Method for

Calculation of small colour differences

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Committees responsible for this British Standard

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British Ceramic Research Ltd.
British Ceramic Tile Council
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Institute of Vitreous Enamellers
International Wool Secretariat
Lighting Industry Federation Ltd.
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Paintmakers' Association of Great Britain Ltd.
Royal Institute of British Architects
Society of British Printing Ink Manufacturers
Society of Dyers and Colourists
Society of Leather Technologists and Chemists
Textile Research Council

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Contents

Foreword

This British Standard has been prepared under the direction of the Multitechnics Council.

In 1976 Commission Internationale d'Eclairage (CIE) published a colour difference formula known as CIELAB (currently given in CIE Publication No. 15.2) which has since become widely used internationally for quantifying the perceived colour difference between surface colours.

It has been established as section J01 of BS 1006 for textiles and as BS 3900-D10 for paints and varnishes. For industry and commerce, however, this formula has a major weakness. Over 50 000 assessments made by 44 professional shade passers in the textile and paint industries against reference specimens of 262 different colours were analysed. It was found that if the decisions had been made using the optimum CIELAB ΔE^* value of the pass/fail boundary for each of the four data sets the number of wrong decisions would have been significantly greater than the average number of wrong decisions made by the assessors.

This weakness has been overcome by dividing CIELAB colour space into an infinite number of ellipsoidal microspaces defined by semi-axes oriented in the directions of lightness, chroma and hue scaled so as to correlate with visual assessment. The relative importance of lightness and chroma differences compared with differences in hue varies in different industries, and to allow for this, relative tolerances I and c are included in the formula. When $I = c = 1$ the formula quantifies the perceptibility (magnitude) of the colour difference. Different values of I and/or c allow for variations in the relative importance of lightness and chroma differences, and when these are used the formula quantifies the acceptability of the colour difference. This formula was devised by the Colour Measurement Committee of the Society of Dyers and Colourists and is known as CMC ($I : c$). The ideas and reasoning behind the modifications made to the JPC79 colour difference formula in developing it into the CMC equation are given by F.J.J. Clarke, R. McDonald and B. Rigg in "Modification to the JPC79 colour-difference formula", *Journal of the Society of Dyers and Colourists*, 1984, **100**, 128–132 and 281–282. Applying CMC (2 : 1) to the four data sets already mentioned gives fewer wrong decisions than would be made by the average observer, and in the only data set giving individual assessments (8 observers), the number of wrong decisions was no greater than that made by the most reliable assessor.

Currently, optimum values of I and c have only been determined for the textile industry. However, work is proceeding on determining optimum values for I and c for other industries, e.g. ceramics, leather, paper, paint and plastics, which will lead to revisions of this standard.

Some test data are given in Appendix A and an example of a simple computer program is given in Appendix B.

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Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 6, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

1 Scope

This British Standard provides a method of calculating the colour difference between two specimens of the same material, measured under the same conditions, such that the numerical value D*E*, the total colour difference, quantifies the perceptibility of the colour difference or the acceptability of the colour match. The latter permits the specification of a maximum value which depends only on the closeness of match required for a given end-use and not on the colour involved nor on the nature of the colour difference.

NOTE 1 Appendix A gives test data for use in checking computer programs, and Appendix B contains a computer program for determining colour difference.

NOTE 2 The titles of the publications referred to in this standard are listed on the inside back cover.

2 Principle

The magnitude of the colour difference between two specimens is calculated by means of a modified version of the internationally recognized CIELAB colour difference formula.

3 Calculation

3.1 CIELAB values

Calculate the CIELAB L^*, C^*_{ab} and h_{ab} values from the X, Y, Z tristimulus values of each specimen as follows:

$L^* = 116(Y/Y_n)^{1/3} - 16$ if $Y/Y_n > 0.008856$

but

$L^* = 903.3(Y/Y_n)$ if $Y/Y_n \leqslant 0.008856$

$a^* = 500\,[f(X/X_n) - f(Y/Y_n)]$

$b^* = 200[f(Y/Y_n) - f(Z/Z_n)]$

where

$f(X/X_n) = (X/X_n)^{1/3}$ if $X/X_n > 0.008856$

but

$f(X/X_n) = 7.787\,(X/X_n) + 16/116$ if $X/X_n \leqslant 0.008856$;

$f(Y/Y_n) = (Y/Y_n)^{1/3}$ if $Y/Y_n > 0.008856$

but

$f(Y/Y_n) = 7.787(Y/Y_n) + 16/116$ if $Y/Y_n \leqslant 0.008856$;

$f(Z/Z_n) = (Z/Z_n)^{1/3}$ if $Z/Z_n > 0.008856$

but

$f(Z/Z_n) = 7.787\,(Z/Z_n) + 16/116$ if $Z/Z_n \leqslant 0.008856$.

For these equations X_n, Y_n and Z_n are the tristimulus values of the illuminant, with those for CIE standard illuminant D_{65} and the 10° observer, the preferred combination, being as given in Table 1, which also states the values for other daylight combinations.

Table 1 — Tristimulus values for four daylight combinations

Combination	Tristimulus value		
	X_n	Y_n	Z_n
D 65/10°	94.811	100.000	107.304
D 65/2°	95.047	100.000	108.883
C/10°	97.285	100.000	116.145
C/2°	98.074	100.000	118.232

$C^* = (a^{*2} + b^{*2})^{0.5}$

$h_{ab} = \arctan b^*/a^*$ expressed on a 0 to 360° scale with the a^* + axis being at 0 and the b^* + axis at 90°.

Calculate the CIELAB colour differences as follows, where subscripts T and R refer to the test specimen and reference specimen respectively:

$$\Delta L^* = L^*_T - L^*_R$$
$$\Delta a^* = a^*_T - a^*_R$$
$$\Delta b^* = b^*_T - b^*_R$$
$$\Delta C^*_{ab} = C^*_{ab,T} - C^*_{ab,R}$$
$$\Delta E^*_{ab} = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{0.5}$$
$$\Delta H^*_{ab} = [(\Delta E^*_{ab})^2 - (\Delta L^*)^2 - (\Delta C^*_{ab})^2]^{0.5}$$

3.2 CMC (*l* : *c*) values

Calculate the ellipsoid semi-axes from the L^*, C^*_{ab} and h_{ab} values of the reference specimen as follows:

$$S_L = \frac{0.040975\,L^*}{1 + 0.01765\,L^*}$$

unless $L^* < 16$, when $S_L = 0.511$

$$S_C = \frac{0.0638\,C^*_{ab}}{1 + 0.0131\,C^*_{ab}} + 0.638$$

$$S_H = (FT + 1 - F)\,S_C$$

where

$$F = \left(\frac{(C^*_{ab})^4}{(C^*_{ab})^4 + 1900}\right)^{0.5}$$

and

$T = 0.36 + \text{abs}\,[0.4\cos(35 + h_{ab})]$

unless h is between 164° and 345° when

$T = 0.56 + \text{abs}\,[0.2\cos(168 + h_{ab})]$

and for the last two equations "abs" indicates the absolute, i.e. positive value, of the term inside the square brackets.

When there is no reference specimen, for example, in shade sorting, calculate the semi-axes from the L^*, C^*_{ab} and h_{ab} values corresponding to the arithmetic mean values of L^*, a^* and b^* of all the specimens involved.

Calculate the colour differences in CMC ($I:c$) units using the following equation:

$$DE = \left[\left(\frac{\Delta L^*}{lS_L}\right)^2 + \left(\frac{\Delta C^*_{ab}}{cS_C}\right)^2 + \left(\frac{\Delta H^*_{ab}}{S_H}\right)^2\right]^{0.5}$$

NOTE When $I = c = 1$ the formula quantifies the perceptibility of colour differences. Optimum values of I and c for quantifying the acceptability of a colour match have so far only been determined for the textile industry, and these values are $I = 2$ and $c = 1$.

4 Interpretation of results

4.1 General

The colour difference between two specimens can be partitioned into components of lightness, chroma and hue, which are discussed in **4.2** to **4.4**.

4.2 Lightness component

The lightness component is calculated for use in the equation for DE (see **3.2**) as $\Delta L^*/IS_L$, and is the difference in lightness between the test specimen and the reference specimen; it is designated DL. The nature of any lightness difference can be derived from the following equation (see **3.1**):

$$\Delta L^* = L^*_T - L^*_R$$

If ΔL^* is positive, the test specimen is lighter than the reference specimen.

If ΔL^* is negative, the test specimen is darker than the reference specimen.

If ΔL^* is zero, there is no difference in lightness.

4.3 Chroma component (see Figure 1)

The chroma component is calculated for use in the equation for DE (see **3.2**) as $\Delta C^*_{ab}/cS_c$ and is the difference in chroma between the test specimen and the reference specimen; it is designated DC. The nature of any chroma difference can be derived from the following equation (see **3.1**):

$$\Delta C^*_{ab} = C^*_{ab,T} - C^*_{ab,R}$$

If ΔC^*_{ab} is positive, the test specimen is of more chroma than the reference specimen.

If ΔC^*_{ab} is negative, the test specimen is of less chroma than the reference specimen.

If ΔC^*_{ab} is zero, i.e. the test specimen lies on the isochroma line passing through the position of the reference specimen, there is no difference in chromatic strength.

4.4 Hue component

The hue component is calculated for use in the equation for DE (see **3.2**) as $\Delta H^*_{ab}/S_H$ and is the difference in hue between the test specimen and the reference specimen; it is designated DH. The nature of any hue difference can be derived in the following way.

Rotate the chroma axis passing through the a*b* coordinates of the reference specimen so that it passes through the a*b* coordinates of the test specimen, minimizing angular displacement. The hue associated with the first major axis to be crossed during rotation, continuing if necessary, will describe the nature of the hue difference unless the reference specimen is of the same hue: to describe a test specimen as being redder than a reference specimen which is red, for example, is meaningless. To overcome this limitation the rotation of the chroma axis is continued until the next major axis is crossed. The hue associated with this axis is also quoted, expressed in brackets. One of the two hue difference terms will be valid, the other meaningless.

If desired the value of DH may be designated + if the chroma axis rotation is anticlockwise and – if clockwise.

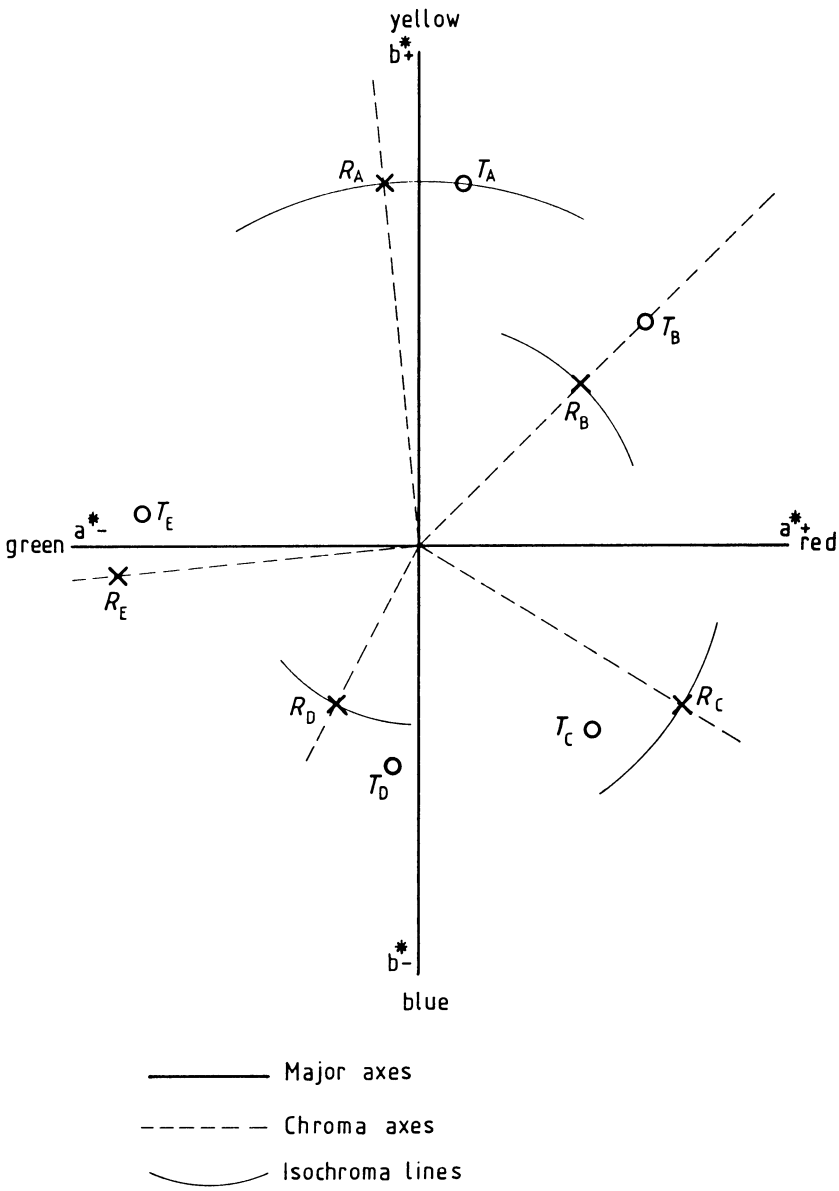
Examples of the derivation of chroma and hue difference descriptors are given in Figure 1.

4.5 Colour difference of white, grey and black (achromatic) specimens

Although the total colour difference in CMC ($I:c$) units is valid for achromatic specimens the method of partitioning this difference is valid for lightness differences only when $C^*_{ab} \leqslant 4.0$, since the chroma and hue difference components often do not correspond with visual assessments.

5 Test report

The total colour difference (see **3.2**) and the component colour differences (see clause **4**) shall be described as being in CMC ($I:c$) units and the values of I and c shall be stated. If any combination of illuminant and colour matching functions other than D 65/10° have been used in the calculations it shall be stated. In the case of achromatic specimens (see **4.5**), which are defined as having $C^*_{ab} \leqslant 4.0$, only the total colour difference DE and the lightness difference shall be reported.



NOTE 1 The following information can be deduced from the figure.

a) Compared with reference specimen R_A, test specimen T_A is equal in chroma and yellower (redder).

b) Compared with reference specimen R_B, test specimen T_B is of more chroma and equal in hue.

c) Compared with reference specimen R_C, test specimen T_C is of less chroma and bluer (greener).

d) Compared with reference specimen R_D, test specimen T_D is of more chroma and bluer (redder).

e) Compared with reference specimen R_E, test specimen T_E is of less chroma and greener (yellower).

NOTE 2 See also K. McLaren and P.F. Taylor "The derivation of hue-difference terms from CIELAB coordinates", *Color Research and Application*, 1981, **6**, 76.

Figure 1 — Derivation of chroma and hue difference descriptors

Appendix A Test data

To help check computer programs giving D*E* values from the CMC formula some test data are given in Table 2. The data are for illuminant D 65 and the 10° observer, using X_n = 94.811, Y_n = 100.000 and Z_n = 107.304. An attempt has been made to include differences covering the various possible paths in the program.

Table 2 — Test data for the CMC (1 : 1) formula

Pair	Tristimulus values			Lightness component, $\Delta L^*/lS_L$	Chroma component, $\Delta C_{ab}/cS_c$	Hue component, $\Delta H_{ab}/S_H$	Total colour difference, D*E* [CMC (1 : 1)]
	X	*Y*	*Z*				
1	19.41	28.41	11.5766				
	19.5525	28.64	10.5791	0.1715	0.9638	– 1.04	1.4282
2	22.48	31.6	38.48				
	22.5833	31.37	36.7901	– 0.1572	– 0.7608	– 0.9855	1.2549
3	28.995	29.58	35.75				
	28.7704	29.74	35.6045	0.1153	– 1.0770	– 1.3981	1.7686
4	4.14	8.54	8.03				
	4.4129	8.51	8.6453	– 0.0675	– 1.7214	1.0659	2.0258
5	4.96	3.72	19.59	0.4662			
	4.6651	3.81	17.7848		– 2.2106	– 2.1041	3.0872
6	15.60	9.25	5.02				
	15.9148	9.15	4.3872	– 0.2087	1.3552	1.0856	1.7489
7	73.0	78.05	81.80				
	73.9351	78.82	84.5156	0.2449	– 1.1188	1.5171	1.9009
8	73.995	78.32	85.306				
	69.1762	73.4	79.713	– 1.5994	0.1221	– 0.5709	1.7026
9	0.704	0.75	0.972				
	0.613873	0.65	0.851025	– 1.7677	– 0.2679	0.2344	1.8032
10	0.22	0.23	0.325				
	0.093262	0.1	0.145292	– 2.2980	– 0.8213	– 0.2092	2.4493

NOTE The test data use values of $l = c = 1$.

Appendix B Computer program

An example of a simple computer program, written in BASIC, that has been used to calculate colour differences is given in Figure 2.

```
10 'CMC(L:C) COLOUR DIFFERENCE FORMULA
20 '##########################################
30 'Input data and print results
40 '##########################################
50 INPUT "Input CMC(l:c) weighting factors 'l','c' ";L,C
60 INPUT  "Input X,Y,Z of standard";X(1),X(2),X(3):LPRINT "X,Y,Z of standard ";X
(1);X(2);X(3):GOSUB 160:L1=CL:A1=CA:B1=CB
70 INPUT "Input X,Y,Z of sample";X(1),X(2),X(3):LPRINT "X,Y,Z of sample   ";X(1)
;X(2);X(3):GOSUB 160:L2=CL:A2=CA:B2=CB
80 GOSUB 230
90 LPRINT "L,a,b,Hue angle of standard ";L1;A1;B1;H1
100 LPRINT "L,a,b,Hue angle of sample   ";L2;A2;B2;H2
110 LPRINT "DL/lSl     DC/cSc     DH/Sh     DE CMC(";L;":"C")"
120 LPRINT DL;DC;DH;DE:LPRINT:GOTO 60
130 '##########################################
140 'Calculate L,a,b values (Ill.D65, 10 degree observer)
150 '##########################################
160 X(1)=X(1)/94.811:X(2)=X(2)/100:X(3)=X(3)/107.304
170 FOR I=1 TO 3:IF X(I)<8.856001E-03 THEN FX(I)=7.787*X(I) ELSE FX(I)=X(I)^(1/3
)-16/116
180 NEXT
190 CL=116*FX(2):CA=500*(FX(1)-FX(2)):CB=200*(FX(2)-FX(3)):RETURN
200 '##########################################
210 'Calculate CMC colour difference
220 '##########################################
230 DL=L2-L1:C1=SQR(B1*B1+A1*A1):C2=SQR(B2*B2+A2*A2):DC=C2-C1
240 S1=DL*DL+(A2-A1)*(A2-A1)+(B2-B1)*(B2-B1)
250 DH=0:AA=S1-DL*DL-DC*DC:IF AA<=0 THEN 260 ELSE DH=SQR(AA)
260 IF (A2*B2)=0 THEN 280 ELSE H2=180-SGN(B2)*90-ATN(A2/B2)*57.3
270 GOTO 300
280 BB2=SGN(ABS(B2)):AA2=SGN(A2+B2)
290 H2=90*(BB2-AA2+1)
300 IF (A1*B1)=0 THEN 320 ELSE H1=180-SGN(B1)*90-ATN(A1/B1)*57.3
310 GOTO 340
320 BB1=SGN(ABS(B1)):AA1=SGN(A1+B1)
330 H1=90*(BB1-AA1+1)
340 IF H1<=164 OR H1>=345 THEN 350 ELSE GOTO 360
350 T=.36+ABS(.4*COS((H1+35)/57.3)):GOTO 370
360 T=.56+ABS(.2*COS((H1+168)/57.3))
370 SL=.040975*L1/(1+.01765*L1):IF L1<16 THEN LET SL=.511
380 SC=.0638*C1/(1+.0131*C1)+.638:F=SQR(C1^4/(C1^4+1900)):SH=SC*(T*F+1-F):DL=DL/
(L*SL):DC=DC/(C*SC):DH=DH/SH:DA=H2-H1:IF DA<0 THEN DH=-DH
390 DE=SQR(DL*DL+DC*DC+DH*DH)
400 RETURN
```

Figure 2 — Computer program for colour differences

blank

Publications referred to

BS 1006, *Methods of test for colour fastness of textiles and leather*[1].

BS 3900, *Methods of test for paints*[1].

BS 3900-D10, *Determination of colour and colour difference: calculation*.

CIE Publication No. 15.2 Colorimetry, 2nd edition, 1986[2].

F.J.J. Clarke, R. McDonald and B. Rigg, "Modification to the JPC79 colour difference formula", *Journal of the Society of Dyers and Colourists*, 1984, **100**, 128–132 and 281–282.

K. McLaren and P.F. Taylor, "The derivation of hue-difference terms from CIELAB coordinates", *Color Research and Application*, 1981, **6**, 76.

[1] Referred to in the foreword only.
[2] Available from the Hon. Librarian (National Illumination Committee), c/o Thorn Lighting Limited, Great Cambridge Road, Enfield, Middlesex EN1 1UL.

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