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BS 5701-2:2003

Guide to quality control and performance improvement using qualitative (attribute) data —

Part 2: Fundamentals of standard attribute charting for monitoring, control and improvement

ICS 03.120.30

Committees responsible for this British Standard

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Association for Road Traffic Safety and Management (ARTSM)
BAE Systems
British Standards Society (BSS)
Clay Pipe Development Association (CPDA)
Federation of Small Businesses (FSB)
Gauge and Tool Makers Association (GTMA)
General Domestic Appliances Ltd.
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Contents

Foreword

BS 5701-1 demonstrates the business benefits, and the versatility and usefulness, of a very simple, yet powerful, pictorial control chart method for monitoring and interpreting qualitative data. This is done in a practical and largely non-statistical manner. BS 5701-2:2003 partially supersedes BS 5701:1980 and BS 2564:1955 and all four parts of BS 5701 together supersede BS 5701:1980 and BS 2564:1955, which are withdrawn.

This qualitative data can range from overall business figures such as percentage profit to detailed operational data, such as percentage absenteeism, individual process parameters and product/service features. The data can either be expressed sequentially in yes/no, good/bad, present/absent, success/failure format, or as summary measures (e.g. counts of events and proportions). For measured data control charting refer to BS 5702-1.

BS 5701-2 continues to focus on the application of standard control charts to the monitoring, control and improvement of business processes. However, it deals with the application of standard attribute charting at a technical level more suitable for practitioners. Its aim is still to be readily comprehensible to an extensive range of prospective users and so facilitate widespread communication, and understanding, of the method. As such, it focuses on a practical statistical treatment of the charting of qualitative data presenting examples of construction and application using a mainly simple pictorial approach.

BS 5701-3 provides a more rigorous, statistical approach to process control and improvement using qualitative data. BS 5701-4 deals with measuring and improving the quality of decision making in the classification process itself.

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Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 27 and a back cover.

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1 Scope

BS 5701-2 describes the fundamentals necessary for the successful application, by practitioners, of standard attribute charting for monitoring, control and improvement of business processes.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

BS EN ISO 9000:2000, Quality management systems — Fundamentals and vocabulary.

BS ISO 3534-1, Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.

BS ISO 3534-2, Statistics — Vocabulary and symbols — Part 2: Applied statistics.

3 Terms, definitions and symbols

For the purposes of this part of BS 5701, the terms, definitions and symbols given in BS ISO 3534-1, BS ISO 3534-2 and BS EN ISO 9000:2000, Clause **3** apply.

4 The four types of attribute control charts

Qualitative data are divided for convenience, into two categories, *classified data* and *count data*. In conventional statistical process control, classified and countable data are handled using a:

1) "*p*" chart for proportion of entities (e.g. units or items), having a particular attribute (e.g. non-conforming), particularly when the sample size is variable;

2) "*np*" chart for numbers of entities (e.g. units or items), having a particular attribute (e.g. non-conforming), from samples of constant size;

3) "*u*" chart for number of events (e.g. non-conformities) per sample, particularly when the sample size is variable;

4) "*c*" charts for number of events (e.g. non-conformities) for samples of constant size.

Figure 1 illustrates the appropriate standard chart to choose to monitor different types of qualitative data.

NOTE 1 Refer to BS 5703 for cumulative sum (CUSUM) methods for handling both discrete and continuous data.

Whilst the four standard discrete data charts of Figure 1 have peculiar names or labels (i.e. "*c*", "*u*" etc.), these labels are in standard use throughout the world. The charts are extremely easy to set up and apply and provide a pictorial window of the behaviour of the chosen characteristic readily understandable to all.

NOTE 2 See Annex A.

5 The construction and use of standard control charts for attribute data

5.1 The "*c*" chart

5.1.1 *General*

The "*c*" chart is one of the simplest control charts to construct and use. Samples of a constant size are taken from a process and the number of non-conformities present within each sample counted. These observations (known as "*c*" numbers) can be recorded and plotted onto a control chart and so provide a way of monitoring the process for statistical control. A procedure for setting up such a control chart is given in **5.1.2** to **5.1.11**.

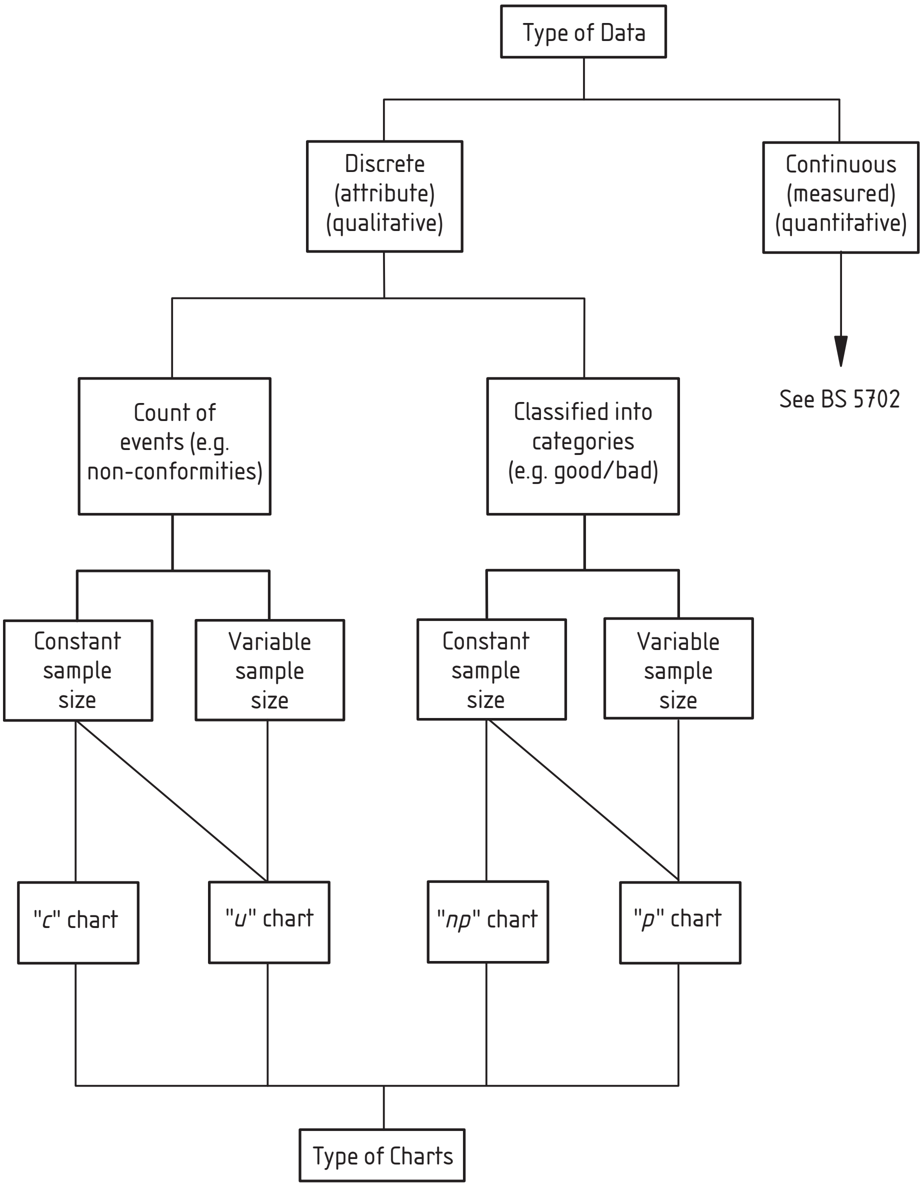


Figure 1 — Standard charts to monitor different types of qualitative data

5.1.2 ***Step 1 – Select the sample size and frequency***

Select a sample size that is large enough so that the expected number of non-conformities (c) observed per sample is more than four.

A useful source of information to assist this selection is any recent history of the process. It can be in the form of inspection records. From such or similar information it can be possible to establish a level of occurrence of the non-conformity, e.g. the total number of non-conformities recorded divided by the total number of items inspected. From this value, it would be possible to estimate a sample size that would have an expected number of non-conformities greater than four.

For example, if past records indicate that of the most recent inspections 2 545 items had been taken and 140 non-conformities had been observed amongst them. The level of occurrence of the non-conformity can be estimated as 0.055, i.e. 140/2 545. Using this estimate, if a sample of 75 were selected, the expected number of non-conformities per sample would be 4.125, i.e. 0.055 × 75. The minimum sample size required is 4/0.005 5 = 72.7, rounded up to 73. After running the chart for a while, a review should be undertaken to check the appropriateness of the selected sample size and adjust it as necessary.

If there is no information, it is suggested that an audit is carried out on the process. During this audit, a random sample is taken from the output of the process and inspected for the non-conformity. The result is used as described above.

The selection of the frequency of sampling should be determined according to those events within the process that are likely to have an impact on the occurrence of the non-conformity, e.g. a change of operator or a shift change. This is sometimes known as the assessment of process dominance. The sampling scheme should be selected so that the full effect of such process events will be observed. The sampling frequency also needs to take account of any production rate. It would be nonsense to specify a sample size so large that the production rate would not produce that quantity in the time allowed.

A further consideration about the sampling frequency is how quickly information about the process will be collected. Clearly, a shorter time interval with a smaller sample size will provide more rapid information but will not be as powerful at detecting lesser special causes. A larger sample taken over a longer time interval is more powerful but is not so rapid.

Figure 2 illustrates where this information can be recorded on a control chart. See "Step 1" in the figure.

5.1.3 ***Step 2 – Record the sample information***

In the appropriate rows and columns, record the information concerning each sample. The chart requires the date, sample size (n) and the number of non-conformities (c). In the case of a "c" chart, the sample size is a constant and so it can be entered as shown in Figure 2.

All subsidiary information relevant to the process should also be collected. For example, if it is thought important to record information such as a batch number or which shift was working at a particular time then other rows can be added to the chart.

5.1.4 ***Step 3 – Select a scale for the control chart***

The vertical scale of the chart should be labelled according to the non-conformity that the samples are inspected for. The scaling should be selected to begin at zero and extent to about three times the expected average number of non-conformities $(\bar{c})$.

This scale should be enough to enable control lines to be drawn and most sample observations to be plotted.

5.1.5 ***Step 4 – Plot the number of non-conformities***

Plot the number of non-conformities (c) for each sample.

Ensure that every plotted point is joined to its neighbours by straight lines. This is so that trends can be more easily identified and should any points be missing, they will be more obvious.

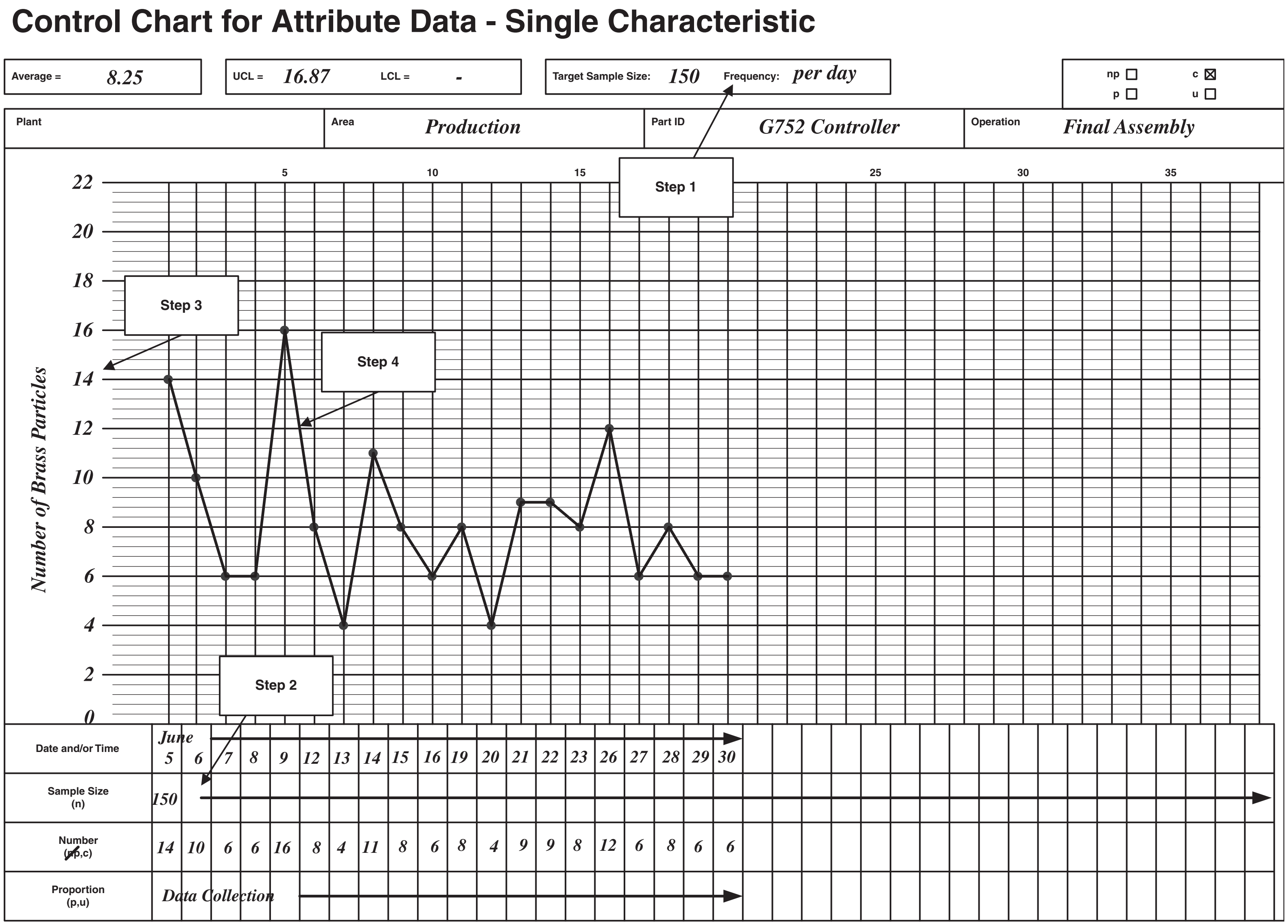


Figure 2 — Single characteristic "c" chart

5.1.6 ***Step 5 – Calculate the process average***

To establish the level of the process average, data gathering should be done over a suitably long enough period. This period should be at least 20 samples long and the period should be indicated on the control chart. In Figure 2, it has been written at the bottom of the chart.

The average number of non-conformities $(\bar{c})$ can be found by summing all of the non-conformities observed during the data collection period. Divide this sum by the number of samples (k) taken. For example, from the data given in Figure 2:

$$\bar{c} = \frac{\sum c}{k} = \frac{14 + 10 + \cdots + 6}{20} = \frac{165}{20} = 8.25$$

where $k = 20$, here.

This calculation can be recorded at the top of the chart, as shown in Figure 3.

5.1.7 ***Step 6 – Calculate the control limits***

Annex B provides the formulae for the control limits. The use of the formulae is illustrated in Figure 3.

$$\mathrm{UCL_c} = \bar{c} + 3\sqrt{\bar{c}} = 8.25 + 3\sqrt{8.25} = 16.87$$

$$\mathrm{LCL_c} = \bar{c} - 3\sqrt{\bar{c}} = 8.25 - 3\sqrt{8.25} < 0$$

Here, the calculated value for the lower control limit (LCL) is less than zero. Since this is an illogical result, because the number of non-conformities can never be less than zero, the lower control limit (LCL) is not plotted. The technical reasons for this are explained in BS 5701-3.

5.1.8 ***Step 7 – Draw the control and centre lines onto the chart***

Draw the centre and control lines as follows:

— *centre line* – draw the process average $(\bar{c})$ onto the chart using a dashed horizontal line. Some practitioners prefer to draw this line in a green colour if a colour coding system is chosen.

— *control lines* – draw the upper (and lower, if applicable) control line onto the chart using a solid horizontal line. This line is often drawn in red.

5.1.9 ***Step 8 – Establish the control limits for on-going control***

Review the chart for any out-of-control signals for the data gathered during the data collection period. If there are no out-of-control signals during this period, the calculated limits can be used for the future control of the process. The centre line and the control lines can be drawn forwards on the control chart as shown in Figure 4.

Should some of the plotted points gathered during the data collection period produce out-of-control signals, it will be necessary to investigate the reason. Depending on the outcome, a further data collection period might be necessary. If such a reason is found, and should action be taken to prevent a future occurrence of the same, then the control limits can be recalculated excluding the sample that contained the "special cause". If the remaining samples now indicate an "in-control" pattern within the revised limits, these revised limits can then be used for on-going control of the process.

A control chart showing a number of out-of-control signals indicates a rather unstable process. A prudent approach would then be to examine the process very thoroughly and to fully correct that which is wrong rather than remove many samples from the calculation of the control limits. It should be possible to eliminate up to three samples without affecting the statistical efficiency of the estimation of the process average too much.

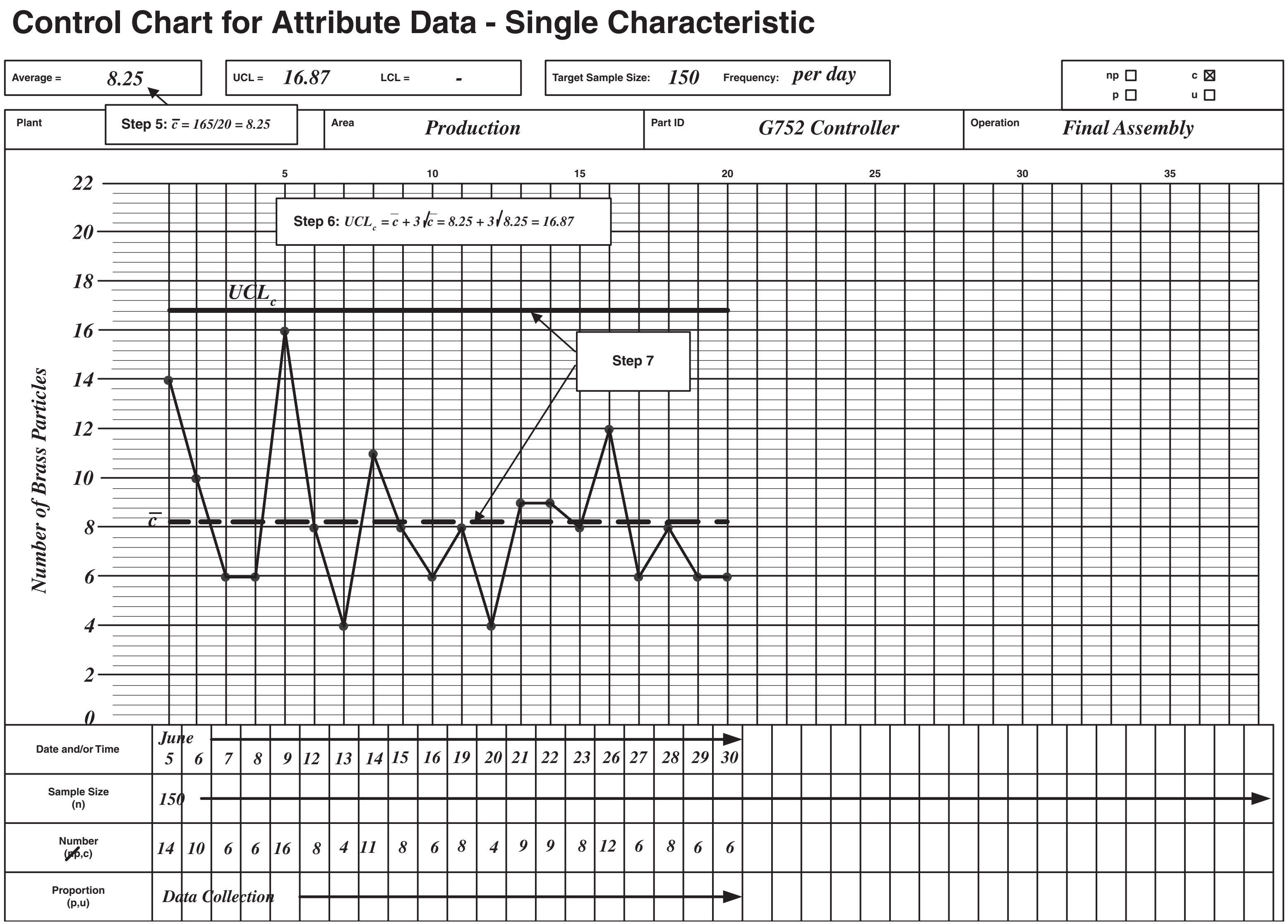


Figure 3 — "c" chart set up

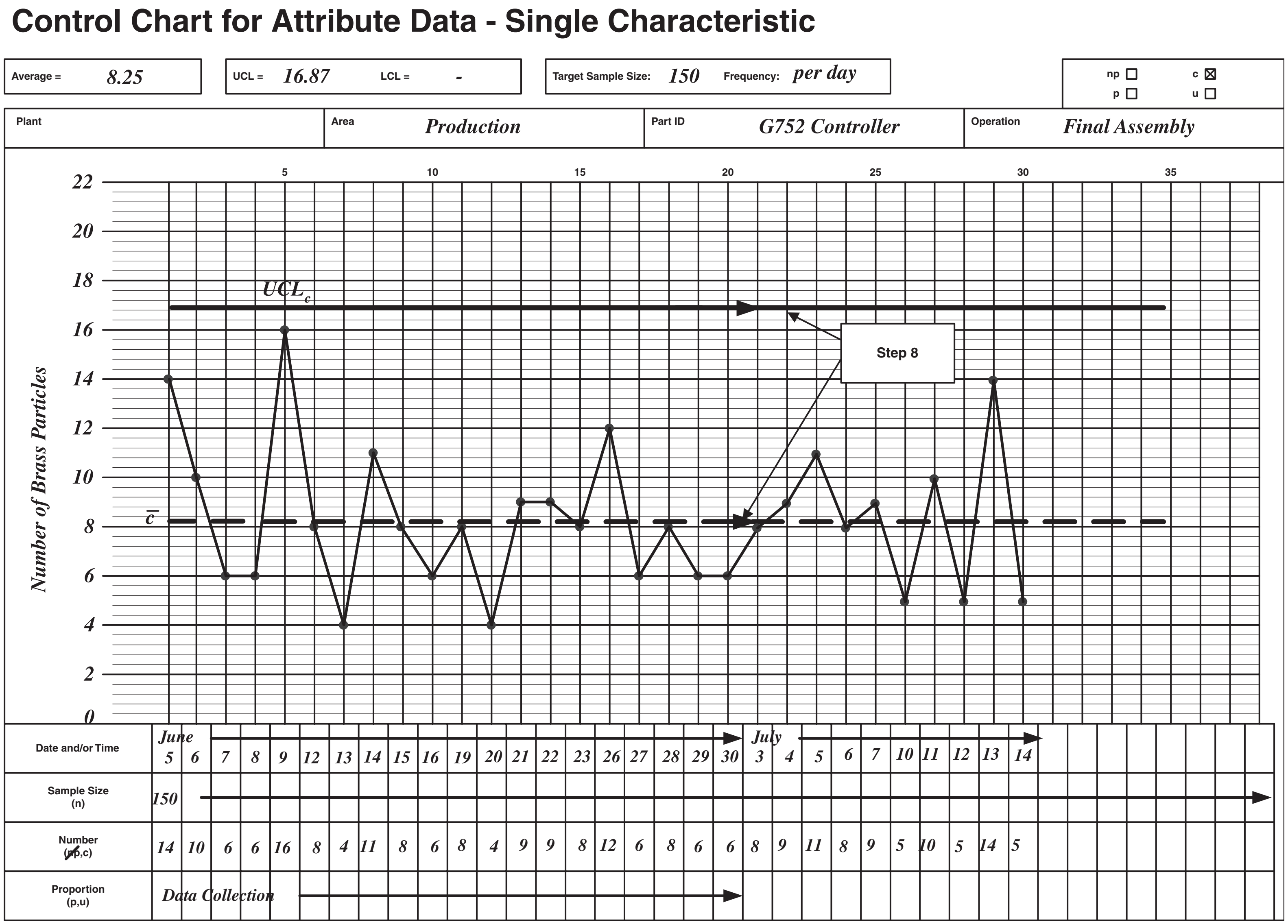


Figure 4 — Extension of control limits on “c” chart

5.1.10 ***Step 9 – Arrangements for revised processes – revised control limits***

Figure 14 shows that a change has been made to the process. The effect of this change is to reduce the number of non-conformities. Consequently, the established control limits have become inappropriate. At this time, a new data collection period should be started to gather information about the new level of the process average.

The data collection period shown in Figure 5 is only partially complete and so the calculations shown in the figure are only provisional. They can be confirmed only after the full data collection period is complete and the full data collected.

5.1.11 ***Step 10 – Arrangements for revised processes – draw revised limits onto the chart***

Once revised control limits have been calculated, the control lines can be drawn onto the chart, as shown in Figure 5. The same procedure as given in **5.1.8** should then be applied.

Ensure all process events are clearly marked onto the control chart as this will aid any retrospective process analysis. An example of this is shown in Figure 5.

5.2 The "*u*" chart

5.2.1 ***General***

When inspecting for non-conformities, if it is not possible to maintain a constant sample size, the "*c*" chart will require modification since a changing sample size will lead to different expected numbers of non-conformities. If no account were taken of this, it would lead to spurious out-of-control signals on the chart as well as missing genuine signals.

The way to overcome the problem is to plot the number of non-conformities per unit instead of plotting the observed number of non-conformities. This requires calculation of the quantity c/n for each sample. This ratio is named u. Figure 6 gives an example of this using the same data that was used for the "*c*" chart example.

5.2.2 ***Calculation of the process average***

The process average ($\overline{u}$) is calculated as follows:

$$\overline{u} = \frac{\sum c}{\sum n} = \frac{14 + 10 + \cdots + 6}{143 + 125 + \cdots + 137} = \frac{165}{3\ 001} = 0.055$$

This is the only valid approach for varying sample sizes. The summing and averaging of all of the individual u values gives rounding errors associated with each calculation of u.

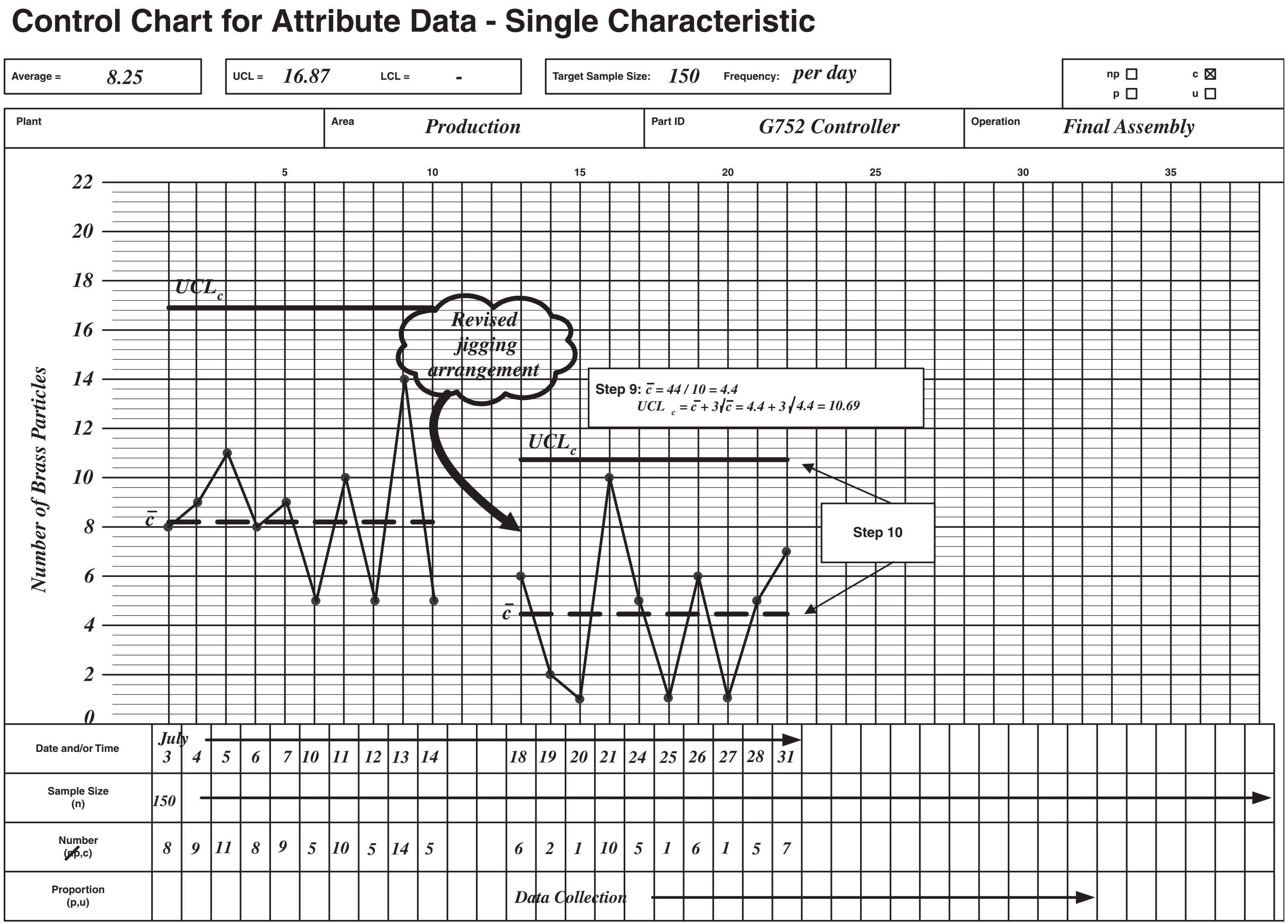


Figure 5 — "*c*" chart illustrating a significant change in performance

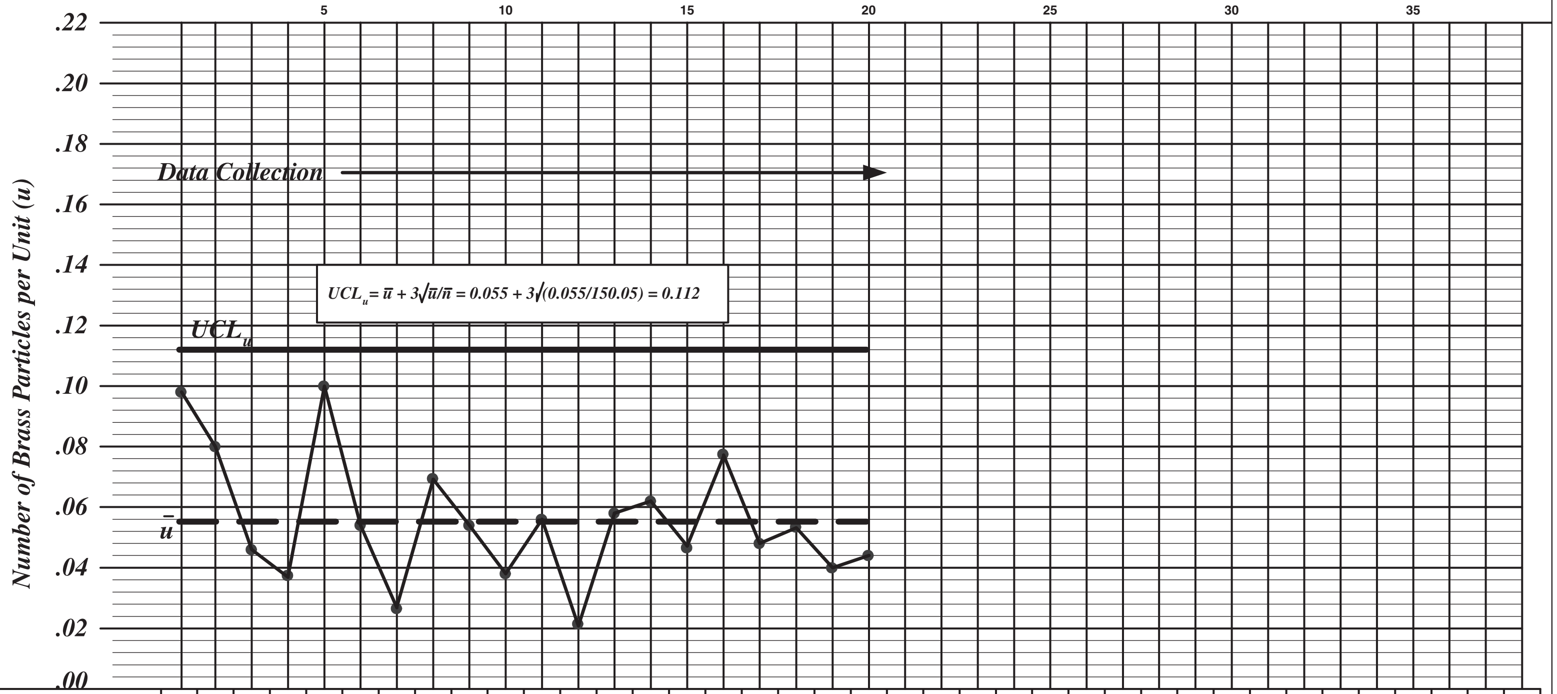
Control Chart for Attribute Data - Single Characteristic

Average = *.055* UCL = *.112* LCL = *-* Target Sample Size: *150* Frequency: *per day*

np ☐ c ☐ p ☐ u ☒

$\bar{u}$ = 165/3001 = .055

Plant | Area: *Production* | Part ID: *G752 Controller* | Operation: *Final Assembly*



Date and/or Time	June 5	6	7	8	9	12	13	14	15	16
Sample Size (n)	143	125	130	161	160	148	149	159	149	157
Number (~~np~~,c)	14	10	6	6	16	8	4	11	8	6
Proportion (~~p~~,u)	.098	.080	.046	.037	.100	.054	.027	.069	.054	.038

Date and/or Time	19	20	21	22	23	26	27	28	29	30
Sample Size (n)	142	191	154	145	169	156	124	151	151	137
Number (~~np~~,c)	8	4	9	9	8	12	6	8	6	6
Proportion (~~p~~,u)	.056	.021	.058	.062	.047	.077	.048	.053	.040	.044

Figure 6 — "*u*" chart

5.2.3 ***Calculation of the control limits***

The formulae for the calculation of the control limits for the "*u*" chart are (see Annex B) as follows:

$$\mathrm{UCL}_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

$$\mathrm{LCL}_u = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

This will give a different position of the control lines for every different sample size. If the chart is to be done manually and without the aid of a computer, this is a very onerous task. Therefore, for manual charts, the above formulae are modified to incorporate the average sample size $(\bar{n})$ and so give approximate limits:

$$\mathrm{UCL}_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

$$\mathrm{LCL}_u = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

These lead to a single approximate position for the controls and make the job of managing the chart much easier. Therefore, in Figure 6, the limits are:

$$\bar{n} = \frac{\sum n}{k} = \frac{3001}{20} = 150.05$$

$$\mathrm{UCL}_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}} = 0.055 + 3\sqrt{\frac{0.055}{150.05}} = 0.112$$

$$\mathrm{LCL}_u = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}} = 0.055 - 3\sqrt{\frac{0.055}{150.05}} < 0$$

5.2.4 ***Adjustment of the control limits for relatively large or small sample sizes***

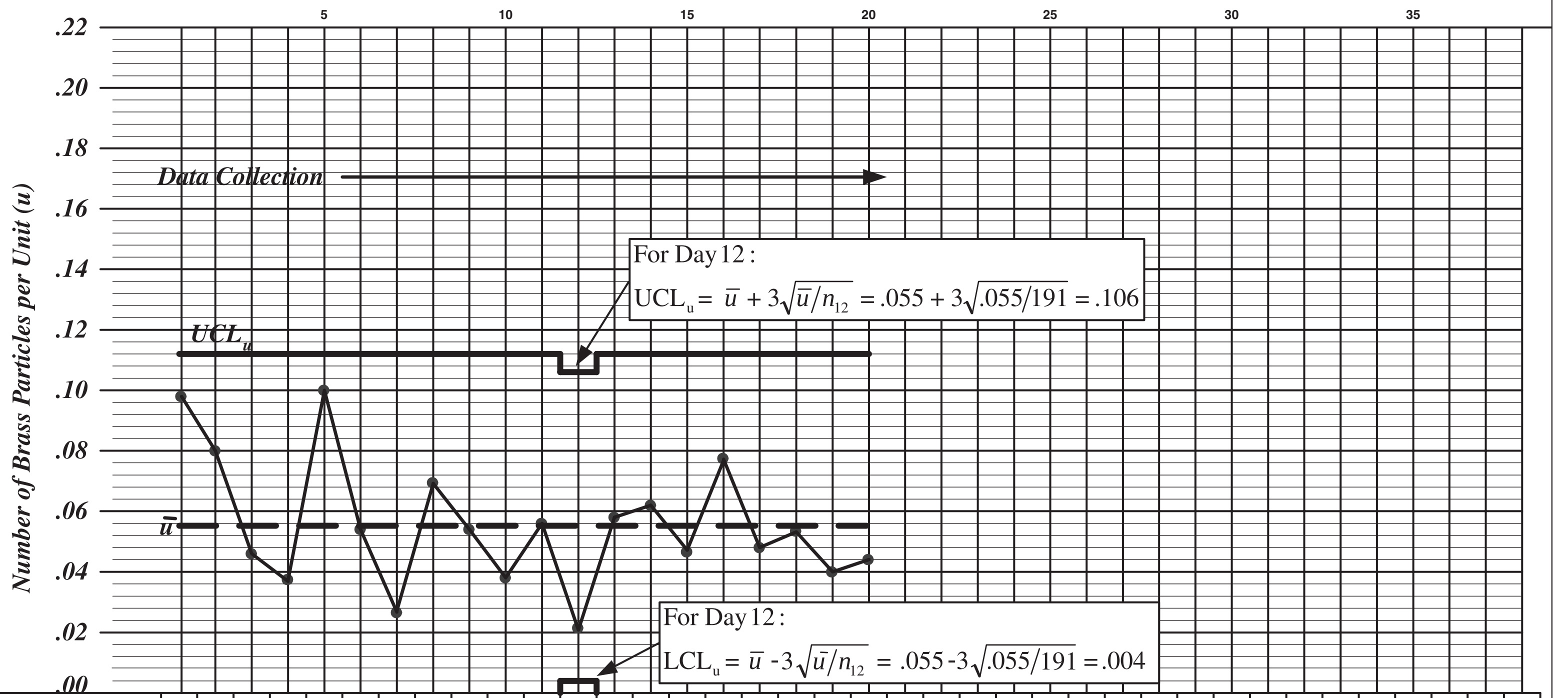
The method of using the average sample size to calculate the control limits works well as long as the sample sizes remain reasonably constant. If they deviate by more than about ±25 % of the average sample size, the limits should be recalculated for those particular samples sizes and the control lines suitably adjusted.

It was shown in **5.2.3** that the average sample size was 150.05. The ±25 % tolerance indicates that for samples of 188 and larger or of 112 and smaller the limits should be recalculated. In Figure 6, the 12th day has a sample size of 191 and so requires this adjustment. Figure 7 illustrates the recalculation.

Control Chart for Attribute Data - Single Characteristic

Average = .055	UCL = .112 LCL = -	Target Sample Size: 150 Frequency: per day	np ☐ c ☐ p ☐ u ☒

Plant	Area	Part ID	Operation
	Production	G752 Controller	Final Assembly



Date and/or Time (June)	5	6	7	8	9	12	13	14	15	16	19	20	21	22	23	26	27	28	29	30
Sample Size (n)	143	125	130	161	160	148	149	159	149	157	142	191	154	145	169	156	124	151	151	137
Number (~~np~~,c)	14	10	6	6	16	8	4	11	8	6	8	4	9	9	8	12	6	8	6	6
Proportion (~~p~~,u)	.098	.080	.046	.037	.100	.054	.027	.069	.054	.038	.056	.021	.058	.062	.047	.077	.048	.053	.040	.044

Figure 7 — Example of adjustment of control limit for variation in sample size

If a computer program is used, the calculation of the exact limits is not a problem. When drawn, they can resemble the battlement of a castle. Figure 8 has been produced in this way.

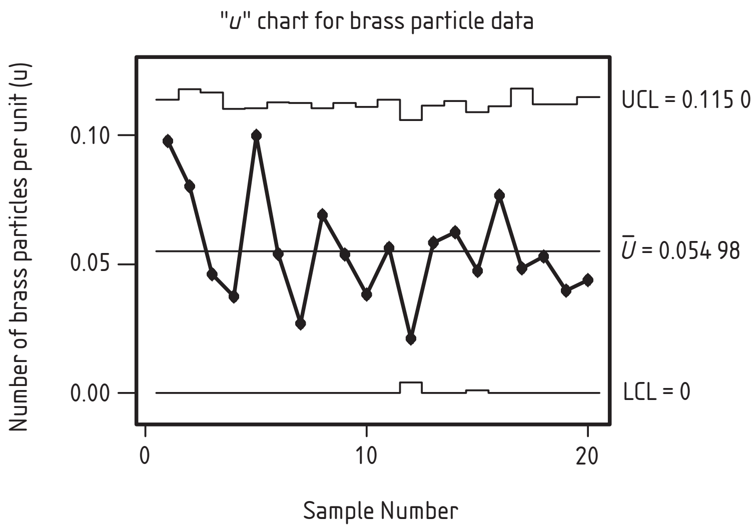


Figure 8 — Computer-generated chart for unequal sample size

5.3 The multiple characteristic chart

5.3.1 *General*

In many circumstances, it is important to monitor and control not only a single characteristic but several, at the same time. It is quite possible to have a separate chart for each of these. However, the obvious consequence of this will be more charts to manage. The multiple characteristic chart is designed to permit the control of multiple characteristics all on one page. An example of such a chart is given in Figure 9. This type of chart can be a "*c*", "*u*", "*np*" or "*p*" chart as appropriate. The example given in Figure 9 is a "*c*" chart.

5.3.2 *Set-up the control chart*

The initial steps in setting up the multiple characteristics chart are identical with those described in **5.1.2**, **5.1.3** and **5.1.4**.

5.3.3 *Step 4 – Plot the total number of non-conformities*

Before plotting, it is necessary to sum for each sample the total number of non-conformities. It is this sum that is plotted on the graph portion of the control chart.

5.3.4 *Calculate the controls and draw them onto the chart*

The same methods given in **5.1.6**, **5.1.7** and **5.1.8** should be applied to calculate the controls and to draw them onto the chart. These are shown in the example in Figure 9.

Control Chart for Attribute Data - Multiple Characteristics

Average = *4.5* UCL = *10.86* LCL = *-* Target Sample Size: *2000* Frequency: *per shift*

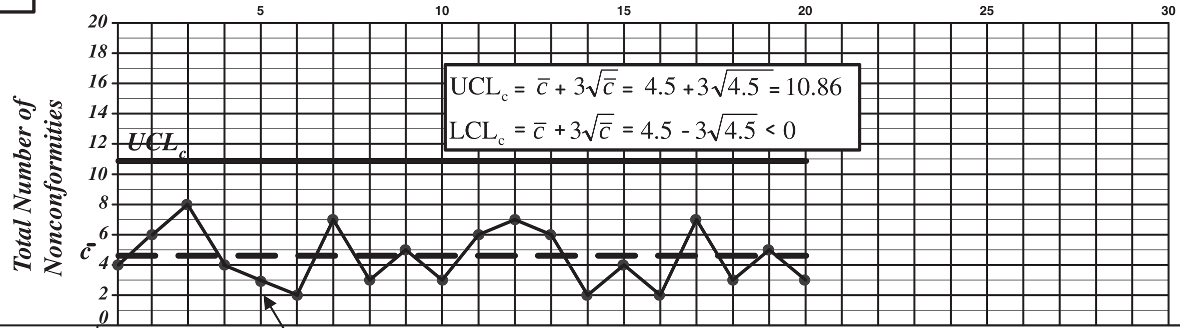
np ☐ c ☒ p ☐ u ☐

$$\bar{c} = \frac{\sum c}{k} = \frac{90}{20} = 4.5$$

Plant	Area	Part ID	Operation
	Flexo	*Wrapper MC354*	*Colour Printing*

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} = 4.5 + 3\sqrt{4.5} = 10.86$$

$$LCL_c = \bar{c} + 3\sqrt{\bar{c}} = 4.5 - 3\sqrt{4.5} < 0$$



No.	Checking Characteristic	Frequency of Nonconforming Units / Nonconformities																				f	%
1	*Out of Register*			2		2		2			1	1	1	2		1	1	2		1		16	18
2	*Scratches*	2	2	3	3	1	1	2	1	1	2	2	3	2	1			3	1	4	2	36	40
3	*Drag Lines*	1	1						1				1	1				1				6	7
4	*Streaks*			1			1	2		2		1					1		1		1	10	11
5	*Dirty Print*	1	2	1	1			1		2		1	1	1	1	2		1	1			16	18
6	*PVDC Omissions*		1	1					1			1	1			1						6	7
Number (~~np~~,c)		4	6	8	4	3	2	7	3	5	3	6	7	6	2	4	2	7	3	5	3		
Proportion (p,u)		*Data Collection* →																					
Sample Size (n)		*2000* →																					
Date and/or Time		*October* 7	7	7	8	8	8	9	9	9	10	10	10	11	11	11	13	14	14	15	15		

Step 4: 'c = 2+1 = 3'

Figure 9 — Multiple characteristic control chart

5.3.5 *Pareto analysis*

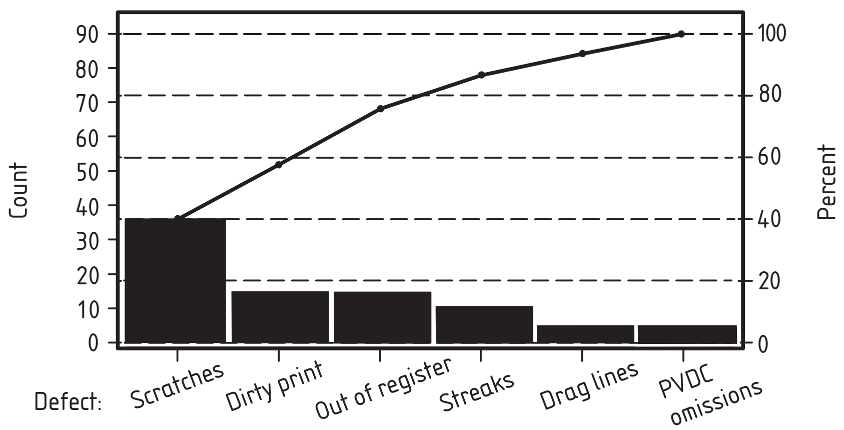
A direct benefit of the use of a multiple characteristic chart is the ability to produce a Pareto diagram easily from the data. To the right hand side of the data region of the chart there are two summary columns headed "*f*" and "%".

The values in the "*f*" column are the frequencies of the nominated characteristics. The example in Figure 9 has 16 observations for "Out of Register", 36 for scratches, etc.

These frequencies are expressed as percentages in the column "%". They are percentages of the total number of non-conformities. In the example there are 90 non-conformities in total. It is common practice for these percentages to be given to the nearest whole percent. This can result in these percentages not summing to 100 due to rounding errors. This will be precise enough for most applications.

These percentages can then be converted into a Pareto diagram as shown in Figure 10.

Diagrams such as these can be used to establish improvement initiatives.



Count:	36	16	16	10	6	6
Percent:	40.0	17.8	17.8	11.1	6.7	6.7
Cum.%:	40.0	57.8	75.6	86.7	93.3	100.0

NOTE The cumulative percentages do not sum to exactly 100 % due to software rounding to one decimal place.

Figure 10 — Pareto diagram to establish priorities for action

5.4 The "*np*" chart

5.4.1 *General*

This type of chart is suitable for data where items in the sample are classified as either "pass" or "reject" and the sample size is constant. The number of non-conforming items are usually recorded and then plotted. Figure 11 illustrates such a chart. The example is of the despatch operation within an organization. Every week, 50 despatches are monitored for their adherence to schedule and a non-conforming despatch is one that is either late, early or contains the wrong quantity of product to be shipped.

5.4.2 *Set up the control chart*

The initial steps in setting up the "*np*" chart are identical with those described in **5.1.2**, **5.1.3**, **5.1.4** and **5.1.5**.

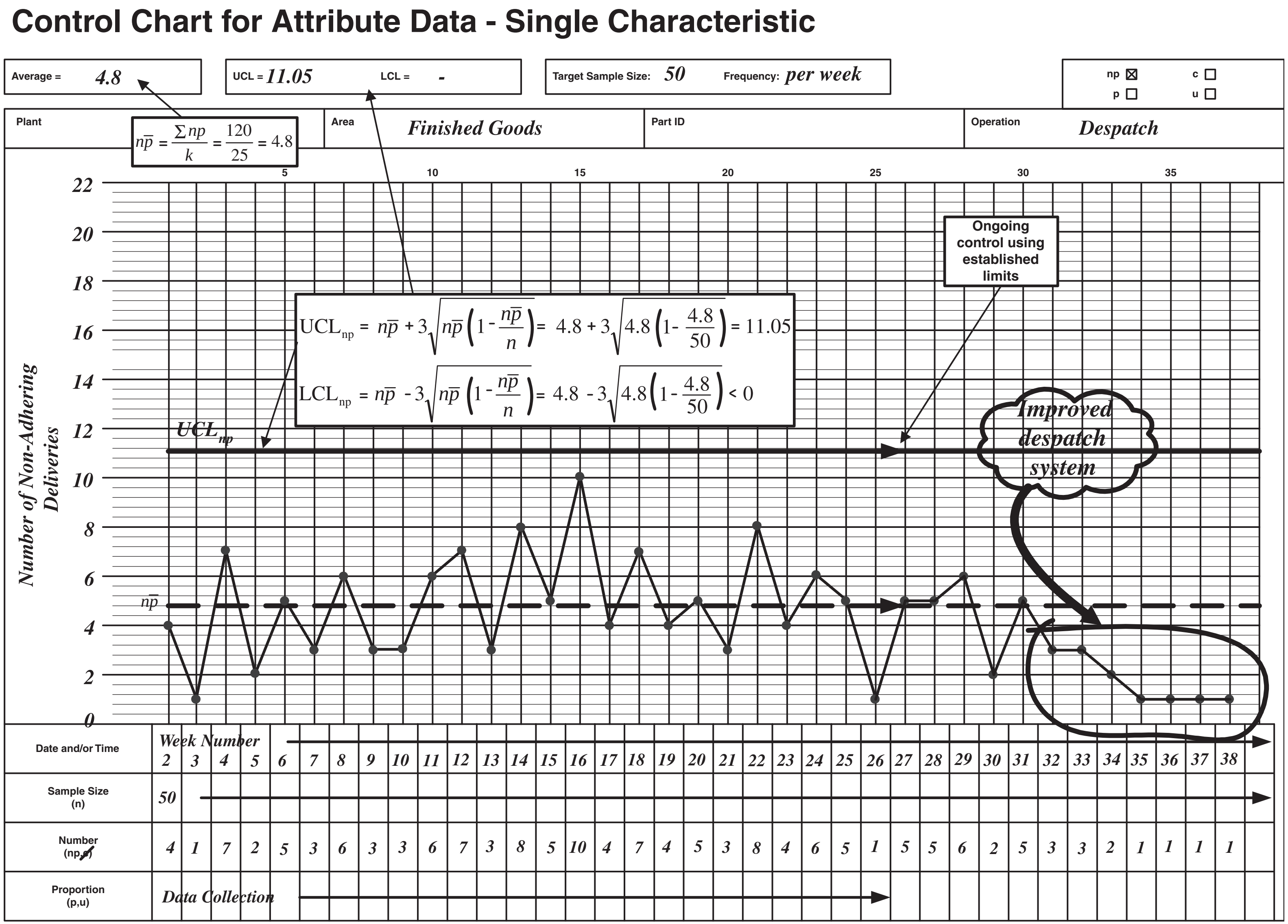


Figure 11 — "*np*" control chart

5.4.3 ***Step 5 – Calculate the process average***

The example in Figure 11 shows that a period of 25 samples has been taken as the data collection period and so the process average $(n\bar{p})$ is calculated as:

$$n\bar{p} = \frac{\sum np}{k} = \frac{4+1+\cdots+1}{25} = \frac{120}{25} = 4.8$$

5.4.4 ***Step 6 – Calculate the control limits***

Annex B shows the formulae for the control limits for the "*np*" chart as:

$$\mathrm{UCL}_{np} = n\bar{p} + 3\sqrt{n\bar{p}\left(1-\bar{p}\right)}$$

and

$$\mathrm{LCL}_{np} = n\bar{p} - 3\sqrt{n\bar{p}\left(1-\bar{p}\right)}$$

A more convenient form of these equations is to express them in terms of $n\bar{p}$ and n, thus:

$$\mathrm{UCL}_{np} = n\bar{p} + 3\sqrt{n\bar{p}\left(1-\frac{n\bar{p}}{n}\right)}$$

and

$$\mathrm{LCL}_{np} = n\bar{p} - 3\sqrt{n\bar{p}\left(1-\frac{n\bar{p}}{n}\right)}$$

In the example, it will be seen that these equations have been used:

$$\mathrm{UCL}_{np} = n\bar{p} + 3\sqrt{n\bar{p}\left(1-\frac{n\bar{p}}{n}\right)} = 4.8 + 3\sqrt{4.8\left(1-\frac{4.8}{50}\right)} = 11.05$$

$$\mathrm{LCL}_{np} = n\bar{p} - 3\sqrt{n\bar{p}\left(1-\frac{n\bar{p}}{n}\right)} = 4.8 - 3\sqrt{4.8\left(1-\frac{4.8}{50}\right)} < 0$$

5.4.5 ***On-going control***

The example in Figure 11 indicates a stable process during the data collection period and so the controls are established for on-going control. The detailed steps here are covered in **5.1.8** and **5.1.9** (Steps 7 and 8).

5.4.6 ***Process changes***

Figure 11 indicates that, from about week 32, a change to the process was identified. An improvement to the despatch system was noted as the reason for the reduction of the number of non-adhering deliveries. Now that the chart has indicated this to be a significant change and assuming the change to be permanent, the controls will require repositioning. To do this, a new data collection period should be begun from week 32. The approach is explained in **5.1.10** and **5.1.11** (Steps 9 and 10).

5.5 The "*p*" chart

5.5.1 *General*

If it is not possible to maintain a constant sample size, the "*np*" chart described in **5.4** requires modification. A changing sample size will lead to different expected numbers of non-conforming items and, if no account were taken of this, it would lead to spurious out-of-control signals on the chart as well as occasionally missing the real signals.

The way to overcome the problem is to plot the proportion of non-conforming items instead of the observed number non-conforming. This requires the calculation of the quantity *np*/*n* for each sample. This ratio is named *p*. Figure 12 gives an example of this chart type.

The steps required to create this chart are the same as those for the "*u*" chart. See **5.2**.

5.5.2 *Calculation of control limits*

Annex B gives the formula for the control limits as:

$$\mathrm{UCL}_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

and

$$\mathrm{LCL}_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

This requires the limits to be recalculated for every sample size. If the chart is to be filled in manually this leads to an onerous task and, if done, produces not a single horizontal control line, but a series of lines at different levels. Figure 13 has been produced using a statistical program that calculates the limits in this way. The data are the first 25 samples shown in Figure 12.

If the sample size does not vary by more or less than 25 % of the average sample size it is considered acceptable to calculate the controls using the average sample size. Therefore, the formulae are modified to be:

$$\mathrm{UCL}_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$$

and

$$\mathrm{LCL}_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$$

The example in Figure 13 has had the control limits calculated in this way and this has lead to a single horizontal line. Should the sample size deviate by an amount more than the 25 % tolerance, the actual control limit for the sample size in question should be calculated and plotted so the proper interpretation of the data can take place.

5.6 Control chart rules for out-of-control

5.6.1 *General*

There are many different rules for determining out-of-control on a control chart. For the purposes of this Standard, only four will be introduced. The four "types" of rule are intended to detect the following four process events:

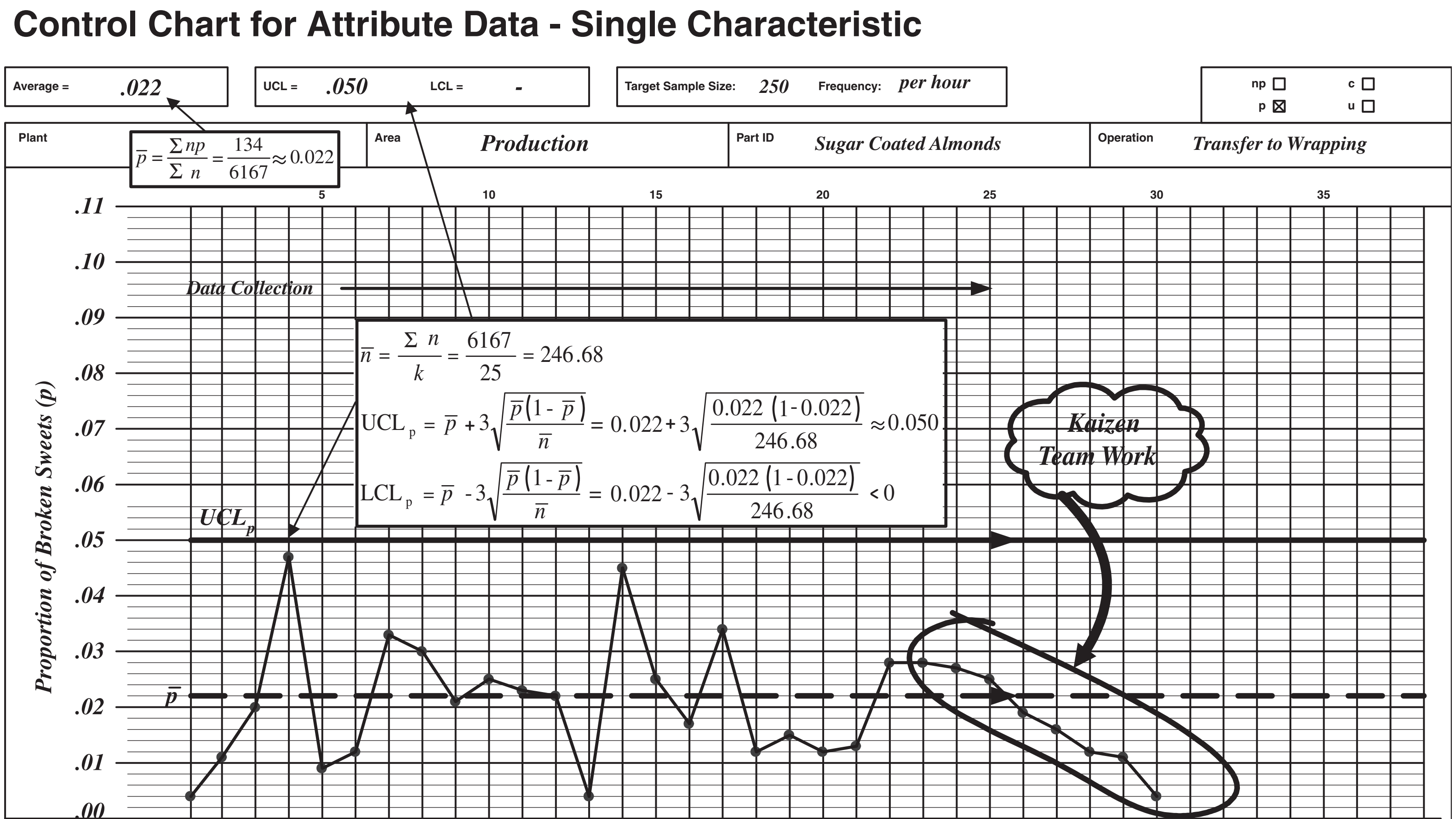
1) a sudden (large) shift in the process average;

2) a shift to running at a different process average level;

3) a gradual change over time (drifting) of the process average;

4) cyclic behaviour within the process.

Control Chart for Attribute Data - Single Characteristic



Date and/or Time	06:00	07:00	08:00	09:00	10:00	11:00	12:00	13:00	14:00	15:00	16:00	17:00	18:00	19:00	20:00
Sample Size (n)	229	269	249	253	233	242	241	265	238	237	266	226	254	244	243
Number (np/c)	1	3	5	12	2	3	8	8	5	6	6	5	1	11	6
Proportion (p/u)	.004	.011	.020	.047	.009	.012	.033	.030	.021	.025	.023	.022	.004	.045	.025

Date and/or Time	21:00	22:00	23:00	00:00	01:00	02:00	03:00	04:00	05:00	06:00	07:00	08:00	09:00	10:00	11:00
Sample Size (n)	235	237	259	264	259	226	250	246	259	243	267	245	256	271	237
Number (np/c)	4	8	3	4	3	3	7	7	7	6	5	4	3	3	1
Proportion (p/u)	.017	.034	.012	.015	.012	.013	.028	.028	.027	.025	.019	.016	.012	.011	.004

25 June → 26 June →

Figure 12 — "p" chart

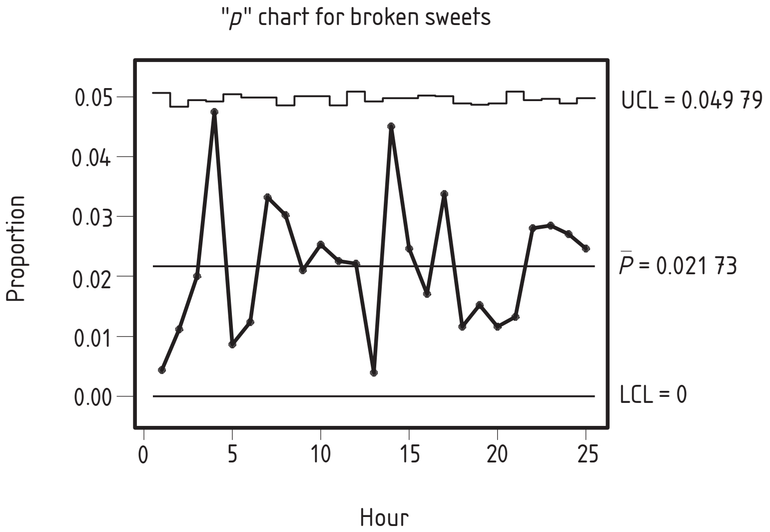


Figure 13 — Computer-generated "*p*" chart

5.6.2 ***Rule 1 to detect a sudden (large) shift in the process average***

Rule 1 is "any plotted point beyond a control line".

Standard control limits are calculated to be three standard deviations above and below the process average level and so are designed to identify large shifts only. An example of such a shift can be seen in Figure 14.

The reason for the out-of-control signal should be sought. A decision should be taken about what to do about the process and the process output.

One or more points beyond either control limit indicates process instability and the presence of a special cause of variation. The special cause can be unfavourable or favourable. Either way, as this is a primary decision rule for any control chart, it requires immediate visual identification (e.g. by encircling) and investigation.

A point above the upper control limit of a fault chart could indicate that:

a) the plot point or control limit is incorrect;

b) the process performance has deteriorated;

c) the conformance criteria have been tightened.

A point below the lower control limit of a fault chart could indicate that:

d) the plot point or control limit is incorrect;

e) the process performance has improved (this should be investigated with a view to achieving a permanent improvement);

f) the conformance criteria have been relaxed.

5.6.3 ***Rule 2 to detect a shift to running at a different process average level***

Rule 2 is "seven plotted points all consecutively above or all consecutively below the process average".

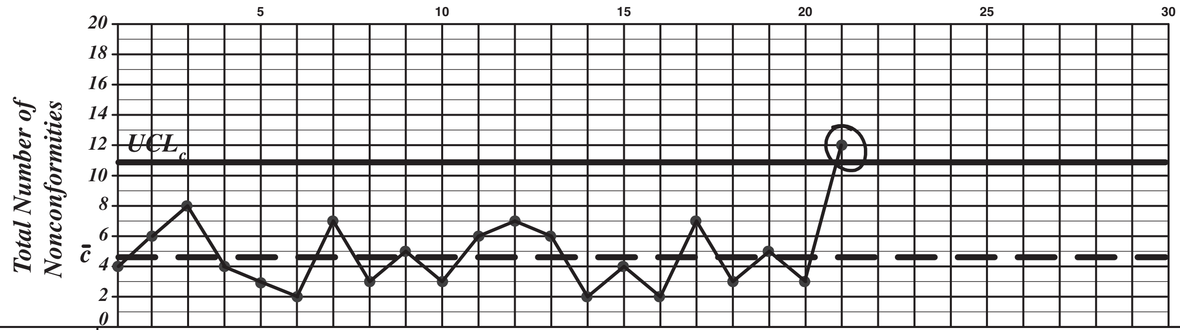
This rule is based on the assumption that, were the process average to have remained unchanged, the probability of seven consecutive points all on one side of the process average is very small – less than 1 %. The encircled points in Figure 11 provide an example of this rule.

Runs below the process average on a fault chart indicate that either the process performance has improved or the conformance criteria have become less stringent. Runs above the process average on a fault chart indicate that either the process performance has worsened or the conformance criteria have become more stringent.

Control Chart for Attribute Data - Multiple Characteristics

Average = *4.5* | UCL = *10.86* LCL = *-* | Target Sample Size: *2000* Frequency: *per shift* | np ☐ p ☐ c ☒ u ☐

Plant	Area *Flexo*	Part ID *Wrapper MC354*	Operation *Colour Printing*



No.	Checking Characteristic	Frequency of Nonconforming Units / Nonconformities																														f	%
1	*Out of Register*			*2*		*2*		*2*			*1*	*1*	*1*	*2*		*1*	*1*	*2*		*1*		*2*											
2	*Scratches*	*2*	*2*	*3*	*3*	*1*	*1*	*2*	*1*	*1*	*2*	*2*	*3*	*2*	*1*			*3*	*1*	*4*	*2*	*3*											
3	*Drag Lines*	*1*	*1*						*1*				*1*	*1*				*1*				*1*											
4	*Streaks*			*1*			*1*	*2*		*2*		*1*					*1*		*1*		*1*	*2*											
5	*Dirty Print*	*1*	*2*	*1*	*1*			*1*		*2*		*1*	*1*	*1*	*1*	*2*		*1*	*1*			*3*											
6	*PVDC Omissions*		*1*	*1*					*1*			*1*	*1*			*1*						*1*											
Number (~~np~~,c)		*4*	*6*	*8*	*4*	*3*	*2*	*7*	*3*	*5*	*3*	*6*	*7*	*6*	*2*	*4*	*2*	*7*	*3*	*5*	*3*	*12*											
Proportion (p,u)		*Data Collection* →																															
Sample Size (n)		*2000* →																															
Date and/or Time		*October* → *7*	*7*	*7*	*8*	*8*	*8*	*9*	*9*	*9*	*10*	*10*	*10*	*11*	*11*	*11*	*13*	*14*	*14*	*15*	*15*	*15*											

Figure 14 — Sudden shift in process average

5.6.4 ***Rule 3 to detect a gradual change over time (drifting) of the process average***

Rule 3 is "any consecutive seven intervals all drifting in the same direction".

This rule operates in a similar way to Rule 2 in so much that it is based on the fact that if the process average is unchanged, the probability of seven consecutive movements in one direction is low. The encircled points in Figure 12 provide an example of this rule.

Runs up, on a fault chart, generally signify that the process performance is worsening or the conformance criteria is getting progressively more rigorous. Runs down, on a fault chart, generally signify that the process performance is improving or the conformance criteria is getting progressively less rigorous.

5.6.5 ***Rule 4 to detect cyclic behaviour within the process***

Rule 4 is "any obvious non-random pattern".

This rule frequently requires a large amount of data for this rule to be really effective. Statistical tests can be employed to assist in this examination.

Non-random patterns can result from:

a) calculation or plotting errors;

b) stratified sampling where subgroups contain observations from process streams that have different averages;

c) data that has been edited.

5.6.6 ***Non-random patterns***

For a process in statistical control, the plotted points on the control chart should represent random behaviour. The data that come from the situations described earlier in Clause **5** will follow either the Poisson distribution (see BS 5701-3) (in the case of the "c" and "u" charts) or the binomial distribution (see BS 5701-3) (in the case of the "np" and "p" charts). However, standard charts exploit their approximations to the normal distribution and thereby produce symmetrical equations for the control limits. Using the properties of the normal distribution, it can be broadly expected that approximately two thirds of the plotted points will lie in the middle third of the region between the control lines.

If a control chart has either far more or has far fewer than this amount, it can be taken as an indication of non-randomness. In general, a large amount of data will need to be gathered before this examination has much statistical significant. For example, for a data set of 25 samples, there would have to be more than 90 % or fewer than 40 % of the plotted points in the "middle third" before the assertion of non-randomness can be made. For larger data sets these percentages become closer together.

5.7 Capability and performance

5.7.1 ***General***

The capability and performance measures of attribute processes are expressed as either non-conformities per given quantity, e.g. per hundred, or the proportion conforming to specification.

5.7.2 ***Capability conditions***

Capability conditions apply only when the process can be shown to be in a state of statistical control. This can be demonstrated by no out-of-control indication on a control chart when the rules of **5.6** are applied. This would allow some prediction to be made about the future state of the process and what might be expected from it.

5.7.3 ***Performance conditions***

Performance conditions apply when a process has not been demonstrated to be in a state of statistical control. This means that only a retrospective analysis can be made about how the process has been and no prediction can be made about its future state.

5.7.4 *"c" and "u" charts*

Capability or performance can be expressed as either the process averages, e.g. ($\bar{c}$ or $\bar{u}$), or as some rate of occurrence such as "non-conformities per hundred units" (NHU). For example, consider the data given in Figure 3. As the process is in a state of statistical control, the following capability measures are applicable:

$$\bar{c} = 8.25$$

or

$$\text{NHU} = \frac{100 \times \bar{c}}{n} = \frac{100 \times 8.25}{150} = 5.5$$

As a second example, from the "*u*" chart in Figure 6, the capability would be:

$$\bar{u} = 0.055$$

or

$$\text{NHU} = 100 \times \bar{u} = 100 \times 0.055 = 5.5$$

The NHU is the same in each case because the charts were based on the same data.

5.7.5 *"np" and "p" charts*

For these charts, capability or performance can be expressed as a proportion or a percentage of output that conforms to specification and is sometimes named "first run capability" (FRC) or just "yield".

As an example, consider the data gathered during the data collection period of the "*np*" chart in Figure 11. The chart shows $n\bar{p}$ to be 4.8 and represents the average number of non-adhering deliveries. The sample size is 50, so the average number of *adhering* deliveries, i.e. the number that satisfy specification, is 50 – 4.8 = 45.2. Thus, FRC = 100(45.2/50) = 90.4 %. As a formula:

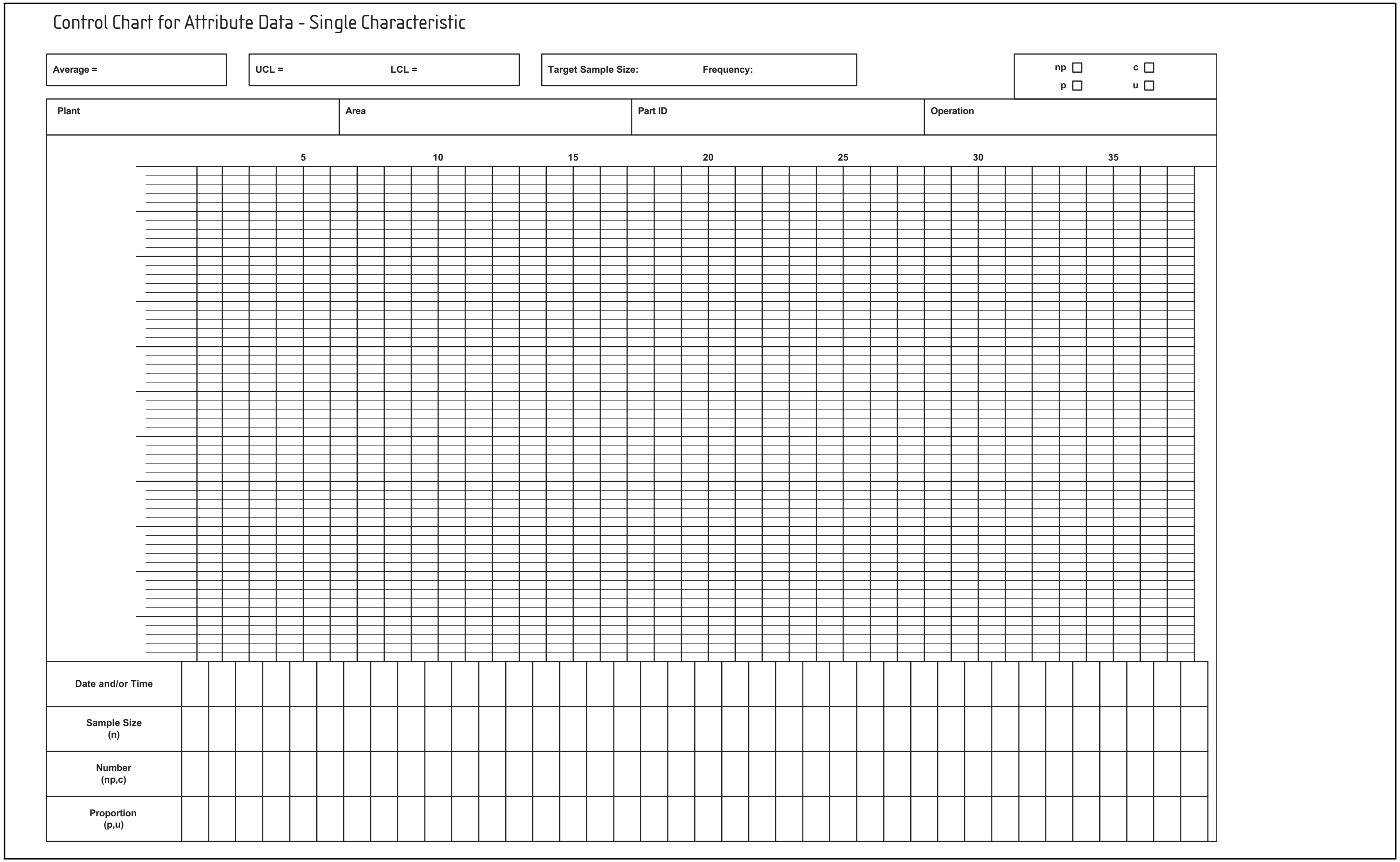
$$\text{FRC} = \frac{100(n - n\bar{p})}{n} = \frac{100(50 - 4.8)}{50} = 90.4\,\%$$

If a "*p*" chart is used as in Figure 12, the calculation based on the first 25 samples is:

$$\text{FRC} = 100(1 - \bar{p}) = 100(1 - 0.022) = 97.8\,\%$$

Annex A (normative) Specimen attribute control charts

A.1 Specimen single characteristic attribute control chart



A.2 Specimen multiple characteristic attribute control chart

Control Chart for Attribute Data - Multiple Characteristics

Average =	UCL = LCL =	Target Sample Size: Frequency:	np ☐ c ☐ p ☐ u ☐

Plant	Area	Part ID	Operation

5 10 15 20 25 30

No.	Checking Characteristic	Frequency of Nonconforming Units / Nonconformities	f	%

Number (np,c)	
Proportion (p,u)	
Sample Size (n)	
Date and/or Time	

Annex B (normative) Formulae for the control limits

Plot points, centre-lines and control limits for standard control charts for discrete data

Table B.1 — Formulae for the control limits

Chart	events: non-conformities		non-conforming units	
	constant sample size: "*c*" chart	variable sample size: "*u*" chart	constant sample size: "*np*" chart	variable sample size: "*p*" chart
plot point	c	u	np	p
centre-line (average)	$\bar{c}$	$\bar{u}$	$n\bar{p}$	$\bar{p}$
UCL	$\bar{c}+3\sqrt{\bar{c}}$	$\bar{u}+3\sqrt{\frac{\bar{u}}{n}}$	$n\bar{p}+3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{p}+3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$
LCL	$\bar{c}-3\sqrt{\bar{c}}$	$\bar{u}-3\sqrt{\frac{\bar{u}}{n}}$	$n\bar{p}-3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{p}-3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

Bibliography

Standards publications

BS 5701-1, *Guide to quality control and performance improvement using qualitative (attribute) data — Part 1: Uses and value of attribute charts in business, industry, commerce and public service.*

BS 5701-3, *Guide to quality control and performance improvement using qualitative (attribute) data — Part 3: Technical aspects of attribute charting: special situation handling.*

BS 5701-4, *Guide to quality control and performance improvement using qualitative (attribute) data — Part 4: Attribute inspection performance control and improvement.*

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