Measurement procedure

6.1 General

The set of measured values made on the workpiece constitute the data upon which calculations are carried out to determine position, size and departure from nominal form. Data collection may be under manual or automatic control.

To obtain reliable results the gathered data should be representative of the geometric feature. The use of too few data points or data points inappropriately distributed may provide an unreliable reference. The provision by the manufacturer of well-defined and explicit measurement procedures (appropriate for the computations to be carried out) should be of value in this respect to the user.

Example. As a result of the machining process used, a nominally cylindrical component may have an elliptical cross-section. The use of too few data points, e.g. three in any position on a profile, or four in positions on a profile that are symmetric with respect to the axes of the ellipse, on the profiles of a number of cross sections may be insufficient to permit a reliable estimate of the true departure from an ideal cylinder to be made.

Documentation should accompany the manufacturer's software to give advice on the number and locations of measuring points. Where possible the manufacturer should guide the user through decision processes that lead to the selection of a suitable measurement procedure. Alternatively, the software itself should assist in this respect. No absolute guidance can be given because the points chosen should take account of the nature of the machining process and the intended function of the workpiece. Some tests of the adequacy of a set of data points should be made by appropriate software provided by the manufacturer.

The user of a coordinate measuring machine may know the likely deformations of a particular workpiece. The measurement procedure should take account of that knowledge. The procedures recommended in **6.3** and **6.4** are not intended to be substitutes for such "in-house" procedures. Rather, they should be regarded as a minimal requirement.

6.2 Coordinate systems

Any, well-defined, coordinate system may be used. Common coordinate systems are:

rectangular x, y, z;
cylindrical r, θ, z;
spherical r, θ, ϕ.

Rectangular axes should usually form a right-handed Cartesian set. (In the rectangular set xyz, the x-axis rotated to y in the xy-plane would cause a right-hand screw to progress along and in the direction of the positive z-axis.)

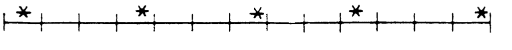
A left-handed set is sometimes convenient when workpieces occur in mirror-imaged pairs.

For the purposes of a particular assessment, there will usually be at least two defined coordinate systems: one fixed in the CMM, the other fixed in the workpiece. By their nature, measurements are made in terms of the CMM's coordinate system. Computed results should be quoted in terms of a workpiece coordinate system, which is normally defined in a calculated reference. It is often convenient and generally numerically desirable for the calculations to be performed in other coordinate systems (see **11.5**).

6.3 Distribution of points

6.3.1 *General.* The distribution of measured data points should normally aim for a uniform coverage of the workpiece. This will help to ensure that the points provide a genuine representation of the geometric feature. However, the distribution should not be so regular that it is possible for it to follow systematic or periodic deformations. For example, if a "circle" has three equal lobes, a distribution of six points equally spaced around the circle may fail completely to detect the lobing effect. A certain amount of randomness and lack of regularity in the distribution of points is therefore generally desirable.

Where periodic distortions (lobes) are suspected, the measured points should not approximate the same position in each period. For a closed feature, e.g. a circle, this can be avoided by having no common factor in the number of measured points, N, and the likely number of periods. In particular, the use of a prime number for N should be satisfactory.



Example. N = 5.

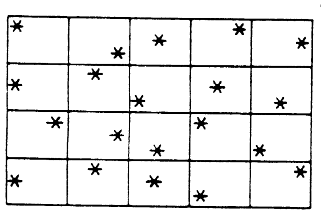
NOTE One point is chosen at random in each subinterval marked with an asterisk.

Figure 1 — A distribution of points on a line

6.3.2 *Lines*. To achieve a nearly uniform distribution of N points on a line segment, the line segment should be divided into N subintervals of equal length and one point placed in each subinterval. If the "line" is likely to suffer a periodic distortion the chosen points should not conform to a regular pattern. One way to ensure this is to choose the point in each subinterval at a "random" position. A uniform pseudorandom number generator should be used for this purpose. To avoid the possibility of having points too close together this process should be refined by dividing the line segment into say $3N - 2$ subintervals of equal length and choosing a random point in each of the 1st, 4th, 7th, ..., $(3N - 2)$nd subintervals (see Figure 1).

6.3.3 *Planes*. To achieve a nearly uniform distribution of (approximately) N points on a rectangular segment of a plane, the rectangle should be divided into $N_1 \times N_2$ sub-rectangles by a regular mesh of lines, where $N_1 N_2$ is approximately equal to N, and one point placed in each sub-rectangle (see Figure 2). The sub-rectangles should be as near to square as convenient. If the "plane" is likely to suffer a periodic distortion the chosen points should not conform to a regular pattern. This can be achieved by an extension of the device described in **6.3.2**. If it is more convenient to gather data on straight lines across the plane, then these should ideally be irregularly spaced and the points on each line spaced according to **6.3.2**.

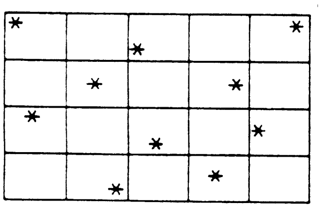
If only a small number of points is to be measured the number of sub-rectangles should be doubled and the points distributed in alternate sub-rectangles in a "chess board" fashion (see Figure 3).



Example. N = 20: choose $N_1 = 4$, $N_2 = 5$ to give 4 × 5 subrectangles.

NOTE One point is chosen at random in every subrectangle.

Figure 2 — A distribution of points in a plane



Example. N = 10: choose $N_1 = 4$, $N_2 = 5$ to give 4 × 5 subrectangles and "chess board" distribution of points.

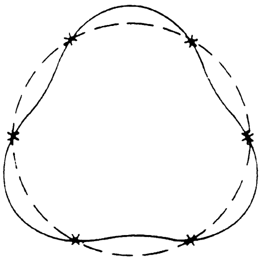
NOTE One point is chosen at random in each subrectangle marked with an asterisk.

Figure 3 — A "chess board" distribution of points in a plane

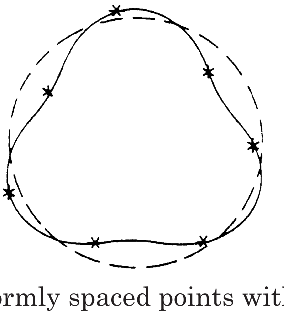
6.3.4 *Circles*. To achieve a nearly uniform distribution of N points the circle should be divided into N equal arcs and one point placed on each arc. If the "circle" is likely to be lobed, a regular distribution should not be used. If it is known that there are likely to be q lobes, N should be chosen so that N and q have no common factor. (N should always be chosen to be greater than q.) If N is divisible by q the information gathered from the measurements may be severely limited.

Example. Six points equally spaced on a 3-lobed "circle" may completely fail to detect the lobing.

Seven points equally spaced on such a circle will detect at least 79 % of the amplitude of the lobing (see Figure 4).



(a) six uniformly spaced points with complete failure to detect lobing



(b) seven uniformly spaced points with at least 79 % of the lobing detected

Figure 4 — A distribution of points on a lobed circle

6.3.5 *Spheres*. The following strategy achieves a nearly uniform coverage with (approximately) N points on the surface of the sector of a sphere of radius r enclosed between two parallel planes which are a distance h apart.

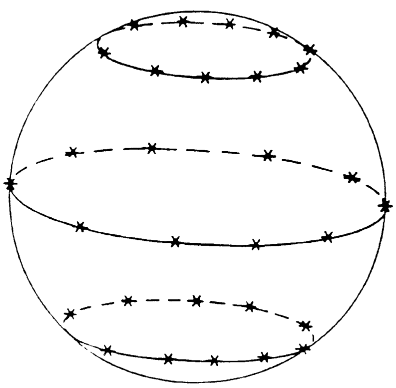
n_c should be determined as an integer close to $\sqrt{\{Nh/(2\pi r)\}}$ and n_p as an integer close to N/n_c. For each of n_c approximately uniformly spaced planes parallel to and including the given planes, n_p approximately uniformly spaced measurements should be taken at (or near) the intersection of the plane and the sphere.

NOTE 1 The strategy is based on the fact that the surface area of the sector of the sphere is equal to that of a cylinder of radius r and height h.

NOTE 2 For a complete sphere, $h = 2r$, in which case n_c is determined as an integer close to $\sqrt{(N/\pi)}$, with a single point at each pole.

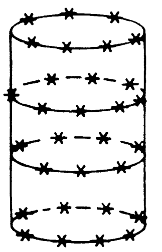
6.3.6 *Cylinders*. Achieving a nearly uniform distribution of N points on a cylinder of height h and radius r is similar to producing a nearly uniform distribution of N points on a rectangular plane segment of length h and breadth $2\pi r$. Thus the distribution for such a plane, as given in **6.3.3**, may be used for the cylinder by "wrapping the plane around the cylinder".

Alternatively, the points can be placed on parallel circles on the cylinders, with the circles roughly uniformly spaced. n_c should be determined as an integer close to $\sqrt{\{Nh/(2\pi r)\}}$ and n_p as an integer close to N/n_c. For each of n_c approximately uniformly spaced planes approximately perpendicular to the cylinder axis, n_p approximately uniformly spaced measurements should be taken at the intersection of the plane and the cylinder.



Example. For a sphere where r = 100 mm, h = 150 mm and N = 30 choose n_c = 3 and n_p = 10.

Figure 5 — A distribution of points on a sphere



Example. For a cylinder where h = 30 mm, r = 10 mm and N = 30 choose n_c = 4, n_p = 7 or 8.

Figure 6 — A distribution of points on a cylinder

It should be beneficial for the number of points to alternate between odd and even on the circles, e.g. in the example, seven points on the first, eight on the second, seven on the third, etc. This will help detect any lobing effect on the circular cross section.

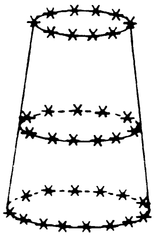
If the straightness of the cylinder is important more circles should be used with fewer points on each circle. If the circularity of the cross section is more important more points on each of a smaller number of circles should be used.

If it is convenient for the probe to move along a helix then substitute "circuit" for "circle" in the above paragraphs.

6.3.7 *Cones*. A nearly uniform distribution of N points on (a frustum of) a cone can be produced in much the same way as on a cylinder with points placed on parallel circles. However, the number of points on the circles should decrease towards the vertex of the cone. If the cone is of height h, side l and radii r_1 and r_2 ($r_2 > r_1$) at its ends, then $l = \sqrt{\{h^2 + (r_2 - r_1)^2\}}$. n_c should be determined as an integer close to $\sqrt{[lN/\{\pi(r_1 + r_2)]}$ and s as an integer close to $2\pi\ (r_2 - r_1)/l$. For each of n_c approximately uniformly spaced planes, approximately perpendicular to the cone axis, approximately uniformly spaced measurements should be taken at the intersection of the plane and the cone. The number of measurements on successive planes decreases by s for a plane nearer the vertex of the cone.

If few points are to be measured the number of circles should be doubled and the number of points on the circles halved, e.g. in Figure 7 circles with five, five, six, six, seven, seven points on the circles could be taken, with the points conforming to a "chess board" pattern.

If the circularity of the cone is important, more points on the circles should be used. If the straightness or angle of the cone is more important, more circles should be used.



Example. For a cone where r_1 = 10 mm, r_2 = 15 mm, h = 20 mm and N = 35, so that l = 20.6 mm, choose n_c = 3, s = 2 and take, successively, 10, 12 and 14 measurements on the three circles.

Figure 7 — A distribution of points on a cone

6.4 Number of points

There is a mathematical minimum number of points necessary to determine each geometric element, e.g. for a circle three points are needed. To gain information about departure from nominal form more points are required. Increasing the total number of measurements can be expected to have a statistically beneficial effect. This point is particularly important if the error of measurement is comparable to the machining error.

Table 1 shows the mathematical minimum number n and the recommended minimum number of points that should be used for the various geometric objects, taking into account the recommendations of **6.3**.

It cannot be overemphasised that the greater the number of appropriately distributed measured points the more reliable the assessment is likely to be.

6.5 Environment

This standard does not provide detailed guidance on methods for treating data gathered in an unstable environment. Either environmental conditions such as temperature and humidity should be held sufficiently constant for their effects on the computed results to be negligible, or appropriate corrections should be made to the raw data as part of the pre-processing stage (see clause **7**).

7 Data pre-processing

7.1 General

If the gathered data is considered of sufficiently high quality for purposes of the assessment it should be left unaltered. Alternatively, if it contains random or systematic errors that, it is judged, would adversely affect the results of the assessment, the data should be pre-processed. Pre-processing can be used to remove outliers, to reduce data errors by smoothing, to operate on data according to the functional requirements of the workpiece under tests, to account for flexing of the probe support arm and the finite dimensions of the probe, and to make corrections for the effects of temperature, humidity and vibration.

For example, the presence of dirt on the surface of the workpiece may yield erroneous, unrepeatable, measurements. Carefully constructed pre-processing software should normally be able to detect those data points which are so affected, and to make appropriate corrections. The provision of such software by the manufacturer should be accompanied by diagnostic information which indicates the data modifications and deletions made.

Table 1 — Minimum number of points (see **6.3**)

Element	Minimum number of points		Comment
	Mathematical (n)	Recommended	
Line	2	5	
Plane	3	9	Approximately three lines of three
Circle	3	7	To detect up to six lobes
Sphere	4	9	Approximately three circles of three in parallel planes
Cylinder	5	12	Circles in four parallel planes for information on straightness
		15	Five points on each circle for information on roundness
Cone	6	12	Circles in four parallel planes for information on straightness
		15	Five points on each circle for information on roundness

7.2 Techniques

The manufacturer should provide techniques for pre-processing the measured data. He should also provide sufficient information to allow the user of the coordinate measuring machine to implement his own techniques; in particular, a complete description of data storage should be given.

8 Establishing a reference

8.1 General

The manufacturer should provide software for computing the reference which gives position and size of the geometric element and which is to be used in the assessment of the workpiece.

Example. The reference for circularity should normally be the centre coordinates and radius of a computed circle. For instance, this circle may be the smallest circle enclosing the data points.

It is possible to compute the reference in any one of a number of ways. In any case the criterion by which the reference has been computed should be stated clearly and unambiguously.

The manufacturer should employ computer implementations of mathematical algorithms in determining the reference. The conditions under which the software will work correctly should be clearly stated.

Example. A particular cylindricity assessment procedure might require a statement of the following form.

"The method used to compute the reference for a cylinder assumes that the first three and the last three data points lie respectively on two approximately circular sections, and the remaining data points lie on these sections or sections between these."

NOTE 1 The knowledge that the first and the last few measured points lie at the opposite ends of the cylindrical object permits an initial estimate of the cylindrical element to be computed rapidly by the software. Such an estimate will speed convergence of some algorithms in computing the best-fit cylinder.

NOTE 2 Some problems have multiple solutions, all of which are mathematically correct. A restriction such as that above may be necessary to ensure that the physically correct solution is selected (see **11.3** and **11.4**).

8.2 Computation of the reference

The reference is defined by the parameters of the corresponding geometric element that best fits the measured points. The fit is represented by the values of its parameters, e.g. radius and centre coordinates of a circle. Many different criteria for specifying the best fit are possible. In general, the criterion should be to make some combination of the residuals as small as possible. In mathematical terms, the reference is obtained by optimizing the chosen combination of the residuals with respect to the parameters.

Examples. Examples of criteria for specifying best fit are:

least squares $\min \Sigma_i \, res_i^2$; and

minimax $\min \max_i \, |res_i|$

Here, the residual, res_i, is a measure of the departure of the ith point from the fit. The residual is conventionally defined as the distance of the point from the reference. However, when calculating a reference circle by least squares, a particularly simple algorithm can be obtained if the residual is taken to be the difference between the squared distance of the point from the circle centre and the squared radius of the circle (see example in **10.2**).

Not all criteria are of this general form, e.g. the minimum circumscribing circle (or sphere) is given by:

minimize radius r subject to
$res_i \leqslant 0, \; i = 1, 2, ..., N$

Frequently used criteria are least squares, minimax, maximum inscribed and minimum circumscribed.

In all cases, the criterion used to compute the reference and the manner in which the residuals are defined should be stated.

9 Departure from nominal form

9.1 General

Once the reference has been determined, the deviation of the measured workpiece from nominal form can be assessed. The departure from nominal form is defined as the spread of the measured data about the reference. First, the deviation, e_i, of a single measured point from the reference should be taken as the distance from the point to the reference. (Where appropriate, the distance is given a sign according to which side of the reference the data point lies.) The spread of the deviations is then computed from these e_i (see **9.2**).

Since the departure from nominal form is derived from the reference and the data points, points that have been modified or deleted in the pre-processing stage of the assessment process should be included in assessing departure. The manufacturer should provide a clear indication of which points have been included.

9.2 Measures of spread

The output from assessment software should explicitly state the definition of spread used. For a given reference, different definitions of spread will in general give rise to different values, even for the same data. Also, using the same definition can give different values if different criteria are used to determine the reference. The choice is essentially independent of the criterion used to determine the reference figure, but some definitions will be more appropriate to a particular criterion. For example, where measurement errors are significant, least squares should be suitable when computing the reference and in these circumstances the root mean square residual or some other related measure of departure should be appropriate. If the minimum zone criterion is used to obtain the reference, the range is immediately available without further computation.

Example. This example is about assessing circularity in a predefined plane. Let r denote the computed radius of the reference circle and r_i the distance of the ith data point from the centre of the reference circle; then $e_i = r_i - r$. The spread of the deviations may be taken as:

$$\max_i e_i - \min_i e_i \equiv \max_i r_i - \min_i r_i ,$$

i.e. the (positive) difference between the distance of the point furthest from the centre of the reference circle and that of the point nearest to the centre of the reference circle. Alternatively, the spread may be taken as the root mean square deviation:

$$\sqrt{\{\Sigma_i e_i^2/(N-3)\}} = \sqrt{\{\Sigma_i (r_i - r)^2/(N-3)\}}$$

where

N is the number of data points and has to be greater than three.

9.3 Statistics

If manufacturers provide statistics associated with the computed results, the assumptions under which they have been calculated should be stated.

10 Information to be provided by an assessment

10.1 General

Once the data has been processed on the computer to obtain an assessment of a geometric feature, information relating to the assessment should be provided to the user. Subclauses **10.2** and **10.3** recommend ways in which this can be done.

10.2 Statement of results

An assessment of geometric form is valid only if the nature of the assessment is explicitly stated. A single number, e.g. 0.035 mm used to represent circularity, is by itself meaningless. Conversely, if it is explicitly described and mathematically sound, any convenient assessment should be acceptable.

The definition of the residuals and the fitting criterion used in determining the reference should both be clearly stated, as should the measure used in calculating the departure from nominal form.

Example. Consider the determination of a reference by fitting a circle to a set of data points by least squares. The use of the term "least squares" by itself is ambiguous. It could, for instance, refer to solutions obtained in (at least two) different ways.

The so-called linear least-squares solution may be obtained by applying least squares to the "standard" circle equation:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Another solution is the circle obtained by applying least squares to the equation:

$$\sqrt{\{(x - x_0)^2 + (y - y_0)^2\}} - r = 0$$

Both forms of circle are valid mathematical representations. However, in general, different least-squares circles and hence different geometric assessments will be produced by their use. There are circumstances in which either of these forms is valid.

10.3 Information

10.3.1 *General.* Information made available by assessment software should be as full as possible consistent with economy of use. Even if the application is within an automated process, details of the assessment should be available for human interpretation. In general, the information provided need not be immediately visible to the user but should be readily accessible. It may be given in any of a number of forms including printed computer output, displays on VDU screens, and filed computer records.

Two "levels" of information can be identified. The first is "essential" information, which should always be provided where appropriate.

10.3.2 *Essential information.* The following information should always be provided:

a) nature of the assessment;

b) identification of the workpiece (e.g. serial number, type);

c) details of assessment (parameters of reference, departure from form, etc.).

NOTE If the software fails to complete the assessment, the details of the assessment should contain a statement to that effect together with a reason for failure, e.g. too few data points provided.

10.3.3 *Optional information*. The following information is optional and should be provided where appropriate:

a) job title;

b) place, date and time;

c) environmental details;

d) user;

e) any other information considered relevant.

Example. An example containing both essential and optional information is:

Circularity assessment

Workpiece No. 123

Edinburgh Works, 3.00 pm, 31 December 1999

20 °C, 50 % humidity

User: A. B. Clark

No pre-processing applied

10 points approximately uniformly spaced around the workpiece

Smallest circumscribing circle

Centre coordinates – 357.653 mm, 484.922 mm

Radius 24.993 mm

Out of roundness (greatest departure of data points from circle) 0.035 mm

NOTE Ideally the coordinates of the points used should also be included.

11 Numerical considerations

11.1 General

Numerical software inaccuracies in providing an assessment can arise from a number of sources. Adoption by software writers of the recommendations given in **11.2** to **11.7** should help to avoid many of these.

11.2 Data errors

The data points that are taken as representative of an object under test contain two types of error.

a) *Form error*. This error is in the object itself and is a consequence of the inability of the machining process to produce a workpiece that is "perfect", e.g. cylindrical in a mathematical sense.

b) *Measurement error*. This error is due to the inability of the coordinate measuring machine to provide exact values of the coordinates of points on the object.

The measured data may have errors which are dominated by one of the above types or it may contain errors of comparable size from both sources. The way in which the assessment is undertaken should take this into account. For example, if the measuring accuracy is much greater than the machining accuracy, the measurements can be taken as accurate information about the form of the component and it is recommended that methods based on largest inscribed, smallest circumscribed or minimum zone forms be used. However, if the opposite applies then much of the information about form will be obscured by measurement error. In this case, least-squares analysis of the data is recommended. For cases between these extremes it is recommended that a combination of the least-squares and other approaches is used. The data should initially be smoothed as part of the pre-processing stage (see clause **7**) and then a reference computed (see clause **8**).

This standard does not advocate a single approach, but urges the use of methods which are appropriate to the task in hand. In particular (see **10.2** and **10.3**), any software that carries out geometric form assessment should, in addition to the usual numerical output, provide information indicating clearly the method of analysis used.

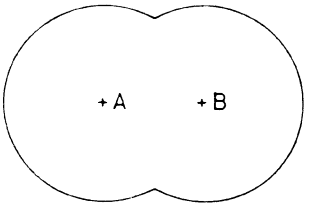


Figure 8 — Example of a profile with two equal maximum inscribed circles, centred at A and B

11.3 Nonuniqueness of reference

For a particular set of measured data and a given criterion for specifying the reference, there may not be a unique best fit reference. That is, the criterion can be satisfied by more than one set of parameter values.

Example. Data symmetrically placed on the (exaggerated) profile shown in Figure 8 gives rise to two distinct circles with (the same) maximum radius: one centred at A, the other at B.

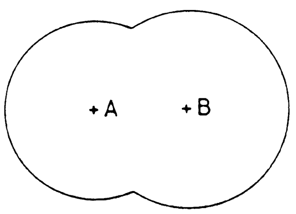
Under these circumstances, the same reference will not necessarily be calculated by different implementations, depending on, for example, the choice of starting values in an iterative computation. This is particularly important if the fit is to be used subsequently in a secondary calculation, when some means should be found to choose between the alternative solutions. An algorithm should indicate when there is more than one solution and, where applicable, the method used to select a particular solution.

11.4 Local optima

Many algorithms use iterative processes that can converge to solutions that are only locally best. That is to say that although small changes to the computed parameter values will worsen the fit, larger changes may achieve a better optimum value.

Example. An algorithm seeking the maximum inscribed circle to data on the (exaggerated) profile shown in Figure 9, and starting from an initial estimate near A for the centre would be likely to converge to a circle centred at A, whereas a larger circle, centred at B, can be inscribed.

It is recommended that manufacturers' software indicate possible difficulties of this type.



NOTE The global maximum is centred at B.

Figure 9 — Example of a profile with a locally maximum inscribed circle, centred at A

11.5 Data transformations

Software for computing a reference should, where appropriate, first carry out simple transformations of the data such as shifting, scaling and rotation. Such operations serve the purpose of making the magnitudes of the numerical values involved of more manageable size. As a result the risk of computer underflow and overflow is reduced, and loss of precision in the computations due to unnecessary correlations and common leading digits is avoided. After the reference has been computed account should be taken of the transformations carried out in order to refer the results to a coordinate system defined in the workpiece (see **6.2**).

NOTE One of the most valuable transformations is a simple shift or translation involving the centroid of the measured data. If the measured coordinates are (x_i, y_i, z_i), $i = 1, 2, ..., N$, then $x_i - \bar{x}$, $y_i - \bar{y}$ and $z_i - \bar{z}$, where $\bar{x} = \Sigma_i x_i/N$, $\bar{y} = \Sigma_i y_i/N$ and $\bar{z} = \Sigma_i z_i/N$, should be used in place of x_i, y_i and z_i, respectively. Once the reference has been computed, the relevant reference parameters are adjusted accordingly. For example, in the case of a sphere, the computed centre (x_0, y_0, z_0) would be replaced by $(x_0 + \bar{x}, y_0 + \bar{y}, z_0 + \bar{z})$.

11.6 Self-validation

In the case of any particular assessment, most of the work is involved in computing the reference. In some cases it should be possible for the software itself to determine whether it has functioned correctly on this stage of the computation. This should be carried out by testing whether the computed reference satisfies appropriate properties.

Example. For the smallest circumscribing circle let S denote the measured set of data points and C the circumscribing circle for S. C is unique and satisfies the following conditions:

a) all points in S lie in or on C; and

b) either

i) there are three points of S which lie on C and which form an acute-angled triangle; or

ii) there are two points of S which lie on C such that the line joining them is a diameter of C.

A circle satisfying these conditions is the circumscribing circle for S. Conversely, a circle not satisfying these conditions is not the circumscribing circle for S.

NOTE Account should be taken of computer rounding errors in validating a computed reference.

11.7 Diagnostic information

The manufacturer should ensure that an algorithm, if necessary, reports that it is unable to carry out the computation required rather than produce an incorrect result or fail in an unexpected way.

Additionally, the algorithm should inform the operator that a computed solution is not unique in cases where this circumstance arises.

Appendix A Formulae for distance of a point to a geometric element

A.1 Introduction

In computing a reference or in assessing departure from form it is necessary to make use of appropriate formulae for the distance of a point from a geometric element. The following formulae, given in terms of the recommended parametrizations of geometric elements, are recommended for this purpose.

The point is denoted by (x_p, y_p) in two dimensions and (x_p, y_p, z_p) in three dimensions, and the distance of the point from the geometric element is denoted by d.

A.2 Line specified by one point and the direction cosines (see 5.2.1.2)

The distance d of a point from a line specified as in **5.2.1.2** by one point and the direction cosines can be expressed by the following formulae:

a) for a point lying in the plane containing L:

$$d = b(x_p - x_0) - a(y_p - y_0) \quad (1)$$

b) for a point not lying in the plane containing L:

$$d = \sqrt{[\{b(x_p - x_0) - a(y_p - y_0)\}^2 + z_p^2]} \quad (2)$$

A.3 Line specified by two points on the line (see 5.2.1.3)

The distance d of a point from a line specified as in **5.2.1.3** by two points on the line can be expressed by the following formulae:

i) for a point lying in the plane containing L:

$$d = \frac{(y_2 - y_1)(x_p - x_1) - (x_2 - x_1)(y_p - y_1)}{\sqrt{\{(x_2 - x_1)^2 + (y_2 - y_1)^2\}}} \quad (3)$$

or, equivalently, formula 3 with $x_p - x_1$ replaced by $x_p - x_2$ and $y_p - y_1$ by $y_p - y_2$;

ii) for a point not lying in the plane containing L:

$$d = \sqrt{\left[\frac{\{(y_2 - y_1)(x_p - x_1) - (x_2 - x_1)(y_p - y_1)\}^2}{(x_2 - x_1)^2 + (y_2 - y_1)^2} + z_p^2\right]} \quad (4)$$

or, equivalently, formula 4 with $x_p - x_1$ replaced by $x_p - x_2$ and $y_p - y_1$ by $y_p - y_2$.

A.4 Line in three dimensions specified by one point and the direction cosines (see 5.2.2.2)

The distance d of a point from a line in three dimensions specified as in **5.2.2.2** by one point and the direction cosines can be expressed by the following formula:

$$d = \sqrt{(u^2 + v^2 + w^2)} \quad (5)$$

where

$$u = c(y_p - y_0) - b(z_p - z_0) \quad (6)$$
$$v = a(z_p - z_0) - c(x_p - x_0) \quad (7)$$
$$w = b(x_p - x_0) - a(y_p - y_0) \quad (8)$$

A.5 Line in three dimensions specified by two points on the line (see 5.2.2.3)

The distance d of a point from a line in three dimensions specified as in **5.2.2.3** by two points on the line can be expressed by the following formula:

$$d = \frac{\sqrt{(u^2 + v^2 + w^2)}}{\sqrt{\{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\}}} \quad (9)$$

where

$$u = (z_2 - z_1)(y_p - y_1) - (y_2 - y_1)(z_p - z_1) \quad (10)$$
$$v = (x_2 - x_1)(z_p - z_1) - (z_2 - z_1)(x_p - x_1) \quad (11)$$
$$w = (y_2 - y_1)(x_p - x_1) - (x_2 - x_1)(y_p - y_1) \quad (12)$$

or, equivalently, formulae 10, 11 and 12 with $x_p - x_1$ replaced by $x_p - x_2$, $y_p - y_1$ by $y_p - y_2$ and $z_p - z_1$ by $z_p - z_2$.

A.6 Plane specified by a point on the plane and the direction cosines of the normal (see **5.2.3.2**)

The distance d of a point from a plane specified as in **5.2.3.2** by a point on the plane and the direction cosines of the normal to the plane can be expressed by the following formula:

$$d = a(x_p - x_0) + b(y_p - y_0) + c(z_p - z_0). \qquad (13)$$

A.7 Plane specified by a point on the plane and a point on the normal (see **5.2.3.3**)

The distance d of a point from a plane specified as in **5.2.3.3** by a point on the plane and a point on the normal to the plane can be expressed by the following formula:

$$d = \frac{(x_1 - x_0)(x_p - x_0) + (y_1 - y_0)(y_p - y_0) + (z_1 - z_0)(z_p - z_0)}{\sqrt{\{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2\}}} \qquad (14)$$

A.8 Circle specified by its centre and radius (see **5.2.4.1**)

The distance d of a point from a circle specified by its centre and radius can be expressed by the following formulae:

i) for a point lying in the plane containing C:

$$d = \sqrt{\{(x_p - x_0)^2 + (y_p - y_0)^2\}} - r \qquad (15)$$

ii) for a point not lying in the plane containing C:

$$d = \sqrt{\{z_p{}^2 + (f - r)^2\}} \qquad (16)$$

where

$$f = \sqrt{\{(x_p - x_0)^2 + (y_p - y_0)^2\}} \qquad (17)$$

A.9 Circle in three dimensions specified by its centre and radius and the direction cosines of the normal or a point on the normal (see **5.2.5.2**)

The distance d of a point from a circle in three dimensions specified as in **5.2.5.2** by its centre and radius and the direction cosines of the normal to the plane containing the circle or a point on the normal can be expressed by the following formulae:

$$d = \sqrt{\{g^2 + (f - r)^2\}} \qquad (18)$$

where

a) for a circle specified by its centre and radius and the direction cosines:

$$g = a(x_p - x_0) + b(y_p - y_0) + c(z_p - z_0) \qquad (19)$$

$$f = \sqrt{(u^2 + v^2 + w^2)} \qquad (20)$$

$$u = c(y_p - y_0) - b(z_p - z_0) \qquad (21)$$

$$v = a(z_p - z_0) - c(x_p - x_0) \qquad (22)$$

$$w = b(x_p - x_0) - a(y_p - y_0) \qquad (23)$$

b) for a circle specified by its centre and radius and a point on the normal to the plane containing C:

$$g = \frac{(x_1 - x_0)(x_p - x_0) + (y_1 - y_0)(y_p - y_0) + (z_1 - z_0)(z_p - z_0)}{\sqrt{\{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2\}}} \qquad (24)$$

$$f = \frac{\sqrt{(u^2 + v^2 + w^2)}}{\sqrt{\{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2\}}} \qquad (25)$$

$$u = (z_1 - z_0)(y_p - y_0) - (y_1 - y_0)(z_p - z_0) \qquad (26)$$

$$v = (x_1 - x_0)(z_p - z_0) - (z_1 - z_0)(x_p - x_0) \qquad (27)$$

$$w = (y_1 - y_0)(x_p - x_0) - (x_1 - x_0)(y_p - y_0) \qquad (28)$$

A.10 Sphere specified by its centre and radius (see **5.2.6**)

The distance d of a point from a sphere specified as in **5.2.6** by its centre and radius can be expressed by the following formula:

$$d = \sqrt{\{(x_p - x_0)^2 + (y_p - y_0)^2 + (z_p - z_0)^2\}} - r \quad (29)$$

A.11 Cylinder specified by its axis and radius (see **5.2.7**)

The distance d of a point from a cylinder specified as in **5.2.7** by its axis and radius can be expressed by the following formulae:

a) for a cylinder axis specified as in **5.2.2.2**:

$$d = \sqrt{(u^2 + v^2 + w^2)} - r \quad (30)$$

where

$$u = c(y_p - y_0) - b(z_p - z_0) \quad (31)$$

$$v = a(z_p - z_0) - c(x_p - x_0) \quad (32)$$

$$w = b(x_p - x_0) - a(y_p - y_0) \quad (33)$$

b) for a cylinder axis specified as in **5.2.2.3**:

$$d = \frac{\sqrt{(u^2 + v^2 + w^2)}}{\sqrt{\{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\}}} - r \quad (34)$$

where

$$u = (z_2 - z_1)(y_p - y_1) - (y_2 - y_1)(z_p - z_1) \quad (35)$$

$$v = (x_2 - x_1)(z_p - z_1) - (z_2 - z_1)(x_p - x_1) \quad (36)$$

$$w = (y_2 - y_1)(x_p - x_1) - (x_2 - x_1)(y_p - y_1) \quad (37)$$

or, equivalently, formulae 35, 36 and 37 with $x_p - x_1$ replaced by $x_p - x_2$, $y_p - y_1$ by $y_p - y_2$ and $z_p - z_1$ by $z_p - z_2$.

A.12 Cone specified by its angle, axis and the position of the cone on its axis (see **5.2.8**)

The distance d of a point from a cone specified as in **5.2.8** by its angle, axis and the position of the cone on its axis can be expressed by the following formula:

$$d = f \cos\frac{\psi}{2} + g \sin\frac{\psi}{2} - s \quad (38)$$

where

a) for a cone axis specified as in **5.2.2.2**;

$$g = a(x_p - x_0) + b(y_p - y_0) + c(z_p - z_0) \quad (39)$$

$$f = \sqrt{(u^2 + v^2 + w^2)} \quad (40)$$

$$u = c(y_p - y_0) - b(z_p - z_0) \quad (41)$$

$$v = a(z_p - z_0) - c(x_p - x_0) \quad (42)$$

$$w = b(x_p - x_0) - a(y_p - y_0) \quad (43)$$

b) for cone axis specified as in **5.2.2.3**;

$$g = \frac{(x_2 - x_1)(x_p - x_0) + (y_2 - y_1)(y_p - y_0) + (z_2 - z_1)(z_p - z_0)}{\sqrt{\{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\}}} \quad (44)$$

$$f = \frac{\sqrt{(u^2 + v^2 + w^2)}}{\sqrt{\{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\}}} \quad (45)$$

$$u = (z_2 - z_1)(y_p - y_1) - (y_2 - y_1)(z_p - z_1) \quad (46)$$

$$v = (x_2 - x_1)(z_p - z_1) - (z_2 - z_1)(x_p - x_1) \quad (47)$$

$$w = (y_2 - y_1)\,(x_p - x_1) - (x_2 - x_1)\,(y_p - y_1) \qquad (48)$$

or, equivalently, formulae 46, 47 and 48 with $x_p - x_1$ replaced by $x_p - x_2$, $y_p - y_1$ by $y_p - y_2$ and $z_p - z_1$ by $z_p - z_2$. In the formulae for g, it is assumed that the vectors (a, b, c) and $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$ point along the cone axis in the direction of decreasing radius.

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